

# Lecture 3

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## 1 Sample Spaces

The set of all possible outcomes of a random experiment is called a *sample space*

## 2 Events

An event  $E$ , is a set of some outcomes of a probability experiment, i.e. a subset of the sample

$$E \subseteq S$$

If an event  $E$  contains no outcomes, then  $E$  is an impossible event.

## 3 Probability

$$P(E) = \frac{N(E)}{N(S)}$$

$$0 \leq P(E) \leq 1; P(S) = 1$$

**Example 3.1.** A card is drawn from a standard deck. Find the probabilities of the following events.

- Getting a queen.
- Getting a club.
- Getting a number.

**Solution 3.1.**

$$|S| = 52$$

$$E1 \text{ is the event of getting a queen, } P(E1) = \frac{|E1|}{|S|} = \frac{4}{52} = \frac{1}{13}$$

$$E2 \text{ is the event of getting a club, } P(E2) = \frac{|E2|}{|S|} = \frac{13}{52} = \frac{1}{4}$$

$$E3 \text{ is the event of getting a number, } P(E3) = \frac{|E3|}{|S|} = \frac{40}{52} = \frac{10}{13}$$

## 4 Counting Rules

### 4.1 Multiplication Rule

In a sequence of two experiments, if the first experiment can occur in  $m$  different ways and the second one can occur in  $n$  different ways, then the whole sequence can occur in  $m \times n$  different ways.

**Example 4.1.** Consider the manufacturing of: number-plates consisting of two letters followed by four digits

1. How many plates are possible?

$$26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6760000$$

2. How many plates are possible, if no letter or digit can be repeated?

$$26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3276000$$

**Example 4.2.** In a class of 10 students, 6 are to be chosen and seated in a row for a picture. How many different pictures are possible?

$$10 \times 9 \times 8 \times 7 \times 6 \times 5 \text{ different pictures.}$$

**Tip** what if they were to sit in a circle?

### 4.2 Permutation

An arrangement of  $n$  objects A SPECIFIC ORDER using  $k$  objects at a time is called a permutation and it is denoted by  ${}^nP_k$ . The number of repetition-free permutations (linear arrangements) of size  $k$  from a set of  $n$  distinct objects is given by:

$${}^nP_k = \frac{n!}{(n-k)!}; \quad 0 \leq k \leq n$$

### 4.3 Combination

A selection of  $k$  distinct objects WITHOUT REGARD TO ORDER out of  $n$  objects is called a combination and it is denoted by  ${}^nC_k$ . The number of repetition-free combinations of size  $k$  from  $n$  distinct objects is given:

$${}^nC_k = \frac{{}^nP_k}{k!} = \frac{n!}{(n-k)!k!}; \quad 0 \leq k \leq n$$

## 4.4 Problems

### Example 4.3.

**Example 4.4.** In a class of 10 students, three are to be chosen to represent the class in a competition. How many selections are possible?

Note that, here, the students are not selected in any specific order.

The number of selections is  $^{10}C_3$

**Example 4.5.** A student is taking a Math-401 test in which 7 questions out of 10 are to be answered. In how many ways can the student answer the exam if :

1. Any 7 questions may be selected.

The student can answer the exam in  $^{10}C_7$

2. The first 2 questions must be selected.

The student can answer the exam in  $^8C_5$

3. The student must choose 3 questions from the first 5 and 4 questions from the

The student can answer the exam in  $^5C_3 \times ^5C_4 = 50$

**Example 4.6.** What is the number of (possibly meaningless) words that are made up of all the letters in the word **chemistry**

The number of such words is  $^9P_9$

**Example 4.7.** What is the number of permutations of the letters in the word **ball**?

Note that there are repeated letter so it's not 4 distinct objects.

The number of permutations is  $\frac{^4P_4}{2!} = 12$

**Example 4.8.** What is the number of permutations of the letters in the word **Pepper**?

The number of permutations is  $\frac{^6P_6}{2!3!}$

**Example 4.9.** The owner of a pizzeria prepares every pizza by always combining 4 different ingredients. How many ingredients does he need, at least, if he would like to offer 30 different pizzas in the menu?

we need to find  $n$  such that  $^nC_4 = 30$

$$\frac{n!}{(n-4)!4!} = 30$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 30 \times 4!$$

$$n(n-1)(n-2)(n-3) - 720 = 0$$

By solving the equation  $n = 6.8 \Rightarrow$  he needs at least 7 ingredients

**Example 4.10.** You play a simple card game. You draw 3 cards. If any of them is a King or the three cards are of the same suit, you win otherwise you lose. How many hands are you winning?

A = draw at least one king in a hand of 3 cards

$$N(A) = \text{One King or Two or Three} = \\ ({}^4C_1 \times {}^{48}C_2) + ({}^4C_2 \times {}^{48}C_1) + ({}^4C_3 \times {}^{48}C_0)$$

B = A hand draw 3 cards with same suit

$$N(B) = {}^4P_1 \times {}^{13}P_3$$

$A \cap B$  = a hand with draw 3 cards with a king and the same suit.

$$N(A \cap B) = {}^4C_1 \times {}^1C_1 {}^{12}C_2$$

$$N(A \cup B) = N(A) + N(B) - N(A \cap B) = 5684.$$

**Example 4.11.** In how many ways you can arrange the word “ORANGE”, if:

- 2 vowels and 2 consonants are used to make 4-letter words.

$$\text{No.of ways} = {}^3C_2 \times {}^3C_2 \times 4!$$

- 2 vowels and 3 consonants are used to make 5-letter words.

$$\text{No.of ways} = {}^3C_2 \times {}^3C_3 \times 5!$$

**Example 4.12.** There are 8 men and 9 women on a committee selection pool. A committee consisting of: a president, a vice-president, and 3 coordinators; is to be formed. In how many ways can exactly three women be on the committee?

$$N = {}^9C_3 \times {}^8C_2 \times \left[ \frac{5!}{3!} \right]$$