

## ← Problems in Combinatorics

Quiz, 7 questions



1. Alice has 7 different textbooks. She would like to lend three books to Bob for a weekend. How many ways does she have to do it?

1 / 1  
point

35

### Correct Response

That's right! Alice has to pick 3 books out of 7 books she has. The order does not matter and she cannot pick the same book twice. So we have combinations of size 3 out of 7 objects. There are  $7 \times 6 \times 5 / (3 \times 2 \times 1) = 7 \times 5 = 35$  of them.



2. Alice has 7 textbooks and Bob has 5 textbooks. All textbooks are different. They would like to exchange three books each between each other for a weekend. That is, Alice gives Bob three of her books and Bob gives Alice three of his books. How many ways do they have to do it?

1 / 1  
point

350

### Correct Response

This is correct. When each of them pick three books among their  $N$  books ( $N=7$  for Alice and  $N=5$  for Bob), we are dealing with combinations: unordered selection without repetitions. Thus Alice can pick three books in  $7 \times 6 \times 5 / (3 \times 2 \times 1) = 7 \times 5 = 35$  ways and Bob can pick three books in  $5 \times 4 \times 3 / (3 \times 2 \times 1) = 10$  ways. Since we want *both* Alice and Bob to choose books, we are dealing with pairs of choices, so we have to multiply variants for both people.



3. Five married couples are planning a barbecue. They need to pick three couples who will be responsible for bringing food. How many ways do they have to do it?

1 / 1  
point

10

### Correct Response

This is correct. We pick three couples out of five without repetitions and order. These are combinations of size three out of five options.



4. Five married couples are planning a barbecue. They need to hold a meeting dedicated to the planning. The meeting should consist of five people, one from each couple. How many possible ways do they have to pick people for the meeting?

1 / 1  
point

32

### Correct Response

This is correct! Indeed, in each couple they have to pick one person. There are two ways to do this. Since this have to be repeated five times separately, we have to apply the product rule and multiply 2 by itself five times. Note, that we are dealing with tuples here: we pick a sequence of length 5, where each position in the sequence correspond to one of the couple, and we have two options for each position, two members of the couple.



5. Five married couples gathered for a barbecue. They need to pick three people among them who will be responsible for preparing the table. But they do not want to pick two people from the same couple for this (this would not be fair). How many ways do they have to pick three people satisfying this requirement?

1 / 1  
point

80

### Correct Response

This is correct! Indeed, due to the restriction we have to pick three persons from different couples. For this we first have to pick three couples, from which we are going to choose people for our task. And then we have to pick one person from each of the three couples. We can make the first choice in 10 ways (compare with Question 3 of

the test) and then we can pick one person in each of the three couples. we can do it in 8 ways (compare with Question 4 of this test). Each pair of choices on two stages above result in a different pick of people to prepare the table. So by the product rule in total we have  $10 \times 8 = 80$  possibilities.



6. In a 6 number lottery one is trying to guess an unordered subset of 6 numbers among 44 numbers without repetitions. For this one picks 6 numbers out of 44 himself. How many ways are there to do this? You can use wolfram alpha to compute the exact number.

1 / 1  
point

7059052

**Correct Response**

This is correct! We have to pick a subset of size 6 out of 44 options. These are combinations and the answer is a binomial coefficient.



7. In a 6 number lottery one is trying to guess an unordered subset of 6 numbers among 44 numbers without repetitions. After the lottery the organisers decided to count how many possible ways are there to guess correctly exactly three numbers. What is the answer to this question? You can use wolfram alpha to compute the exact number.

1 / 1  
point

168720

**Correct Response**

This is correct! Since the lottery has already happened, the set of winning numbers is fixed. So to guess exactly three winning numbers one has to pick three numbers among 6 winning numbers and pick 3 other numbers among remaining  $44 - 6 = 38$  numbers. Choices for both cases can be computed as binomial coefficients and then we have to multiply the results by the product rule.

Also note that if we divide this number by the answer to the previous problem we get approximately 0.024. As we will discuss next week this is *the probability* to guess exactly three numbers correctly, that is the chances are 2.4%.