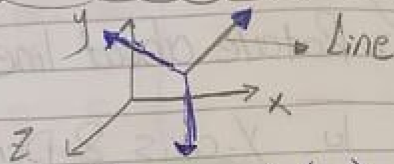
Section [3]

★ Rotation around line in The Space

Method



1) Translate To Given point (−)
2)- Generate New system (u, v, w) on old system using line Direction

$u \; v \; w$
$\downarrow \; \downarrow \; \downarrow$
$X \; Y \; Z$

- if The Direction are Given Directly Assume u or v or w The Direction

- if we have 2 points

★ Direction = $P_2 - P_1$

① - Normalize $V(a, b, c)$

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \quad \frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$\frac{c}{\sqrt{a^2+b^2+c^2}}$$

- $u = (a, b, c)$
- $v = (-b, a, 0)$
- $w = u \times v$

Ex:
$u = (2, 1, 1)$
$v = (-1, 2, 0)$

2: $u \cdot v = 0$

Dot product
(ضرب عادي)

$u = (a, b, c)$
$v = (-b, a, 0)$

$$\boxed{-ab + ab + 0 = 0}$$

3 $W = u \times v$ Cross product

$$W = \begin{array}{c} u \\ v \end{array} \begin{bmatrix} a & b & c \\ -b & a & 0 \end{bmatrix}$$

علشان نجيب قيمة عنصر بنحذف العمود الاول
ونضرب العمودين الباقيين مقص (cross)

Ex
$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & 0 \end{bmatrix} = (-1, 2, 5)$$

$u = (1, 2, 1)$

$v = (-2, 1, 0)$

$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

$W = u \times v$ Cross product

• هنحذف اول عمود

$(2 * 0) - (1 * 1) = -1$

هنحذف العمود التاني

$(1 * 0) - (1 * -2) = 2$

القيمه دى لتكون سالب

$(+ * -) = -$ لو الناتج طلع +

$(- * -) = +$ لو طلع -

هنحذف العمود التالت

$(1 * 1) - (2 * -2) = 1 + 4 = 5$

$\therefore W = (-1, -2, 5)$

Ex: $u = (3, 1, 5)$

$v = (-1, 3, 0)$

$W = u \times v$

$$\begin{bmatrix} 3 & 1 & 5 \\ -1 & 3 & 0 \end{bmatrix}$$

نحذف العمود الاول

$(1 \times 0) - (3 \times 5) = -15$

نحذف العمود الثاني

$(3 \times 0) - (-1 \times 5) = 5$

-5 نقلب المكونه

نحذف العمود الثالث

$(3 \times 3) - (-1 \times 1) = 10$

$$W = (-15, -5, 10)$$

3) Rotat New system to old system

$$R_1 = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4) if we conside The line U Rotate about X-axis

if $v \longrightarrow$ Rotate about y-axis
if $w \longrightarrow$ Rotate about z-axis

R_2

5) Rotate from old system to New system

$R_3 = R_1^T \longrightarrow$ Transpose دوران المصفوفة

6) Translate To Given point (+)

Final Matrix: $(((T_2 * R_3) * R_2) * R_1) * T_1$

Ex) Rotate about line having The Direction (2, 1, 1) & passing through (4, 1, 6)

- 1) Translate To (-4, -1, -6)

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & \boxed{-4} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- 2) Generate new system (u, v, ω)

Note: let $u = (2, 1, 1)$ | $u = (a, b, c)$, $v = (-b, a, 0)$

$u \cdot v = 0$ & $v = (-1, 2, 0)$

$W = u \times v$

$$W = \begin{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix}$$

$W = (-2, -1, 5)$.

- Normalize u, v, w

$$\frac{a}{\sqrt{a^2+b^2+c^2}}$$

$u = (2, 1, 1)$

$$\frac{2}{\sqrt{-2^2+1^2+1^2}} = \frac{2}{\sqrt{6}}$$

$$\frac{1}{\sqrt{-2^2+1^2+1^2}} = \frac{1}{\sqrt{6}}$$

$$\frac{1}{\sqrt{-2^2+1^2+1^2}} = \frac{1}{\sqrt{6}}$$

$$\therefore \left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

$v = (-1, 2, 0)$ | $\frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0$

$\therefore \left(\frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right)$

$w = (-2, -1, 5)$

$$\frac{-2}{\sqrt{2^2+(-1)^2+(5)^2}} = \frac{-2}{\sqrt{30}}, \frac{-1}{\sqrt{30}}, \frac{5}{\sqrt{30}}$$

$\therefore \left(\frac{-2}{\sqrt{30}}, \frac{-1}{\sqrt{30}}, \frac{5}{\sqrt{30}} \right)$

- Rotate to New Syste.

$$R_1 = \begin{array}{c} \\ u \\ v \\ w \\ 1 \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & 1 \end{array} \\ \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 & 0 \\ \frac{-2}{\sqrt{30}} & \frac{-1}{\sqrt{30}} & \frac{5}{\sqrt{30}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Note (3) we let u Direction

The Rotate about X-axis.

(11)

Ex

Rotate about line passing Through $(3,2,1)$ & $(5,6,7)$ $(\theta = 90^\circ)$

1) Translate about one of Two points $(-3,-2,-1)$

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2) Create New system

1. Direction = $P_2 - P_1$

النقطة الثانية ← P_2 ، P_1 → النقطة الاولى

Direction = $(5,6,7) - (3,2,1)$

$= (2,4,6)$

let $u = (2,4,6)$

$v = (-4,2,0)$

$$w = \begin{bmatrix} 2 & 4 & 6 \\ -4 & 2 & 0 \end{bmatrix} = (-12, -24, 20)$$

* Normalize u, v, w

$$u = \left(\frac{2}{\sqrt{56}}, \frac{4}{\sqrt{56}}, \frac{6}{\sqrt{56}}\right)$$

$$v = \left(\frac{-4}{\sqrt{20}}, \frac{2}{\sqrt{20}}, 0\right)$$

$$w = \left(\frac{12}{\sqrt{1120}}, \frac{-24}{\sqrt{1120}}, \frac{-20}{\sqrt{1120}}\right)$$

* Rotate To New System

u, v, w

$$R_1 = \begin{bmatrix} \frac{2}{\sqrt{50}} & \frac{4}{\sqrt{50}} & \frac{6}{\sqrt{50}} & 0 \\ \frac{-4}{\sqrt{20}} & \frac{2}{\sqrt{20}} & 0 & 0 \\ \frac{12}{\sqrt{1120}} & \frac{24}{\sqrt{1120}} & \frac{-20}{\sqrt{1120}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3) We let $U \rightarrow$ Direction so Rotate about X-axis

$\cos 90 = 0 \quad \sin 90 = 1$

$$R_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4) Rotate From old To New System

$$R_3 = R_1^T$$

$$R_3 \begin{bmatrix} \frac{2}{\sqrt{56}} & \frac{-4}{\sqrt{20}} & \frac{12}{\sqrt{1120}} & 0 \\ \frac{4}{\sqrt{56}} & \frac{2}{\sqrt{20}} & \frac{-24}{\sqrt{1120}} & 0 \\ \frac{6}{\sqrt{56}} & 0 & \frac{-20}{\sqrt{1120}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5) Translate To (3, 2, 1)

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Matrix: $T_2 * R_3 * R_2 * R_1 * T_1$

Ex Rotate about line passing through

$(2, 4, 6), (3, 5, 7)$ $(\theta = 45^\circ)$

Rotate the following polygon about line passing through $(3, 2, 1)$ with $\theta = 90^\circ$ & having the direction $(4, 5, 6)$

Polygon: $(5, 55, 25), (50, 45, 15)$

$(60, 55, 35), (70, 40, 5)$