Bode Diagram

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Bode Plot

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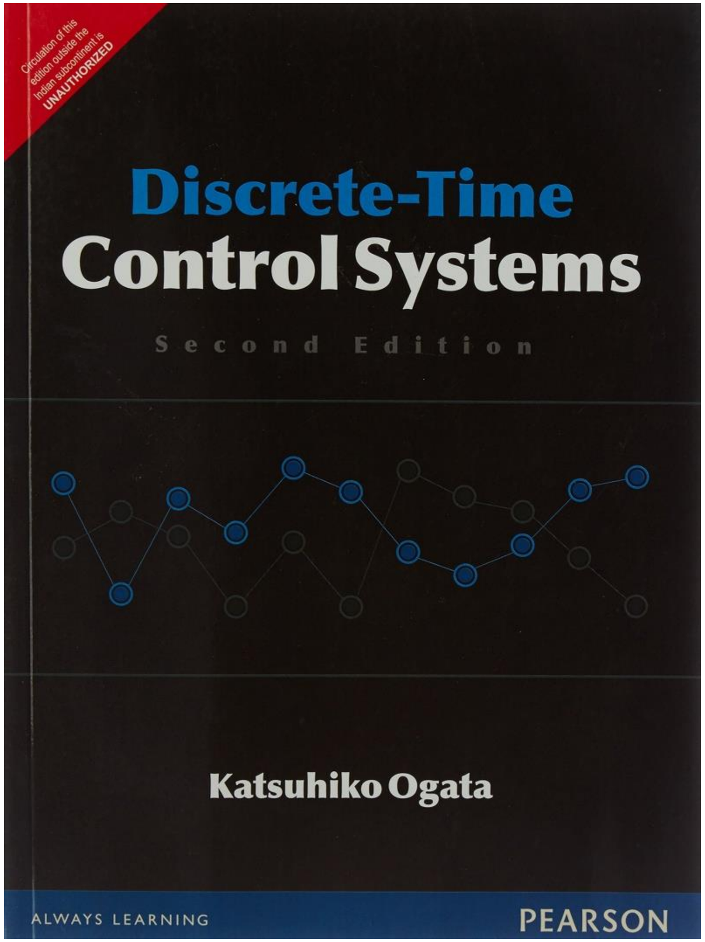
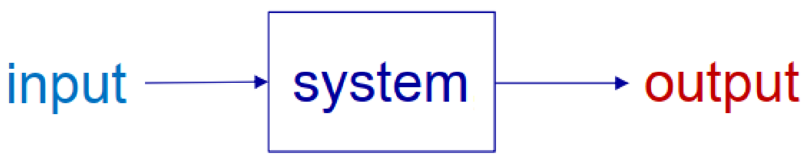


Frequency Response Definition

What is frequency response of a system?

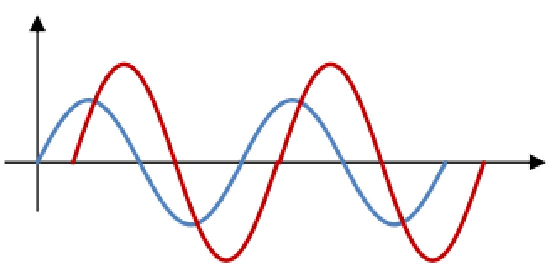
- The response of a system can be partitioned into both the transient response and the steady state response.
  - We can find the transient response by using Fourier integrals.
  - The **steady state response of a system** for an input sinusoidal signal is known as the **frequency response**.

- The sinusoid is a unique input signal, and the resulting output signal for a linear system, as well as signals throughout the system, is sinusoidal in the steady-state.



$$r(t) = A_1 sin(\omega t)$$

$$y(t) = A_2 sin(\omega t + \phi)$$



Input vs. output

Same frequency

Different amplitude & phase angle

Bode Plot Definition

What is Bode Plot?

- Bode Plot is a (semi log) plot of the transfer function magnitude and phase angle as a function of frequency.

- The Bode plot or the Bode diagram consists of two plots: −
  - **Magnitude plot**
  - **Phase plot**

- In both the plots, **x-axis** represents **angular frequency** (logarithmic scale).

- Whereas, **y-axis** represents **the magnitude** (linear scale) of open loop transfer function in **the magnitude plot** and

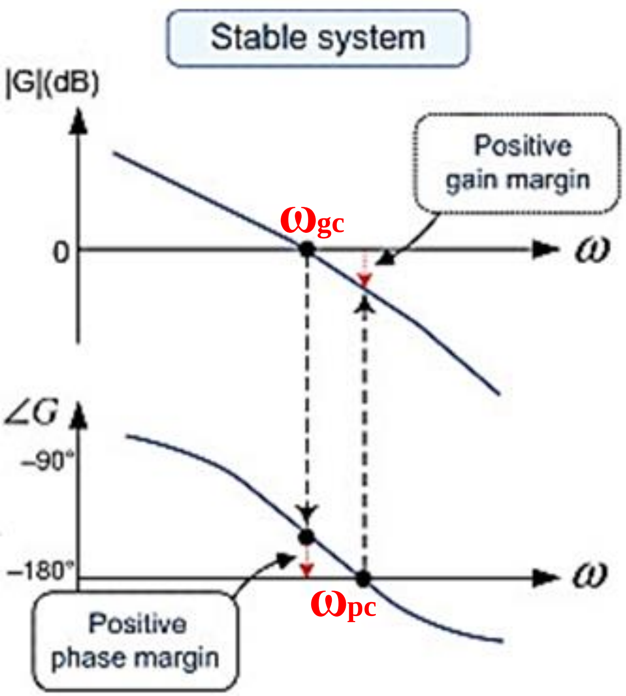
- **the phase angle** (linear scale) of the open loop transfer function in **the phase plot**.

- The magnitude is expressed in dB and the frequency is generally plotted in log scale.

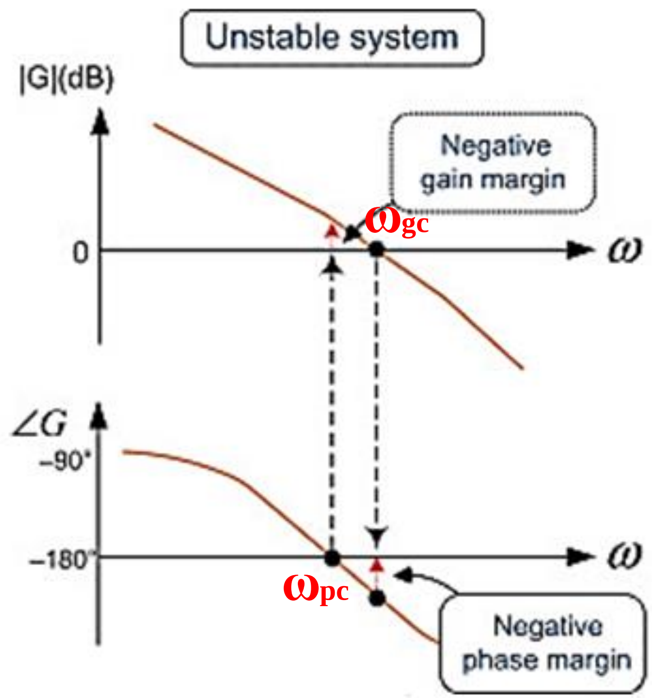
Stability Analysis using Bode Plots

- From the Bode plots, we can say whether the control system is stable, marginally stable or unstable based on the values of these parameters.
  - Gain cross over frequency and phase cross over frequency
  - Gain margin and phase margin
- **Phase Cross over Frequency**
  - The frequency at which the phase plot is having the phase of -180^0 is known as **phase cross over frequency**. It is denoted by ω_{pc}.
  - The unit of phase cross over frequency is **rad/sec**.
- **Gain Cross over Frequency**
  - The frequency at which the magnitude plot is having the magnitude of zero dB is known as **gain cross over frequency**. It is denoted by ω_{gc}.
  - The unit of gain cross over frequency is **rad/sec**.
- **The stability of the control system based on the relation between the phase cross over frequency and the gain cross over frequency is listed below.**
  - If the phase cross over frequency ω_{pc} is greater than the gain cross over frequency ω_{gc}, then the control system is **stable**.
  - If the phase cross over frequency ω_{pc} is equal to the gain cross over frequency ω_{gc}, then the control system is **marginally stable**.
  - If the phase cross over frequency ω_{pc} is less than the gain cross over frequency ω_{gc}, then the control system is **unstable**.

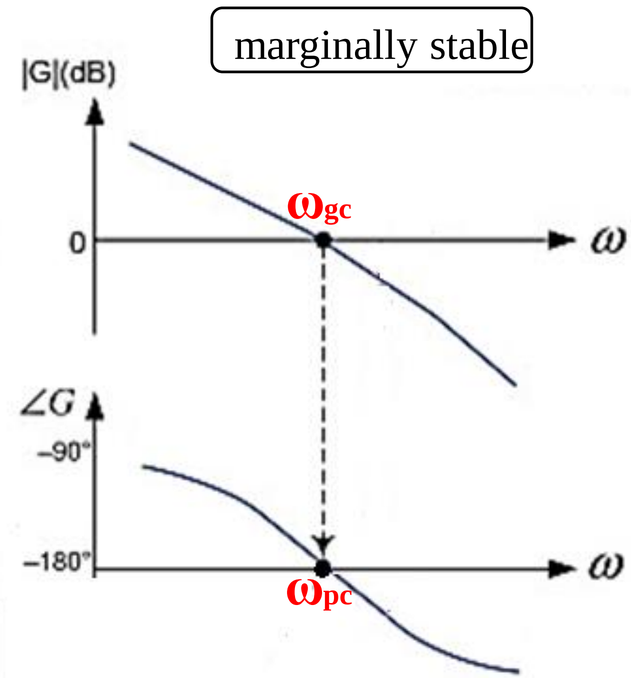
Stability Analysis using Bode Plots



- GM +ve
- PM +ve



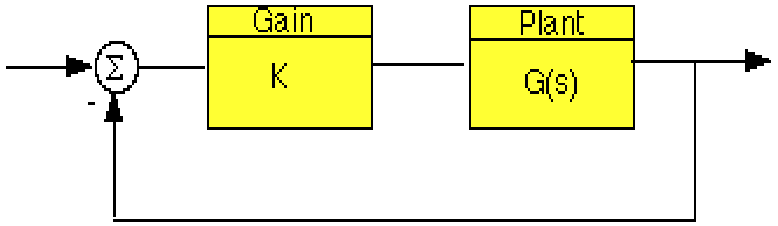
- GM -ve
- PM -ve



- GM zero
- PM zero

Gain and Phase Margin

Let's say that we have the following system:



where K is a variable (constant) gain and G(s) is the plant under consideration

The gain and phase margin are two metrics to tell you how close the system is to oscillation (instability).

Gain Margin: :

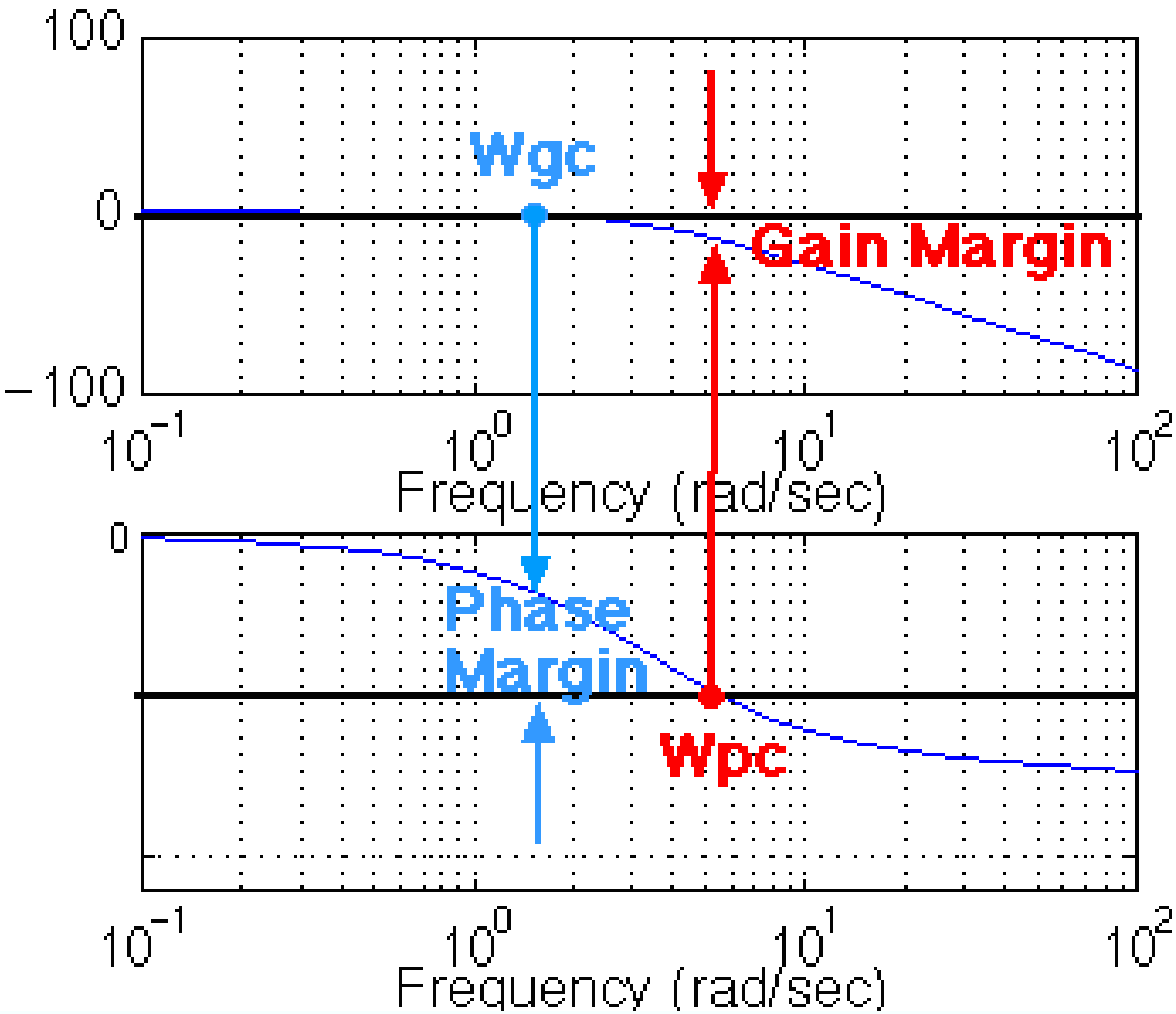
- The greater the **Gain Margin** (GM), the greater the stability of the system. The gain margin refers to the amount of gain, which can be increased or decreased without making the system unstable. (i.e., after varying the gain up to a certain threshold, the system becomes marginally stable and then further variation of gain leads to instability).
- It is usually expressed as a magnitude in dB.
- We can usually read the gain margin directly from the Bode plot by calculating the vertical distance between the magnitude curve (on the Bode magnitude plot) and the x-axis at the frequency where the Bode phase plot = 180°. This point is known as the **phase crossover frequency.**

Gain and Phase Margin

Phase margin:

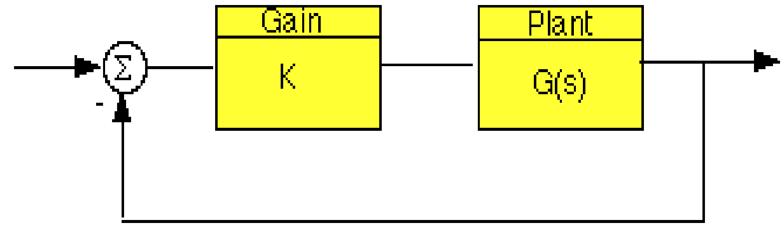
- The greater the Phase Margin (PM), the greater will be the stability of the system. The phase margin refers to the amount of phase, which can be increased or decreased without making the system unstable (i.e., after varying the phase up to a certain threshold, the system becomes marginally stable and then further variation of phase leads to instability).
- It is usually expressed as a phase in degrees.
- We can usually read the phase margin directly from the Bode plot by calculating the vertical distance between the phase curve (on the Bode phase plot) and the x-axis at the frequency where the Bode magnitude plot = 0 dB. This point is known as the **gain crossover frequency**.

Gain and Phase Margin



Construction rules for bode plot

- For the following discrete control systems:



$$G\ (z) = \frac{b_0 z^m + b_1 z^{m-1} + \cdots + b_m}{z^n + a_1 z^{n-1} + \cdots + a_n}$$

- **Step1:** Using the bilinear Transformation:

$$z = \frac{1+\left(\frac{T}{2}\right)w}{1-\left(\frac{T}{2}\right)w}$$

$$G(w) = \frac{k\left(1+\frac{w}{w_1}\right)\left(1+\frac{w}{w_2}\right)\ \ldots}{w^N\left(1+\frac{w}{w_a}\right)\left(1+\frac{w}{w_b}\right)\ \ldots}$$

- **Step 2:** Draw bode diagram for the system.

1. **Magnitude plot:**
   - **Step 2a: Corner frequencies:**
     - Poles: w_a , w_b , …
     - Zeros: w_1 , w_2 , …
   - **Step 2b: The initial Line:**
     - Slope = -20 N , where N is the system type.
     - At w=1: mag_value =20 Log k dB.

Construction rules for bode plot

- **Step 2c:**
  - **If line meet a pole, slope decrease by 20 dB/decade.**
  - **If line meet a zero, slope increase by 20 dB/decade.**

2. Phase plot:

$$\theta = \sum \theta_z - \sum \theta_p$$

$$= [\tan^{-1}(\frac{w}{w_1}) - \tan^{-1}(\frac{w}{w_2})] - [90N + \tan^{-1}(\frac{w}{w_a}) + \tan^{-1}(\frac{w}{w_b})]$$

- Initial value at v=0 ➔ θ = -90 N
- Initial value at v=∞ ➔ θ = -90(n_p-n_z)

- **Notice**: w = jv

v	0	w1	…	wa	w2	…	wb	∞
θ	-90N	√	√	√	√	…	√	-90(n_p-n_z)

Construction rules for bode plot

❑ Example:

- **Sketch the bode plots and determine the gain cross-over and phase-cross-over frequency for:**

$$G(z)= \frac{-k(0.01873z+0.01752)}{1000(z^2-1.8187z+0.8187)}$$

- The sampling time T= 0.2sec, k =2/3.

❑ Solution:

- **Step1:**

Using the bilinear Transformation:

$$z=\frac{1+\left(\frac{T}{2}\right)w}{1-\left(\frac{T}{2}\right)w}=\frac{1+0.1w}{1-0.1w}$$

T= 0.2sec

$$G(w)= \frac{-k(0.01873\times\frac{1+0.1w}{1-0.1w}+0.01752)}{1000\left(\left(\frac{1+0.1w}{1-0.1w}\right)^2-1.8187\left(\frac{1+0.1w}{1-0.1w}\right)+0.8187\right)} = \frac{-k(\overbrace{-0.000333\,w^2+0.09633\,w+0.9966})}{1000(w^2+0.9969\,w)}$$

$$w1=-300$$
$$w2=10$$

$$=\frac{-k(w+300)(w-10)}{1000w(w+1)}$$

Construction rules for bode plot

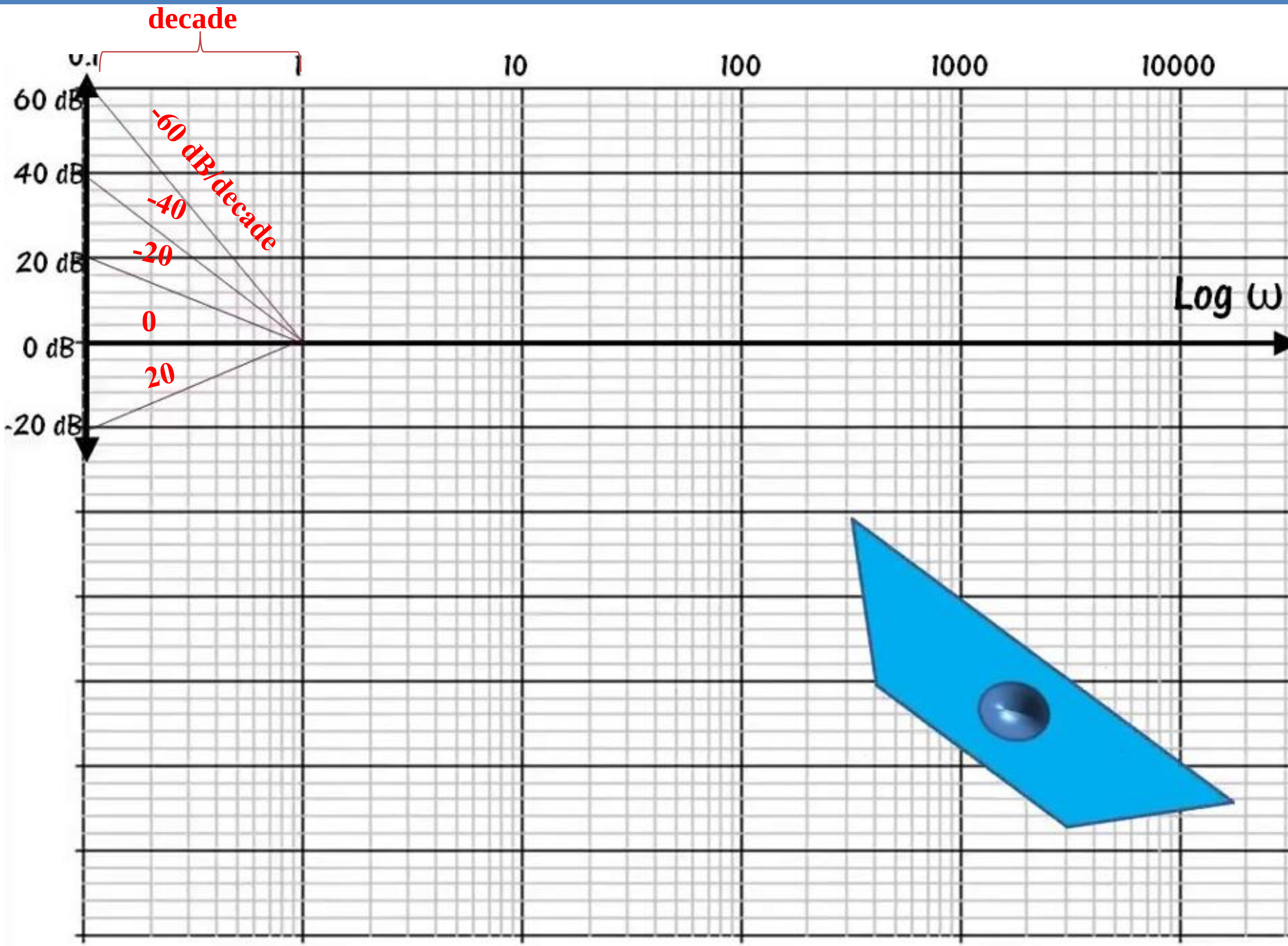
$$G(w)=\frac{3k(1+\frac{w}{300})(1-\frac{w}{10})}{w(1+\frac{w}{1})}$$

- **Step 2:** **Draw bode diagram for the system.**
- **<u>Magnitude plot:</u>**
  - **Step 2a: Corner frequencies:**
    - **Poles: 1**
    - **Zeros: 300, 10**

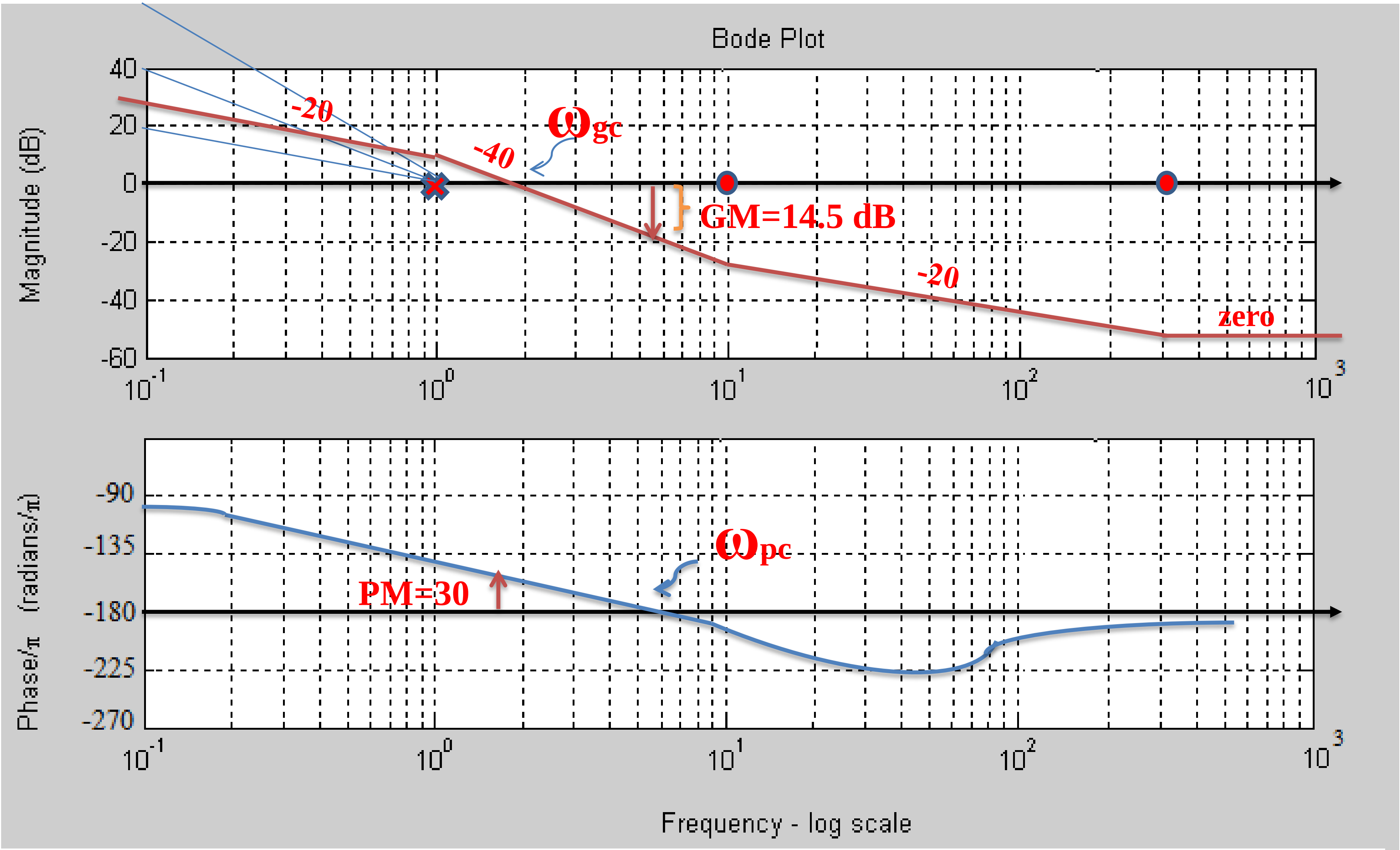
  - **Step 2b: The initial Line:**
    - **Slope = -20 N = -20*1 = -20 dB/decade, where N is the system type.**
    - **At w=1: mag_value =20 Log K = 20 Log 3k = 20 Log 2 = 6 dB.**

  - **Step 2c:**
    - **If line meet a pole, slope decrease by 20 dB/decade.**
    - **If line meet a zero, slope increase by 20 dB/decade.**

Semi-log Paper for Bode Plots



Construction rules for bode plot



Construction rules for bode plot

➢ Phase plot:

$$\theta = \sum \theta_z - \sum \theta_p$$

$$= [\tan^{-1}(\frac{v}{300}) - \tan^{-1}(\frac{v}{10})] - [90 + \tan^{-1}(\frac{v}{1})]$$

- Initial value at v=0 ➔ $\theta =$ -90n = -90
- Initial value at v=∞ ➔ $\theta =$ -90(n_p-n_z) = -180

- **Notice**: w = jv

v	0	1	2	3	10		300		∞
θ	-90	√	√	√	√		√		-180

- **From figure:**
- the phase cross over frequency ω_{pc} is greater than the gain cross over frequency ω_{gc}, then the control system is **stable**.