

# Unit 7: Synchronization and Matched Filtering

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EL-GY 6013: DIGITAL COMMUNICATIONS

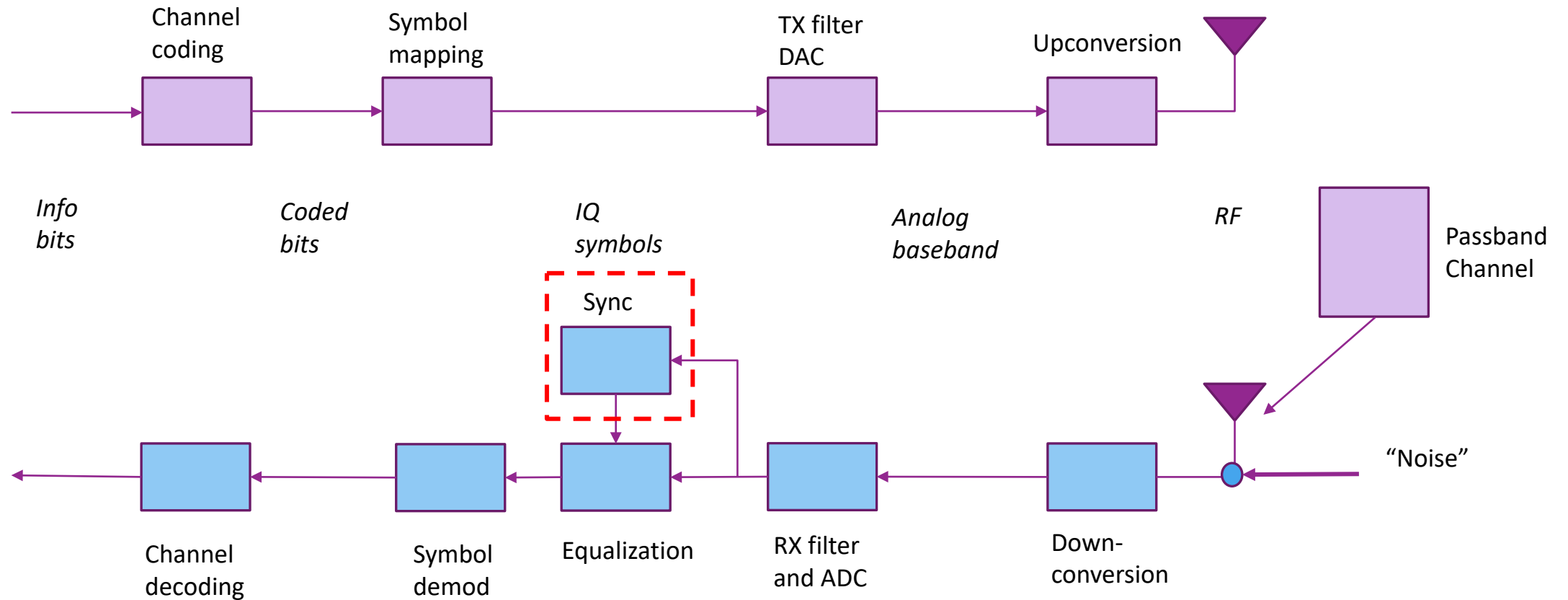
PROF. SUNDEEP RANGAN

# Learning Objectives

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- ❑ Describe the synchronization mechanisms in common commercial standards
- ❑ Formulate binary decision tasks as hypothesis testing problems
- ❑ Compute the LRT detector for a hypothesis testing problem
- ❑ Compute error probabilities and optimize the threshold
- ❑ Formulate signal detection as a hypothesis test
- ❑ Describe and analyze the matched filter detector
- ❑ Analyze various non-idealities including clock offset, auto-correlation and multi-path
- ❑ Simulate the MF detector for real systems

# This Unit



# Outline

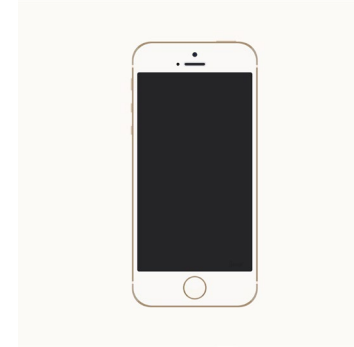
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## Detection and Synchronization Problem

- ☐ Hypothesis Testing
- ☐ Match Filtering for Detection
- ☐ Match Filtering Convolution
- ☐ Automatic Gain Control (AGC)
- ☐ Matched Filtering as a Generalized Likelihood Ratio Test

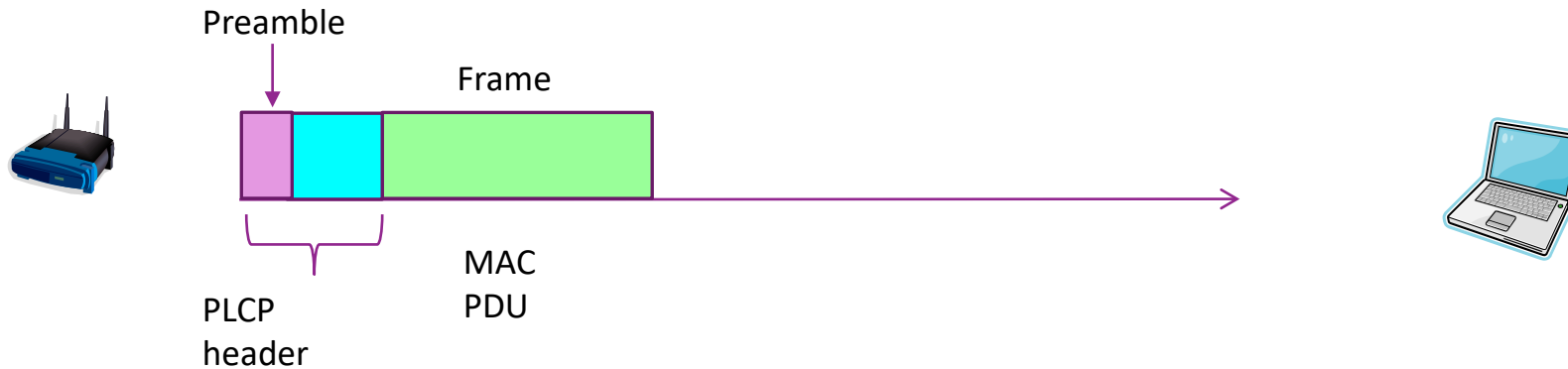
# Synchronization and Detection Problem

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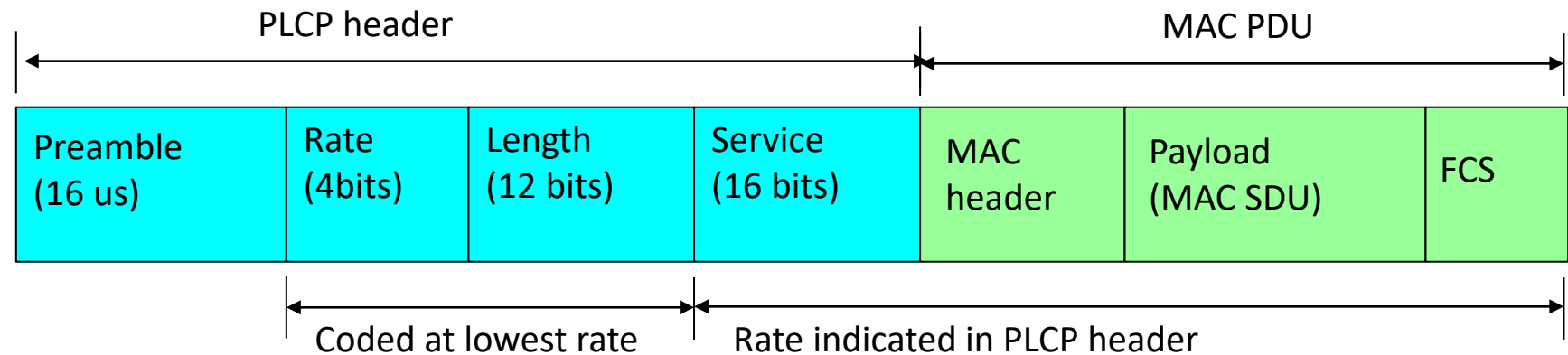
- ❑ Two key problems in most communication receivers:
  - **Detect** if a transmitter is **present**
  - **Synchronize** to the transmitter
- ❑ Basic first step in any communication process
- ❑ Assumes the transmitter broadcasts a signal
- ❑ Receiver must detect and synchronize to it

# Ex 1: 802.11g Transmission



- ❑ All data is transmitted in **frames**
- ❑ Frames may arrive at any time
- ❑ Each frame begins with known **preamble**
  - Common to all frames
- ❑ RX station listens for preamble to detect:
  - Presence of frame.
  - If frame is present, determines timing delay of the remaining frame

# 802.11g PLCP Header Details



## □ PLCP header details:

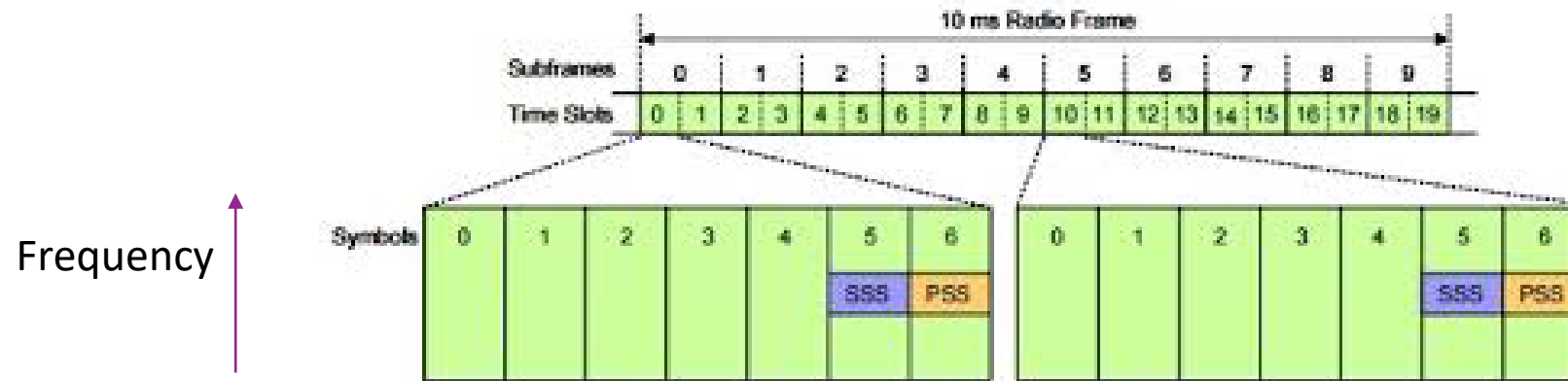
- Preamble: Used for initial detection, synchronization, channel estimation
- Rate: Signals MCS for service bits & MAC PDU
- Length: Number of OFDM symbols in frame
- Service: Scrambler sync

## □ MAC header: Contains MAC layer control info

- Segmentation, MAC addresses, ...

## □ MAC FCS: frame check sum (used to detect errors)

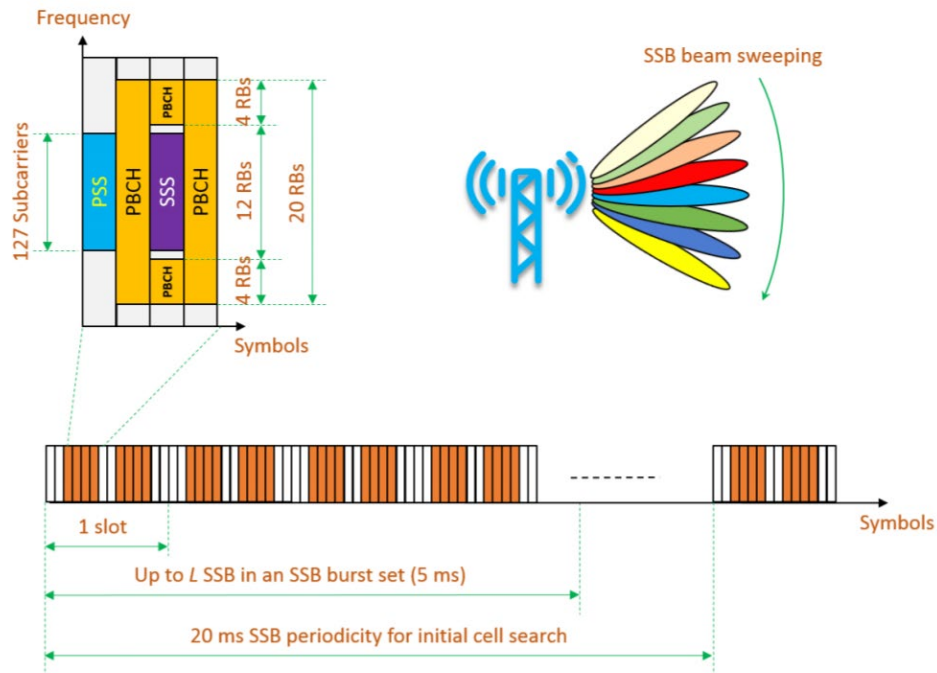
# Ex 2: LTE Downlink Primary Sync Signal (PSS)



- ❑ Each cell transmits periodically PSS
  - Narrowband, short (71.4  $\mu$ s x 1.08 MHz)
  - One of 3 PSS signals
- ❑ Once PSS is detected, mobile (UE) knows frame timing
  - Decodes subsequent signals SSS, broadcast, ...



# Ex. 3. 5G New Radio Beam Sweeping



- ❑ Directional synchronization for mmWave
- ❑ Transmit multiple SS Burst
  - One in each direction
- ❑ MmWave typically use 120 kHz subcarrier spacing
- ❑ With 120 kHz SCS:
  - SSB = 4 OFDM symbols =  $35.7 \mu s$
  - Each SSB, contains a PSS
  - PSS time duration = 1 OFDM symbol =  $8.92 \mu s$
  - Bandwidth = 127 SC = 15.24 MHz
  - Up to 64 SS Bursts / burst period
  - Typical SSB periodicity = 20 ms
  -

# Simple Synchronization Model

- ❑ TX sends a preamble / synchronization signal

- $x[n]$ ,  $n = 0, 1, 2, \dots, N - 1$
- Complex baseband samples.
- Sample rate  $\frac{1}{T}$

- ❑ If signal is present at RX:

$$y[n] = hx[n - k] + w[n]$$

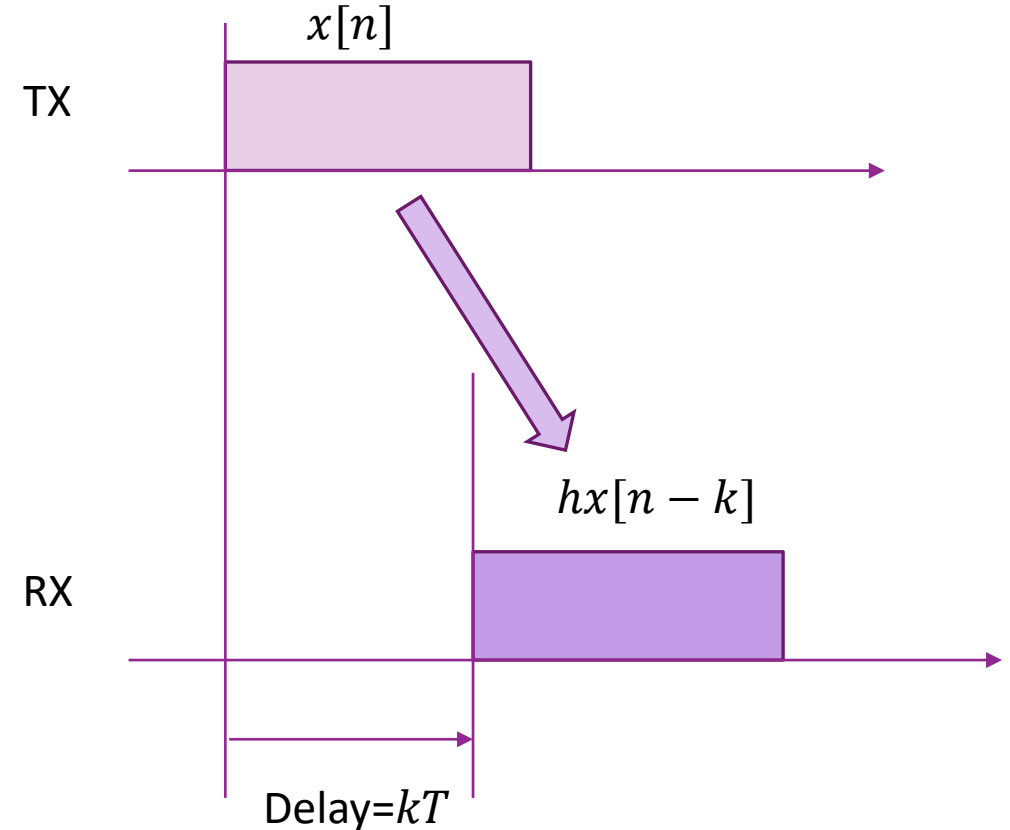
- $h$ : Complex channel gain
- $k$ : Integer delay

- ❑ Problem detect if signal is present or not.

- If so, what is the delay

- ❑ For now, we assume:


- Integer delays, no multipath
- Will address these issues later



# Outline

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☐ Detection and Synchronization Problem

 ☐ Hypothesis Testing

☐ Match Filtering for Detection

☐ Match Filtering Convolution

☐ Automatic Gain Control (AGC)

☐ Matched Filtering as a Generalized Likelihood Ratio Test

# Hypothesis Testing

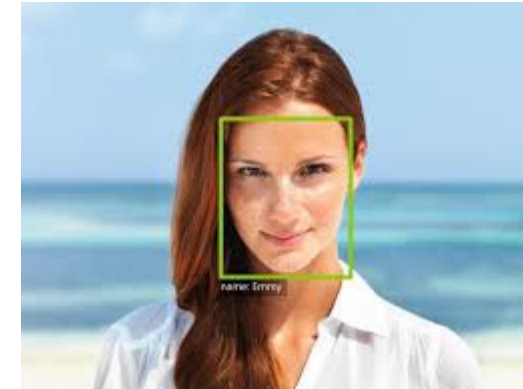
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- ❑ Classic problem in statistics or decision theory
- ❑ Observe data  $y$
- ❑ Two possible hypotheses for data
  - $H_0$ : Null hypothesis
  - $H_1$ : Alternate hypothesis
- ❑ Model statistically:
  - $p(y|H_i), i = 0,1$
  - Assume some distribution for each hypothesis
  - Each density is the **likelihood** of  $y$
- ❑ **Problem**: Determine which hypothesis is true given data  $y$

# Applications

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- ❑ Many applications
- ❑ Pattern recognition:
  - Does this image contain a face or not?
  - Is this person X?
- ❑ Detection:
  - Is the transmitted bit 0 or 1?
- ❑ This lecture: Is a signal present or not?



# Simple Example

## □ Scalar Gaussian

- $H_0: y = -A + w$
- $H_1: y = A + w,$
- $w \sim N(0, \sigma^2)$

## □ In this case:

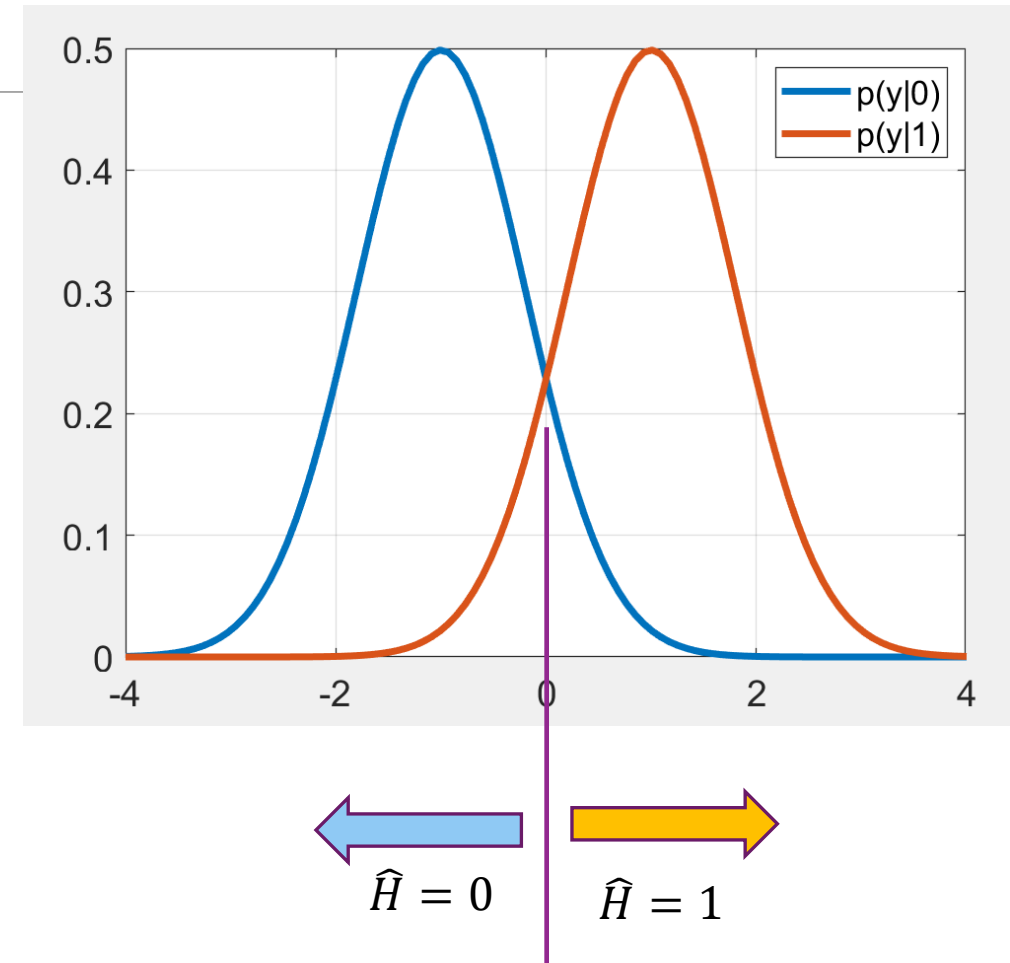
- $p(y|H_0) = N(y| -A, \sigma^2)$
- $p(y|H_1) = N(y| A, \sigma^2)$

## □ Saw this earlier in BPSK transmissions

## □ Max likelihood detector from earlier

- Selects the most likely hypothesis
- In this case

$$\hat{H} = \arg \max_j p(y|H = j) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$



# Types of Errors

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- ❑ For binary detection problems, there are two errors:
  - **Type I error** (False alarm): Decide  $H_1$  when  $H_0$
  - **Type II error** (Missed detection): Decide  $H_0$  when  $H_1$
- ❑ In many problems, the consequences of these errors is different
- ❑ Example: Medical diagnosis
  - False alarm: You tell the patient he is ill, when he is fine
  - Missed detection: You miss the illness
  - Consequences are different
- ❑ Given detector, we define two error probabilities:
  - False alarm probability:  $P_{FA} = P(\hat{H} = 1|H = 0)$
  - Missed detection probability:  $P_{MD} = P(\hat{H} = 0|H = 1)$

# Likelihood Ratio Test

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□ We can tradeoff the error probabilities with a likelihood ratio test:

□ Likelihood ratio test (LRT)

$$\hat{H} = 1 \Leftrightarrow \frac{p(x|H_1)}{p(x|H_0)} \geq \gamma$$

- $\gamma$  is an adjustable **threshold**
- Increasing  $\gamma \Rightarrow$  Lowers  $P_{FA}$ , but lowers  $P_D$

□ Often performed in log domain

$$\hat{H} = 1 \Leftrightarrow L^*(x) = \log \frac{p(x|H_1)}{p(x|H_0)} \geq \gamma'$$

□ Note that  $\gamma = 0$  corresponds to maximum likelihood detector



# Gaussian Example

## □ Scalar Gaussian case:

- $p(y|H_0) = N(y|-A, \sigma^2) = C \exp(-\frac{(y+A)^2}{2\sigma^2})$
- $p(y|H_1) = N(y|A, \sigma^2) = C \exp(-\frac{(y-A)^2}{2\sigma^2})$

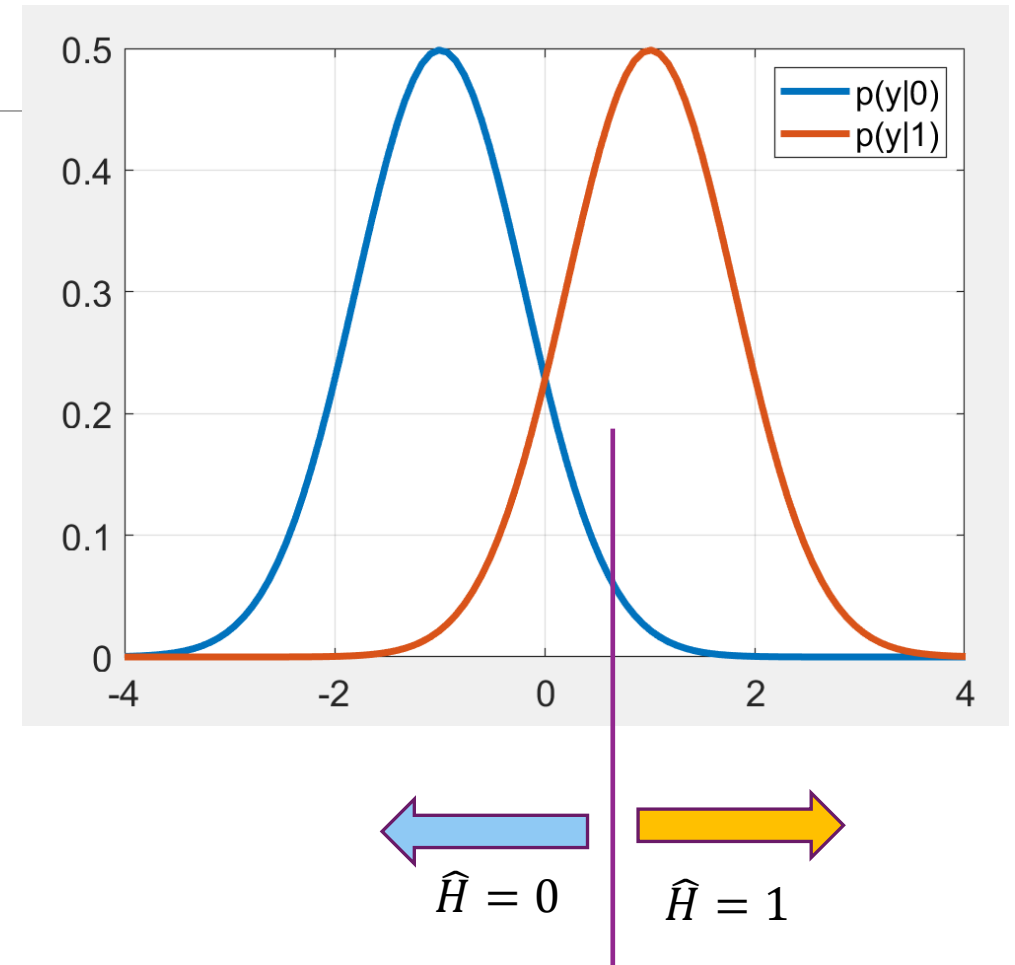
## □ Log likelihood ratio:

$$\begin{aligned} L(y) &:= \ln \frac{p(y|H_1)}{p(y|H_0)} \\ &= \frac{1}{2\sigma^2} [(y+A)^2 - (y-A)^2] = \frac{2Ay}{\sigma^2} \end{aligned}$$

## □ LRT: $\hat{H} = 1$ if and only if

$$L(y) \geq \gamma \Leftrightarrow y \geq t = \frac{\gamma\sigma^2}{2A}$$

- $t$  is an adjustable threshold



# Computing Error Probabilities

From previous slide, LRT detector is:

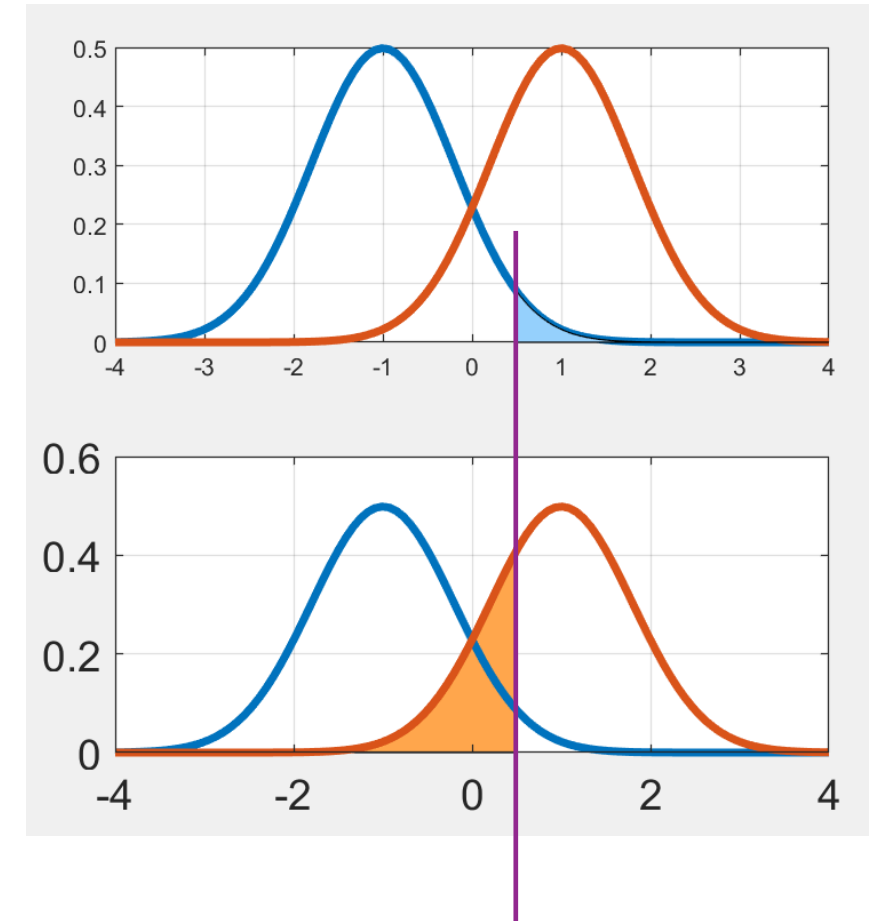
$$\hat{H} = \begin{cases} 1 & y \geq t \\ 0 & y < t \end{cases}$$

FA probability:

- $P_{FA} = P(\hat{H} = 1|H = 0) = P(y \geq t|H = 0) = \int_t^{\infty} p(y|0)dy$
- This is the area under the curve (blue)
- For Gaussian:  $P_{FA} = Q\left(\frac{t-A}{\sigma}\right)$

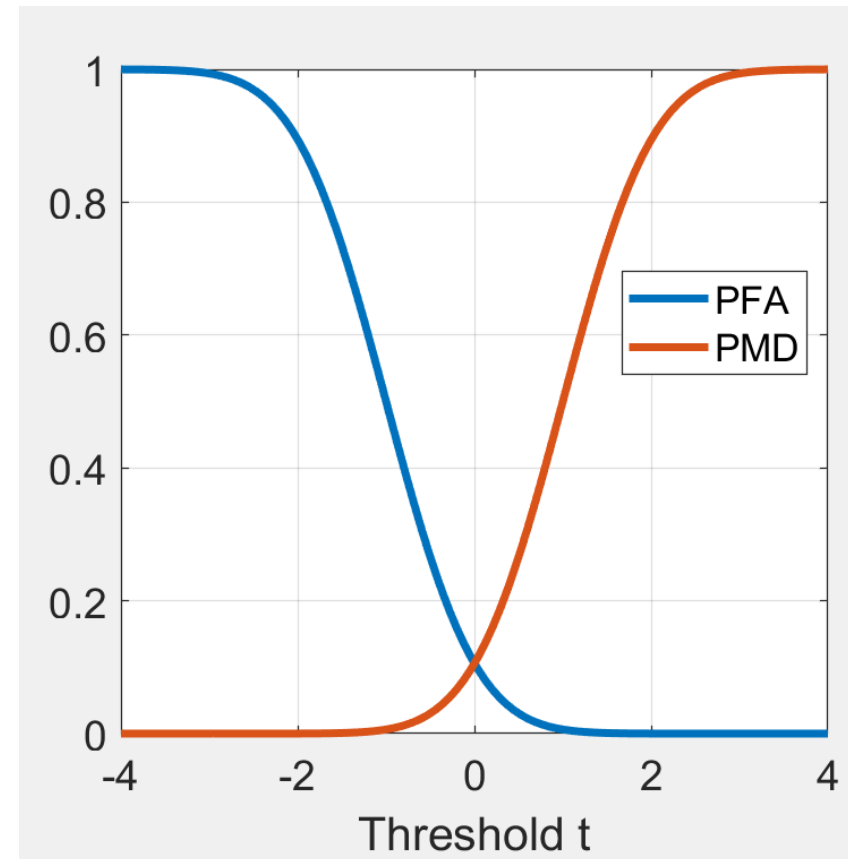
MD probability

- $P_{MD} = P(\hat{H} = 0|H = 1) = P(y < t|H = 1) = \int_{-\infty}^t p(y|1)dy$
- This is the area under the curve (orange)
- For Gaussian:  $P_{MD} = 1 - Q\left(\frac{t-A}{\sigma}\right)$



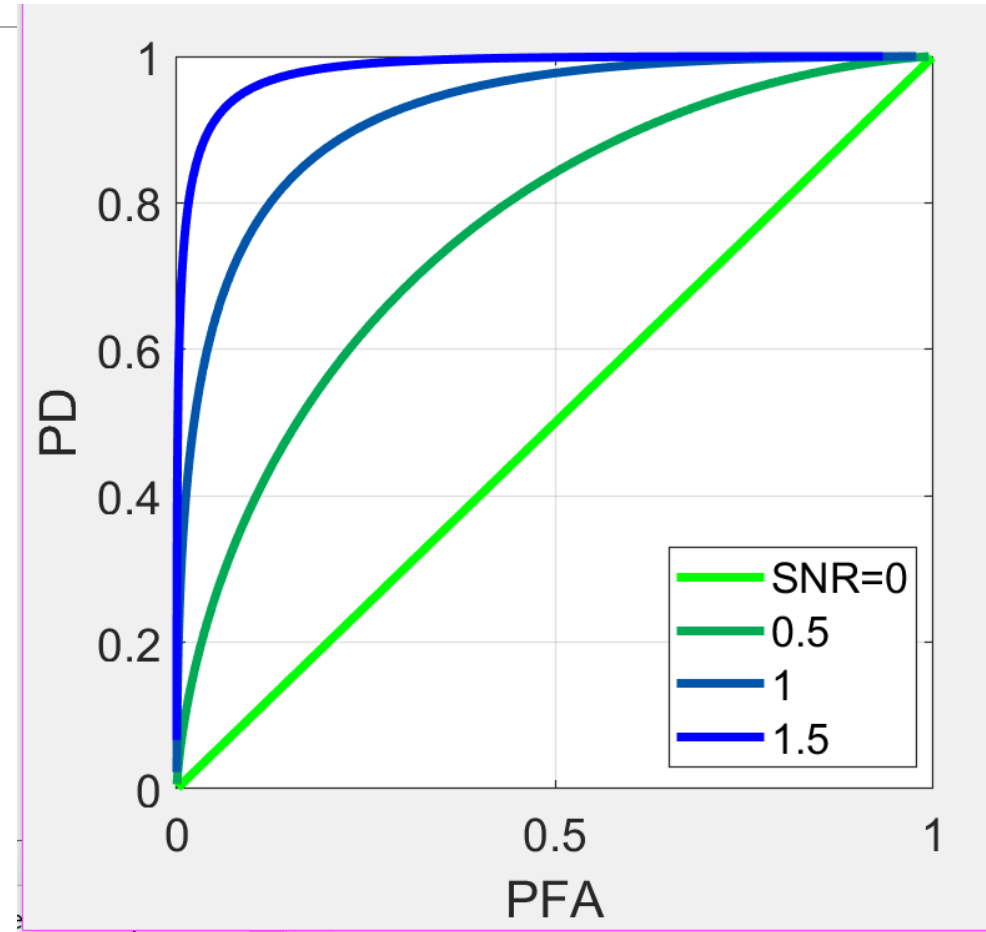
# Tradeoff

- ❑ Tradeoff between  $P_{FA}$  and  $P_{MD}$ 
  - $P_{FA} = Q\left(\frac{t+A}{\sigma}\right)$
  - $P_{MD} = 1 - Q\left(\frac{t-A}{\sigma}\right)$
- ❑ Increasing threshold  $t$ :
  - Decreases false alarms
  - But increases missed detections
- ❑ Selection of optimal threshold
  - Depends on the application
  - What are the relative costs of these errors?



# ROC Curve

- ❑ Receiver operating characteristic
- ❑ Plot of  $P_D$  vs.  $P_{FA}$
- ❑ Trace out:  $(P_{FA}(\gamma), P_D(\gamma))$
- ❑ Random guessing achieves:  
$$P_D + P_{FA} = 1$$
- ❑ Higher the line is better



# Neyman-Pearson Theorem

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- **Theorem:** Suppose that an LRT obtains  $P_{FA} = \alpha$ .  
Then any other test with  $P_{FA}$  will have a  $P_D$  less than or equal to the LRT.
- LRT is the most powerful test
- Obtains best  $P_{FA}$  vs.  $P_D$  performance

# In Class Exercise

## Synchronization In-Class Exercises

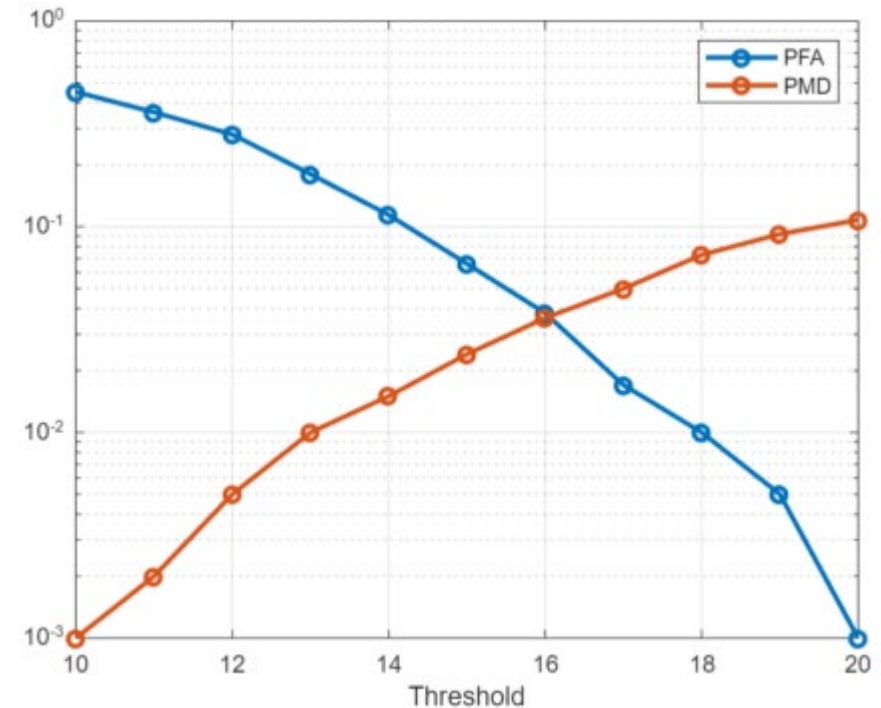
### Hypothesis Testing for Poisson Random Variables

We will simulate hypothesis testing for discriminating between two Poisson distributions. This type of detector occurs in optical systems where the receiver counts the number of photons. The unknown variable is  $x=0$  or  $1$ . We receive a discrete random variable  $y$  with conditional probability:

- $P(y|x=0)$  is Poisson with rate  $\lambda_{00}$ .  $P(x=0)=p_0$
- $P(y|x=1)$  is Poisson with rate  $\lambda_{01}$ .  $P(x=1)=p_1$


The parameters are below

```
lam0 = 10; % Rate when x=0
lam1 = 20; % Rate when x=1
p0 = 0.8; % P(x=0)
p1 = 1-p0; % P(x=1)
```



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- Sample rate  $\frac{1}{T}$

- ❑ If signal is present at RX:

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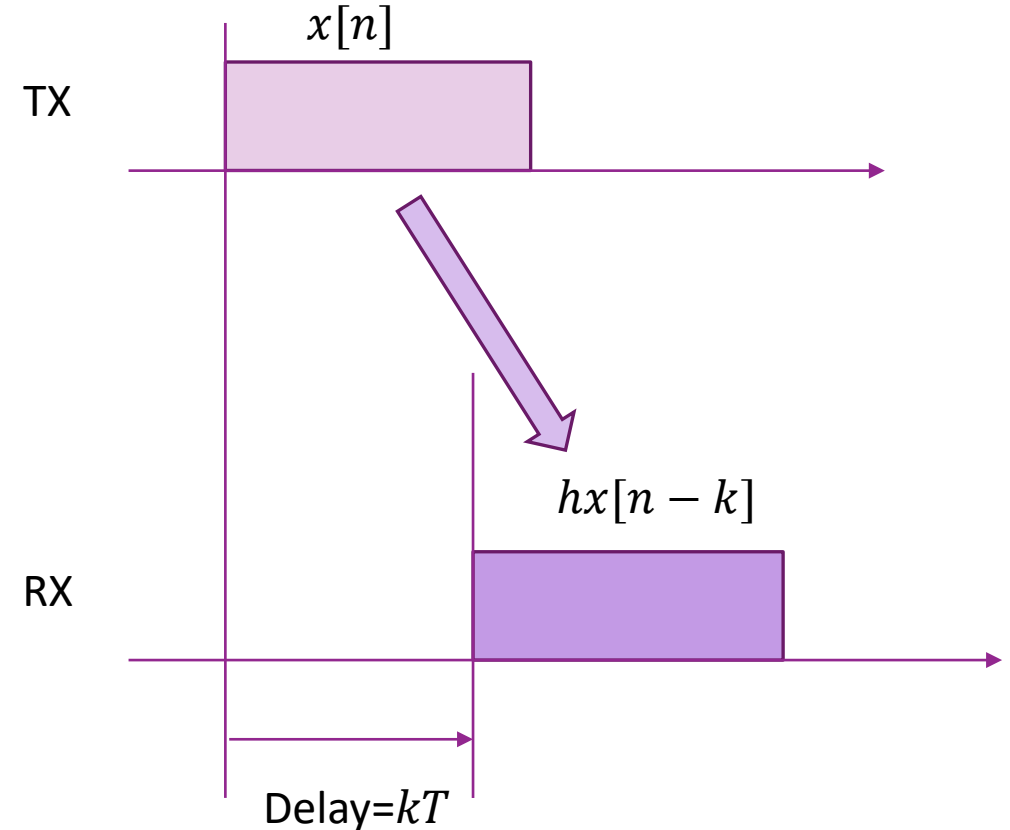
- $h$ : Complex channel gain
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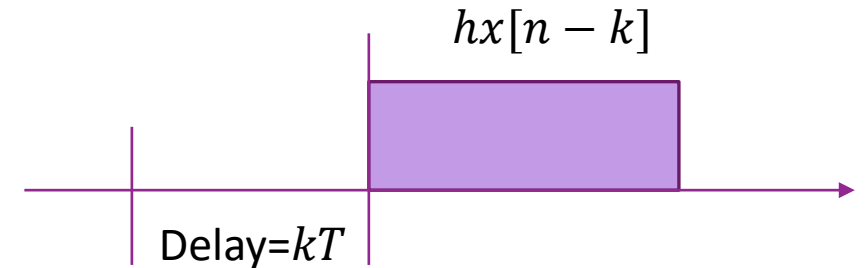
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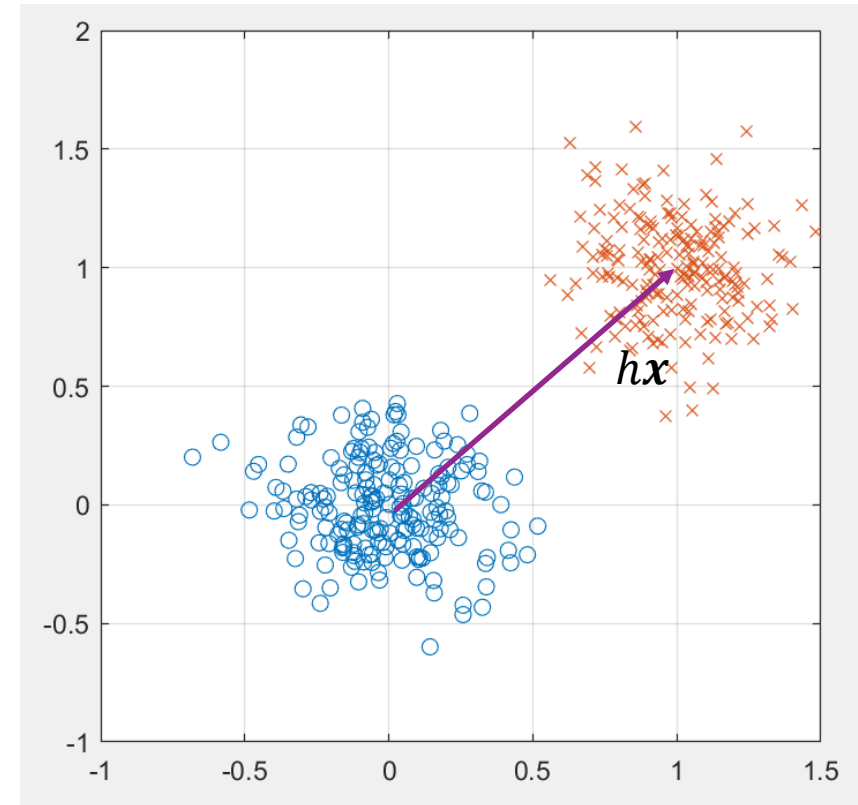
# Detect as a Hypothesis Test

- At each delay  $k$ , we consider two hypotheses:
- $H_1$ : Signal is present:
$$r[n] = hx[n - k] + w[n],$$
  - $h$  is a complex, baseband channel gain
  - Recall that we are assuming a single path channel (for now)
- $H_0$ : Signal is absent:
$$r[n] = w[n]$$
- In both cases, assume  $w[n]$  is white noise:
  - $w[n] \sim \mathcal{CN}(0, N_0)$



# Hypothesis Test in Vector Form

- Without loss of generality, consider delay  $k = 0$
- Let  $\mathbf{r}$  be the vector of RX samples:
  - $\mathbf{r} = [r[0], \dots, r[N - 1]]^T$
- Write two hypotheses in vector form:
  - $H_1: \mathbf{r} = h\mathbf{x} + \mathbf{w}$  [Signal present]
  - $H_0: \mathbf{r} = \mathbf{w}$  [Signal absent]
- Geometrically:



# Match Filter Detector

## □ Hypotheses:

- $H_1$ :  $\mathbf{r} = h\mathbf{x} + \mathbf{w}$  [Signal present]
- $H_0$ :  $\mathbf{r} = \mathbf{w}$  [Signal absent]

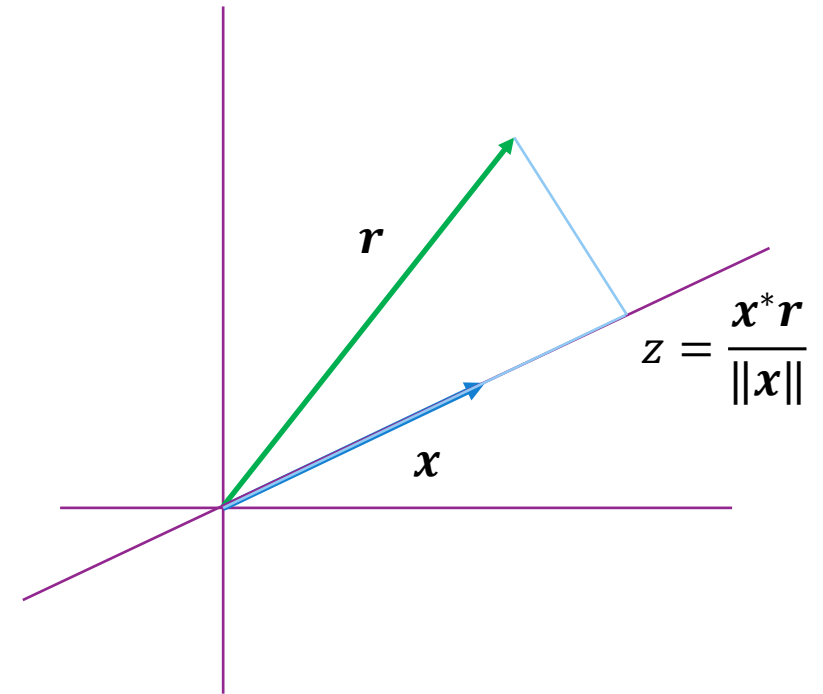
## □ Match filter energy detector:

- Project RX signal to TX waveform

$$z = \frac{\mathbf{x}^* \mathbf{r}}{\|\mathbf{x}\|}$$

- Measure energy:  $y = |z|^2$
- Declare  $\hat{H} = \begin{cases} 1 & y \geq t \\ 0 & y < t \end{cases}$
- $t$  is a threshold

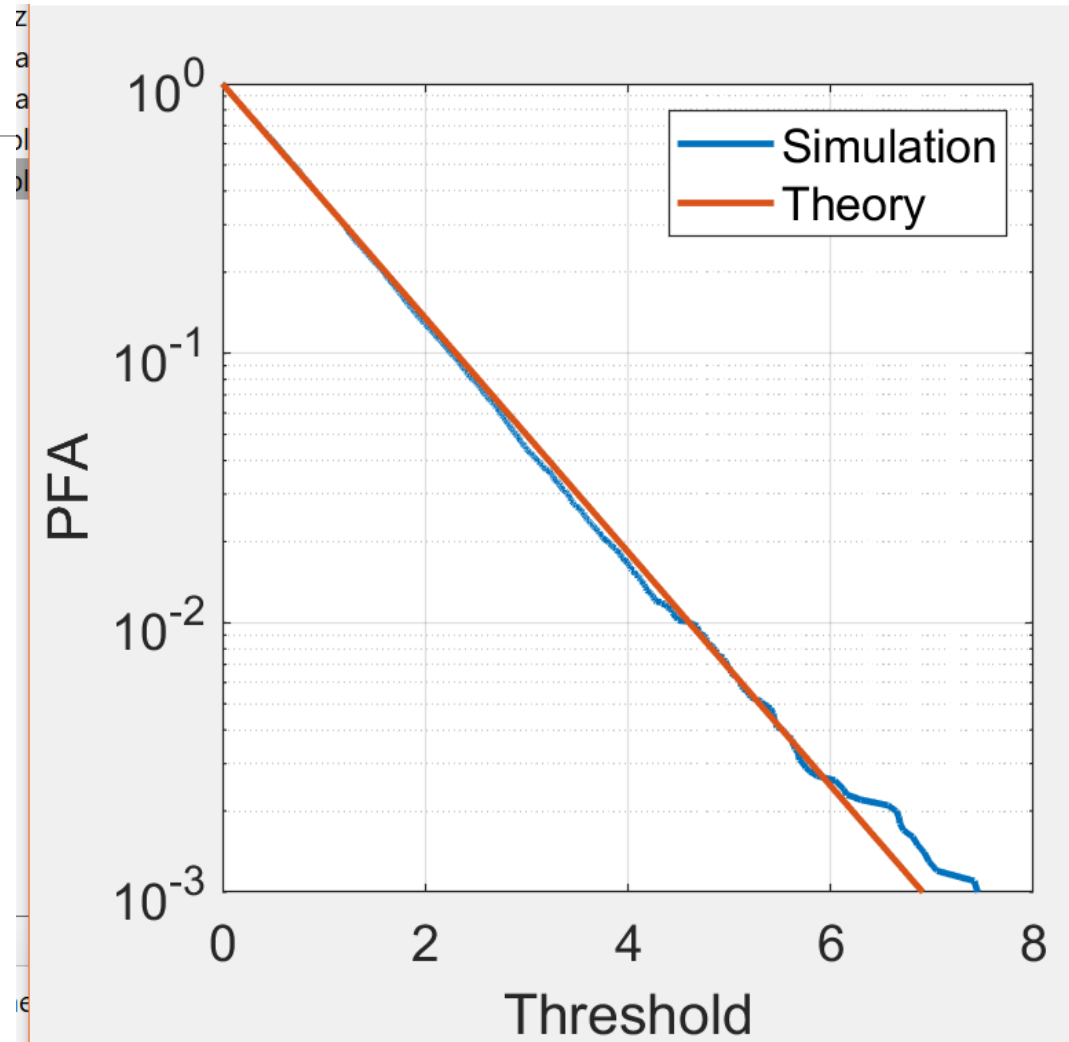
## □ Later we will show this is the optimal hypothesis test



# False Alarm

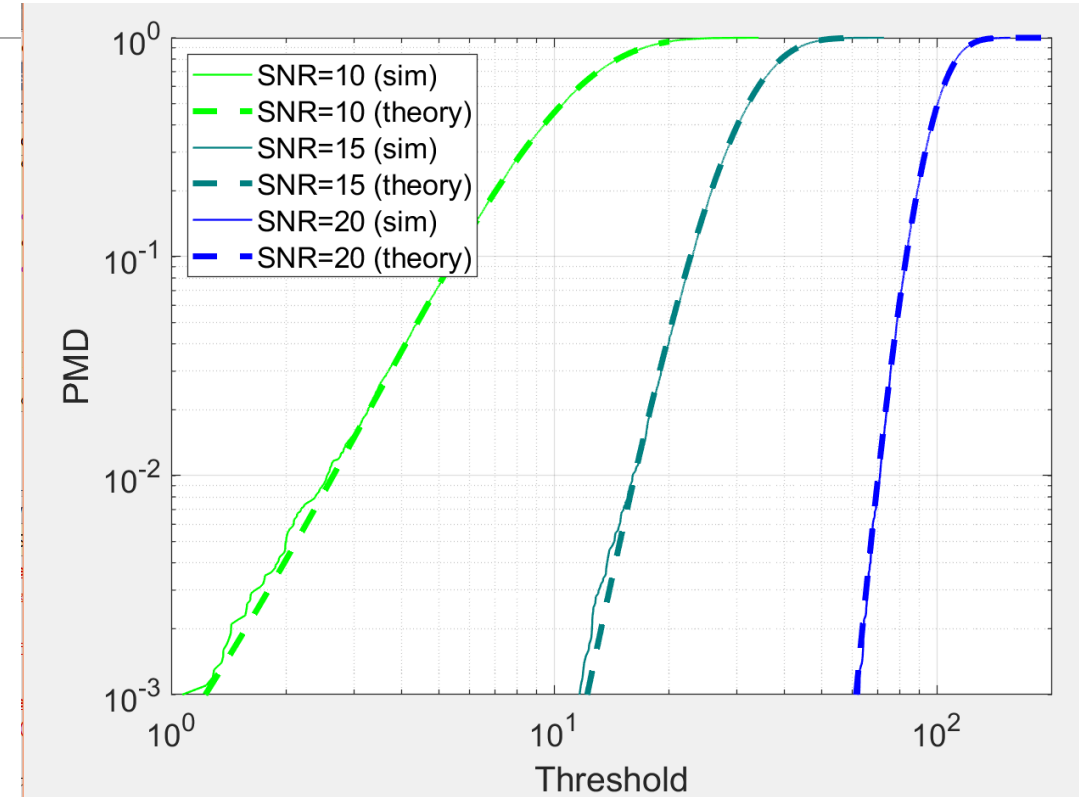
## False alarm

- Under  $H_0: \mathbf{r} = \mathbf{w}$ ,  $\mathbf{w} \sim \mathcal{CN}(0, N_0 \mathbf{I})$
- Statistic  $z = \frac{\mathbf{x}^* \mathbf{r}}{\|\mathbf{x}\|} = \frac{\mathbf{x}^* \mathbf{w}}{\|\mathbf{x}\|}$
- This is a linear function of a Gaussian
- $E(z) = \frac{\mathbf{x}^* E(\mathbf{w})}{\|\mathbf{x}\|} = 0$ ,
- $E|z|^2 = \frac{\mathbf{x}^* E(\mathbf{w} \mathbf{w}^*) \mathbf{x}}{\|\mathbf{x}\|^2} = N_0 \frac{\mathbf{x}^* \mathbf{x}}{\|\mathbf{x}\|^2} = N_0$
- Hence,  $z \sim \mathcal{CN}(0, N_0)$
- Hence  $y = |z|^2$  is exponential with  $E(y) = N_0$
- $P_{FA} = P(y \geq t) = e^{-t/N_0}$



# Missed Detection

- Under  $H_0: \mathbf{r} = h\mathbf{x} + \mathbf{w}$ ,  $\mathbf{w} \sim \mathcal{CN}(0, N_0 \mathbf{I})$
- Statistic  $z = \frac{\mathbf{x}^* \mathbf{r}}{\|\mathbf{x}\|} = \frac{\mathbf{x}^* \mathbf{w}}{\|\mathbf{x}\|}$
- Similar to FA calculation:  $z \sim \mathcal{CN}(A, N_0)$ ,  $A = h\|\mathbf{x}\|$
- Can show:  $y = |z|^2 \sim \frac{N_0}{2} \nu$ 
  - $\nu$  is a **non-central chi squared** with 2 degrees of freedom
  - Non-centrality parameter  $\lambda = \frac{2|h|^2\|\mathbf{x}\|^2}{N_0} = 2 \text{ SNR}$



# Simulation

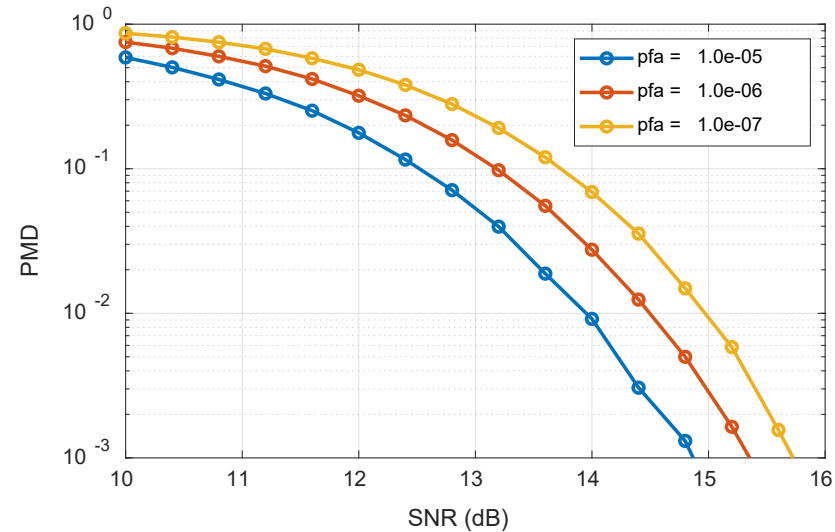
```
% FA targets to test|
pfaTest = [1e-5,1e-6,1e-7];
nfa = length(pfaTest);
legstr = cell(nfa,1);
for ifa = 1:nfa
    % Compute FA target
    pfaTgt = pfaTest(ifa);
    t = -log(pfaTgt);

    % Measure PMD
    ntest = 1e5;
    snrTestTheory = linspace(10,18,21)';
    nsnr = length(snrTestTheory);
    pmdTheory = zeros(nsnr,1);

    for isnr = 1:nsnr
        snr = snrTestTheory(isnr);
        A = 10.^(0.05*snr);
        z = A + (randn(ntest,1)+1i*randn(ntest,1))/sqrt(2);
        rho = abs(z).^2;
        pmdTheory(isnr) = mean(rho < t);
    end

    semilogy(snrTestTheory, pmdTheory, 'o-', 'Linewidth', 2);
    hold on;

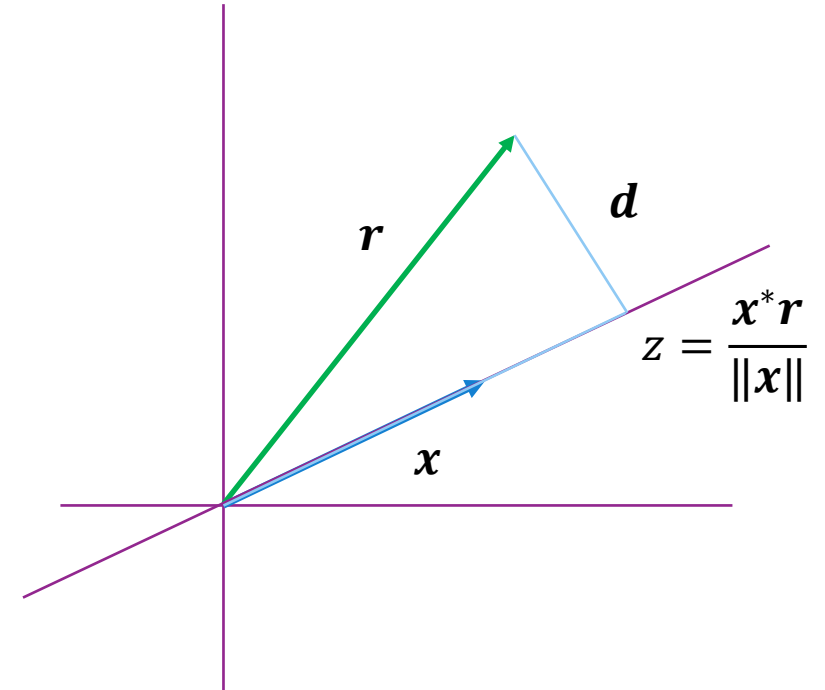
    legstr{ifa} = sprintf('pfa = %9.1e', pfaTgt);
end
```



- ☐ Theoretically calculated threshold based on PFA target
- ☐ Simulate PMD based on SNR

# Noise Estimation

- ❑ Threshold  $t = -N_0 \ln P_{FA}^{TGT}$
- ❑ Requires we know noise energy  $N_0$
- ❑ How do we estimate this?
- ❑ Suppose  $\mathbf{r} = h\mathbf{x} + \mathbf{w}$
- ❑ Consider residual signal:  $\mathbf{d} = \mathbf{r} - z \frac{\mathbf{x}}{\|\mathbf{x}\|}$ ,  $z = \frac{\mathbf{x}^* \mathbf{r}}{\|\mathbf{x}\|^2}$ 
  - Component of  $\mathbf{r}$  not spanned by  $\mathbf{x}$
  - $\mathbf{d}$  is the projection of  $\mathbf{w}$  onto an  $N - 1$  dim space
  - Can show that  $E\|\mathbf{d}\|^2 = (N - 1)N_0$
  - Take noise estimate:  $\hat{N}_0 = \frac{1}{N-1} \|\mathbf{d}\|^2$



# Noise Estimation 2

□ Use threshold with estimate noise  $t = -\hat{N}_0 \ln P_{FA}^{TGT}$

□ Detector takes  $\hat{H} = 1$  if

$$|z|^2 \geq t = -\hat{N}_0 \ln P_{FA}^{TGT} = c \|\mathbf{d}\|^2,$$

- $c = -\frac{\ln P_{FA}^{TGT}}{N-1}$

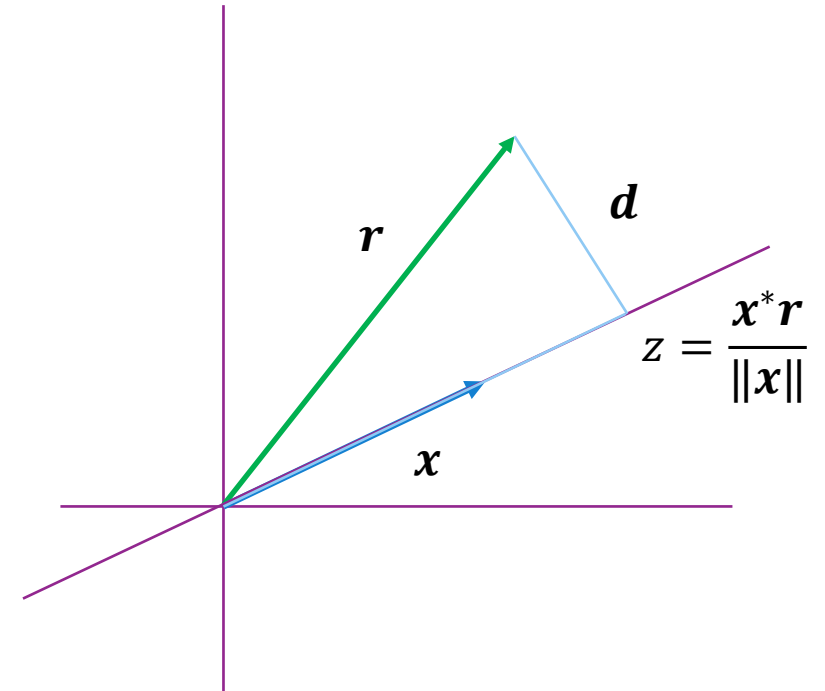
- But  $\|\mathbf{d}\|^2 = \|\mathbf{r}\|^2 - |z|^2$

- So test is equivalent to:

$$\frac{|z|^2}{\|\mathbf{r}\|^2 - |z|^2} \geq c \Leftrightarrow \rho = \frac{|z|^2}{\|\mathbf{r}\|^2} \geq \frac{c}{1+c} = \gamma$$

- Note  $\rho = \frac{|z|^2}{\|\mathbf{r}\|^2} = \frac{|\mathbf{x}^* \mathbf{r}|^2}{\|\mathbf{r}\|^2 \|\mathbf{x}\|^2}$  = fraction of energy in direction  $\mathbf{x}$

□ Conclusion: With noise estimation MF is equivalent to





# In Class Exercise

## Signal Detection

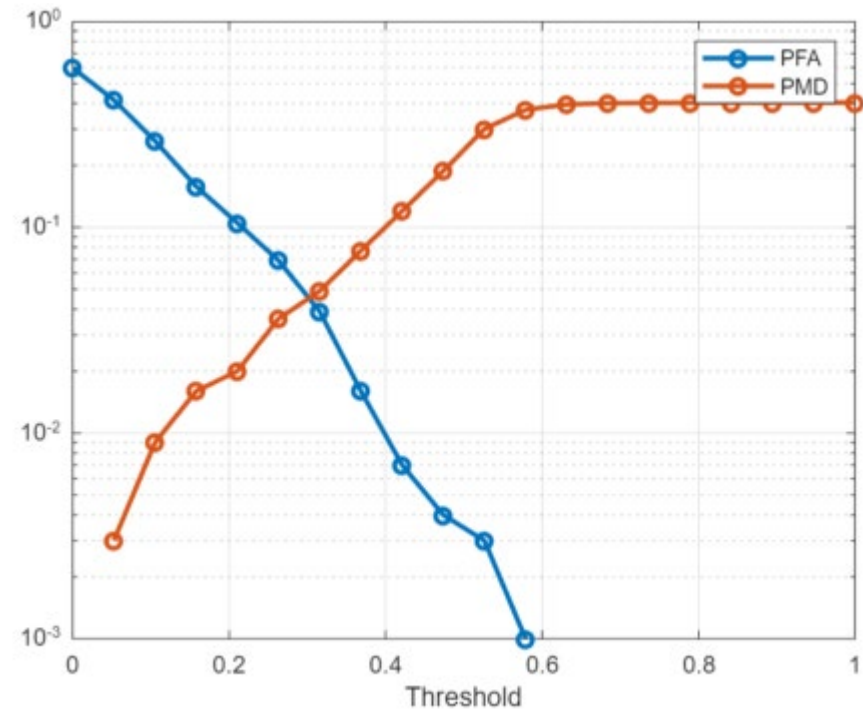
In this section, we will simulate a simple signal detection. We suppose we have a complex baseband signal of length  $n_s$ . The unknown variable is  $u=0$  or  $1$  depending if a signal is present. The RX signal,  $r$ , can then be modeled as:

$$r(j) = u \cdot h \cdot x(j) + w(j), \quad j = 1, \dots, n_s$$

where the channel gain and noise are modeled as complex Gaussians:


$$w(j) \sim \text{CN}(0, w_{\text{var}}), \quad h \sim \text{CN}(0, h_{\text{var}})$$

```
p0 = 0.6; % P(u=0)=1-P(u=1)
ns = 8; % Signal length
snr = 10; % hvar/wvar
hvar = 1; % Mean channel gain
wvar = db2pow(-snr); % Noise energy per sample
ntrial = 1000; % Number of trials to test
x0 = exp(1i*rand(ns,1)*2*pi); % Random true signal
```



# Outline

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- ☐ Detection and Synchronization Problem
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- ☐ Matched Filtering as a Generalized Likelihood Ratio Test

# Match Filtering with Unknown Delay

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- ❑ Synchronization signal  $x[n], n = 0, 1, \dots, N - 1$
- ❑ RX signal at delay  $k$ :
  - $r[n] = hx[n - k] + w[n]$
- ❑ Problem: Detect if signal is present. If so, what is the delay  $k$ ?
- ❑ Match filter (without normalization) at delay  $k$  is:
$$z[k] = \sum_n r[n + k]x^*[n]$$
- ❑ Hypothesis test:
  - $|z[k]|^2 \geq t \Rightarrow$  Detect signal at delay at  $k$

# Further Analysis Details

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- ❑ We need to examine three key practical issues that degrade performance
  - ❑ Preamble auto-correlation
  - ❑ Multi-path
  - ❑ Carrier offset

# Match Filtering as a Convolution

- Match filter (without normalization) at delay  $k$  is:

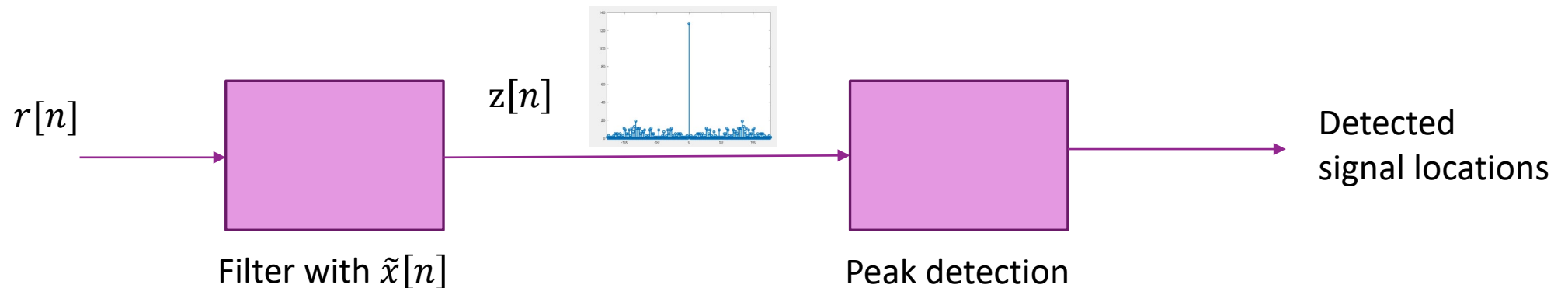
$$z[k] = \sum_n r[n+k]x^*[n]$$

- Define **adjoint** signal:  $\tilde{x}[n] = x^*[-n]$

- Complex conjugate and time reversal

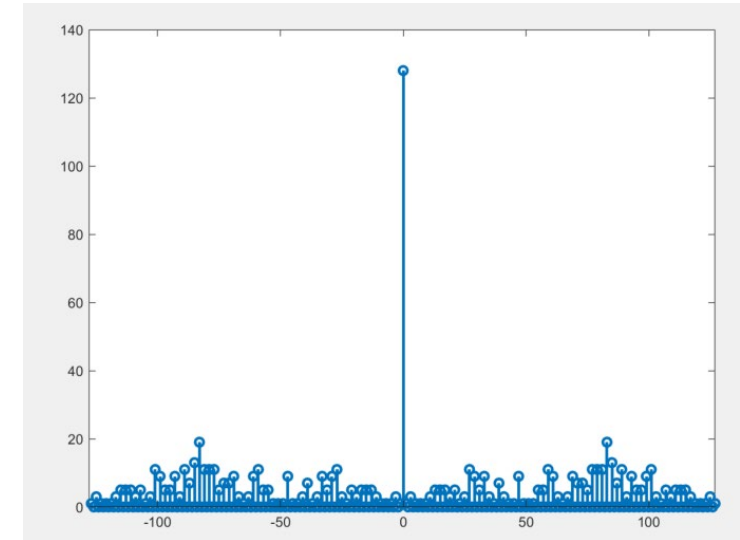
- MF output can be computed via a convolution:

- $z[k] = \sum_n r[n+k]x^*[n] = \sum_n r[n+k]\tilde{x}[-n] = \sum_n r[k-n]\tilde{x}[n] = (r * \tilde{x})[k]$



# Signal Auto-Correlation

- ❑ Consider what happens with no noise:
  - $r[n] = hx[n - k_0]$ ,  $k_0$  = “True” delay
- ❑ Run match filter:  $z[k] = (r * \tilde{x})[k]$
- ❑ Can show output is:  $z[k] = hR_x[k - k_0]$ 
  - $R_x[\ell]$  = autocorrelation of transmitted signal
  - $R_x[\ell] = \sum_n x[n]x^*[n - \ell]$
- ❑ Since we want  $z[k]$  small for  $k \neq k_0$ , we want:  
 $R_x[\ell] \approx 0$  for  $\ell \neq 0$
- ❑ Many sequences with low auto-correlation
  - Golay, Walsh, ....



Auto-correlation of Golay 128 sequence  
Used in 802.11ad preamble

# Multipath

- Up to now we have assumed that there is a single path:

$$r[n] = hx[n - k_0]$$

- But, in reality there is often multipath:

$$r[n] = \sum_k h[k]x[n - k]$$

- Due to multi-path in channel and pulse shape filtering

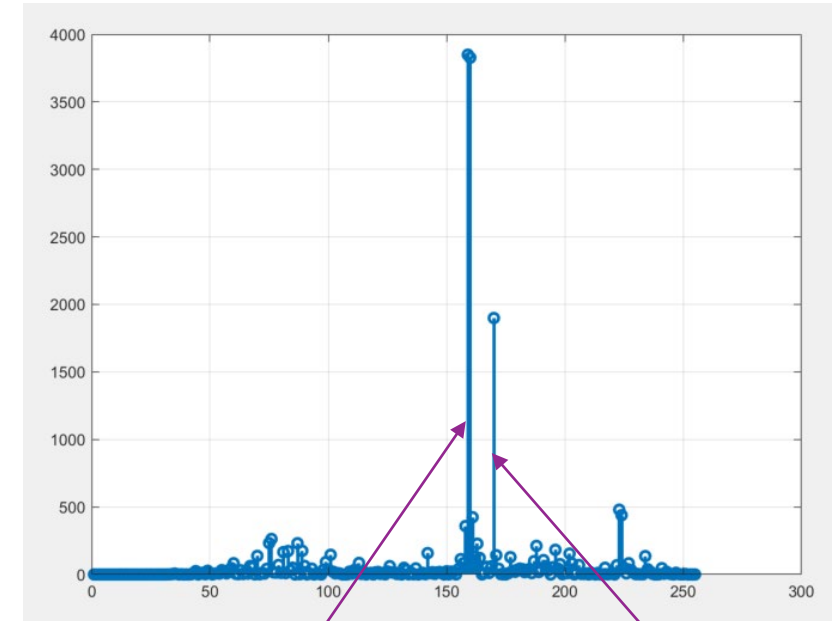
- Match filter has delayed copies of auto-correlation:

$$z[n] = \sum h[k]R_x[n - k]$$

- One peak in MF output for each path

Ex: Two path channel

$$h[n] = \text{sinc}(n - 0.5) + 0.5\text{sinc}(n - 10.2)$$



Path at  $k = 0.5$       Path at  $k = 10.2$

# Frequency Offsets

❑ When initially searching for a preamble, there may be a significant carrier offset

❑ Causes a phase rotation in samples:

$$r[n] = e^{i\theta n} h x[n - k] + w[n]$$

- $\theta$  is the phase rotation per sample
- $\theta = \Delta f T$ ,  $\Delta f$  = frequency error,  $T$  = sampling rate

❑ Must integrate over range where phase does not change significantly

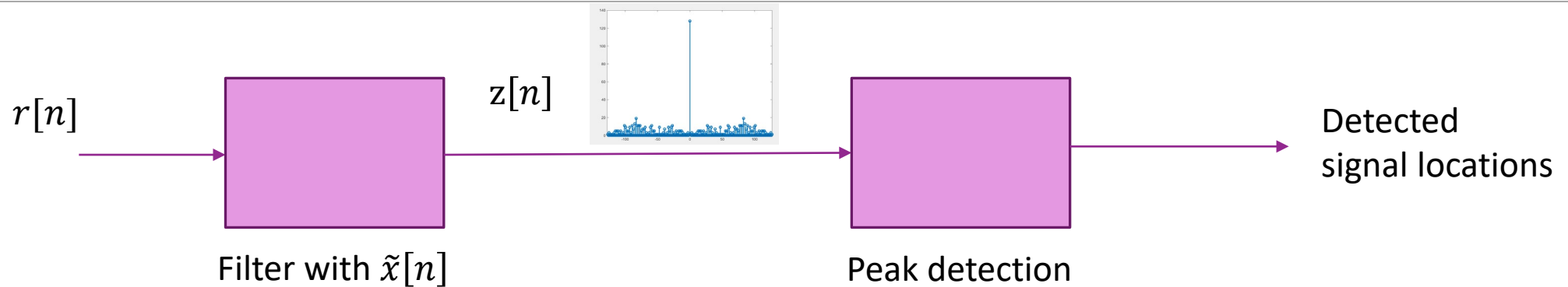
- Pre-amble length must be  $N \ll \frac{1}{\Delta f T}$

❑ Example: Suppose the carrier offset = 10 ppm,  $f_c = 60$  GHz and  $\frac{1}{T} = 1.76$  Gs/s

- $\Delta f T = \frac{(10)^{-5}(60)(10)^9}{1.76(10)^9} = 3.4(10)^{-4} \Rightarrow \frac{1}{\Delta f T} \approx 2.9(10)^3$  samples
- In time duration, this is  $\frac{1}{\Delta f} = 1.67$   $\mu$ s
- A very short time before the signal is completely rotated



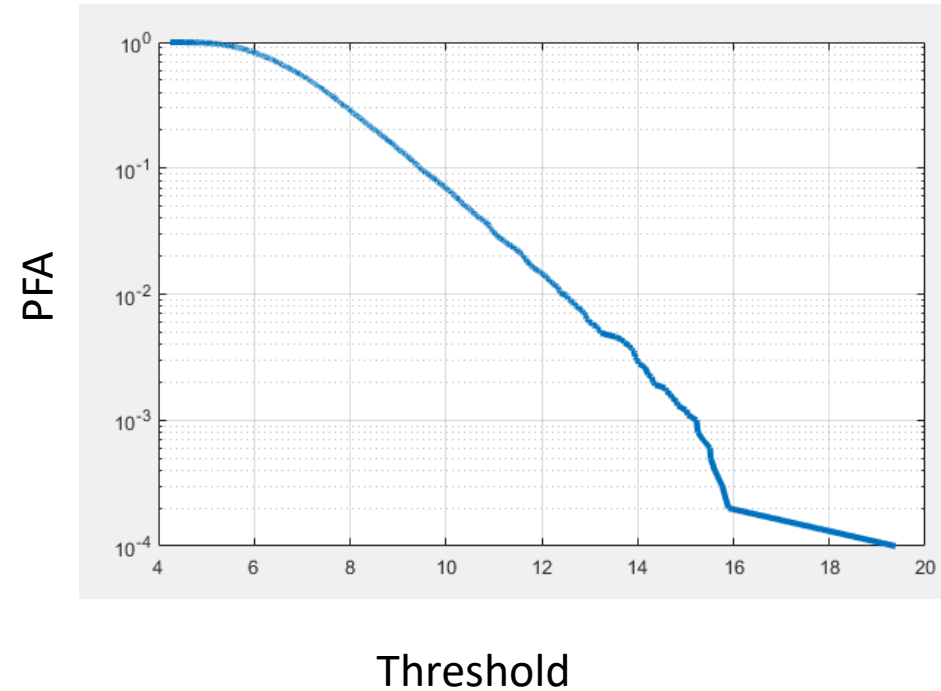
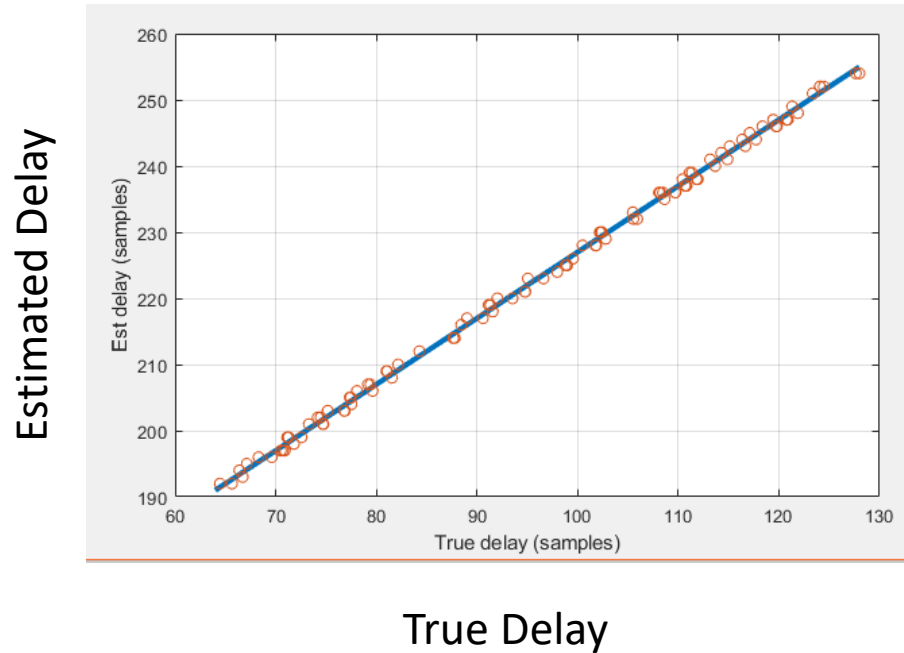
# Detailed Simulation Example



- ❑ Transmit 128 length Golay pre-amble
- ❑ Filter through channel with single (possibly fractional) delay
  - $r[n] = h[n] * x[n] + w[n]$ ,  $h[n] = \text{sinc}(n - \frac{\tau}{T})$
- ❑ Set threshold for FA target of  $10^{-3}$  per 1000 samples
- ❑ Measure MD probability as a function of the SNR

# Calibration

- ❑ Need to calibrate the FA probability and delay offset



# Missed Detection

```
for isnr = 1:nsnr
    % Get the SNR
    snr = snrTest(isnr);
    wvar = 10.^(-0.1*snr)*npre;

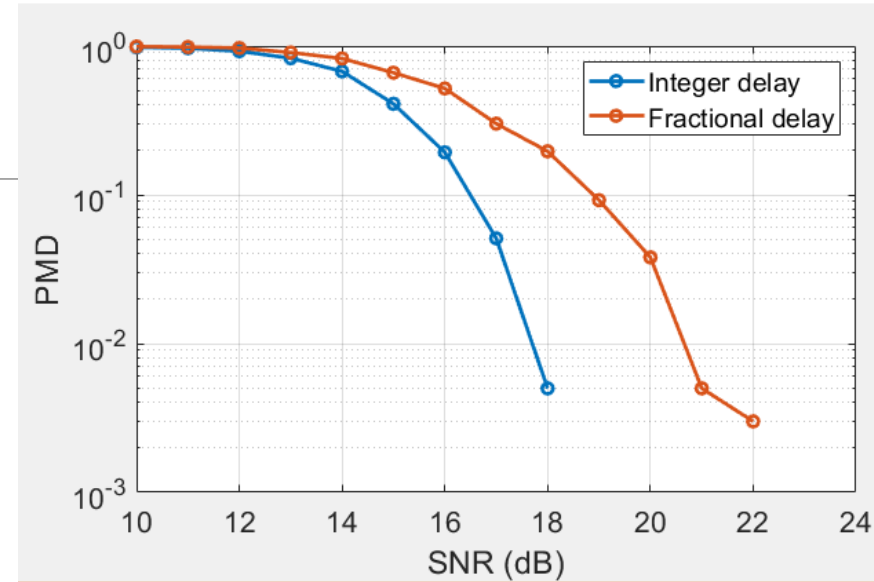
    dly0 = unifrnd(64,128,ntest,1);
    dlyEst = zeros(ntest,1);
    rhoMax = zeros(ntest,1);

    for it = 1:ntest
        % Create a random delay
        gain = exp(1i*2*pi*rand(1));
        x = delaysig(xpre,gain,dly0(it),nsamp);

        % Add noise
        w = (randn(nsamp,1) + 1i*randn(nsamp,1))*sqrt(wvar/2);
        r = x + w;

        % Estimate the delay
        [rhom, im, ~] = predetect(r,xpre,maxdly);
        rhoMax(it) = rhom;
        dlyEst(it) = im - dlyOff;
    end

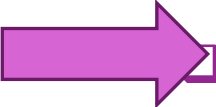
    I = (rhoMax > tfa);
    pmd(isnr) = 1-mean(I);
    dlyerr(isnr) = sqrt(mean((dlyEst(I) - dly0(I)).^2));
    fprintf(1,'SNR = %12.4e PMD=%12.4e dly=%12.4e\n', ...
        snr, pmd(isnr), dlyerr(isnr));
end
```



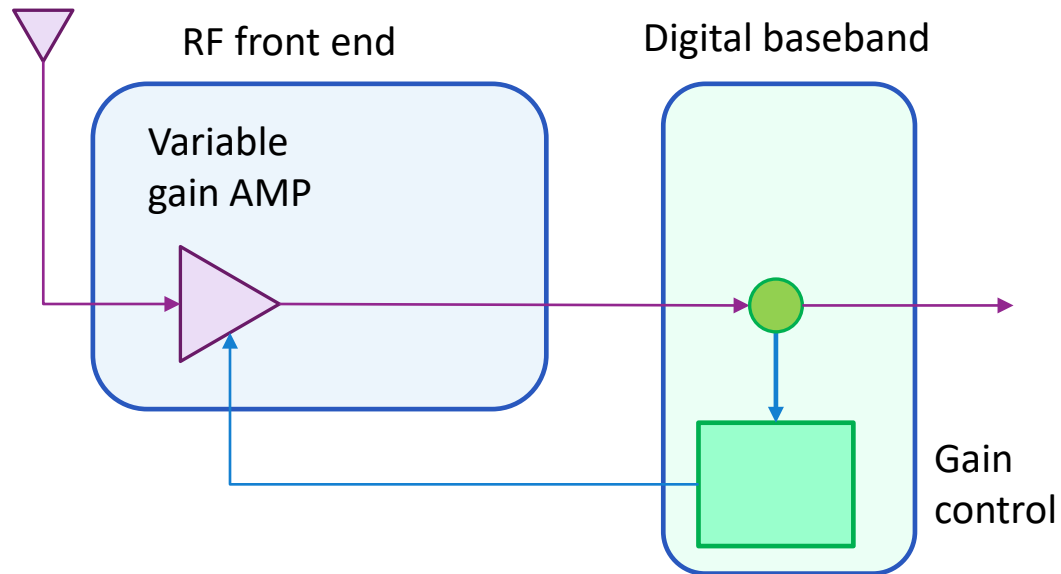
- ❑ Loss of about 3dB with fractional delay offset
- ❑ Signal energy is split in two samples
- ❑ Need to use over-sampling to compensate
  - See lab

# Outline

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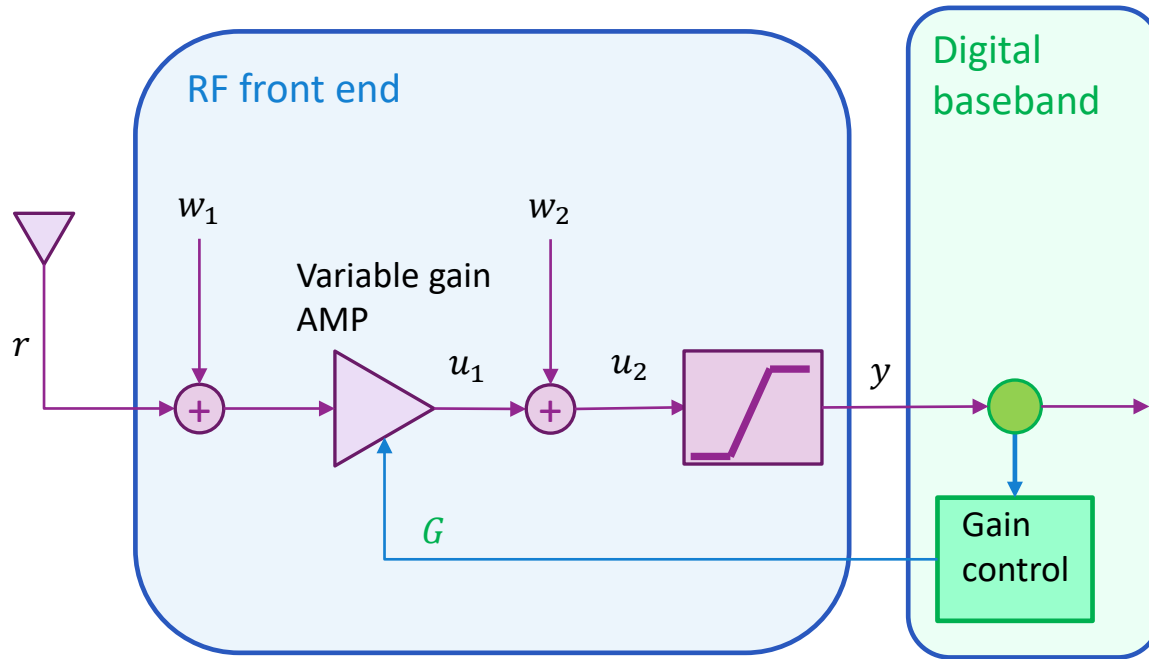
- ☐ Detection and Synchronization Problem
- ☐ Hypothesis Testing
- ☐ Match Filtering for Detection
- ☐ Match Filtering Convolution
-  ☐ Automatic Gain Control (AGC)
- ☐ Matched Filtering as a Generalized Likelihood Ratio Test

# Automatic Gain Control



- ❑ AGC goal: Bring RX signal to “correct” level
- ❑ Tradeoff of two factors
- ❑ Want high gain: Overcome noise after AMP
- ❑ But gain too high  $\Rightarrow$  saturates RX
- ❑ AGC finds optimal level

# Mathematical Model



## RF front-end

- Linear gain with noise:

$$u_1(t) = \sqrt{G}r(t) + w_1(t)$$

- Noise after AMP

$$u_2(t) = u_1(t) + w_2(t)$$

- Memoryless nonlinearity:

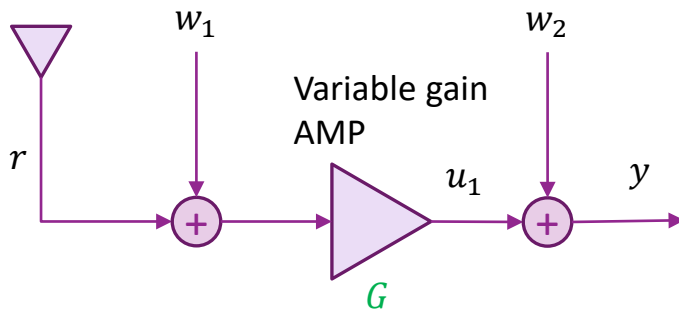
$$y(t) = \phi(u_2(t))$$

- $w_1(t)$  and  $w_2(t)$  have PSD  $N_1$  and  $N_2$

## Digital baseband

- Measures RX power or SNR
- Controls gain  $G$

# Analysis with No Nonlinearity



□ With no nonlinearity:

- $y = \sqrt{G}(r + w_1) + w_2 = \sqrt{G}r + v$
- $v = \sqrt{G}w_1 + w_2$

□ PSD of signal component  $\sqrt{G}r$  in output:  $GS_r$

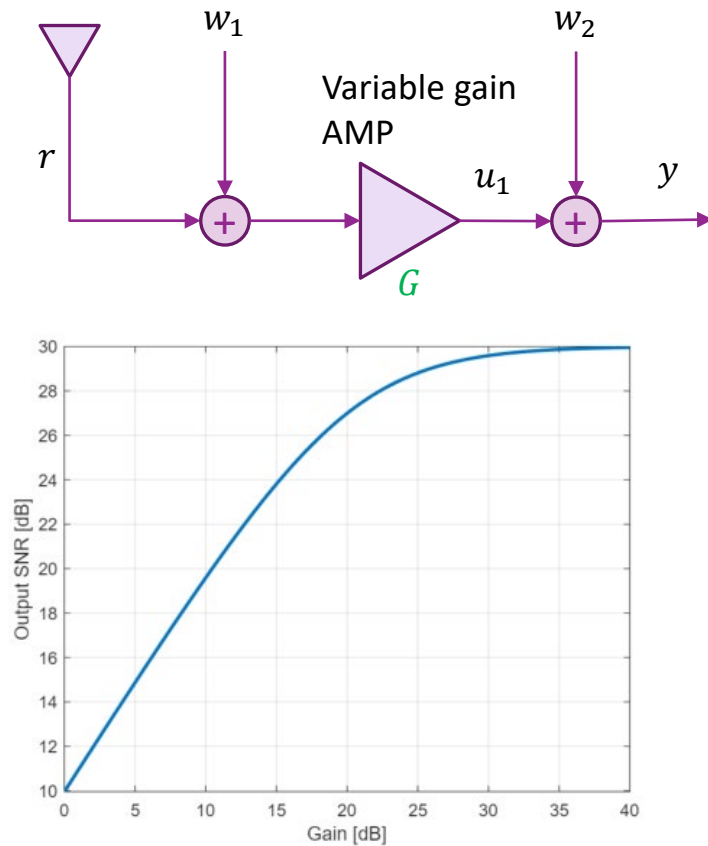
□ PSD of noise in output:  $S_v = GN_1 + N_2$

□ Output SNR:

$$\gamma_{out} = \frac{GS_R}{GN_1 + N_2} = \frac{\gamma_1}{1 + \frac{\gamma_1}{G\gamma_2}}$$

- $\gamma_1 = \frac{S_r}{N_1}, \quad \gamma_2 = \frac{S_r}{N_2}$

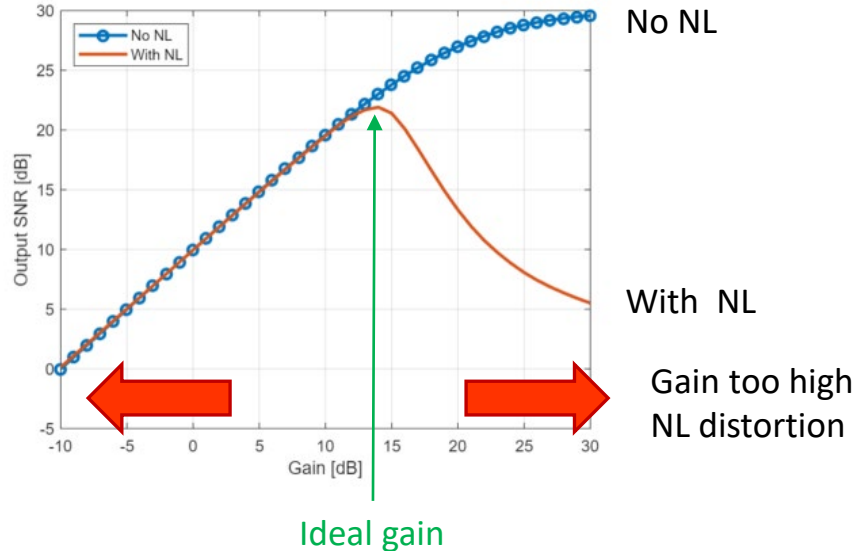
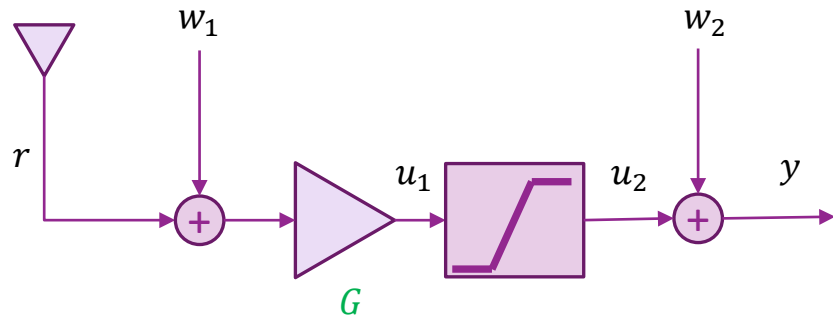
# Analysis with No Nonlinearity Example



- ❑ Output SNR from previous slide:  $\gamma_{out} = \frac{\gamma_1}{1 + \frac{\gamma_1}{G\gamma_2}}$
- ❑ Fig to left:  $\gamma_1 = 30$  dB,  $\gamma_2 = 10$  dB
- ❑ Observations with no nonlinearity:
  - Increasing gain always improve output SNR
  - Saturates at  $\gamma_{out} \rightarrow \gamma_1$  as  $G \rightarrow \infty$
- ❑ Design principle:
  - Have high SNR (low noise) in first stage
  - Increase gain to overcome noise in later stage
- ❑ Typical RF design starts with a low noise amplifier (LNA)



# Example with Saturation Nonlinearity



□ With high gain  $G$  :

- Signal  $u_1$  becomes large
- May saturate component  $\Rightarrow$  distortion

□ Saturation occurs in many RF components

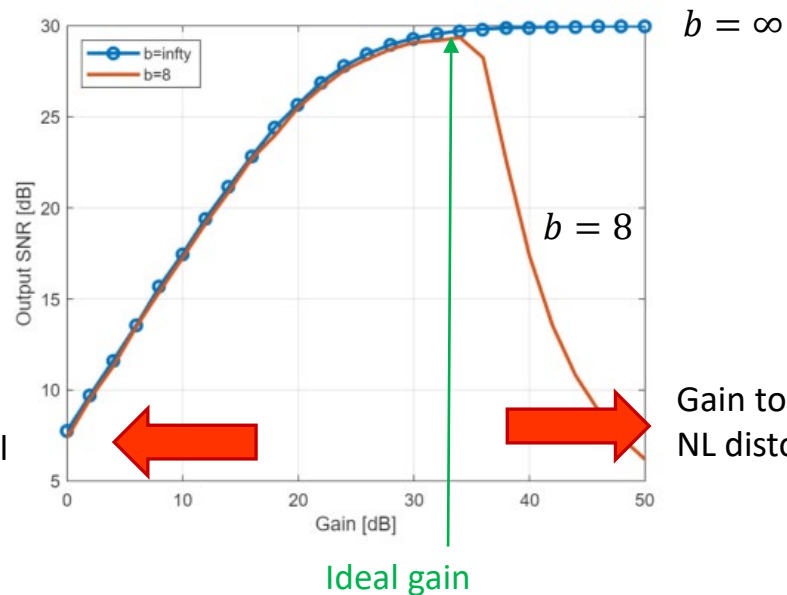
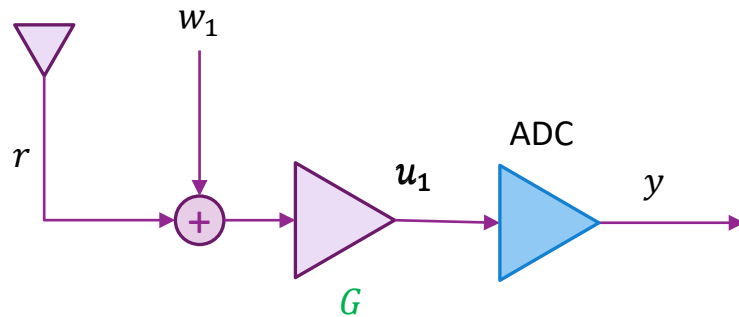
- E.g., mixer, LNA

□ Example:

- $\gamma_1 = \frac{E|r|^2}{E|w_1|^2} = 30 \text{ dB}$ ,  $\gamma_2 = \frac{E|r|^2}{E|w_2|^2} = 10 \text{ dB}$
- Nonlinearity is a saturation: Clips at  $\pm A$
- Saturation level  $\frac{|A|^2}{E|w_1|^2} = 50 \text{ dB}$

□ We see ideal gain trades off gain and NL distortion

# Example with ADC

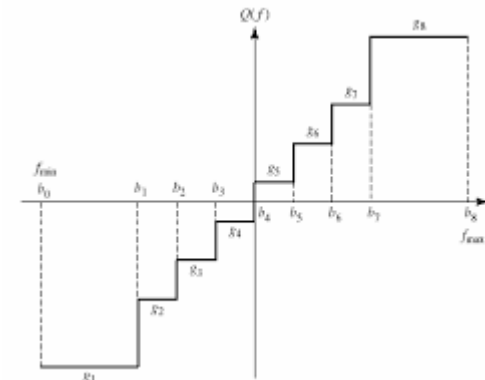


## Example ADC:

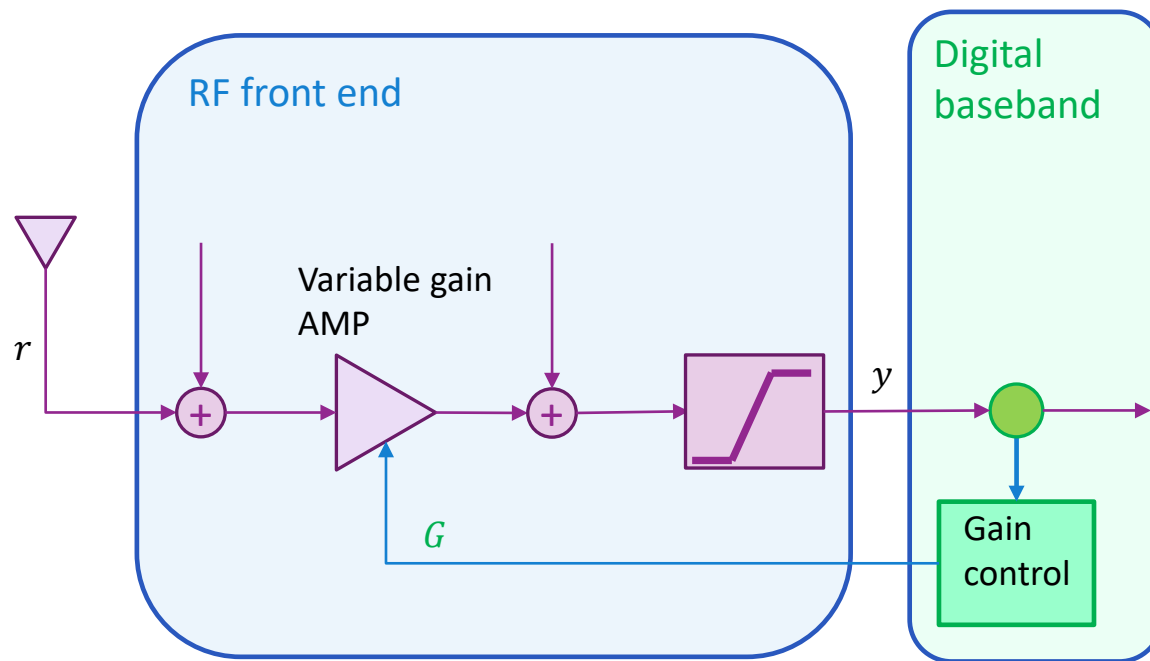
- Similar to quantization + saturation
- Saturates at  $y \in [-2^{b-1}, 2^{b-1} - 1]$  with  $b$  bits

## With finite bit width

- Set gain to not overflow ADC



# Practical Gain Control



## □ Measure RX power

- $E_{rx}[n] = \sum_{i=0}^{L-1} |y[n-i]|^2$
- RX power over last  $L$  symbols

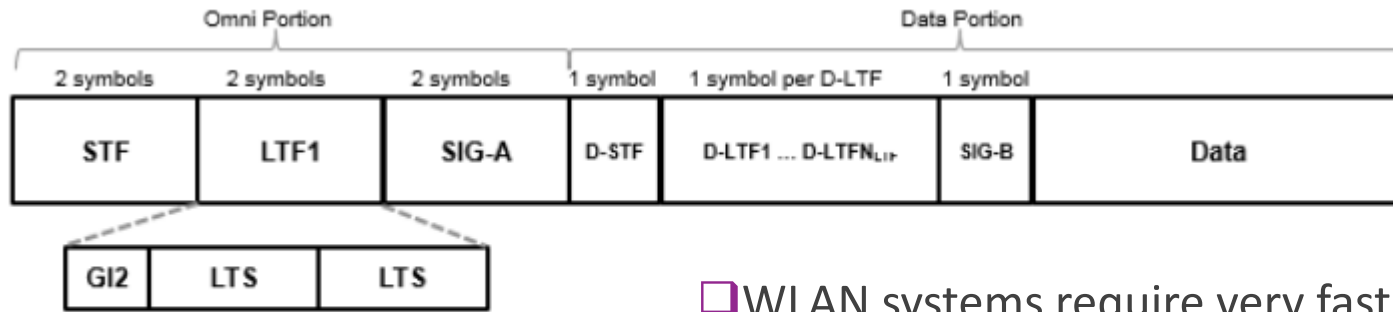
## □ Adjust gain:

- $E_{rx}[n] > t \Rightarrow$  Decrease gain
- $E_{rx}[n] \leq t \Rightarrow$  Increase gain

## □ Threshold $t$

- Set to ensure signal is not saturating

# Gain Control in 802.11



- ❑ WLAN systems require very fast AGCs
- ❑ STF (short training field). Duration is  $8 \mu s$ 
  - Used for detection *and* AGC
  - Typically set AGC in first  $\sim 2 \mu s$
  - Perform detection in remaining  $6 \mu s$
- ❑ Cannot detect STF until AGC has run
  - Otherwise, signal will be too low in SNR or too distorted
- ❑ AGC is implemented in hardware

# The 9361 Integrated Circuit

RX

- Gain performed in at least three stages
  - LNA gain control at RF
  - TIA (transimpedance AMP) gain at IF
  - Digital gain during digital filtering
- Single gain setting selects all three gains
  - Ensure all intermediate points do not overflow

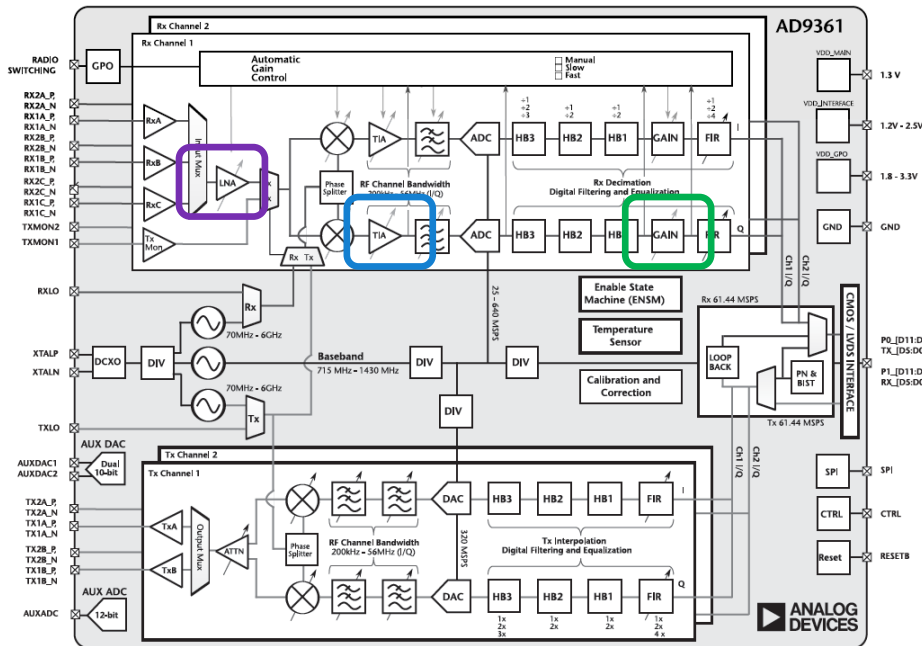


Figure 1.8 Integrated ZIF architecture used in the Pluto SDR.

TX

# AGC in the Pluto

**sdr\_rx**

Create receiver System object for radio hardware



## GainSource — Gain source

'AGC Slow Attack' (default) | 'AGC Fast Attack' | 'Manual'

Gain source, specified as one of the following:

- 'AGC Slow Attack' — For signals with slowly changing power levels
- 'AGC Fast Attack' — For signals with rapidly changing power levels
- 'Manual' — For setting the gain manually with the Gain property

**Data Types:** char | string



## Gain — Radio receiver gain

10 (default) | scalar

Radio receiver gain in dB, specified as a scalar from -4 to 71.

☐ MATLAB interface gain control

☐ AGC Fast Attack

- AGC automatically performed in HW
- Ideal for WLAN with fast gain
- But host has no visibility to gain selected

☐ Manual

- User can manually select gain
- We will use this in lab
- But it is very slow
- Just for education

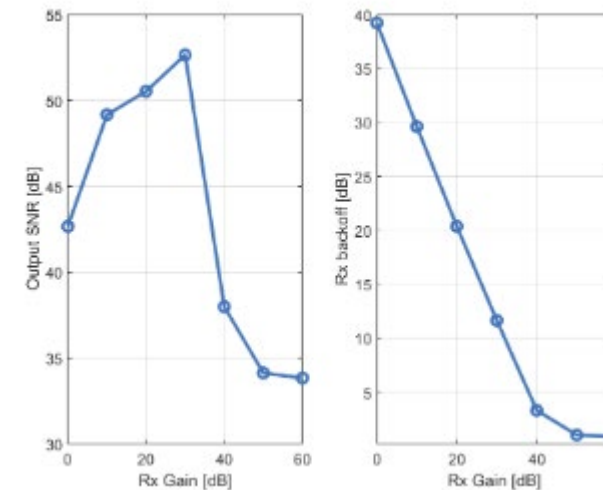
# SDR Lab

## Lab 5: Gain Control and Building a Simple AGC

Gain control is a fundamental operation in a receiver to adjust signal levels to a correct value. In this lab, you will learn to:


- Mathematically model a receiver with variable gain and saturation
- Simulate the effect of gain and nonlinearities on the output SNR
- Manually control the gain on the SDR
- Measure the RX backoff of a received signal
- Build a simple AGC to maintain a target RX backoff
- Measure the RX power with gain

[https://github.com/sdrangan/sdrlab/tree/main/lab05\\_gain](https://github.com/sdrangan/sdrlab/tree/main/lab05_gain)



# Outline

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- ☐ Detection and Synchronization Problem
- ☐ Hypothesis Testing
- ☐ Match Filtering for Detection
- ☐ Match Filtering Convolution
- ☐ Automatic Gain Control (AGC)
-  ☐ Matched Filtering as a Generalized Likelihood Ratio Test



# Likelihood Ratio Test

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□ In vector form:

- $H_1: \mathbf{r} = h\mathbf{x} + \mathbf{w}$  [Signal present]
- $H_0: \mathbf{r} = \mathbf{w}$  [Signal absent]

□ Likelihoods:

- $p(\mathbf{r}|H_0, \sigma^2) = \frac{c}{\sigma^{2N}} \exp\left(-\frac{\|\mathbf{r}\|^2}{\sigma^2}\right),$
- $p(\mathbf{r}|H_1, \sigma^2, h) = \frac{c}{\sigma^{2N}} \exp\left(-\frac{\|\mathbf{r}-h\mathbf{x}\|^2}{\sigma^2}\right)$
- Cannot apply regular LRT since parameters are unknown
- GLRT
- $\Lambda_0(\mathbf{r}, \sigma^2) := -\frac{1}{N} \ln p(\mathbf{r}|H_0) = \frac{1}{N} \ln \sigma^2 + \frac{\|\mathbf{r}\|^2}{N\sigma^2}$

# Generalized Likelihood Ratio Test

## □ Null hypothesis

- $\Lambda_0(r, \sigma^2) := -\frac{1}{N} \ln p(r|H_0) = \frac{1}{N} \ln \sigma^2 + \frac{\|r\|^2}{N\sigma^2}$
- $\bar{\Lambda}_0(r) := \min_{\sigma^2} \frac{1}{N} \ln \sigma^2 + \frac{\|r\|^2}{N\sigma^2} = \frac{1}{N} \ln \frac{\|r\|^2}{N} + 1$

## □ Present hypothesis:

- $\Lambda_1(r, \sigma^2, h) := -\frac{1}{N} \ln p(r|H_1) = \frac{1}{N} \ln \sigma^2 + \frac{\|r-hx\|^2}{N\sigma^2}$
- Minimize over  $h$ :  $\min_h \|r-hx\|^2 = \|r\|^2 - \frac{|x^*r|^2}{\|x\|^2}$
- $\bar{\Lambda}_1(r) := \min_{\sigma^2, h} \ln p(r|H_1) = \frac{1}{N} \ln \frac{1}{N} \left[ \|r\|^2 - \frac{|x^*h|^2}{\|x\|^2} \right] + 1$

## □ GLRT: $L(r) := \bar{\Lambda}_1(r) - \bar{\Lambda}_0(r) = -\ln[1 - \rho]$ , $\rho = \frac{|x^*h|^2}{\|x\|^2 \|r\|^2}$

## □ Details in clas