

Automata and Language Theory

Chapter 3(Context Free Grammar)

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Chapter 3- Context Free Grammar

Definition 3.1.1

A **context-free grammar** is a quadruple (V, Σ, P, S) where V is a finite set of variables, Σ (the alphabet) is a finite set of terminal symbols, P is a finite set of rules, and S is a distinguished element of V called the start symbol. The sets V and Σ are assumed to be disjoint.

A **rule**, often called a *production*,

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$G: S \rightarrow a S b / ab / A$

$A \rightarrow a A / a$

$\Sigma = \{ a, b \}$

$N = V = \{ S, A \}$

$P = \{ S \rightarrow a S b, S \rightarrow a b, S \rightarrow A, \\ A \rightarrow a A, A \rightarrow a \}$

$S = S : \text{Start Symbol}$

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Derivation

G: $S \rightarrow a S b / SS / ab$

$S \Rightarrow ab$

*$S \Rightarrow aSb$
 $\Rightarrow aabb$*

*$S \Rightarrow aSb$
 $\Rightarrow aaSbb$
 $\Rightarrow \dots aa \dots bb$*

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Let G be the grammar : $S \rightarrow AA$

$A \rightarrow AAA / bA / Ab / a$

Construct the left most and right most derivation of the string *baba*

Left most derivation

$S \Rightarrow AA$
 $\Rightarrow bAA$
 $\Rightarrow baA$
 $\Rightarrow ba bA$
 $\Rightarrow baba$

Right most derivation

$S \Rightarrow AA$
 $\Rightarrow Aa$
 $\Rightarrow Aba$
 $\Rightarrow baA$
 $\Rightarrow baba$

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$$G = (V, \Sigma, P, S)$$

$$V = \{S, A\}$$

$$\Sigma = \{a, b\}$$

$$P: S \rightarrow AA$$

$$A \rightarrow AAA \mid bA \mid Ab \mid a$$

$$S \Rightarrow AA$$

$$\Rightarrow aA$$

$$\Rightarrow aAAA$$

$$\Rightarrow abAAA$$

$$\Rightarrow abaAA$$

$$\Rightarrow ababAA$$

$$\Rightarrow ababaA$$

$$\Rightarrow ababaa$$

(a)

$$S \Rightarrow AA$$

$$\Rightarrow AAAA$$

$$\Rightarrow aAAA$$

$$\Rightarrow abAAA$$

$$\Rightarrow abaAA$$

$$\Rightarrow ababAA$$

$$\Rightarrow ababaA$$

$$\Rightarrow ababaa$$

(b)

$$S \Rightarrow AA$$

$$\Rightarrow Aa$$

$$\Rightarrow AAAa$$

$$\Rightarrow AAbAa$$

$$\Rightarrow ABabaa$$

$$\Rightarrow AbAbaa$$

$$\Rightarrow Ababaa$$

$$\Rightarrow ababaa$$

(c)

$$S \Rightarrow AA$$

$$\Rightarrow aA$$

$$\Rightarrow aAAA$$

$$\Rightarrow aAAa$$

$$\Rightarrow abAAa$$

$$\Rightarrow abAbAa$$

$$\Rightarrow ababAa$$

$$\Rightarrow ababaa$$

(d)

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Derivation

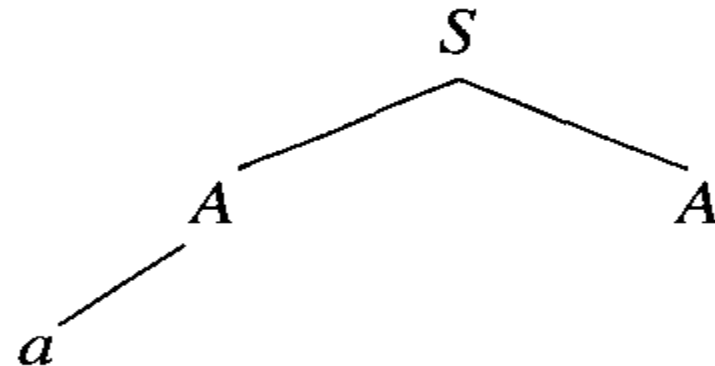
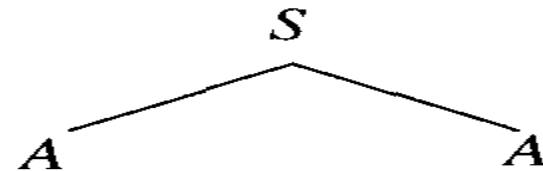
S

$\Rightarrow AA$

$\Rightarrow aA$

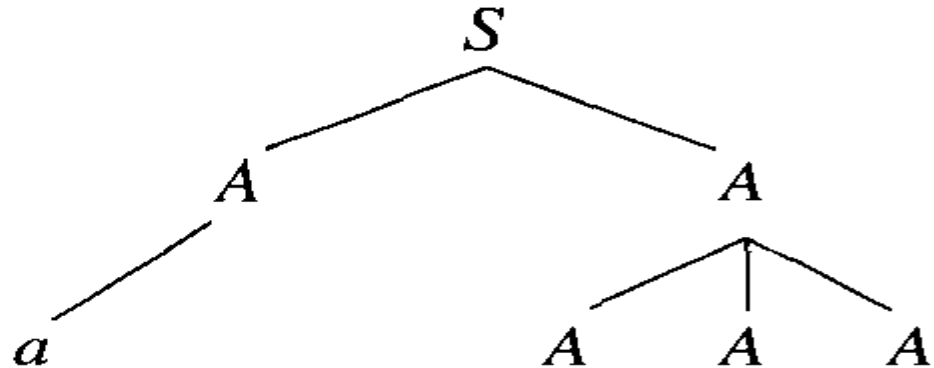
Tree

S

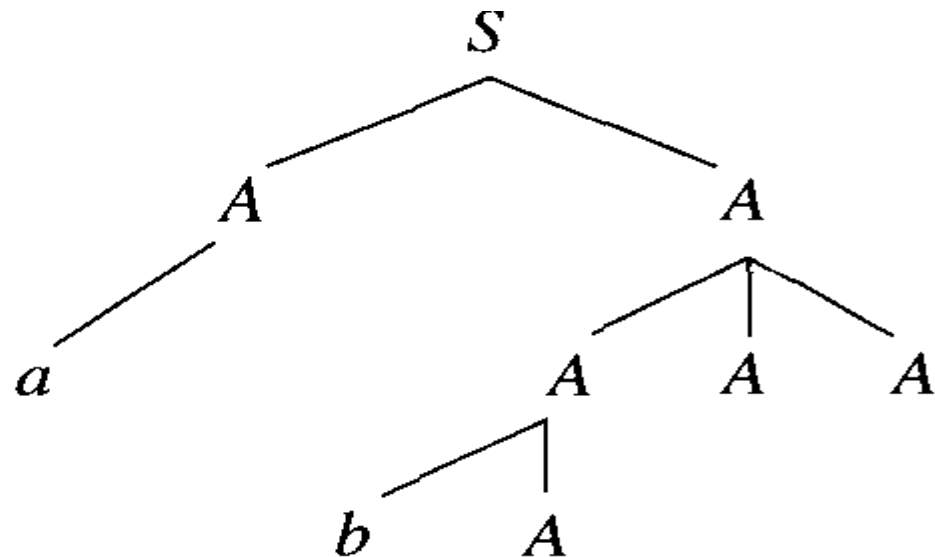


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$\Rightarrow aAAA$

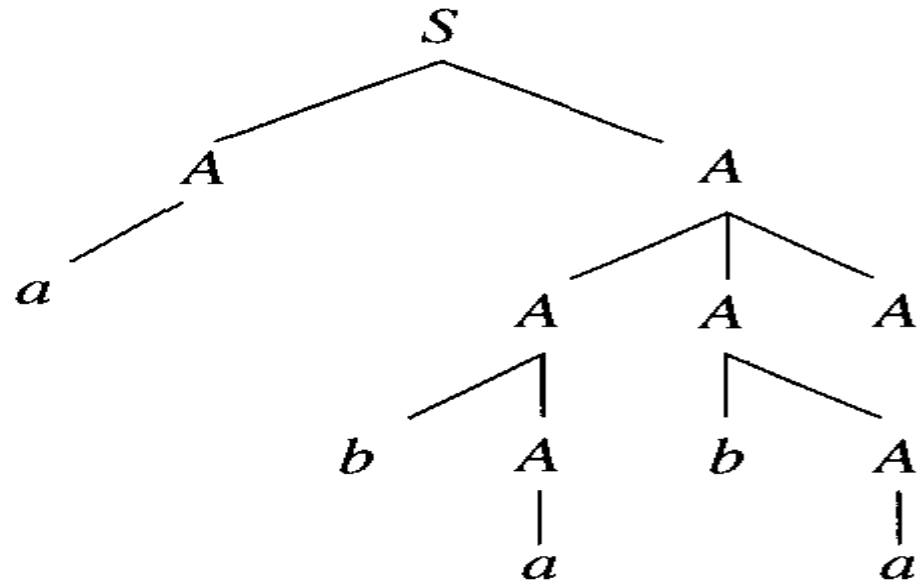


$\Rightarrow abAAA$

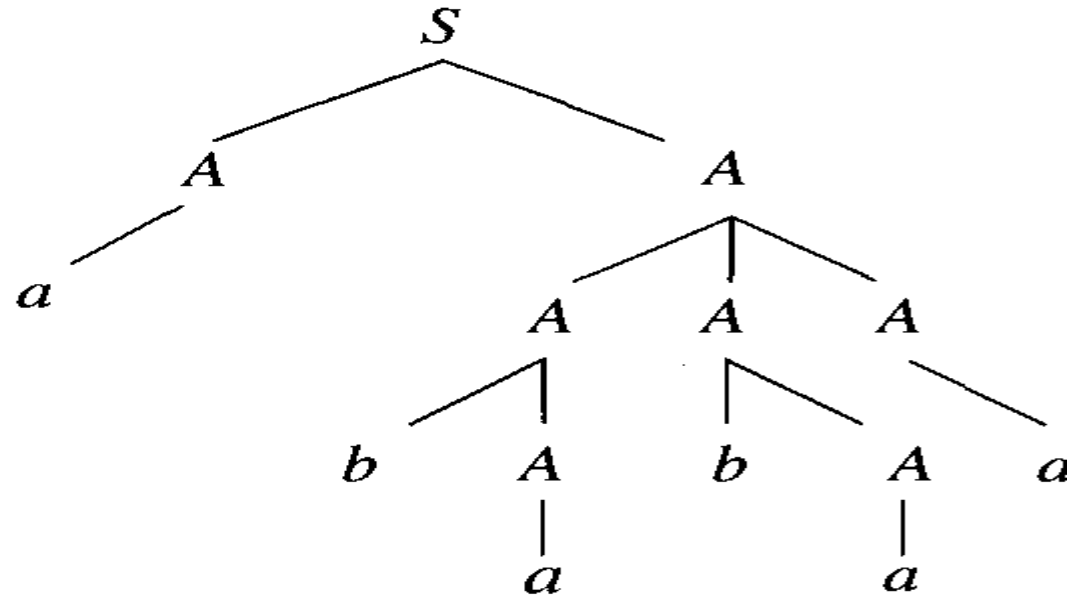


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$\Rightarrow ababaA$



$\Rightarrow ababaa$



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For the $S \rightarrow S + S \mid S S \mid (S) \mid S * \mid a$ with string $(a + a) * a$:

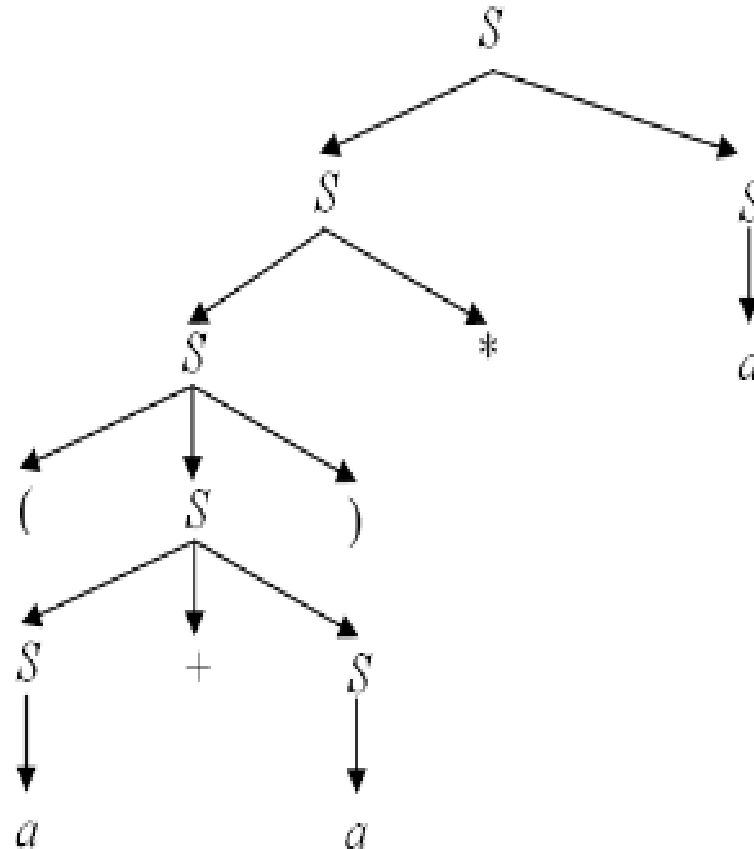
- i) Give a leftmost derivation for the string.
- ii) Give a rightmost derivation for the string.
- iii) Give a parse tree for the string.

$$\begin{aligned} \text{i) } S &\Rightarrow S S \Rightarrow S * S \Rightarrow (S) * S \Rightarrow (S + S) * S \\ &\Rightarrow (a + S) * S \Rightarrow (a + a) * S \Rightarrow (a + a) * a. \end{aligned}$$

$$\begin{aligned} \text{ii) } S &\Rightarrow S S \Rightarrow S a \Rightarrow S * a \Rightarrow (S) * a \Rightarrow (S + S) * a \\ &\Rightarrow (S + a) * a \Rightarrow (a + a) * a. \end{aligned}$$

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iii)



Parse tree