1 GIBBS

Vectors :

$$\vec{r}_1 = -294.32\hat{\imath} + 4265.1\hat{\jmath} + 5986.7\hat{k}\ \texttt{Km}$$
$$\vec{r}_2 = -1365.5\hat{\imath} + 3637.6\hat{\jmath} + 6346.8\hat{k}\ \texttt{Km}$$
$$\vec{r}_3 = -2940.3\hat{\imath} + 2473.7\hat{\jmath} + 6555.8\hat{k}\ \texttt{Km}$$

Magnitude of the vector:

$$r_1 = \sqrt{-294.32^2 + 4265.1^2 + 5986.7^2} = 7356.513\ \texttt{Km}$$
$$r_2 = \sqrt{-1365.5^2 + 3637.6^2 + 6346.8^2} = 7441.68\ \texttt{Km}$$
$$r_3 = \sqrt{-2940.3^2 + 2473.7^2 + 6555.8^2} = 7598.886\ \texttt{Km}$$

Step2:$Find\vec{C}_{12}, \vec{C}_{23}$ and $\vec{C}_{31}$

$$\vec{C}_{12} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -294.32 & 4265.1 & 5986.7 \\ -1365.5 & 3637.6 & 6346.8 \end{vmatrix} = 5292516.76i + -6306848.67j + 4753375.62k(\texttt{Km}^2)$$

$$\vec{C}_{23} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1365.5 & 3637.6 & 6346.8 \\ -2940.3 & 2473.7 & 6555.8 \end{vmatrix} = 8147298.92i + -9709551.14j + 7317797.93k(\texttt{Km}^2)$$

$$\vec{C}_{31} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2940.3 & 2473.7 & 6555.8 \\ -294.32 & 4265.1 & 5986.7 \end{vmatrix} = -13151842.79i + 15673190.95j + -11812614.15k(\texttt{Km}^2)$$

Step3:The magnitude of $\vec{C}_{23}$:

$$C_{23} = \sqrt{8147298.92^2 + -9709551.14^2 + 7317797.93^2} = 14635710.764(\texttt{Km}^2)$$

The unit vector of $\vec{C}_{23}$:

$$\hat{C_{23}} = \left(\frac{8147298.92i + -9709551.14j + 7317797.93k}{14635710.764}\right) = 0.557i + -0.663j + 0.5k$$

The unit vector of $\vec{r}_1$:

$$\hat{u_{r_1}} = \left(\frac{-294.32i + 4265.1j + 5986.7k}{7356.513}\right) = -0.04i + 0.58j + 0.814k$$

Therefore :

$$\hat{u_{r_1}}.\hat{C_{23}} = -6.118058187509767e - 06$$

This is close enough to or equal zero for our purposes.
The three vectors r_1, r_2, and r_3 are coplanar.

Step4 :

$$\begin{aligned}
\vec{N} &= r_1\vec{C}_{23} + r_2\vec{C}_{31} + r_3\vec{C}_{12} \\
&= 7356.51(8147298.92i + -9709551.14j + 7317797.93k) \\
&+ 7441.68(-13151842.79i + 15673190.95j + -11812614.15k) \\
&+ 7598.89(5292516.76i + -6306848.67j + 4753375.62k) \\
\vec{N} &= (2281140568.33i + -2718596493.52j + 2048144274.9k)(\texttt{Km}^3) \\
\text{N} &= \sqrt{2281140568.33^2 + -2718596493.52^2 + 2048144274.9^2} \\
&= 4097470458.447(\texttt{Km}^3)
\end{aligned}$$

$$\begin{aligned}
\vec{D} &= \vec{C}_{12} + \vec{C}_{23} + \vec{C}_{31} \\
&= (5292516.76i + -6306848.67j + 4753375.62k) \\
&+ (-13151842.79i + 15673190.95j + -11812614.15k) \\
&+ (8147298.92i + -9709551.14j + 7317797.93k) \\
\vec{D} &= (287972.89i + -343208.86j + 258559.4k)(\texttt{Km}^2) \\
\text{D} &= \sqrt{287972.89^2 + -343208.86^2 + 258559.4^2} \\
&= 517275.237(\texttt{Km}^2)
\end{aligned}$$

$$\begin{aligned}
\vec{S} &= (\vec{r}_1)(r_2 - r_3) + (\vec{r}_2)(r_3 - r_1) + (\vec{r}_3)(r_1 - r_2) \\
&= (-294.32i + 4265.1j + 5986.7k)(7441.68 - 7598.886) \\
&= (-1365.5i + 3637.6j + 6346.8k)(7598.886 - 7356.513) \\
&= (-2940.3i + 2473.7j + 6555.8k)(7356.513 - 7441.68) \\
\vec{S} &= (-34275.77i + 478.57j + 38810.21k)(\texttt{Km}^2)
\end{aligned}$$

Step5 :

$$\vec{v}_2 = \sqrt{\left(\frac{\mu}{ND}\right)}\left(\frac{\vec{D} \times \vec{\mathrm{r}}_2}{\mathrm{r}_2} + \vec{S}\right)$$

$$\begin{aligned}
\vec{v}_2 = &\sqrt{\frac{398,600}{(4097470458.45)(517275.24)}} \\
&\times \left[\frac{\begin{vmatrix} \text{i} & \text{j} & \text{k} \\ \text{-294.32} & \text{4265.1} & \text{5986.7} \\ \text{-1365.5} & \text{3637.6} & \text{6346.8} \end{vmatrix}}{7441.68} + (-34275.77i + 478.57j + 38810.21k)\right] \\
&= (-6.22i + -4.01j + 1.6k)\ \ \texttt{(Km/s)}
\end{aligned}$$

Calculate the orbital elements
we have $\vec{r}$ and $\vec{v}$

$$\vec{r} = (-1365.5)\hat{\imath} + (3637.6)\hat{\jmath} + (6346.8)\hat{k} \;\; \texttt{(Km)}$$

$$\vec{v} = (-6.22)\hat{\imath} + (-4.01)\hat{\jmath} + (1.6)\hat{k} \;\; \texttt{(Km/s)}$$

Step1

$$r = \sqrt{\vec{r}\ .\ \vec{r}} = \sqrt{((-1365.5))^2 + ((3637.6))^2 + ((6346.8))^2} = 7441.68 \;\; \texttt{(Km)}$$

Step2

$$v = \sqrt{\vec{v}\ .\ \vec{v}} = \sqrt{((-6.22))^2 + ((-4.01))^2 + ((1.6))^2} = 7.57 \;\; \texttt{(Km/s)}$$

Step3

$$v_r = \frac{\vec{v}.\vec{r}}{r} = \frac{((-6.22)).((-1365.5)) + ((-4.01)).((3637.6)) + ((1.6)).((6346.8))}{7441.68} = 0.54 \;\; \texttt{(Km/s)}$$

Since $v_r > \ 0$, the satellite is flying away from perigee.
Step4

$$\vec{h} = \vec{r} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ (-1365.5) & (3637.6) & (6346.8) \\ (-6.22) & (-4.01) & (1.6) \end{vmatrix} = 31280.88\hat{\imath} + -37277.19\hat{\jmath} + 28095.03\hat{k} \;\; \texttt{(Km}^2\texttt{/s)}$$

Step5

$$h = \sqrt{\vec{h}\ .\ \vec{h}} = \sqrt{(31280.88)^2 + (-37277.19)^2 + (28095.03)^2} = 56190.86 \;\; \texttt{(Km}^2\texttt{/s)}$$

Step6

$$i = \cos^{-1}\frac{h_z}{h} = \cos^{-1}\left(\frac{28095.03}{56190.86}\right) \Rightarrow 60.0^\circ$$

Since $0 > i < \ 90$, this is prograde orbit.
Step7

$$\vec{N} = \vec{\hat{K}} \times \vec{h} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 1 \\ 31280.88 & -37277.19 & 28095.03 \end{vmatrix} = 37277.19\hat{\imath} + 31280.88\hat{\jmath} + -0.0\hat{k} \;\; \texttt{(Km}^2\texttt{/s)}$$

Step8

$$N = \sqrt{\vec{N}\ .\ \vec{N}} = \sqrt{(37277.19)^2 + (31280.88)^2 + (-0.0)^2} = 48662.95 \;\; \texttt{(Km}^2\texttt{/s)}$$

Step9

we know that $N_Y \geq\ 0$; therefore Ω must lie in the first or second quadrant

$$\Omega = \cos^{-1}\frac{N_x}{N} = \cos^{-1}\left(\frac{31280.88}{48662.95}\right) \Rightarrow 40.0^\circ$$

Step10

$$\vec{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\vec{r} - rv_r\vec{v}\right]$$

$$\begin{aligned}\vec{e} = &\frac{1}{398600}\left[\left(7.57^2 - \frac{398600}{7441.68}\right)\right]((-1365.5)\hat{\imath} + (3637.6)\hat{\jmath} + (6346.8)\hat{k}) \\ &- 7441.68 * 0.54\ ((-6.22)\hat{\imath} + (-4.01)\hat{\jmath} + (1.6)\hat{k}) \\ \vec{e} = &(0.05\hat{\imath} + 0.07\hat{\jmath} + 0.04\hat{k})\end{aligned}$$

Step11

$$e = \sqrt{\vec{e}\ .\ \vec{e}} = \sqrt{(0.05)^2 + (0.07)^2 + (0.04)^2} = 0.1$$

Clearly, the orbit is an ellipse.

Step12

We know that $e_z \geq\ 0$; therefore ω must lie in the first or second quadrant

$$\omega = \cos^{-1}\frac{\vec{N}.\vec{e}}{Ne} = \cos^{-1}\left[\frac{(37277.19)(0.05) + (31280.88)(0.07) + (-0.0)(0.04)}{(48662.95)(0.1)}\right] \Rightarrow 30.07^\circ$$

Step13

We know that $v_r \geq\ 0$, which means $0^\circ \geq \theta <\ 180^\circ.Therefore,$

$$\theta = \cos^{-1}\frac{\vec{e}.\vec{r}}{er} = \cos^{-1}\left[\frac{(0.05)(-1365.5) + (0.07)(3637.6) + (0.04)(6346.8)}{(0.1)(7441.68)}\right] \Rightarrow 49.93^\circ$$

Having found the six orbital elements, we can go on to compute other parameters

The perigee and apogee radii are

$$r_p = \frac{h^2}{\mu}\frac{1}{1 + e\cos(0^\circ)} = \left(\frac{56190.86^2}{398600}\right)\left(\frac{1}{1 + 0.1 * 1.0}\right) = 7200.46\ \texttt{(Km)}$$

$$r_a = \frac{h^2}{\mu}\frac{1}{1 + e\cos(180^\circ)} = \left(\frac{56190.86^2}{398600}\right)\left(\frac{1}{1 + 0.1 * -1.0}\right) = 8802.41\ \texttt{(Km)}$$

From these it follows that the semimajor axis of the ellipse is

$$a = \frac{1}{2}\left(r_p + r_a\right) = 8001.44\ \texttt{(Km)}$$