The Heat Equation Example:

By using crank- Nicolson method solve:

$$\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0$$

Given $\mathbf{u(x,0) = 0}$ **,** $\mathbf{u(0,t) = 0}$**,** $\mathbf{u(1,t) = 200t}$ **, h=0.25,** $\boldsymbol{x} \in [\mathbf{0}, \mathbf{1}]$**,** $\boldsymbol{t} = [\mathbf{0}, \mathbf{1}]$

$$\frac{\partial^2 u}{\partial x^2} - a\frac{\partial u}{\partial t} = 0$$ Where a=16

Based on the finite difference method

1. **Divide the interval into subintervals of width h.**
2. **Divide the interval into subintervals of width k.**
3. **Replace the first and second partial derivatives with them**

Solution

First: $k = ah^2 = 16(0.25) = 1$

first equation is:

$$\boldsymbol{U_1 = 1/4(0 + 0 + 0 + U_2)}$$

$$4U_1 = U_2$$

$$4U_1 - U_2 = 0$$

Second equation is:

$$\boldsymbol{U_2 = 1/4(0 + 0 + U_1 + U_3)}$$

$$4U_2 = U_1 + U_3$$

$$-U_1 = +4U_2 - U_3 = 0$$

Third equation is:

$$\boldsymbol{U_3 = 1/4(0 + 0 + U_2 + 200)}$$

$$4U_3 = U_2 + 200$$

$$-U_2 = +4U_3 = 200$$

So, the three equations are:

- $4U_1 - U_2 = 0$
- $-U_1 = +4U_2 - U_3 = 0$
- $-U_2 = +4U_3 = 200$

The solution of the PDE when t =0.25 sec is the solution of the following tridiagonal system of equations

$$\begin{matrix} 4 & -1 & 1 \\ -1 & 4 & -3 \\ -1 & 4 & 2 \end{matrix} \begin{bmatrix} U_{1,1} \\ U_{2,1} \\ U_{3,1} \end{bmatrix} = \begin{matrix} 3.5714 \\ 14.2857 \\ 53.5714 \end{matrix}$$

The solution of the PDE when t =0.5 sec is the solution of the following tridiagonal system of equations

$$\begin{matrix} 4 & -1 & 1 \\ -1 & 4 & -3 \\ -1 & 4 & 2 \end{matrix} \begin{bmatrix} U_{1,1} \\ U_{2,1} \\ U_{3,1} \end{bmatrix} = \begin{matrix} 3.1243 \\ 12.2753 \\ 42.127 \end{matrix}$$

The solution of the PDE when t =0.75 sec is the solution of the following tridiagonal system of equations

$$\begin{bmatrix} 4 & -1 & 1 \\ -1 & 4 & -3 \\ -1 & 4 & 2 \end{bmatrix}\begin{bmatrix} U_{1,1} \\ U_{2,1} \\ U_{3,1} \end{bmatrix} = \begin{bmatrix} 2.2125 \\ 10.2436 \\ 50.5312 \end{bmatrix}$$

The solution of the PDE when t =1 sec is the solution of the following tridiagonal system of equations

$$\begin{bmatrix} 4 & -1 & 1 \\ -1 & 4 & -3 \\ -1 & 4 & 2 \end{bmatrix}\begin{bmatrix} U_{1,1} \\ U_{2,1} \\ U_{3,1} \end{bmatrix} = \begin{bmatrix} 1.3148 \\ 9.1832 \\ 47.2315 \end{bmatrix}$$