$\boxed{1}$

$\boxed{\text{sec 1}}$

Complex numbers
- Caretsian form
- polar form

$z = x + iy \longrightarrow$ Caretsian form

$x \longrightarrow$ Real part $(Re(z))$

$+y \longrightarrow$ imaginary part $(Im(z)) \longrightarrow$ كدا اي جمب ال i

$z = 3 + 5i$

$R(z) = 3$, $img(z) = +5$

Notes:

$\rightarrow i = \sqrt{-1}$, $i^2 = -1$

$\rightarrow i^3 = i^2 \cdot i = -1 \cdot i = -i$, $i^4 = i^3 \cdot i = -i \cdot i = -i^2 = 1$

or $i^4 = (i^2)^2 = (-1)^2 = 1$

$\longrightarrow i^5 = i^3 \cdot i^2 = -i \cdot (-1) = i$ or $i^5 = i^4 \cdot i = 1 \cdot i = i$

$\longrightarrow i^6 = i^4 \cdot i^2 = 1 \cdot -1 = -1$

$\longrightarrow$ [illegible] $i^{10} =$ # اشوف اقرب رقم يقبل القسمة على ال 4 وهو $\boxed{8}$

$10 - 8 = 2$

$i^{10} = i^2 = -1$

$\longrightarrow i^{26} =$ اقرب رقم يقبل القسمة على 4 هي $\boxed{24}$

$26 - 24 = 2$

$i^{26} = i^2 = -1$

→ $i^{15} =$, $i^{15} = i^{3} = -i$

$15 - 12 = 3$

Complex plane (Argand digram):

$z = x + iy$

$r = |z| = \sqrt{x^2 + y^2}$

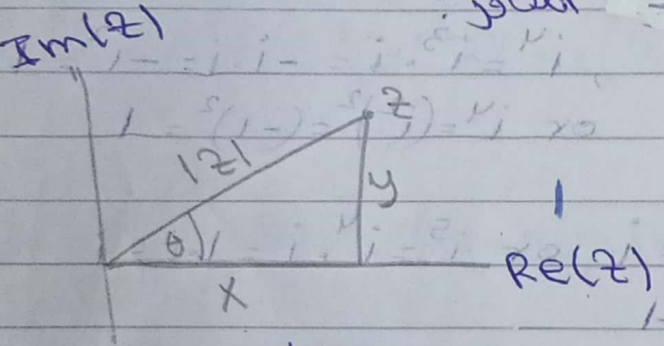
Real   Im.

img

$\angle z = \theta = \tan^{-1}\left(\frac{y}{x}\right)$

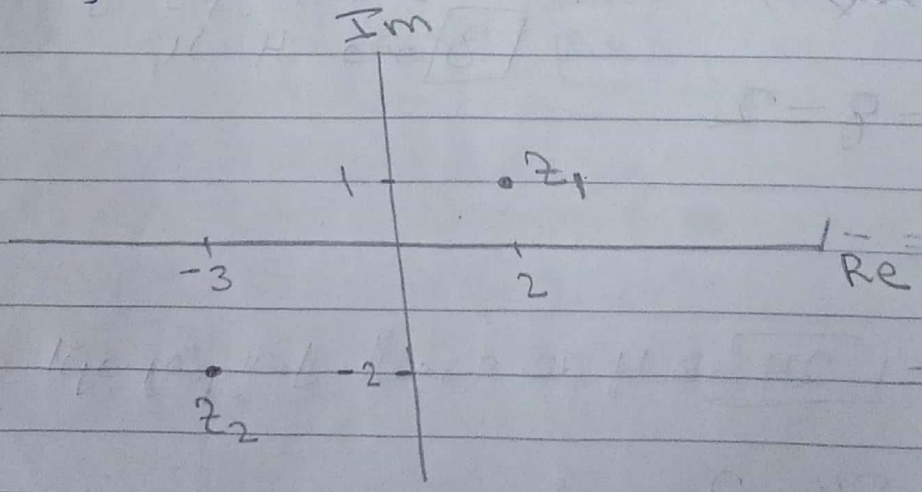
real

تمثيل الرقم علي المحاور:



EX: represent the complex numbers:

$z_1 = 2 + i$, $z_2 = -3 - 2i$



Addition and subtraction

$$z_1 \pm z_2 = (x_1 \pm x_2) + i(y_1 \pm y_2)$$

نجمع ال real مع ال real و ال imag مع ال img

ونفس الكلام في الطرح

Ex: $z_1 = 3+5i$, $z_2 = 4+2i$

a) $z_1 + z_2 = (3+4) + i(5+2) = 7+7i$

b) $z_2 - z_1 = (4-3) + i(2-5) = 1-3i$

$z_1 = 2+3i$, $z_2 = 3-i$

a) $z_1 + z_2 = (2+3) + i(3+(-1)) = 5+2i$

b) $z_1 - z_2 = (2-3) + i(3-(-1)) = -1+4i$

Notes:

$z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

$z_1 = z_2 \xrightarrow{\text{يعني}} x_1 = x_2 , \; y_1 = y_2$

$z_1 = 0 \longrightarrow x_1 = 0 \; \boxed{\text{and}} \; y_1 = 0$

multiplication and division

Ex: $z_1 = 2+3i$ $z_2 = 3-i$

a) $z_1 z_2 = (2+3i)(3-i)$

$$= 6 - 2i + 9i - 3\underbrace{i^2}_{-1} = 6+3-2i+9i = 9+7i$$

b) $\frac{z_1}{z_2} = \frac{2+3i}{3-i}$

في القسمة دائماً بنعمل ضرب في المرافق عشان نتخلص من المقام.

← المرافق ← بنشيل كل i ونحطها -i

$\frac{z_1}{z_2} = \frac{2+3i}{3-i} * \frac{3+i}{3+i}$

$= \frac{6+2i+9i+3i^2}{9+3i-3i-i^2} = \frac{3+11i}{10} = \frac{3}{10} + \frac{11}{10}i$

$Re\left(\frac{z_1}{z_2}\right) = \frac{3}{10}$ $Im\left(\frac{z_1}{z_2}\right) = \frac{11}{10}$

ضرب العدد في المرافق بتاعه دائماً بيطلع الناتج جزء Real بس

EX: Express the complex numbers in form a+ib

a) $\frac{(6+2i)-(1+3i)}{-1+i-2} = \frac{(6-1)+i(2-3)}{-3+i}$

$= \frac{5-i}{-3+i} * \frac{-3-i}{-3-i} = \frac{-15-5i+3i+i^2}{9+3i-3i-i^2}$

$= \frac{-16-2i}{10} = -\frac{16}{10} - \frac{2}{10}i = -\frac{8}{5} - \frac{1}{5}i$

c

$\boxed{5}$

$\cancel{b})$ $(1+i)^{20} = \left((1+i)^2\right)^{10} = (1+2i+\underbrace{\cancel{i^2}}_{-1})^{10} = (2i)^{10} = 2^{10} \cdot \underbrace{i^{10}}_{-1} = -2^{10}$

$i^{10} = i^{10-8} = i^2 = -1$

c) $\frac{1+i}{1-i} * \frac{1+i}{1+i} = \frac{1+\cancel{i}+\cancel{i}+\cancel{i^2}}{1+i-i-\underbrace{i^2}_{1-1}} = \frac{2i}{2} = i$

$\cancel{d})$ $(1+i)(i-2)(i+3)$

$= (i-2+\underbrace{i^2}_{-1}-2i)(i+3) = (-3-i)(i+3)$

$= -3i-9-\underbrace{i^2}_{-1}-3i = -8-6i$

$\cancel{e})$ $i^{26}-3i^7+i^6(1-i^3)-(-i)^{18}$

$= i^{26-24}-3i^{7-4}+i^{6-4}(1-(-i))-(-i)^{18-16}$

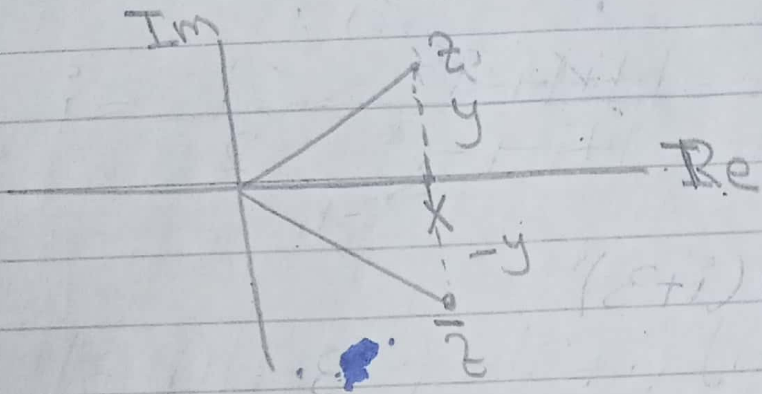
$= i^2-3i^3+i^2(1+i)-(-i)^2 = -1+3i+i^2+i^3-i^2$

$= -\cancel{1}+3i-1-i+\cancel{1} = -1+2i$

Complex Conjugation:

$z = x + iy \longrightarrow \bar{z} = x - iy$

نشيل كل i أو نحط بدلها -i



Notes:

1) $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$

2) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

3) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

4) $\bar{\bar{z}} = z$

$z = x + iy \qquad \bar{z} = x - iy$

$\rightarrow z + \bar{z} = (x + x) + i(y - y) = 2x$

$x = \frac{z + \bar{z}}{2}$

5) $Re(z) = \frac{z + \bar{z}}{2} \longrightarrow$ لو عايز نجيب الجزء $Re(z)$

$\to z-\bar{z} = (x-x)+i(y-(-y)) = i(2y)$

$y = \frac{z-\bar{z}}{2i}$

(b) $Im(z) = \frac{z-\bar{z}}{2i} \to Im(z)$ لو عاوز تجيب جزء

Ex: Find the conjugate of:

(a) $1+i \to 1-i$

(b) $(1+i)^{i} \to (1-i)^{-i}$

(c) $(i)^{2+i} \to (-i)^{2-i}$

(d) $(-i)^{2i} \to (-(-i))^{-2i} = (i)^{-2i}$

sheet 35 ص

(a) $(1+i)i(2-i) = (i+i^2)(2-i) = 2i - \underbrace{i^2}_{-1} + 2\underbrace{i^2}_{-1} - \underbrace{i^3}_{-i}$

$= 2i+1-2+i = 3i-1 = -1+3i$

c) $\frac{2+i}{2-i} * \frac{2+i}{2+i} = \frac{4+2i+2i+\cancel{i^2}^{-1}}{4+2i-2i-\cancel{i^2}_{-1}} = \frac{3+4i}{5}$

$= \frac{3}{5} + \frac{4}{5} i$

d) $\frac{1+i}{i} + \frac{i}{1-i}$

$\frac{1+i}{i} * \frac{-i}{-i} = \frac{-i-\cancel{i^2}^{-1}}{-i^2} = \frac{\cancel{1}-i}{1} = 1-i$

$\frac{i}{1-i} * \frac{1+i}{1+i} = \frac{i+i^2}{1+i-i-i^2} = \frac{-1+i}{2} = -\frac{1}{2} + \frac{1}{2} i$

$1-i + \frac{-1}{2} + \frac{1}{2} i = \left(1+\left(-\frac{1}{2}\right)\right) + i\left(-1+\frac{1}{2}\right) = \frac{1}{2} - \frac{1}{2} i$

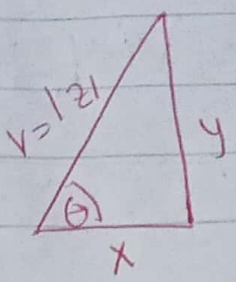
g) $(1+i)^{100} = \left((1+i)^2\right)^{50} = (1+2i+\cancel{i^2}_{-1})^{50} = (2i)^{50}$

$2^{50} i^{50} = 2^{50} i^{50-48} = 2^{50} i^{2} = -2^{50}$

Modulus of complex number (Absolute value)

$z = x + iy$

$r = |z| = \sqrt{\underbrace{x^2}_{\text{real}} + \underbrace{y^2}_{\text{img}}}$



properties:

1) $|z_1 z_2| = |z_1|\,|z_2|$

2) $|z| \geq 0 \rightarrow$ الناتج يكون دائما (+)

$|z| = 0$ if $z = 0 \rightarrow x = 0$ and $y = 0$

3) $|z| = |\bar{z}| = r = \sqrt{x^2+y^2}$

$\bar{z} = x - iy \rightarrow r = \sqrt{x^2+(-y)^2} = \sqrt{x^2+y^2}$

4) $z\bar{z} = (x+iy)(x-iy) = x^2 - \cancel{ixy} + \cancel{iyx} - \underbrace{i^2}_{-1} y^2 = x^2+y^2$

$z\bar{z} = |z|^2$

5) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

6) $|z_1 - z_2| = |z_2 - z_1|$

$$|z_1 - z_2| = |(x_1 + iy_1) - (x_2 + iy_2)| = |(x_1 - x_2) + i(y_1 - y_2)|$$

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

وهي تساوي $\leftarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

7) $|z_1 + z_2| \leq |z_1| + |z_2|$ (Proof)

8) $|z_1 - z_2| \geq \big||z_1| - |z_2|\big|$ (Proof)

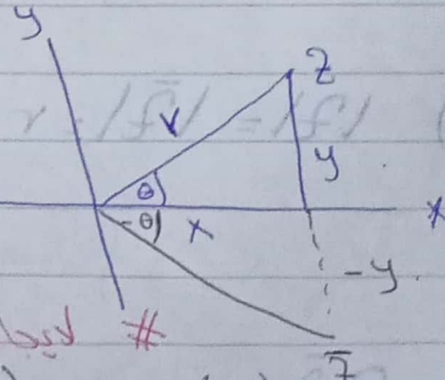
Argument of complex number:

$z = x + iy$

$\arg(z) = \theta \quad , \quad |z| = r$

$\tan\theta = \frac{\text{المقابل}}{\text{المجاور}} = \frac{y}{x} \rightarrow \theta = \tan^{-1}\left|\frac{y}{x}\right|$

اعتماداً على المحور



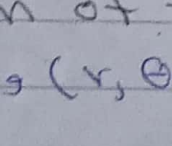
لإيجاد r او |z| :

$r = \sqrt{x^2 + y^2} \longrightarrow$ دايماً "+ve" , Note: $\arg(\bar{z}) = -\arg(z)$

Polar form of z:

في الـ polar تكون الاحداثيات (r, θ) ومطلوب كتابة z في صورة (r, θ) :

$z = x + iy$



$\cos\theta = \frac{x}{r} \longrightarrow x = r\cos\theta$

$\sin\theta = \frac{y}{r} \longrightarrow y = r\sin\theta$

$z = r\cos\theta + ir\sin\theta$

$= r[\cos\theta + i\sin\theta]$

$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$

$\therefore z = re^{i\theta} \longrightarrow$ in polar form

$r = \sqrt{x^2 + y^2} \quad , \quad \theta = \tan^{-1}\left|\frac{y}{x}\right|$

$z = r[\cos\theta + i\sin\theta] = re^{i\theta}$

in polar form