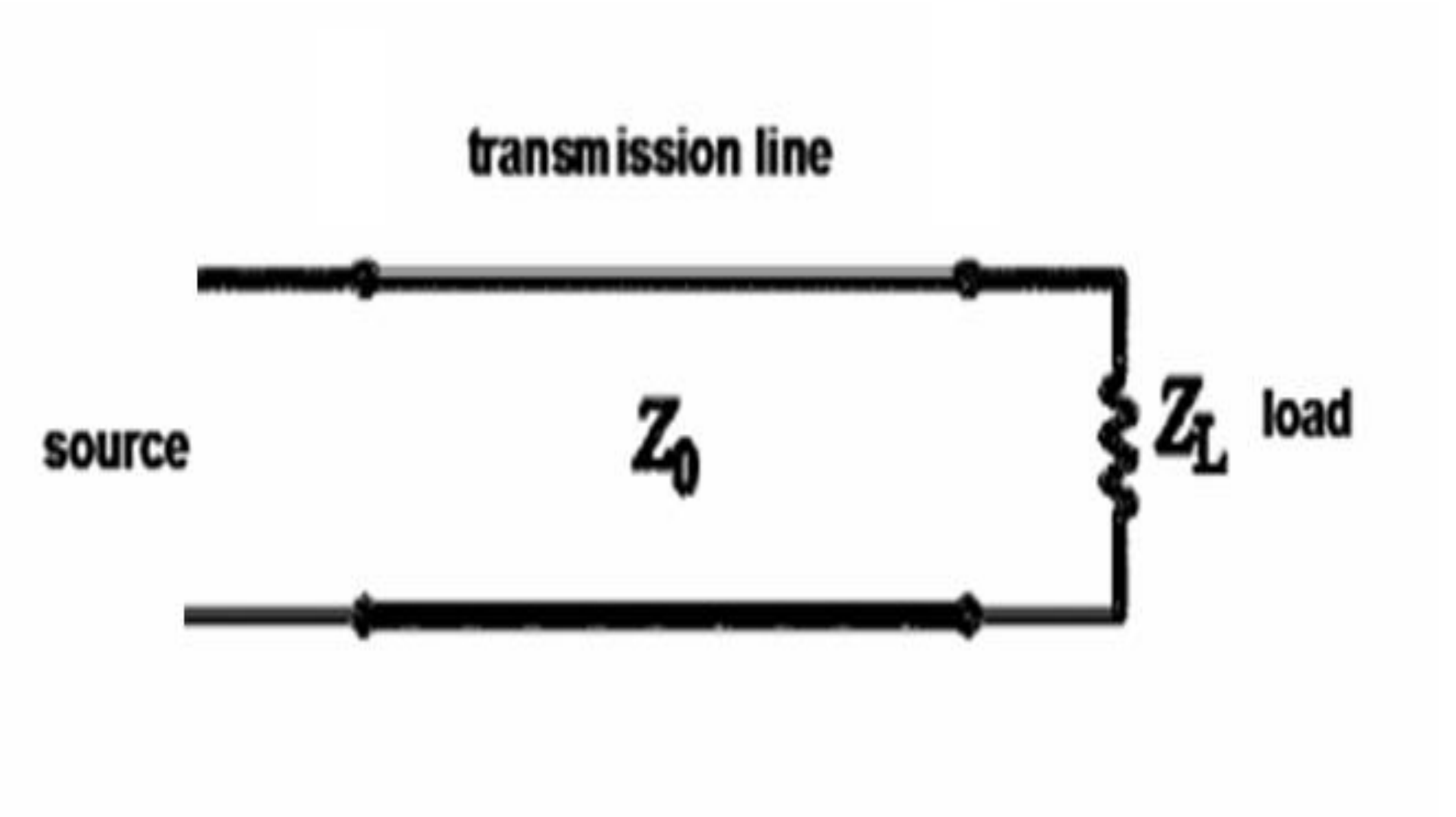
Transmission lines (T.L.) performance

Topics:

- Voltage reflection coefficient (Γ)
- Voltage standing wave ratio (VSWR)
- Input impedance of transmission line of length L

Transmission line voltage reflection coefficient



Voltage reflection coefficient (Γ) definition:

Consider a transmission line of length L connected to load impedance z_l and it is characterized by γ and z_0 characteristic impedance. . When mismatch occurs (i.e. $Z_L \neq Z_0$) if a wave is incident, then a reflected wave exist and travel back towards the sending end. Voltage reflection coefficient (Γ) is defined as the ratio between the amplitude of the reflected (or backward) wave to the amplitude of incident (or forward) wave at the load end. It is a measure of mismatch at the end of the line.

Voltage reflection coefficient (Γ) is given by

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma = \frac{V''}{V'} = |\Gamma| \angle \phi$$

Where

V' is incident wave

V'' is reflected wave

$|\Gamma|$ is the magnitude of the reflection coefficient

$\angle \phi$ is the phase of the reflection coefficient

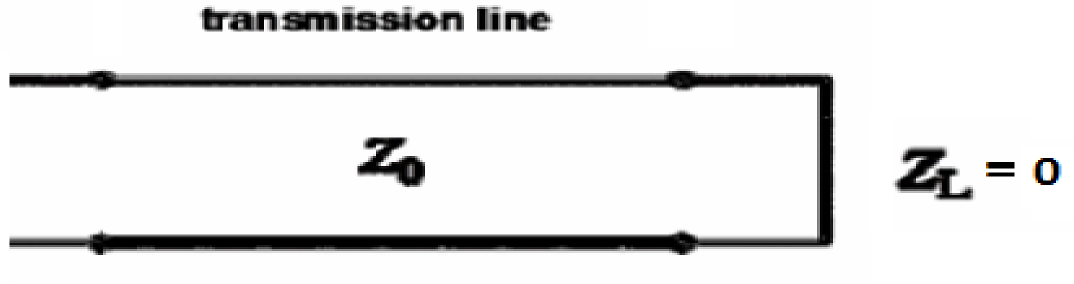
Its limits

$-1 \le \Gamma < 1$

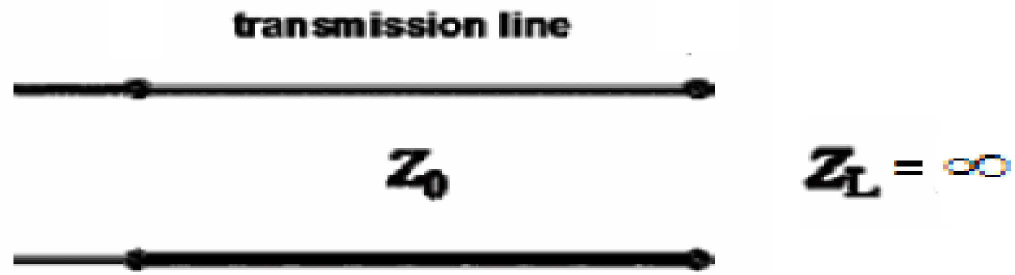
Its units: dimensionless

Voltage reflection coefficient special cases

short circuit



open circuit



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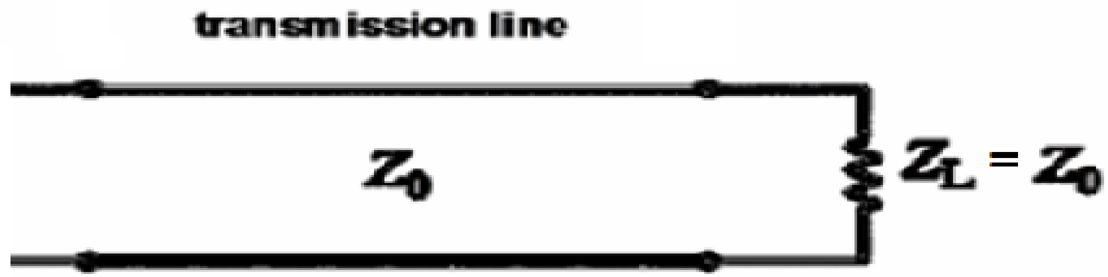


Voltage reflection coefficient (Γ) special cases

(1)In case of open circuit (O.C.)

$Z_L = \infty$ $\Gamma = 1$

$$\Gamma = \frac{\frac{Z_L}{Z_L} - \frac{Z_o}{Z_L}}{\frac{Z_L}{Z_L} + \frac{Z_o}{Z_L}} = \frac{1-0}{1+0} = 1$$

(2) In case of short circuit (S.C.)

$Z_L = 0$ $\Gamma = -1$

$$\Gamma = \frac{0 - Z_o}{0 + Z_o} = -1$$

(3) In case of matching

$Z_L = Z_o$ $\Gamma = 0$

Voltage standing wave ratio (VSWR)

When an incident wave travels on transmission line and mismatch occurs (i.e. $Z_L \neq Z_0$). Then, reflected wave occurs. The incident and the reflected waves combine and form voltage standing wave pattern has maximum and minimums at fixed distances.

Voltage standing wave ratio (VSWR) definition

Voltage standing wave ratio (VSWR) is defined as the ratio of maximum voltage to minimum voltage

$$VSWR = \frac{|V_{\max}|}{|V_{\min}|}$$

Its limits

$$1 \le VSWR < \infty$$

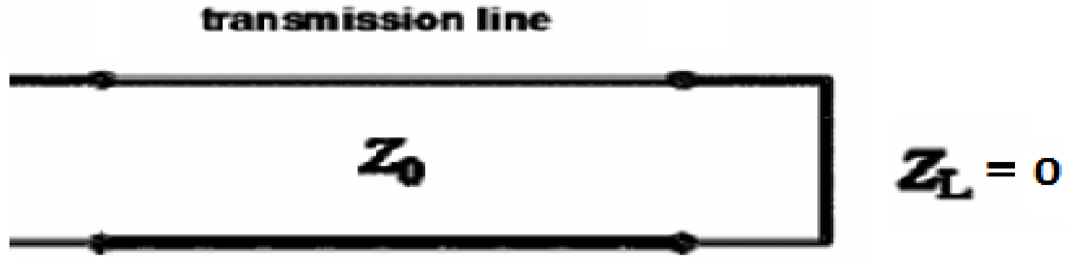
Its units: dimensionless

Relation between voltage reflection coefficient and voltage standing wave ratio

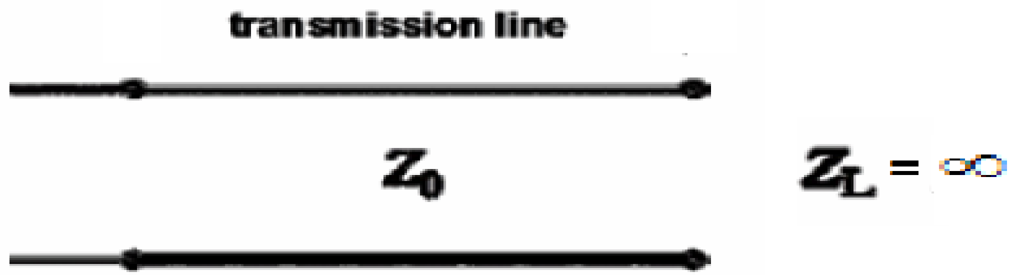
$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

VSWR special cases

short circuit



open circuit



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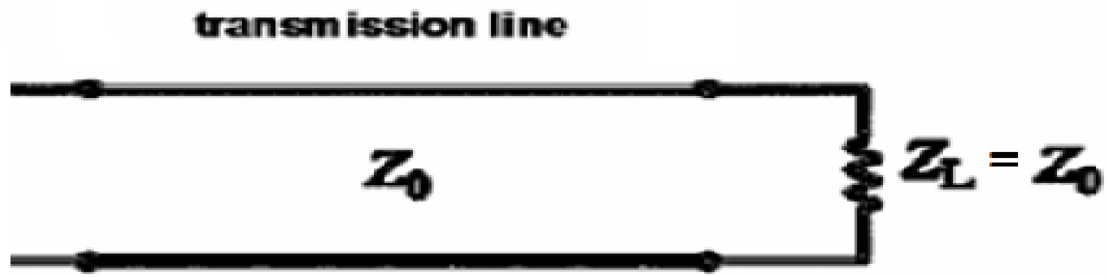


Voltage standing wave ratio (VSWR) special cases

(1)In case of short circuit (S.C.)

$$Z_L = 0$$

$$\Gamma = -1$$

$$VSWR = \frac{1+|-1|}{1-|-1|} = \frac{2}{0} = \infty$$

(2) In case of open circuit (O.C.)

$$Z_L = \infty$$

$$\Gamma = +1$$

$$VSWR = \frac{1+|1|}{1-|1|} = \frac{2}{0} = \infty$$

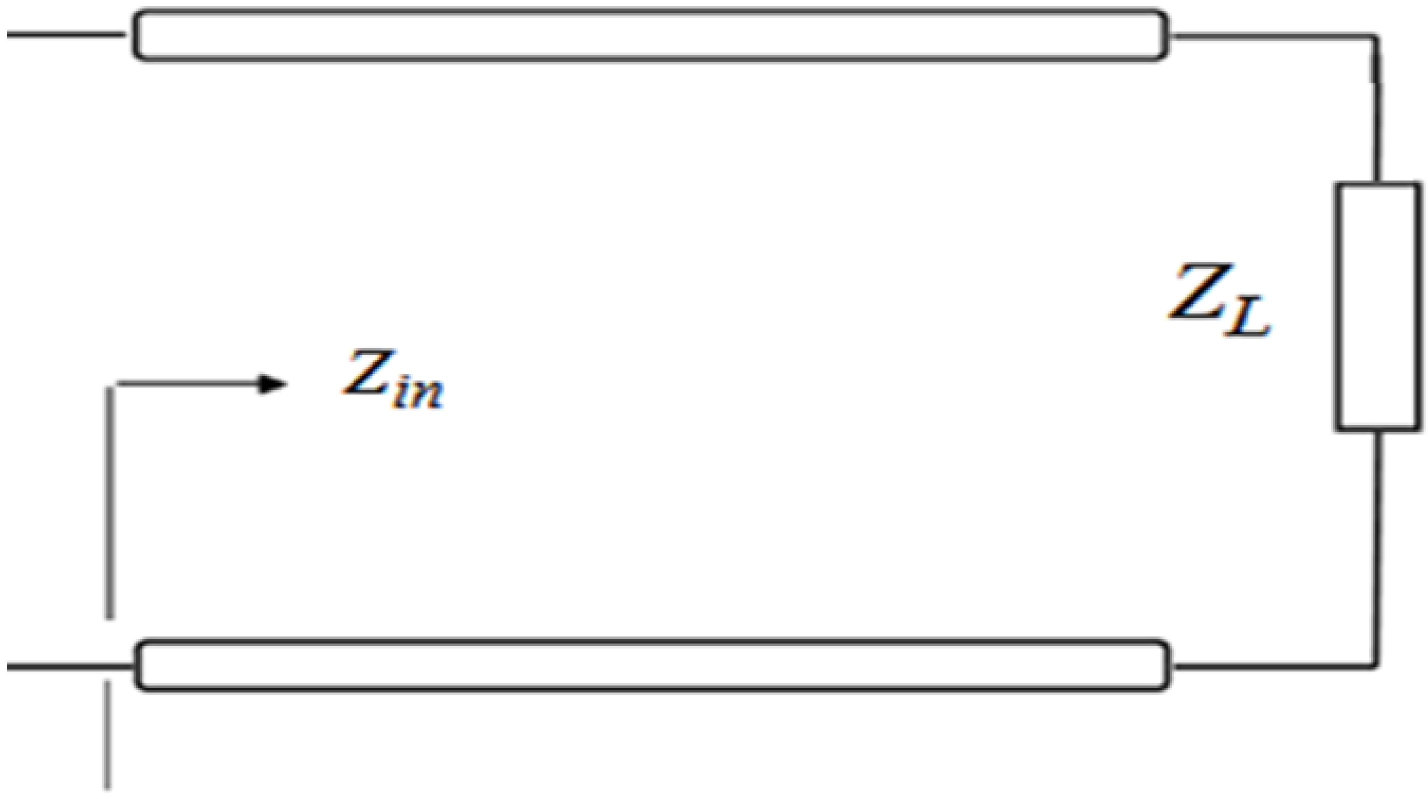
(3)In case of matching

$$Z_L = Z_O$$

$$\Gamma = 0$$

$$VSWR = \frac{1+0}{1-0} = 1$$

Transmission line input impedance



Input impedance

The input impedance of lossy transmission line of length L

$$Z_{in} = Z_O \frac{Z_L + Z_O \tanh(\gamma L)}{Z_O + Z_L \tanh(\gamma L)}$$

Input impedance of lossy transmission line special cases

(1)In case of short circuit (S.C.)

$Z_L = 0$ Then, $Z_{in} = Z_{SC} = Z_O \tanh(\gamma L)$

(2) In case of open circuit (O.C.)

$Z_L = \infty$ Then, $Z_{in} = Z_{OC} = \dfrac{Z_O}{\tanh(\gamma L)} = Z_O \coth(\gamma L)$

Thus,

$$Z_{SC}.Z_{OC} = Z_O \tanh(\gamma L).Z_O \coth(\gamma L) = Z_O^2$$

$$Z_O = \sqrt{Z_{SC}.Z_{OC}}$$

Input impedance of lossless transmission line

$\alpha = 0$ Then, $\gamma = \alpha + j\beta = j\beta$

$\tanh(\gamma L) = \tanh(\alpha + j\beta)L = \tanh(j\beta L) = j\tan(\beta L)$

Thus,

$$Z_{in} = Z_O \frac{Z_L + jZ_O \tan(\beta L)}{Z_O + jZ_L \tan(\beta L)}$$

Input impedance of lossless transmission line special cases

(1)In case of short circuit (S.C.)

$Z_L = 0$ Then, $Z_{in} = Z_{SC} = jZ_O \tan(\beta L)$

(2) In case of open circuit (O.C.)

$Z_L = \infty$ Then, $Z_{in} = Z_{OC} = \dfrac{Z_O}{j\tan(\beta L)} = -jZ_O \cot(\beta L)$

βL is called the electrical length of transmission line. Its unit is either degree or radians

Notes

$$coh(x) = \frac{e^x + e^{-x}}{2}$$

$$Sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$Tanh(x) = \frac{Sinh(x)}{Coh(x)}$$

Comparison between lossy and lossless T.L. input impedance of length L

	Lossy T.L.	Lossless T.L.
General case	$Z_{input} = Z_0 \dfrac{Z_L + Z_0 \tanh(\gamma L)}{Z_0 + Z_L \tanh(\gamma L)}$	$Z_{input} = Z_0 \dfrac{Z_L + jZ_0 \tan(\beta L)}{Z_0 + jZ_L \tan(\beta L)}$
Short circuit $Z_L = 0$	$Z_{SC} = Z_0 \tanh(\gamma L)$	$Z_{input} = jZ_0 \tan(\beta L)$
Open circuit $Z_L = \infty$	$Z_{OC} = Z_0 \coth(\gamma L)$	$Z_{input} = -jZ_0 \cot(\beta L)$