Lecture 05: Morphological Image Processing

Swakkhar Shatabda

CSE 4883: Digital Image Processing, Spring 2022
Department of Computer Science and Engineering
United International University

Outline

Morphological Image Processing

- Morphology commonly denotes a branch of biology that deals with the form and structure of animals and plants.
- Extracting image components that are useful in the representation and description of region shape, such as boundaries, skeletons, and the convex hull.
- pre- or postprocessing, such as morphological filtering, thinning, and pruning.
- Morphological operations are defined in terms of sets.
    - objects (sets of foreground pixels)
    - structuring elements (SE) (both foreground and background pixels.)

Sets

Because the images with which we work are rectangular arrays, and sets in general are of arbitrary shape, applications of morphology in image processing require that sets be embedded in rectangular arrays.

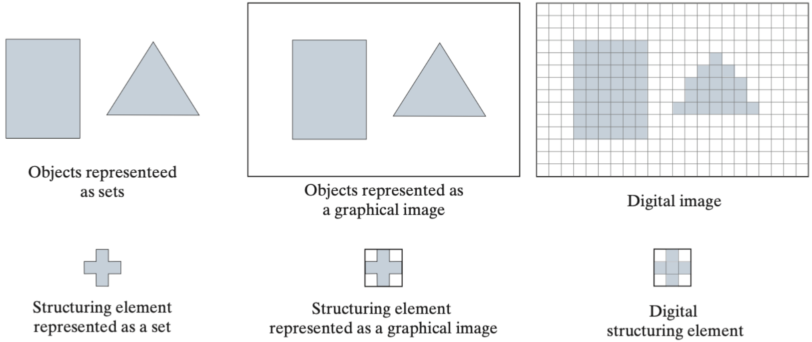


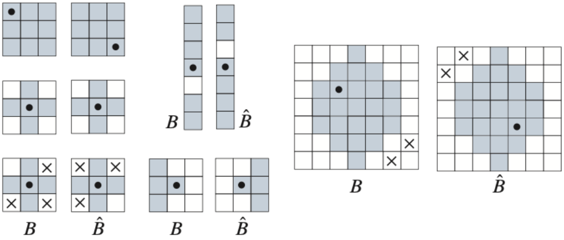
FIGURE 9.1 Top row. *Left:* Objects represented as graphical sets. *Center:* Objects embedded in a background to form a graphical image. *Right:* Object and background are digitized to form a digital image (note the grid). Second row: Example of a structuring element represented as a set, a graphical image, and finally as a digital SE.

Reflection

The reflection of a set (structuring element) B about its origin, denoted by $\hat{B}$, is defined as:

$$\hat{B} = \{w | w = -b, \text{ for } b \in B\}$$

FIGURE 9.2 Structuring elements and their reflections about the origin (the ×'s are don't care elements, and the dots denote the origin). Reflection is rotation by 180° of an SE about its origin.



Translation

The translation of a set B by point $z = (z_1, z_2)$, denoted $(B)_z$, is defined as:

$$(B)_z = \{c | c = b + z, \text{ for } b \in B\}$$

Erosion

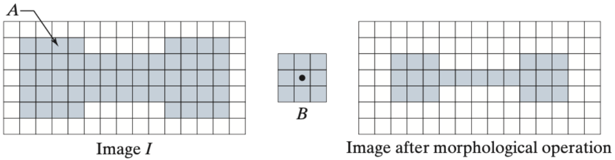
With A and B as sets in Z^2 , the erosion of A by B, denoted $A \ominus B$, is defined as:

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

where A is a set of foreground pixels, B is a structuring element, and the z's are foreground values (1's).

$$I \ominus B = \{z | (B)_z \subseteq A \text{ and } A \subseteq I\} \cup \{A^c | A^c \subseteq I\}$$

Erosion - Example



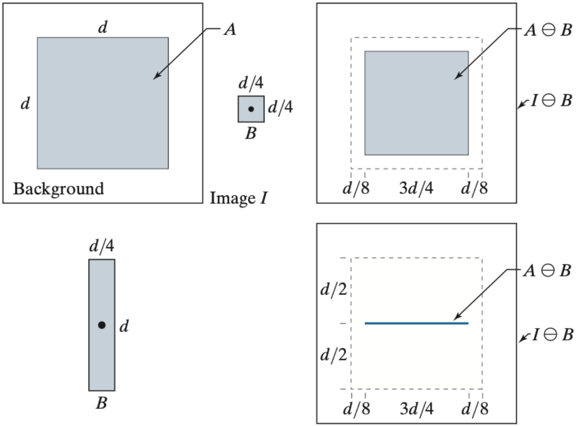
a b c

FIGURE 9.3
(a) A binary image containing one object (set), A. (b) A structuring element, B. (c) Image resulting from a morphological operation (see text).

Erosion - Example

a b c
d e

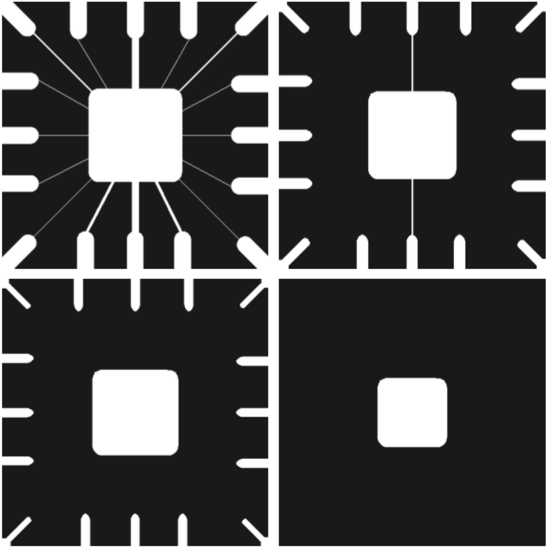
FIGURE 9.4
(a) Image I, consisting of a set (object) A, and background.
(b) Square SE, B (the dot is the origin).
(c) Erosion of A by B (shown shaded in the resulting image).
(d) Elongated SE.
(e) Erosion of A by B. (The erosion is a line.) The dotted border in (c) and (e) is the boundary of A, shown for reference.



Erosion - Example

a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask in which foreground pixels are shown in white. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45 elements, respectively, all valued 1.



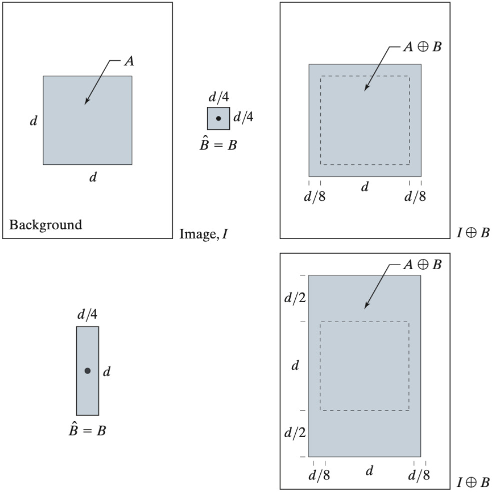
Dilation

With A and B as sets in Z^2, the dilation of A by B,denoted as $A \oplus B$, is defined as:

$$A \oplus B = \{z | [(\hat{B})_z \cap A] \subseteq A\}$$

- This equation is based on reflecting B about its origin and translating the reflection by z, as in erosion.
- The dilation of A by B then is the set of all displacements, z, such that the foreground elements of $\hat{B}$ overlap at least one element of A.

Dilation - Example



a b c
d e

FIGURE 9.6
(a) Image I, composed of set (object) A and background. (b) Square SE (the dot is the origin). (c) Dilation of A by B (shown shaded). (d) Elongated SE. (e) Dilation of A by this element. The dotted line in (c) and (e) is the boundary of A, shown for reference.

Dilation - Example

a c
b

FIGURE 9.7
(a) Low-resolution text showing broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

1	1	1
1	1	1
1	1	1

Duality

Erosion and dilation are duals of each other with respect to set complementation and reflection.

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

Opening

The opening of set A by structuring element B, denoted by $A \circ B$, is defined as:

$$A \circ B = (A \ominus B) \oplus B$$

Thus, the opening A by B is the erosion of A by B, followed by a dilation of the result by B.

- The opening of A by B is the union of all the translations of B so that B fits entirely in A.

Opening - Example

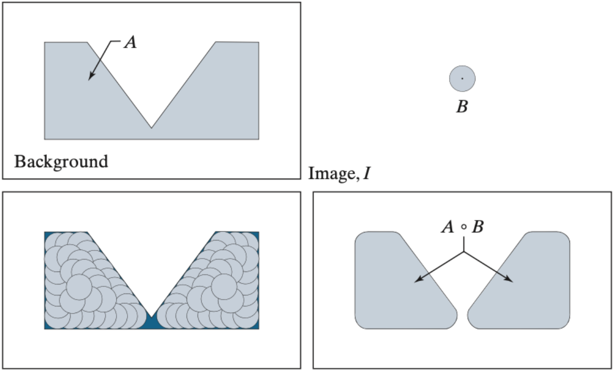


FIGURE 9.8
(a) Image I, composed of set (object) A and background.
(b) Structuring element, B.
(c) Translations of B while being contained in A. (A is shown dark for clarity.)
(d) Opening of A by B.

Closing

The closing of set A by structuring element B, denoted $A \bullet B$, is defined as:

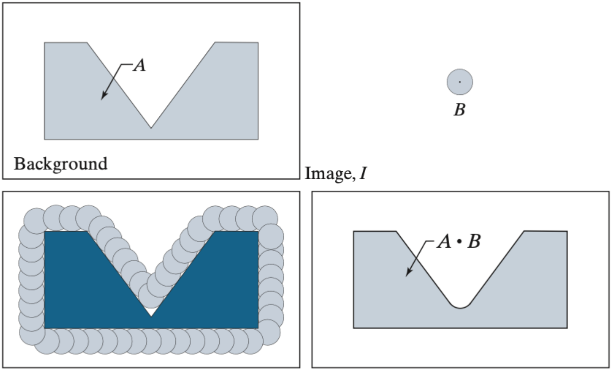
$$A \bullet B = (A \oplus B) \ominus B$$

which says that the closing of A by B is simply the dilation of A by B, followed by erosion of the result by B.

Closing - Example

a	b
c	d

FIGURE 9.9
(a) Image I, composed of set (object) A, and background.
(b) Structuring element B.
(c) Translations of B such that B does not overlap any part of A. (A is shown dark for clarity.)
(d) Closing of A by B.



Morphologcal Processing - Example

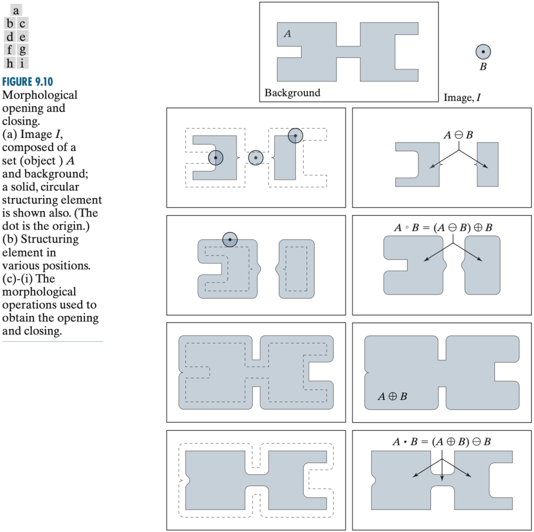
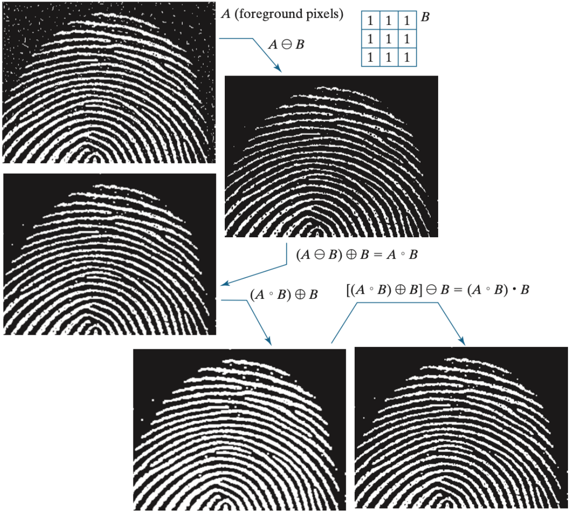


FIGURE 9.10 Morphological opening and closing. (a) Image I, composed of a set (object) A and background; a solid, circular structuring element is shown also. (The dot is the origin.) (b) Structuring element in various positions. (c)-(i) The morphological operations used to obtain the opening and closing.

Morphologcal Processing - Example



a b c d e f

FIGURE 9.11 (a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Dilation of the erosion (opening of A). (e) Dilation of the opening. (f) Closing of the opening. (Original image courtesy of the National Institute of Standards and Technology.)

Duality and others

$$(A \circ B)^c = (A \bullet \hat{B})$$

$$(A \bullet B)^c = (A \circ \hat{B})$$

1. $A \circ B$ is a subset of A
2. If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
3. $(A \circ B) \circ B = A \circ B$

1. A is a subet of $A \bullet B$
2. If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
3. $(A \bullet B) \bullet B = A \bullet B$

hit-or-miss transform (HMT)

Two structuring elements: B1 , for detecting shapes in the foreground, and B2 , for detect- ing shapes in the background. The HMT of image I is defined as

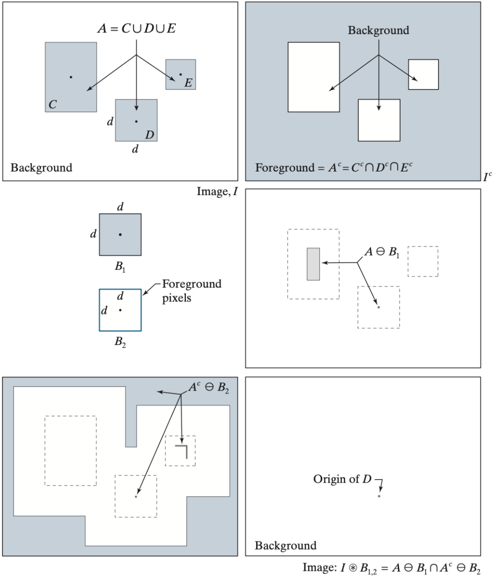
$$I \circledast B_{1,2} = \{z | (B_1)_z \subseteq A \text{ and } (B_2)_z \subseteq A^c\}$$

$$I \circledast B_{1,2} = (A \ominus B_1) \cap (A^c \ominus B_2)$$

HMT - Example

a b
c d
e f

FIGURE 9.12
(a) Image consisting of a foreground (1's) equal to the union, A, of set of objects, and a background of 0's.
(b) Image with its foreground defined as A^c.
(c) Structuring elements designed to detect object D.
(d) Erosion of A by B_1.
(e) Erosion of A^c by B_2.
(f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origin of their respective components. Each dot is a single pixel.



HMT - Example

$$I \circledast B = \{z | (B)_z \subseteq I\}$$

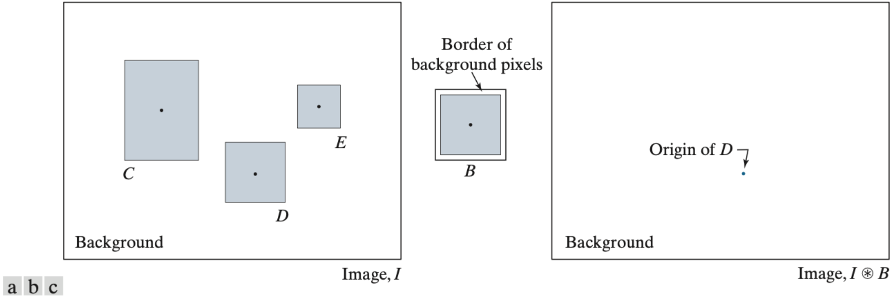


FIGURE 9.13 Same solution as in Fig. 9.12, but using Eq. (9-17) with a single structuring element.

HMT - Example

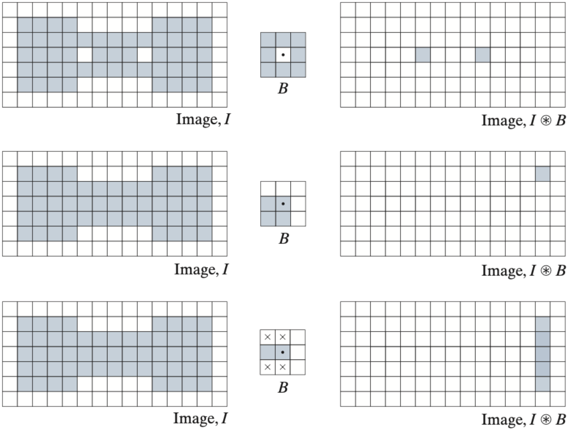


FIGURE 9.14 Three examples of using a single structuring element and Eq. (9-17) to detect specific features. First row: detection of single-pixel holes. Second row: detection of an upper-right corner. Third row: detection of multiple features.

Boundary Extraction

$$\beta(A) = A - (A \ominus B)$$

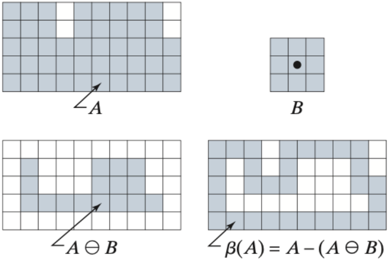


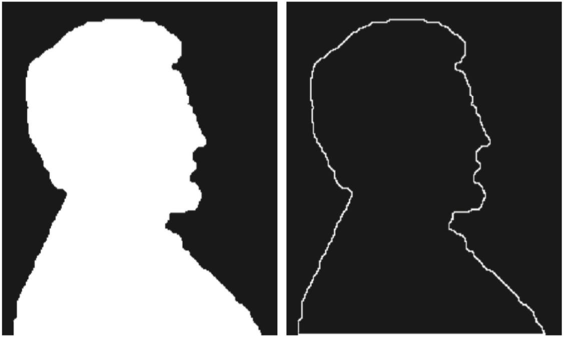
FIGURE 9.15
(a) Set, A, of foreground pixels.
(b) Structuring element.
(c) A eroded by B.
(d) Boundary of A.

Boundary Extraction

$$\beta(A) = A - (A \ominus B)$$

a b

FIGURE 9.16
(a) A binary image.
(b) Result of using Eq. (9-18) with the structuring element in Fig. 9.15(b).

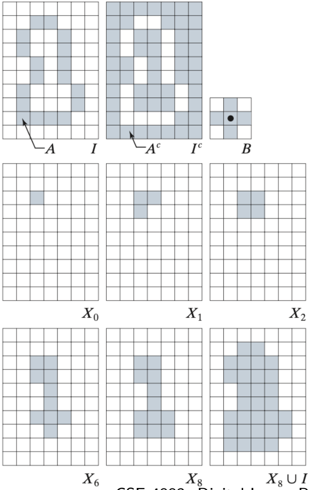


Hole Filling

$$X_k = (X_{k-1} \oplus B) \cap I^c \text{ for } k = 1, 2, 3, \cdots$$

a	b	c
d	e	f
g	h	i

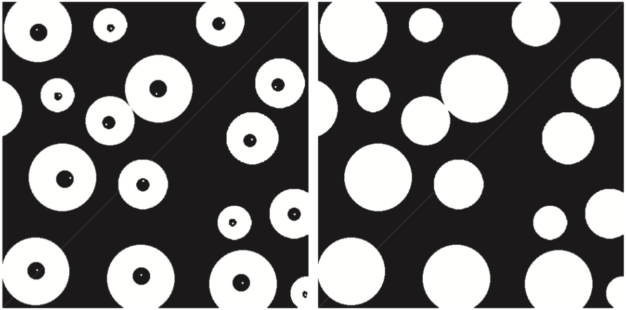
FIGURE 9.17 Hole filling. (a) Set A (shown shaded) contained in image I. (b) Complement of I. (c) Structuring element B. *Only* the foreground elements are used in computations (d) Initial point inside hole, set to 1. (e)–(h) Various steps of Eq. (9-19). (i) Final result [union of (a) and (h)].



Hole Filling

a b

FIGURE 9.18 (a) Binary image. The white dots inside the regions (shown enlarged for clarity) are the starting points for the hole-filling algorithm. (b) Result of filling all holes.



Connected Components

$$X_k = (X_{k-1} \oplus B) \cap I \text{ for } k = 1, 2, 3, \cdots$$

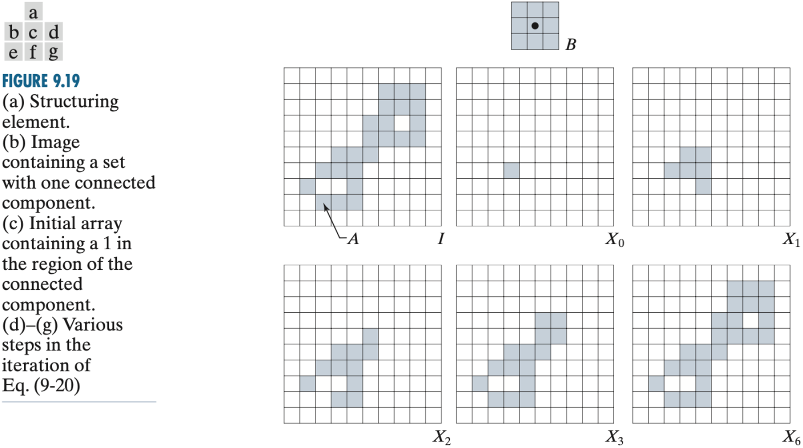


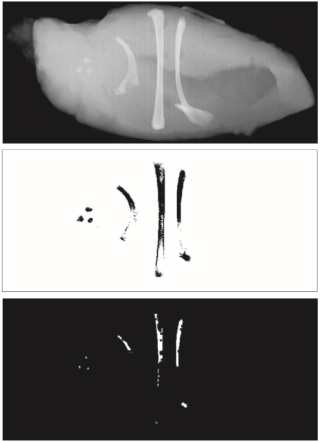
FIGURE 9.19
(a) Structuring element.
(b) Image containing a set with one connected component.
(c) Initial array containing a 1 in the region of the connected component.
(d)–(g) Various steps in the iteration of Eq. (9-20)

Connected Components

$$X_k = (X_{k-1} \oplus B) \cap I \text{ for } k = 1, 2, 3, \cdots$$

a
b
c d

FIGURE 9.20
(a) X-ray image of a chicken filet with bone fragments.
(b) Thresholded image (shown as the negative for clarity).
(c) Image eroded with a 5×5 SE of 1's.
(d) Number of pixels in the connected components of (c).
(Image (a) courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex Hull

$$X_k^i = (X_{k-1}^i \circledast B^i) \cup X_{k-1}^i, \text{ for } i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3, \cdots, X_0^i = I$$

Convex Hull

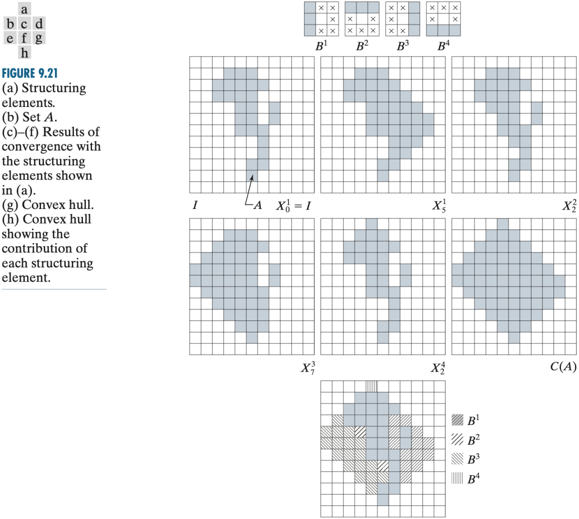
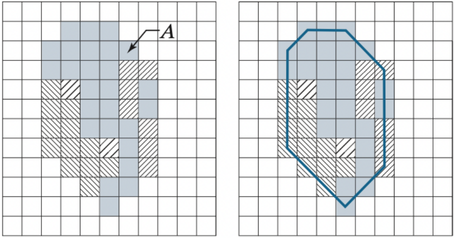


FIGURE 9.21
(a) Structuring elements.
(b) Set A.
(c)–(f) Results of convergence with the structuring elements shown in (a).
(g) Convex hull.
(h) Convex hull showing the contribution of each structuring element.

Convex Hull

a b

FIGURE 9.22
(a) Result of limiting growth of the convex hull algorithm.
(b) Straight lines connecting the boundary points show that the new set is convex also.



Thinning

$$A \otimes B = A - (A \circledast B)$$

$$A \otimes \{B\} = ((\cdots((A \otimes B^1) \otimes B^2) \cdots) \otimes B^n)$$

Thinning

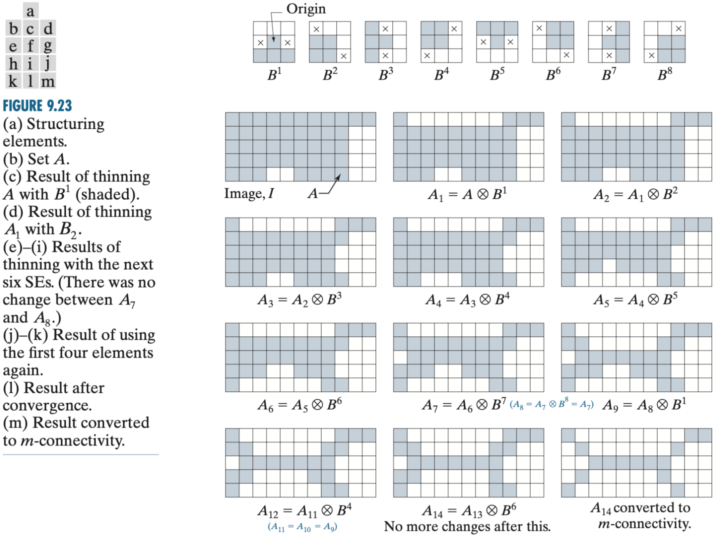


FIGURE 9.23
(a) Structuring elements.
(b) Set A.
(c) Result of thinning A with B^1 (shaded).
(d) Result of thinning A_1 with B_2.
(e)–(i) Results of thinning with the next six SEs. (There was no change between A_7 and A_8.)
(j)–(k) Result of using the first four elements again.
(l) Result after convergence.
(m) Result converted to m-connectivity.

Thickening

$$A \odot B = A - (A \circledast B)$$

$$A \odot \{B\} = ((\cdots((A \odot B^1) \odot B^2) \cdots) \odot B^n)$$

However, a separate algorithm for thickening is seldom used in practice. Instead, the usual procedure is to thin the background of the set in question, then complement the result.

Thickening

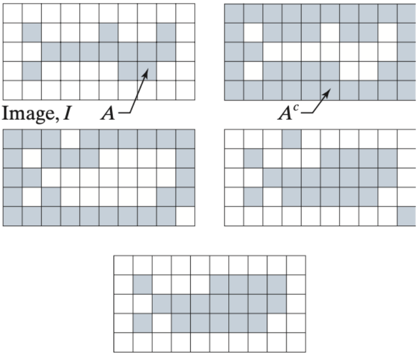


FIGURE 9.24
(a) Set A.
(b) Complement of A.
(c) Result of thinning the complement.
(d) Thickened set obtained by complementing (c).
(e) Final result, with no disconnected points.

Skeleton

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus KB) \circ B$$

$$K = \max\{k | (A \ominus kB) \neq \varnothing\}$$

Skeleton

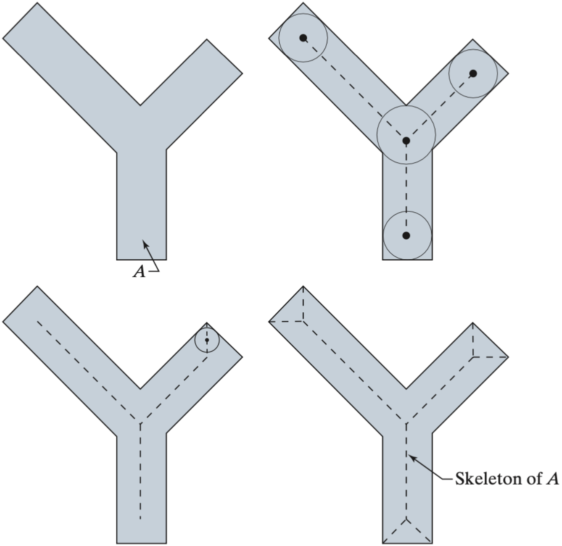
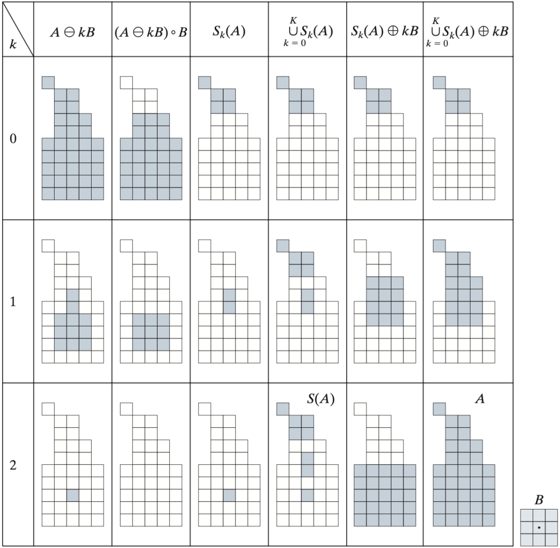


FIGURE 9.25
(a) Set A.
(b) Various positions of maximum disks whose centers partially define the skeleton of A.
(c) Another maximum disk, whose center defines a different segment of the skeleton of A.
(d) Complete skeleton (dashed).

Skeleton

FIGURE 9.26 Implementation of Eqs. (9-28) through (9-33). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.



Grayscale Erosion/Dilation

The grayscale erosion of f by a flat structuring element b at location (x, y) is defined as the minimum value of the image in the region coincident with $b(x, y)$ when the origin of b is at (x, y).

$$[f \ominus b](x, y) = \min_{(s,t)\in b}\{f(x + s, y + t)\}$$

$$[f \oplus b](x, y) = \max_{(s,t)\in b}\{f(x - s, y - t)\}$$

Grayscale Erosion/Dilation

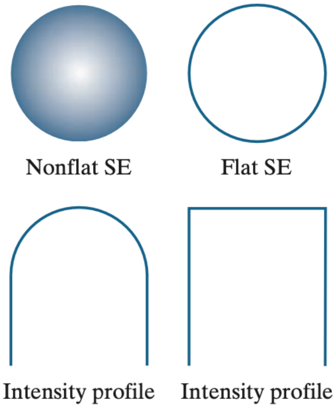
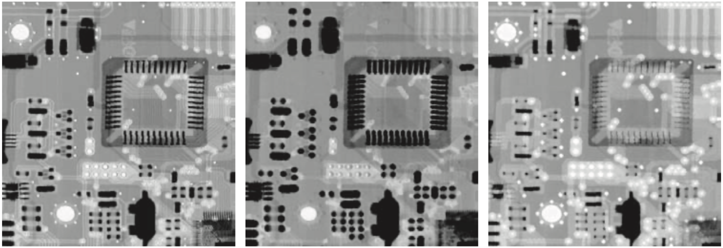


FIGURE 9.36 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their centers. All examples in this section are based on flat SEs.

Grayscale Erosion/Dilation

a b c

FIGURE 9.37
(a) Gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of 2 pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)



Thats it!

Thank you