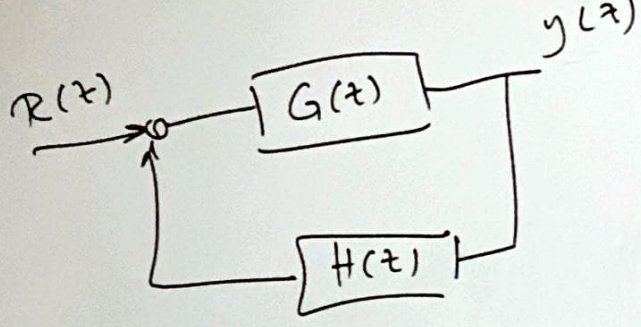
* Stability analysis of closed loop system in z-plane :



- for the following closed loop system

the Transfer function can be given by:

$$T.f = \frac{G(z)}{\underbrace{1+G(z)H(z)}} = \frac{q(z)}{p(z)}$$

the stability of the system defined by the C/cs eqn:

$$p(z) = 1 + G'H(z) = 0$$

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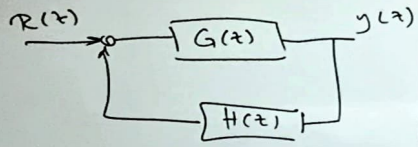


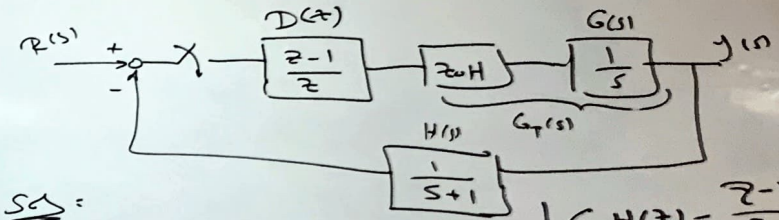
$$T.f = \underbrace{\frac{G(z)}{1+G(z)H(z)}}_{} = \frac{Y(z)}{R(z)}$$

the stability of the system defined by the C/c eqn:

$$P(z) = 1 + G'H(z) = 0$$

Eg: check the stability for the following closed loop system

$T = 1$ sec



Sol:

$$T.f = \frac{Y(z)}{R(z)} = \frac{D(z)\cdot G_T(z)}{1+D(z)G_TH(z)}$$

$$G_T(z) = \frac{z-1}{z}\left\{\frac{G(s)}{s}\right\}$$

$$= \frac{z-1}{z}\left\{\frac{1}{s^2}\right\} \quad t$$

$$= \frac{\cancel{z-1}}{\cancel{z}}\cdot\frac{\cancel{z}T}{(z-1)^{\cancel{2}}}$$

$$\boxed{G_T(z) = \frac{1}{z-1}}$$

$$D(z)\,G_T(z) = \frac{1}{\cancel{z-1}}\cdot\frac{\cancel{z-1}}{z}$$

$$\boxed{D(z)G_T(z) = \frac{1}{z}}$$

$$G_TH(z) = \frac{z-1}{z}\left\{\frac{G(s)\cdot H(s)}{s}\right\}$$

$$= \frac{z-1}{z}\left\{\frac{1}{s^2(s+1)}\right\}$$

$$= \frac{z-1}{z}\left\{\frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1}\right\}$$

$$A = \lim_{s\to 0}\frac{1}{s+1} = 1$$

$$B = \lim_{s\to 0}\frac{d}{ds}\left(\frac{1}{s+1}\right) = \lim_{s\to 0}\frac{-1}{(s+1)^2} = -1$$

$$C = \lim_{s\to -1}\frac{1}{s^2} = 1$$

$$= \frac{z-1}{z} Z \left[\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1} \right]$$

$\frac{1}{s^2} \to t$, $\frac{1}{s} \to u$, $\frac{1}{s+1} \to e^{-t}$

$e^{-1} = 0.36$

$$= \frac{z-1}{z} \left[\frac{Tz}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-T}} \right]$$

$$D(z)\,G_T H(z) = \frac{z-1}{z} \cdot \frac{z-1}{\cancel{z}} \left[\frac{T\cancel{z}}{(z-1)^2} - \frac{\cancel{z}}{z-1} + \frac{\cancel{z}}{z-0.36} \right]$$

$$= \frac{(z-1)^2}{z} \left[\frac{1}{(z-1)^2} - \frac{1}{z-1} + \frac{1}{z-0.36} \right]$$

$$= \left[\frac{1}{z} - \frac{z-1}{z} + \frac{(z-1)^2}{z(z-0.36)} \right]$$

$$= \left[\frac{-z+2}{z} + \frac{(z-1)^2}{z(z-0.36)} \right]$$

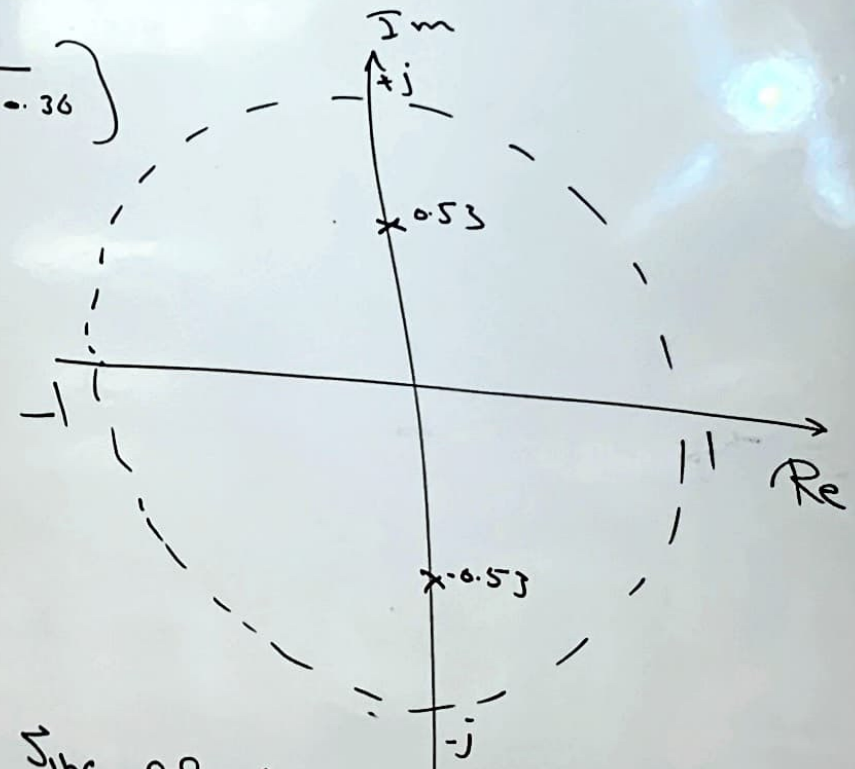
$$D(z)\,G_T H(z) = \left[\frac{0.36z + 0.28}{z(z-0.36)} \right]$$

$$T.F = \frac{z-0.36}{z^2+0.28} = \frac{q(z)}{p(z)}$$

C/Cs eqn:

$$p(z) = z^2 + 0.28 = 0$$

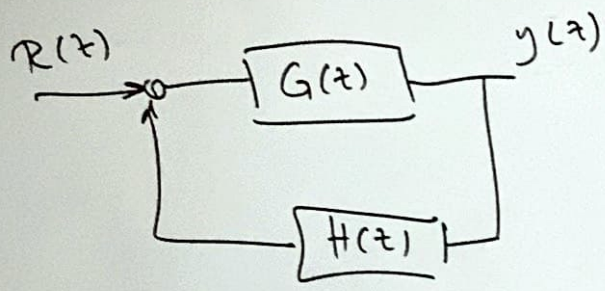
$$z = \pm 0.53i$$



Since poles of the closed loop system lie inside the unit circle then system is stable

* Stability analysis of closed loop system in z-plan :

- for the following closed loop system the Transfer function can be given by:



$$T.f = \underbrace{\frac{G(z)}{1+G(z)H(z)}}_{} = \frac{q(z)}{p(z)}$$

the stability of the system defined by the C/cs eqn:

$$p(z) = 1 + G'H(z) = 0$$

$$p(z) = \underbrace{az^3 + bz^2 + cz^1 + dz^0}_{} = 0$$

$z_1 = 0.5$

$z_2 = +0.75j$

$z_3 = -0.75j$

$$= (z_1 - 0.5)(z_2 + 0.75j)(z_3 - 0.75j) = 0$$

$$P(z) = a_0 z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_{n-1} z + a_n$$

x) Conditions for stability :

1. $|a_n| < a_0$

2. $P(z)\Big|_{z=1} > 0$

3. $P(z)\Big|_{z=-1}$ → for n even $P(-1) > 0$
   → for n odd $P(-1) < 0$

4 - Construct Jury Table

$|b_{n-1}| > |b_0|$

$|c_{n-2}| > |c_0|$

$\vdots$

$|q_{+2}| > |q_0|$ no. of coefficient equal '3' stop

Z^0	Z^1	Z^2	Z^3	...	Z^{n-1}	Z^n
a_n	a_{n-1}	a_{n-2}	a_{n-3}	...	a_1	a_0
a_0	a_1	a_2	a_3	...	a_{n-1}	a_n
b_{n-1}	b_{n-2}	b_{n-3}	...	b_1	b_0	
b_0	b_1	b_2	...	b_{n-2}	b_{n-1}	
c_{n-2}	c_{n-3}	...	c_1	c_0		
c_0	c_1	...	c_{n-3}	c_{n-2}		
q_2	q_1	q_0				

no. of coefficient equal '2' stop

* iff all conditions satisfied then system is stable.

a_0	a_1	a_2	a_3	$\cdots$	a_{n-1}	a_n
b_{n-1}	b_{n-2}	b_{n-3}	$\cdots$	b_1	b_0	
b_0	b_1	b_2	$\cdots$	b_{n-2}	b_{n-1}	
c_{n-2}	c_{n-3}	$\cdots$	c_1	c_0		
c_0	c_1	$\cdots$	c_{n-3}	c_{n-2}		
q_2	q_1	q_0				

no. of coefficient equal '3' stop

iff all conditions satisfied then system is stable.

$$b_{n-1} = \begin{vmatrix} a_n & a_0 \\ a_0 & a_n \end{vmatrix}, \quad b_{n-2} = \begin{vmatrix} a_n & a_1 \\ a_0 & a_{n-1} \end{vmatrix}, \quad \cdots, \quad b_0 = \begin{vmatrix} a_n & a_{n-1} \\ a_0 & a_1 \end{vmatrix}$$

$c_{n-2} = |b_{n-1} \quad b_{n-2}|$ $c_{n-3} = |b_{n-1} \quad b_{n-3}|$ $\cdots$

Q) Construct the Jury Table and check the stability of the char. eqn

$$P(z) = z^4 - 1.2z^3 + 0.07z^2 + 0.3z - 0.08 = 0$$

Sol:

- Conditions of stability

1. $|a_0| < a_n$, $|-0.08| < 1$ satisfy

2. $P(1) = (1)^4 - 1.2(1)^3 + 0.07(1)^2 + 0.3(1) - 0.08 = 0.09 > 0$ Satisfy

3. $P(-1) = (-1)^4 - 1.2(-1)^3 + 0.07(-1)^2 + 0.3(-1) - 0.08 = 1.55 > 0$

$n = 4$ (even) & $P(-1) > 0$ satisfy

- Construct Jury Table

z^0	z^1	z^2	z^3	z^4
-0.08	0.3	0.07	-1.2	1
1	-1.2	0.07	0.3	-0.08

Q1 Construct the Jury Table and check the stability of the C/Cs eqn

$$P(z) = z^4 - 1.2z^3 + 0.07z^2 + 0.3z - 0.08$$

Sol:

+ Conditions of stability

1- $|a_n| < a_0$, $|-0.08| < 1$ Satisfy

2- $P(z)|_{z=1} = (1)^4 - 1.2(1)^3 + 0.07(1)^2 + 0.3(1) - 0.08 = 0.09 > 0$

Satisfy

3- $P(z)|_{z=-1} = (-1)^4 - 1.2(-1)^3 + 0.07(-1)^2 + 0.3(-1) - 0.08 = 1.89 > 0$

+ Conditions of stability

1- $|a_n| < a_0$, $|-0.08| < 1$ Satisfy

2- $P(z)|_{z=1} = (1)^4 + 1.2(1)^3 + 0.07(1)^2 + 0.3(1) - 0.08 = 0.09 > 0$

Satisfy

3- $P(z)|_{z=-1} = (-1)^4 - 1.2(-1)^3 + 0.07(-1)^2 + 0.3(-1) - 0.08 = 1.89 > 0$

$n = 4$ (even) && $P(z)|_{z=-1} > 0$ Satisfy

4- Construct Jury Table:

Z^0 Z^1 Z^2 Z^3 Z^4

Z^0	Z^1	Z^2	Z^3	Z^4
-0.08	0.3	0.07	-1.2	1
1	-1.2	0.07	0.3	-0.08
-0.994	1.176	-0.756	-0.204	
-0.204	-0.756	1.176	-0.994	
0.946	-1.184	0.315		

Coefficients equal '3' Stop

$$T.F = \frac{G(z)}{1+G(z)} = \frac{P(z)}{P(z)}$$

$\therefore$ the stability of the system is given by the c/cs equation:

$$P(z) = 1 + G(z) = 0$$

$$\therefore P(z) = 1 + \frac{k(0.3679z + 0.2642)}{(z-0.3679)(z-1)} = 0$$

$$P(z) = (z-0.367)(z-1) + k(0.3679z + 0.2642) = 0$$

$$P(z) = z^2 - 1.367z + 0.367 + 0.3679kz + 0.2642k = 0$$

$$P(z) = z^2 + (0.3679k - 1.367)z + 0.2642k + 0.367 = 0$$

$$P(z) = z^2 + (0.3679k - 1.367)z + 0.2642k + 0.367 = 0$$

<u>Conditions of stability</u>

$$1 - |a_n| < a_0 \Rightarrow |0.2642k + 0.367| < 1$$

$$-1 < 0.264k + 0.367 < 1$$

$$-1.367 < 0.264k < 0.63$$

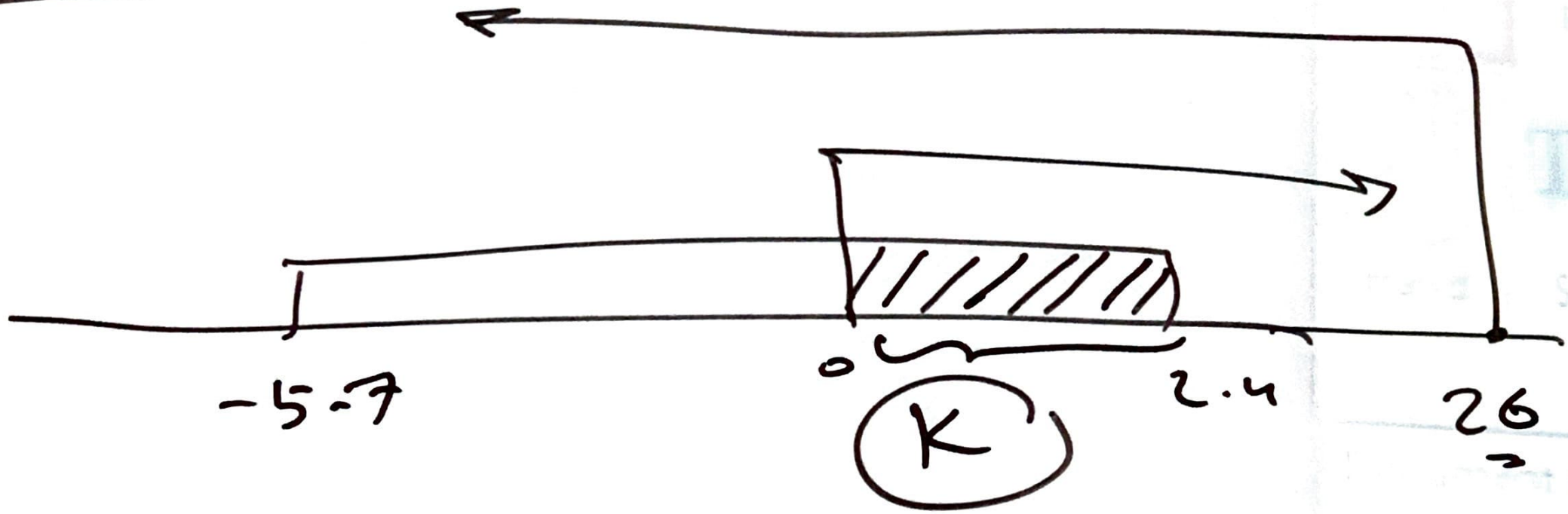
$$-5.1775 < k < 2.3925 \quad (1)$$

$$k > 0 \quad \text{--- (3)}$$

3- $P(z)\Big|_{z=-1} > 0$ for n even

$$\therefore P(z)\Big|_{z=-1} = (-1)^2 + (0.3679k - 1.367)(-1) + 0.2642k + 0.367 > 0$$

$$k < 26.382$$

we select the Range that satisfy the three conditions (the intersection between them).

$e^{-1} = 0.36$