Stability analysis using Bode plots

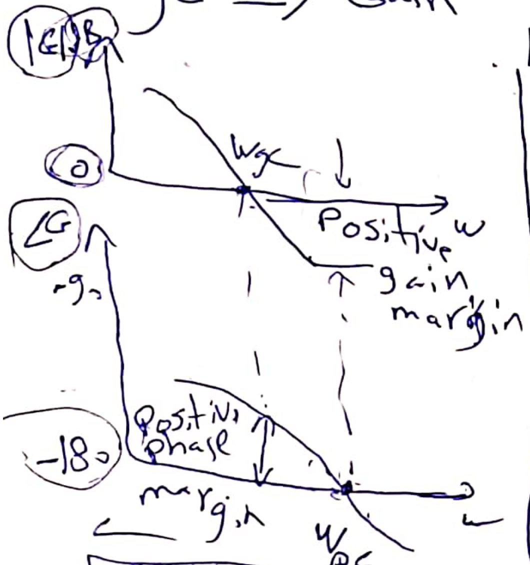
⇒ $\angle G(z)$ ⇒ magnitude and phase angle as a function of frequency

(ω) angular frequency → $|G|$ dB, → $\angle G$

① ω_{pc} ⇒ phase cross over frequency (-180°) rad/sec

ω_{gc} ⇒ Gain // // // (Zero dB) rad/sec

semi log $|G|$ dB

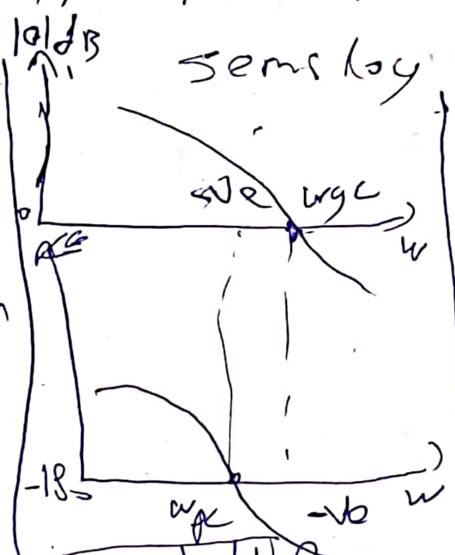


Stable

① $\omega_{pc} > \omega_{gc}$

or

② GM +ve
PM +ve

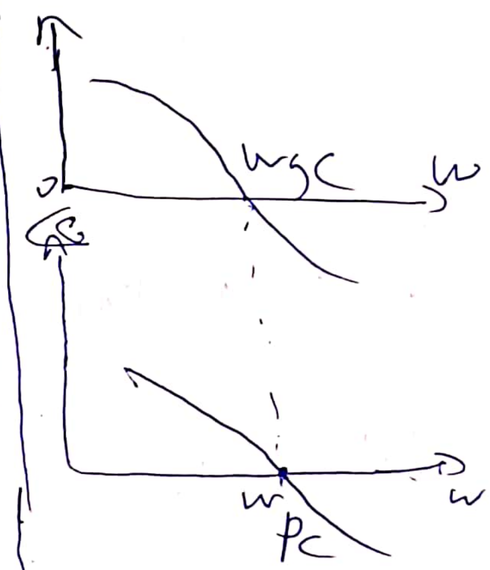


unstable

① $\omega_{pc} < \omega_{gc}$

or

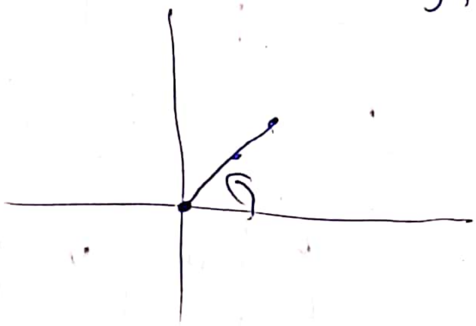
② GM -ve
PM -ve



① $\omega_{pc} = \omega_{gc}$

② GM = zero
PM = zero

marginally stable



bode plot drawing $\boxed{z = e^{jT}}$

$s = j w$

① $\Rightarrow G(w) = \dfrac{K\left(1 + \frac{w}{w_1}\right)\left(1 + \frac{w}{w_2}\right) \cdots}{w^N \left(1 + \frac{w}{w_a}\right)\left(1 + \frac{w}{w_b}\right) \cdots}$

$\Rightarrow G(z) = \dfrac{b_0 z^m + b_1 z^{m-1} + \cdots + b_m}{z^n + a_1 z^{n-1} + \cdots + a_n}$

bilinear transformation

$\Rightarrow z = \dfrac{1 + \left(\frac{T}{2}\right) w}{1 - \left(\frac{T}{2}\right) w}$

② ⓐ magnitude corner frequency

Poles : w_a , w_b

ⓑ Zero : w_1, w_2

$\Rightarrow$ initial $20 \log K - 20 \log w^N \Rightarrow$ or

$w = 1 \Rightarrow 20 \log K$ dB

ⓒ pole -20 dB / decade

zero 20 "

③ phase plot

$\phi = \sum \theta_z - \sum \theta_p$

w	0	w_1	$\cdots$	w_2	w_a	w_b	∞
ϕ							

- sketch the bode plots and determine the gain cross over and phase cross-over frequency

$$G(z) = \frac{-K(0.01873z + 0.01752)}{1000(z^2 - 1.8187z + 0.8187)}$$

$T = 0.2\,sec$ $K = \frac{2}{3}$

$1 \Longrightarrow$ $z = \frac{1 + \frac{T}{2}w}{1 - \frac{T}{2}w} = \frac{1 + 0.1w}{1 - 0.1w}$

$$G(w) = \frac{-K\left(0.01873 \times \frac{1+0.1w}{1-0.1w} + 0.01752\right)}{1000\left(\left(\frac{1+0.1w}{1-0.1w}\right)^2 - 1.8187\left(\frac{1+0.1w}{1-0.1w}\right) + 0.8187\right)} \times \frac{(1-0.1w)^2}{(1-0.1w)^2}$$

$$\Rightarrow = \frac{-K(-0.00033w^2 + 0.09633w + 0.9966)}{1000(w^2 + 0.9969w)}$$

$$= \frac{-K(w + 300)(w - 10)}{1000w(w+1)}$$

semi

$$\Rightarrow \boxed{= \frac{3K\left(1 + \frac{w}{300}\right)\left(1 - \frac{w}{10}\right)}{w\left(1 + \frac{w}{1}\right)}}$$

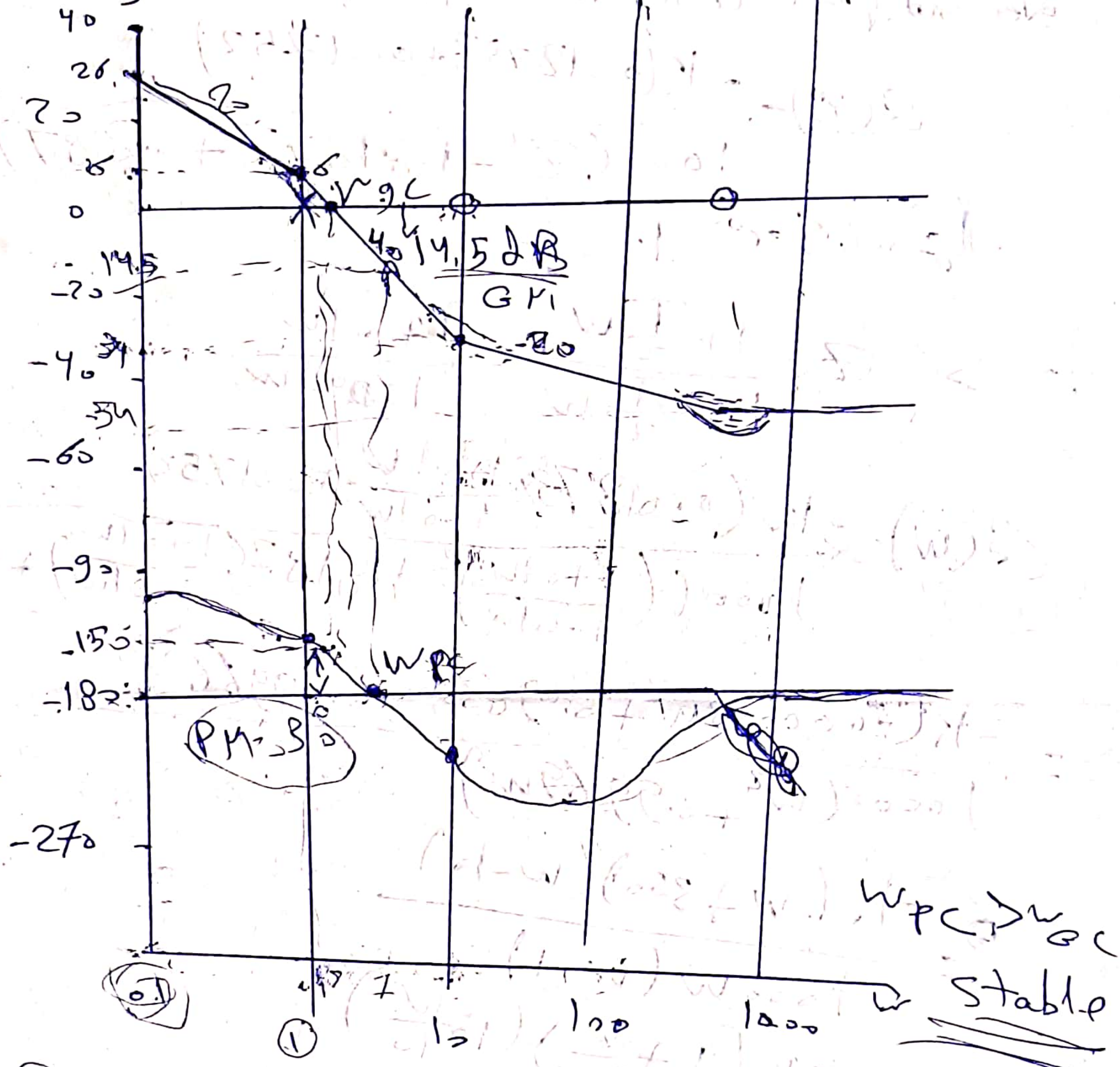
$2 \Longrightarrow$ corner frequency

$P_1 = 1$, $z_1 = 10$, $z_2 = 300$

factors

$= 20\log 3K - 20\log w = 20\log 2 - 20\log w^{-1}$

$F_c = .328$



$\theta = \sum \theta_z - \sum \theta_p$

$= \left(\tan^{-1}\left(\frac{w}{300}\right) + \tan^{-1}\frac{w}{10}\right) - \left[90 + \tan^{-1}\frac{w}{1}\right)$

W	0	0.1	1	10	100	300	500		∞
θ	-90	-96	-140	-217					-180

$$1 - G(z) = \frac{0.3679K(z+0.7181)}{100(z-1)(z-0.3679)}$$

Sol:-

$$1 \Longrightarrow \boxed{z = \frac{1+(\frac{T}{2})w}{1-(\frac{T}{2})w}} \longrightarrow T = 0.1 sec$$

$$z = \frac{1+0.05w}{1-0.05w}$$

$$G(w) = \frac{0.3679K\left(\frac{1+0.05w}{1-0.05w}+0.7181\right)}{100\left(\frac{1+0.05w}{1-0.05w}-1\right)\left(\frac{1+0.05w}{1-0.05w}-0.3679\right)}$$

$$\times (1-0.05w)^2$$

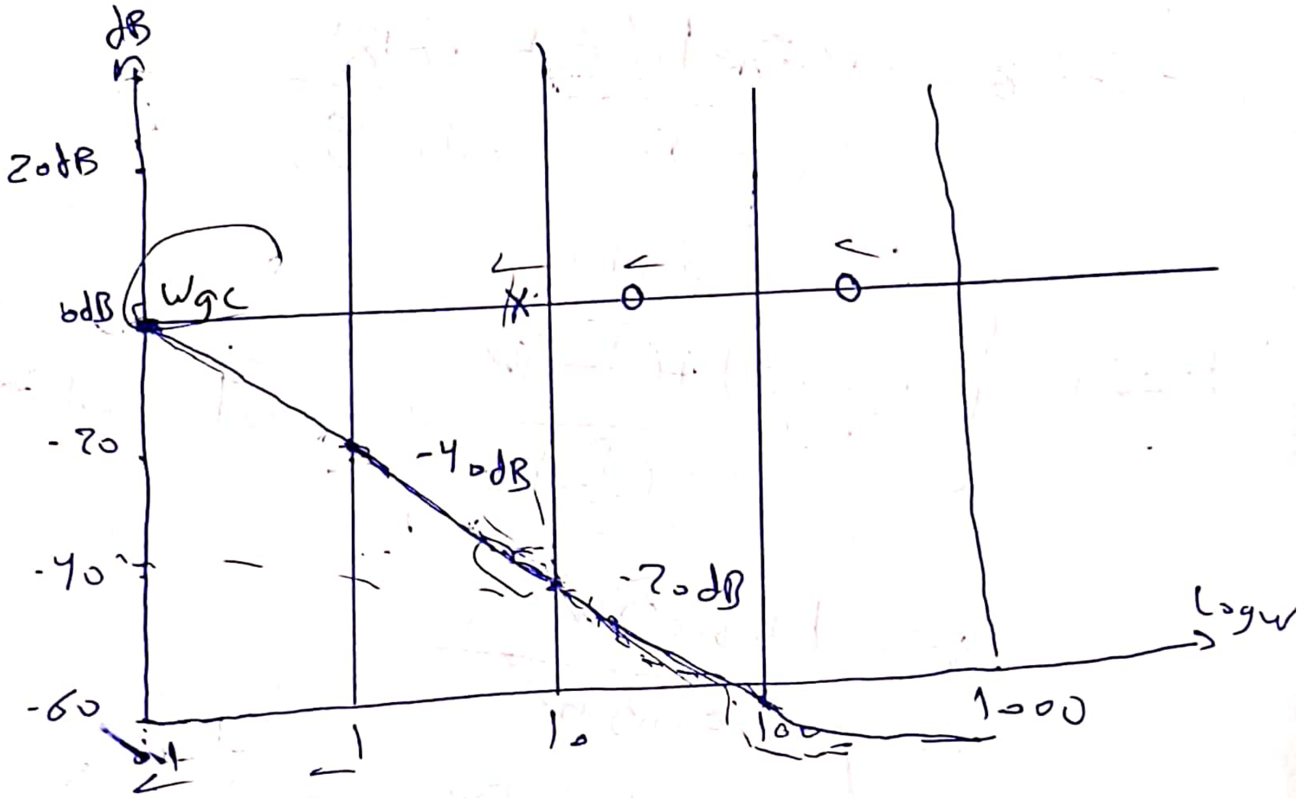
$$= \frac{0.1K(1-0.05w)(0.0082w+1)}{w(0.1082w+1)}$$

$$\boxed{= \frac{0.1K\left(1-\frac{1}{20}w\right)\left(\frac{1}{121.94}w+1\right)}{w\left(\frac{1}{9.2421}w+1\right)}}$$

$2 \Longrightarrow$ corner frequency $z_1 = 20$, $z_2 = 121.94$

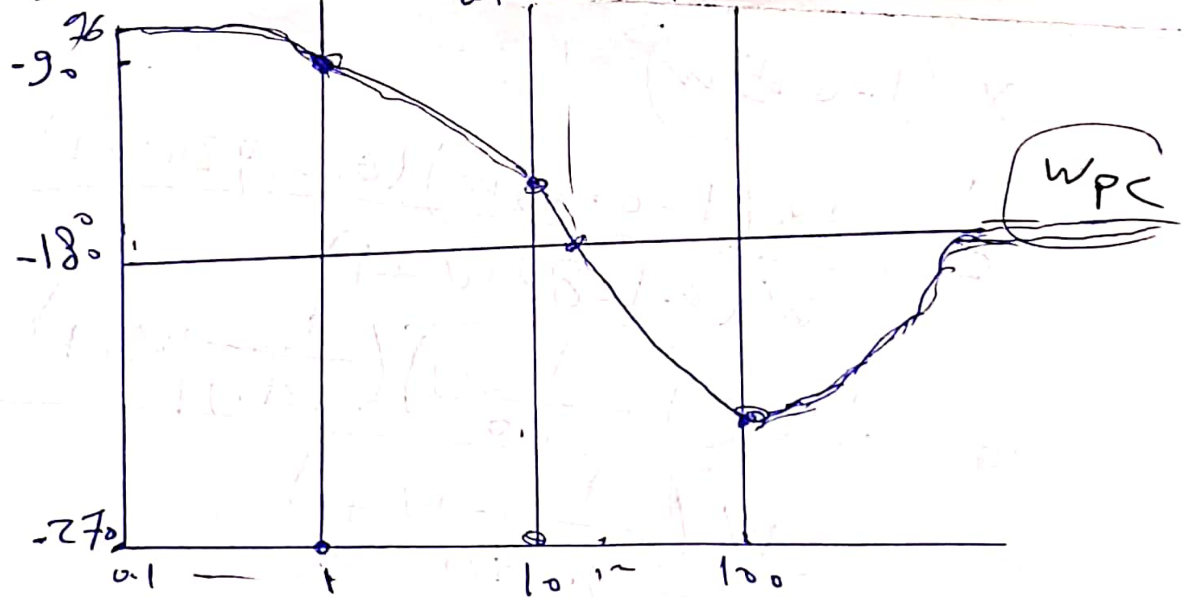
$\boxed{P_1 = 9.2421}$

Ze.



$\boxed{K=1}$

$* \; 20\log(0.1K) - 20\log(w) = 20\log 0.1 - 20\log w$ at $w=0.1 \Rightarrow 0\,dB$, at $w=1 \Rightarrow -20\,dB$



Phase plot $\theta = \sum\theta_z - \sum\theta_p$

$= \left(\tan^{-1}\frac{w}{121.94} - \tan^{-1}\frac{w}{20}\right) - \left[90 + \tan^{-1}\frac{w}{9.2421}\right)$

$w_{pc} > w_{gc}$ stable

w	0	1	2	3	10	11	12	13	100	∞
θ	-90	-98.5			-159.9				-213	-180