Problems: Convolutional Codes

Prof. Sundeep Rangan

1. *Convolutional Encoder.* Consider a rate 1/2 convolutional encoder with polynomials:

$$c_1[t] = b[t] + b[t-1] + b[t-3],$$
$$c_2[t] = b[t] + b[t-1] + b[t-2].$$

   (a) What is the constraint length K?

   (b) What are the generator polynomials, g_1 and g_2, in binary and octal?

   (c) Suppose we wish to encode $\boldsymbol{b} = [1, 0, 1, 1]$. How many tail bits do you add?

   (d) Write the output $c_1[t]$ and $c_2[t]$ for the input bits in part (c).

   (e) For this input, what is rate of the code including the tail bits?

2. *FSM representation of convolutional encoders.* Consider a convolutional encoder

$$c_1[t] = b[t] + b[t-1],$$
$$c_2[t] = b[t] + b[t-2].$$

   We will use the state $x[t] = (b[t-1], b[t-2])$.

   (a) Complete Table 1 to indicate the next state $x[t+1]$ and output $c[t]$ for each current state $x[t]$ and input $b[t]$.

   (b) Given the table in part (a), draw a state diagram:

   - Draw one node for each state.
   - Draw arrows indicating the transitions. Use a different line type (e.g., solid and dashed) for transitions for $b[t] = 0$ and $b[t] = 1$.
   - Draw the output bits $c[t]$ above each transition.

$x[t]$	$x[t+1]$		$c[t] = (c_1[t], c_2[t])$	
	$b[t] = 0$	$b[t] = 1$	$b[t] = 0$	$b[t] = 1$
(0,0)				
(0,1)				
(1,0)				
(1,1)				

Table 1: Problem 2: State transition and output table to be completed.

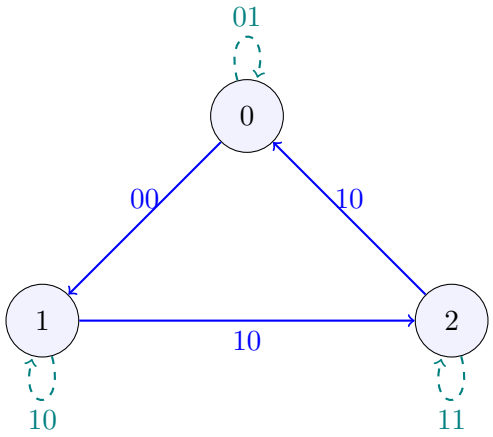

Figure 1: Problem 3. FSM representation of an encoder with three states $x[t] \in \{0, 1, 2\}$. The solid blue lines are the transitions for $b[t] = 1$, and the dashed teal lines are the transitions for $b[t] = 0$.

3. *Viterbi decoding.* Consider a encoder described by the FSM in Fig. 2. This FSM is not from a real convolutional encoder – it is completely made up to make the problem simple. At each time step, the FSM takes a binary input $b[t] \in \{0, 1\}$. There are three states, $x[t] \in \{0, 1, 2\}$. There are two outputs $c[t] = (c_1[t], c_2[t])$. The initial state is $x[0] = 0$.

   (a) Suppose that the information bits are

   $$\boldsymbol{b} = (b[0], b[1], b[2]) = (1, 0, 1).$$

   What is the state sequence $x[t]$ and output sequence $c[t]$?

   (b) Draw the trellis diagram for the states $x[t]$, $t = 0, 1, 2, 3$. On each branch of the trellis:
   - Use a different line type (e.g., solid and dashed) for transitions for $b[t] = 0$ and $b[t] = 1$.
   - Draw the output bits $c[t]$ above each transition.

   (c) Now suppose that the input bits $(b[0], b[1], b[2])$ are not known. To estimate the bits, we maximize the value function:

   $$J(c) = \sum_{i=1}^{6} c_i L_i,$$

   for the LLRs:

   $$L = (L_1, \ldots, L_6) = (-1.5, 1, 0.3, -2, 1.8, 0.5).$$

   On the trellis diagram from part (b), draw the branch metrics above each branch.

   (d) Use the Viterbi algorithm to compute the partial value function at each node. Write the values in the node. Find the sequence $\boldsymbol{b} = (b[0], b[1], b[2])$ that results in the highest value.