

**GUC**  
**German University in Cairo**  
**Faculty of Engineering and Material Science**  
**Department of Mechatronics Engineering**

**Robust and Optimal Control**  
**Assignment #1**

**Due date:**                      **Wednesday 20.2.2019**

Name	
App. No.	
Group No.	

**Note:**

*Assignments not submitted in this form will **NOT** be accepted.*

*Assignments should be submitted in hardcopy.*

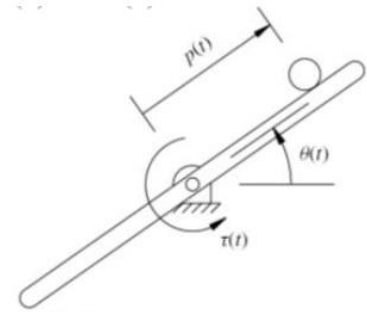
*Cheating will not be tolerated and it is your responsibility to ensure the genuineness of your work.*

## **Problem 1:**

A system is described by the nonlinear model:

$$\left[ \frac{J_b}{r^2} + m \right] \ddot{p}(t) + mg \sin \theta(t) - m p(t) \dot{\theta}(t)^2 = 0$$

$$[m p(t)^2 + J + J_b] \ddot{\theta}(t) + 2 m p(t) \dot{p}(t) \dot{\theta}(t) + mg p(t) \cos \theta(t) = \tau(t)$$



Ball and beam apparatus.

In which  $p(t)$  is the ball position,  $\theta(t)$  is the beam angle and  $\tau(t)$  is the applied torque.

System parameters	Symbol	Value
Mass moment of inertia of the beam	$J$	9.99e-04
Mass of the ball	$m$	0.11 Kg
Radius of the ball	$r$	0.015
Mass moment of inertia of the ball	$J_b$	9.99e-04
Gravitational acceleration constant	$g$	9.81

## **Requirements:**

- 1) Linearize the system around an operating point of your choice.
- 2) Discretize the system for a sampling time  $T_s = 0.1s$ .
- 3) The system analysis: check its stability, controllability and observability.
- 4) Design a finite horizon linear quadratic regulator for the discrete-time system using the recursive approach for two different initial conditions. Choose your own weighing matrices.
- 5) Simulate your controller on the linearized system and on the nonlinear system.
- 6) Plot the optimal cost-to-go from time  $j$  to the end of the time horizon  $N$ .
- 7) Plot the response of the system outputs and control effort.
- 8) Comment on your results.

Note that you can use Matlab for your calculations. You should show all your steps clearly, submit all the codes used and the results.