Math 202 Assignment #1 - Spring 2019

Due on Tuesday 22nd at the beginning of tutorial

Never leave your answer or work with decimals unless asked to. Leave your answers exact and all functions fully evaluated if possible. Show all work in order to receive full credit. Use a combination of English and Mathematical language to describe the solution to a problem.

Review of Math 100/109 and Math 101

Each question is out of 2 marks, making this section out of 10 marks.

1. Find and classify (as relative min, max or neither) all critical points of $f(x) = \sqrt{3}\cos(x) + \sin(x)$ over $[0, 2\pi]$. *Hint: There are two critical points. Also,* $\tan(x) = \sin(x)/\cos(x)$

2. A $216m^2$ rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence?

   Hint: Draw a picture of a rectangle with a line right through the middle. This will help set up the overall perimeter.

3. Solve the initial value problem (differential equation with initial condition)

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1-x^2}}, \qquad y(0) = -1$$

   Hint: It's separable.

4. Find the domain and range of $f(x) = x\arcsin(x - \pi)$.

   Hint: The domain and range is mainly restricted by the arcsin function. How is it further affected by the shift and scaling?

5. Find the equation of the tangent line to $x^2 + xy - y^2 = 1$ at the point $P(2, 3)$.

   Hint: Use implicit differentiation.

New Material

Each question is out of 7 marks, making this section out of 35 marks.

1. Consider the vectors $\mathbf{u} = \langle 4, 10, -8\rangle$ and $\mathbf{w} = \langle 2, -4, -6\rangle$. Any vector can be decomposed as the sum $\mathbf{u} = \mathbf{u}_{||} + \mathbf{u}_{\perp}$ where $\mathbf{u}_{||}$ is parallel to $\mathbf{w}$ and $\mathbf{u}_{\perp}$ is perpendicular to $\mathbf{w}$. Compute this decomposition. *Hint:* $Proj_{\mathbf{w}}\mathbf{u} = \mathbf{u}_{||}$ *and then solve to compute* $\mathbf{u}_{\perp} = \mathbf{u} - \mathbf{u}_{||}$.

2. Using the triple scalar product, find the volume of the parallelepiped with adjacent edges PQ, PR, and PS where $P = (1, 1, 1)$, $Q = (2, 0, 3)$, $R = (4, 1, 7)$, and $S = (3, -1, -2)$.

3. The plane $6x + 3y + 4z = 12$ forms a triangle when graphed in the first octant. Find the surface area of the tetrahedron bounded by this plane in the first octant. *Hint: We covered graphing a plane in the first octant in lecture.*

4. Find the parametric equations for the line of intersection of the planes $x + y + z = 5$ and $3x - y = 4$.

5. Find the scalar equation of the plane that contains the lines L_1: $x(t) = 2 + 2t$, $y(t) = -4 + 3t$, $z(t) = 2 - 5t$ and L_2: $x(s) = 3 + 4s$, $y(s) = -4 + 6s$, $z(s) = 5 - 10s$.