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(PART 1)

(a) $x[n] = \frac{1}{5} n \left(\frac{1}{3}\right)^{n-1} u(n-1)$

Solution:

$$x[n] = \frac{1}{5} n \left[\frac{1}{3}\right]^{n-1} u(n-1) \quad \text{——} (A)$$

In this equation the time shift property is used so to solve this, we ignore the time shift the equation will be

$$x[n] = \frac{1}{5} n \left(\frac{1}{3}\right)^{n} u(n) \quad \text{——} (B)$$

we know that

$$n a^n u[n] \xrightarrow{z} \frac{a z^{-1}}{(1 - a z^{-1})^2}$$

so using this formula on equation B

$$= \frac{1}{5} n \left(\frac{1}{3}\right)^{n} u[n] = \frac{\frac{1}{3} z^{-1}}{(1 - \frac{1}{3} z^{-1})^2}$$

$$X(z) = \frac{1}{3} z^{-1} \Big/ \left(1 - \frac{1}{3} z^{-1}\right)^2$$

Now we know that for shifting property z-transform will

$$x(n-k) = X(z) z^{-k}$$

so here $k = 1$ so the actual transform will be

$$X(z) = \left(\frac{\frac{1}{3} z^{-1}}{(1 - \frac{1}{3} z^{-1})^2}\right) z^{-1}$$

so here z^{-1} is for that delay

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(b) $x[n] = \left(\frac{1}{2}\right)^n \left(U[n] - U[n-10]\right)$

Solution:

we know that

$$a^n U[n] \xrightarrow{Z} \frac{1}{1-az^{-1}} \quad \text{—— (A)}$$

$$x[n] = \left(\frac{1}{2}\right)^n U[n] - \left(\frac{1}{2}\right)^n U[n-10]$$

Z-transform of

$$\left(\frac{1}{2}\right)^n U[n] \xrightarrow{Z} \frac{1}{1-\left(\frac{1}{2}\right)z^{-1}} \quad \text{—— (A1)}$$

Now to find the z-transform of

$$\left(\frac{1}{2}\right)^n U[n-10] \quad \text{—— (B)}$$

To ignore the time shift delay in B so its transform will be

$$\left(\frac{1}{2}\right)^n U[n] \xrightarrow{Z} \frac{1}{1-\frac{1}{2}z^{-1}} \quad \text{—— (A1)}$$

Now at delay 10:

$$\left(\frac{1}{2}\right)^n U[n-10] \xrightarrow{Z} \left(\frac{1}{(1-\frac{1}{2}z^{-1})}\right) z^{-10} \quad \text{—— (A2)}$$

$$X(z) = A1 + A2$$

So $$X(z) = \left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) - \left(\frac{z^{-10}}{1-\frac{1}{2}z^{-1}}\right)$$

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(Part 2)

Solution:

$U = 8, \ T = 5, \ H = 6, \ T_h = 1$

$x[n] = \{U, T, H, T_h\}$

$h[n] = \{T_h, H, T, U\}$

$x[n] = \{8, 5, 6, 1\}$

$h[n] = \{1, 6, 5, 8\}$

Taking z-transform:

$X(z) = 8z^{1} + 5z^{0} + 6z^{-1} + 1z^{-2}$

$H(z) = 1z^{0} + 6z^{-1} + 5z^{-2} + 8z^{-3}$

Convolution property of z-transform:

$Y(z) = X(z)H(z)$

$$Y(z) = (8z^{1} + 5 + 6z^{-1} + z^{-2})(1 + 6z^{-1} + 5z^{-2} + 8z^{-3})$$

$$= 8z^{1} + 48z^{0} + 40z^{-1} + 64z^{-2} + 5 + 30z^{-1} + 25z^{-2} + 40z^{-3} + 6z^{-1} + 36z^{-2} + 30z^{-3} + 48z^{-4} + z^{-2} + 6z^{-3} + 5z^{-4} + 8z^{-5}$$

$$Y(z) = 8z + 53 + 76z^{-1} + 126z^{-2} + 76z^{-3} + 53z^{-4} + 8z^{-5}$$

$$y[n] = \{8, \underset{\uparrow}{53}, 76, 126, 76, 53, 8\}$$

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(Part 3)

$$y[n] = \frac{1}{2} y[n-1] + x[n]$$

solution:

Applying Z transform, we get

$$Z^{+}\{y(n)\} = Z^{+}\{\tfrac{1}{2} y(n-1) + x(n)\}$$

$$\Rightarrow y^{+}(z) = \frac{1}{2} z^{+}\{y(n-1)\} + x^{+}(z)$$

$$Z^{+}\{y(n-1)\} = z^{-1}\{Y^{+}(z) + \sum_{n=1}^{1} y(-n) z^{1}]$$

$$= z^{-1}[Y^{+}(z) + y(-1) z^{1}]$$

$$Z^{+}\{y(n-1)\} = Y^{+}(z) z^{-1} + y(-1)]$$

$$+ \frac{1}{1-(1/3)z^{-1}}$$

$$= Y^{+}(z) = \frac{1}{2} Y^{+}(z) z^{-1} + \frac{1}{2} + \frac{1}{1-(1/3)z^{-1}}$$

$$= Y^{+}(z)\left[1 - \frac{1}{2} z^{-1}\right] = 0.5 + \frac{1}{1-(1/3)z^{-1}}$$

$$Y^{+}(z) = \frac{0.5}{1-(1/2)z^{-1}} + \frac{1}{(1-(1/2)z^{-1})(1-(1/3)z^{-1})}$$

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$$Y^+(z) = \frac{0.5}{1-\frac{1}{2}z^{-1}} + \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{3}z^{-1}}$$

$$A = \frac{\cancel{\left(1-\frac{1}{2}z^{-1}\right)} \times 1}{\cancel{\left(1-\frac{1}{2}z^{-1}\right)}\left(1-\frac{1}{3}z^{-1}\right)}\Bigg|_{z^{-1}=2} = 3$$

$$B = \frac{\left(1-\frac{1}{3}z^{-1}\right) \times 1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}\Bigg|_{z^{-1}=3} = 2$$

$$Y^+(z) = \frac{0.5}{1-\frac{1}{2}z^{-1}} + \frac{3}{1-\frac{1}{2}z^{-1}} - \frac{2}{1-\frac{1}{3}z^{-1}}$$

$$Y^+(z) = \frac{3.5}{1-\frac{1}{2}z^{-1}} - \frac{2}{1-\frac{1}{3}z^{-1}}$$

$$y[n] = 3.5\left(\frac{1}{2}\right)^n u[n] - 2 \times \left(\frac{1}{3}\right)^n u(n)$$

$$y[n] = \left[3.5\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n\right] u(n)$$

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(PART 4)

Solution:

$$x[n] = \{ --, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1 -- \}$$

It is a periodic signal.

(a)

So we know that

$$C_K = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n}$$

$$C_K = \frac{1}{3} \sum_{n=0}^{2} x[n] e^{-jk 2\pi/3 n}$$

$$C_K = \frac{1}{3}\left(1 + e^{-jkK/3}\right)$$

Put $C = 0$

$$C_0 = \frac{1}{3}(1 + e^0) = \frac{2}{3}$$

$C = 1$

$$C_1 = \frac{1}{3}\left(1 + e^{-j2\pi/3}\right)$$

$$= \frac{1}{3}\left(1 - \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}\right)$$

$$C_1 = \frac{1}{3}\left(1 - \cos\frac{2\pi}{3}\right) + \frac{1}{3} j \sin\frac{2\pi}{3}$$

$$C_2 = \frac{1}{3}\left(1 - e^{-j4\pi/3}\right)$$

$$= \frac{1}{3}\left(1 - \cos\frac{4\pi}{3} + j\sin 4\pi/3\right)$$

$$= \frac{1}{3}\left(1 - \cos\pi/3\right) - \frac{1}{3} j \sin 4\pi/3$$

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$|C_0| = \frac{2}{3}$ $|C_1| = 0.608$

$|C_2| = 0.02$

$\theta_{C_0} = 0$

$\theta_{C_1} = \tan^{-1}\left(\frac{0.3 \sin 2\pi/3}{0.3 - 0.3\cos 2\pi/3}\right)$

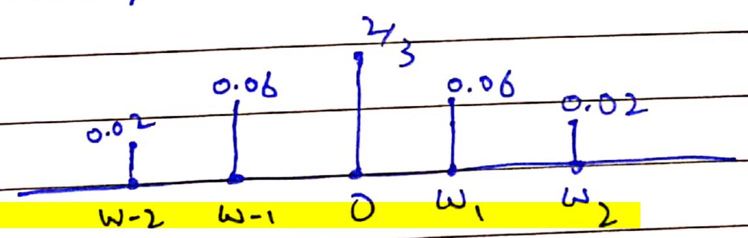
$= \tan^{-1}\left(\frac{0.05}{0.0002}\right)$

$= \pi/2$

$\theta_{C_2} = -\pi/2 = 1 \quad \tan^{-1}\left(\frac{1/3 \sin 4\pi/3}{1/3 - 1/3 \cos 4\pi/3}\right)$

(Part b)

Magnitude spectrum:



Phase spectrum: