

Zero's at the end :-

Aptitude

III) $UDV = 2/4/6/8$

2^{10} की cycle में रहती है; $2^{10} = 1024$ $\left\{ \begin{array}{l} 2^{\text{odd}} = 24 \\ 2^{\text{even}} = 76 \end{array} \right.$

Ex: $2^{41} = (2^{10})^4 \times 2^1 = (2^{10})^{\text{even}} \times 2 = 76 \times 2 = 52$

Ex: $4^{41} = 2^{82} = 2^{82}$

Ex: $6^{41} = 2^{41} \times 3^{41}$

Ex: $8^{41} = 2^{123}$

1

#) 50! में no. of 2's & 8's
 $2's = \text{Prime} = 25 + 12 + 6 + 3 + 1 = 47$
 $8's = \text{Composite} = 2^3 = 47/3 = 15$
 $3's = \text{Prime} = 16 + 5 + 1 = 22$
 $6's = \text{Composite} = 2 \times 3 = 2^{17} \times 3^{22} = 6^{17} \times 3^5$

Zero's at the end) $10's = 0's = 5's = 10 + 2 = 12$

Note: $50!^{27}$ में no. of 0's = 5's = $(12) \times 27$

#) Factorization:

$N = a^p \times b^q \times c^r$
 ① $n = (p+1)(q+1)(r+1)$
 ② $S_n = \frac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)}{(a-1)(b-1)(c-1)}$
 ③ $P_n = N^{1/2}$

$N = \text{no.}$
 $a, b, c \rightarrow \text{Prime factors}$
 $n = \text{no. of factors}$
 $S_n = \text{Sum of factors}$
 $P_n = \text{Product of factors}$

* **Perfect Square or cube**
 $162 \times y$; $y = ?$ if it's perfect cube
 $162 = 2 \times 3^4 \Rightarrow x(2^2 \times 3^2) \Rightarrow y = 36$

* How many factors of N are div. by given no.

$= \frac{a^p \times b^q \times c^r}{x}$; $x \rightarrow \text{Prime factor form of given no.}$
 $= a^j \times b^k \times c^l = (j+1)(k+1)(l+1) \text{ factors}$

#) P & C:
 combination
 (Selection)
 ${}^n C_r$

permutation
 (selection followed by arrangement)
 ${}^n P_r = {}^n C_r \times r!$

Note: ${}^n C_r = {}^n C_{n-r}$
 Ex: No 2 vowels together in GANESHPURI
 (AEIOU) (GNSHPR) $\Rightarrow 6! (2! \times 4!)$

Note: $6^n \times abc \pm = xyz \pm$

Note: In $N!$ if $N \geq 4$, $N!$ is div. by 4
 $\Rightarrow 2^N = (2^2)^{N/2} = 6$

Note: $4^{\text{odd}} = 4$; $4^{\text{even}} = 6$
 $9^{\text{odd}} = 9$; $9^{\text{even}} = 1$

#) No. divisible by: $N = abc$

* 3: $a+b+c$ div. by 3
 $1 \times 2 \times 3 \times 4 \times 5 \times 6 \checkmark$
 9: $a+b+c$ div. by 9

Ex: $N = 1! + 2! + \dots + 20!$
 $N \% 20 = ?$
 $20 = 2^3 \times 5 = 5 \times 4$
 $= 1!/20 + 2!/20 + 3!/2 + 4!/2 + 0 + 0 \dots$
 $= 33/20 = 13 \text{ or } 7$

$X G \times N \times S \times H \times P \times R \Rightarrow 6! (2! \times 4!)$
 Ex: (Subgroup formation)

A	1
B	2

 (i) no cond.
 $({}^1 C_0 + {}^3 C_1 + {}^2 C_2) \cdot ({}^2 C_0 + {}^2 C_1 + {}^2 C_2)$
 or $2^3 \times 2^2$

Last 2 digit:

2) when $UDV = 2$
 Ex: 23861
 \downarrow
 1

#) Straight Lines = Diagonals + Sides
 $n C_2 = \frac{n(n-1)}{2}$; $n = \text{no. of non-collinear points}$

Triangles = $n C_3$

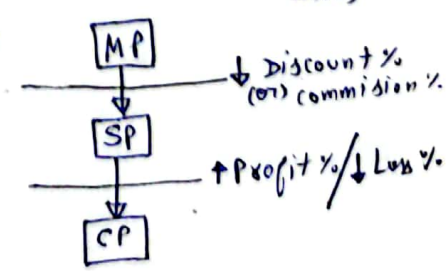
% change = $\frac{FV - IV}{IV} \times 100 \%$

Profit/Gain % = $\frac{SP - CP}{CP} \times 100$

Loss % = $\frac{CP - SP}{CP} \times 100$

Margin % = $\frac{SP - CP}{SP} \times 100$

Profitability = $\frac{\text{Profit}}{\text{Sales}}$



Note: $N! = \dots 0$ when $N \geq 5$

II) when $UDV = 3/9/7$
 UDV 1 का मशीन Power cycle में
 Ex: 3^{13}
 $= (3^4)^3 \times 3$
 $= (81)^3 \times 27$
 $= 01 \times 27$
 $= 27$

#) $E = P \times C$
 $\frac{1}{5} \uparrow 25\% \mid \downarrow \frac{1}{5} = 20\%$

$\frac{2}{3} = 66.67\% \mid \frac{7}{8} = 87.50\%$
 $\frac{3}{8} = 37.50\% \mid \frac{5}{6} = 83.33\%$
 $\frac{5}{8} = 62.50\%$

$\frac{2}{3} \leftarrow \begin{array}{c|c} P & C \\ \hline \uparrow 66.67 & \downarrow \frac{2}{2+3} = \frac{2}{5} = 40\% \\ \downarrow 37.50 & \uparrow \frac{3}{8-3} = \frac{3}{5} = 60\% \end{array}$

#) Geometry/Handshakes/Tournaments/
 Gift exchanges —

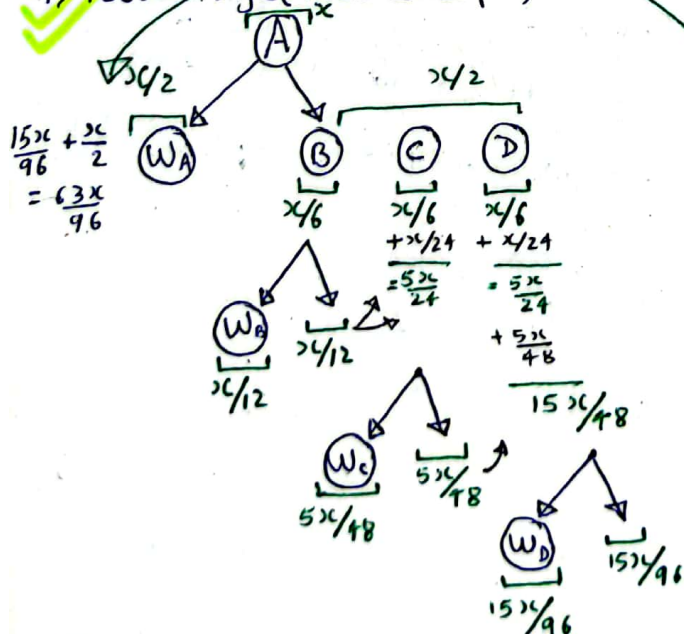
$A \rightarrow B$ $1 \times n_2$ Exchange
 $A \leftrightarrow B$ $2 \times n_2$ Exchange
 $A \leftrightarrow B$ $100 \times n_2$ Exchange

#) Number sum — using digits
 $1/2/3/\dots/n$, exactly once, total sum
 of n -digit numbers that can be formed is
 $\frac{n!}{n} (1+2+3+\dots+n) (10^{n-1} + 10^{n-2} + \dots + 10^0)$

Ex: $1/6/8 \Rightarrow \frac{3!}{3} (4+6+8) (111)$

Ex: $1/1/2/3/4 \Rightarrow \frac{5!}{5 \times 2!} (1+1+2+3+4) (11111)$

#) Percentage (Tree concept) :-



#) Reasoning:

I) Deductive [LN] (logical necessity) (unique soln)
 II) Inductive [LP] (logical possibility) (no unique soln)
 LP + maybe = LN

#) Shopkeeper markup goods:

$MP = 1.5625y$
 $D\% = 20\%$
 $SP = 1.25y = \frac{5}{4}y$
 $P\% = 25\%$
 $CP = y$

\Rightarrow Shopkeeper raised up goods 56.25% more than CP

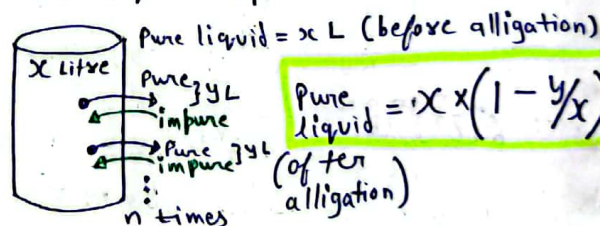
#) Mixture alligation:

n_1, n_2, \dots, n_k } sets of items (ratios)
 A_1, A_2, \dots, A_k } Averages (Price or anything like 5 ₹/kg)

$A_w = \frac{A_1 n_1 + A_2 n_2 + \dots + A_k n_k}{n_1 + n_2 + \dots + n_k}$

$n_1/n_2 = \frac{A_2 - A_w}{A_w - A_1}$

#) Pure Liquid taken out & other liquid replenished:



#) SI/CI:

Amount = Principle + Interest

In SI; Interest = $\frac{P \times t \times r}{100}$

In CI; Amount = $P \left(1 + \frac{r}{100}\right)^n$

CI Interest = $A - P$
 Semi annual compounding
 $r = \frac{20}{2} = 10\%$ 1 साल में 2 बार
 $\Rightarrow P(1.1)^4$
 Quarterly compounding
 $r = \frac{20}{4} = 5\%$ 1 साल में 4 बार
 $\Rightarrow P(1.05)^8$

#) Word in dictionary:-

Ex) DELHI
 Total = $5! = 120$
 Ordered = $\frac{5!}{2!} = 60$

10^{th} word: $D \rightarrow E/H/I \rightarrow 3! = 6$

EHI
 EIH
 HEI
 HIE
 IEH
 IHE
 LDIEH

$\frac{101}{-24 \times 4}$
 $\frac{101}{-96}$
 $\frac{101}{-96}$
 $\frac{101}{-96}$

#) Work & time :-

$\text{A} \rightarrow x \quad \frac{1}{x}$
 $\text{B} \rightarrow y \quad \frac{1}{y}$
 $\text{A+B} \rightarrow \frac{1}{x+y}$

One day work = $\frac{x+y}{x \cdot y}$ Days to complete = $\frac{x \cdot y}{x+y}$

Ex: $\text{A} = 10 \text{ D} \quad \text{B} = 20 \text{ D} \quad \text{C} = 30 \text{ D}$
 Day 1 2 3 ...
 (A+B) (A+C) (A+B) ...

1 pattern $\rightarrow 29\% \text{ WD}$
 (2 days) $\times 3$
 6 days $\rightarrow 87\% \text{ WD}$
 13% WL

7th day $\rightarrow \text{A+B} = \frac{13\%}{15\%} = \frac{13}{15} \text{ day}$

$\Rightarrow 6 \frac{13}{15} \text{ Days}$

#) Distribution of wages -

Ex: ₹ 45
 $\text{A} \rightarrow 15 \text{ D} \Rightarrow \frac{1}{15} = 6.67\% \quad \frac{5}{15} \times 45 = 15 \text{ ₹}$
 $\text{B} \rightarrow 20 \text{ D} \Rightarrow \frac{1}{20} = 5\% \quad \frac{5}{20} \times 45 = 11.25 \text{ ₹}$
 $\text{C} \rightarrow 12 \text{ D} \Rightarrow \frac{1}{12} = 8.33\% \quad \frac{5}{12} \times 45 = 18.75 \text{ ₹}$
 $\text{A+B+C} = 20\% = \frac{1}{5} \Rightarrow 5 \text{ Days}$

#) Pipes & cistern -

Ex: $\text{A} \rightarrow 15 \text{ h} \quad \text{B} \rightarrow 20 \text{ h} \quad \text{C} \rightarrow 30 \text{ h}$
 $\frac{1}{15} = 6.67\%$
 $\frac{1}{20} = 5\%$
 $\frac{1}{30} = 3.33\%$
 $\frac{1}{50} = 2\%$

$6.67 + 8.33 - 5 - 2 = 8\% \Rightarrow \frac{100\%}{8\%} = 12.5 \text{ h}$

$W = D M T E$; $W = \text{work or wage or working hours}$
 $\frac{D_1 M_1 T_1 E_1}{W_1} = \frac{D_2 M_2 T_2 E_2}{W_2}$

#) Logarithms -

$\log_a m = \frac{1}{\log_m a}$
 $\log_a a = 1$
 $\log_{10} 2 = 0.30 \quad 6 = 0.80$
 $3 = 0.50 \quad 7 = 0.85$
 $4 = 0.60 \quad 8 = 0.90$
 $5 = 0.70 \quad 9 = 0.95$
 $10 = 1.00$

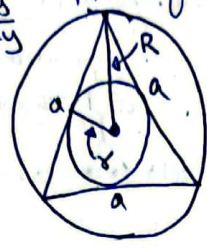
$\log_a m = \frac{1}{b} \cdot \log_a m$
 $\log_x m = p \Rightarrow m = x^p$

#) Geometry :-

interior angle = $\frac{(n-2) \times 180}{n}$

Sum of interior angles = $(n-2) \times 180$

Area of $\Delta = \frac{1}{2} \times B \times H = \frac{1}{2} \times a \times b \times \sin C$

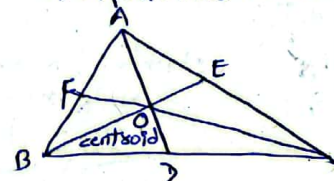


$= \sqrt{s(s-a)(s-b)(s-c)} ; s = \frac{a+b+c}{2}$

$= \frac{abc}{4R} = 4 \cdot S ; R = \text{circumradius}$
 $r = \text{inradius}$

$\frac{R}{r} = \frac{2}{1} \quad R = \frac{a}{\sqrt{3}}$
 $r = \frac{a}{2\sqrt{3}}$

#) Apollonius theorem -



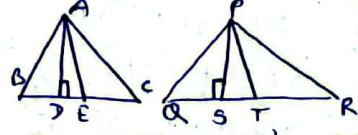
$\frac{AO}{OD} = \frac{BO}{OE} = \frac{CO}{OF} = \frac{2}{1}$

$AB^2 + AC^2 = 2[AD^2 + BD^2]$

$BA^2 + BC^2 = 2[BE^2 + CE^2]$

$CA^2 + CB^2 = 2[CF^2 + AF^2]$

#) Similarity - $\Delta ABC \sim \Delta PQR$



$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{PR} = \left(\frac{AD}{PS}\right) = \left(\frac{AE}{PT}\right)$

$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{PR^2}$

#) Area of sector:



$\text{Area} = \frac{\theta}{360} \times \pi r^2$

$\alpha = x+y$
 Exterior = Sum of two opposite interior

#) Verbal reasoning :-

#) Logical connectives -

I) Either-Or/Or:

- Either P or Q
- P, otherwise Q
- unless P, Q
- Q, unless P

II) If-then:

If P then Q

III) Only if P, then Q

If P then Q
 reverse में लिखा है

$P \text{ AND } Q = \frac{P}{1} \frac{Q}{1}$

$\overline{P \text{ OR } Q} = \overline{P} \text{ AND } \overline{Q}$

$\overline{P \text{ AND } Q} = \overline{P} \text{ OR } \overline{Q}$

$\overline{P} \rightarrow Q$
 $\text{OR } Q \rightarrow P$

only then the compound statement is true

$P \rightarrow Q$
 $\text{OR } \overline{Q} \rightarrow \overline{P}$

$Q \rightarrow P$
 $\overline{P} \rightarrow \overline{Q}$

#) Time & work:

- Days \rightarrow work } Just by inversion
work \rightarrow Days
- Ex: Total work in 30 days
Then, one day work = $\frac{1}{30}$ amt. of work
- Ex: One day work = $\frac{1}{40}$ amt. of work
Then, Total work in 40 days
- \uparrow work $\Rightarrow \downarrow$ time $\Rightarrow \downarrow$ days
 \uparrow men $\Rightarrow \uparrow$ work $\Rightarrow \downarrow$ days

Q: A transporter receives same no. of orders each day. Currently he has some pending orders to be shipped. If he uses 7 trucks, then at the end of 4th day he can clear all the orders. Alternatively if he uses only 3 trucks, then all the orders are cleared at the end of 10th day. What is min.^m no. of trucks required so that there will be no pending order at the end of 5th day.

7T \rightarrow 4d | Daily I get $\rightarrow x$ orders
3T \rightarrow 10d | Pending I've $\rightarrow y$ orders
tT \rightarrow 5d | 1 Truck carries $\rightarrow z$ orders each day

$$\begin{aligned} 7 \times z \times 4 &= 4x + y & \text{--- i} \\ 3 \times z \times 10 &= 10x + y & \text{--- ii} \\ \text{ii} - \text{i} &\Rightarrow 2z = 6x \\ &\Rightarrow z = 3x \\ &\Rightarrow y = 80x \end{aligned}$$

Now,

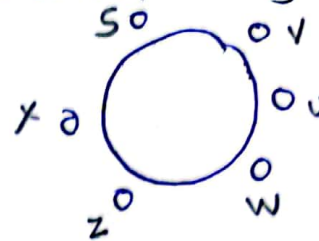
$$\begin{aligned} t \times z \times 5 &= 5x + y \\ t \times 3x \times 5 &= 5x + 80x \\ &\Rightarrow t = 6 \text{ Trucks} \end{aligned}$$

Q: Constr.ⁿ cost is 13,200 ₹.

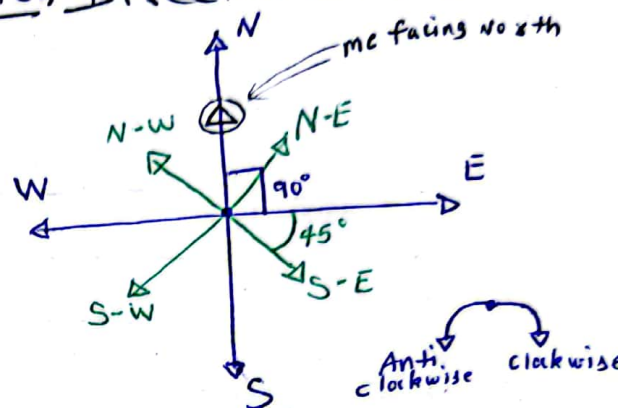
Now, Labour wages per day inc. by $\frac{1}{5}$
Working hours per day dec. by $\frac{1}{24}$
New constr.ⁿ cost?

cost	wages	working hours
13,200	1	1
	$1 + \frac{1}{5}$	$1 - \frac{1}{24}$
$\Rightarrow 13200 \times \frac{6}{5} \times \frac{23}{24}$	$= \frac{6}{5}$	$= \frac{23}{24}$
$= 15,180 \text{ ₹}$		

Note: To Solve Seating arrangement in circular



Note: Direction sense



Note: Revenue = Profit + Expense

Note:

- Sunrise \Rightarrow Morning \Rightarrow Shadow on the west line
- Sunset \Rightarrow Evening \Rightarrow Shadow on the east line
- 12 noon \Rightarrow No shadow \Rightarrow Sun over head

Note: If clock time in mirror is 1:30 then, actual time is 11:30

$$\begin{aligned} &11:30 \\ &- 01:30 \\ &\hline &10:30 \end{aligned}$$

Note: vol. & SA:

$$\begin{aligned} \text{vol} &\rightarrow \text{cyl} = \pi r^2 h & \rightarrow \text{Sphere} = \frac{4}{3} \pi r^3 \\ &\rightarrow \text{cone} = \frac{1}{3} \pi r^2 h \\ \text{cSA} &\rightarrow \text{Cyl} = 2 \pi r h = \text{Lateral SA} & \rightarrow \text{Sphere} = 4 \pi r^2 \\ &\rightarrow \text{cone} = \pi r l \end{aligned}$$

Note: 1 hectare = $100 \times 100 \text{ m}^2$
1 dm = 10 cm

Note: No. of zeros at end of $25 \times 25!$
for $25!$ $\Rightarrow 5 + 1 \Rightarrow 6$
for 25 or 5^2 we have 2 more # of 5's
So, Total = 8

* Work & time questions: —
Q-1) A can work 5 times faster than B and takes 60 days less than B to complete the work. In how many days does A & B individually can complete the work?

Alone
A → n-60 days

B → n days

Also, $A = 5B$

$$\Rightarrow (n-60) = \frac{1}{5} \times n$$

$$n = 75 \text{ days}$$

⇒ A → 15 days Individually
B → 75 days

Together $\frac{xy}{x+y}$

$$= \frac{15 \times 75}{15 + 75}$$

$$= \frac{15 \times 75}{90}$$

$$= 12.5 \text{ days}$$

Q-2) If 24 men can finish a work in 10 days, then find the no. of days required to complete the same work by 30 men.

24 men → 10 days

30 men → ? days

1 day work

$$\frac{1}{10}$$

$$?$$

$$24 \times \frac{1}{10} = 30 \times \frac{1}{?}$$

$$? = \frac{3}{24}$$

$$? = \frac{1}{8}$$

$$\Rightarrow 8 \text{ days}$$

Q-3) A can do a work in 3 days. 6

B can do the same work in 6 days

C can do " " " " 7 days

If they work together, in how many days will they complete the work.

Alone one day work

$$A \rightarrow \frac{1}{3}$$

$$B \rightarrow \frac{1}{6}$$

$$C \rightarrow \frac{1}{7}$$

$$A+B+C \rightarrow \frac{1}{3} + \frac{1}{6} + \frac{1}{7} = \frac{14+7+6}{42}$$

$$= \frac{27}{42} = \frac{9}{14}$$

$$\Rightarrow \text{Days to finish} = \frac{14}{9} = 1\frac{5}{9}$$

Q-4) P & Q can do a work in 12 days.

Q & R can do the same work in 16 days & R and P can do it in 24 days. Find the time in which P, Q and R can finish the work together.

one day work

$$P+Q \rightarrow 12 \text{ days} \quad \frac{1}{12}$$

$$Q+R \rightarrow 16 \text{ days} \quad \frac{1}{16}$$

$$R+P \rightarrow 24 \text{ days} \quad \frac{1}{24}$$

$$\Rightarrow (P+Q+R) \times 2 = \frac{1}{12} + \frac{1}{16} + \frac{1}{24}$$

$$= \frac{4+3+2}{48} = \frac{9}{48} = \frac{3}{16}$$

$$\Rightarrow P+Q+R = \frac{3}{32} \leftarrow \text{one day work}$$

$$\Rightarrow \text{Days to finish} = \frac{32}{3}$$

Q5) P can do a work in 30 days.
Q is 25% more efficient than P in completing the same work.
In how many days will Q complete the work.

P → 30 days 1 day work $\frac{1}{30}$

Q is 25% more efficient than P

P	Q
100%	125%
$\frac{1}{30}$	$1.25 \times \frac{1}{30}$

$$\Rightarrow Q \text{ one day work} = \frac{1.25}{30}$$

$$\begin{aligned} \text{Days to finish} &= \frac{30}{1.25} \\ &= 24 \text{ days} \end{aligned}$$

Q6) If 3 men can do a work in 2 days & 4 boys can do the same work in 6 days. Then, in how many days will the same work be completed by 8 men & 8 boys.

3M	→ 2 d] 3 times	$\frac{1}{2}$
4B	→ 6 d		$\frac{1}{6}$

$$\Rightarrow 3M = 3 \times (4B)$$

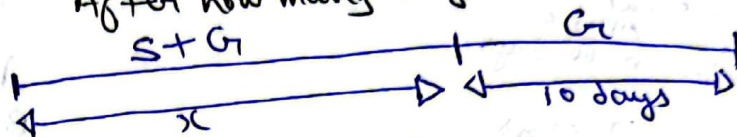
$$1M = 4B$$

$$\text{now, } 8M + 8B \Rightarrow 32B + 8B = 40B$$

$$\begin{aligned} 4B &\rightarrow \frac{1}{6} \\ 40B &\rightarrow ? \Rightarrow ? = \frac{40}{6 \times 4} \\ &\quad ? = \frac{10}{6} \end{aligned}$$

$$\Rightarrow \text{Days to complete} = \frac{6}{10}$$

Q7) Sita & Gita can do a work in 20 days & 25 days. Both begin together but after few days Sita leaves. Then Geeta finished the remaining work in 10 days. After how many days did Sita leave?



S	→ 20 d	one day work $\frac{1}{20}$
G	→ 25 d	$\frac{1}{25}$

Now, 10 days work of G :-

$$= 10 \times \frac{1}{25} = \frac{2}{5} \text{ of the work}$$

\Rightarrow S+G together did $\frac{3}{5}$ of the work in x days

$$S+G \text{ together } \frac{1}{20} + \frac{1}{25}$$

$$= \frac{25+20}{20 \times 25}$$

$$= \frac{45}{20 \times 25} = \frac{9}{100}$$

$$\Rightarrow \text{one day work of } S+G = \frac{9}{100}$$

$$\text{So, } x \text{ days work of } S+G = x \times \frac{9}{100}$$

$$\text{w.k.t } x \times \frac{9}{100} = \frac{3}{5}$$

$$x = \frac{20}{3} \text{ days}$$

Q8) When P alone does a work, he takes 25 days more than the time taken by P & Q working together to complete the work. But Q alone takes 9 days more than the time taken by P & Q working together to complete the work. In what time P & Q together finish this work?

$$P+Q \rightarrow N \text{ days}$$

$$Q \rightarrow N+9 \text{ days}$$

$$P \rightarrow N+25 \text{ days}$$

$$N = \sqrt{\text{Extra of P} \times \text{Extra of Q}}$$

$$= \sqrt{9 \times 25}$$

$$= 15 \text{ days}$$

Q-9) A can complete a work in 12 days and B can complete in 8 days. A works for 8 hours everyday, while B works for 10 hours everyday. If A & B together start working 8 hours per day, in how many days will they complete the work?

$$\begin{aligned} A &\rightarrow 12 \text{ d or } 96 \text{ hrs} & \frac{1}{96} \\ B &\rightarrow 8 \text{ d or } 80 \text{ hrs} & \frac{1}{80} \end{aligned}$$

$$A+B \rightarrow \frac{1}{96} + \frac{1}{80} = \frac{80+96}{96 \times 80} = \frac{176}{96 \times 80}$$

$$\Rightarrow 1 \text{ hr work of } A+B = \frac{176}{96 \times 80}$$

$$\Rightarrow \text{hrs to complete} = \frac{96 \times 80}{176}$$

$$\Rightarrow \text{Days to complete} = \frac{96 \times 80}{176 \times 8} = \frac{60}{11}$$

Q-10) Raj can build a house alone in 16 days but Suraj alone can build it in 12 days. Raj & Suraj work on alternate days. If Raj works on first day, house will be built in how many days?

$$\begin{aligned} R &\rightarrow 16 \text{ d} & \frac{1}{16} \\ S &\rightarrow 12 \text{ d} & \frac{1}{12} \end{aligned}$$

$$1 \text{ pattern} = R, S = \frac{1}{16} + \frac{1}{12} \quad (2 \text{ days})$$

$$= \frac{3+4}{48}$$

$$6 \times 1 \text{ pattern } (2 \text{ days}) = \frac{7}{48} \text{ work done} \times 6$$

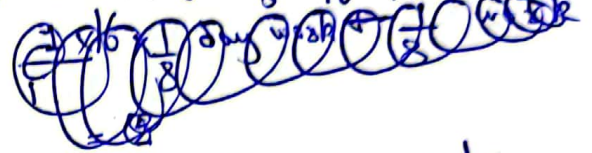
$$12 \text{ days} \rightarrow \frac{42}{48} \text{ work done}$$

but $\frac{6}{48}$ work left
or $\frac{1}{8}$ work left

On 13th day Raj works

$$\Rightarrow \text{1 day work} = \frac{1}{16} \text{ work done}$$

$$\Rightarrow \text{work left} = \frac{1}{8} - \frac{1}{16} = \frac{1}{16} \text{ work left}$$



On 14th day Suraj works

$$1 \text{ day} \rightarrow \frac{1}{12} \text{ work done}$$

$$\frac{1}{16} \times \frac{1}{12} \text{ day} \leftarrow \frac{1}{16} \text{ work done}$$

$$\frac{3}{4} \text{ day} \leftarrow \frac{1}{16} \text{ work done}$$

$$\text{So, Ans: } 13 \frac{3}{4}$$