Normal, Gamma, Beta Distributions Paper

Within the realm of continuous probability distribution, the most used kind is Normal Distribution. Normal Distribution is also known as the Gaussian Distribution and is very important in regards to independent, random variables. It is widely known by its bell-shaped curve in statistical reports. Normal Distribution is symmetrical around its mean, but not all symmetrical distributions are referred to as normal. In normal distribution, similarly to any probability distribution, it describes how the values of a variable are distributed. This specific distribution, however, is the most important probability distribution in statistics because it "accurately describes the distribution of values for many natural phenomena" (Frost 2023).

A real-life example of this would be looking at pizza delivery times. Putting density and minutes on the x and y axis and plotting the points allows for a typical bell curve pattern to ensue. From this, you can easily find the mean and standard deviation. For the Normal Probability Distribution, Definition 4.8 from the textbook states that "A random variable Y is said to have a normal probability distribution if and only if for $\sigma > 0$ and $-\infty < \mu < \infty$, the density function of Y is $f(y) = 1/\sigma\sqrt{2\pi}\ e^{-(y-\mu)^2/(2\sigma^2)}$, $-\infty < y < \infty$"(p. 178) . It is important to note that the normal density function contains two parameters, μ and σ. Theorem 4.7 from the textbook states that "If Y is a normally distributed random variable with parameters μ and σ, then $E(Y) = \mu$ and $V(Y) = \sigma^2$"(p. 178). The results contained in Theorem 4.7 imply that the parameter μ locates the center of the distribution and that σ measures its spread. In regards to this, the normal density function is symmetric around the value μ, so areas need to be tabulated on only one side of the mean. The tabulated areas are to the right of points z, where z is the distance from the mean, measured in standard deviations.

Looking at Gamma Distribution some random variables are always nonnegative and for various reasons yield distributions of data that are skewed (nonsymmetric) to the right. In other words, the density function progressively decreases as y increases, and the majority of the area under the density function is found close to the origin. An example of this would be the lengths of time between malfunctions for aircraft engines since they possess a skewed frequency distribution. “The Gamma distribution is a particular of normal distribution, which describes many life events including predicted rainfall, the reliability of mechanical tools and machines, or any applications that only have positive results” (Metwalli). Unfortunately, these applications are often unbalanced, which explains the Gamma Distribution’s skewed shape.

Definition 4.9 from the textbook describes how “A random variable Y is said to have a *gamma distribution with parameters* $\alpha > 0$ and $\beta > 0$ if and only if the density function of Y is

$f(y) = \{y^\alpha-1e-y/\beta/\beta\alpha(\alpha), \quad 0 \leq y < \infty,$

0, elsewhere, where $(\alpha) = \int_0^\infty y^\alpha-1\ e^{-y}\, dy$”(p. 185). In comparison to this, Theorem 4.8 from the textbook states “If Y has a gamma distribution with parameters α and β, then $\mu = E(Y) = \alpha\beta$ and $\sigma^2 = V(Y) = \alpha\beta^2$”(p. 186). This effectively links the parameters of the gamma distribution to key statistical measures, namely the mean (μ) and variance (σ^2), providing a comprehensive understanding of the distribution's characteristics based on its parameters.

Transitioning from the Gamma Distribution, it's interesting to explore the Beta Probability Distribution, another important distribution in statistics. According to the textbook, “The beta density function is a two-parameter density function defined over the closed interval $0 \leq y \leq 1$”(p. 194). It is often used as a model for proportions. Some real-life examples of this would be looking at the proportion of impurities in a chemical product or the proportion of time that a machine is under repair. When trying to identify Beta Density Functions it may be a bit difficult due to the fact that they assume widely differing shapes for various values of the two parameters α and β. The Beta Distribution has a unique place in the universe of probability distributions. This is because it represents every conceivable combination of probability values. Modeling uncertainty regarding the likelihood of success in a random experiment is among the most popular uses of this distribution. It is utilized in the Beta Distribution three-point approach in project management, for example, to pinpoint uncertainty in project time estimation. Strong quantitative techniques and fundamental statistics are provided to compute confidence intervals for the anticipated completion time.

Throughout this closer look at Normal, Gamma, and Beta Distributions, there were a lot of interesting findings. It revealed the rich and varied landscape of continuous probability distributions, each with its unique characteristics and applications. The depiction of symmetric data around a mean, which is frequently observed in social and natural processes, is made possible by the Normal Distribution. When predicting time-based occurrences like equipment breakdowns, the Gamma Distribution is a valuable tool because it is designed for skewed, nonnegative data. Because of its versatility in representing fixed-interval probabilities, the Beta Distribution is essential in proportionate contexts like project management and quality control. When combined, these distributions provide statisticians with an essential toolset that allows for the sophisticated modeling and comprehension of a broad variety of data kinds and real-world occurrences.

Works Cited

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