

Hill cipher 3x3

Encryption

$$C = K \cdot P \pmod{26}$$

$$K = \begin{bmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{bmatrix}$$

$$P = ACT$$

$$C = \begin{bmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 19 \end{bmatrix}$$

$$= \begin{bmatrix} 67 \\ 222 \\ 319 \end{bmatrix} \pmod{26} = \begin{bmatrix} 15 \\ 14 \\ 7 \end{bmatrix}$$

$$C = \boxed{POH}$$

(2)

Decryption

$$P = K^{-1} \cdot C \pmod{26}$$

How To get inverse of K

$$K^{-1} = \left(\frac{1}{|K|} \right) \text{adj } K$$

$$|K| = ?$$

$$K = \begin{bmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{bmatrix}$$

$$|K| = \begin{vmatrix} 6(16 \times 15 - 17 \times 10) \\ -24(13 \times 15 - 20 \times 10) \\ +1(13 \times 17 - 20 \times 16) \end{vmatrix}$$

$$= 6 \times 70 - 24 \times (-5) + 1 \times -99$$

$$= 441$$

①

obj (1)

$$K = \begin{bmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{bmatrix}$$

- Repeat The first and second Column & row.

$$\begin{bmatrix} 6 & 24 & 1 & 6 & 24 \\ 13 & 16 & 10 & 13 & 16 \\ 20 & 17 & 15 & 20 & 17 \\ 6 & 24 & 1 & 6 & 24 \\ 13 & 16 & 10 & 13 & 16 \end{bmatrix}$$

② Remove P^T column & P^T row

(41)

$$\text{adj}(K) = \begin{bmatrix} 16 \times 15 - 17 \times 10 & 17 \times 1 - 24 \times 15 & 24 \times 10 - 16 \times 1 \\ 10 \times 20 - 13 \times 15 & 15 \times 6 - 20 \times 1 & 1 \times 13 - 6 \times 10 \\ 13 \times 17 - 16 \times 20 & 20 \times 24 - 6 \times 17 & 6 \times 16 - 13 \times 24 \end{bmatrix}$$

$$= \begin{bmatrix} 70 & -343 & 224 \\ 5 & 70 & -47 \\ -99 & 378 & -216 \end{bmatrix}$$

③ Remove (-ve) by adding multiple of 6
(26)

$$-343 \rightarrow 21$$

$$-47 \rightarrow 5$$

$$-99 \rightarrow 5$$

$$-216 \rightarrow 18$$

(5)

$$\text{adj} = \begin{bmatrix} 70 & 21 & 224 \\ 5 & 70 & 5 \\ 5 & 378 & 18 \end{bmatrix}$$

$$K^{-1} = \frac{1}{|K|} \cdot \text{adj}(K)$$

$$\frac{1}{|K|} = \frac{1}{441} \pmod{26}$$

$$n = \frac{1}{441} \pmod{26}$$

$$441 \cdot n = 1 \pmod{26}$$

$$n = 1$$

$$441 \pmod{26} = 25 \quad \times$$

$$n = 2$$

$$441 \times 2 \pmod{26} = 24$$

$$n = 25$$

$$441 \times 25 \pmod{26} = 1 \quad \checkmark$$

$$\frac{1}{441} \bmod 26 = 25$$

$$\frac{1}{K} K^{-1} = \frac{1}{K} \cdot adj(K)$$

$$= 25 \cdot \begin{bmatrix} 70 & 21 & 224 \\ 5 & 70 & 5 \\ 5 & 378 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 1750 & 525 & 5600 \\ 125 & 1750 & 125 \\ 125 & 9450 & 450 \end{bmatrix}$$

$$\bmod 26$$

$$K^{-1} = \begin{bmatrix} 8 & 5 & 10 \\ 21 & 8 & 21 \\ 21 & 12 & 8 \end{bmatrix}$$

(7)

$$P = K^{-1} \cdot P \pmod{26}$$

$$= \begin{bmatrix} 8 & 5 & 10 \\ 21 & 8 & 21 \\ 21 & 12 & 8 \end{bmatrix} \begin{bmatrix} 15 \\ 14 \\ 7 \end{bmatrix} \pmod{26}$$

$$= \begin{bmatrix} 260 \\ 574 \\ 530 \end{bmatrix} \pmod{26} = \begin{bmatrix} 0 \\ 2 \\ 10 \end{bmatrix}$$

$$P = a c t$$