Data structures and algorithms Tutorial 7 - Introduction to graph theory

Amr Keleg

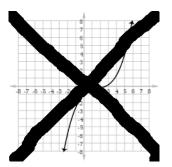
Faculty of Engineering, Ain Shams University

April 10, 2020

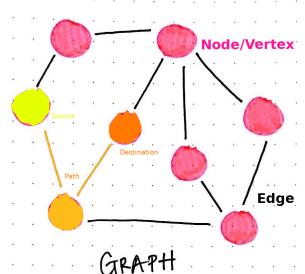
Contact: amr mohamed@live.com

- 1 Basic concepts of graph theory
 - Terminology
 - Types of graphs
 - Degree in graphs
- 2 Adjacency Matrix

Graph? NO



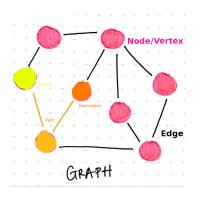
Graph? YES



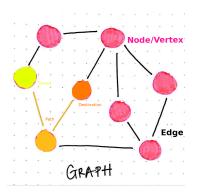
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2 Adjacency Matrix

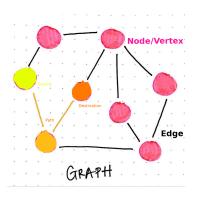
- How to represent a graph using basic data-structures?
- Searching algorithms in graph DFS Depth First Search
- Sheet 4 Question 3
- DFS using stack instead of recursion
- Sheet 4 Question 3 using stack



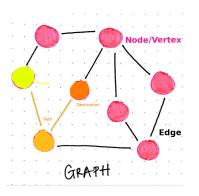
Node



- Node
- Edge:



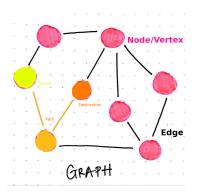
- Node
- Edge: A direct connection between two nodes



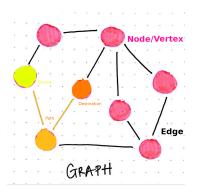
Node

■ Edge: A direct connection between two nodes

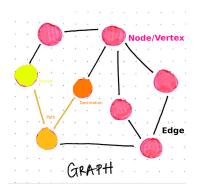
■ Path:



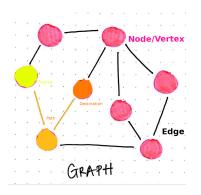
- Node
- Edge: A direct connection between two nodes
- Path: A set of edges connecting source and destination nodes



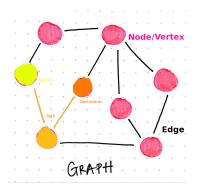
- Node
- Edge: A direct connection between two nodes
- Path: A set of edges connecting source and destination nodes
- Weight:



- Node
- Edge: A direct connection between two nodes
- Path: A set of edges connecting source and destination nodes
- Weight: A value give to each edge



- Node
- Edge: A direct connection between two nodes
- Path: A set of edges connecting source and destination nodes
- Weight: A value give to each edge
- Degree:



- Node
- Edge: A direct connection between two nodes
- Path: A set of edges connecting source and destination nodes
- Weight: A value give to each edge
- Degree: No. of edges for each node

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LTypes of graphs

Undirected vs Directed

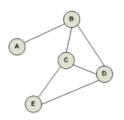


Fig 1. Undirected Graph

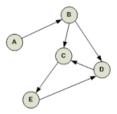
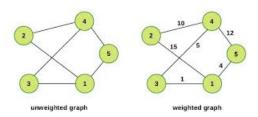


Fig 2. Directed Graph

Unweighted vs Weighted



■ Followers on Twitter

Followers on Twitter (Directed)

- Followers on Twitter (Directed)
- Friends on facebook

- Followers on Twitter (Directed)
- Friends on facebook (Undirected)

- Followers on Twitter (Directed)
- Friends on facebook (Undirected)
- Roads of Google Maps

- Followers on Twitter (Directed)
- Friends on facebook (Undirected)
- Roads of Google Maps (Directed)

The two properties aren't mutually exclusive.

E.G: A graph can be:

- Directed and Unweighted
- Undirected and Unweighted
- Directed and Weighted
- Undirected and Weighted

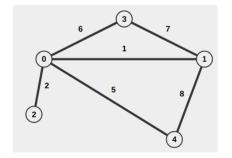
1 Basic concepts of graph theory

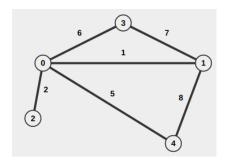
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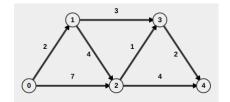
LDegree in graphs





Node	Degree
0	4
1	3
2	1
3	2
4	2

LDegree in graphs



3
2 4 1 2
0 7 2 4 4

Node	In-degree	Out-degree	
0	0	2	
1	1	2	
2	2	2	
3	2	1	
4	2	0	

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For simplicity and without loss of generality, let's assume that the graph isn't dynamic (We won't need to add or delete nodes after the construction of the graph).

How to represent a graph using basic data-structures?

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2 Adjacency Matrix

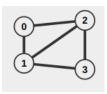
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How to represent a graph using basic data-structures?

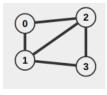
- Wikipedia definition: "In graph theory and computer science, an adjacency matrix is a square matrix used to represent a finite graph. The elements of the matrix indicate whether pairs of vertices are adjacent or not in the graph."
- For a graph of N nodes, build a 2D array (matrix) of size N*N.
- The values stored in the adjacency matrix differs according to the type of the graph.

How to represent a graph using basic data-structures?

Generate the adjacency matrix for the following graph.

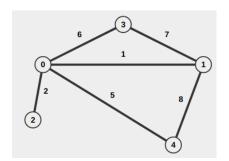


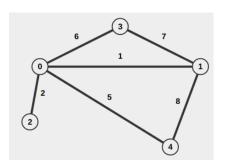
Generate the adjacency matrix for the following graph.



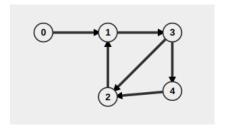
Adjacency Matrix					
	0	1	2	3	
0	0	1	1	0	
1	1	0	1	1	
2	1	1	0	1	
3	0	1	1	0	

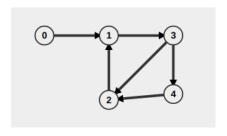
Generate the adjacency matrix for the following graph.



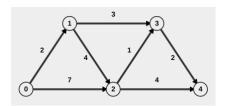


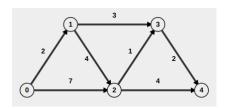
Adjacency Matrix					
	0	1	2	3	4
0	0	1	2	6	5
1	1	0	0	7	8
2	2	0	0	0	0
3	6	7	0	0	0
4	5	8	0	0	0





Adjacency Matrix					
	0	1	2	3	4
0	0	1	0	0	0
1	0	0	0	1	0
2	0	1	0	0	0
3	0	0	1	0	1
4	0	0	1	0	0





Adjacency Matrix					
	0	1	2	3	4
0	0	2	7	0	0
1	0	0	4	3	0
2	0	0	0	1	4
3	0	0	0	0	2
4	0	0	0	0	0

```
// An undirected unweighted graph
3 3 // 3 nodes and 3 edges
```

- 0
- 0 2
- 1 2

```
#include < vector >
int main(){
  // An empty vector of size 0
  vector < int > v1:
  // A vector of size 10
  vector < int > v2(10);
  // A vector of size 10 and initial value 5
  vector < int > v3(10, 5);
```

★ vector - C++ Reference ×	-	_ >
← → C △ ① Not secure 0	cplusplus.com/reference/vector/vector/ 🔍 🖈 🧞 💿 🥦 📠 🛐 📨 🔞 😬 📀 🌃 👼	() :
fx Mei	mber functions	
(const	tructor) Construct vector (public member function)	
(destru	uctor) Vector destructor (public member function)	
operat	tor= Assign content (public member function)	
Iterators	s:	
begin	Return iterator to beginning (public member function)	
end	Return iterator to end (public member function)	
rbegin	Return reverse iterator to reverse beginning (public member function)	
rend	Return reverse iterator to reverse end (public member function)	
cbegin	Return const_iterator to beginning (public member function)	
cend S	Return const_iterator to end (public member function)	
crbegi	Return const_reverse_iterator to reverse beginning (public member function)	
crend	Return const_reverse_iterator to reverse end (public member function)	
Capacit	ty:	
size	Return size (public member function)	
max_s	size Return maximum size (public member function)	
resize	Change size (public member function)	
capaci	ity Return size of allocated storage capacity (public member function)	
empty	Test whether vector is empty (public member function)	
reserv	Request a change in capacity (public member function)	



```
Example
 1 // constructing vectors
 2 #include <iostream>
 3 #include <vector>
 5 int main ()
 6 {
    // constructors used in the same order as described above:
   std::vector<int> first:
                                                          // empty vector of ints
 9 std::vector<int> second (4,100);
                                                          // four ints with value 100
10 std::vector<int> third (second.begin().second.end()): // iterating through second
11 std::vector<int> fourth (third):
                                                          // a copy of third
                                                                                      @ Edit & Run
13 // the iterator constructor can also be used to construct from arrays:
14 int myints[] = {16,2,77,29};
15 std::vector<int> fifth (myints, myints + sizeof(myints) / sizeof(int) );
    std::cout << "The contents of fifth are:";
18 for (std::vector<int>::iterator it = fifth.begin(); it != fifth.end(); ++it)
    std::cout << ' ' << *it:
20 std::cout << '\n';
22 return θ:
23 }
```

Complexity

Output:

Constant for the default constructor (1), and for the move constructors (5) (unless alloc is different from x's allocator).

For all other cases, linear in the resulting container size.

The contents of fifth are: 16 2 77 29

Additionally, if InputIterator in the range constructor (3) is not at least of a forward iterator category (i.e., it is just an input iterator), the new capacity cannot be determined beforehand and the construction incurs in additional logarithmic complexity in size (reallocations while growing).

Tutorial 7

Adjacency Matrix

```
\begin{tabular}{ll} // & \textit{Vector of strings} \\ & \textit{vector} < \textit{string} > \textit{vs} (100 \, , \ "ABC" \, ); \\ \end{tabular}
```

```
// Vector of strings
vector<string>vs(100, "ABC");
// Vector of vectors
vector < vector < int > >
```

```
// Vector of strings
vector<string>vs(100, "ABC");
// Vector of vectors
vector < vector < int > > vi
```

Adjacency Matrix

```
// Vector of strings
vector<string>vs(100, "ABC");
// Vector of vectors
vector< vector<int>> vi(100,
```

```
// Vector of strings
vector<string>vs(100, "ABC");
// Vector of vectors
vector< vector<int>> vi(100, vector<int>(10, -1))
```

```
// Vector of strings vector<string>vs(100, "ABC"); 

// Vector of vectors vector< vector vector vi(100, vector vector vint) vi(100, vector vint); 

// 100 rows and 10 columns 

// vi[0][0] to vi[99][9]
```

```
// Vector of strings
vector<string>vs(100, "ABC");

// Vector of vectors
vector<<rp>vector
vi(100, vector<int>(10, -1));

// 100 rows and 10 columns
// vi[0][0] to vi[99][9]

int * arr; // or int ** arr;
arr = new int*[100];
```

```
// Vector of strings
vector<string>vs(100, "ABC");

// Vector of vectors
vector< vector<int>> vi(100, vector<int>>(10, -1));

// 100 rows and 10 columns
// vi[0][0] to vi[99][9]

int * arr; // or int ** arr;
arr = new int *[100];
for(int i=0; i<100; i++)
{
    arr[i] = new int[10];
    for(int j=0;j<10;j++)
    {
        arr[i][j] = -1;
    }
}</pre>
```

```
// Undirected unweighted
int n, m;
cin >>n>>m;
```

```
// Undirected unweighted
int n, m;
cin>>n>m;
// 1) A static array (not preferable)
int adj_matrix [1000][1000]; // 1000 is just a large no.
// 2) Using dynamic arrays
// without getting our hands dirty
vector<vector<int> > adj_matrix(n, vector<int> (n, 0));
```

```
// Undirected unweighted
int n, m;
cin>>m>>m;
// 1) A static array (not preferable)
int adj_matrix [1000][1000]; // 1000 is just a large no.
// 2) Using dynamic arrays
// without getting our hands dirty
vector<vector<int>> adj_matrix(n, vector<int> (n, 0));
int a, b;
```

```
// Undirected unweighted
int n, m;
cin>>n>m;
// 1 A static array (not preferable)
int adj_matrix [1000][1000]; // 1000 is just a large no.
// 2) Using dynamic arrays
// without getting our hands dirty
vector<vector<int>> adj_matrix(n, vector<int>> (n, 0));
int a, b;
for (int i=0; i<m ;i++){
    cin>a>>b;
    adj_matrix[a][b] = 1;
    adj_matrix[b][a] = 1;
}
```

```
// Directed unweighted
int n, m;
cin>>n>>m;
vector<vector<int>> adj_matrix(n, vector<int> (n, 0));
```

```
// Directed unweighted
int n, m;
cin >>n>>m;

vector<vector<int> > adj_matrix(n, vector<int> (n, 0));
int a, b;
for (int i=0; i<m; i++){
    cin>>a>>b;
    adj_matrix[a][b] = 1;
}
```

```
// Undirected Weighted
int n, m;
cin>>n>>m;
vector<vector<int>> adj_matrix(n, vector<int> (n, 0));
```

```
// Undirected Weighted
int n, m;
cin>>n>>m;

vector<vector<int>> adj_matrix(n, vector<int> (n, 0));

int a, b, c;
for (int i=0; i<m; i++){
    cin>>a>>b>>c;
    adj_matrix[a][b] = c;
    adj_matrix[b][a] = c;
}
```

```
// Directed Weighted
int n, m;
cin >>n>>m;
vector<vector<int>> adj_matrix(n, vector<int> (n, 0));
```

```
// Directed Weighted
int n, m;
cin>>n>>m;

vector<vector<int>> adj-matrix(n, vector<int> (n, 0));
int a, b, c;
for (int i=0; i<m ; i++){
    cin>>a>>b>>c;
    adj-matrix[a][b] = c;
}
```

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```
One of the questions that DFS can solve is:
"Are nodes src and des connected?"
bool dfs(int node, int des,
    const vector < vector < int > > & adj_matrix);
// Return true if there is a path between node and des
```

```
bool dfs(int node, int des,
  const vector<vector<int> > & adj_matrix){
  if (node = des)
    return true:
  for (int next_node=0;
     next_node < adj_matrix [node]. size ();</pre>
     next_node++){
    if (adj_matrix[node][next_node] == 0)
      continue;
    if (dfs(next_node, des, adj_matrix))
      return true;
  return false:
```

```
bool dfs(int node, int des,
  const vector < vector < int > > & adj_matrix ,
  vector < bool > & visited){
  if (node = des)
    return true:
  visited [node] = true;
  for (int next_node=0;
     next_node < adj_matrix [node]. size ();</pre>
     next_node++){
    if (adj_matrix[node][next_node] == 0
      | visited[next_node])
      continue:
    if (dfs(next_node, des, adj_matrix, visited))
      return true;
  return false:
```

```
bool dfs(int node, int des,
  const vector<vector<int>> & adj_matrix ,
  vector < bool > & visited);
int main(){
  int n, m;
  cin >> n >> m:
  vector<vector<int> > adj_matrix(n, vector<int> (n, 0))
  ... // Build the graph
  int src . des :
  cin >> src >> des :
  vector < bool > visited (n, false);
  if ( dfs(src , des , adj_matrix , visited ))
    cout << " Connected" ;</pre>
  else
    cout << "Not _ connected":
```

Outline

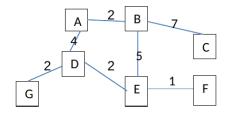
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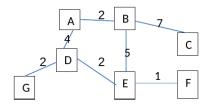
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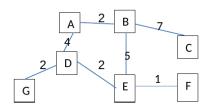
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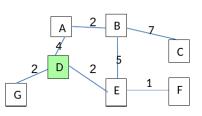
Q3. Apply both depth first and breadth first algorithms starting from vertex D

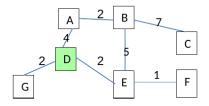


Note: The paths will depend on the way nodes are pushed to the queue/ stack and whether you mark the node on push/ on pop. State your assumptions clearly when you answer such questions unless the rules are stated in the question's statement.

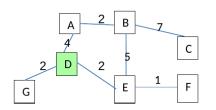


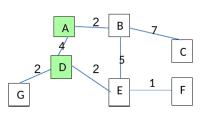


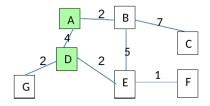


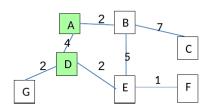


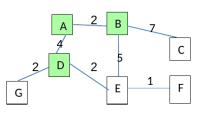
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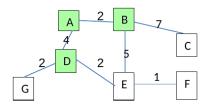


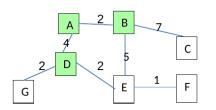


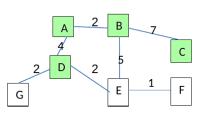


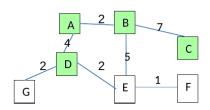


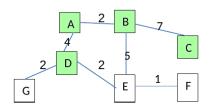
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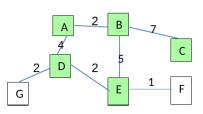


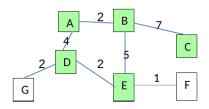


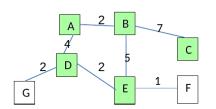


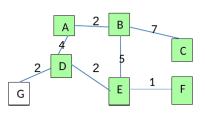


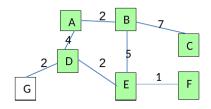


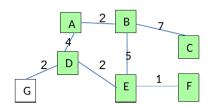


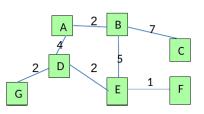












DFS: D - A - B - C - E - F - G

Outline

1 Basic concepts of graph theory

- Terminology
- Types of graphs
- Degree in graphs

2 Adjacency Matrix

- How to represent a graph using basic data-structures?
- Searching algorithms in graph DFS Depth First Search
- Sheet 4 Question 3
- DFS using stack instead of recursion
- Sheet 4 Question 3 using stack

Tutorial 7

```
bool dfs(int node, int des, const vector<vector<int>> & adj_matrix,
  vector<bool> & visited){
  stack<int> nodes_stack:
  nodes_stack.push(node);
  while (! nodes_stack.empty()) {
    int cur_node = nodes_stack.top();
    nodes_stack.pop();
    visited [cur_node] = 1:
    if (cur_node==des)
      return true:
    for (int next_node=0; next_node<adj_matrix[cur_node].size(); next_node++){</pre>
      if (adj_matrix[cur_node][next_node] == 0 || visited[next_node])
        continue;
      nodes_stack.push(next_node);
  return false:
```

Outline

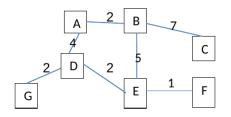
1 Basic concepts of graph theory

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Q3. Apply both depth first and breadth first algorithms starting from vertex D



Note: The paths will depend on the way nodes are pushed to the queue/ stack and whether you mark the node on push/ on pop. State your assumptions clearly when you answer such questions unless the rules are stated in the question's statement.

```
void dfs(int node, const vector<vector<int>> & adj_matrix,
  vector <bool> & visited){
[X] stack<int> nodes_stack;
  nodes_stack.push(node);
  while (! nodes_stack.empty()) {
    int cur_node = nodes_stack.top();
    nodes_stack.pop():
    cout << cur_node << " =" ;
    visited [cur_node] = 1;
    for (int next_node=0; next_node<adj_matrix[cur_node].size(); next_node++){</pre>
      if (adj_matrix[cur_node][next_node] == 0 || visited[next_node])
        continue:
      nodes_stack.push(next_node);
```



```
void dfs(int node, const vector<vector<int>> & adi_matrix.
  vector<bool> & visited){
  stack<int> nodes_stack:
[X] nodes_stack.push(node);
  while (! nodes_stack.empty()) {
    int cur_node = nodes_stack.top();
    nodes_stack.pop();
    cout << cur_node << " _";
    visited[cur_node] = 1;
    for (int next_node=0; next_node<adj_matrix[cur_node].size(); next_node++){</pre>
      if (adj_matrix[cur_node][next_node] == 0 || visited[next_node])
        continue:
      nodes_stack.push(next_node);
```





```
Sheet 4 - Question 3 - using stack
```

```
void dfs(int node, const vector<vector<int>> & adj_matrix ,
  vector <bool> & visited){
  stack<int> nodes_stack;
  nodes_stack.push(node);
  while (! nodes_stack.empty()) {
    int cur_node = nodes_stack.top():
    nodes_stack.pop();
    cout << cur_node << "_";
      visited [cur_node] = 1;
[X]
    for (int next_node=0; next_node<adj_matrix[cur_node].size(); next_node++){</pre>
      if (adj_matrix[cur_node][next_node] == 0 || visited[next_node])
        continue;
      nodes_stack.push(next_node);
```

D



```
void dfs(int node, const vector<vector<int>> & adj_matrix,
  vector <bool> & visited){
  stack<int> nodes_stack:
  nodes_stack.push(node);
  while (! nodes_stack.empty()) {
    int cur_node = nodes_stack.top();
    nodes_stack.pop():
    cout << cur_node << " =" ;
    visited [cur_node] = 1;
    for (int next_node=0; next_node<adj_matrix[cur_node].size(); next_node++){</pre>
      if (adj_matrix[cur_node][next_node] == 0 || visited[next_node])
        continue:
      nodes_stack.push(next_node);
[X]
```

G E A



```
void dfs(int node, const vector<vector<int>> & adj_matrix,
  vector <bool> & visited){
  stack<int> nodes_stack:
  nodes_stack.push(node);
  while (! nodes_stack.empty()) {
    int cur_node = nodes_stack.top();
    nodes_stack.pop():
    cout << cur_node << " =" ;
    visited [cur_node] = 1;
[X]
    for (int next_node=0; next_node<adj_matrix[cur_node].size(); next_node++){</pre>
      if (adj_matrix[cur_node][next_node] == 0 || visited[next_node])
        continue:
      nodes_stack.push(next_node);
```

D-G





```
void dfs(int node, const vector<vector<int>> & adj_matrix ,
  vector<bool> & visited){
  stack<int> nodes_stack:
  nodes_stack.push(node);
  while (! nodes_stack.empty()) {
    int cur_node = nodes_stack.top();
    nodes_stack.pop();
    cout << cur_node << "_":
    visited[cur\_node] = 1;
[X]
    for (int next_node=0; next_node<adj_matrix[cur_node].size(); next_node++){</pre>
      if (adj_matrix[cur_node][next_node] == 0 || visited[next_node])
        continue:
      nodes_stack.push(next_node);
```

D - G - E





```
void dfs(int node, const vector<vector<int>> & adj_matrix,
  vector<bool> & visited){
  stack<int> nodes_stack:
  nodes_stack.push(node);
  while (! nodes_stack.empty()) {
    int cur_node = nodes_stack.top();
    nodes_stack.pop();
    cout << cur_node << "_":
    visited[cur\_node] = 1;
    for (int next_node=0; next_node<adj_matrix[cur_node].size(); next_node++){</pre>
      if (adj_matrix[cur_node][next_node] == 0 || visited[next_node])
        continue:
      nodes_stack.push(next_node);
[X]
}
```

D - G - E





```
void dfs(int node, const vector<vector<int>> & adj_matrix,
  vector <bool> & visited){
  stack<int> nodes_stack;
  nodes_stack.push(node);
  while (! nodes_stack.empty()) {
    int cur_node = nodes_stack.top();
    nodes_stack.pop();
    cout << cur_node << " " ;
    visited[cur_node] = 1;
[X]
    for (int next_node=0; next_node<adj_matrix[cur_node].size(); next_node++){</pre>
      if (adj_matrix[cur_node][next_node] == 0 || visited[next_node])
        continue:
      nodes_stack.push(next_node);
```

D - G - F - F





```
void dfs(int node, const vector<vector<int>> & adj_matrix ,
  vector<bool> & visited){
  stack<int> nodes_stack:
  nodes_stack.push(node);
  while (! nodes_stack.empty()) {
    int cur_node = nodes_stack.top();
    nodes_stack.pop();
    cout << cur_node << "_":
    visited[cur\_node] = 1;
[X]
    for (int next_node=0; next_node<adj_matrix[cur_node].size(); next_node++){</pre>
      if (adj_matrix[cur_node][next_node] == 0 || visited[next_node])
        continue:
      nodes_stack.push(next_node);
```

D - G - F - F - R





```
void dfs(int node, const vector<vector<int>> & adj_matrix,
  vector<bool> & visited){
  stack<int> nodes_stack:
  nodes_stack.push(node);
  while (! nodes_stack.empty()) {
    int cur_node = nodes_stack.top();
    nodes_stack.pop();
    cout << cur_node << "_":
    visited[cur\_node] = 1;
    for (int next_node=0; next_node<adj_matrix[cur_node].size(); next_node++){</pre>
      if (adj_matrix[cur_node][next_node] == 0 || visited[next_node])
        continue:
      nodes_stack.push(next_node);
[X]
```

D - G - E - F





Tutorial 7

- Visit/Mark on pop (as we have done) OR
- Visit/Mark on push (as long as node is pushed to the stack, mark it as visited)

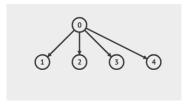
Tutorial 7

What is the complexity of the DFS on adjacency matrix? The graph has N nodes and E edges.

What is the complexity of the DFS on adjacency matrix?

The graph has N nodes and E edges.

$$O(N^2) \tag{1}$$



For a simple graph like this one, DFS using adjacency matrix will make 25(5*5) iterations/checks.

Tutorial 7

To be continued!

Feedback form: https://forms.gle/hTuHcEMs87vLNrUL9