1 GIBBS

Vectors :

$\vec{r}_1 = -294.32\hat{i} + 4265.1\hat{j} + 5986.7\hat{k}(Km)$
$\vec{r}_2 = -1365.5\hat{i} + 3637.6\hat{j} + 6346.8\hat{k}(Km)$
$\vec{r}_3 = -2940.3\hat{i} + 2473.7\hat{j} + 6555.8\hat{k}(Km)$

Magnitude of the vector :

$r_1 = \sqrt{-294.32^2 + 4265.1^2 + 5986.7^2} = 7356.513(Km)$
$r_2 = \sqrt{-1365.5^2 + 3637.6^2 + 6346.8^2} = 7441.68(Km)$
$r_3 = \sqrt{-2940.3^2 + 2473.7^2 + 6555.8^2} = 7598.886(Km)$

Step2:$Find\ \vec{C}_{12}, \vec{C}_{23}\ and\ \vec{C}_{31}\vec{C}_{12} =$

$$\begin{vmatrix} \text{i} & \text{j} & \text{k} \\ \text{-294.32} & \text{4265.1} & \text{5986.7} \\ \text{-1365.5} & \text{3637.6} & \text{6346.8} \\ & & \end{vmatrix} = \text{5292516.76i + -6306848.67j + 4753375.62k(Km}^2\text{)}$$

$\vec{C}_{23} =$

$$\begin{vmatrix} \text{i} & \text{j} & \text{k} \\ \text{-1365.5} & \text{3637.6} & \text{6346.8} \\ \text{-2940.3} & \text{2473.7} & \text{6555.8} \\ & & \end{vmatrix} = \text{8147298.92i + -9709551.14j + 7317797.93k(Km}^2\text{)}$$

$\vec{C}_{31} =$

$$\begin{vmatrix} \text{i} & \text{j} & \text{k} \\ \text{-2940.3} & \text{2473.7} & \text{6555.8} \\ \text{-2940.3} & \text{2473.7} & \text{6555.8} \\ & & \end{vmatrix} = \text{-13151842.79i + 15673190.95j + -11812614.15k(Km}^2\text{)}$$

Step3:The magnitude of$\vec{C}_{23}$:

$$C_{23} = \sqrt{8147298.92^2 + -9709551.14^2 + 7317797.93^2} \quad = 14635710.764$$

The unit vector of $\vec{C}_{23}$:

$$\hat{C_{23}} \quad = \quad \left(\frac{8147298.92i + -9709551.14j + 7317797.93k}{14635710.764}\right) \quad = \quad 0.557i \ + \ -0.663j \ + \ 0.5k$$

The unit vector of $\vec{u}_{r_1}$:

$$\hat{u_{r_1}} = \left(\frac{-294.32i + 4265.1j + 5986.7k}{7356.513}\right) = -0.04i + 0.58j + 0.814k$$

Therefore, :

$$\hat{u_{r_1}}.\hat{C_{23}} = -6.118058187509767e - 06$$

This is close enough to or equal zero for our purposes. The three vectors r_1, r_2, and r_3 are coplanar.

Step4 : $\vec{N} = r_1\vec{C}_{23} + r_2\vec{C}_{31} + r_3\vec{C}_{12}$

$$= 7356.51(8147298.92i + -9709551.14j + 7317797.93k)$$
$$+7441.68(-13151842.79i + 15673190.95j + -11812614.15k)$$
$$+7598.89(5292516.76i + -6306848.67j + 4753375.62k)$$

$$\vec{N} = (2281140568.33i + -2718596493.52j + 2048144274.9k)$$

$$\mathrm{N} = \sqrt{2281140568.33^2 + -2718596493.52^2 + 2048144274.9^2}$$

$$=4097470458.447(\mathrm{Km}^2)$$

$$\vec{D} = \vec{C}_{12} + \vec{C}_{23} + \vec{C}_{31}$$
$$= (5292516.76i + -6306848.67j + 4753375.62k)$$
$$+(-13151842.79i + 15673190.95j + -11812614.15k)$$
$$+(8147298.92i + -9709551.14j + 7317797.93k)$$

$$\vec{D} = (287972.89i + -343208.86j + 258559.4k)$$

$$\mathrm{D} = \sqrt{287972.89^2 + -343208.86^2 + 258559.4^2}$$

$$=517275.237(\mathrm{Km}^2)$$

$$\vec{S} = (\vec{r}_1)(r_2 - r_3) + (\vec{r}_2)(r_3 - r_1) + (\vec{r}_3)(r_1 - r_2)$$
$$= (-294.32i + 4265.1j + 5986.7k)(7441.68 - 7598.886)$$
$$= (-1365.5i + 3637.6j + 6346.8k)(7598.886 - 7356.513)$$
$$= (-2940.3i + 2473.7j + 6555.8k)(7356.513 - 7441.68)$$

$$\vec{S} = (-34275.77i + 478.57j + 38810.21k)$$

Step5 :

$$\vec{v}_2 = \sqrt{\frac{\mu}{\mathrm{N}D}}(\frac{\vec{D}\times\vec{\mathrm{r}}_2}{\mathrm{r}}_2 + \vec{S})$$
$$= \sqrt{\frac{398{,}600}{(4097470458.45)(517275.24)}}\times$$

$$= \left(\frac{\begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ \text{-294.32} & 4265.1 & 5986.7 \\ \text{-1365.5} & 3637.6 & 6346.8 \end{vmatrix}}{7441.6795315305} + (-34275.77i + 478.57j + 38810.21k) \right)$$

=(-6.22i + -4.01j + 1.6k) (Km/s)