

# Automata and Language Theory

## Chapter 3(Context Free Grammar)

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# Chapter 3- Context Free Grammar

## How to Construct Grammar ???

**Example 1:** Construct the grammar for the set of strings over  $\{a, b\}$  that contain exactly 2b's.

$a^* b a^* b a^*$

**$G:S \rightarrow AbAbA$**

**$A \rightarrow aA / \lambda$**

## Chapter 3- Context Free Grammar

**Example 2:** Construct the grammar for the set of strings over  $\{a, b\}$  that contain substring  $bb$ .

$$(a \cup b)^* bb (a \cup b)^*$$

$$G: S \rightarrow A b b A$$

$$A \rightarrow a A / b A / \lambda$$

# Chapter 3- Context Free Grammar

Remark:

$a^*$



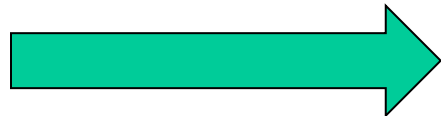
$A \rightarrow aA / \lambda$

$a^+$



$A \rightarrow aA / a$

$(a \cup b)^*$



$A \rightarrow aA / bA / \lambda$

## Chapter 3- Context Free Grammar

**Example 3:** Construct the grammar for the set of strings over  $\{a, b\}$  that contain at least 2b.

$$(a \cup b)^* b (a \cup b)^* b (a \cup b)^*$$

$$G: S \rightarrow AbAbA$$

$$A \rightarrow aA / bA / \lambda$$

# Chapter 3- Context Free Grammar

## Example 4(Chapter 2 : Exercise 12)

Give the regular expression for the set of strings over  $\{a, b, c\}$  which all the a's precede the b's, which in turn precede the c's. It is possible that there are no a's, b's, or c's.

**Solution**

$a^* b^* c^*$

Construct the grammar

G:  $S \rightarrow aS / A$

$A \rightarrow bA / B$

$B \rightarrow cB / \lambda$

OR

G:  $S \rightarrow aS / A / \lambda$

$A \rightarrow bA / B / \lambda$

$B \rightarrow cB / \lambda$

# Chapter 3- Context Free Grammar

## Example 5 (Chapter 2 : Exercise 13)

Give the regular expression for the set of strings over  $\{a, b, c\}$  which all the a's precede the b's, which in turn precede the c's. **without the null string .**

**Solution**

$a^+ b^+ c^+$

**Construct the grammar**

**G:  $S \rightarrow aS / aA$**

**$A \rightarrow bA / bB$**

**$B \rightarrow cB / c$**

# Chapter 3- Context Free Grammar

**Example 6:** Construct the grammar that has a regular expression:  $a^+b^*$

**Solution**

$$G: S \rightarrow aS / aB$$

$$B \rightarrow bB / \lambda$$

OR

$$G: S \rightarrow AB$$

$$A \rightarrow aA / a$$

$$B \rightarrow bB / \lambda$$



## Chapter 3- Context Free Grammar

Construct the language for the grammar

$$G: S \rightarrow aSa / aB a$$

$$B \rightarrow bB / \lambda$$

**Solution**

$$S \Rightarrow aSa$$

$$\Rightarrow aaSaa$$

...

$$\Rightarrow a^n S a^n$$

$$\Rightarrow a^n aBa a^n$$

$$\Rightarrow a^n b^m a^n$$

$$L(G) = \{a^n b^m a^n, n > 0, m \geq 0\}$$

## Chapter 3- Context Free Grammar

Construct the language for the grammar

$$G: S \rightarrow aSa / aBa$$

$$B \rightarrow bB / b$$

**Solution**

$$S \Rightarrow aSa$$

$$\Rightarrow aaSaa$$

...

$$\Rightarrow a^n S a^n$$

$$\Rightarrow a^n aBa a^n$$

$$\Rightarrow a^n b^m a^n$$

$$L(G) = \{a^n b^m a^n, n, m > 0\}$$

## Chapter 3- Context Free Grammar

Construct the language for the grammar

$$G: S \rightarrow aSbb / A$$

$$A \rightarrow cA / c$$

**Solution**

$$S \Rightarrow aSbb$$

$$\dots \Rightarrow aaSbbbb$$

$\dots$

$$\Rightarrow a^n S (bb)^n$$

$$\Rightarrow a^n A (bb)^n$$

$$\Rightarrow a^n c^m (bb)^n$$

$$L(G) = \{ a^n c^m b^{2n}, n \geq 0, m > 0 \}$$