

## Lab 3

Consider a tandem of 2 single server queues. Customers arrive according to a renewal process into server 1. After service completion they join the queue corresponding to the second server, and after service completion they leave the system. Interarrival and service are i.i.d. denoted generically by  $A$  (interarrival times),  $S_1, S_2$  (service times in servers 1 and 2). At time  $t = 0$  there are  $L_1(0) = q$  customers in the first queue (including those in service) and no customers in the second queue ( $L_2(0) = 0$ ). We observe the system during a finite time horizon  $[0, T]$ .

Suppose arrival process is Poisson with rate  $\lambda$  (of the following values) =  $\{1, 5\}$ , and the service times have exponential distributions with rates  $\mu_1 = \{2, 4\}$ ,  $\mu_2 = \{3, 4\}$ .

A. Assume  $q = 0$ . Estimate the time average number of customers in the system

$$\frac{1}{T} \int_0^T (L_1(t) + L_2(t)) dt$$

via simulation.

1. Namely, generate interarrival and service times according to their prescribed distribution.
2. Then generate the implied queue length process and compute the corresponding integral.
3. Perform the simulations several times  $N$  for  $T = 10, 50, 100$  and  $1000$ .
4. Report on the obtained values, averaged over  $N$  and compare your results with the prediction of a steady-state product form formulas for

$$E\pi[L_1 + L_2]$$

where  $\pi$  is the stationary distribution. Include the values of  $N$ .

B. Assume now  $q = 1000$  and  $T = 2000$ . Perform the same experiment as above. Produce plots of  $L_1$  and  $L_2$  for one specific run of the simulation (it suffices to produce plots of  $L_1, L_2$  only at times of arrivals or service completions). How do the two plots compare?

### **Deliverables:**

**\*\* Use your preferred Language of Choice (MATLAB, Python, Java, Etc.).**

**\*\* Choose any type of simulation you feel comfortable with.**

- 1- Detailed Report with tabulated comparison of variation of  $\lambda$ ,  $\mu_1$ ,  $\mu_2$  and  $T$  with each other.
- 2- Your code.
- 3- Teams of 5.
- 4- Any plagiarism will be severely penalized.