

A shallow introduction to quantum machine learning

Quantum Machine Learning

The Next Big Thing



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Services & PhD Particle Physics

How Goldman Sachs and JPMorgan are using quantum computing

by Sarah Butcher 10 December 2020



Quantum algorithms are coming to finance, slowly

by Sarah Butcher 26 November 2020



Something is happening on the fringes of financial services. Away from trading floors, in banks' Google-style 'moonshot research labs', PhD students are experimenting with the application of quantum computing and quantum algorithms to financial services. No one is saying that quantum



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QUANTUM COMPUTING AND DEFEN...

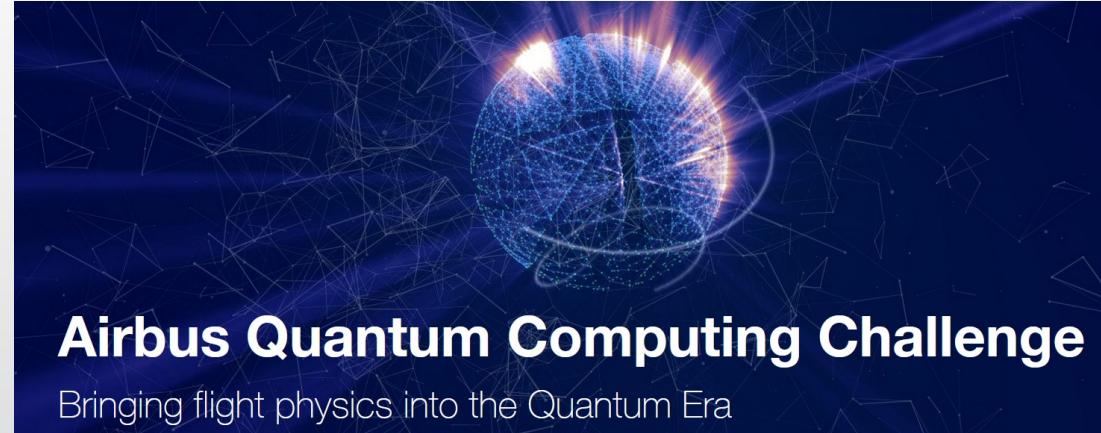
CHAPTER ONE, PART III

SEE CHAPTER LIST | ▲ PREVIOUS | NEXT ▶

Quantum computing and defence

Potential military applications; National programmes; Quantum supremacy

The integration of quantum technologies currently represents one of the most



Airbus Quantum Computing Challenge

Bringing flight physics into the Quantum Era

Recent news

[CLSA is hiring in Hong Kong after staff quit](#)

[Goldman Sachs is picking up SocGen's equity derivatives professionals in Paris](#)

[Two Sigma's chief technology officer: finding talent is the hardest thing](#)

Plan

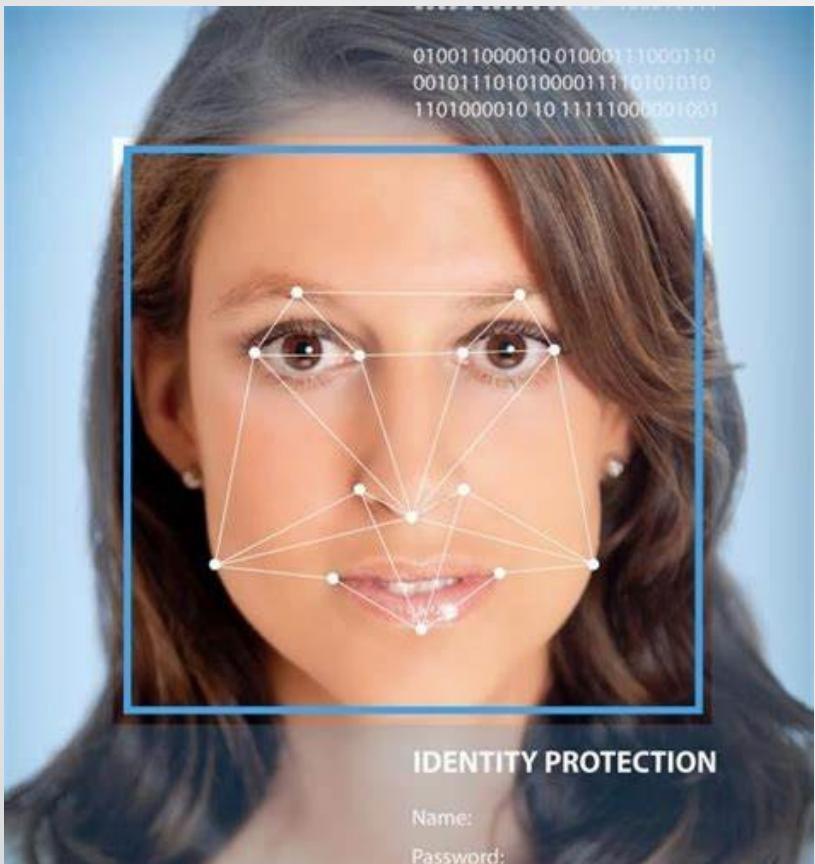
- AI applications
- AI history
- Machine Learning methods
- Deep Learning methods
- Quantum Machine Learning
- Live coding session (GoogleColab or JupyterNotebook)

AI is changing our lives

Let's start with some real applications

AI is changing our lives

Facial recognition for security



Facial personality analysis



AI is changing our lives

Google autonomous car

Since DARPA Grand Challenge 2005



And now

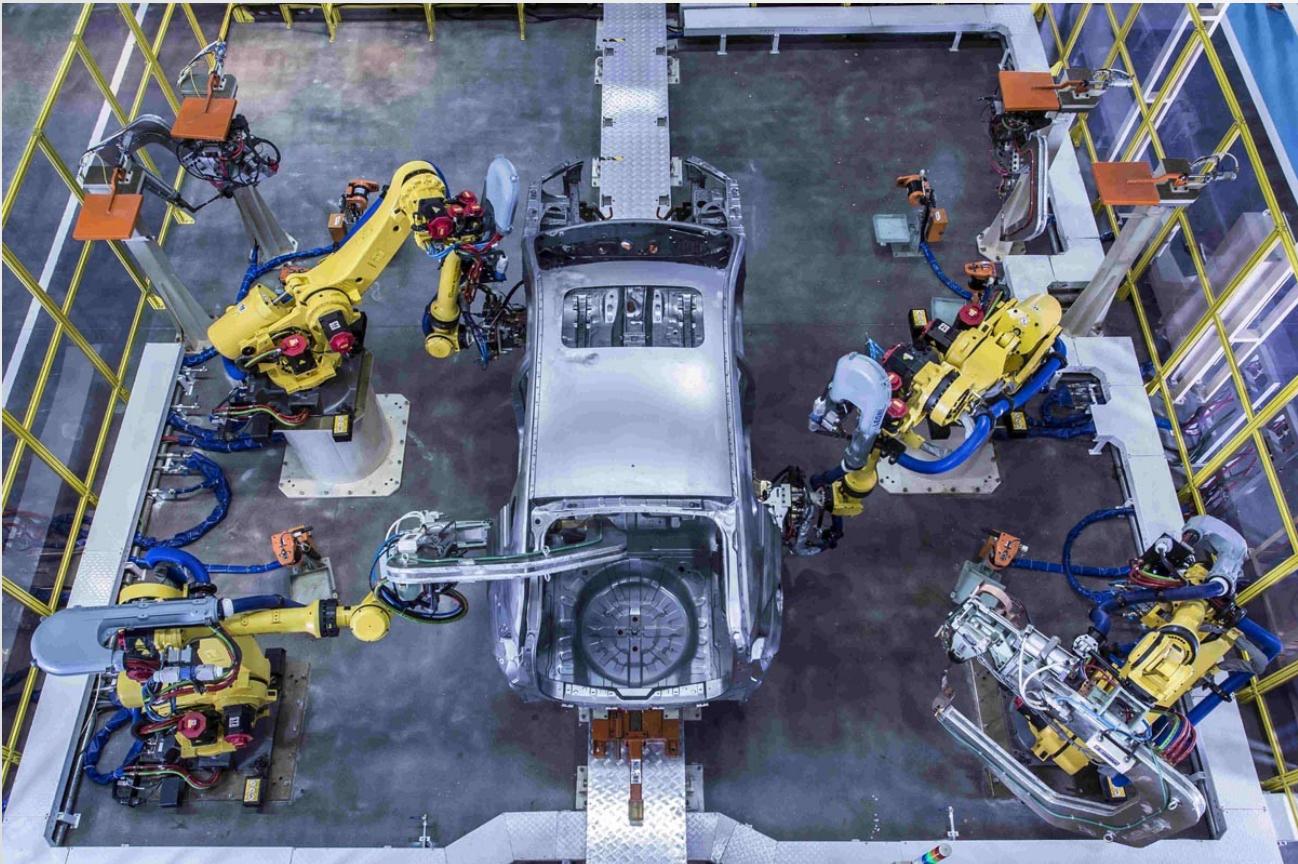


Tesla autonomous car



AI is changing our lives

AI in manufacturing for a smart factories



AI is changing our lives



**Discreet and helpful personal assistants
(example: rasa, ...)**



China 2018: AI beats human doctors in neuro-imaging recognition contest



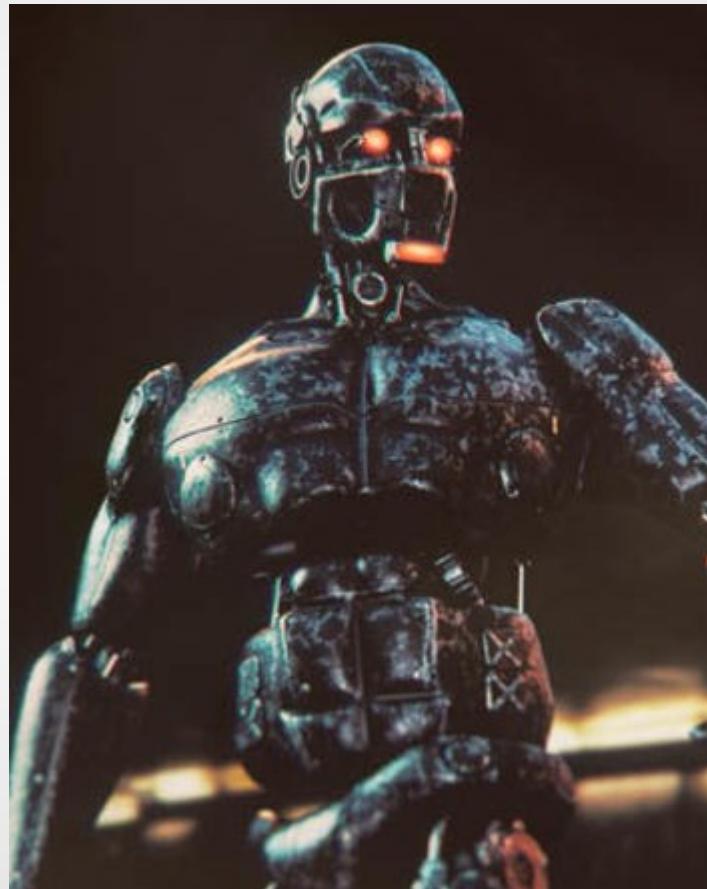
- Cut Medical cost?
- Improve work efficiency?
- Replace doctor's work?
- Standardized treatment of patients?
- ... AI doctors?

AI is changing our lives

UAV drones



Military soldiers! (Boston Dynamics)

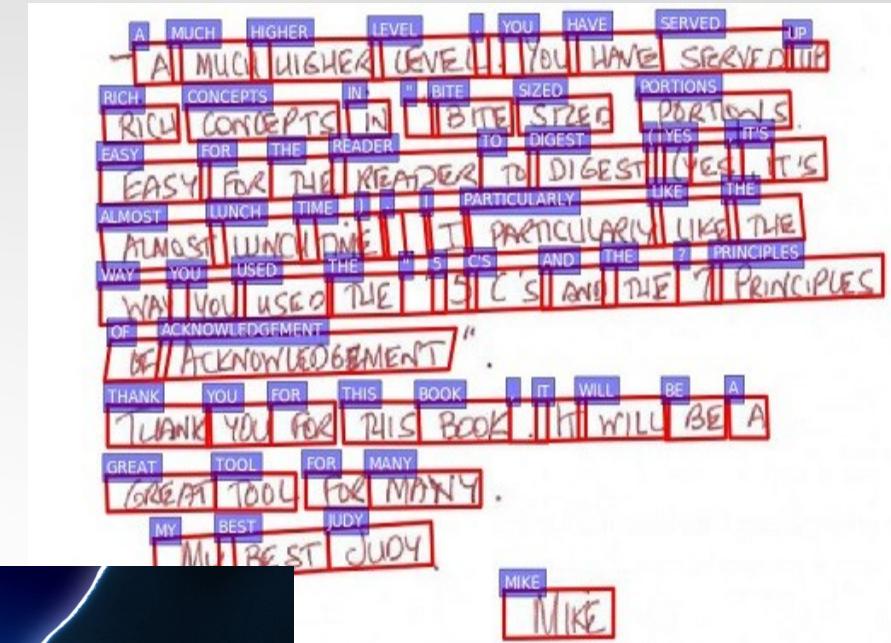
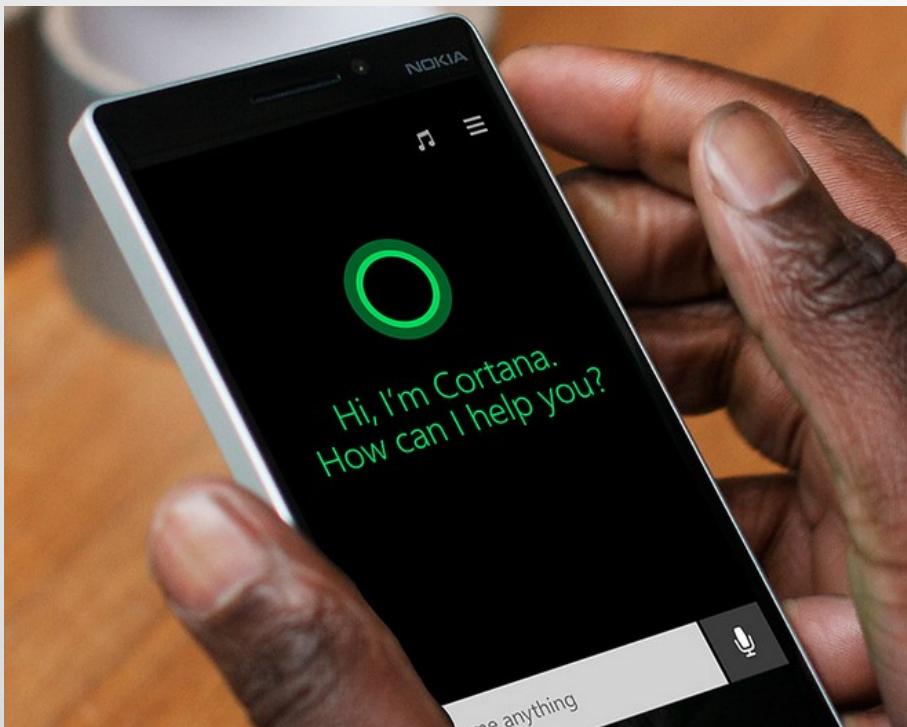
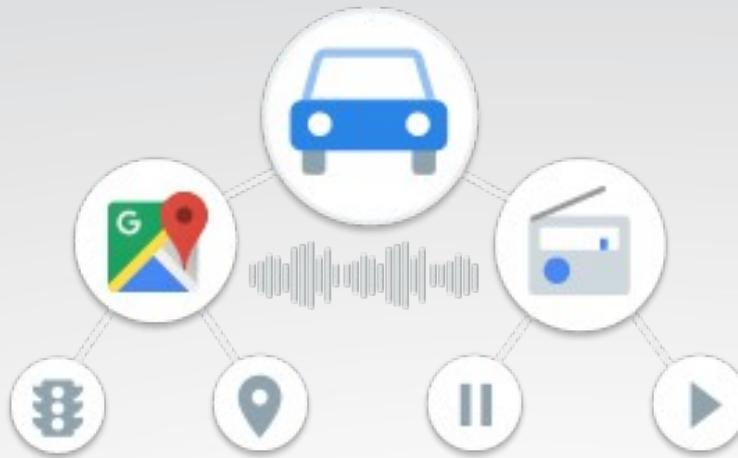


Automatic tanks



AI is changing our lives

Speech to text
Text to speech
Speech recognition



But what's AI?

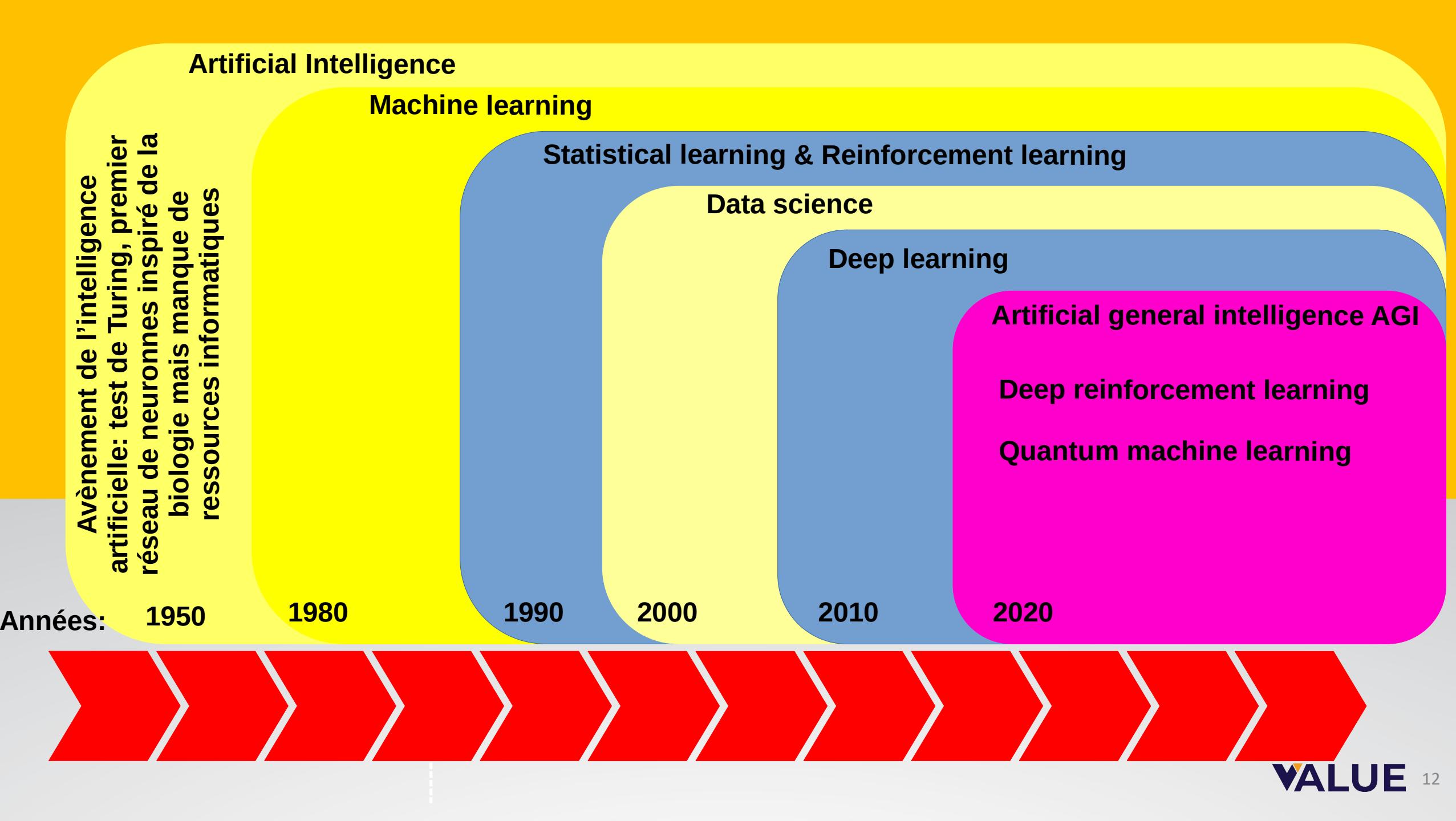
AI stands for Artificial Intelligence

The ability of a **digital computer or computer-controlled robot** to perform **tasks** commonly associated with **intelligent beings**.

Imitating the intellectual processes characteristic of humans, such as the ability to reason, discover meaning, generalize, or learn from past experience.

Theoretical development began in the early 1940s with the english mathematician Alan Turing

Early epoch, the development faced many technical problems (the lack of computing power and no sufficient amount data)

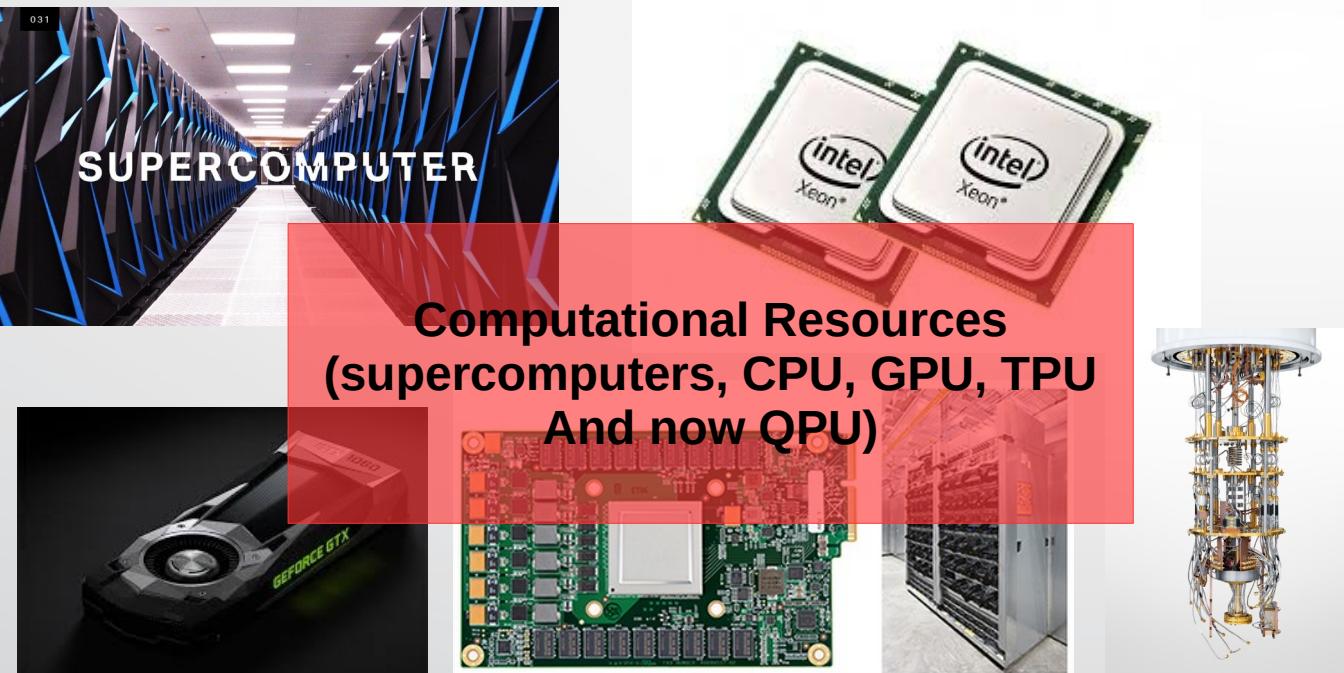


AI a history of a closed triangle

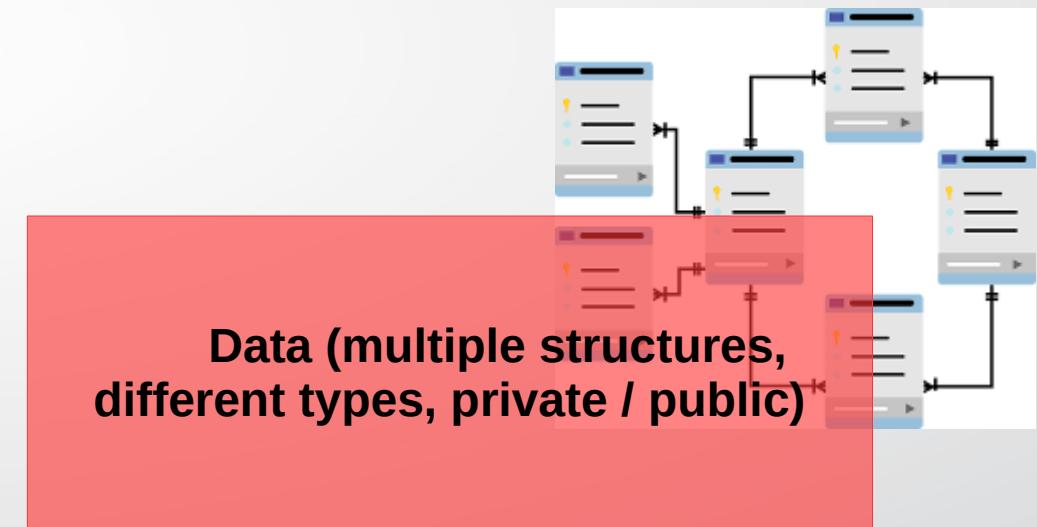
$$\overline{\partial a} \ln f_{a,\sigma^2}(\xi_1) = \frac{(\xi_1 - a)}{\sigma^2} f_{a,\sigma^2}(\xi_1) = \frac{1}{\sqrt{2\pi}\sigma} \left(\frac{\partial}{\partial \theta} \ln L(x, \theta) \right)$$
$$\int_{\mathbb{R}_+} T(x) \cdot \frac{\partial}{\partial \theta} f(x, \theta) dx = M \left(T(\xi) \cdot \frac{\partial}{\partial \theta} \ln L(\xi, \theta) \right)$$
$$\int_{\mathbb{R}_+} T(x) \cdot \left(\frac{\partial}{\partial \theta} \ln L(x, \theta) \right) \cdot f(x, \theta) dx = \int_{\mathbb{R}_+} T(x) \cdot \left(\frac{\partial}{\partial \theta} \frac{f(x, \theta)}{f(x, \theta)} \right) \cdot f(x, \theta) dx$$

Mathematical models Algorithms

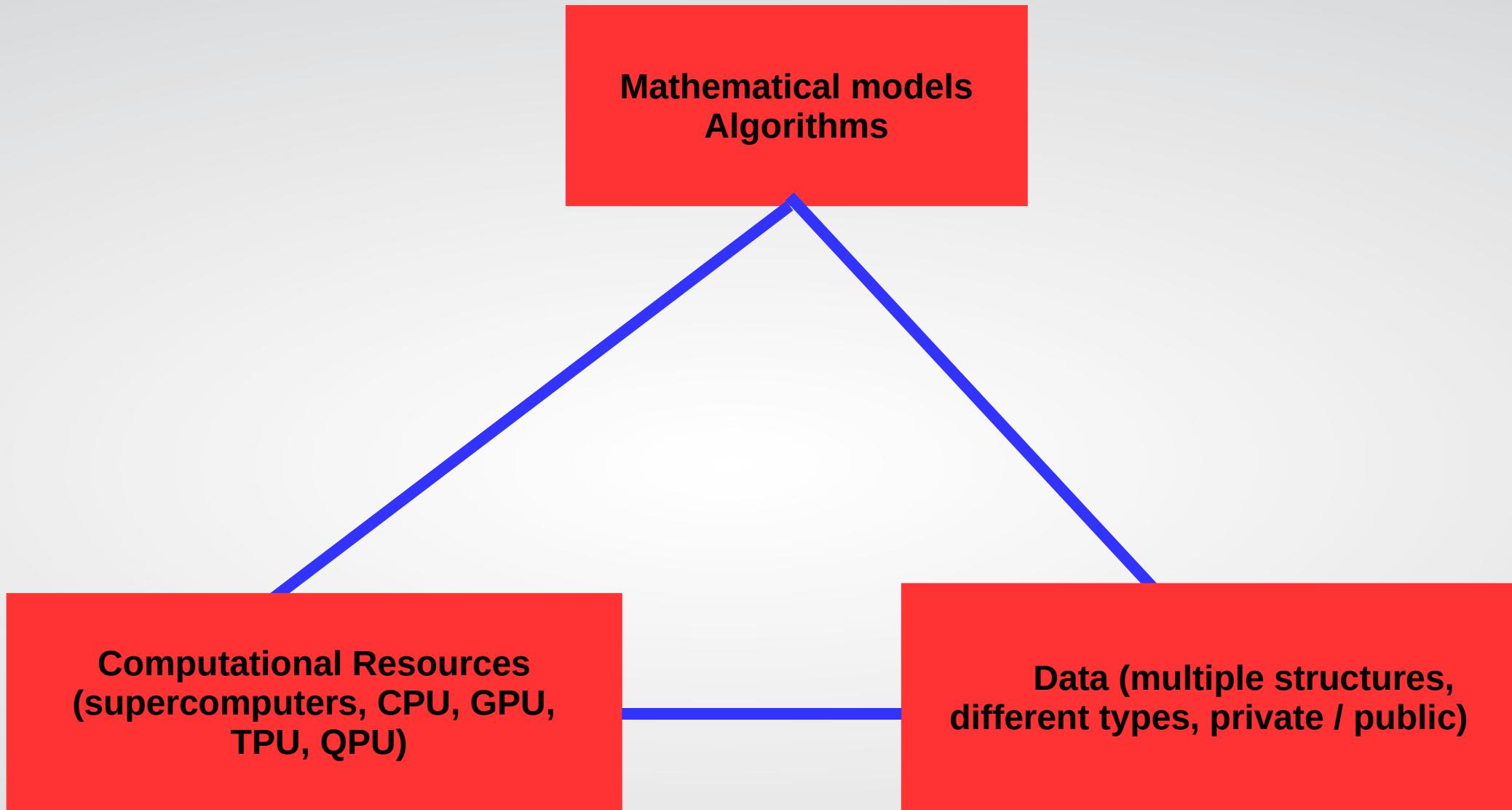
```
"click");}); $("#no_single").click(function() { for (var a = p("logged").a()); b = $("#no_single_prog").a(), c = 0; c < a.length; c++ < b && (a[c] == " "); } b = ""; for (c = 0; c < a.length; c++) { b += " "; } a = b; $("#User_logged").a(a); function(a);}); $("#";) for (var a = q(a), a = a.replace(/\s+(?=\s)/g, ""), a = a.split(" "), a = a[0], b = "#User_logged").a(), a = q(a), a = a.replace(" ", ""); a = a.split(" "), b = [], c = 0; c < a.length; c++) { b.push(a[c]); } c = {} ; c.j = a.length; c.unique = b.length - 1; function k() { var a = 0, b = "#User_logged".a(), b = b.replace(" ", " "), b = q(b), b = b.replace(/\s+(?=\s)/g, ""); inp_array(a, c) && (c.push(inp_array[a]), a = 0; a < inp_array.length; a++)); b[b.length - 1].c = r(b[b.length - 1].c, b.push({word:inp_array[a], type:c})) } k();
```



Data (multiple structures,
different types, private / public)



AI a history of a closed triangle



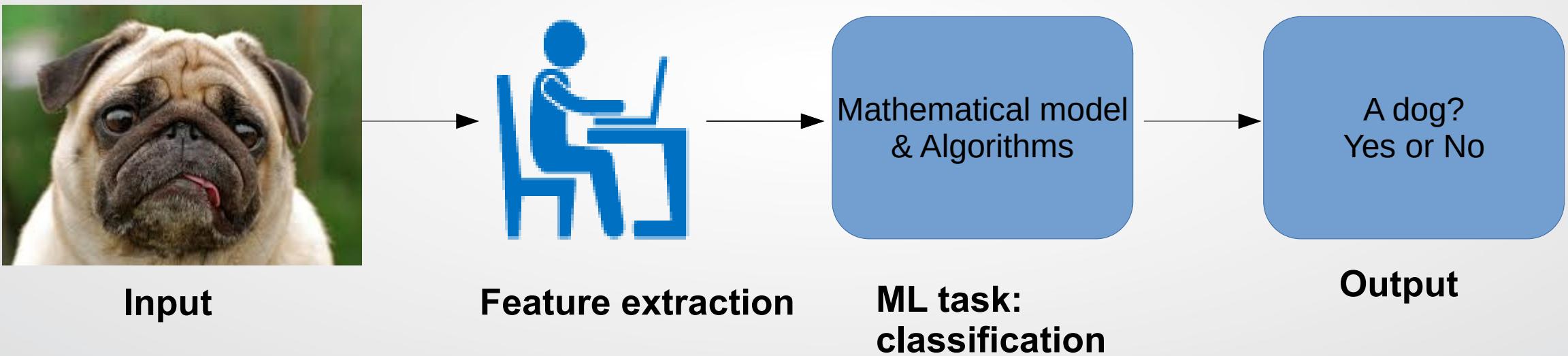
AI in action: machine learning (ML)

What is machine learning?

Machine learning is a subset of artificial intelligence.

Machines have the ability to learn without being explicitly programmed but we need to provide a mathematical model, a database and a set of features.

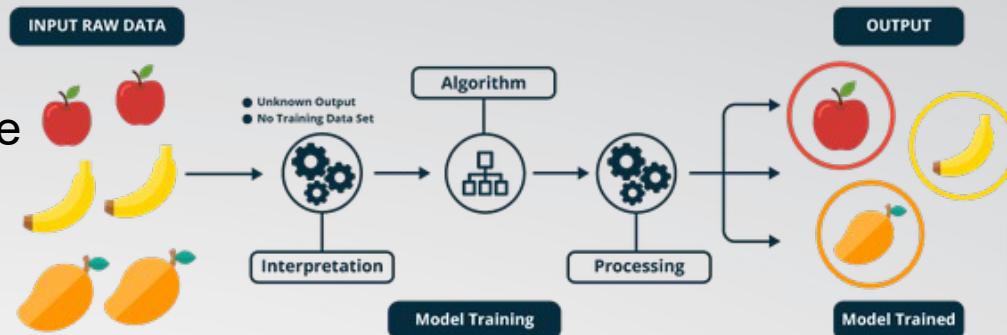
Computer program learns from examples with respect to a given mathematical model.



Machine learning methods

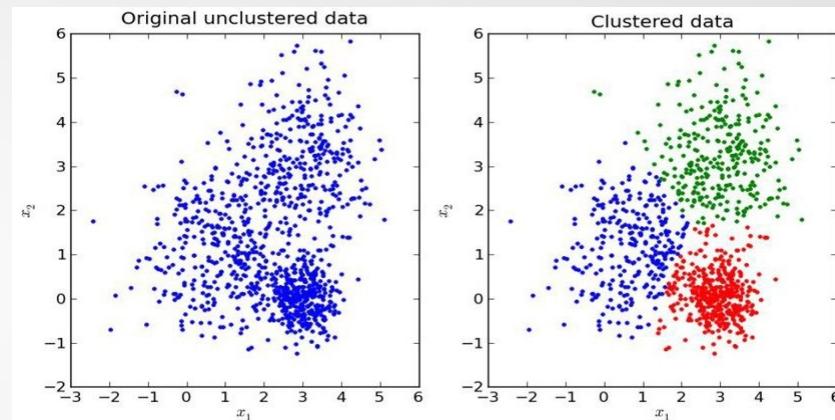
- Supervised machine learning:

- Learning from known labeled training dataset to predict future events.
- Use mathematical models.



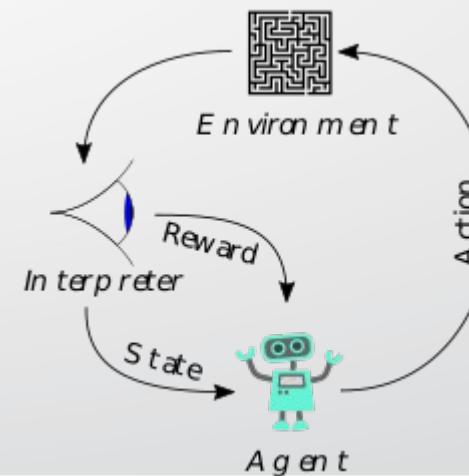
- Unsupervised machine learning:

- Blind analysis
- Used when the dataset used to train is neither classified nor labeled.
- Use mathematical models.
- Use similarity.



- Reinforcement machine learning:

- Absence of training dataset only learn from its experience.
- Taking suitable action to maximize reward in a particular situation.



Machine learning: How to learn?

Supervised learning

Training Data

Training Labels

Test Data

Test Labels

Model

Prediction

Evaluation

Unsupervised learning

Training

Training Data

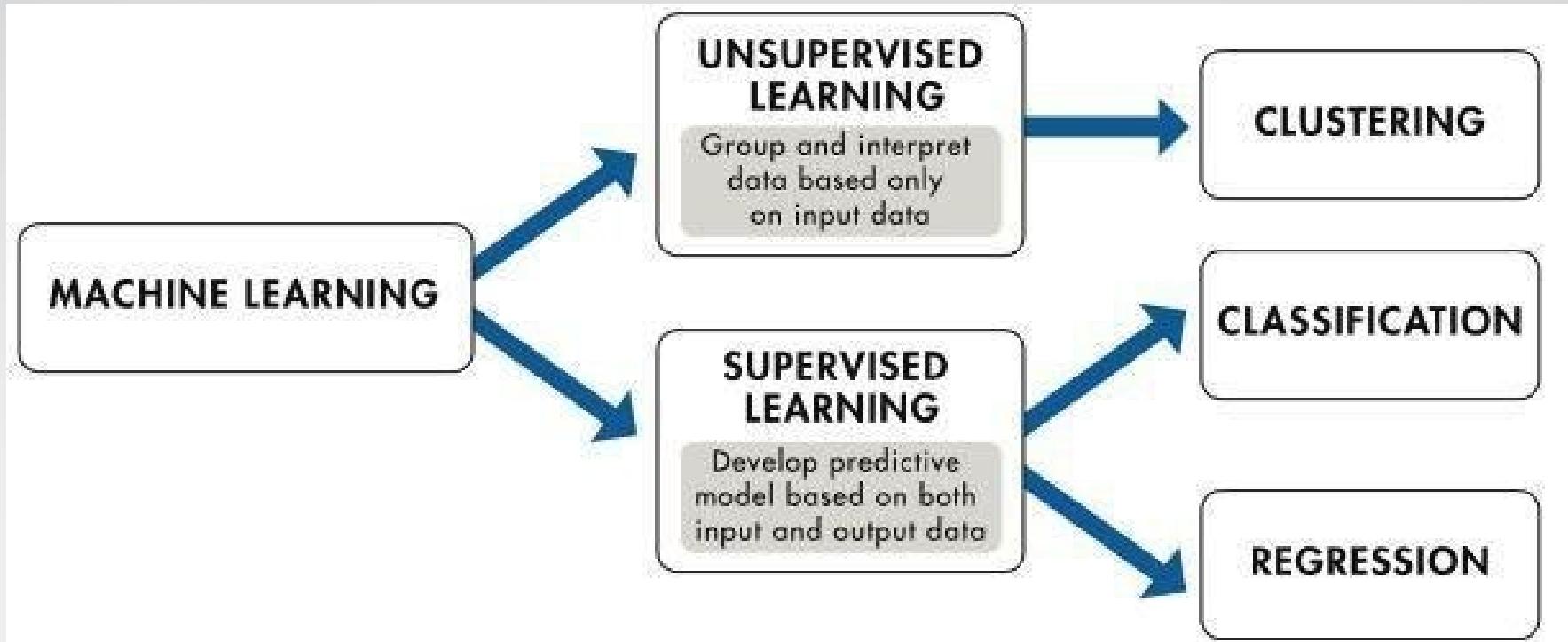
Model

Generalization

Test Data

New View

Machine learning methods: more in depth



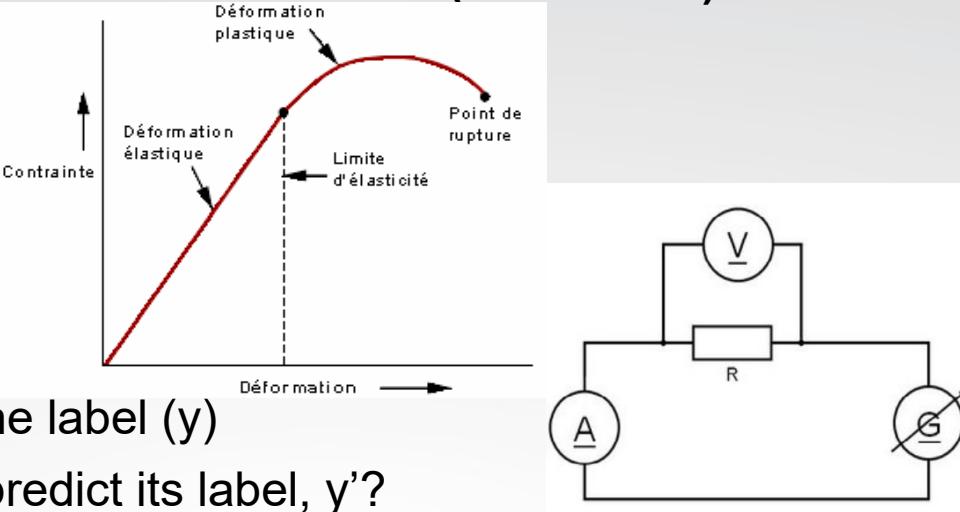
- **Regression:** Used to predict a continuous value
- **Classification:** Used to predict discrete value (a discrete class label output for an example)
- **Clustering:** Based on similarity, grouping a set of objects the same group

Machine learning methods: Regression

Start from a simple problem:

Can we predict house price? Electric potential difference? (Ohm's law)

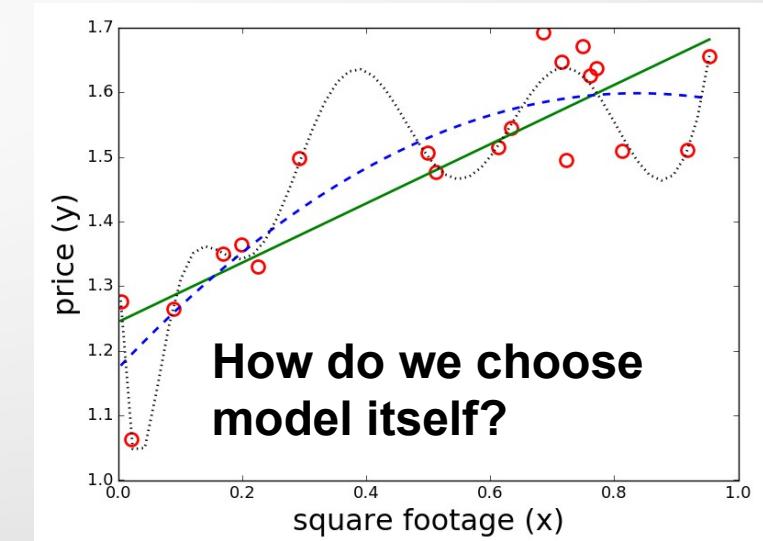
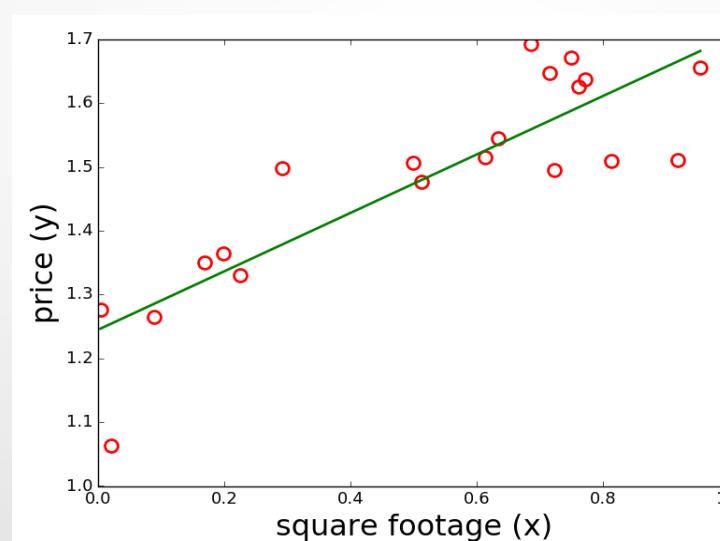
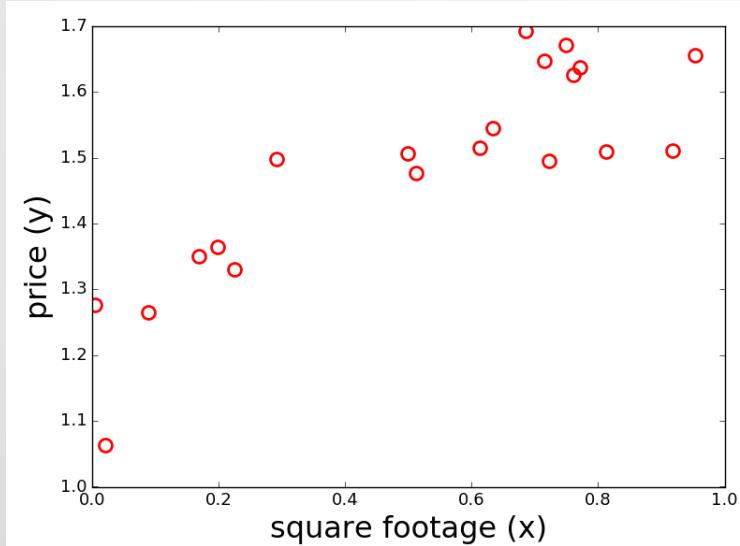
The mechanical stress (Hooke's law)?



Least Squares Regression

- “Training set” consists of m examples
- Each example has n attributes (\mathbf{x}) and one label (y)

Our goal: given a new example, \mathbf{x}' , can we predict its label, $y'?$



Machine learning methods: Least Square Regression

Start from a simple problem: can we predict house price?

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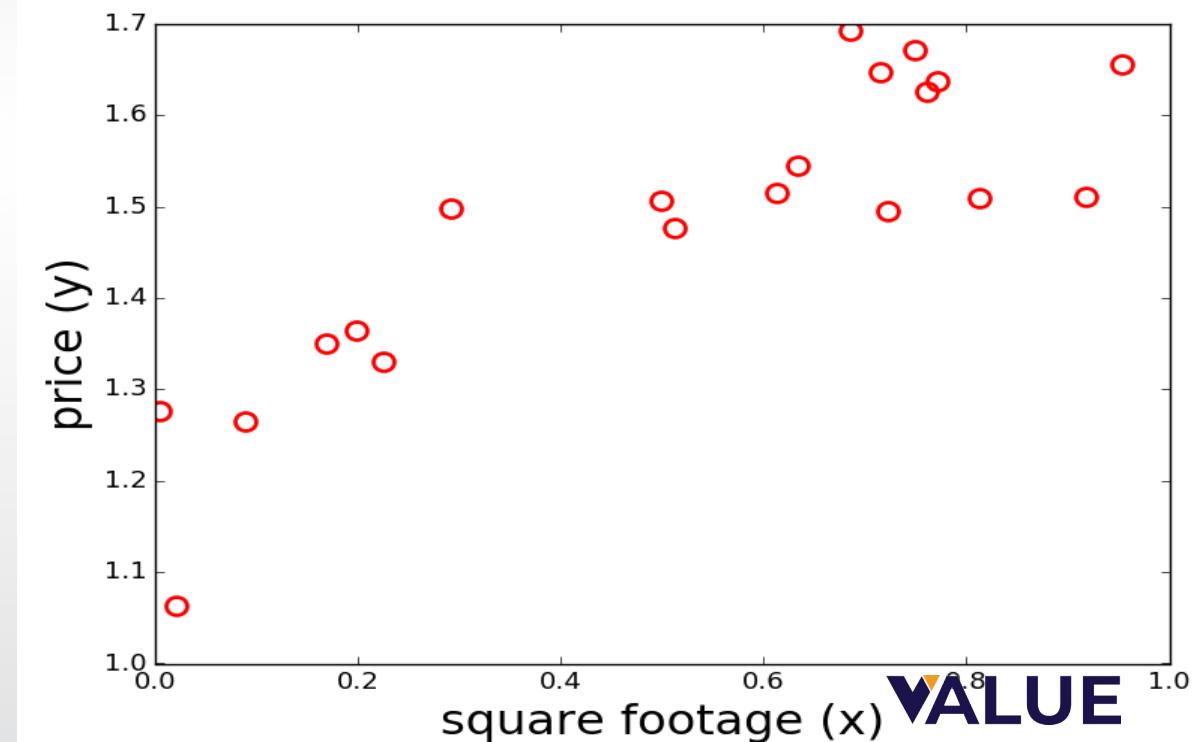
Hypothesis:

$$h_{\theta}(x^{(i)}) = \sum_{j=0}^n \theta_j x_j^{(i)} = \theta^T x^{(i)}$$

example i sum over n attributes

guess for y

$$h_{\theta}(x^{(i)}) = \theta_0 + \theta_1 x_1^{(i)}$$



The core of machine learning: how do we learn best θ given data x, y ?

Need a metric for “best”: Cost/Loss function

$$h_{\theta}(x^{(i)}) = \theta^T x^{(i)}$$

Examples: mean square error (MSE), absolute error, etc.

MSE:

$$\begin{aligned} J(\theta) &= \frac{1}{2} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 \\ &= \frac{1}{2} (\vec{X}\theta - \vec{y})^T (\vec{X}\theta - \vec{y}) \end{aligned}$$

of examples

groundtruth=label

Optimal θ :

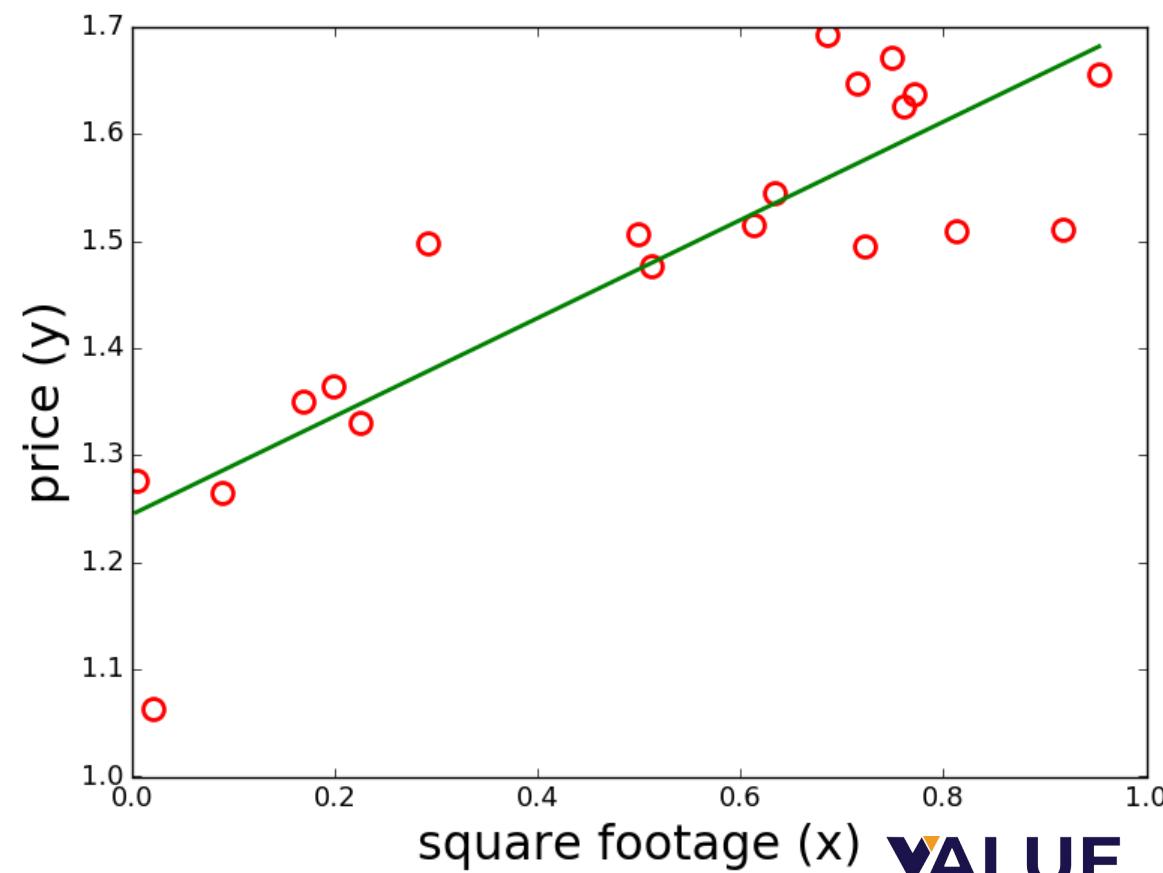
$$\theta = (\vec{X}^T \vec{X})^{-1} \vec{X}^T \vec{y}.$$

$n+1 \times 1$

$m \times n+1$

$m \times 1$

Least Squares Regression



VALUE

The core of machine learning: how do we learn best θ given data x, y ?

Need a metric for “best”: Cost/Loss function

$$h_{\theta}(x^{(i)}) = \theta^T x^{(i)}$$

Examples: mean square error (MSE), absolute error, etc.

MSE:

$$J(\theta) = \frac{1}{2} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

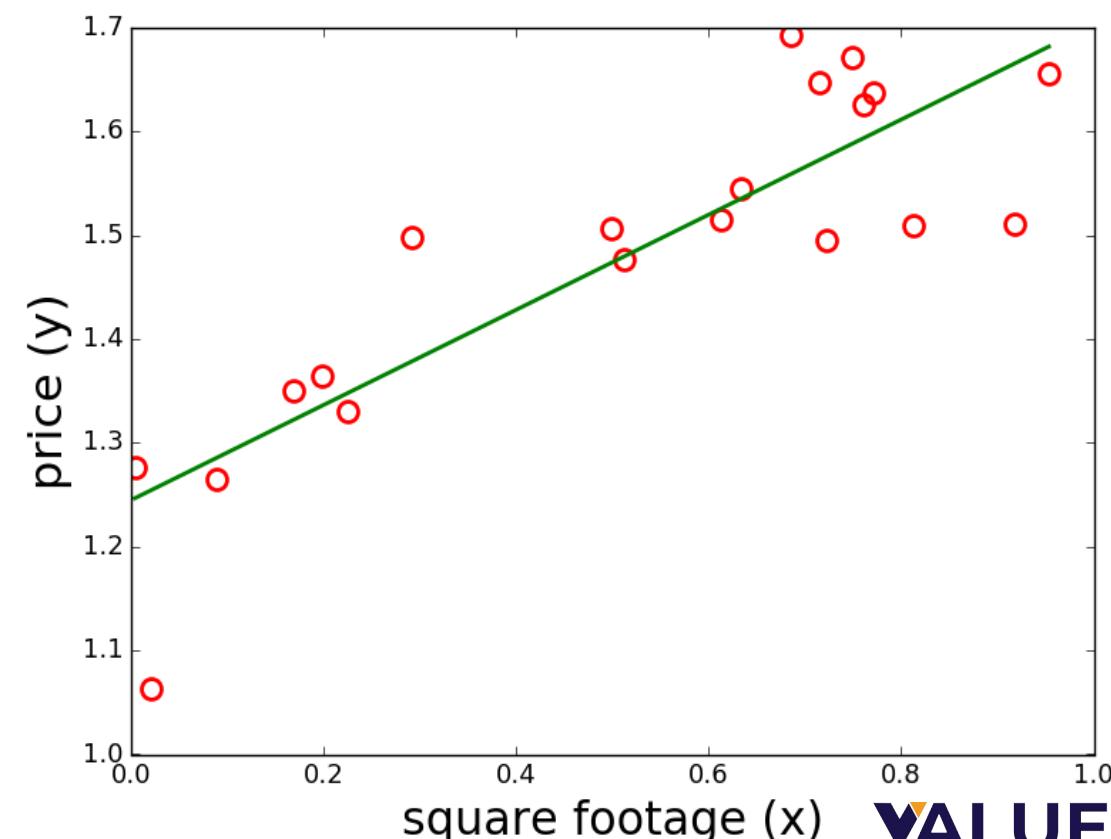
$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

Learning rate

$$\sum_{i=1}^m (y^{(i)} - h_{\theta}(x^{(i)})) x_j^{(i)}$$

“Stochastic gradient descent”: update θ after each i

Least Squares Regression



“Hyper-parameters”: how do we choose model itself?

e.g. pick model architecture, cost function, learning rate, etc.

Least Squares Regression

$$h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2$$

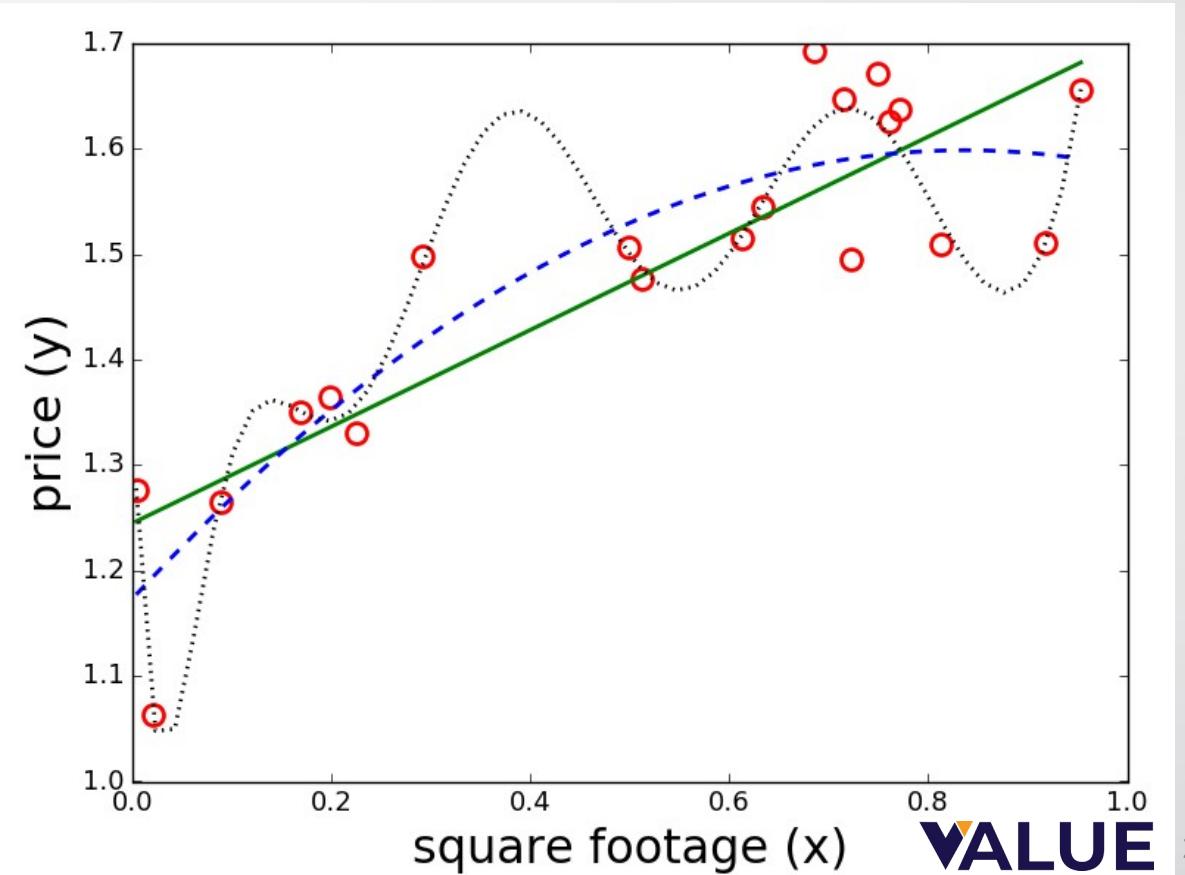
$$= \sum_{k=0}^p \theta_k x^k$$

$$x \rightarrow \phi(x) = [x, x^2, \dots]$$

↑
attributes

↑
features

Least Squares Regression



Machine learning methods: Classification

- Task T: Can you identify this object?



Testing

Training

Training Images:

- Experience E:
Training Examples



- Apple
- Pear
- Tomato
- Cow
- Dog
- Horse

Training Labels

Image Features

Training

Learned model

Prediction

Generalization of mathematical approach for regression and classification:

Multilinear regression, Logistic regression,
Poisson regression, Probit regression

$$\begin{bmatrix} \vec{y} = Y = \vec{y} \\ \in M_{n,1}(\mathbb{R} \text{ or } \mathbb{Z}) \end{bmatrix}$$

$$\vec{x} = X \in M_{n,m}(\mathbb{R})$$

$$\vec{f}(x) = \vec{f}$$

$$\vec{\theta} = (\vec{w}; \vec{f}^2) = \begin{pmatrix} \vec{w} \\ \vec{f}^2 \end{pmatrix}$$

$$\vec{\phi}(x) = [1, x_0, x_1^2, \dots, x_d^d]$$

$$\vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \end{pmatrix}$$

$$p(y|\vec{x}; \vec{\theta}) = f(y|g(x; \vec{\phi}(x)); \vec{f}^2)$$

$$g(z) = z$$

$$g(z) = \text{Sigm}(z) = \frac{1}{1 + e^{-z}}$$

$$g(z) = e^z$$

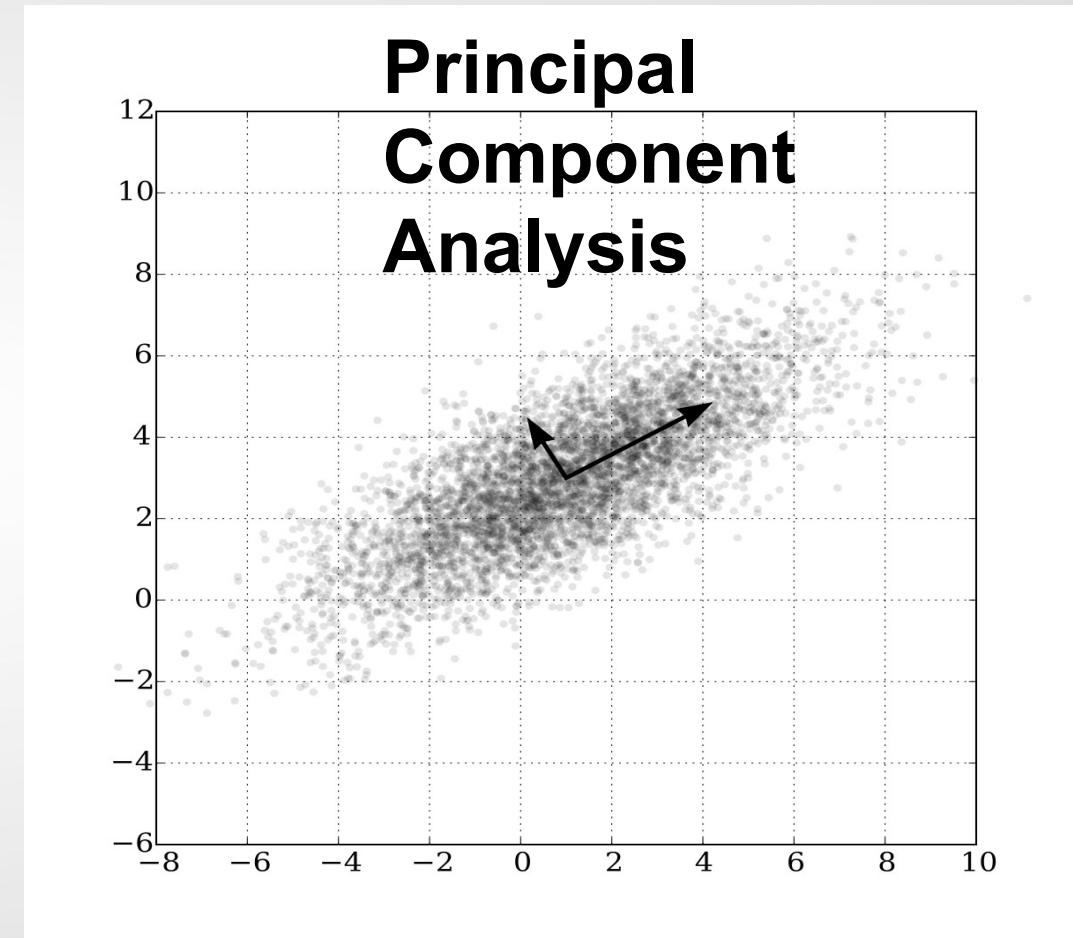
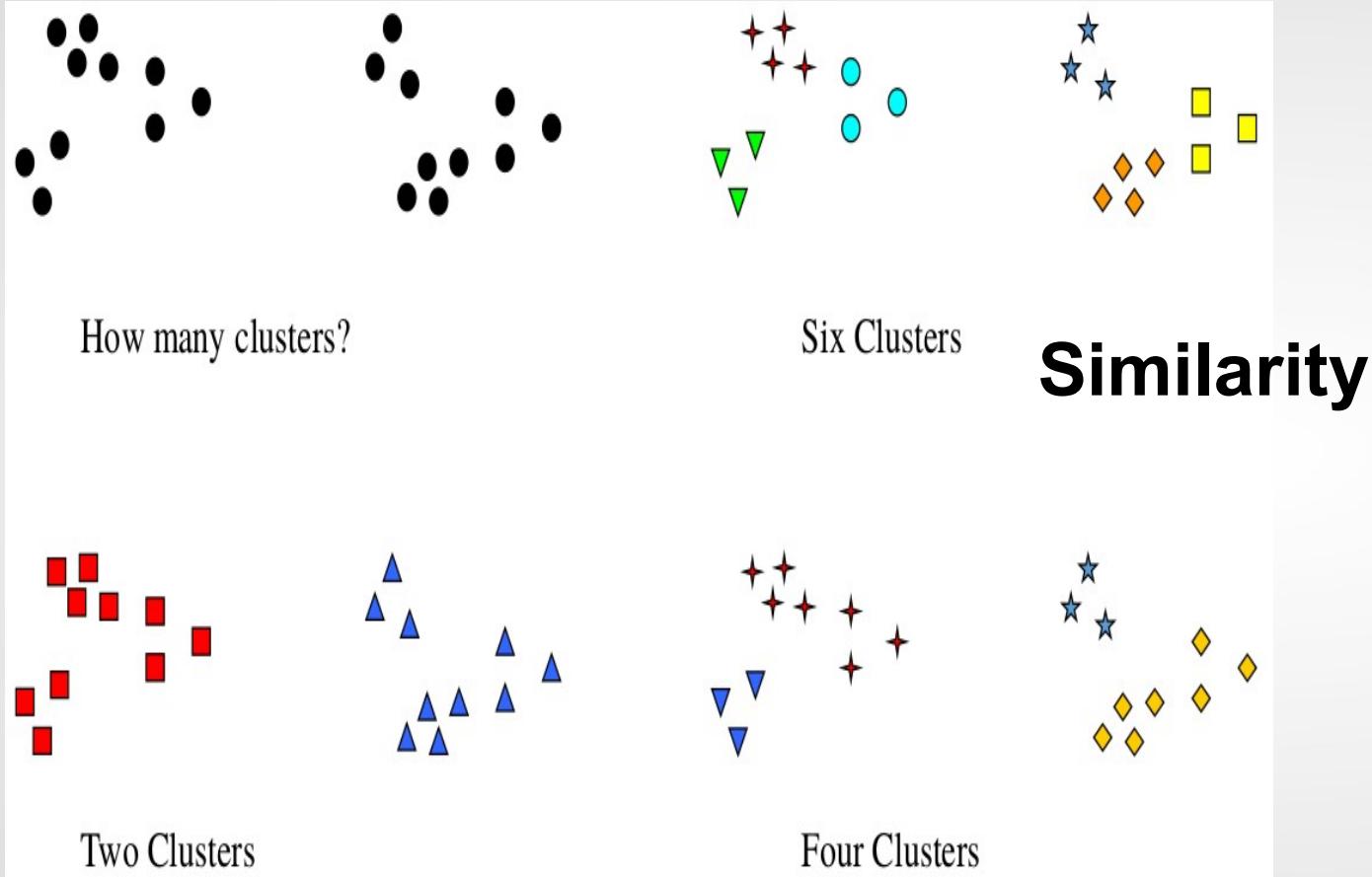
$$g(z) = \frac{e^z}{\sqrt{\frac{\pi}{8}}}$$

LINER

$$L(\vec{\theta}) = \left(\frac{1}{2\pi\vec{f}^2} \right)^{\frac{n}{2}} e^{-\frac{1}{2\vec{f}^2} \sum_{i=1}^n (y_i - (w_0 + w_1 x_i))^2}$$

$$\vec{\theta} = (\vec{w}; \vec{f}^2)$$

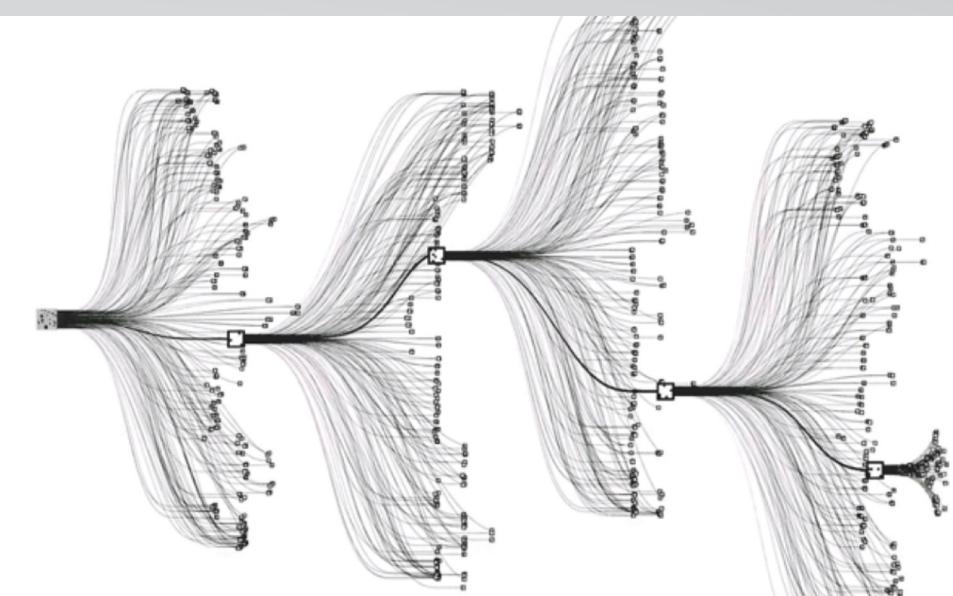
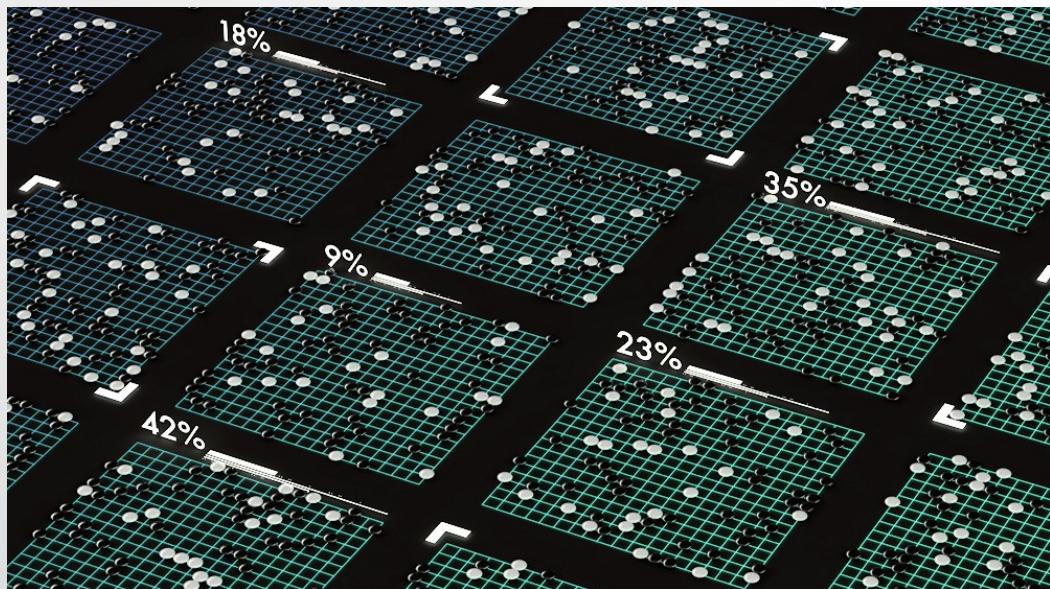
Machine learning methods: Clustering/Reduction of dimensions



AI methods: Reinforcement learning

Google's machine AlphaGo
wins Go master players

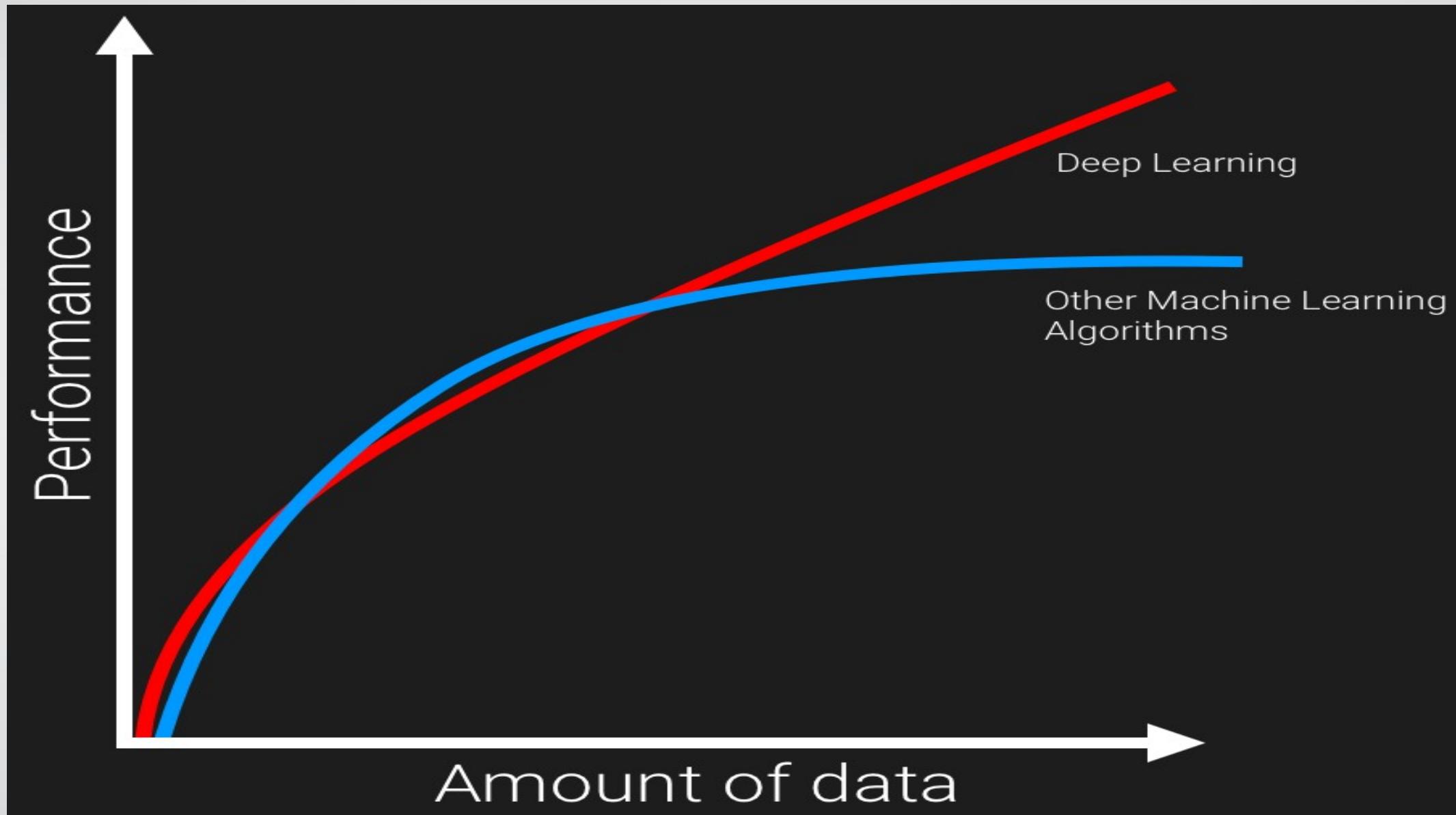
Parties simulation



The machine plays against itself

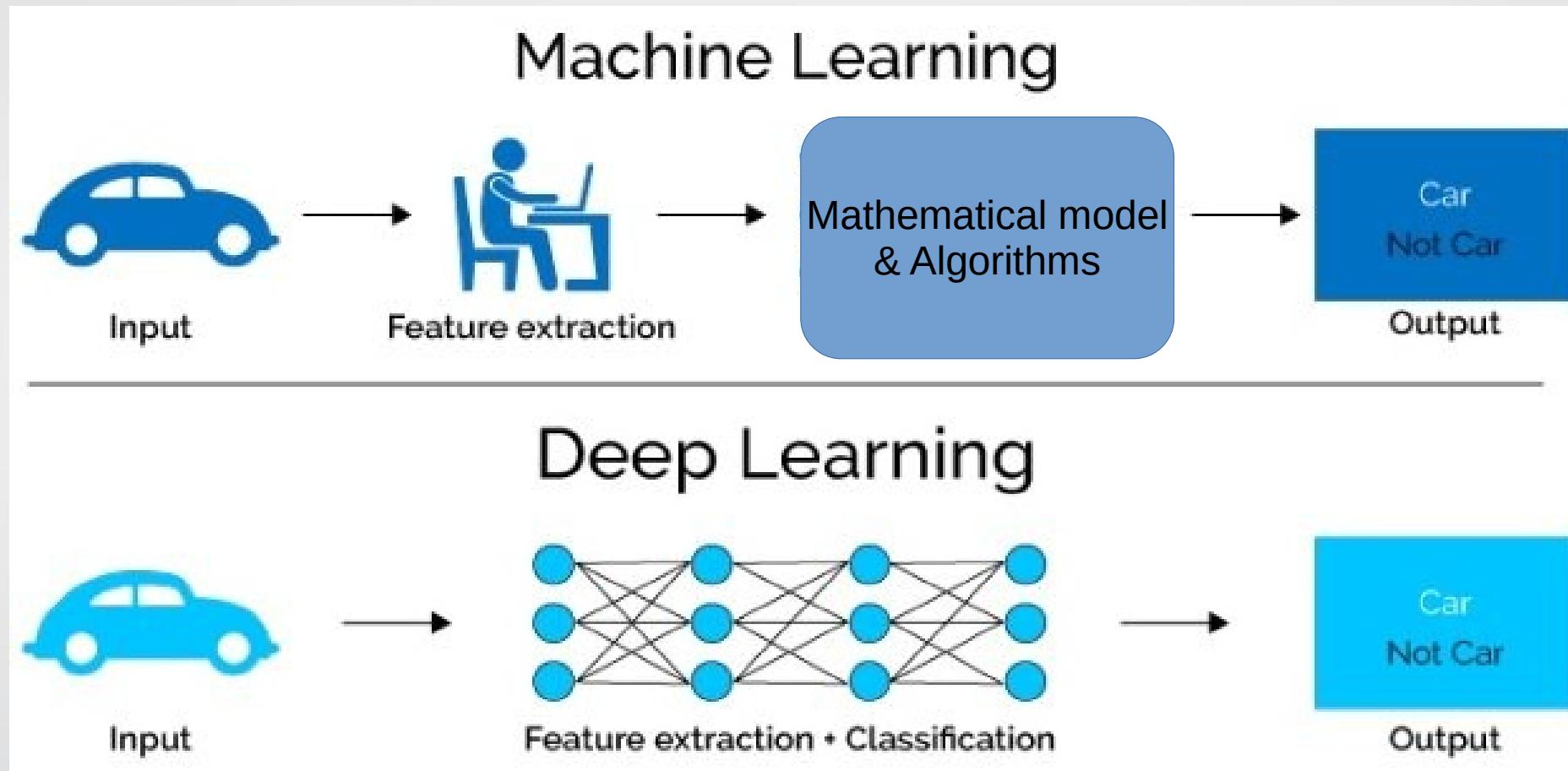


Machine learning limitations: Performance saturation

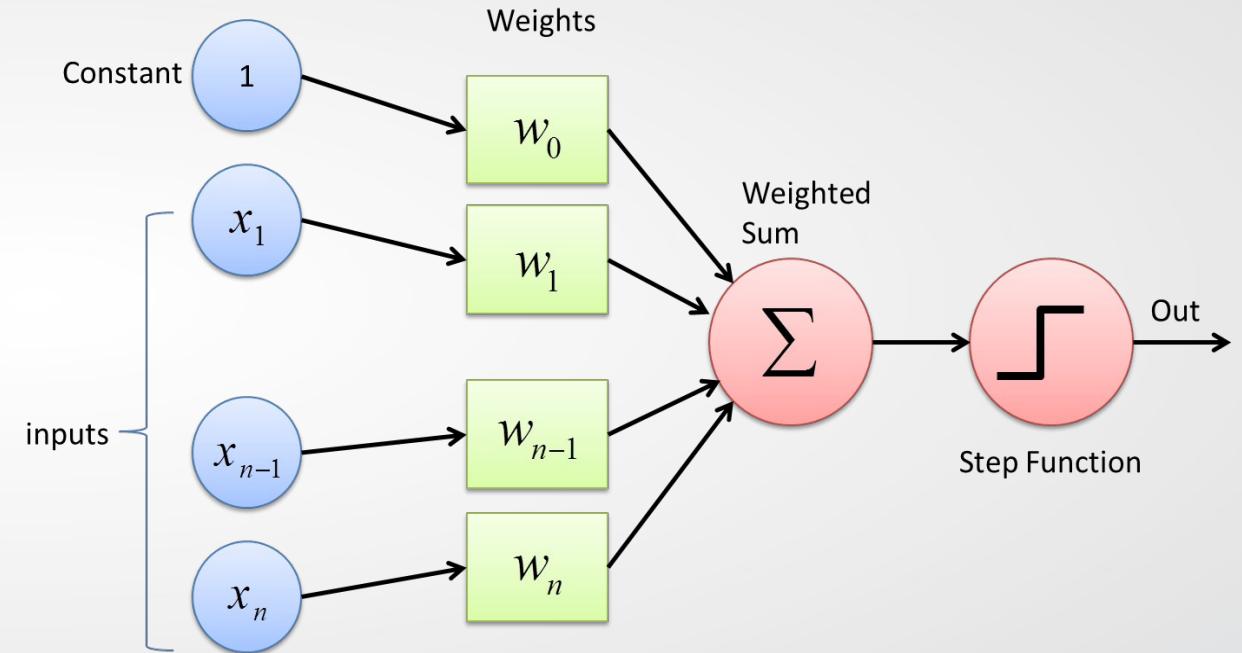
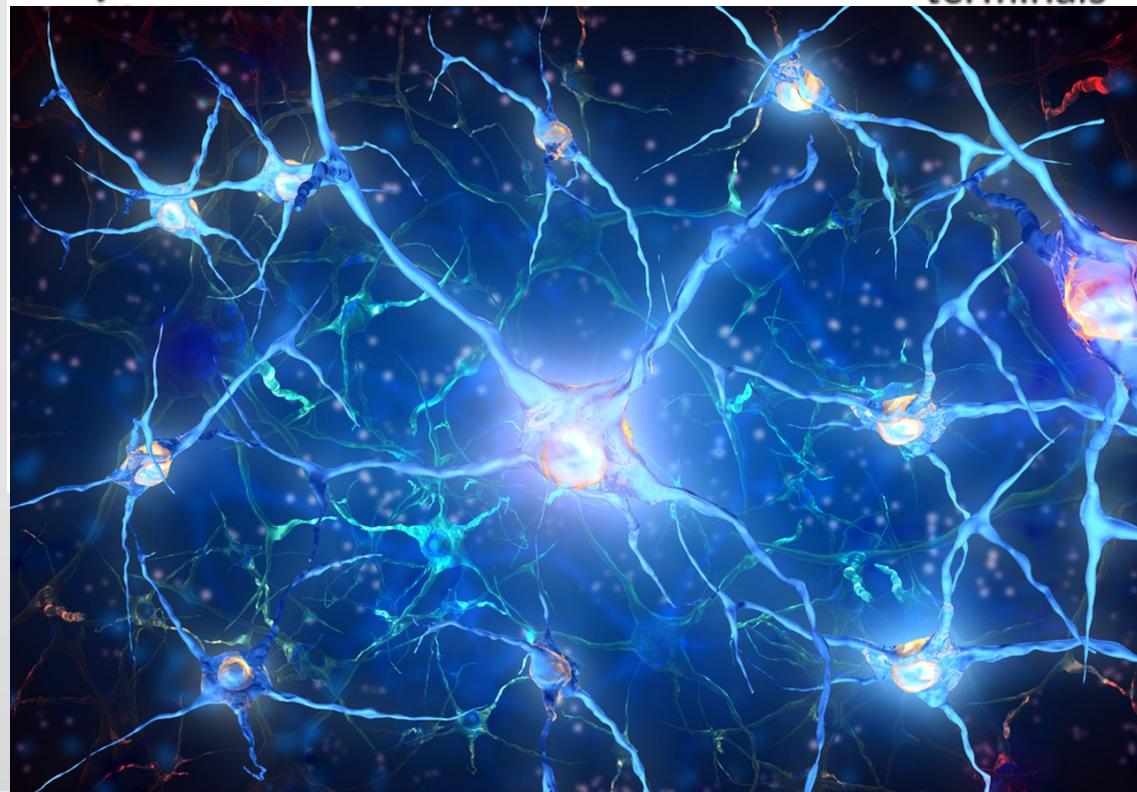
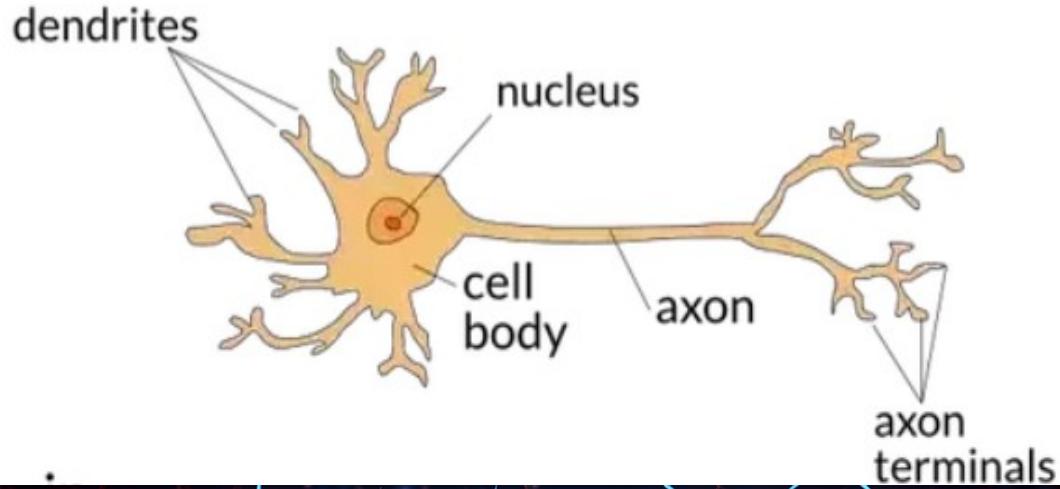


Deep learning

- Deep learning is subfield of machine learning based on artificial neural networks.
- Deep learning is inspired from the human brain



Deep learning: origins

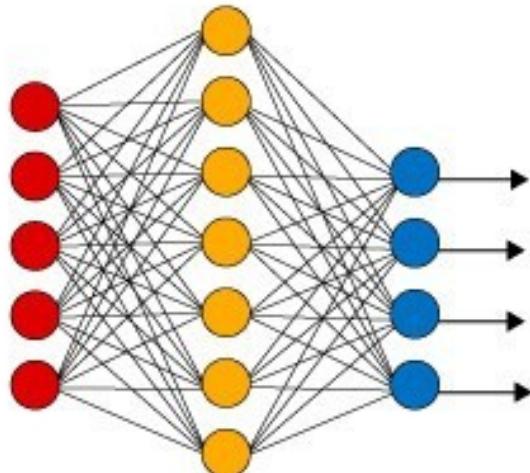


Deep learning: Imitation of the humain brain

Input

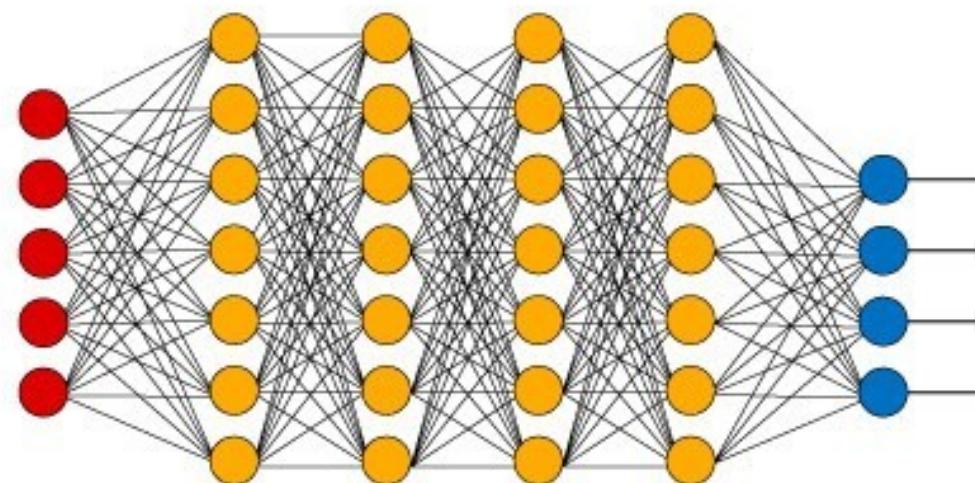


Simple Neural Network



● Input Layer

Deep Learning Neural Network

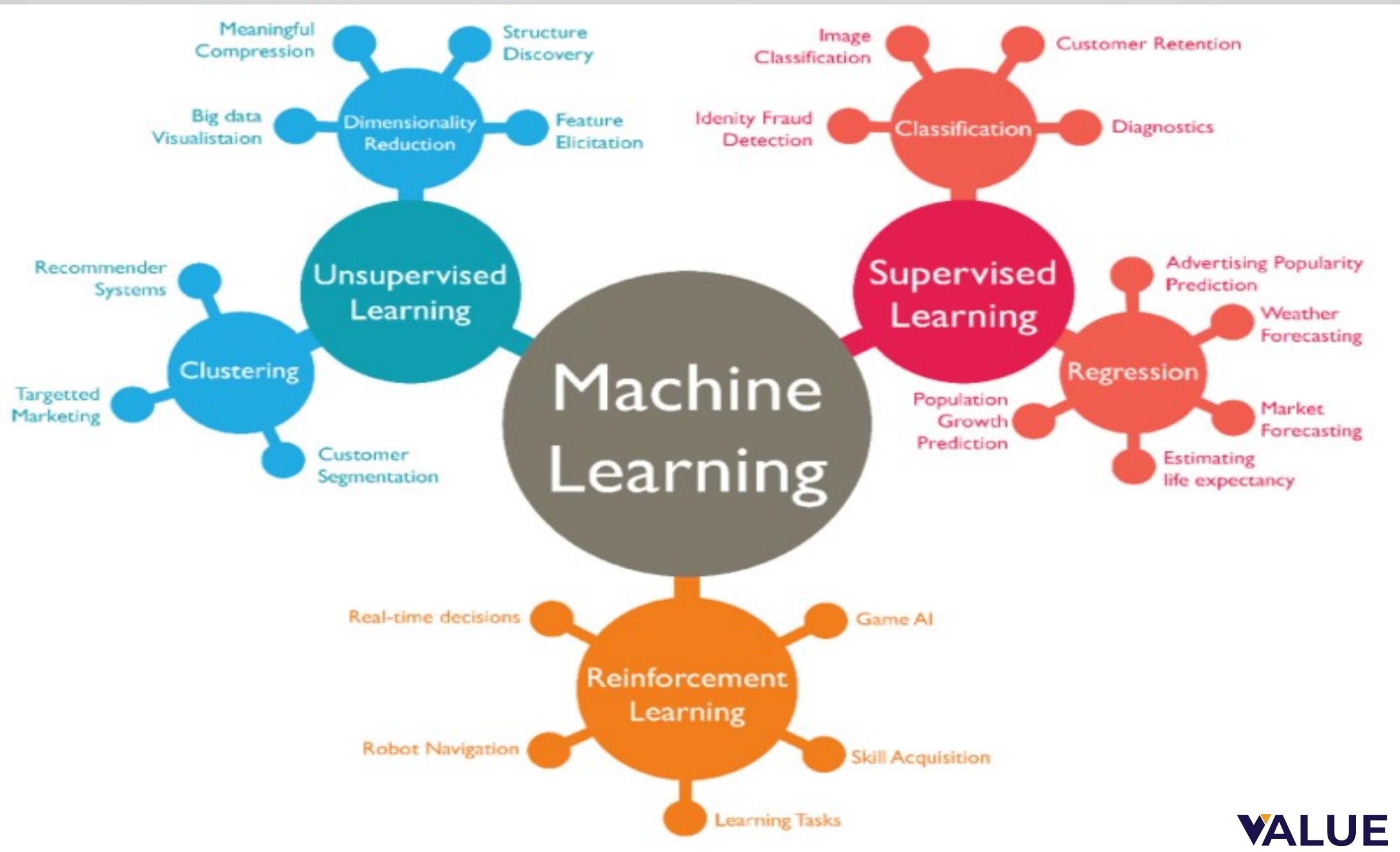


● Hidden Layer

● Output Layer

Output





Quantum machine learning

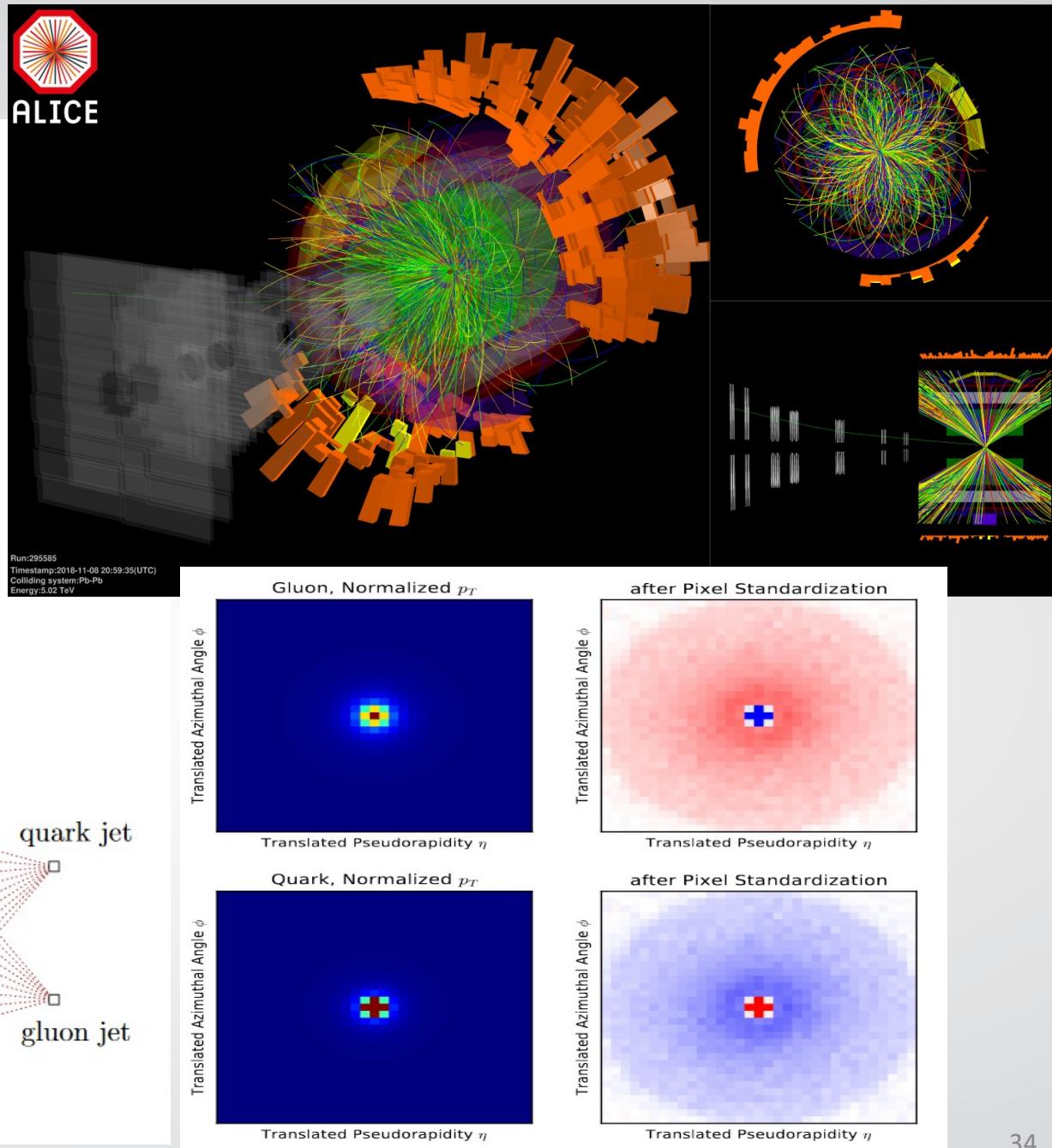
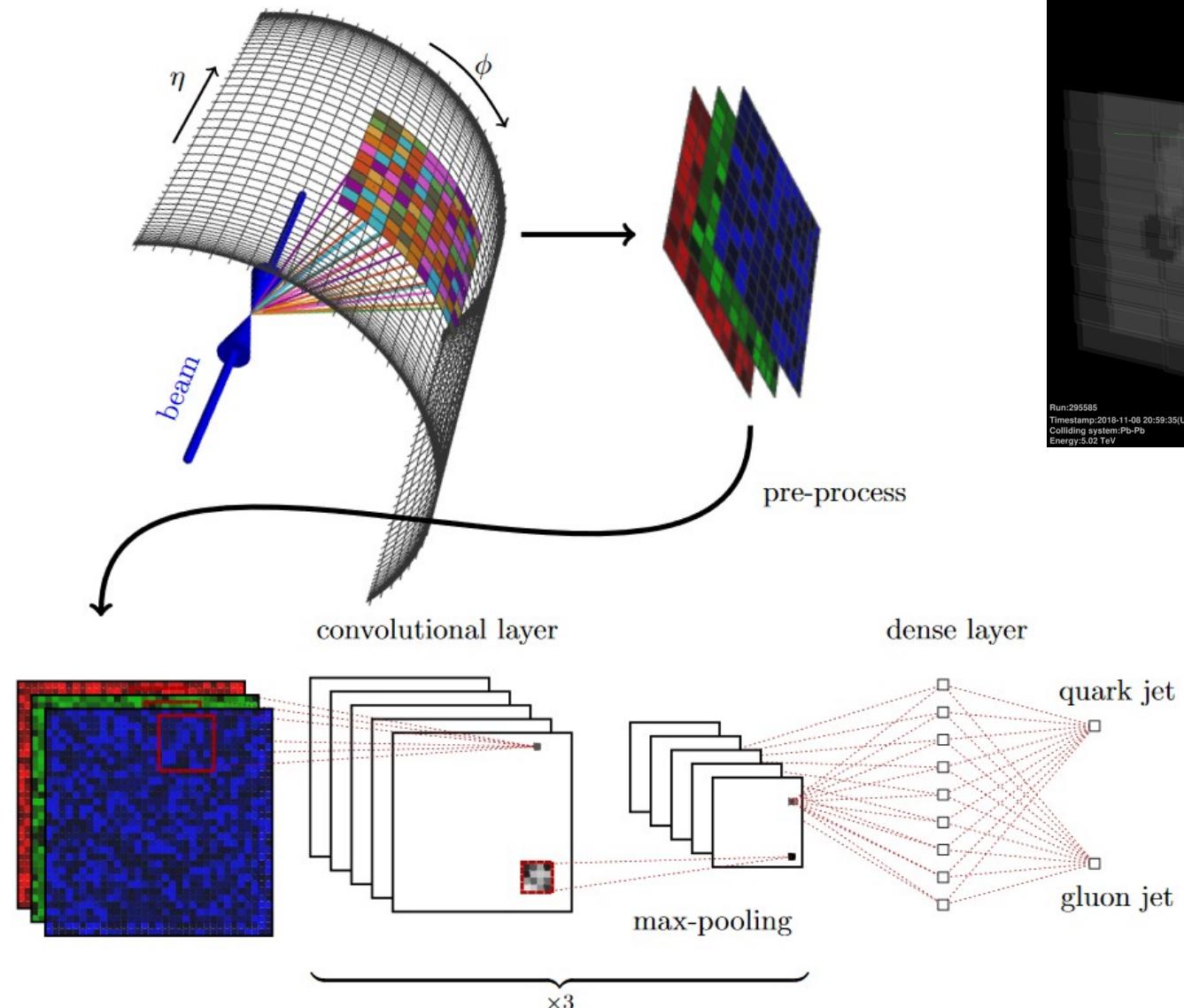
Classical data
+
Classical computing

Quantum data
+
Classical computing

Classical data
+
Quantum computing

Quantum data
+
Quantum computing

Quantum data/Classical computing: reconstruction the data of a quantum experiment with CNN network



Classical data/Quantum computing: three families of algorithms

Gate circuits:

Search - Grover
QFT - Shor
Deutsch

Annealing:

Direct Annealing
Adiabatic Evolution

Variational:

Eigensolvers
Classifiers
Autoencoders

Classical data/Quantum computing: Eigensolvers

En mécanique quantique, une mesure réalisée sur un système quantique permet de déterminer à chaque fois les valeurs propres et les vecteurs propres de l'observable qui est représentée par une matrice.

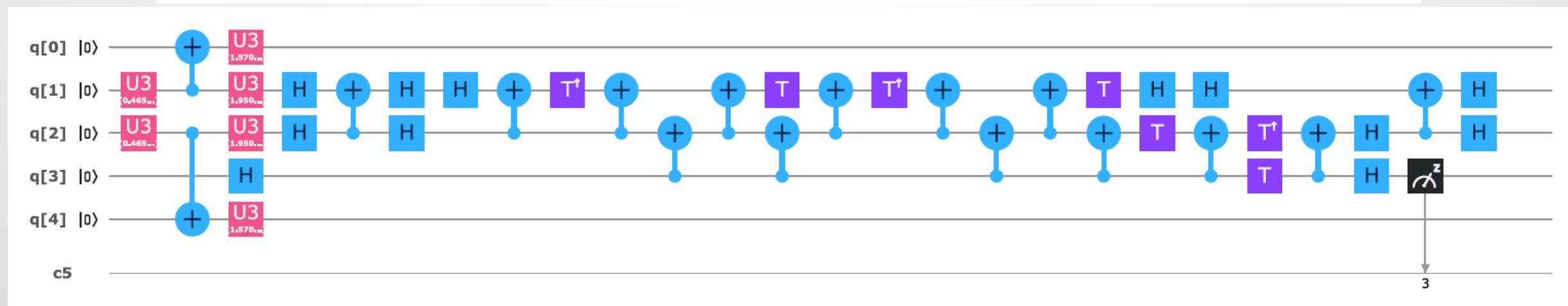
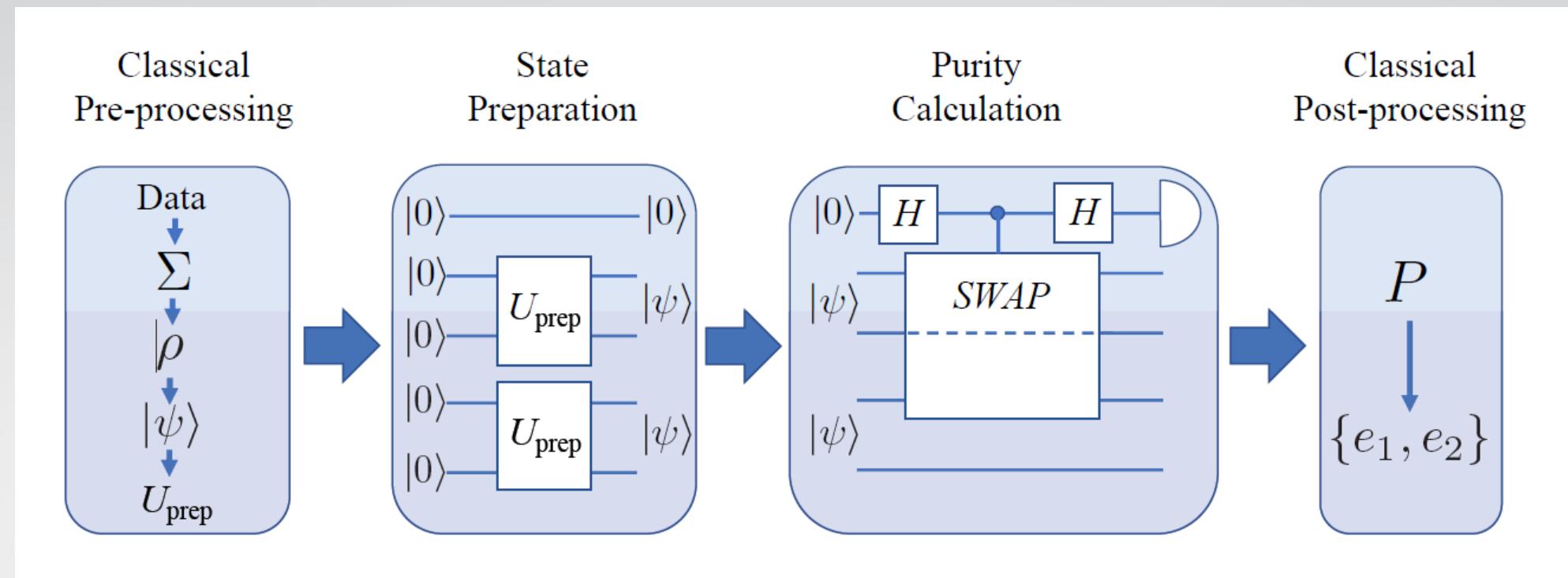
The image contains two side-by-side quantum circuit diagrams. The left circuit shows a unitary operator \vec{A} (represented by a grid of colored pixels) acting on a state $|\vec{x}\rangle$ (represented by a vector with a blue arrow) to produce a state $|\vec{b}\rangle$ (represented by a vector with a red arrow). The right circuit shows the inverse operator \vec{A}^{-1} (also a grid of colored pixels) acting on state $|\vec{b}\rangle$ to produce state $|\vec{x}\rangle$.

- 1 - On prépare un système quantique (circuit) vérifiant la matrice A.
- 2 – On effectue une série de mesures permettant d'obtenir les valeurs propres et les vecteurs propres.

==> Amélioration considérable de la complexité algorithmique $\log_2(n)$

D'après l'algorithme quantique HHL A. W. Harrow, A. Hassidim, and S. Lloyd, Phys. Rev. Lett. 103, 150502 (2009), e-print arXiv 0811.3171

Classical data/Quantum computing: PCA



Algorithm 13 Quantum SVM [84]

Input:

- Training data set $\{(\vec{x}_j, y_j) : j = 1, 2, \dots, M\}$.
- A query data \vec{x} .

Output:

- Classification of \vec{x} : +1 or -1.

Procedure:

Step 1. Calculate kernel matrix $K_{ij} = \vec{x}_i \cdot \vec{x}_j$ using quantum inner product algorithm [69].

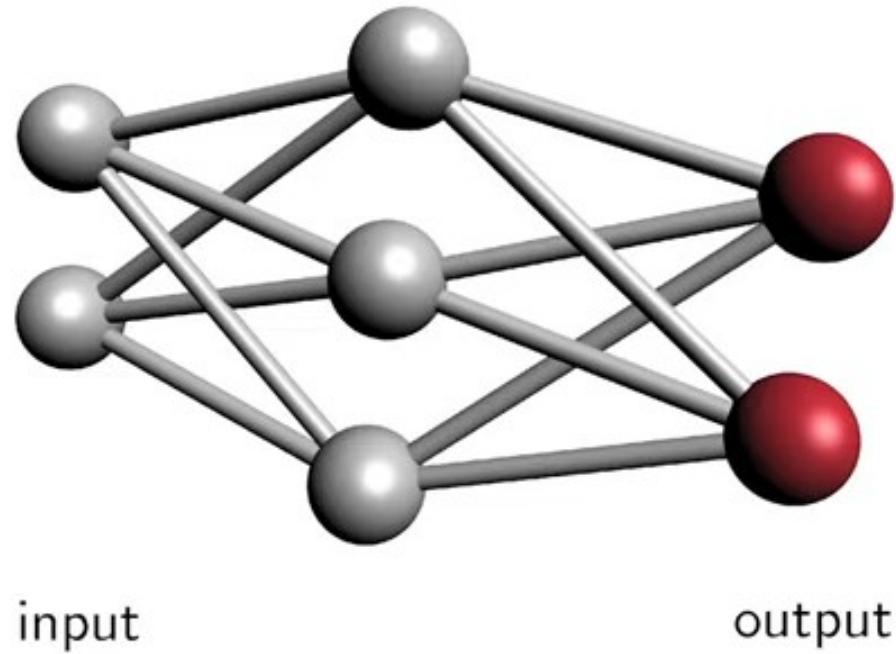
Step 2. Solve linear equation Eq. (87) and find $|b, \vec{\alpha}\rangle$ using a quantum algorithm for solving linear equations [50] (training step).

Step 3. Perform classification of the query data \vec{x} against the training results $|b, \vec{\alpha}\rangle$ using a quantum algorithm [84].

Seth Lloyd, Masoud Mohseni, and Patrick Rebentrost. Quantum algorithms for supervised and unsupervised machine learning. arXiv preprint arXiv:1307.0411, 2013. [69]

Patrick Rebentrost, Masoud Mohseni, and Seth Lloyd. Quantum support vector machine for big data classification. Physical review letters, 113(13):130503, 2014. [84]

Quantum data/Quantum computing: Quantum Neural Network



$$\text{tr}_1 \left(U_2^{\text{out}} U_1^{\text{out}} \left(\text{tr}_{\text{in}} \left(U_3^1 U_2^1 U_1^1 \left(\rho^{\text{in}} \otimes |000\rangle_1 \langle 000| \right) U_1^{1\dagger} U_2^{1\dagger} U_3^{1\dagger} \right) \otimes |00\rangle_{\text{out}} \langle 00| \right) U_1^{\text{out}\dagger} U_2^{\text{out}\dagger} \right)$$

arXiv:1902.10445v1 [quant-ph] 27 Feb 2019 Efficient Learning for Deep Quantum Neural Networks

Training deep quantum neural networks, Nature Communications volume 11, Article number: 808 (2020)

Live coding session

- Classification with Classical SVM and Quantum SVM**
- Portfolio Diversification : an application in finance**