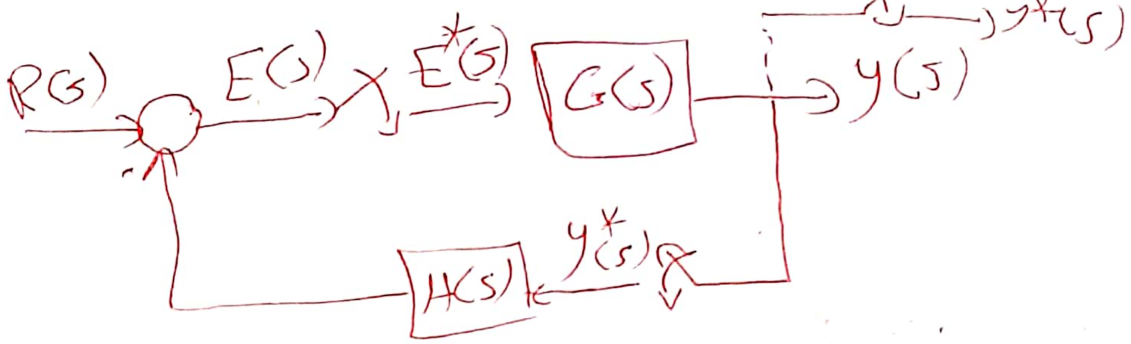
Transfer function of closed Loop



$$Y(s) = G(s)\,E^*(s)$$

$$E(s) = R(s) - H(s)\,y^*(s)$$

$$E^*(s) = R^*(s) - H^*(s)\,y^*(s)$$

$$y(s) = G(s) * \left[R^*(s) - H^*(s)\,y^*(s)\right]$$

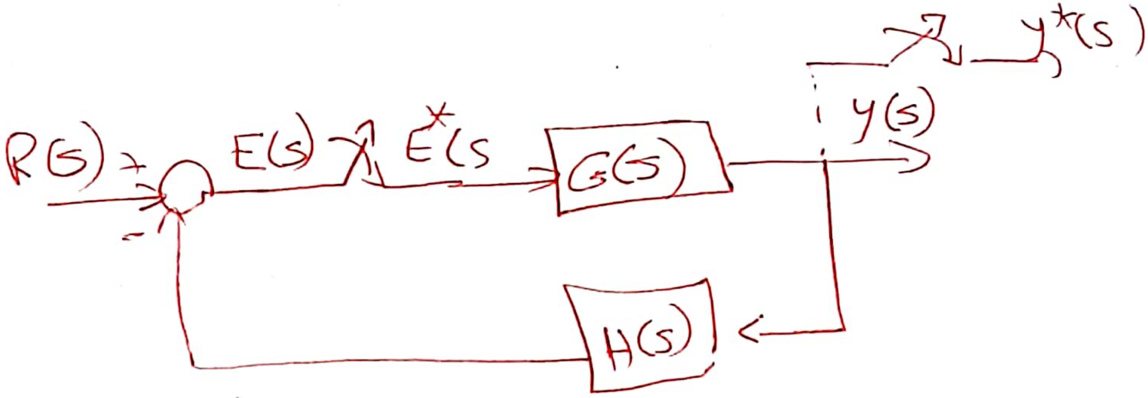
$$y(s) = G(s)\,R^*(s) - H^*(s)\,y^*(s)\,G(s)$$

$$y^*(s) = G^*(s)\,R^*(s) - G^*(s)\,H^*(s)\,y^*(s)$$

$$y^*(s)\left[1 + G^*(s)\,H^*(s)\right] = G^*(s)\,R^*(s)$$

$$y^*(s) = \frac{G^*(s)}{1 + G^*(s)\,H^*(s)}\,R^*(s)$$

$$y(z) = \frac{G(z)}{1 + G(z)\,H(z)} * R(z)$$



$$Y(s) = G(s)E^*(s)$$

$$E(s) = R(s) - H(s)Y(s)$$

$$E(s) = R(s) - H(s)G(s)E^*(s)$$

$$E^*(s) = R^*(s) - GH^*(s)E^*(s)$$

$$E^*(s)\left[1 + GH^*(s)\right] = R^*(s)$$

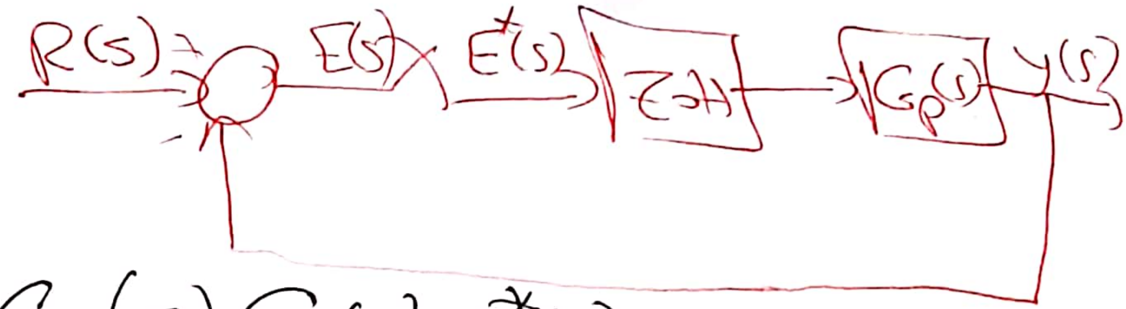
$$E^*(s) = \frac{R^*(s)}{1 + GH^*(s)}$$

$$Y(s) = G(s)\frac{R^*(s)}{1 + GH^*(s)}$$

$$Y^*(s) = \frac{G^*(s)}{1 + GH^*(s)}R^*(s)$$

$$Y(z) = \frac{G(z)}{1 + GH(z)}R(z)$$

ex9:-



$$Y(s) = G_{ZOH}(s)\, G_p(s)\, E^*(s)$$

$$E(s) = R(s) - Y(s) \Longrightarrow E^*(s) = R^*(s) - Y^*(s)$$

$$Y(s) = G_{ZOH} G_p(s)\left(R^*(s) - Y^*(s)\right)$$

$$Y(s) = G_{ZOH}(s)\, G_p(s)\, R^*(s) - G_{ZOH} G_p(s)\, Y^*(s)$$

$$Y^*(s) = G_{ZOH} G_p^*(s)\, R^*(s) - G_{Zoh} G_p^*(s)\, Y^*(s)$$

$$Y^*(s)\left[1 + G_{Zoh} Y^*(s)\right] = G_{Zoh} G_p(s)\; R^*(s)$$

$$Y^*(s) = \frac{G_{Zoh} G_p^*(s)}{1 + G_{Zoh} G_p^*(s)} R^*(s)$$

$$Y(Z) = \frac{G_{Zoh} G_p(Z)}{1 + G_{Zoh} G_p(Z)} R(Z)$$

$$\frac{Y(Z)}{R(Z)} = \frac{G_{Zoh} G_p(Z)}{1 + G_{Zoh} G_p(Z)}$$

$$G_{zoh}(s) = \frac{1-e^{-Ts}}{s} \qquad G_p(s) = \frac{1}{s+1}$$

$$G_{zoh}G_p(s) = \frac{1-e^{-Ts}}{s} \times \frac{1}{s+1} = \frac{1-e^{-Ts}}{s(s+1)}$$

$$= (1-e^{-Ts})\left(\frac{1}{s} - \frac{1}{s+1}\right)$$

$$G_{zoh}G_p(z) = (1-z^{-1})\, Z\left\{\frac{1}{s} - \frac{1}{s+1}\right.$$

$$= \frac{z-1}{z} \times \left(\frac{z}{z-1} - \frac{z}{z-e^{-T}}\right)$$

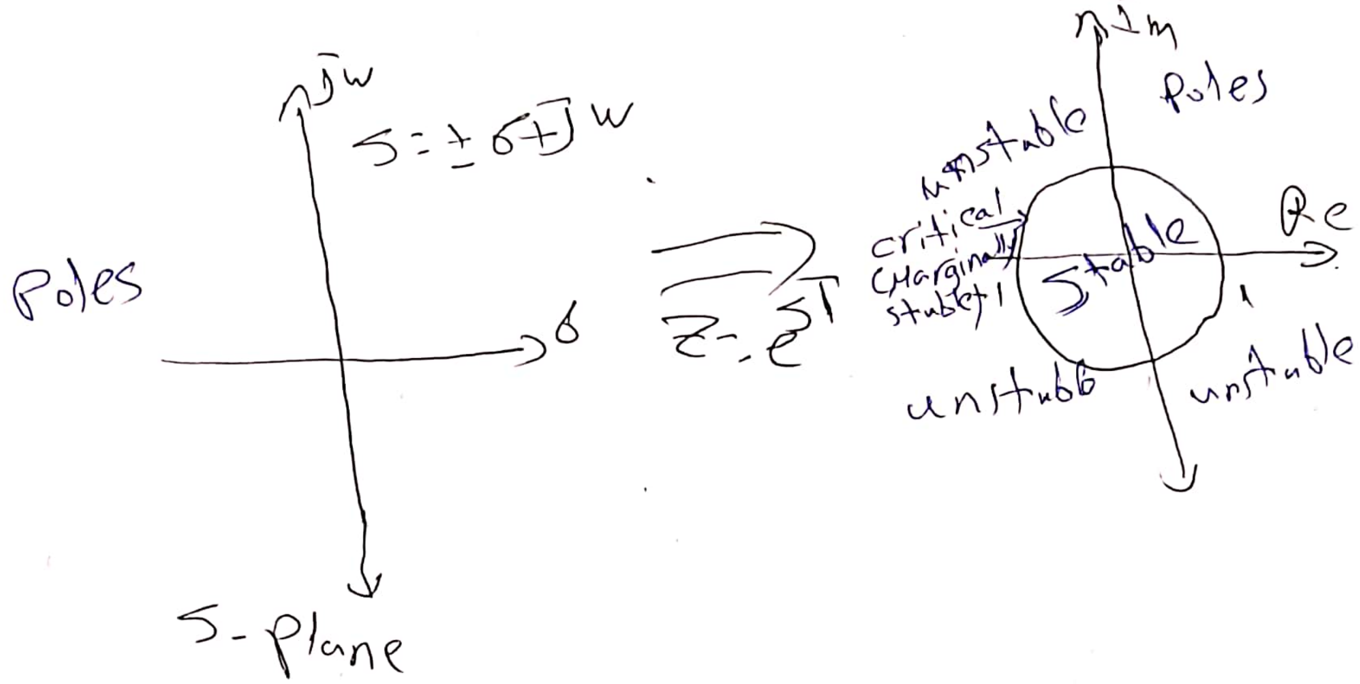
$$G_{zoh}G_p(z) = \frac{1-e^{-T}}{z-e^{-T}}$$

$$\frac{Y(z)}{R(z)} = \frac{G_{zoh}G_p(z)}{1+G_{zoh}G_p(z)}$$

$$1+G_{zoh}G_p(z) = 1 + \frac{1-e^{-T}}{z-e^{-T}} = \frac{z+1-2e^{-T}}{z-e^{-T}}$$

$$\frac{Y(z)}{R(z)} = \frac{1-e^{-T}}{z+1-2e^{-T}}$$

* stability of digital Control systems



① $H(z) = \dfrac{4(z-0.2)}{(z-0.2)(z-0.1)}$ ⟹ stable

② $H(z) = \dfrac{4(z-2)}{(z-2)(z-0.1)}$ ⟹ unstable

③ $H(z) = \dfrac{8(z+0.2)}{(z-0.8)(z-1)}$ ⟹ marginally stable

④ $H(z) = \dfrac{4(z-0.2)}{(z-0.2)(z-0.1)}$ ⟹ stable

Jury test

- the stability of z-domain polynomials directly as the routh test for s-domain polynomials

$$f(z) = a_n z^{n} + a_{n-1} z^{n-1} + \dots + a_1 z + a_0 = 0$$

* no. of rows $= 2n-3$

Row	z^0	z^1	z^2	...	z^{n-2}	z^{n-1}	z^n
1	a_0	a_1	a_2		a_{n-2}	a_{n-1}	a_n
2	a_n	a_{n-1}	a_{n-2}		a_2	a_1	a_0
3	b_0	b_1	b_2		b_{n-2}	b_{n-1}	
4	b_{n-1}	b_{n-2}	b_{n-3}		b_1	b_0	
.	c_0	c_1	c_2		c_{n-2}		
.							
$2n-3$							

$$b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix}, \quad k = 0, 1, n-1$$

$$c_k = \begin{vmatrix} b_0 & b_{n-1-k} \\ b_{n-1} & b_k \end{vmatrix}, \quad k = 0 \to n-2$$

[R]

* The necessary conditions

No. of conditions is = (n+1)

① $f(1) > 0$

② $f(-1) > 0$ if n is even

$f(-1) < 0$ " n is odd

③ $|a_0| < a_n$

④ $|b_0| > |b_{n-1}|$

⑤ $|c_0| > |c_{n-2}|$

⋮

(n+1) $|r_0| > |r_2|$

(ex1) $f(z) = z^2 - z + 0.632$

order = n = 2

# of rows = 2n-3 = 4-3 = 1

* stability conditions = n+1 = 3

① $f(1) = 0.632 > 0$ ←

② $f(-1) = 2.632 > 0$ ←

③ $|a_0| < a_2$ $0.632 < 1$ ←

Row	z^0	z^1	z^2
1	0.632	-1	1

stabl

3

Ex2 $f(z) = z^3 + 3.3z^2 + 4z + 0.8$

$n = 3$

No. of rows $= 2n - 3 = 3$,

No. of stability Conditions $n + 1 = 4$;

Row	z^0	z^1	z^2	z^3
1	a_0 0.8	a_1 4	3.3 a_2	1 a_3
2	1 a_3	a_2 3.3	4 a_1	0.8 a_0
3	$b_0 =$ -0.36	$b_1 =$ -0.1	$b_2 =$ -1.36	

$$b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix} = \begin{vmatrix} a_0 & a_{3-k} \\ a_3 & a_k \end{vmatrix}$$

$$b_0 = \begin{vmatrix} a_0 & a_3 \\ a_3 & a_0 \end{vmatrix} = \begin{vmatrix} 0.8 & 1 \\ 1 & 0.8 \end{vmatrix} = -0.36$$

$$b_1 = \begin{vmatrix} a_0 & a_2 \\ a_3 & a_1 \end{vmatrix} = -6.1$$

(1) $f(1) > 0$ $\quad f(1) = 9.1$ ✓

(2) $f(-1) < 0$ $\quad f(-1) = -0.9$ ✓

(3) $|a_0| < a_3$ $\quad |a_0| = 0.8,\ a_3 = 1$ ✓

(4) $|b_0| > |b_2|$ $\quad |b_0| = 0.36,\ |b_2| = 1.36$ ✗

unstable

[4]