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SPICE Model of Thermoelectric Elements Including Thermal Effects

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Abstract

An electrical model for a Peltier cell, based on the analogy between thermal and electrical variables, is proposed. The use of thermal models allows the global performance of thermocooling circuit and signal system to be checked by using electrical circuit analysis programs such as SPICE. The maximum error in the steady state between measured and simulated temperatures are less than 0.3 ^{o}C for hot temperature and less than 0.2 ^{o}C for cold temperature with 31.7 ^{o}C of temperature change.

1. Introduction

Thermoelectric coolers (TEC) are versatile temperature control devices. Among the many profits provided by the TEC devices it can found the following:

- They don't have moving parts.
- They are used in applications where space limitations and reliability are paramount.
- They don't have CFC's.
- They may be used for heating or cooling by reversing the direction of current flow.

Presently, we are working in the development of a low-cost pollutant gas detector based on infrared optical absorption spectroscopy [1][2]. We use a PbSe array of 64 pixels [3] as sensor element, this array incorporate a TEC device. The TEC element is necessary because dark current characteristics of photoconductors have a big dependence on the temperature. In our case, if temperature increases 1 ^{o}C, around 298 K, then the PbSe dark resistance increases close to 3% [4][5][6]. The variation of characteristics may be misunderstood as a variation of the concentration of pollutant gases.

The thermoelectric cooling cell that includes the PbSe array has a heat pumping capacity of 0.97 W. We need a cold temperature in the TEC around -20 ^{o}C and a stability of temperature around 0.5 ^{o}C for a resolution of 2 ppm in the calculation of the gas concentration.

Development of a TEC model for PSPICE circuit simulation program allows the TEC cells to be included in the computer-aided design of accurate control circuits.

A SPICE model is proposed for thermoelectric elements that are based in the Peltier effect [7][8]. The model has been validated and the error between measured and simulated temperatures in the steady state is less than 0.5 ^{o}C.

2. Physical phenomena

Four basic physical phenomena can be associated with the operation of thermoelectric devices: The Seebeck effect, the Peltier effect, the Thomson effect, and the Joule effect. The Seebeck effect is the voltage generated when a temperature change is maintained between the two sides of a TEC. The Peltier effect is the heating or cooling effect observed when an electrical current is passed through two dissimilar junctions. The Thomson effect is heating or cooling effect in a homogeneous conductor observed when an electrical current is passed in the direction of a temperature gradient. The Joule effect is the heating effect observed in a conductor as an electrical current is passed through the conductor.

A typical TEC module consists of two ceramic plates with several p- and n-type semiconductor (SC) material connected electrically in series and thermally in parallel, see in Fig. 1.

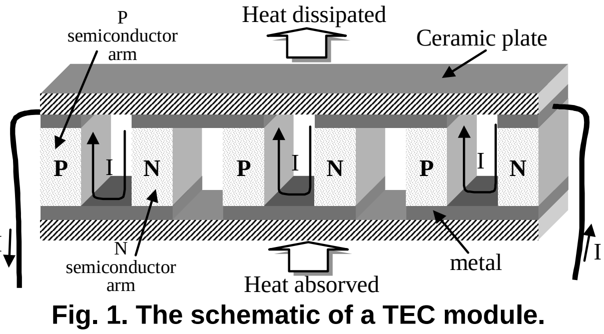

Fig. 1. The schematic of a TEC module.

Firstly, we only consider a SC bar having a different temperature at each end through which the current is flowing, see Fig. 2. Where

T_c is the cold side temperature,
T_h is the hot side temperature,
q_c is the absorbed heat in the cold side,
q_h is the generated heat in the hot side, and
I is the electrical current.

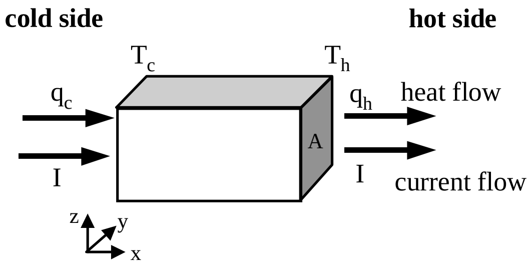

Fig. 2. Definition of current and heat flows in a homogeneous bar.

Under steady state conditions, the contribution to the energy flow through a unit volume of the phenomena associated with thermoelectric devices can be described by the following differential equation:

$$TJ\,\frac{d\alpha}{dx} + \tau J\,\frac{dT}{dx} - \rho J^2 - \frac{d}{dx}\left(k\,\frac{dT}{dx}\right) = 0 \qquad (1)$$

Where
T is the absolute temperature (K),
J is the electrical current density (A/cm^2),
α is the Seebeck coefficient (V/K),
τ is the Thomson coefficient (V/K),
ρ is the electrical resistivity (Ω cm) , and
k is the thermal conductivity of the material.

If we consider a thermoelectric device as a couple of two dissimilar semiconductors (N and P) and we assume averaged transport properties, then we can solve numerically (1) for the N arm as

$$k_N\,\frac{d^2T}{dx^2} - \tau_N J\,\frac{dT}{dx} + \rho_N J^2 = 0 \qquad (2)$$

where k_N, τ_N and ρ_N are averaged properties.

The same equation will be applied for the P arm, but with different properties and an opposite direction of current. The equation corresponding to heat flow at the junction of two dissimilar conductors is at the cold side:

$$q_c = \alpha T_c\, I + \frac{1}{2}\tau_m\, I\,\Delta T - \frac{1}{2} I^2 R_m - K_m \Delta T \qquad (3)$$

and at the hot side:

$$q_h = \alpha T_h\, I - \frac{1}{2}\tau_m\, I\,\Delta T + \frac{1}{2} I^2 R_m - K_m \Delta T \qquad (4)$$

where τ_m, R_m and K_m are average properties of a couple.

The electrical power is equal to the difference between heat flow at the hot side and heat flow at the cold side:

$$P_e = q_h - q_c = \alpha(T_h - T_c)\, I - \tau_m\, I\,\Delta T + I^2 R_m \qquad (5)$$

The electrical behavior is only due to Seebeck and Joule effects. Thus, the voltage at the thermoelectric terminals is:

$$V_p = \alpha(T_h - T_c) + IR_m \qquad (6)$$

In the next section, we will obtain the SPICE model from the described equations of thermoelectric phenomena.

3. Electric model of a Peltier device

A Peltier device can be modeled by a three port system: two thermal ports and an electrical port as it can be seen in Fig. 3. The voltage of thermal ports corresponds to the temperature of the cooled surface, T_c, and the temperature of the heated surface, T_h. The current corresponds to the thermal power absorbed from the device being cooled and from heat generated because of Joule effect.

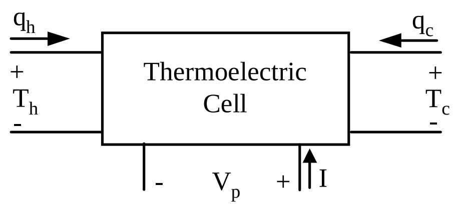

Fig. 3. Three port model for a TEC element

3.1 Thermal ports

Table I lists the analogies between electrical and thermal variables that will be used.

Table I. Analogies between electrical and thermal variables

Thermal variable	Electrical variable
Heat flow (W)	current flow (A)
Temperature difference (K)	Voltage (V)
Thermal conductivity (Wm^{-1}K^{-1})	Electrical conductivity (Ω^{-1}m^{-1})
Thermal mass (J/K)	Electrical capacity (F)

According to Table I, an electrical current source models a heat flow source and a voltage source models a temperature source. A resistor with value $l\, k_m^{-1} A^{-1}$ represents a thermal loss, where l (m) is the length of the material and A (m^2) is the section. And, a capacitor with value mc_o is a thermal mass, where m (g) is the mass of the material and c_o (Jg^{-1}K^{-1}) is the specific heat.

Thus, according to the thermal-electrical analogies and the thermal equations (see (3) and (4)) the proposed thermal model results in the circuit shown by Fig. 4.

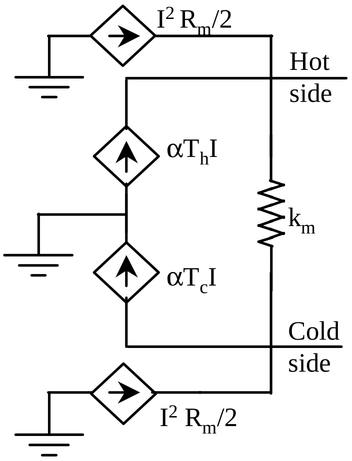

Fig. 4. Thermal model proposed

The proposed model is equivalent to the model presented in Fig. 5 where we have take into account the electrical input power (P_e) and the heat flow absorbed at the cold surface.

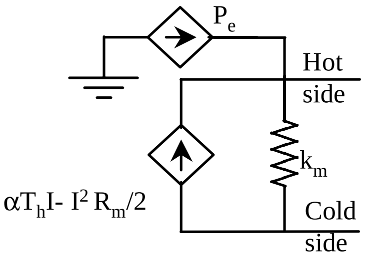

Fig. 5. Equivalent model for the thermal circuit

Now, we can add two capacitors to take into account the thermal mass of every side of the TEC module. Fig. 6 shows the complete model, C_h is the capacitor for the hot side and C_c is the capacitor for the cold side. Measurements have shown that the capacitance values C_h and C_c are around 2 J/K.

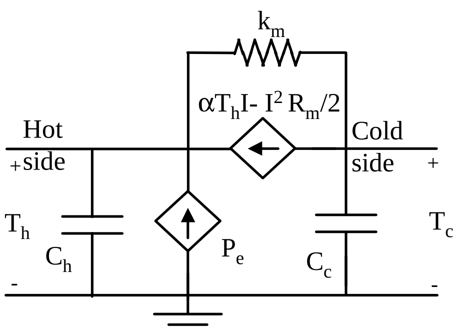

Fig. 6. Modification of the thermal circuit to take into account the thermal mass of the cold side, C_c, and the hot side, C_h

The thermal circuit consists of two voltage-controlled current sources, a thermal resistance and two thermal capacitors. Current sources model the heat flow between cold surface and hot surface.

3.2 Electrical port

The electrical behavior (see (6)) can be modeled by a voltage source depending on temperature difference between hot and cold surfaces, and a resistance for the Joule effect. Fig. 7 shows the proposed electrical circuit, where V_α is the Seebeck voltage produced by two dissimilar conductors [7]:

$$V_\alpha = \alpha (T_h - T_c) \qquad (7)$$

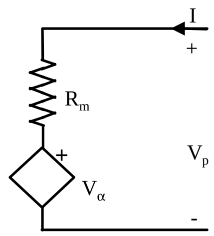

Fig. 7. Proposed electrical model.

3.3 Complete model

As it can be seen, Fig. 8 shows the structure of the equivalent circuit proposed for a TEC element, consisting of a thermal and an electric circuit. All temperatures are in Kelvin degrees.

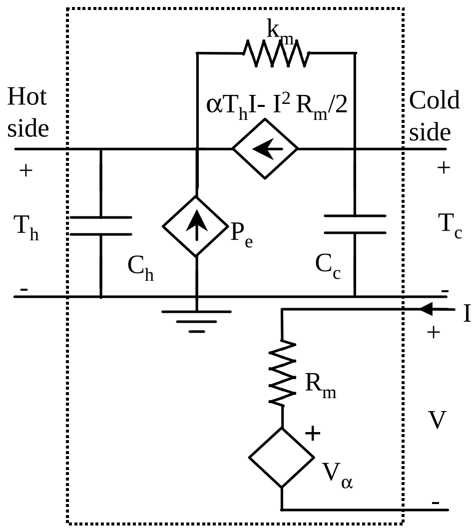

Fig. 8. Proposed complete model.

4. Measurement and simulation results

The structure of the measuring system is shown in Fig. 9. The array of 64 photoconductors of PbSe is connected to a big aluminum mass that is joined to the cold surface of the TEC device. The hot surface of TEC device is connected to a heat sink. The electrical model of the measuring system without photoconductor array is shown in Fig. 10, where

- k_{rad} is thermal resistance of the heat sink (0.34 K/W),
- k_{conint} is the thermal resistance of the thermal mass between the TEC and the photoconductor array (3.1 K/W),
- k_{sil} is the thermal resistance of silicone used between the TEC and the thermal mass, and between the TEC and the heat sink (0.143 K/W),
- C_{rad} is de thermal capacitance of heat sink (340 J/K),
- C_{conint} is the thermal capacitance of the thermal mass (304 J/K), and
- T_{amb} is the ambient temperature (296.4 K).

The TEC is a K1-127-1.4/1.5 module whose characteristics are shown in Table II.

Table II. Characteristics of K1-127-1.4/1.5 TEC

Maximum current	6 A
Maximum voltage	15.4 V
Temperature change	73 °C
Maximum heat load	53 W
Size	40 mm x 40 mm x 3.9 mm

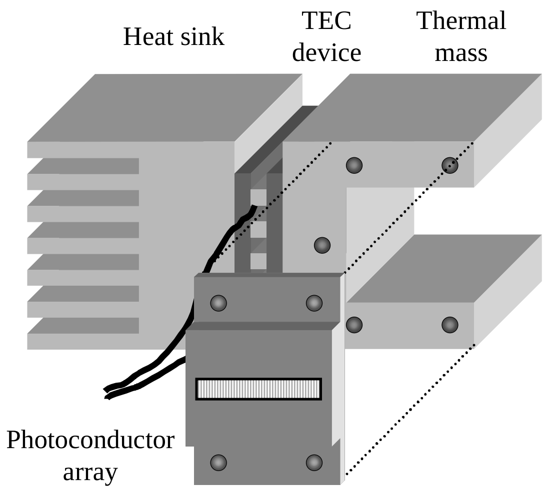

Fig. 9. Structure of the measuring system.

The measurements and simulations corresponding to a drive current of 2.1 A are shown in Fig. 11. The temperatures of the cold and the hot surfaces stabilize around –0.2 °C and 31.5 °C, respectively.

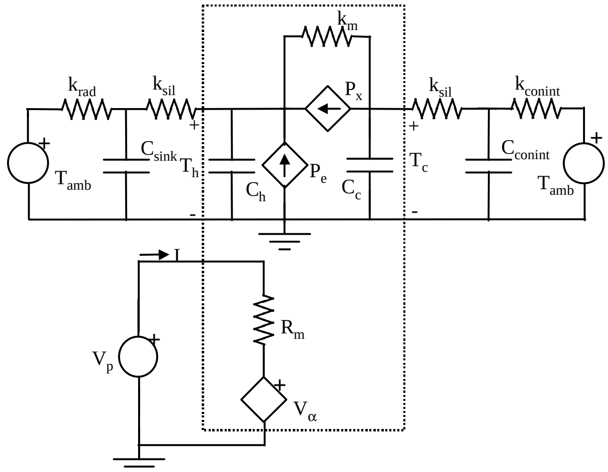

Fig. 10. Model of the measuring system.

As it can be seen, there is a huge resemblance between measured and simulated temperatures. Fig. 12 shows the absolute error between measured and simulated temperatures. The maximum error of hot temperature in the steady state is less than 0.3 °C and the maximum error of cold temperature is less than 0.2 °C.

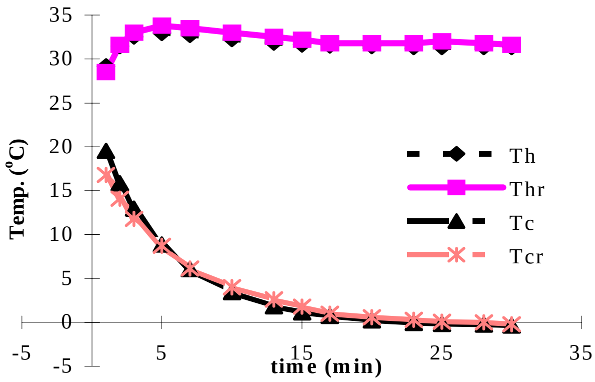

Fig. 11 Measured (Thr and Tcr) and simulated (Th and Tc) temperatures for a drive current of 2.1 A

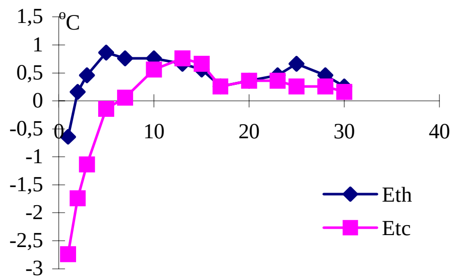

Fig. 12. Absolute error between measured and simulated temperatures for a drive current of 2.1 A

The SPICE code used for the simulations is:

```
***************************************
* SIMULATION OF MEASURING SYSTEM
* TAMB = AMBIENT TEMPERATURE
* SE = SEEBECK CONSTANT
***************************************

.PARAM TAMB=296.4, SE=0.05292, RM=1.806

*********************************
******** THERMAL CIRCUIT ********
*********************************

*** HEAT SINK ***

VTAMB 3 0 DC {TAMB}
RKRAD 4 3 0.34
CRAD 4 0 340 IC={TAMB}
RSILH 4 1 0.143

*** THERMAL PELTIER MODEL ***

CH 1 0 2 IC={TAMB}
GPE 0 1 VALUE={I(VPOS)*(I(VPOS)*RP+SE*
+(V(1)-V(2)))}
RKM 1 2 1.768
GPX 2 1 VALUE={I(VPOS)*
+(SE*V(2)-0.9*I(VPOS))}
CC 2 0 2 IC={TAMB}

*** THERMAL MASS ***

RSILC 5 2 0.143
CCONINT 5 0 304 IC={TAMB}
RCONINT 5 3 3.1

************************************
******** ELECTRICAL CIRCUIT ********
************************************

*** ELECTRICAL PELTIER MODEL ***

VPOS 11 13 DC 0
RM 13 12 1.8
EALPHA 12 0 VALUE = {SE*(V(1)-V(2))}

***** EXTERNAL CURRENT SOURCE *****

IPOS 0 11 2.1

.OPTIONS RELTOL=5U
.TRAN 1 2K UIC
.END
*****************************************
```

5. Conclusions

A three port SPICE model for thermoelectric coolers has been proposed. The model allows complete simulations of electrical and thermal behavior in circuits with TEC modules. The accuracy of TEC model has been demonstrated, for a temperature change of 31.7 ^{o}C the maximum error between measured and simulated temperatures is less than 0.3 ^{o}C in the steady state.

6. References

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