Unit 1. Passband Modulation

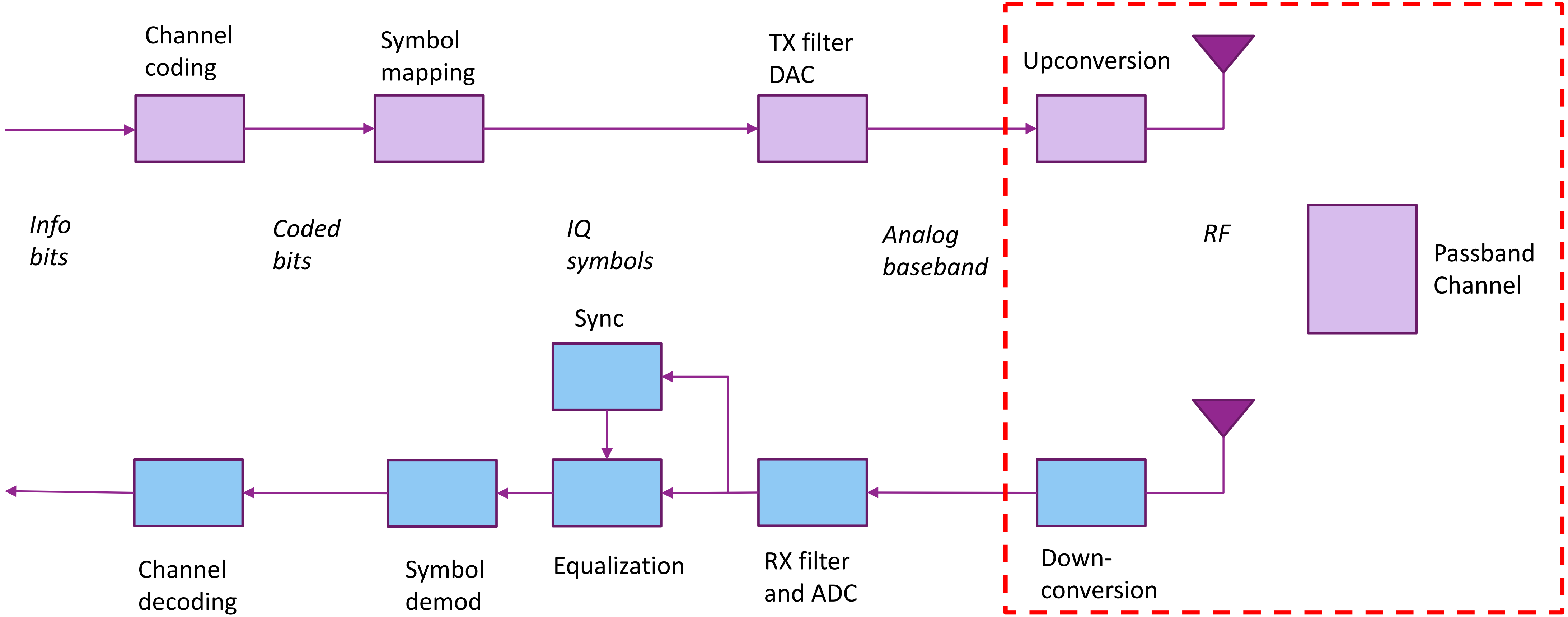
EL-GY 6013: DIGITAL COMMUNICATIONS

PROF. SUNDEEP RANGAN

Learning Objectives

- ❑ Determine if a system is real passband or baseband
- ❑ Mathematically describe upconversion and downconversion
  - In time-domain and frequency-domain
- ❑ Compute simple continuous-time Fourier transforms (Review)
- ❑ Select parameters and analyze low-pass filter in down conversion
- ❑ Determine if a signal is a power or energy signal
  - Convert power in dBm
- ❑ Compute the effective baseband filter given a passband filter
- ❑ Model impairments such as time and frequency offsets

This Unit


Channel coding
Symbol mapping
TX filter DAC
Upconversion
Info bits
Coded bits
IQ symbols
Analog baseband
RF
Passband Channel
Sync
Channel decoding
Symbol demod
Equalization
RX filter and ADC
Down-conversion

Outline

- Time-Domain Relationships
- Fourier Transform Review
- Frequency-Domain Relationships
- Power and Energy Spectra
- Baseband Equivalent Filters
- Wireless channels

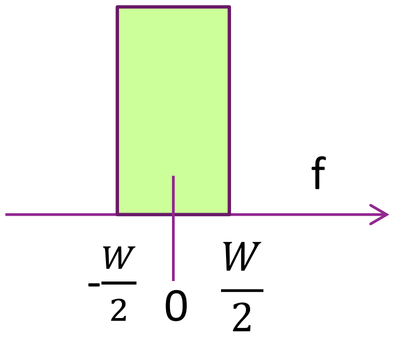
Signals in Communications

- Signal: Any quantity that varies in time
  - Can be continuous time $x(t)$
  - Or discrete time $x[n]$
  - Real or complex valued

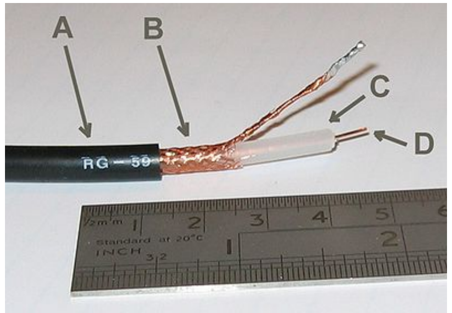
- Signals in communications:
  - $v(t)$ = Voltage at a particular point / place in a circuit (relative to ground)
  - $E_z(t)$ = Electric field strength in a particular direction
    Note: electric field is a vector quantity $E(t) = [E_x(t), E_y(t), E_z(t)]$
  - A digital sample of a signal
  - An intermediate value used in processing a signal

Real Baseband Systems

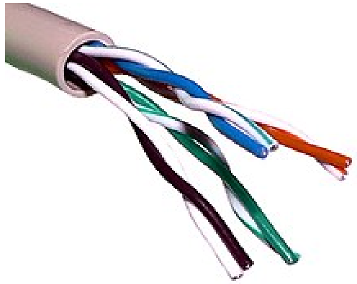
- Real baseband communication systems:
  - Communicate with lowpass real-valued signals
  - $X(f) \approx 0$ for $|f| \leq \frac{W}{2}$



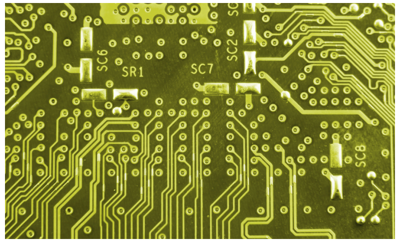
- Examples



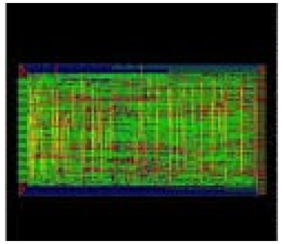
Coaxial cable



Twisted pair



PCB traces
e.g. microstrip
or stripline

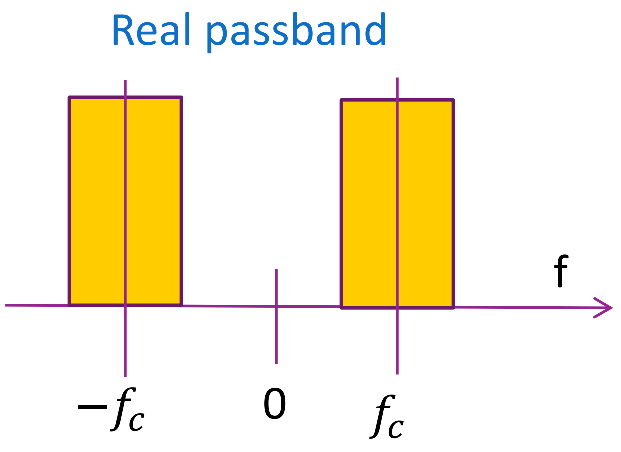


ASIC metal
traces

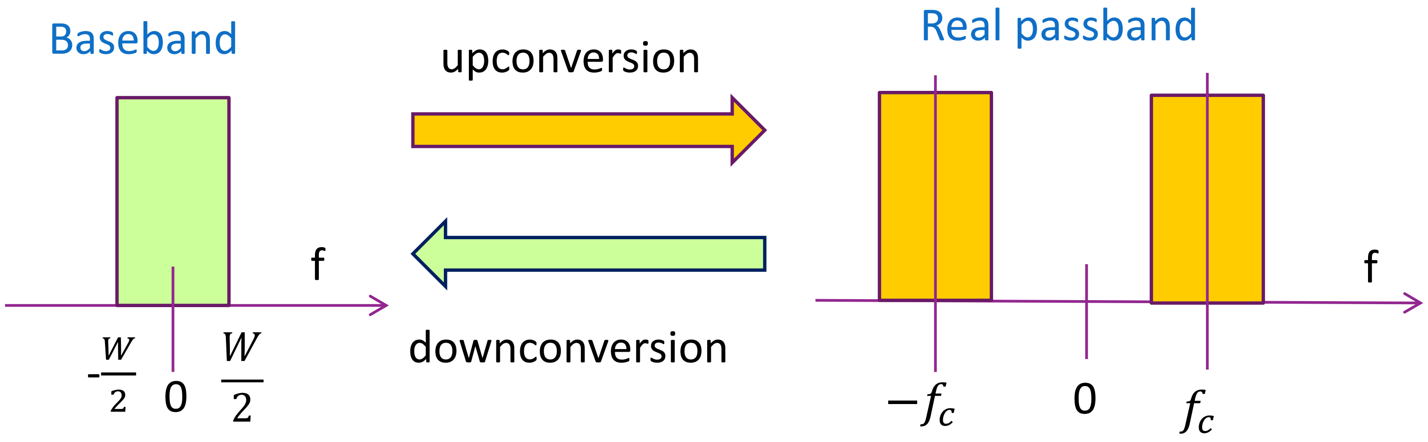
Real Passband Communications

- Real passband communication systems
  - Transmit around a carrier frequency f_c
  - f_c is sometimes called the center frequency
  - $X(f) \neq 0$ for $|f - f_c| < W$ and $|f + f_c| < W$

- Mostly radio frequency communication
  - Often wireless
  - Transmissions are restricted to bandwidth
  - Also, RF propagation is limited to certain bands
  - RF communication also occurs over cables



Up- and Downconversion



- Up and downconversion: Shift center frequency of signals
- Used for all passband communications systems
  - Information occurs or is processed in baseband
  - Transmitted and received in real passband

Upconversion in Time Domain

- Baseband signals: $u_i(t)$ and $u_q(t)$,
  - Also called "in-phase" and "quadrature" (I and Q)
  - Real-valued. Typically bandlimited to $|f| < \frac{W}{2}$ ($\frac{W}{2}$ = Single-sided bandwidth)
  - Sometimes called the "cosine" and "sine" part.

- Carrier frequency f_c
  - Also called the "center" frequency

- Create real passband signal:

$$u_p(t) = u_i(t)\cos(2\pi f_c t) - u_q(t)\sin(2\pi f_c t)$$

- Upconversion is also called modulation
  - But, we will use that term for something later.

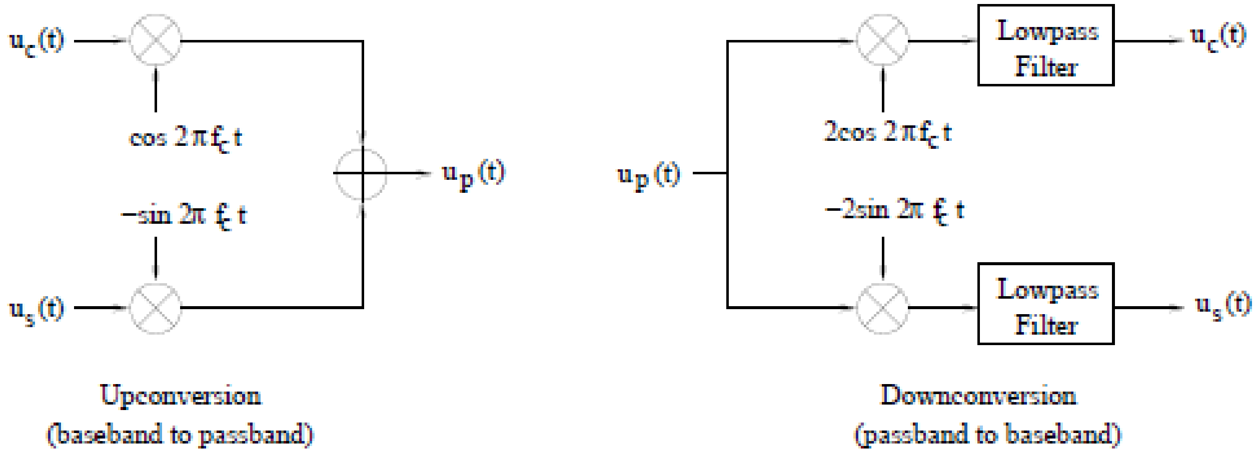
Downconversion

- Can recover I part from multiplication by sinusoid:
- Recovery of the I part:
  - $v_i(t) = 2u_p(t)\cos(2\pi f_c t) = u_i(t) +$ high freq terms
  - $u_i(t) = \mathrm{LPF}(v_i(t))$
- Recovery of the Q part:
  - $v_q(t) = -2u_p(t)\sin(2\pi f_c t) = u_q(t) +$ high freq terms
  - $u_q(t) = \mathrm{LPF}(v_q(t))$
- Can derive relations using

$$\sin(2x) = 2\sin(x)\cos(x) \quad 2\cos^2(x) = 1 + \cos(2\mathrm{x})$$

- Note gain of 2 and sign.

Up and Downconversion Block Diagram



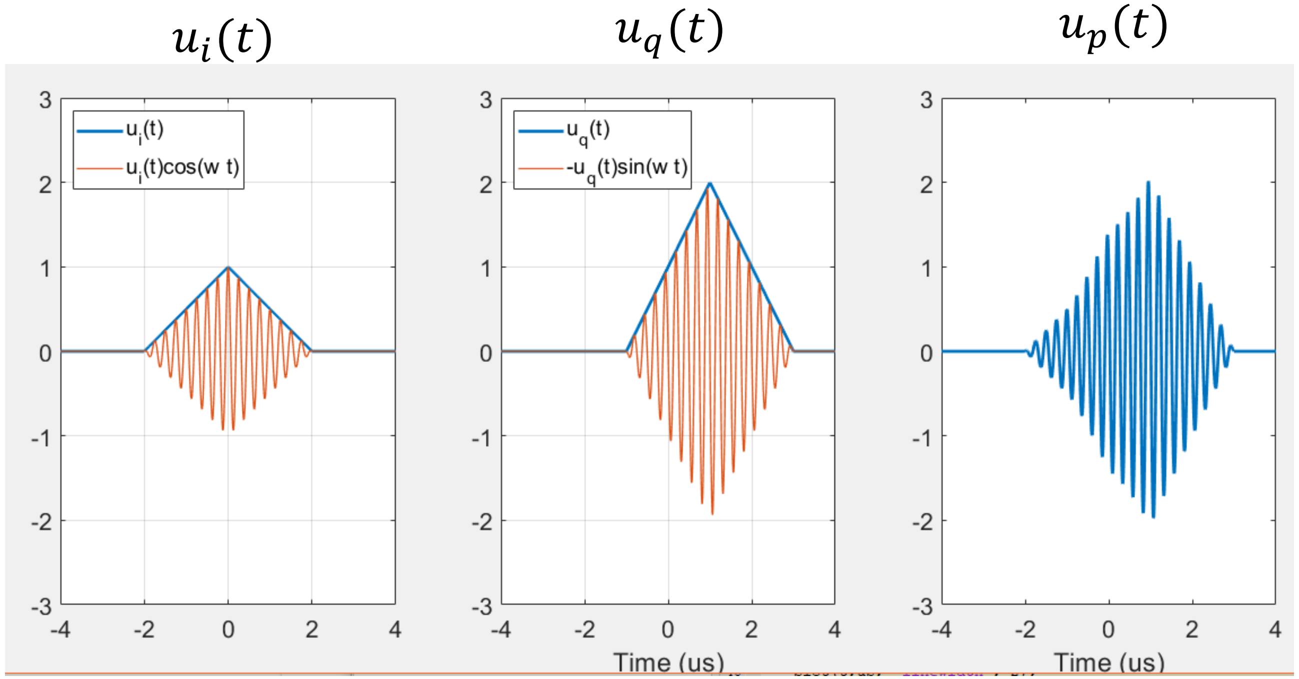
- Fig. 2.28 from Madhow
- Implementation with multipliers

Example

- Suppose
  - $u_i(t) = Tri\left(\frac{t}{T}\right),\ u_q(t) = 2Tri\left(\frac{t}{T} - 0.5\right), Tri(s) := \max(0,1-|s|)$
  - $T = 2\ \mu s,\ f_c = \frac{8}{T} = 4$ MHz

```
% Create the baseband signals
nt = 1024;
T = 2.0;
t = linspace(-2*T,2*T,nt)';
f0 = 8/T;
ui = max(1-abs(t/T),0);
uq = 2*max(1-abs(t/T-0.5),0);


% Modulate the I and Q components
uicos = ui.*cos(2*pi*f0*t);
uqsin = -uq.*sin(2*pi*f0*t);
up = uicos + uqsin;
```



Actual IQ Mixer

http://www.markimicrowave.com/Mixers/IQ_Quadrature-IF_Double-Balanced/IQ-0318.aspx

- LO = "local oscillator" = square or sine wave at f_c
- I1, I2 = I and Q inputs.
  - Generally, lowpass
- RF = passband output centered at f_c

Datasheet	RF [GHz]	LO [GHz]	IF [MHz]	Conversion Loss [dB]	Image Rejection [dB]	Amplitude Deviation [dB]	Phase Deviation [Degrees]	Isolations L-R [dB]	Isolations L-I [dB]
IQ-0318	3 to 18	3 to 18	DC to 500	7	22	0.75	10	40	20

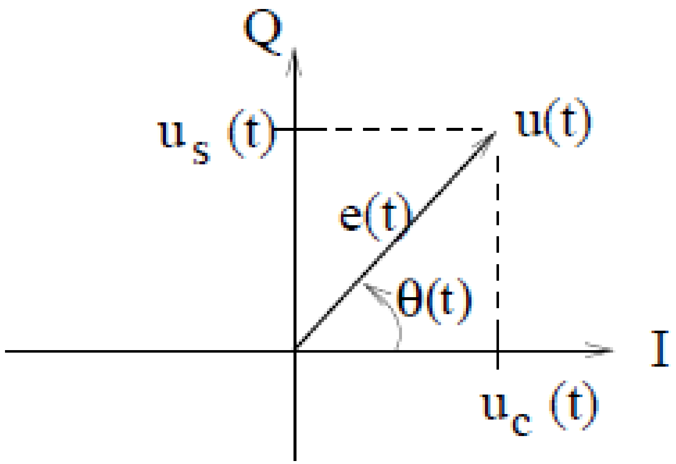
Complex Envelope

- Complex envelope:

$$u(t) = u_i(t) + ju_q(t)$$

- Magnitude and phase:
  - $e(t) = \sqrt{u_c^2(t) + u_s^2(t)}$
  - $\theta(t) = \tan^{-1}(u_s(t)/u_c(t))$

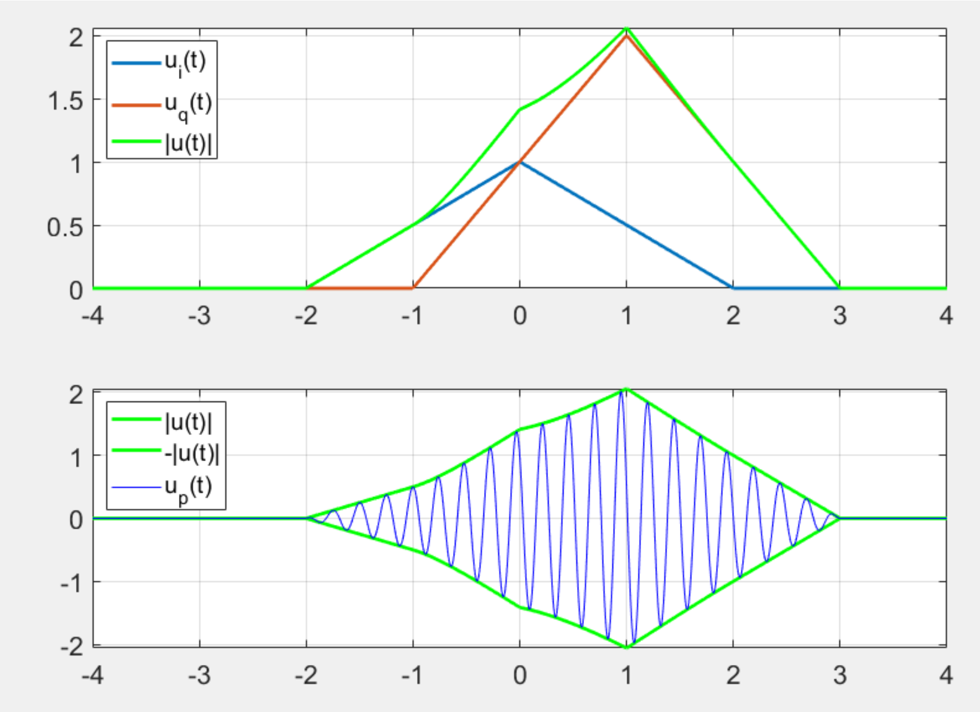
Fig 2.29, Madhow



- Can recover passband signal:

$$u_p(t) = e(t)\cos(2\pi f_c t + \theta(t))$$

Visualizing the Convex Envelope



- Observe passband signal:
  - Oscillates between $-|u(t)|$ and $|u(t)|$
  - Is much higher frequency than baseband

```
%% Plot the convex envelope
umag = abs(ui + 1i*uq);

subplot(2,1,1);
plot(t,[ui uq],'Linewidth', 2);
hold on;
plot(t,umag,'g-','Linewidth',2);
hold off;
grid on;
legend('u_i(t)', 'u_q(t)', '|u(t)|','Location','NorthWest');
set(gca,'Fontsize',16);

subplot(2,1,2);
plot(t,[umag -umag],'g-','Linewidth',2);
hold on;
plot(t,up,'b-');
hold off;
legend('|u(t)|','-|u(t)|', 'u_p(t)','Location','NorthWest');
grid on;
set(gca,'Fontsize',16);
```

Complex Baseband has all Information

- Can recover passband signal from alternate form:

$$u_p(t) = Re\left[u(t)e^{2\pi j f_c t}\right]$$

  - Derive on board

- All information content of the signal is in $u(t)$
  - The passband signal is the complex baseband with a fast, but predictable offset

Complex Notation for Downconversion

- Upconversion: $u_p(t) = Re(u(t)e^{j\omega_c t})$
- Downconversion:
  - $v(t) = 2u_p(t)e^{-j\omega_c t} = u(t) +$ High freq terms
  - $u(t) = H_{LPF}(v(t))$
- Proof:
  - $u_p(t) = Re\left(u(t)e^{j\omega_c t}\right) = \frac{1}{2}(ue^{j\omega_c t} + u^* e^{-j\omega_c t})$
  - $2u_p(t)e^{-j\omega_c t} = u(t) + \text{HFT}$
  - HFT = high frequency terms

Sample Problem (Soln on Board)

- Suppose that $T = 1\ \mu s$ and

$$u(t) = \begin{cases} 1+j & t \in [0,T) \\ 1-j & t \in [T,2T) \\ 0 & \text{else} \end{cases}$$

- What are $u_i(t)$ and $u_q(t)$? Draw them.
- Write an equation for $u_p(t)$ with a carrier frequency $f_c = 4\ \text{MHz}$
- Draw $u_p(t)$ for $t \in [1,2]\ \mu s$.

Outline

- ❑Time-Domain Relationships
- ❑Fourier Transform Review
- ❑Frequency-Domain Relationships
- ❑Power and Energy Spectra
- ❑Baseband Equivalent Filters
- ❑Wireless channels

Fourier Transform

- $s(t)$: real or complex continuous-time signal
- Fourier Transform: time-domain to frequency domain

$$S(f) = \int_{-\infty}^{\infty} s(t)e^{-2\pi ift}\, dt$$

- Inverse Fourier transform:

$$s(t) = \int_{-\infty}^{\infty} S(f)e^{2\pi ift}\, df$$

- Represents signals in their frequency components

FT in Angular Frequency

- Angular frequency: $\omega = 2\pi f$
- Fourier Transform: time-domain to frequency domain

$$S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-i\omega t}\, dt$$

- Inverse Fourier transform:

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{i\omega t}\, df$$

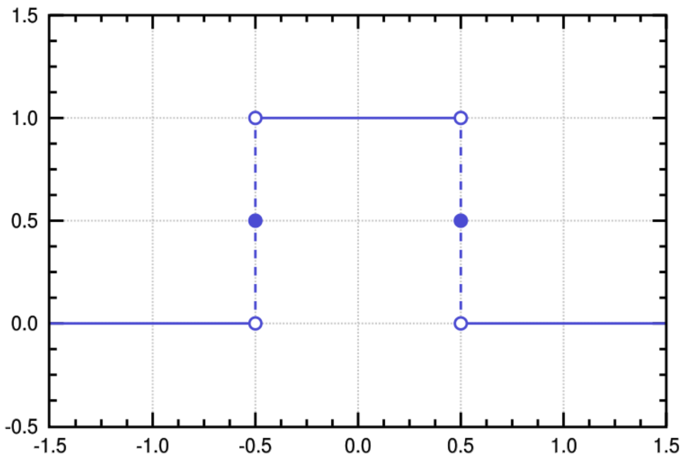
- Note scaling
- Some texts use other scalings

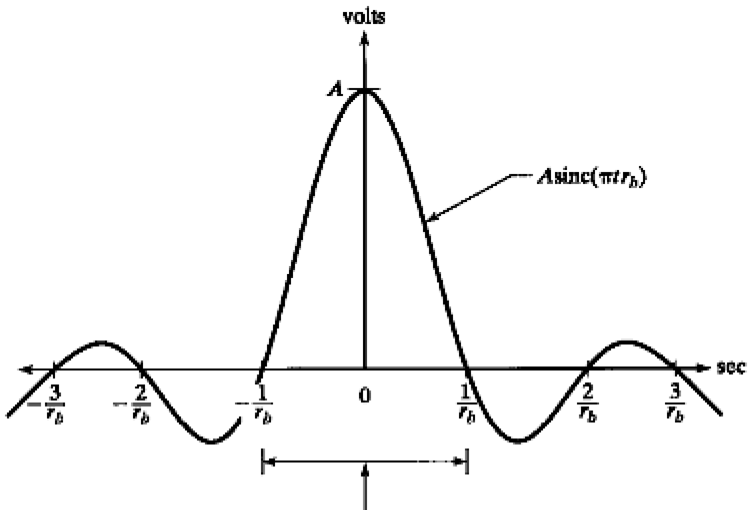
Rect and Sinc

- $rect(at) \leftrightarrow \frac{1}{|a|} sinc\left(\frac{f}{a}\right) = \frac{\sin(\pi f/|a|)}{\pi f}$
- $sinc(at) \leftrightarrow \frac{1}{|a|} rect\left(\frac{f}{a}\right)$

Height = $\frac{1}{|a|}$

Main lobe f $= \pm|a|$



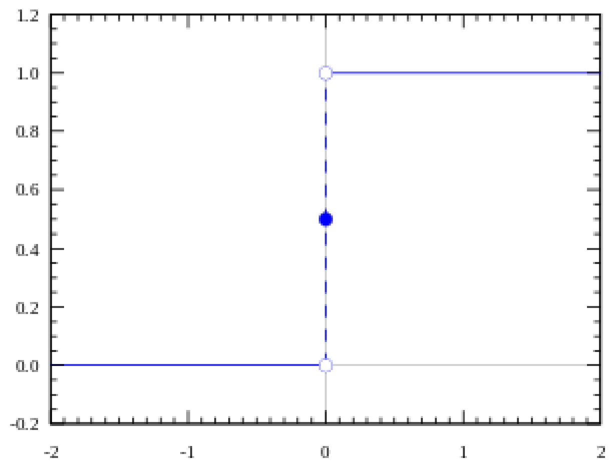


Width of the main lobe is set to $\frac{2}{r_b}$. Why? We'll see soon.

Unit Steps

- Unit step

$$u(t) = \begin{cases} 1 & t > 0 \\ 1/2 & t = 0 \\ 0 & t < 0 \end{cases}$$



- FT:
  - $e^{-\alpha t}u(t) \leftrightarrow \frac{1}{\alpha+2\pi i f},\ Re(\alpha) > 0$
  - $u(t) \leftrightarrow \frac{1}{2}\delta(f) + \frac{1}{2\pi i f}$

Table (From WikiPedia)

	Function	Fourier transform unitary, ordinary frequency
	$f(x)$	$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-2\pi i x\xi}\,dx$
201	$\mathrm{rect}(ax)$	$\frac{1}{\lvert a\rvert}\cdot \mathrm{sinc}\left(\frac{\xi}{a}\right)$
202	$\mathrm{sinc}(ax)$	$\frac{1}{\lvert a\rvert}\cdot \mathrm{rect}\left(\frac{\xi}{a}\right)$
203	$\mathrm{sinc}^2(ax)$	$\frac{1}{\lvert a\rvert}\cdot \mathrm{tri}\left(\frac{\xi}{a}\right)$
204	$\mathrm{tri}(ax)$	$\frac{1}{\lvert a\rvert}\cdot \mathrm{sinc}^2\left(\frac{\xi}{a}\right)$
205	$e^{-ax}u(x)$	$\frac{1}{a+2\pi i\xi}$
206	$e^{-\alpha x^2}$	$\sqrt{\frac{\pi}{\alpha}}\cdot e^{-\frac{(\pi\xi)^2}{\alpha}}$
207	$e^{-a\lvert x\rvert}$	$\frac{2a}{a^2+4\pi^2\xi^2}$

$f(x)$	$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-2\pi i x\xi}\,dx$
1	$\delta(\xi)$
$\delta(x)$	1
e^{iax}	$\delta\left(\xi - \frac{a}{2\pi}\right)$
$\cos(ax)$	$\frac{\delta\left(\xi-\frac{a}{2\pi}\right)+\delta\left(\xi+\frac{a}{2\pi}\right)}{2}$
$\sin(ax)$	$\frac{\delta\left(\xi-\frac{a}{2\pi}\right)-\delta\left(\xi+\frac{a}{2\pi}\right)}{2i}$

$\mathrm{sgn}(x)$	$\frac{1}{i\pi\xi}$
$u(x)$	$\frac{1}{2}\left(\frac{1}{i\pi\xi}+\delta(\xi)\right)$
$\sum_{n=-\infty}^{\infty}\delta(x-nT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(\xi-\frac{k}{T}\right)$

Other Properties

- $s(t-a) \leftrightarrow e^{-2\pi iaf}S(f)$
- $e^{2\pi iat}s(t) \leftrightarrow S(f-a)$
- $s(at) \leftrightarrow S(f/a)/|a|$
- $d^n s(t)/dt^n \leftrightarrow (2\pi if)^n S(f)$
- $t^n s(t) \leftrightarrow d^n S(f)/df^n$
- $s^*(t) \leftrightarrow S^*(-f)$
- $s(t) \leftrightarrow S(f) \Rightarrow S(t) \leftrightarrow s(-f)$

Problems

2.10 Determine the Fourier transform of each of the following signals (α is positive).

1. $x(t) = \frac{1}{1+t^2}$
2. $\Pi(t-3) + \Pi(t+3)$
3. $\Lambda(2t+3) + \Lambda(3t-2)$
4. $\text{sinc}^3 t$
5. $t \,\text{sinc}\, t$
6. $t \cos 2\pi f_0 t$
7. $e^{-\alpha|t|} \cos(\beta t)$
8. $te^{-\alpha t} \cos(\beta t)$

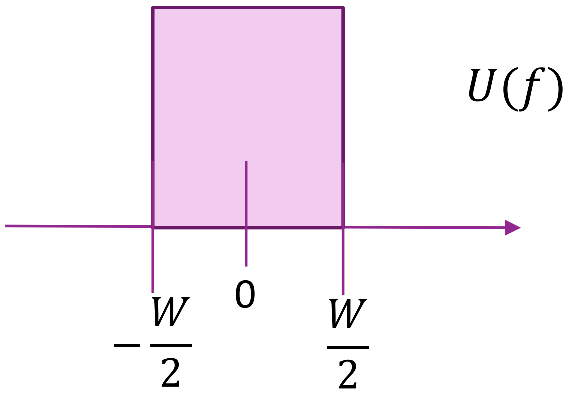
- Solutions on board

Outline

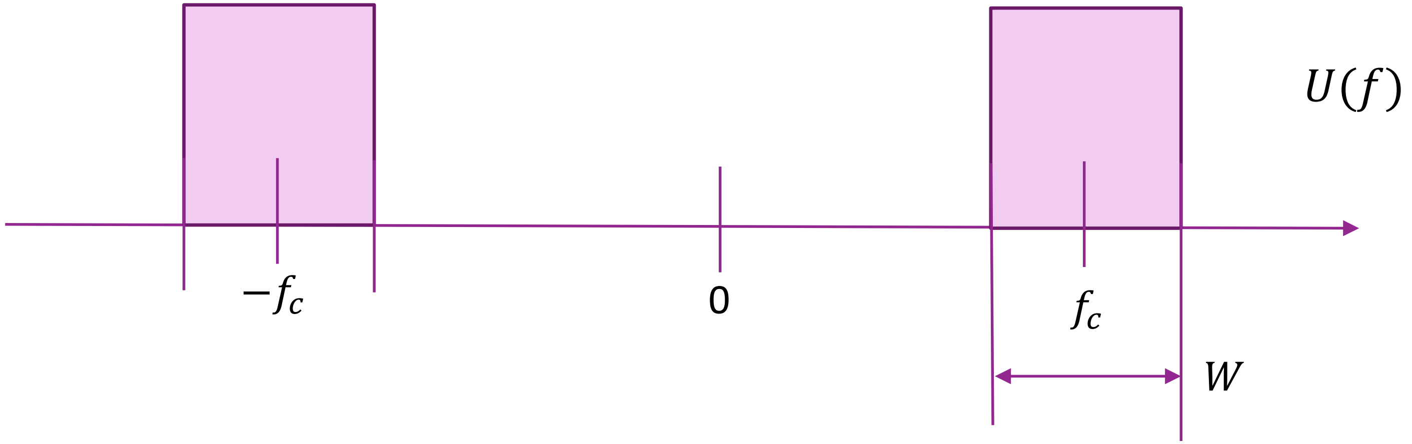
- ❑Time-Domain Relationships
- ❑Fourier Transform Review
- ❑Frequency-Domain Relationships
- ❑Power and Energy Spectra
- ❑Baseband equivalent filters
- ❑Wireless channels

Bandwidth Terminology

- Baseband signals
  - Centered around $f = 0$, complex
  - $\frac{W}{2} =$ single sided bandwidth
  - $W =$ two sided bandwidth
  - Band-limited to $|f| \leq \frac{W}{2}$

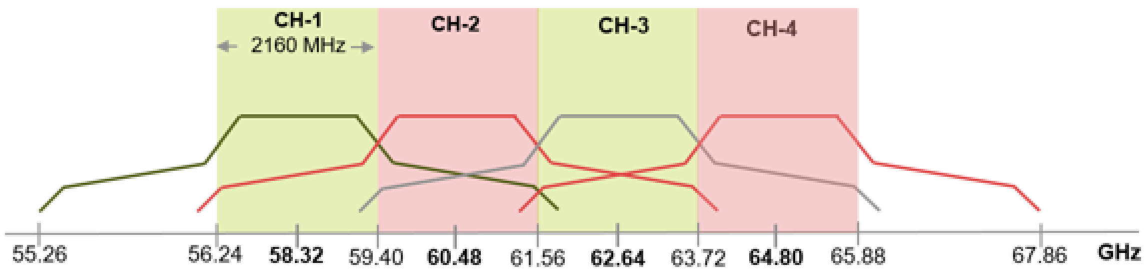


- Passband signals
  - Centered around $f = f_c$, real
  - $W =$ bandwidth (per side or image)
  - Band-limited to $|f - f_c| \leq \frac{W}{2}$



Importance of Bandwidth

- Data rate generally scales linearly in bandwidth
  - If the transmit power and bandwidth increase by $N \Rightarrow$ the communication rate increase by N
  - We will see this in detail later
- Ex: Compare GSM (2G) and LTE (4G)
  - Single channel of GSM system = 200 kHz
  - Single channel of LTE = 20 MHz
  - If power scales sufficiently, LTE would in general have 100x data rate
  - LTE, in fact, can have even more capacity due to other improvements
- Figure to the right: 802.11ad channels
  - The channels are > 2 GHz



Frequency Domain Relationships

Baseband to Passband

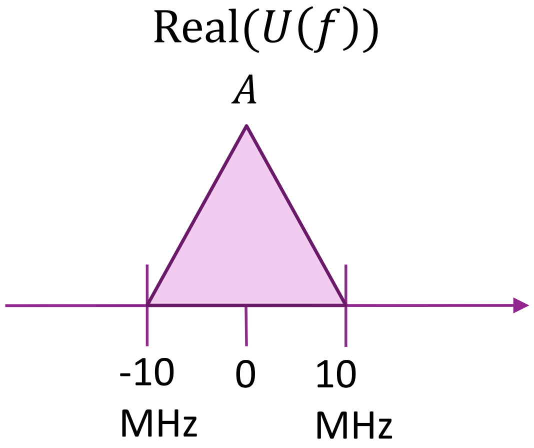
- Suppose that $U(f)$ is bandlimited to $[-W, W]$ and $f_c > W$

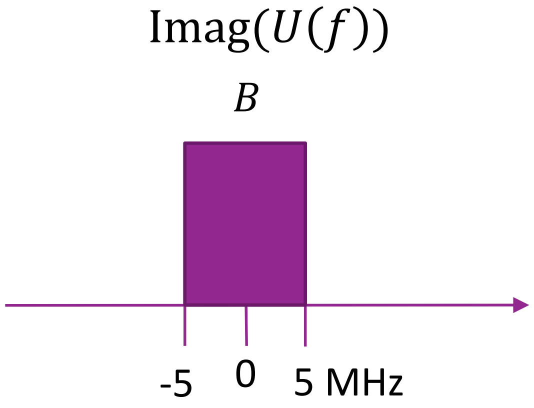
$$U_p(f) = \frac{1}{2}[U(f - f_c) + U^*(-f - f_c)]$$

- Use notation:
  - $U^+(f) \coloneqq \frac{1}{2} U(f - f_c)$: This is $U(f)$ shifted to the right by f_c and scaled by $\frac{1}{2}$
  - $U^-(f) \coloneqq \frac{1}{2} U^*(-f - f_c)$: Flip $U^+(f)$ around y axis and take negative of the imaginary part
- Proof:
  - Let $c(t) = u(t)e^{2\pi j f_c t} \leftrightarrow C(f) = U(f - f_c)$
  - $u_p(t) = Re\big(c(t)\big) = \frac{1}{2}\big(c(t) + c^*(t)\big)$
  - Now use conjugate symmetry $c^*(t) \leftrightarrow C^*(-f)$

Example Problem

- Suppose baseband signal is as drawn:
- What is the:
  - Single-sided bandwidth?
  - Two-sided bandwidth?
- Write an equation for $u_i(t)$
  - You do not need to evaluate the integral.
- Draw the passband frequency response if $f_c = 2$ GHz
  - Draw both the positive and negative images



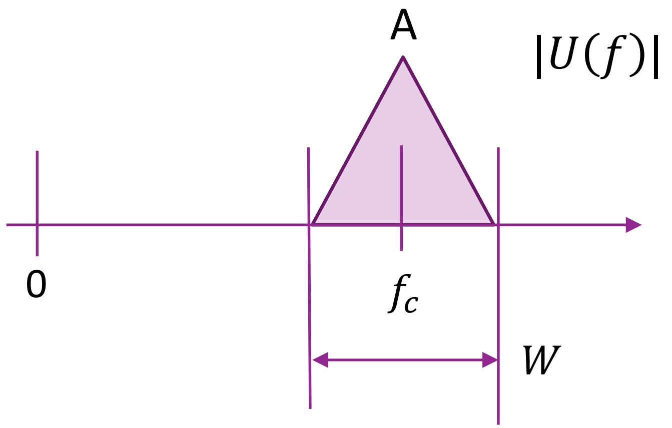


Frequency Domain Relationships
Passband to Baseband

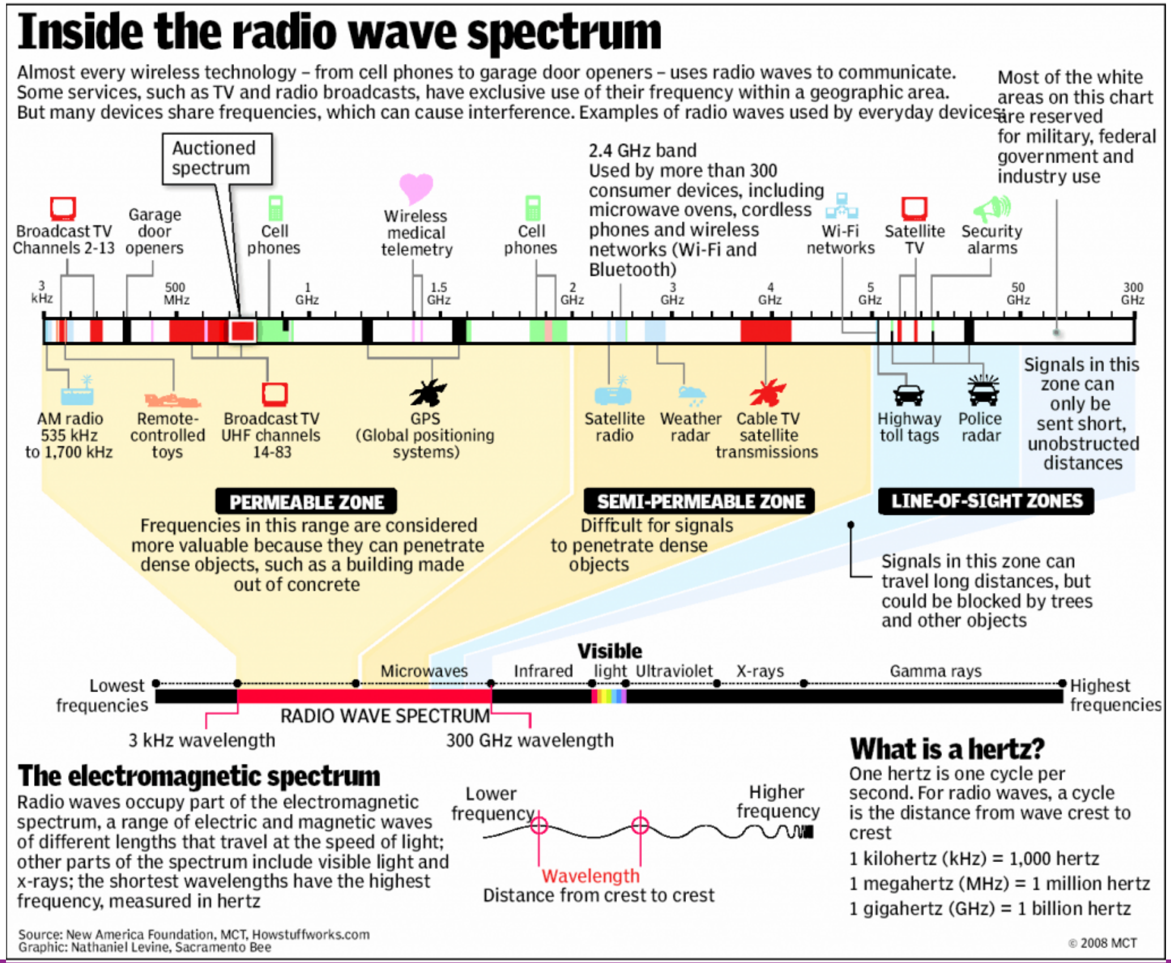
- Downcoversion in time-domain: $v(t) = 2u_p(t)e^{-j\omega_c t},\ u(t) = h_{LPF}(t) * v(t)$
- In frequency-domain: $U(f) = 2U_p(f + f_c)H_{LPF}(f)$
  - Shift to left, scale by 2 and filter
- Ideal filtering:
  - Suppose $U_p(f)$ has bandwidth W around f_c
  - Then typically have: $H_{LPF}(f) = 1$ for $|f| \le \frac{W}{2}$ and $H_{LPF}(f) = 0$ for $|f| > f_c - \frac{W}{2}$
  - $U(f) = 2U_p(f + f_c)1_{\{|f| \le W\}}$
  - Shift to the left and remove left image.
- Pictures on board

Example Problem

- Suppose right image of passband is as shown:
  - $W = 4$ MHz, $f_c = 800$ MHz
- Assume a LPF $H_{LPF}(f) = Rect\left(\frac{f}{f_0}\right)$
- Draw magnitude spectrum of down-converted signal
  - When $f_0 = 5$ MHz
  - When $f_0 = 3$ MHz
- What range of values f_0 will:
  - Keep the low-pass component
  - Reject the high frequency component
- Solution on board



Radio Spectrum

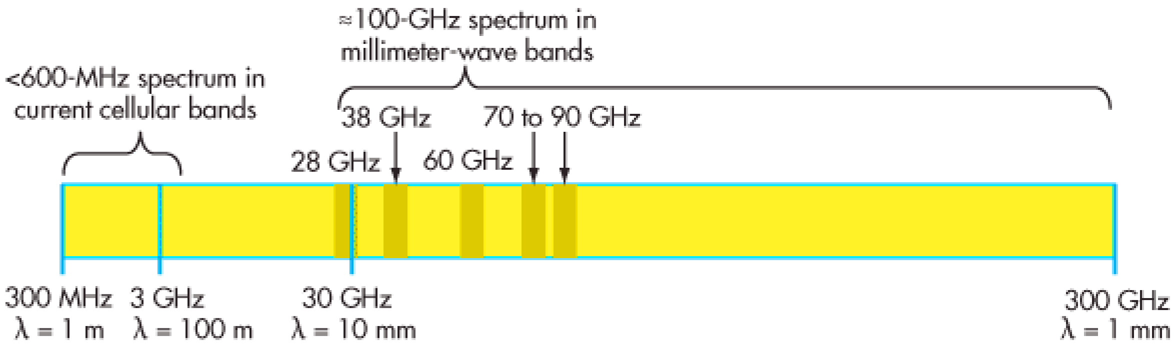


Bandwidth and Center Frequencies Examples

System	Duplex	Center freq (MHz)	Bandwidth
GSM	FDD	GSM-850: 824-849 (UL), 869-894 (DL) GSM-900: 890-914 (UL), 935-959 (DL) GSM-1800: 1710–1784(UL), 1805.2–1879(DL) GSM-1900: 1850–1910(UL), 1930–1990(DL)	200 kHz per channel
UMTS	FDD	GSM + other bands ~2100 and ~1900	5 MHz per carrier
LTE	Mostly FDD	Mostly in 2100 to 2600 MHz	1.4 to 20 MHz, 10 MHz typical
802.11abg	TDD	2.4 GHz (ISM band) and 5 GHz (U-NII band)	20 MHz
802.11n			20, 40 MHz
802.11ac			20-160 MHz
802.11ad	TDD	60 GHz (millimeter wave spectrum)	2.16 GHz

Millimeter Wave

- New bands for 5G
  - 100x more bandwidth than conventional bands below 6 GHz
  - Bands at 28 GHz and 38 GHz opened up by FCC
  - 5G systems operating in these bands are coming soon



Outline

- ❑Time-Domain Relationships
- ❑Fourier Transform Review
- ❑Frequency-Domain Relationships
- ❑Power and Energy Spectra
- ❑Baseband equivalent filters
- ❑Wireless channels

Energy and Power Signals

- Instantaneous power: $|x(t)|^2$
  - Why squared?
- Energy:
  - $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$
  - Signal is called an “energy signal” if $E_x < \infty$
- Power:
  - $P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$
  - Energy per unit time
  - Signal is called a “power signal” if limit P_x exists and is finite

Power of a Periodic Signal
Time-Domain Method

- Suppose $x(t)$ is periodic, period T
- Theorem: $x(t)$ is a power signal and power can be computed from any one period

$$P_x = \frac{1}{T}\int_0^T |x(t)|^2 dt = \frac{1}{T}\int_{t_0}^{t_0+T} |x(t)|^2 dt$$

  - Proof on board

Example: Done on board

- Suppose that $x(t)$ has period T

$$x(t) = a + bt, t \in [0, T]$$

  - a, b are real
- Draw $x(t)$
- What is P_x
- What if a, b were complex?

Power of a Periodic Signal
Fourier Series Method

- Suppose that $x(t)$ is periodic with period T
- Then has Fourier Series

$$x(t) = \sum c_n e^{2\pi j f_n t}, \qquad f_n = n/T$$

- Theorem: Power of $x(t)$ is:

$$P_x = \sum |c_n|^2$$

- Note that if $x(t) = \sum g(t - nT)$, then $c_n = G(f_n)$
  - Can compute power from Fourier transform of $g(t)$

Example: On board

- Suppose $T = 10\ \mu s$,

$$x(t) = \sum g(t - nT), \qquad g(t) = \begin{cases} 2 & t \in [0, T/4) \\ -1 & t \in [T/4, T) \\ 0 & \text{else} \end{cases}$$

- Draw $x(t)$
- What is the FT $G(f)$?
- What is the FT $X(f)$?
- What is the power of $\mathrm{x(t)}$?
- What fraction of power of $\mathrm{x(t)}$ is in the $|f| \leq 250$ kHz?

Energy Density

- Energy of signal: $E_x = \int |x(t)|^2 dt$

- From Parseval's identity: $E_x = \int |X(f)|^2 df$
  - Can compute energy in frequency-domain

- Energy density: $G_x(f) = |X(f)|^2$
  - Density of energy around frequency f

Power Spectral Density (PSD)

- Three equivalent ways to define PSD
- Definition 1: via windowing in time
- Definition 2: via filtering
- Definition 3: via auto-correlation for a random process
  - More advanced.
  - We will cover this in the next unit

PSD: Time-Windowing Definition

- Let $x(t)$ be a power signal
- Define windowed signal:

$$x_T(t) = \begin{cases} x(t) & |t| \le T \\ 0 & |t| > T \end{cases}$$

- PSD is defined as:

$$S_x(f) \coloneqq \lim_{T\to\infty} \frac{1}{2T} |X_T(f)|^2$$

- Similar to energy signal, but with averaging over time.
- Can show power is given by:

$$P_x = \int_{-\infty}^{\infty} S_x(f)df$$

  - $S_x(f)$ represents power per unit frequency

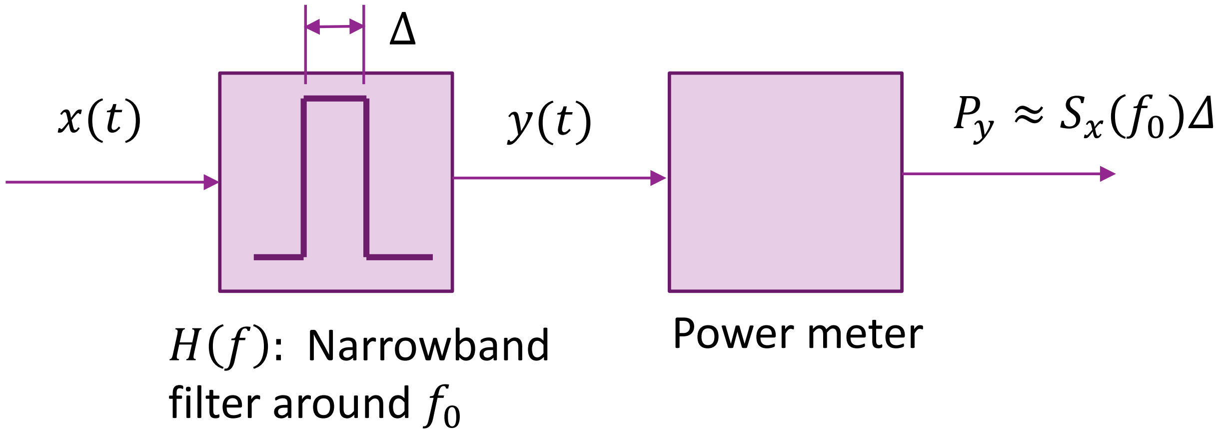
PSD: Filtering Definition

- Let $x(t)$ be a power signal
- Select frequency f_0 to measure PSD
- Filter with narrowband filter
  - $y(t) = h(t) * x(t)$
  - $H(f) = 1$ for $|f - f_0| \leq \Delta/2$
- Measure power P_y

- PSD at f_0 is defined as

$$S_x(f_0) \coloneqq \lim_{\Delta \to 0} \frac{1}{\Delta} P_y$$

- Can show this is equivalent to window definition
- Reveals how much power is in a certain frequency



Spectrum Analyzer

- Measures PSD in real time
- Uses averaging of FT
  - But proper averaging is quite tricky
- Lab 2: Use MATLAB function pwelch

Units

- Energy signals:
  - E_x: Joules
  - $G(f) = |X(f)|^2$: Joules / Hz

- Power signals (much more common):
  - P_x: Joules / sec = Watts
  - $S_x(f)$: Watts / Hz = Joules

Power: Linear and decibel scale

- Receive or transmit antenna energy per unit time
  - Measured in Watts (W) or mW
  - Power values in W or mW called *linear scale*
  - Use notation $P_{|W}$ or $P_{|mW}$ when units need to be specified

- Power often measured in dB scale:
  - $P_{|dBW} = 10\log_{10}(P_{|W} / 1W)$
  - $P_{|dBm} = 10\log_{10}(P_{|mW} / 1mW)$

- Example: P = 250 mW (typical max mobile transmit power)
  - $P_{|dBW} = 10\log_{10}(0.25W / 1W)$
  - $P_{|dBm} = 10\log_{10}(250mW / 1mW)$

Some important dB values

- Some conversions don't need a calculator:
  - 10log10(2) = 3 [Most important: Doubling power = 3dB]
  - 10log10(3) =4.7 ~5
  - 10log10(10) = 10

- You can cascade these.
- Ex: If the power is increased by 50 in linear scale, what is the increase in dB? Answer:

$$10\log_{10}(50) = 10\log_{10}(10^2/2)$$
$$= 2\times 10\log_{10}(10) - 10\log_{10}(2) = 2\times 10 - 3 = 17\ \mathrm{dB}$$

PSD and Linear Filters

- Suppose $y(t) = h(t) * x(t)$
- Then: $Y(f) = H(f)X(f)$
  - $S_y(f) = |H(f)|^2 S_x(f)$
- Transfer function $|H(f)|^2$ power gain at frequency f:

$$|H(f)|^2 = \frac{S_y(f)}{S_x(f)} \frac{\text{output power at } f}{\text{input power at } f}$$

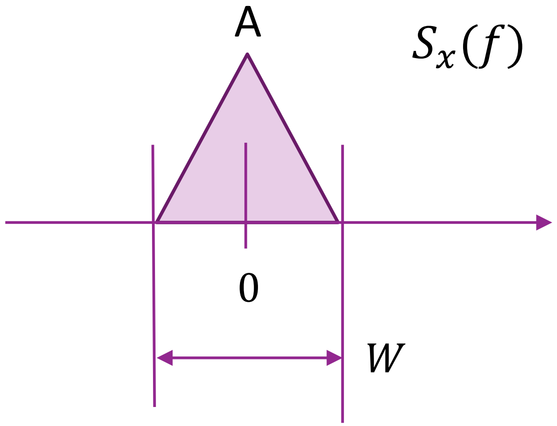
  - Dimensionless quantity
  - Often expressed in dB

Typical Wireless Power Transmit Levels

- 100 kW = 80 dBm: Typical FM radio transmission with 50 km radius
- 1 kW = 60 dBm: Microwave oven element (most of this doesn't escape)
- ~300 W = 55 dBm: Geostationary satellite
- 250 mW = 24 dBm: Cellular phone maximum power (class 2)
- 200 mW = 23 dBm: WiFi access point
- 32 mW = 15 dBm: WiFi transmitter in a laptop
- 4 mW = 6 dBm: Bluetooth 10 m range
- 1 mW = 0 dBm: Bluetooth, 1 m range

Example 1:

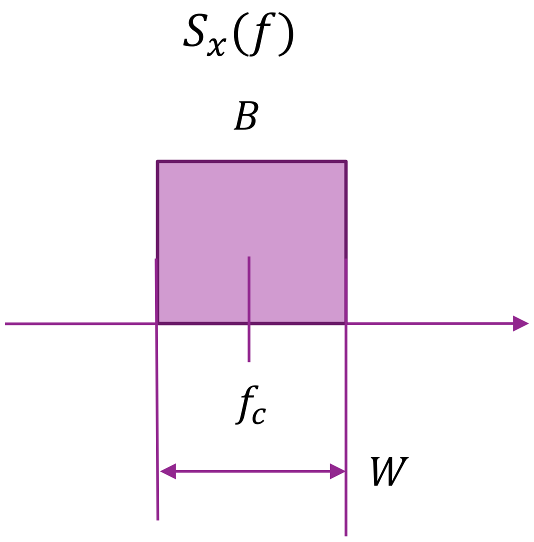
- $S_x(f)$ is as shown.
- What is the power (in linear scale) in terms of W, A?
- Suppose the power is $P_x = 20$ dBm, $W = 20$ MHz, $f_c = 2$ GHz
  - What is A?
  - What are the units of A?



- Answer: on board

Example 2

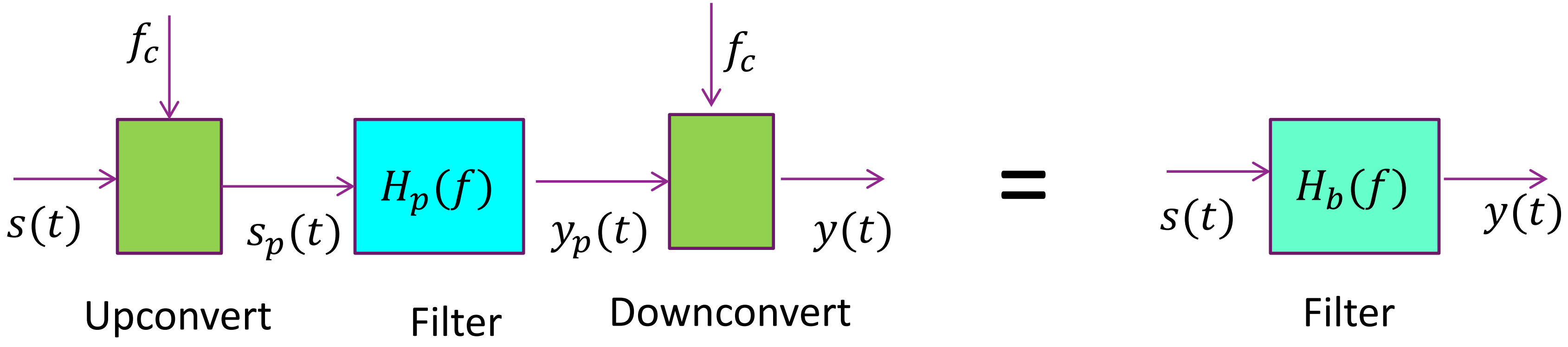
- $S_x(f)$ is shown for $f > 0$. Assume $x(t)$ is real. $W = 20$ MHz, B=$2(10)^{-8}$ mW/Hz, $f_c = 2$ GHz
- What is P_x? (Linear and in dBm).
- Suppose $y(t) = h(t) * x(t)$ with $H(f) = f_0/(2\pi jf + f_0)$
- What is $S_y(f)$? Draw it.
- Assuming $f_c \gg f_0$ what is P_y?
- What is the attenutation in dB?



Outline

- ❑Time-Domain Relationships
- ❑Fourier Transform Review
- ❑Frequency-Domain Relationships
- ❑Power and Energy Spectra
- ❑Baseband equivalent filters
- ❑Wireless channels

Filtering



- Filtering at passband equivalent to complex baseband filter
- Assuming downconversion filter is ideal (see next slide):
  - $H_b(f) = H_p(f + f_c)$ for $|f| \leq \frac{W}{2}$
  - Simply shift $H_p(f)$ to the left by f_c.

Proof of Result

- Using the conversions from passband:
- Downconversion formula: $Y(f) = 2Y_p(f + f_c)H_{LPF}(f)$
- Filtering in passband: $Y(f) = 2H(f + f_c)U_p(f + f_c)H_{LPF}(f)$
- Using upconversion formula:

$$Y(f) = H(f + f_c)\{U^*(-f - 2f_c) + U(f)\}H_{LPF}(f)$$

- Assume:
  - $U(f)H_{LPF}(f) \approx U(f)$ Filtering preserves baseband image
  - $U^*(-f - 2f_c)H_{LPF}(f) \approx 0$ Filtering removes image around $-2f_c$
- Then $Y(f) = H(f + f_c)U(f)$

Delay

- Important special case: Suppose that $h_p(t) = A\delta(t-\tau)$
  - A = gain
  - τ = delay
- Passband frequency response is: $H_p(f) = Ae^{-2\pi j f \tau}$
- Baseband frequency response:

$$H_b(f) = H_p(f + f_c) = Ae^{-2\pi j (f_c + f)\tau}$$

- Equivalent impulse response:

$$h_b(t) = Ae^{-2\pi j f_c \tau}\delta(t-\tau)$$

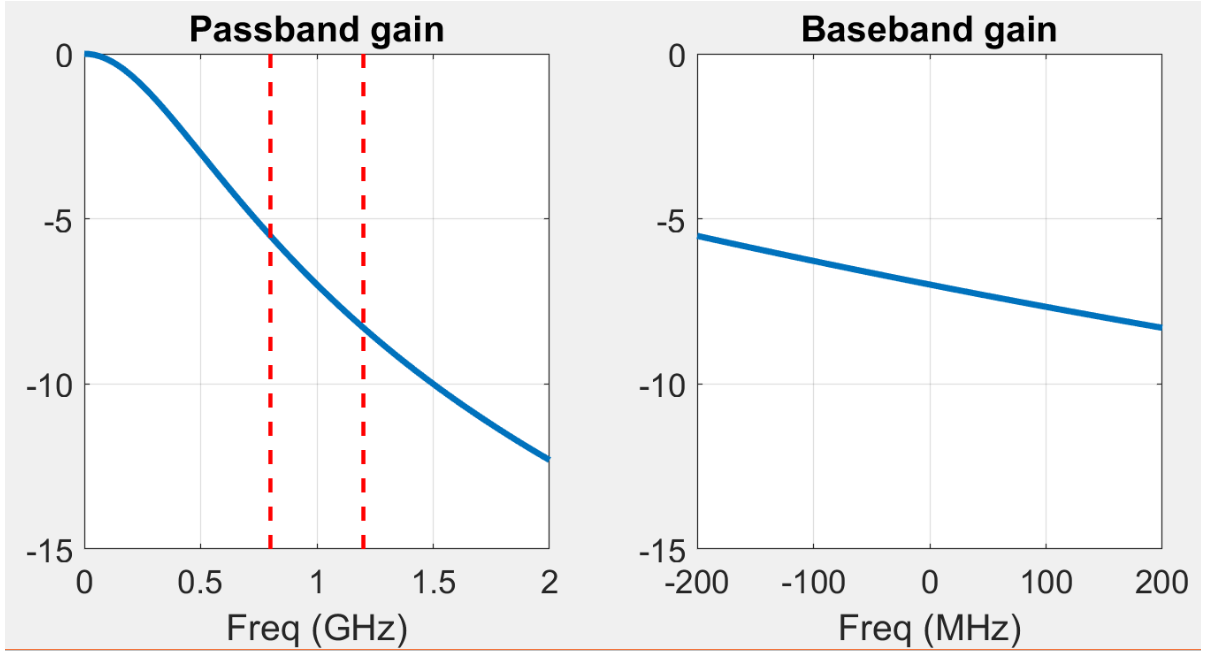
- Delay adds a constant phase rotation

Example: First Order Filter

- Passband: $H_p(\omega) = \frac{1}{1+j\omega/\omega_0}$
- Effective baseband: $H(\omega) = \frac{1}{1+j(\omega+\omega_c)/\omega_0}$
- Observe baseband response is:
  - Almost flat
  - Not symmetric around $f = 0$

```
fp = 1e9*linspace(0,2,128)';
Hp = freqs(G0,[1/w0 1], 2*pi*fp);
plot(fp/1e9, 20*log10(abs(Hp)), 'Linewidth', 3);
```

```
fb = linspace(-2e8,2e8,128)';
Hb = freqs(G0, [1/w0, 1+1i*fc/f0], 2*pi*fb);
plot(fb/1e6, 20*log10(abs(Hb)), 'Linewidth', 3);
```

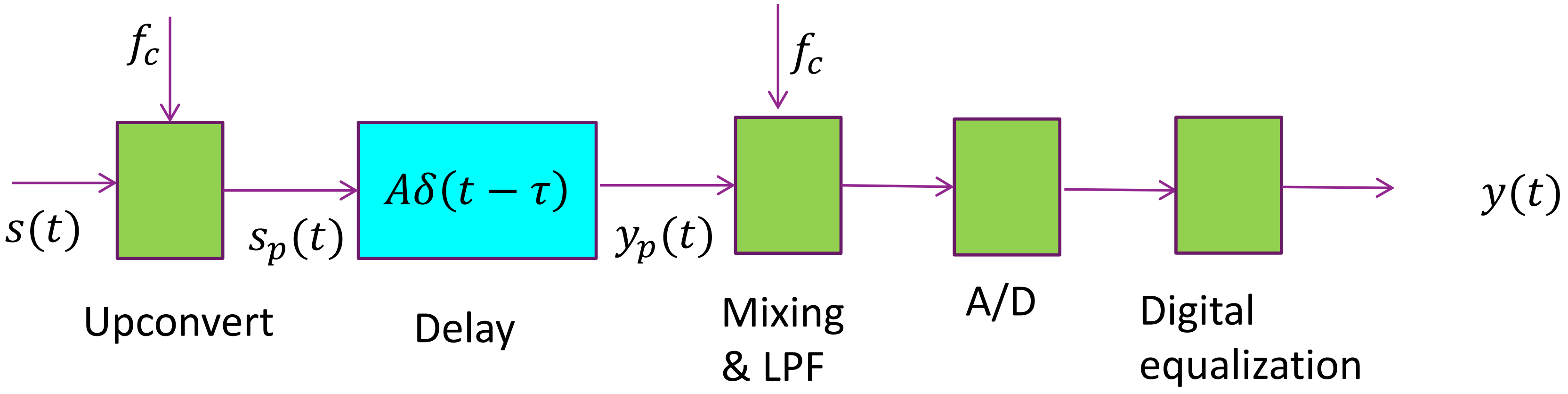


Passband cutoff freq $f_0 = 0.5$ GHz
Carrier freq $f_c = 1$ GHz

Frequency Errors

- LO on TX and RX are often slightly mismatched
- Suppose
  - Upconversion: $u_p(t) = Re\left(u_{TX}(t)e^{j\omega_0 t+\theta_0}\right)$
  - Downcoversion: $u_{RX}(t) = 2u_p(t)e^{-(j\omega_1 t+\theta_1)}$ + LPF
- Then:
  - $u_{RX}(t) = u_{TX}(t)e^{j((\omega_0-\omega_1)t+(\theta_0-\theta_1))}$
- Causes a phase rotation

Delay and Synchronization



- Two methods to compensate for delay at the RX
- Method 1: Correct in analog by adjusting phase of LO
- Method 2: Correct digital by inverting the gain $Ae^{2\pi j f_c \tau}$
  - This is a special case of equalization

Parts Per Million

- Oscillator error often measured in parts per million (ppm):

$$\Delta(\text{ppm}) \coloneqq \frac{|f_c - f_c'|}{f_c}(10)^6$$

  - f_c = desired carrier frequency
  - $f_c{}'$ = actual carrier frequency

- Example:
  - f_c = 2.5 GHz, $\Delta = 10$ ppm (typ value for low-cost oscillator)
  - Then,

$$|f_c - f_c'| = (2.5)(10)^9(10)(10)^6 = 25 \text{ kHz}$$

  - Very large frequency shift!

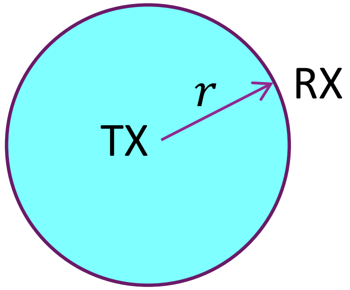
Outline

- ❑Time-Domain Relationships
- ❑Fourier Transform Review
- ❑Frequency-Domain Relationships
- ❑Power and Energy Spectra
- ❑Baseband equivalent filters
- ❑Wireless channels

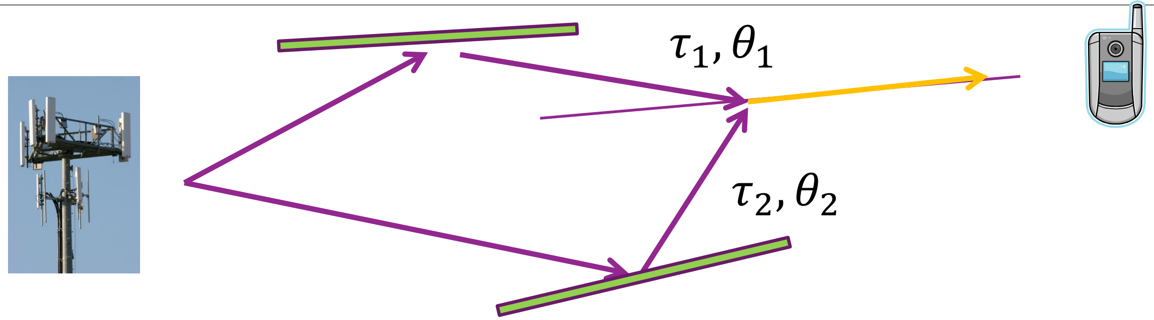
Freespace Wireless Channels

- Free space propagation
  - No obstacles
  - Isotropic (equal power in all directions)
- Power decreases as r^{-2}
  - $\Rightarrow$ Gain = Ar^{-1} for some A
- Delay is $\tau = r/c, c = 3(10)^8$ m/s
- Hence, baseband channel is:

$$h(t) = \frac{A}{r} e^{\frac{j2\pi f_c r}{c}} \delta(t - r/c)$$



Multipath

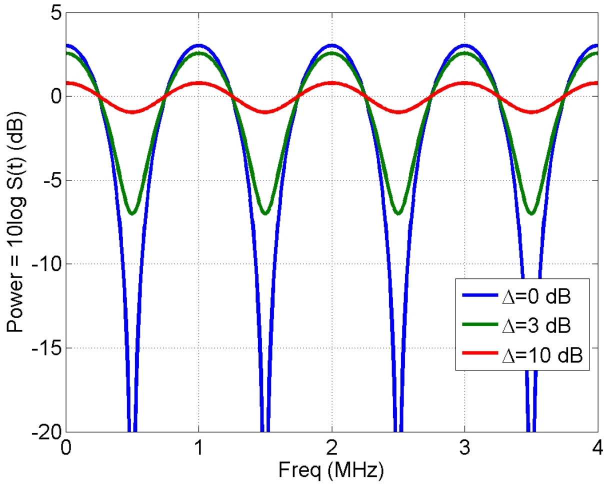


- ❑ Wireless signals can arrive in many directions
  - Reflections, diffraction, transmission, …
- ❑ Each path will have different gain and delay
- ❑ Receiver sees the combined total

$$h(t) = \sum_{k=1}^{K} h_k e^{-2\pi j f_c \tau_k} \delta(t - \tau_k)$$

  - h_k = complex gain of each path

Two-Path Example



Magnitude response

$$S(f) = |H(f)|^2 = \left|h_1 e^{2\pi i f \tau_1} + h_2 e^{2\pi i f \tau_2}\right|^2$$

Plot shows:

$$\tau_2 - \tau_1 = 1\ \mu s,\ |h_1|^2 + |h_2|^2 = 1,\ |h_2|^2 = 10^{0.1\Delta}|h_1|^2$$

- Rate of variation in frequency depends on delay spread: $\tau_2 - \tau_1$
- Size of variation depends on spread of path gains:
  - Average $S(t) = |h_1|^2 + |h_2|^2$
  - Min $S(t) = (|h_1| - |h_2|)^2$, Max $S(t) = (|h_1| + |h_2|)^2$

NYU TANDON SCHOOL OF ENGINEERING

Example Problem (On board)

- A wireless channel has 2 paths:
  - Path 1: Power gain of -80 dB, travels 100m
  - Path 2: Power gain of -83 dB, travels 120m
- What are the amplitude gains of the two paths, h_1, h_2?
- What are the two delays of the paths: τ_1, τ_2?
- What is the average, minimum and maximum power?