Assignment No : 2

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Properties of determinats

• What is determinant?

A scaler value that is calculated from the elements of a square matrix is called determinent.
The vertical lines are known as columns and the horizontal lines are known as rows
If determinent is of order 'n' then it has n number of rows and n number of columns.

Determinent of 2x2 matrix

Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Determinent of 3x3 matrix

Matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \Rightarrow \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei-fh) - b(di-fg) - c(dh-eg)$$

Properties of determinents

→ There will be no change in the value of determinant if rows and coloums are interchanged

→ If rows and coloums are interchanged then its sign changes as well.

→ Same rows and colums of determinents (same mean same value) then the determinant will be equal to zero.

→ If any row or column of a determinant is multiplied with any variable say 'K' then its value is also multiplied with K

→ If some or all elements of row or columns are expressed as the sum of two or more terms then the determinant can be expressed as the sum of Two or more determinents.

Detailed properties of determinants.

1: Reflection Property

The value of determinants remain unchanged if rows and columns are interchanged.

Example

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} \Rightarrow \begin{vmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{vmatrix}$$

Determinant 1:

$$= 2[(0)(-7) -(4)-(5)] - (-3)[(6)(-7) -(4)(1)] + 5[(6)(5) -(1)(0)]$$

$$= 2[0-20] - (-3)[-42-4) + 5[30-0]$$

$$= 2(-20) - (-3)(-46) + 150$$

$$= -40 - 138 + 150$$

$$= -28$$

Determinant 2:

$$= 2[(0)(-7) -(5)(4)] - 6[(-3)(-7) -(5)(5)] + 1[(-3)(4) -(0)(5)]$$

$$= 2[0-20] - 6[(2)-25 + 1(-12)]$$

$$= 2(-20) - 6 - (4) + 1(-12)$$

$$= -40 + 24 - 12$$

$$= -28$$

Hence the answer of both determinants are same.

2: Switching Property :

If any two rows or coloums of a determinent are interchanged then sign of determinant changes as well.

Example

$$\begin{vmatrix} 2 & -3 & 4 \\ 6 & 0 & 5 \\ 1 & 6 & -7 \end{vmatrix} \Rightarrow \text{Interchanging } R_1 \text{ and } R_3$$

$$\begin{vmatrix} 1 & 6 & -7 \\ 6 & 0 & 5 \\ 2 & -3 & 4 \end{vmatrix} \Rightarrow \text{Resultant determinant}$$

Determinant 1,

$$= 2[(0)(-7) - (5)(6)] - (-3)[(6)(-7) - (5)(1)] + 4[(6)(6) - (0)(1)]$$

$$= 2[0-30] - (-3)[-42-5] + 4[36]$$

$$= 2(-30) - (-3)(-47) + 4(36)$$

$$= -60 - 141 + 144$$

$$= -57$$

Determinant 2:

$$= 1[(0)(4) - 5(-3)] - 6[(6)(4) - (5)(2)] - 7[(6)(-3) - 2(0)]$$

$$= 1[0+15] - 6(24-10) - 7[-18]$$

$$= 1(15) - 6(14) - 7(-18)$$

$$= 15 - 84 + 126$$

$$= +57$$

Hence the sign of determinants are changed

3. All zero property

If all the elements of a coloumn or a row then determinant is also 0

Example :

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 9 & 7 & 4 \end{vmatrix} = 0$$

4 : Repetition / Proportionality Property

If any two rows or coloumns of a determiant are same , value of the determiant is zero

Example :

$$\begin{vmatrix} 4 & 6 & 4 \\ 4 & 6 & 4 \\ 9 & 6 & 4 \end{vmatrix} = 0$$

5 : Scalar Multiple Property:

If the element of row or coloumn of a determiant is multiplied by any non-zero constant Then the delemiant also get multiplied by the same constant

$$\begin{vmatrix} 102 & 18 & 36 \\ 13 & 4 & 1 \\ 17 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 6(17) & 6(3) & 6(6) \\ 13 & 4 & 1 \\ 17 & 3 & 6 \end{vmatrix}$$

$$= 6 \times \begin{vmatrix} 17 & 3 & 6 \\ 13 & 4 & 1 \\ 17 & 3 & 6 \end{vmatrix}$$

6: Sum Property :

If elements of row or a column of a determinant are expressed as the sum of two (or more) terms, then the determinant can be expressed as sum of (or more) determinants.

Example

$$\begin{vmatrix} a & b & c \\ a+dx & b+dy & c+dz \\ x & y & z \end{vmatrix} = \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ dx & dy & dz \\ x & y & z \end{vmatrix}$$

7: Property of invariance :

Suppose any scalar multiples of corresponding elements of other rows and coloumn are added to every element of any row or coloumn of a determinant

Example

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + Kb_1 & a_2 + Kb_2 & a_3 + Kb_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

8: Factor Property.

If a determinant becomes zero when we insert $x = \alpha$ then $(x-\alpha)$ is a factor of Δ. This is called the factor property.

Example

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \Rightarrow \Delta_1 = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$$

9: Triangle Property.

If the elements of a determinant above or below the main diagonal consist of zero then the determinant is equal to the product of diagonal element.

Example

$$\begin{vmatrix} 4 & 5 & 1 \\ 8 & 9 & 2 \\ 0 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 0 & 0 \\ 5 & 9 & 0 \\ 1 & 2 & 3 \end{vmatrix} = (4)(9)(3) = 108$$

10: Determinants of co-factor matrix

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \quad \Delta' = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{33} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$$

Where c_{ij} denotes the co-factor of the elements a_{ij} in Δ.