Problems: Passband Modulation

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1. *Passband conversion in time-domain.* Suppose that the complex baseband signal in time-domain is

$$u(t) = (a + bj)e^{-\alpha t}, \quad t \geq 0,$$

where a, b, α are real.

(a) What at the I and Q components?

(b) What is the real passband signal after upconversion with a carrier frequency f_c?

2. *Passband conversion in frequency domain.* Let $u(t)$ be a complex baseband signal with the real and imaginary parts of the spectrum (Fourier Transform) shown in Fig. 1. The constants are $f_0 = 5$ MHz, $f_1 = 10$ MHz, $A = 8$ and $B = 10$.

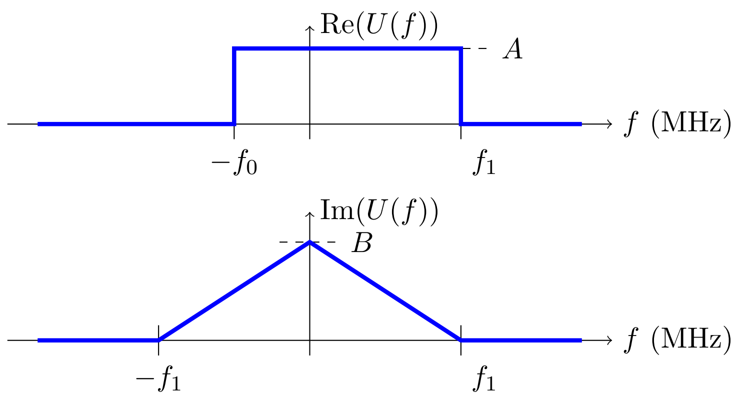

Figure 1: Real and imaginary parts of complex baseband signal $U(f)$

(a) Suppose that we create a real passband signal $u_p(t) = \mathrm{Re}(u(t)e^{2\pi i f_c t})$ for a carrier frequency $f_c = 800$ MHz. Draw the spectrum of $U_p(f)$. Show both the real and imaginary parts and show both the positive and negative frequencies.

(b) Is $u(t)$ an energy signal or power signal? What is its energy or power (in linear scale)? Leave your answer in terms of A, B, f_0 and f_1. You do not need to convert to dB scale.

(c) A receiver attempts to downcovert the signal with a two step process:

$$v(t) = 2u(t)e^{-2\pi i f_c t}, \quad \hat{u}(t) = h_{LPF}(t) * v(t),$$

where $h_{LPF}(t)$ has a frequency response,

$$H_{LPF}(f) = \begin{cases} C & \text{if } |f| < f_{LPF} \\ 0 & \text{if } |f| \geq f_{LPF}. \end{cases}$$

For what values of C and f_{LPF} is $\hat{u} = u(t)$?

3. *Baseband equivalent filter.* Consider a communication system with three steps:

   - A complex baseband signal $u(t)$ is upconverted $u_p(t) = \mathrm{Re}(u(t)e^{2\pi i f_c t})$ for some f_c.
   - The real passband channel is passed through a linear filter,

     $$\frac{dy_p(t)}{dt} = bu_p(t) - ay_p(t),$$

     with constants a and $b > 0$.
   - The received signal is downconverted, $v(t) = 2y_p(t)e^{-2\pi i f_c t}$ and $y(t) = h_{\mathrm{LPF}}(t) * v(t)$ where $h_{\mathrm{LPF}}(t)$ is an ideal low-pass filter.

   (a) What is the real passband frequency response, $H_p(f) = \frac{Y_p(f)}{U_p(f)}$?

   (b) What is the effective baseband frequency response $H(f) = \frac{Y(f)}{U(f)}$?

   (c) Find a_1 and b_1 such that

   $$\frac{dy(t)}{dt} = b_1 x(t) - a_1 y(t).$$

   (d) Suppose that $2\pi f_c \gg a$, what is the power gain of $H(0)$ in dB?

4. *PSD and RX filtering.* Suppose that a real passband signal has two components:

   $$x(t) = x_0(t) + x_1(t),$$

   where $x_0(t)$ is a desired signal, and $x_1(t)$ is an interfering signal. They have PSD $S_i(f) = A_i\, Rect((f - f_i)/W_i)$, $i = 0, 1$ with parameters:

   - Desired signal: $f_0 = 2.50$ GHz, $W_0 = 20$ MHz, total receive power $P_0 =$ -100 dBm.
   - Interfering signal: $f_1 = 2.53$ GHz, $W_1 = 10$ MHz, total receive power $P_1 =$ -80 dBm.

   (a) Find A_i from P_i using reasonable approximations. State the units of A_i.

   (b) Draw $S_0(f)$ and $S_1(f)$.

   (c) A signal is downconverted with mixing $v(t) = 2x(t)e^{2\pi i f_c t}$ and $u(t) = h(t) * v(t)$. Find f_c and a filter magnitude response $|H(f)|^2$ such that:

   - The component from desired signal is centered at 0 and amplified to -60 dBm.
   - The component from interfering signal attenuated to below -110 dBm.

   There is no single correct answer. Draw $|H(f)|^2$ and the PSD of $u(t)$.

5. *Circuit implementations.* Fig. 2 shows the circuit schematic of the RF front-end a SiGe (silicon Germanium) bipolar transceiver presented in:

   Floyd, Brian A., et al. "SiGe bipolar transceiver circuits operating at 60 GHz." *IEEE journal of solid-state circuits* 40.1 (2005): 156-167.

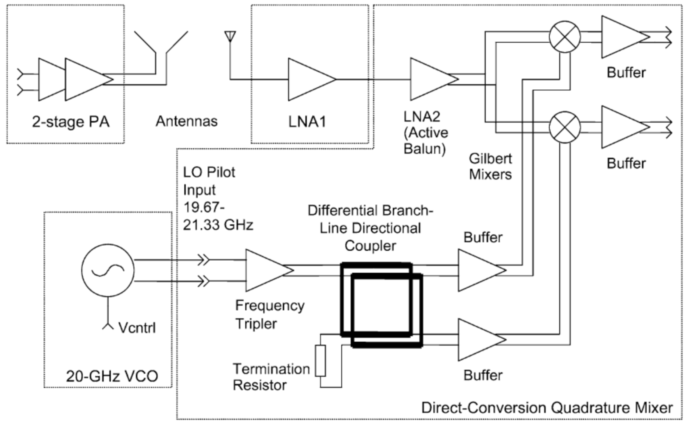

Figure 2: Schematic diagram of a SiGe RF front-end from Floyd, *et. al*, 2005.

Feel free to look up terms in the paper or any other source to answer the following questions:

(a) Which block is the local oscillator?

(b) What is the carrier frequency? Why?

(c) What is the role of the "Differential branch line directional coupler"?

(d) How is the frequency tuned?