**Question 1: K-means Clustering for 8 Examples into 3 Clusters**

**Data Points**: A1=(2,10), A2=(2,5), A3=(8,4), A4=(5,8), A5=(7,5), A6=(6,4), A7=(1,2), A8=(4,9)

**Solution Method**:

* **Centroids**:
  + C1 = A1 = (2,10)
  + C2 = A3 = (8,4)
  + C3 = A7 = (1,2)
* **Iteration 1**:
  + Compute Euclidean distances from each point to each centroid:
    - For A1=(2,10): Dist(C1)=0, Dist(C2)=√((2-8)²+(10-4)²)=8.49, Dist(C3)=√((2-1)²+(10-2)²)=8.06 → Assign to C1
    - For A2=(2,5): Dist(C1)=√((2-2)²+(5-10)²)=5, Dist(C2)=√((2-8)²+(5-4)²)=6.08, Dist(C3)=√((2-1)²+(5-2)²)=3.16 → Assign to C3
    - For A3=(8,4): Dist(C1)=√((8-2)²+(4-10)²)=8.49, Dist(C2)=0, Dist(C3)=√((8-1)²+(4-2)²)=7.28 → Assign to C2
    - For A4=(5,8): Dist(C1)=√((5-2)²+(8-10)²)=3.61, Dist(C2)=√((5-8)²+(8-4)²)=5, Dist(C3)=√((5-1)²+(8-2)²)=7.21 → Assign to C1
    - For A5=(7,5): Dist(C1)=√((7-2)²+(5-10)²)=7.07, Dist(C2)=√((7-8)²+(5-4)²)=1.41, Dist(C3)=√((7-1)²+(5-2)²)=6.71 → Assign to C2
    - For A6=(6,4): Dist(C1)=√((6-2)²+(4-10)²)=7.21, Dist(C2)=√((6-8)²+(4-4)²)=2, Dist(C3)=√((6-1)²+(4-2)²)=5.39 → Assign to C2
    - For A7=(1,2): Dist(C1)=√((1-2)²+(2-10)²)=8.06, Dist(C2)=√((1-8)²+(2-4)²)=7.28, Dist(C3)=0 → Assign to C3
    - For A8=(4,9): Dist(C1)=√((4-2)²+(9-10)²)=2.24, Dist(C2)=√((4-8)²+(9-4)²)=6.40, Dist(C3)=√((4-1)²+(9-2)²)=7.62 → Assign to C1
  + Clusters:
    - Cluster 1 (C1): {A1, A4, A8}
    - Cluster 2 (C2): {A3, A5, A6}
    - Cluster 3 (C3): {A2, A7}
  + Update Centroids:
    - Cluster 1: Mean of (2,10), (5,8), (4,9) = ((2+5+4)/3, (10+8+9)/3) = (3.67, 9)
    - Cluster 2: Mean of (8,4), (7,5), (6,4) = ((8+7+6)/3, (4+5+4)/3) = (7, 4.33)
    - Cluster 3: Mean of (2,5), (1,2) = ((2+1)/2, (5+2)/2) = (1.5, 3.5)
    - New centroids: C1=(3.67,9), C2=(7,4.33), C3=(1.5,3.5)
* **Iteration 2**:
  + Recalculate distances and reassign points:
    - A1: Dist(C1)=√((2-3.67)²+(10-9)²)=1.67, Dist(C2)=√((2-7)²+(10-4.33)²)=7.43, Dist(C3)=√((2-1.5)²+(10-3.5)²)=6.50 → C1
    - A2: Dist(C1)=√((2-3.67)²+(5-9)²)=5.27, Dist(C2)=√((2-7)²+(5-4.33)²)=5.05, Dist(C3)=√((2-1.5)²+(5-3.5)²)=1.50 → C3
    - A3: Dist(C1)=√((8-3.67)²+(4-9)²)=6.66, Dist(C2)=√((8-7)²+(4-4.33)²)=1.20, Dist(C3)=√((8-1.5)²+(4-3.5)²)=6.52 → C2
    - A4: Dist(C1)=√((5-3.67)²+(8-9)²)=1.62, Dist(C2)=√((5-7)²+(8-4.33)²)=4.04, Dist(C3)=√((5-1.5)²+(8-3.5)²)=5.70 → C1
    - A5: Dist(C1)=√((7-3.67)²+(5-9)²)=5.37, Dist(C2)=√((7-7)²+(5-4.33)²)=0.75, Dist(C3)=√((7-1.5)²+(5-3.5)²)=5.70 → C2
    - A6: Dist(C1)=√((6-3.67)²+(4-9)²)=5.38, Dist(C2)=√((6-7)²+(4-4.33)²)=1.20, Dist(C3)=√((6-1.5)²+(4-3.5)²)=4.53 → C2
    - A7: Dist(C1)=√((1-3.67)²+(2-9)²)=7.42, Dist(C2)=√((1-7)²+(2-4.33)²)=6.20, Dist(C3)=√((1-1.5)²+(2-3.5)²)=1.80 → C3
    - A8: Dist(C1)=√((4-3.67)²+(9-9)²)=0.36, Dist(C2)=√((4-7)²+(9-4.33)²)=5.46, Dist(C3)=√((4-1.5)²+(9-3.5)²)=6.10 → C1
  + Clusters remain the same:
    - Cluster 1: {A1, A4, A8}
    - Cluster 2: {A3, A5, A6}
    - Cluster 3: {A2, A7}
  + Update Centroids: Same as previous, indicating convergence.
* **Final Clusters**:
  + Cluster 1: {A1=(2,10), A4=(5,8), A8=(4,9)}
  + Cluster 2: {A3=(8,4), A5=(7,5), A6=(6,4)}
  + Cluster 3: {A2=(2,5), A7=(1,2)}

**Question 2: K-means Clustering for 7 Individuals into 3 Clusters**

**Data Points**:

|  |  |  |
| --- | --- | --- |
| **Subject** | **A** | **B** |
| 1 | 1.0 | 1.0 |
| 2 | 1.5 | 2.0 |
| 3 | 3.0 | 4.0 |
| 4 | 5.0 | 7.0 |
| 5 | 3.5 | 5.0 |
| 6 | 4.5 | 5.0 |
| 7 | 3.5 | 4.5 |

**Solution Method**:

* **Centroids**:
  + C1 = S1 = (1.0,1.0)
  + C2 = S4 = (5.0,7.0)
  + C3 = S5 = (3.5,5.0)
* **Iteration 1**:
  + Compute Euclidean distances from each point to each centroid:
    - For S1=(1.0,1.0): Dist(C1)=0, Dist(C2)=√((1-5)²+(1-7)²)=7.21, Dist(C3)=√((1-3.5)²+(1-5)²)=4.72 → Assign to C1
    - For S2=(1.5,2.0): Dist(C1)=√((1.5-1)²+(2-1)²)=1.12, Dist(C2)=√((1.5-5)²+(2-7)²)=6.10, Dist(C3)=√((1.5-3.5)²+(2-5)²)=3.61 → Assign to C1
    - For S3=(3.0,4.0): Dist(C1)=√((3-1)²+(4-1)²)=3.61, Dist(C2)=√((3-5)²+(4-7)²)=3.61, Dist(C3)=√((3-3.5)²+(4-5)²)=1.12 → Assign to C3
    - For S4=(5.0,7.0): Dist(C1)=√((5-1)²+(7-1)²)=7.21, Dist(C2)=0, Dist(C3)=√((5-3.5)²+(7-5)²)=2.83 → Assign to C2
    - For S5=(3.5,5.0): Dist(C1)=√((3.5-1)²+(5-1)²)=4.72, Dist(C2)=√((3.5-5)²+(5-7)²)=2.83, Dist(C3)=0 → Assign to C3
    - For S6=(4.5,5.0): Dist(C1)=√((4.5-1)²+(5-1)²)=5.41, Dist(C2)=√((4.5-5)²+(5-7)²)=2.24, Dist(C3)=√((4.5-3.5)²+(5-5)²)=0.71 → Assign to C3
    - For S7=(3.5,4.5): Dist(C1)=√((3.5-1)²+(4.5-1)²)=4.53, Dist(C2)=√((3.5-5)²+(4.5-7)²)=3.04, Dist(C3)=√((3.5-3.5)²+(4.5-5)²)=0.50 → Assign to C3
  + Clusters:
    - Cluster 1 (C1): {S1, S2}
    - Cluster 2 (C2): {S4}
    - Cluster 3 (C3): {S3, S5, S6, S7}
  + Update Centroids:
    - Cluster 1: Mean of (1.0,1.0), (1.5,2.0) = ((1.0+1.5)/2, (1.0+2.0)/2) = (1.25, 1.5)
    - Cluster 2: (5.0, 7.0) (unchanged, single point)
    - Cluster 3: Mean of (3.0,4.0), (3.5,5.0), (4.5,5.0), (3.5,4.5) = ((3.0+3.5+4.5+3.5)/4, (4.0+5.0+5.0+4.5)/4) = (3.625, 4.625)
    - New centroids: C1=(1.25,1.5), C2=(5.0,7.0), C3=(3.625,4.625)
* **Iteration 2**:
  + Recalculate distances and reassign points:
    - S1: Dist(C1)=√((1.0-1.25)²+(1.0-1.5)²)=0.56, Dist(C2)=√((1.0-5.0)²+(1.0-7.0)²)=7.02, Dist(C3)=√((1.0-3.625)²+(1.0-4.625)²)=4.30 → C1
    - S2: Dist(C1)=√((1.5-1.25)²+(2.0-1.5)²)=0.50, Dist(C2)=√((1.5-5.0)²+(2.0-7.0)²)=5.83, Dist(C3)=√((1.5-3.625)²+(2.0-4.625)²)=3.13 → C1
    - S3: Dist(C1)=√((3.0-1.25)²+(4.0-1.5)²)=3.25, Dist(C2)=√((3.0-5.0)²+(4.0-7.0)²)=3.61, Dist(C3)=√((3.0-3.625)²+(4.0-4.625)²)=0.74 → C3
    - S4: Dist(C1)=√((5.0-1.25)²+(7.0-1.5)²)=6.96, Dist(C2)=0, Dist(C3)=√((5.0-3.625)²+(7.0-4.625)²)=2.66 → C2
    - S5: Dist(C1)=√((3.5-1.25)²+(5.0-1.5)²)=4.30, Dist(C2)=√((3.5-5.0)²+(5.0-7.0)²)=2.83, Dist(C3)=√((3.5-3.625)²+(5.0-4.625)²)=0.46 → C3
    - S6: Dist(C1)=√((4.5-1.25)²+(5.0-1.5)²)=4.92, Dist(C2)=√((4.5-5.0)²+(5.0-7.0)²)=2.24, Dist(C3)=√((4.5-3.625)²+(5.0-4.625)²)=0.56 → C3
    - S7: Dist(C1)=√((3.5-1.25)²+(4.5-1.5)²)=4.07, Dist(C2)=√((3.5-5.0)²+(4.5-7.0)²)=3.04, Dist(C3)=√((3.5-3.625)²+(4.5-4.625)²)=0.25 → C3
  + Clusters remain the same:
    - Cluster 1: {S1, S2}
    - Cluster 2: {S4}
    - Cluster 3: {S3, S5, S6, S7}
  + Update Centroids: Same as previous, indicating convergence.
* **Final Clusters**:
  + Cluster 1: {S1=(1.0,1.0), S2=(1.5,2.0)}
  + Cluster 2: {S4=(5.0,7.0)}
  + Cluster 3: {S3=(3.0,4.0), S5=(3.5,5.0), S6=(4.5,5.0), S7=(3.5,4.5)}

Q3:

A paper with writing on it

AI-generated content may be incorrect.