sec2

- Partial Fraction Method. table
- Long division "

$$F(z) = \sum_{k=0}^{\infty} f(kT) z^{-k}$$

Partial fraction $f(z) = \frac{N(z)}{D(z)}$ $N(z) < D(z)$

ex:1 $F(z) = \frac{z}{(z+0.1)(z+0.2)(z+0.3)}$ $\div z$

Sol;

$$\frac{f(z)}{z} = \frac{1}{(z+0.1)(z+0.2)(z+0.3)} = \frac{A}{z+0.1} + \frac{B}{z+0.2} + \frac{C}{z+0.3}$$

$$A = \lim_{z \to -0.1} (z+0.1)\frac{f(z)}{z} = \frac{1}{0.1 \times 0.2} = 50$$

$$B = \lim_{z \to -0.2} (z+0.2)\frac{f(z)}{z} = \frac{1}{(z+0.1)(z+0.3)} = \frac{1}{-0.1 \times 0.1} = -100$$

$C = 50$

$$\frac{f(z)}{z} = \frac{50}{z+0.1} - \frac{100}{z+0.2} + \frac{50}{z+0.3}$$

$$f(z) = \frac{50z}{z+0.1} - \frac{100z}{z+0.2} + \frac{50z}{z+0.3}$$

$$f(kT) = 50(-0.1)^k - 100(-0.2)^k + 50(-0.3)^k \quad k \geq 0$$

ex2: $f(z) = \dfrac{1+2z^{-1}}{(1-0.2z^{-1})(1+0.6z^{-1})} = \dfrac{z^2+2z}{(z-0.2)(z+0.6)}$

$$\frac{f(z)}{z} = \frac{z+2}{(z-0.2)(z+0.6)} = \frac{A}{z-0.2} + \frac{B}{z+0.6}$$

$A = 2.75$, $B = -1.75$

$$\frac{f(z)}{z} = \frac{2.75}{z-0.2} - \frac{1.75}{z+0.6} \Rightarrow f(z) = \frac{2.75z}{z-0.2} - 1.75\frac{z}{z+0.6}$$

$$f(kT) = 2.75*(0.2)^k - 1.75(-0.6)^k \quad k \geq 0$$

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ex3: $f(z) = \dfrac{z+1}{z^2+0.3z+0.02} = \dfrac{z+1}{(z+0.1)(z+0.2)}$

$$\frac{f(z)}{z} = \frac{z+1}{z(z+0.1)(z+0.2)} = \frac{A}{z} + \frac{B}{(z+0.1)} + \frac{C}{z+0.2}$$

$A = 50$, $B = -90$, $C = 40$

$$\frac{f(z)}{z} = \frac{50}{z} - \frac{90}{(z+0.1)} + \frac{40}{(z+0.2)}$$

$$f(z) = 50 - \frac{90z}{(z+0.1)} + \frac{40z}{(z+0.2)}$$

$$f(kT) = 50\delta(k) - 90(-0.1)^k + 40(-0.2)^k$$

- Long division method

ex4

$$f(z) = \frac{0.5z}{z^2 - 1.6z + 0.6} = \frac{0.5z^{-1}}{1 - 1.6z^{-1} + 0.6z^{-2}}$$

$$0.5z^{-1} + 0.8z^{-2} + 0.98z^{-3} + \cdots$$

$$1 - 1.6z^{-1} + 0.6z^{-2} \;\big)\; 0.5z^{-1}$$

$$\ominus 0.5z^{-1} \oplus - 0.8z^{-2} \ominus + 0.3z^{-3}$$

$$0.8z^{-2} - 0.3z^{-3}$$

$$\ominus 0.8z^{-2} \oplus - 1.28z^{-3} \ominus + 0.48z^{-4}$$

$$0.98z^{-3} - 0.48z^{-4}$$

$$\ominus 0.98z^{-3} \oplus - 1.568z^{-4} \ominus + 0.588z^{-5}$$

$$1.088z^{-4} - 0.588z^{-5}$$

$$f(z) = 0.5z^{-1} + 0.8z^{-2} + 0.98z^{-3} + 1.088z^{-4} + \cdots$$

$$= 0 + 0.5z^{-1} + 0.8z^{-2} + 0.98z^{-3} + 1.088z^{-4} \cdots$$

$$f(kT) = \{0,\ 0.5,\ 0.8,\ 0.98,\ \cdots\}$$

difference equations

$$y(k+n)+a_{n-1}\,y(k+n-1)+\dots+a_1\,y(k+1)+a_0\,y(k)$$
$$=b_n\,u(k+n)+b_{n-1}\,u(k+n-1)$$

$$y(k+n)\Rightarrow z^n y(z)-z^n y(0)-z^{n-1}y(1)-z^{n-2}y(2)$$
$$-z^1\,y(n-1)$$

discrete function	Z-transform
$X(k+4)$	$z^4X(z)-z^4X(0)-z^3X(1)-z^2X(2)-zX(3)$
$X(k+3)$	$z^3X(z)-z^3X(0)-z^2X(1)-zX(2)$
$X(k+2)$	$z^2X(z)-z^2X(0)-zX(1)$
$X(k+1)$	$zX(z)-zX(0)$
$X(k)$	$X(z)$
$X(k-1)$	$z^{-1}X(z)$
$X(k-2)$	$z^{-2}X(z)$
$X(k-3)$	$z^{-3}X(z)$
$X(k-4)$	$z^{-4}X(z)$

$x_0 ?$

ex5: $y(k+2)-5y(k+1)+6y(k)=1(k)$

$y(0)=y(1)=0$

$$\left[z^2y(z)-\cancel{z^2y(0)}-\cancel{zy(1)}\right]-5\left[zy(z)-\cancel{zy(0)}\right]$$

$$+6y(z)=\frac{z}{z-1}$$

$$(z^2-5z+6)y(z)=\frac{z}{z-1}$$

$$y(z)=\frac{z}{(z-1)(z^2-5z+6)}=\frac{z}{(z-1)(z-2)(z-3)}$$

Hint: $z \to$ Advance, $\frac{1}{z} \to$ delay

ex6: ~~z y(z) - y(z)~~ $y(k+1)-y(k)=u(k+1)$

$$zy(z)-y(z)=zu(z)$$

$$\boxed{\frac{y(z)}{u(z)}=\frac{z}{z-1}} \to \text{transfer function}$$

find the response to sample unit step

$$u(z)=\frac{z}{z-1}$$

$$y(z)=\frac{z}{z-1}*\frac{z}{z-1}$$

$$y(z)=\frac{z^2}{(z-1)^2}=\left(\frac{z}{(z-1)^2}\right)z$$

$$y(k)=k+1$$

5

ex7:

$y(k+2)+3y(k+1)+2y(k)=u(k)$

find the transfer function

$z^2 y(z)+3z\,y(z)+2y(z)=u(z)$

$(z^2+3z+2)\,y(z)=u(z)$

$$\frac{y(z)}{u(z)}=\frac{1}{z^2+3z+2}$$

Assignment

- Consider the following difference equation

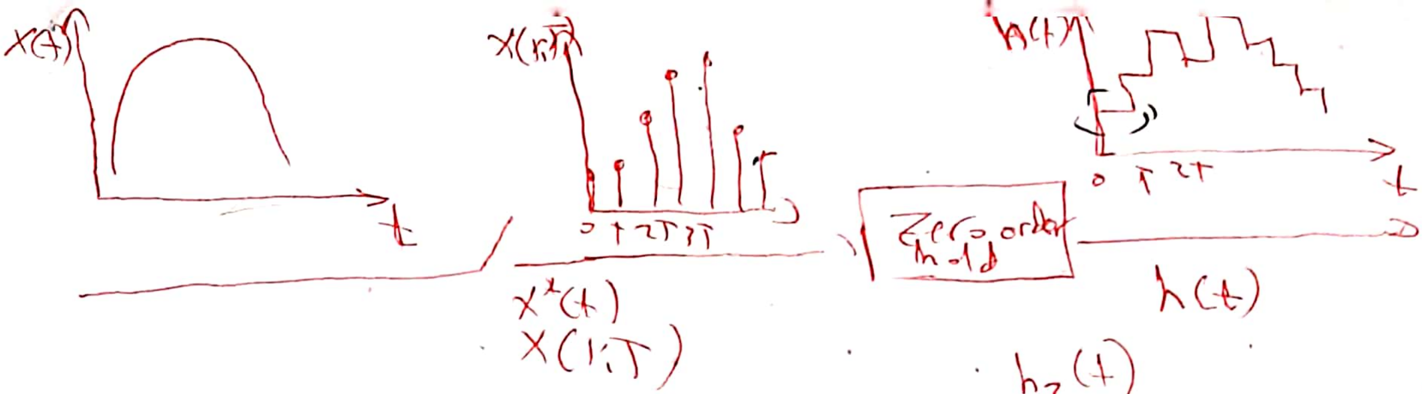
$8y(k+2)-6y(k+1)+y(k)=g(k)$,

$y(0)=1, \quad y(1)=1.5$

a) determine the output $y(k)$

b) plot $y(k)$ for the first (4) samples

c) find the initial and final values



$$h(t) = x(0)\overbrace{[1(t)-1(t-T)]}^{h_1(t)} + x(T)\overbrace{[1(t-T)-1(t-2T)]}^{h_2(t)}$$

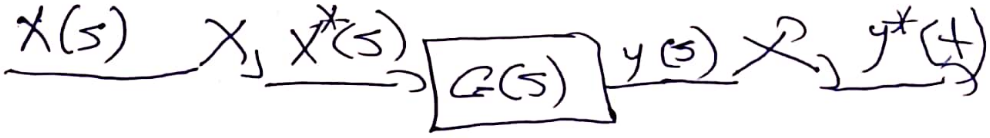
$$L[\overbrace{1(t)-1(t-T)}^{h_1(t)}]$$

$$L[1(t)] = \frac{1}{s} \quad , \quad L[1(t-T)] = \frac{e^{-sT}}{s}$$

$$L[h_1(t)] = \frac{1}{s} - \frac{e^{-sT}}{s} = \frac{1-e^{-sT}}{s} \quad \#$$

$$h(t) = \frac{1-e^{-sT}}{s} \sum_{k=0}^{\infty} x(kT)\, e^{-kTs}$$

$$G_{h_0}(s) = \frac{1-e^{-Ts}}{s} \quad \#$$ T.f ZOH



$$y(s) = G(s)\,X^*(s)$$

$$y^*(s) = [G(s)\,X^*(s)]^* = [G^*(s)\,X^*(s)]$$

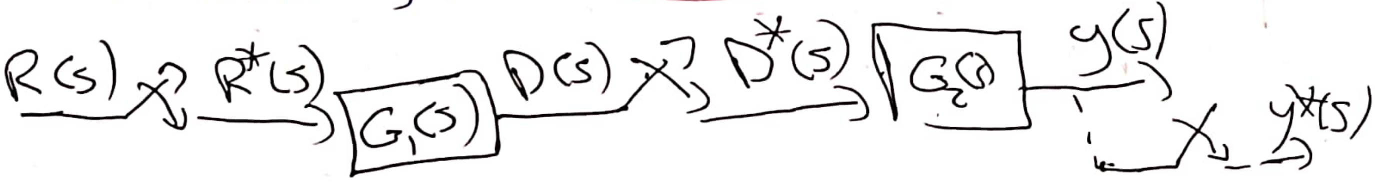
Apply Z-Transform

$$y(z) = G(z)\,X(z)$$

$$G(z) = \frac{y(z)}{X(z)}$$

$$[A^*(s)B(s)C(s)D^*(s)]^* = [A^*(s)D^*(s)\,BC^*(s)]$$

- separated by a sampler



$$y(s) = G_2(s)\,D^*(s)$$

$$D(s) = G_1(s) * R^*(s) \longrightarrow D^*(s) = G_1^*(s) * R^*(s)$$

$$y(s) = G_2(s) * G_1^*(s) * R^*(s)$$

$$y^*(s) = [G_2(s) * G_1^*(s) * R^*(s)]^* = [G_1^*(s)\,R^*(s) * G_2^*(s)]$$

$$y(z) = G_1(z)\,G_2(z)\,R(z)$$

$$\frac{y(z)}{R(z)} = G_1(z)\,G_2(z)$$

ex7

$$R(s) \xrightarrow{\times} R^*(s) \longrightarrow \boxed{\frac{1}{s}} \longrightarrow D(s) \xrightarrow{\times} D^*(s) \longrightarrow \boxed{\frac{a}{s+a}} \longrightarrow Y(s)$$

$G_1 = \frac{1}{s}$, $G_2(s) = \frac{a}{s+a}$

find $Y(z)$ when input is a unit step

Sol.:

$$Y(z) = G_1(z)\, G_2(z)\, R(z)$$

$$G_1(s) = \frac{1}{s} \xrightarrow{L^{-1}} 1 \xrightarrow{z} G_1(z) = \frac{z}{z-1}$$

$$G_2(s) = \frac{a}{s+a} \xrightarrow{L^{-1}} a e^{-at} \xrightarrow{z} G_2(z) = \frac{az}{z - e^{-aT}}$$

$$Y(z) = \frac{z}{z-1} * \frac{az}{z - e^{-aT}} * R(z)$$

$$R(z) = \frac{z}{z-1}$$

$$Y(z) = \frac{z}{z-1} * \frac{az}{z - e^{-at}} * \frac{z}{z-1}$$

$$= \frac{az^3}{(z-1)^2 (z - e^{-aT})}$$

- without sampler directly Connected

$$Y(s) = G_2(s) D(s) = G_2(s) G_1(s) R^*(s)$$

$$y^*(s) = [G_2(s) G_1(s) R^*(s)]^*$$

$$= [G_1G_2^*(s) * R^*(s)]$$

$$Y(z) = G_1G_2(z) * R(z) \Rightarrow \frac{Y(z)}{R(z)} = G_1G_2(z)$$

EX8 $G_1(s) = \frac{1}{s}$ $G_2(s) = \frac{a}{s+a}$, $\underline{R(s) = 1}$

using directly Connected

sol:-

$$Y(z) = G_1G_2(z) R(z)$$

$$G_1G_2(s) = \frac{a}{s(s+a)} = \frac{A}{s} + \frac{B}{s+a}$$

$$A = 1, \quad B = -1$$

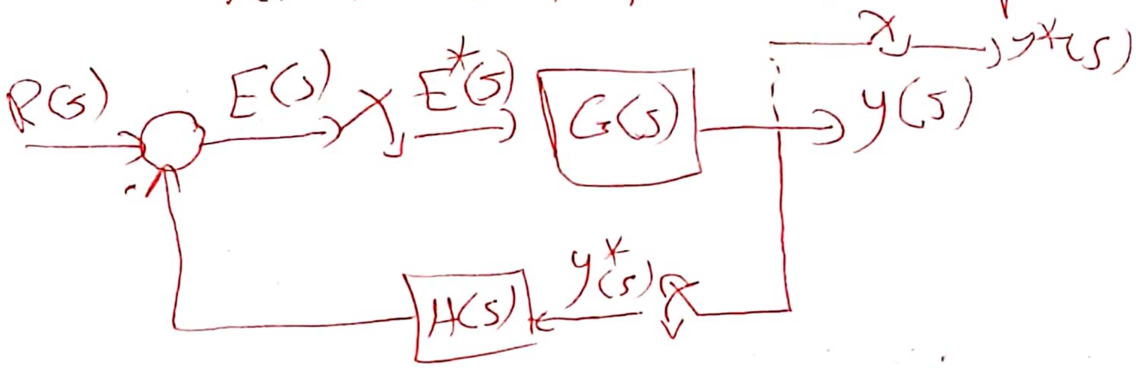
$$\frac{a}{s(s+a)} = \frac{1}{s} - \frac{1}{s+a} \xrightarrow{L^{-1}} 1 - e^{-at} \xrightarrow{Z} \frac{z}{z-1} - \frac{z}{z-e^{-aT}}$$

$$Y(z) = \frac{z}{z-1} * \left[\frac{z}{z-1} - \frac{z}{z-e^{-aT}}\right]$$

$$Y(z) = \frac{z^2(1-e^{-aT})}{(z-1)^2(z-e^{-aT})}$$

$$G_1G_2(z) \neq G_1(z) G_2(z)$$

Transfer function of closed Loop



$$Y(s) = G(s)\,E^*(s)$$

$$E(s) = R(s) - H(s)\,Y^*(s)$$

$$E^*(s) = R^*(s) - H^*(s)\,Y^*(s)$$

$$Y(s) = G(s) * \left[R^*(s) - H^*(s)\,Y^*(s)\right]$$

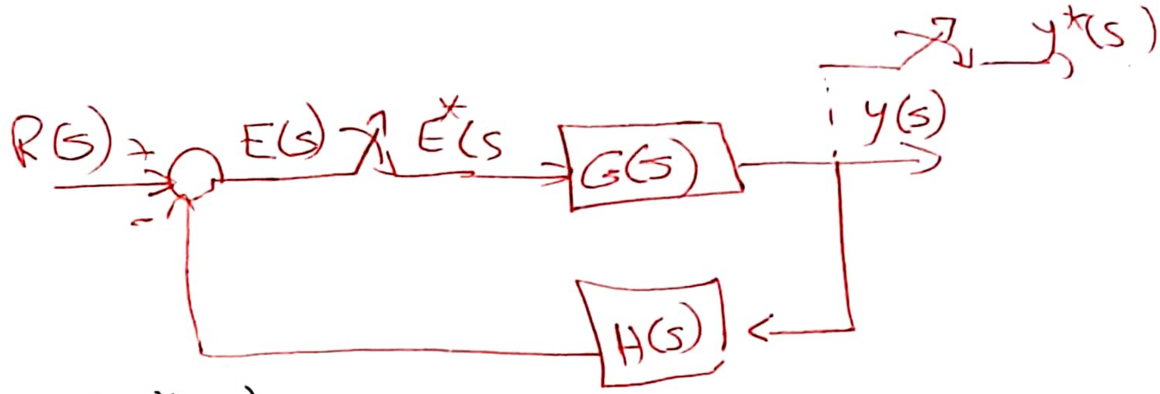
$$Y(s) = G(s)\,R^*(s) - H^*(s)\,Y^*(s)\,G(s)$$

$$Y^*(s) = G^*(s)\,R^*(s) - G^*(s)\,H^*(s)\,Y^*(s)$$

$$Y^*(s)\left(1 + G^*(s)\,H^*(s)\right) = G^*(s)\,R^*(s)$$

$$Y^*(s) = \frac{G^*(s)}{1 + G^*(s)\,H^*(s)}\,R^*(s)$$

$$Y(z) = \frac{G(z)}{1 + G(z)\,H(z)} * R(z)$$



$$Y(s) = G(s)\,E^*(s)$$

$$E(s) = R(s) - H(s)\,Y(s)$$

$$E(s) = R(s) - H(s)\,G(s)\,E^*(s)$$

$$E^*(s) = R^*(s) - GH^*(s)\,E^*(s)$$

$$E^*(s)\left[1 + GH^*(s)\right] = R^*(s)$$

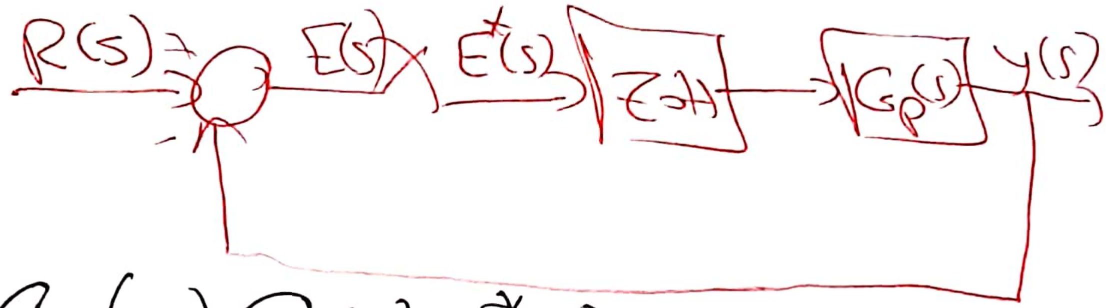
$$E^*(s) = \frac{R^*(s)}{1 + GH^*(s)}$$

$$Y(s) = G(s)\,\frac{R^*(s)}{1 + GH^*(s)}$$

$$Y^*(s) = \frac{G^*(s)}{1 + GH^*(s)}\,R^*(s)$$

$$Y(z) = \frac{G(z)}{1 + GH(z)}\,R(z)$$

exg!-



$$Y(s) = G_{ZOH}(s)\, G_p(s)\, E^*(s)$$

$$E(s) = R(s) - Y(s) \Longrightarrow E^*(s) = R^*(s) - Y^*(s)$$

$$Y(s) = G_{ZOH}\, G_p(s)\left(R^*(s) - Y^*(s)\right)$$

$$Y(s) = G_{ZOH}(s)\, G_p(s)\, R^*(s) - G_{ZOH}\, G_p(s)\, Y^*(s)$$

$$Y^*(s) = G_{ZOH}\, G_p^*(s)\, R^*(s) - G_{ZOH}\, G_p^*(s)\, Y^*(s)$$

$$Y^*(s)\left[1 + G_{ZOH}\, Y^*(s)\right] = G_{ZOH}\, G_p(s)\, R^*(s)$$

$$Y^*(s) = \frac{G_{ZOH}\, G_p^*(s)}{1 + G_{ZOH}\, G_p^*(s)}\, R^*(s)$$

$$Y(z) = \frac{G_{ZOH}\, G_p(z)}{1 + G_{ZOH}\, G_p(z)}\, R(z)$$

$$\frac{Y(z)}{R(z)} = \frac{G_{ZOH}\, G_p(z)}{1 + G_{ZOH}\, G_p(z)}$$

$$G_{zoh}(s) = \frac{1-e^{-Ts}}{s} \qquad G_p(s) = \frac{1}{s+1}$$

$$G_{zoh}G_p(s) = \frac{1-e^{-Ts}}{s} \times \frac{1}{s+1} = \frac{1-e^{-Ts}}{s(s+1)}$$

$$= (1-e^{-Ts})\left(\frac{1}{s} - \frac{1}{s+1}\right)$$

$$G_{zoh}G_p(z) = (1-z^{-1})\, Z\left\{\frac{1}{s} - \frac{1}{s+1}\right\}$$

$$= \frac{z-1}{z} \times \left(\frac{z}{z-1} - \frac{z}{z-e^{-T}}\right)$$

$$G_{zoh}G_p(z) = \frac{1-e^{-T}}{z-e^{-T}}$$

$$\frac{Y(z)}{R(z)} = \frac{G_{zoh}G_p(z)}{1+G_{zoh}G_p(z)}$$

$$1+G_{zoh}G_p(z) = 1 + \frac{1-e^{-T}}{z-e^{-T}} = \frac{z+1-2e^{-T}}{z-e^{-T}}$$

$$\frac{Y(z)}{R(z)} = \frac{1-e^{-T}}{z+1-2e^{-T}}$$