Problems: OFDM Equalization

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1. *OFDM Wirless LAN.* Use the Internet or any other source to find the following parameters for a 40 GHz 802.11n system.
   (a) Find the sample rate, the subcarrier spacing, FFT size, number of samples in the CP, number of occupied subcarriers and number of pilot sub-carriers.
   (b) Why is the DC carrier a null carrier?
   (c) Suppose you use 16-QAM on all data subcarriers (sub-carriers that are not used for pilots). What is the data rate?

2. *OFDM Numerology.* Find the parameters of an OFDM system with the following properties. The parameter should include the sample rate, the subcarrier spacing, FFT size, number of samples in the CP, and number of occupied subcarriers.
   (a) The maximum delay spread is 500 ns
   (b) The CP overhead is approximately 25%
   (c) The occupied bandwidth is approximately 40 MHz.
   (d) The FFT should be a power of 2.

3. *Effective SNR.* Suppose that

$$y[n] = hx[n] + w[n], \quad w[n] \sim \mathcal{C}N(0, N_0), \quad |x[n]|^2 = E_x,$$

   where h is a constant channel gain. Suppose the first N symbols are reference symbols, and we estimate the channel gain h via a simple average:

$$\widehat{h} = \frac{1}{N}\sum_{n=0}^{N-1} \frac{y[n]}{x[n]}.$$

   (a) Find the bias and variance of $\widehat{h}$ as a function of N.
   (b) For any subsequent symbol we can write,

$$y[n] = \widehat{h}x[n] + v[n], \quad v[n] = (h - \widehat{h})x[n] + w[n],$$

   where $v[n]$ is the *effective noise* including channel estimation error. Assuming $x[n]$ is independent of $w[n]$ what is the $E|v[n]|^2$, the effective noise variance.
   (c) What is the degradation in SNR in terms of the number of symbols N you use for reference signals?

4. *MSE error with a correlation.* Suppose that

$$y[n] = hx[n] + w[n], \quad w[n] \sim \mathcal{C}N(0, N_0), \quad |x[n]|^2 = 1.$$

To estimate $h[n]$, reference signals are placed once every N sub-carriers at locations $n = 0, N, 2N, \ldots$. On the reference signals, we compute the raw estimate: $\hat{h}_0[n] = y[n]/x[n]$. Suppose that we can model $h[n]$ as a stationary random process with $E(h[n]) = 0$ and $E(h[n]h^*[n-m]) = E_0 e^{-\alpha|m|}$ for some constants E_0 and m.

(a) Find the MSE of the raw channel estimate $\hat{h}_0[n] = y[n]/x[n]$ on the sub-carriers $n = 0, N, 2N, \cdots$. The MSE is $\mathbb{E}|\hat{h}_0[n] - h[n]|^2$.

(b) Suppose that for other sub-carriers, we estimate $\hat{[}n] = \hat{h}_0[kN]$ where kN is the closest reference sub-carrier to n. Find the MSE of this nearest neighbor estimate $\hat{[}n]$ as a function of the sub-carrier index n. This will be a periodic function in n.

(c) Suppose that the SNR E_0/N_0 is 20 dB and we want that the worst case MSE is bounded by:

$$E|\hat{h}[n] - h[n]|^2 \leq 2N_0,$$

for all sub-carriers n. That is, we want that the estimation error to be no more than 3dB the MSE on the reference sub-carriers. What is N in terms of α. That is, how closely do we need to place the reference sub-carriers as a function of the rate of variation of the random process.

5. *Bias and Variance with a frequency rotation.* Suppose that true OFDM channel is $h[n] = e^{in\theta}$ where θ is the frequency rotation per sub-carrier. We get measurements of the form,

$$y[n] = h[n]x[n] + w[n], \quad w[n] \sim \mathcal{C}N(0, N_0), \quad |x[n]|^2 = E_x.$$

(a) Find the bias and variance of the raw channel estimate $\hat{h}_0[n] = y[n]/x[n]$.

(b) Now consider a raw channel estimate,

$$\hat{h}[n] = \sum_{\ell=-L}^{L} \hat{h}_0[n].$$

Find the bias and variance as a function of L.

(c) Suppose that a channel delay $\tau = 0.5\mu$s relative to the beginning of the FFT window, and the sub-carrier spacing is Δf=15 kHz and the SNR $E_x/N_0 = 10$ dB. Use MATLAB to plot the bias, variance and MSE as a function of the window length L.