Data structures and algorithms

Tutorial 8 - Part 1

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April 17, 2020

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Outline

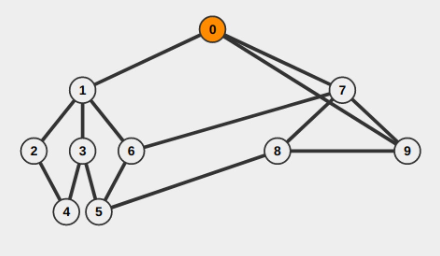
1 Adjacency List
- How to represent a graph using basic data-structures?
- First option - An array(vector) of linked list(lists)
- Second option - An array(vector) of arrays(vectors)
- Searching algorithms in graph - DFS - Depth First Search

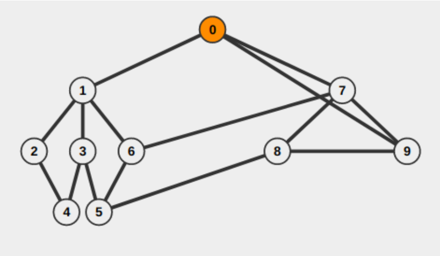
2 Breadth First Search (BFS)

3 Sheet 4 Questions

Outline

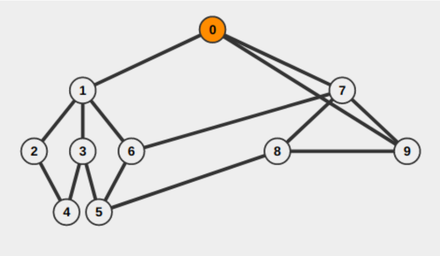
- Adjacency matrix isn't suitable for sparse graphs.
- Adjacency matrix can only represent a single edge between any two nodes.
- Instead of using an adjacency matrix, We will use an adjacency list. For each node, store **only** information about its adjacent nodes.


0
1
7
2
3
6
8
9
4
5



Adjacency Matrix

	0	1	2	3	4	5	6	7	8	9
0	0	1	0	0	0	0	0	1	0	1
1	1	0	1	1	0	0	1	0	0	0
2	0	1	0	0	1	0	0	0	0	0
3	0	1	0	0	1	1	0	0	0	0
4	0	0	1	1	0	0	0	0	0	0
5	0	0	0	1	0	0	1	0	1	0
6	0	1	0	0	0	1	0	1	0	0
7	1	0	0	0	0	0	1	0	1	1
8	0	0	0	0	0	1	0	1	0	1
9	1	0	0	0	0	0	0	1	1	0



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2	0	1	0	0	1	0	0	0	0	0
3	0	1	0	0	1	1	0	0	0	0
4	0	0	1	1	0	0	0	0	0	0
5	0	0	0	1	0	0	1	0	1	0
6	0	1	0	0	0	1	0	1	0	0
7	1	0	0	0	0	0	1	0	1	1
8	0	0	0	0	0	1	0	1	0	1
9	1	0	0	0	0	0	0	1	1	0

Adjacency List

0:	1	7	9	
1:	0	2	3	6
2:	1	4		
3:	1	4	5	
4:	2	3		
5:	3	6	8	
6:	1	5	7	
7:	0	6	8	9
8:	5	7	9	
9:	0	7	8	

How to read a description of the graph and load it into an adjacency matrix?

```
// An undirected unweighted graph

3 3
0 1
0 2
1 2
```

Outline

Outline

The complexity of adding a node at the end of STL's list is O(1): http://www.cplusplus.com/reference/list/list/push_back/

```
// For an Undirected Unweighted graph
int n, m;
cin>>n>>m;
// Initialize the rows as empty vectors.
vector<list<int> > adj_list(n, list<int>());
int a, b;
for (int i=0; i<m ;i++){
  cin>>a>>b;
  // Push node b to the end of
  // the linked list for node a
  adj_list[a].push_back(b);
  adj_list[b].push_back(a);
}
```

```
// For an Directed Unweighted graph
int n, m;
cin>>n>>m;
// Initialize the rows as empty vectors.
vector<list<int> > adj_list(n, list<int>());
int a, b;
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class Connection{
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    int destination;
    int weight;
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First option: OOP approach (used in the slides)

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class Connection{
  public:
    int destination;
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int main(){
  int n; // No. of Nodes
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  vector<list<Connection> > adj_list(n);
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```

Second option: Using STL class (Pair)[1]

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```
#include<utility>

int main(){
  pair<int, int> p1(10, 1);
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Second option: Using STL class (Pair)[1]

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#include<utility>

int main(){
  pair<int, int> p1(10, 1);
  cout<<p1.first<<" "<<p1.second;
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Second option: Using STL class (Pair)[1]

```
#include<utility>

int main(){
  pair<int, int> p1(10, 1);
  cout<<p1.first<<" "<<p1.second;

  pair<string, int> p2("Apple", 30);
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Second option: Using STL class (Pair)[1]

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#include<utility>

int main(){
  pair<int, int> p1(10, 1);
  cout<<p1.first<<" "<<p1.second;

  pair<string, int> p2("Apple", 30);
  cout<<p2.first<<" "<<p2.second;

  pair<int, int> p3;
  p3 = {10, 20};
  cout<<p3.first<<" "<<p3.second;
```

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#include<utility>

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  pair<int, int> p1(10, 1);
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  p3 = {10, 20};
  cout<<p3.first<<" "<<p3.second;

  // Building the graph using pair
  int n; // No. of Nodes
  cin>>n;
```

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  //Change this: vector<list<Connection> > adj_list(n);
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  p3 = {10, 20};
  cout<<p3.first<<" "<<p3.second;

  // Building the graph using pair
  int n; // No. of Nodes
  cin>>n;

  //Change this: vector<list<Connection> > adj_list(n);
  vector<list<pair<int,int> > > adj_list(n);
}
```

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- C++ knows how to sort them containers of those classes.
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```
#include <vector> // vector
#include <utility> // pair
#include <algorithm> // sort
#include <iostream> // cout
using namespace std;

int main(){
  vector<pair<int, int> > v;
  v.push_back({2, 2});
  v.push_back({1, 1});
  sort(v.begin(), v.end());

  for(int i=0; i< v.size(); i++)
    cout<<v[i].first <<" "<<v[i].second<<endl;

  /* The output is:
  1 1
  2 2
  */

  return 0;
}
```

```
#include <vector> // vector
#include <algorithm> // sort
#include <iostream> // cout
using namespace std;

class CustomPair{
public:
  int first;
  int second;
  CustomPair(int f, int s):first(f), second(s){}
};

int main(){
  vector<CustomPair > v;
  v.push_back(CustomPair(2, 2));
  v.push_back(CustomPair(1, 1));
  sort(v.begin(), v.end());

  for(int i=0; i< v.size(); i++)
    cout<<v[i].first<<" "<<v[i].second<<endl;

  return 0;
}
```

OUTPUT?

```
#include <vector> // vector
#include <algorithm> // sort
#include <iostream> // cout
using namespace std;

class CustomPair{
public:
  int first;
  int second;
  CustomPair(int f, int s):first(f), second(s){}
};

int main(){
  vector<CustomPair > v;
  v.push_back(CustomPair(2, 2));
  v.push_back(CustomPair(1, 1));
  sort(v.begin(), v.end());

  for(int i=0; i< v.size(); i++)
    cout<<v[i].first<<" "<<v[i].second<<endl;

  return 0;
}
```

OUTPUT?

C++ does not know how to compare items of CustomPair.

```
amr:[~]: g++ custom_pair.cpp                                                                                    [133/136]
In file included from /usr/include/c++/7/bits/stl_algobase.h:71:0,
                 from /usr/include/c++/7/vector:60,
                 from custom_pair.cpp:1:
/usr/include/c++/7/bits/predefined_ops.h: In instantiation of 'constexpr bool __gnu_cxx::__ops::_Iter_less_iter::operator()(_Iterator$
, _Iterator2) const [with _Iterator1 = __gnu_cxx::__normal_iterator<CustomPair*, std::vector<CustomPair> >; _Iterator2 = __gnu_cxx::_$
normal_iterator<CustomPair*, std::vector<CustomPair> >]':
/usr/include/c++/7/bits/stl_algo.h:81:17:   required from 'void std::__move_median_to_first(_Iterator, _Iterator, _Iterator, _Iterato$
, _Compare) [with _Iterator = __gnu_cxx::__normal_iterator<CustomPair*, std::vector<CustomPair> >; _Compare = __gnu_cxx::__ops::_Iter$
less_iter]'
/usr/include/c++/7/bits/stl_algo.h:1921:34:   required from '_RandomAccessIterator std::__unguarded_partition_pivot(_RandomAccessIter$
tor, _RandomAccessIterator, _Compare) [with _RandomAccessIterator = __gnu_cxx::__normal_iterator<CustomPair*, std::vector<CustomPair>
>; _Compare = __gnu_cxx::__ops::_Iter_less_iter]'
/usr/include/c++/7/bits/stl_algo.h:1953:38:   required from 'void std::__introsort_loop(_RandomAccessIterator, _RandomAccessIterator,
_Size, _Compare) [with _RandomAccessIterator = __gnu_cxx::__normal_iterator<CustomPair*, std::vector<CustomPair> >; _Size = long int;
_Compare = __gnu_cxx::__ops::_Iter_less_iter]'
/usr/include/c++/7/bits/stl_algo.h:1968:25:   required from 'void std::__sort(_RandomAccessIterator, _RandomAccessIterator, _Compare)
[with _RandomAccessIterator = __gnu_cxx::__normal_iterator<CustomPair*, std::vector<CustomPair> >; _Compare = __gnu_cxx::__ops::_Iter$
less_iter]'
/usr/include/c++/7/bits/stl_algo.h:4836:18:   required from 'void std::sort(_RAIter, _RAIter) [with _RAIter = __gnu_cxx::__normal_ite$
ator<CustomPair*, std::vector<CustomPair> >]'
custom_pair.cpp:17:25:   required from here
/usr/include/c++/7/bits/predefined_ops.h:43:23: error: no match for 'operator<' (operand types are 'CustomPair' and 'CustomPair')
       { return *__it1 < *__it2; }
                ~~~~~~~^~~~~~~~
In file included from /usr/include/c++/7/bits/stl_algobase.h:67:0,
                 from /usr/include/c++/7/vector:60,
                 from custom_pair.cpp:1:
/usr/include/c++/7/bits/stl_iterator.h:891:5: note: candidate: template<class _IteratorL, class _IteratorR, class _Container> bool __$
nu_cxx::operator<(const __gnu_cxx::__normal_iterator<_IteratorL, _Container>&, const __gnu_cxx::__normal_iterator<_IteratorR, _Contai$
er>&)
     operator<(const __normal_iterator<_IteratorL, _Container>& __lhs,
     ^~~~~~~~
/usr/include/c++/7/bits/stl_iterator.h:891:5: note:   template argument deduction/substitution failed:
In file included from /usr/include/c++/7/bits/stl_algobase.h:71:0,
```

```
#include <vector> // vector
#include <algorithm> // sort
#include <iostream> // cout
using namespace std;

class CustomPair{
public:
  int first;
  int second;
  CustomPair(int f, int s):first(f), second(s){}
  bool operator <(const CustomPair & other_pair){
    return (this-> first < other_pair.first)
         (this-> first == other_pair.first &&
        this->second <=other_pair.second);
  }
};

int main(){
  vector<CustomPair > v;
  v.push_back(CustomPair(2, 2));
  v.push_back(CustomPair(1, 1));
  sort(v.begin(), v.end());

  for(int i=0; i< v.size(); i++)
    cout<<v[i].first<<" "<<v[i].second<<endl;

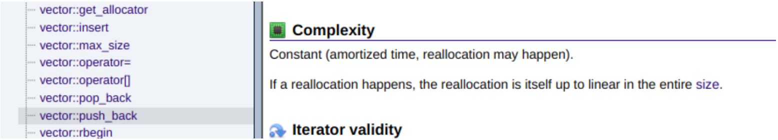
  return 0;
}
```

```
// For an Undirected Weighted graph
int n, m;
cin>>n>>m;
// Initialize the rows as empty vectors.
vector<list<pair<int,int> > > adj_list(n, list<pair<int,int> >());
int a, b, c;
for (int i=0; i<m ;i++){
  cin>>a>>b>>c;
  // Push node b to the end of
  // the linked list for node a
  adj_list[a].push_back({b, c});
  adj_list[b].push_back({a, c});
}
```

```
// For an Directed Weighted graph
int n, m;
cin>>n>>m;
// Initialize the rows as empty vectors.
vector<list<pair<int,int> > > adj_list(n, list<pair<int,int> >());
int a, b, c;
for (int i=0; i<m ;i++){
  cin>>a>>b>>c;
  // Push node b to the end of
  // the linked list for node a
  adj_list[a].push_back({b, c});
}
```

Outline

```
// For an Undirected Unweighted graph
int n, m;
cin>>n>>m;
// Initialize the rows as empty vectors.
vector<vector<int> > adj_list(n, vector<int> ());
int a, b;
for (int i=0; i<m ;i++){
  cin>>a>>b;
  adj_list[a].push_back(b);
  adj_list[b].push_back(a);
}
```



vector::get_allocator
vector::insert
vector::max_size
vector::operator=
vector::operator[]
vector::pop_back
vector::push_back
vector::rbegin

Complexity

Constant (amortized time, reallocation may happen).

If a reallocation happens, the reallocation is itself up to linear in the entire size.

Iterator validity

http://www.cplusplus.com/reference/vector/vector/push_back/

```
// For an Directed Unweighted graph
int n, m;
cin>>n>>m;
vector<vector<int> > adj_list(n, vector<int> ());
int a, b;
for (int i=0; i<m ;i++){
  cin>>a>>b;
  adj_list[a].push_back(b);
}
```

```
// For an Undirected Weighted graph
int n, m;
cin>>n>>m;
vector<vector<pair<int,int> > > adj_list(n, vector<pair<int,int> > ());
int a, b, c;
for (int i=0; i<m ;i++){
  cin>>a>>b>>c;
  adj_list[a].push_back({b, c});
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// For an Directed Weighted graph
int n, m;
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vector<vector<pair<int,int> > > adj_list(n, vector<pair<int,int> > ());
int a, b, c;
for (int i=0; i<m ;i++){
  cin>>a>>b>>c;
  adj_list[a].push_back({b, c});
}
```

Outline

```
bool dfs(int node, int des, const vector<vector<int> > & adj_matrix,
  vector<bool> & visited){

  if (node ==des)
    return true;

  visited[node] = true;

  for (int next_node=0; next_node<adj_matrix[node].size(); next_node++){

    if (adj_matrix[node][next_node] == 0 || visited[next_node])
      continue;

    if (dfs(next_node, des, adj_matrix, visited))
      return true;
  }

  return false;
}
```

```
// Using an array of arrays for adjacency list.

bool dfs(int node, int des, const vector<vector<int> > & adj_list,
  vector<bool> & visited){

  if (node ==des) return true;

  visited[node] = true;

  for (int next_node_index=0; next_node_index<adj_list[node].size(); next_node_index++){

    int next_node = adj_list[node][next_node_index];

    if (visited[next_node])
      continue;

    if (dfs(next_node, des, adj_list, visited))
      return true;
  }

  return false;
}
```

```
// Using an array of lists for adjacency list.

bool dfs(int node, int des, const vector<list<int> > & adj_list,
  vector<bool> & visited){

  if (node ==des) return true;

  visited[node] = true;

  for (list<int >::iterator it = adj_list[node].begin(); it!=adj_list[node].end(); it++){

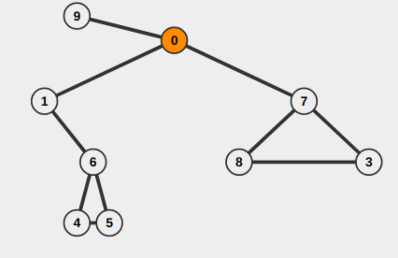
    int next_node = *it;

    if (visited[next_node])
      continue;

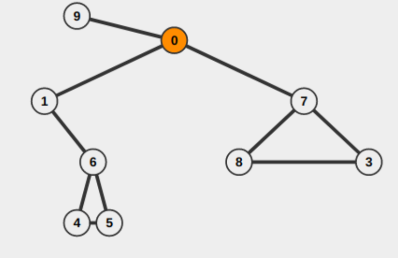
    if (dfs(next_node, des, adj_list, visited))
      return true;
  }

  return false;
}
```

Outline

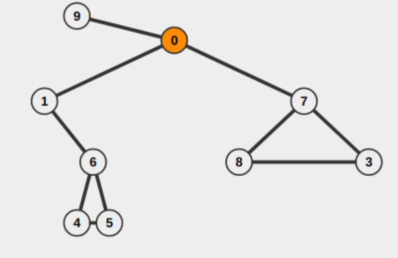


BFS is a traversing algorithm where you should:



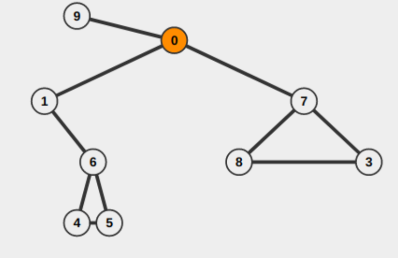
BFS is a traversing algorithm where you should:

- start traversing from a selected node (source or starting node).



BFS is a traversing algorithm where you should:

- start traversing from a selected node (source or starting node).
- traverse the graph layerwise thus exploring the neighbour nodes (nodes which are directly connected to source node).



BFS is a traversing algorithm where you should:

- start traversing from a selected node (source or starting node).
- traverse the graph layerwise thus exploring the neighbour nodes (nodes which are directly connected to source node).
- after exploring all the directly connected neighbour nodes, start exploring the next-level neighbour nodes.

Outline

```
bool bfs(int node, int des, const vector<vector<int> > & adj_mat,
  vector<bool> & visited){

  queue<int> nodes_q;
  nodes_q.push(node);

  while(!nodes_q.empty()){
    node = nodes_q.front();
    nodes_q.pop();
    visites[node] = true;
    if (node==des)
      return true;

    // Loop over neighbouring nodes
    for (int next_node = 0; next_node<adj_mat[node].size(); next_node++){

      if (visited[next_node] || adj_mat[node][next_node] == 0)
        continue;

      nodes_q.push(next_node);
    }
  }

  return false;
}
```

Outline

```
// Using an array of lists for adjacency list.

bool bfs(int node, int des, const vector<list<int> > & adj_list,
  vector<bool> & visited){

  queue<int> nodes_q;
  nodes_q.push(node);
  visited[node] = true;
  while(!nodes_q.empty()){
    node = nodes_q.front();
    nodes_q.pop();
    visited[node] = true;
    if (node==des)
      return true;

    // Loop over neighbouring nodes
    for (list<int>::iterator it = adj_list[node].begin(); it!=adj_list[node].end(); it+

      int next_node = *it;

      if (visited[next_node])
        continue;

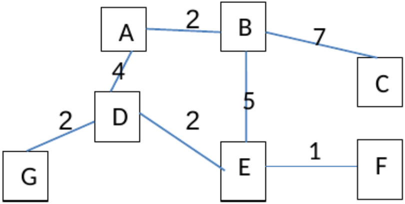
      nodes_q.push(next_node);
    }
  }

  return false;
}
```

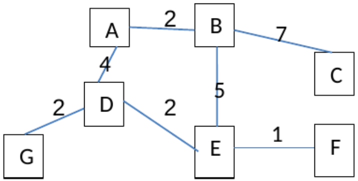
Outline

1. Adjacency List
   - How to represent a graph using basic data-structures?
   - First option - An array(vector) of linked list(lists)
     - Unweighted graphs
     - Weighted graphs
   - Second option - An array(vector) of arrays(vectors)
   - Searching algorithms in graph - DFS - Depth First Search
2. Breadth First Search (BFS)
   - Adjacency Matrix
   - Adjacency List
   - Sheet 4 - Question 3
   - One important application of BFS
3. Sheet 4 Questions
   - Question 2
   - Question 1

Q3. Apply both depth first and breadth first algorithms starting from vertex D



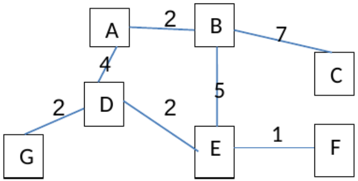
Note: The paths will depend on the way nodes are pushed to the queue/ stack and whether you mark the node on push/ on pop. State your assumptions clearly when you answer such questions unless the rules are stated in the question's statement.



Visited Array:

A	B	C	D	E	F	G

Queue:



Visited Array:

A	B	C	D	E	F	G

Queue:

Answer: BFS: D - A - E - G - B - F - C

Outline

What does the first path found by BFS represent?

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In case of unweighted graphs, the first path found by BFS is actually the shortest one.

Outline

Outline

1. Adjacency List
   - How to represent a graph using basic data-structures?
   - First option - An array(vector) of linked list(lists)
     - Unweighted graphs
     - Weighted graphs
   - Second option - An array(vector) of arrays(vectors)
   - Searching algorithms in graph - DFS - Depth First Search
2. Breadth First Search (BFS)
   - Adjacency Matrix
   - Adjacency List
   - Sheet 4 - Question 3
   - One important application of BFS
3. Sheet 4 Questions
   - Question 2
   - Question 1

Q2. Create an algorithm to determine whether an undirected graph is connected or not (i.e. any node/vertex can be reached from any other node/vertex)

- Start from any node.
- Apply DFS or BFS.
- Count the number of visited nodes.
- If all the nodes where visited then the graph is connected. Otherwise, the graph isn't connected.

```
void dfs(int src, vector<bool> &vis, ....);

int main(){
  int n,m;
  .....
  vector<bool> visited(n, false);
  dfs(0, visited, ...);
  int vis_count = 0;
  for(int i=0;i<n;i++)
    vis_count += visited[i];

  if (vis_count==n)
    cout<<"Connected";
  else
    cout<<"Not connected";
}
```

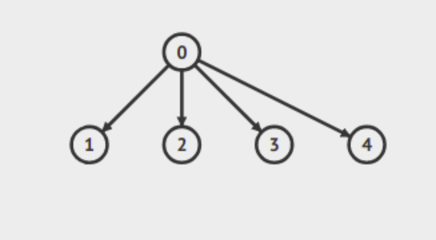
Comments:

- Connectivity in directed graphs is different.

[2] https://en.wikipedia.org/wiki/Strongly_connected_component

Comments:

- Connectivity in directed graphs is different.
- Is this graph connected?



[2] https://en.wikipedia.org/wiki/Strongly_connected_component

Comments:

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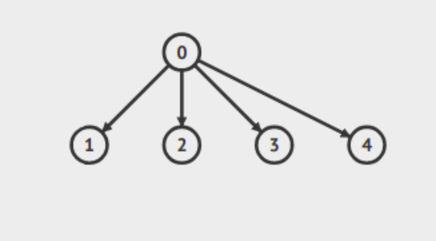

- We define another term (Strongly connected component)[2] and another harder algorithm is needed to check if the graph is strongly connected or not.

[2] https://en.wikipedia.org/wiki/Strongly_connected_component

Outline

Q1. A degree of a vertex is the number of other vertices connected to it. Show that the sum of all of the graph vertex degrees is always even.

- Assume that initially the graph has no edges (Summation of degree $= 0$ "even").
- For each new edge, the degree of two nodes will be increased by one.
- Thus the summation of the degree will increase by two (will still be even).

To be continued!