TSKS14

Multiple Antenna Communications

Lecture 5, 2020

Emil Björnson

LINKÖPING UNIVERSITY

Outline of this lecture

- Practical issues with point-to-point MIMO
- Introduction to multi-user MIMO
  - Uplink and downlink
  - Orthogonal access versus spatial multiplexing
- Capacity region
  - Operating points
  - Uplink capacity region

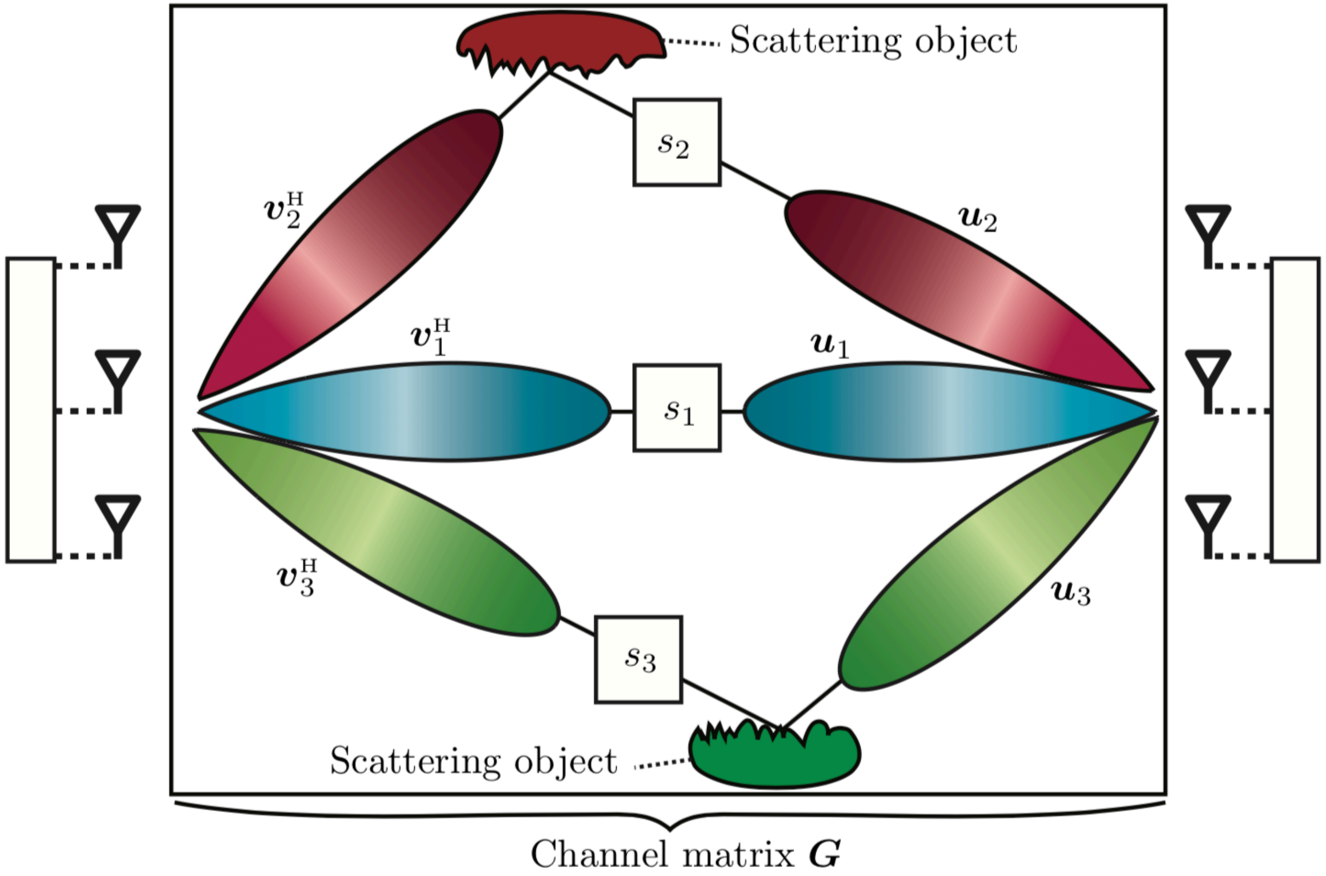
Recall: Point-to-Point MIMO Capacity

- Compute SVD of channel matrix:

$$\boldsymbol{G} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H = \sum_{k=1}^{S} s_k \boldsymbol{u}_k \boldsymbol{v}_k^H$$

  - $\boldsymbol{\Sigma}$ "diagonal" with $s_1, \dots s_S$
  - $\boldsymbol{U} = [\boldsymbol{u}_1 \ \dots \ \boldsymbol{u}_M]$
  - $\boldsymbol{V} = [\boldsymbol{v}_1 \ \dots \ \boldsymbol{v}_K]$

Decompose the channel into S parallel channels



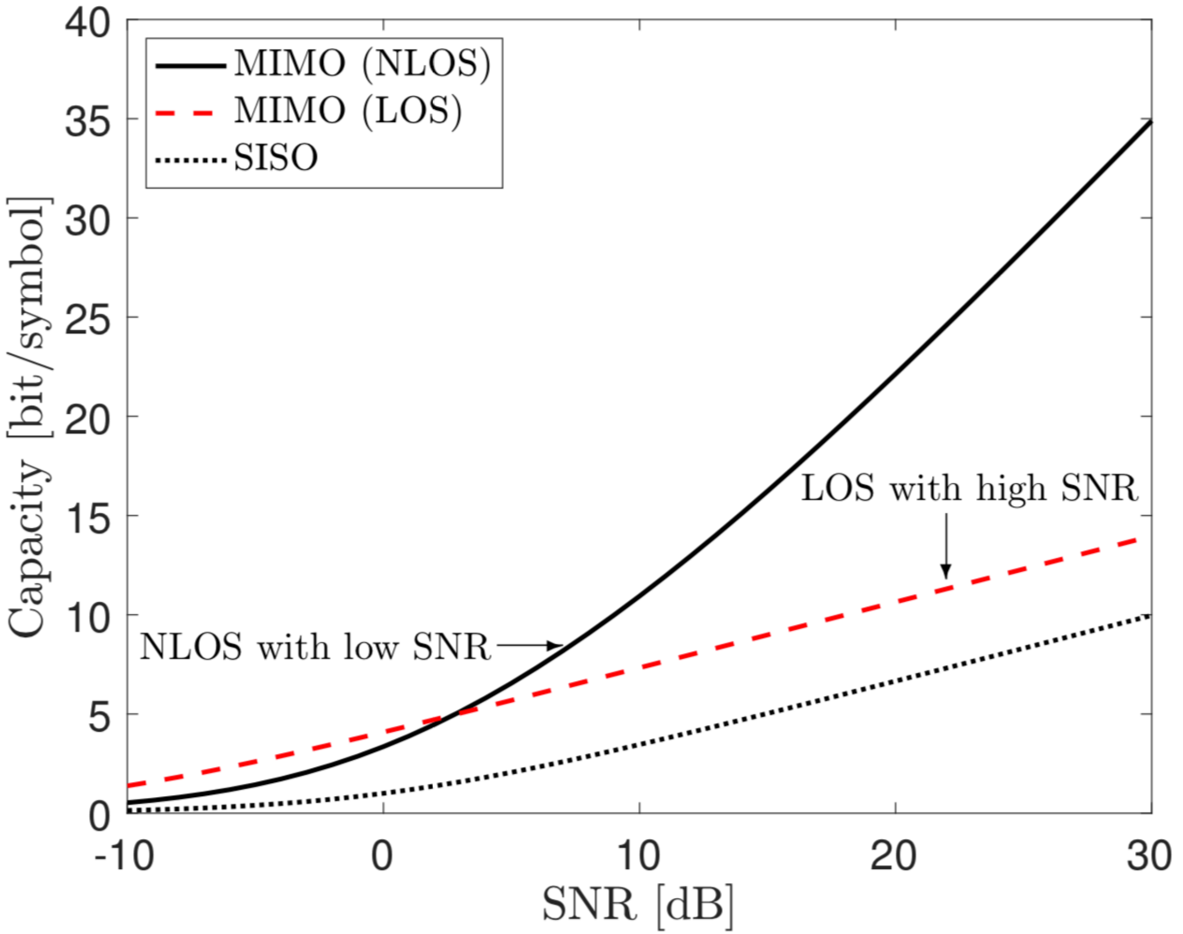
Problems with point-to-point MIMO

- Multiplexing gain: $S = \text{rank}(\boldsymbol{G})$
  - Line-of-sight: $S \approx 1$
  - Non-line-of-sight: $S = \min(M, K)$

Mainly beamforming gain:

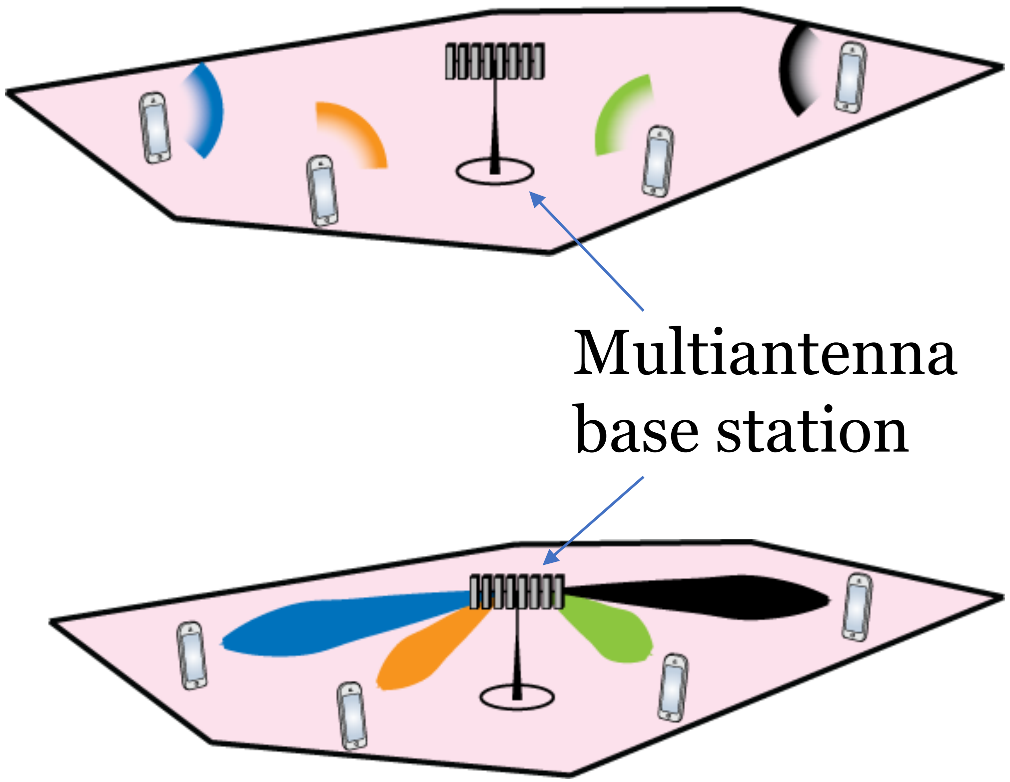
- High SNR:
  Likely to be in line-of-sight (LOS)
- Low SNR:
  Likely to be non-line-of-sight (NLOS)

Not scalable:
User devices are small, cannot fit many antennas



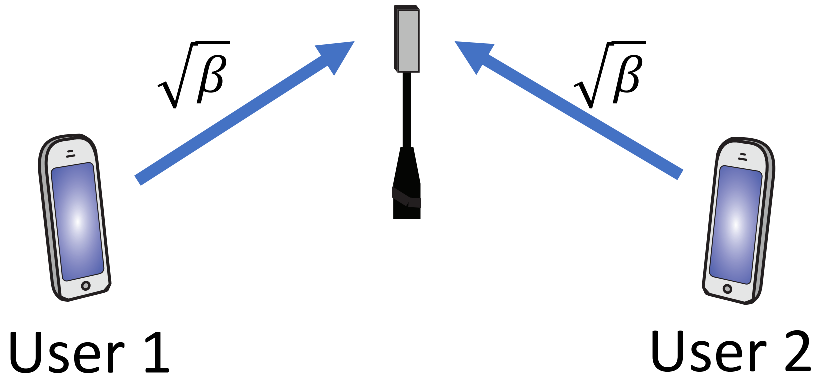
Multiuser MIMO Communication

- Uplink
  - From users to base station
  - Multipoint-to-point MIMO
- Downlink
  - From base station to users
  - Point-to-multipoint MIMO



Orthogonal multiple access

- Two users want to communicate with base station
  - Power per user: P
  - Bandwidth: B
  - Noise power spectral density: N_0



- Divide bandwidth: αB to user 1, $(1-\alpha)B$ to user 2

$$R_1 = \alpha B \log_2\left(1 + \frac{P\beta}{\alpha B N_0}\right)$$

$$R_2 = (1-\alpha) B \log_2\left(1 + \frac{P\beta}{(1-\alpha) B N_0}\right)$$

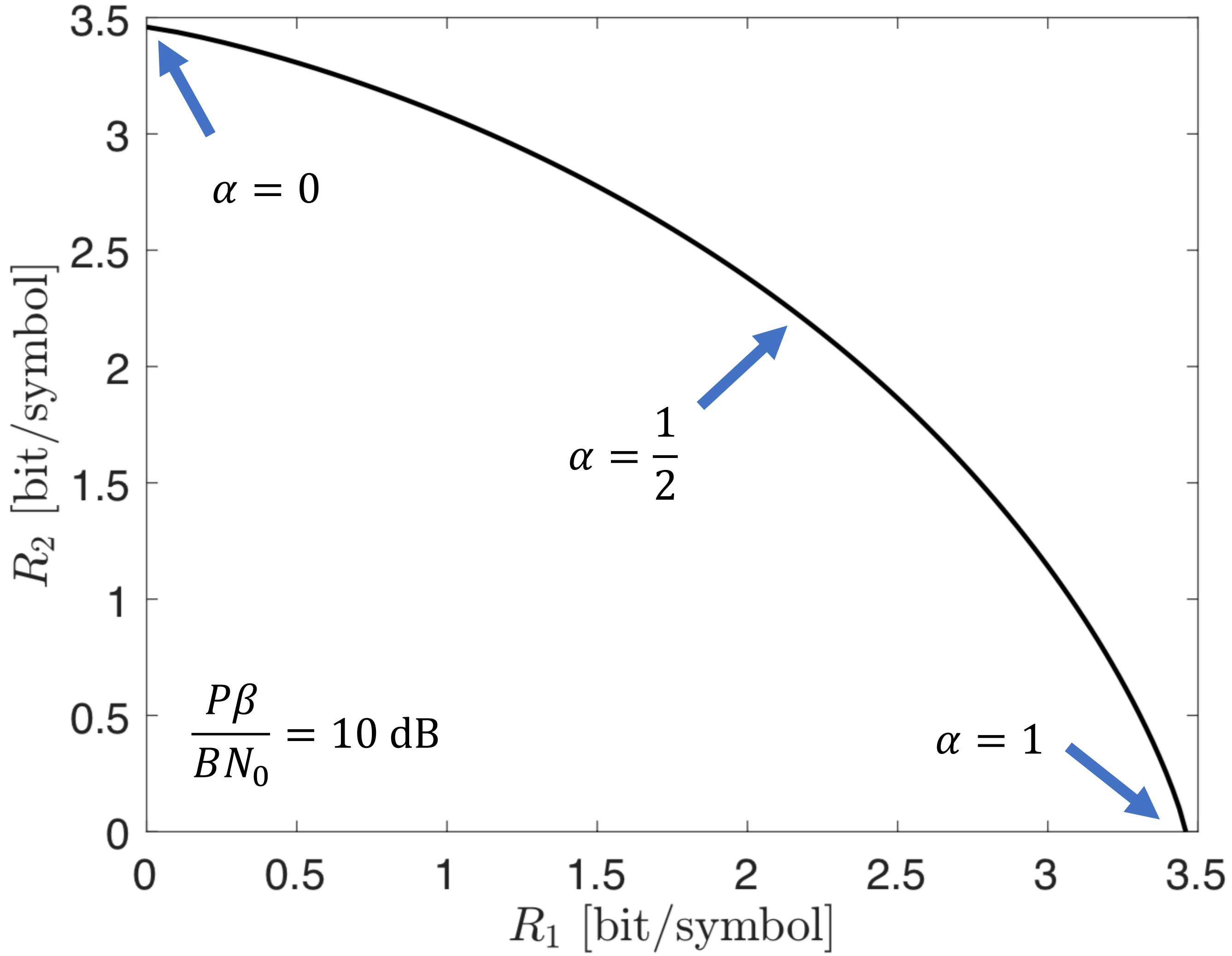
Orthogonal multiple access: Rate region

- Rates depend on α:

$$R_1 = \alpha B \log_2\left(1 + \frac{P\beta}{\alpha B N_0}\right)$$

$$R_2 = (1-\alpha) B \log_2\left(1 + \frac{P\beta}{(1-\alpha) B N_0}\right)$$

What is the preferred operating point?



Non-orthogonal multiple access

- Let both users transmit simultaneously:

$$y = x_1 + x_2 + n$$

Received signal

From user 1 $x_1 \sim CN(0, P)$

From user 2 $x_2 \sim CN(0, P)$

Noise (power BN_0)

- Strategy:
  1. Decode signal from user 1, treat interference as noise
$$R_1 = \log_2\left(1 + \frac{P}{P + BN_0}\right)$$
  2. Subtract x_1: $y - x_1 = x_2 + n$. Decode signal from user 2:
$$R_2 = \log_2\left(1 + \frac{P}{BN_0}\right)$$

Called: Successive interference cancelation

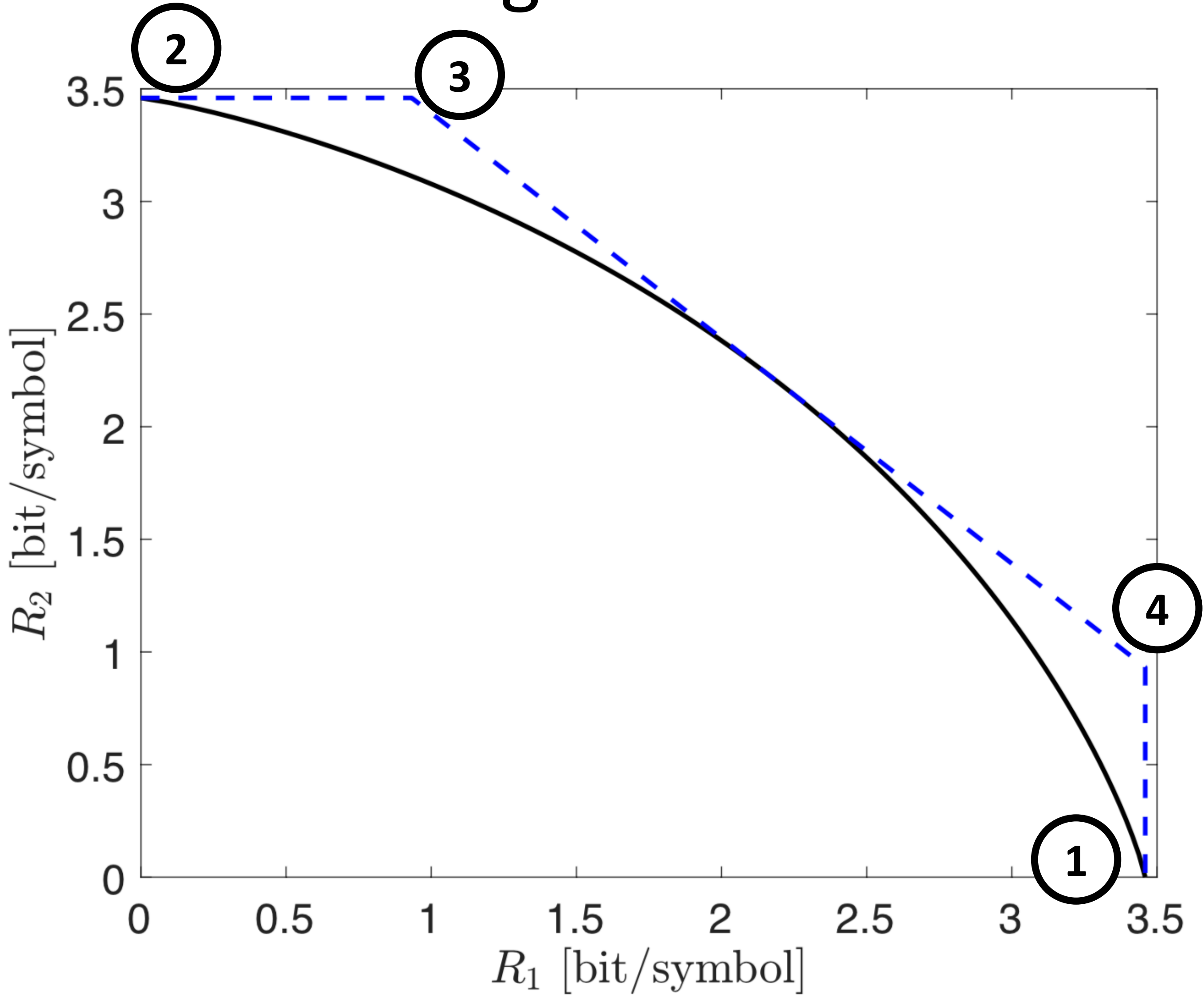
We can change the user order

Non-orthogonal multiple access: Rate region

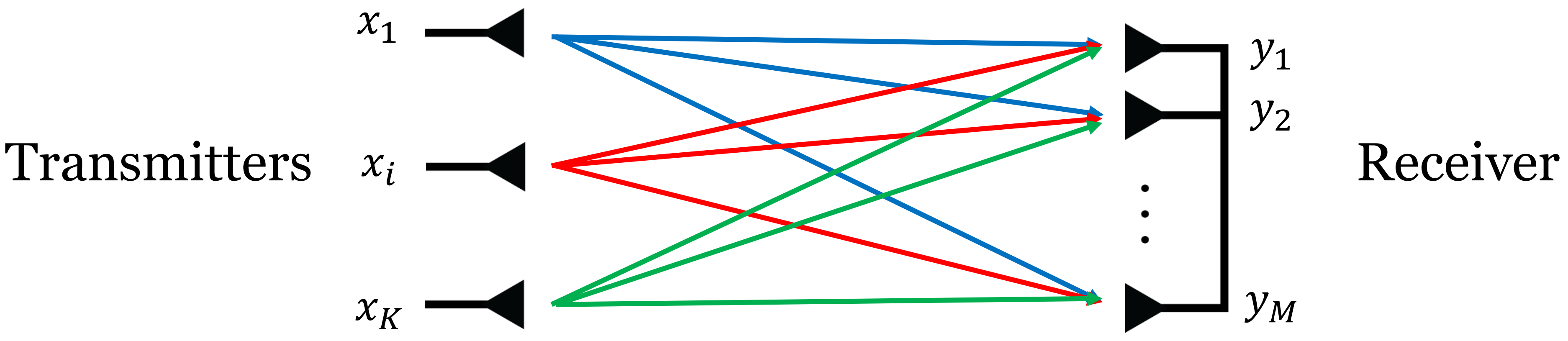
Four operating points (R_1, R_2):

1. $\left(\log_2\left(1+\frac{P}{BN_0}\right), 0\right)$
2. $\left(0, \log_2\left(1+\frac{P}{BN_0}\right)\right)$
3. $\left(\log_2\left(1+\frac{P}{P+BN_0}\right), \log_2\left(1+\frac{P}{BN_0}\right)\right)$
4. $\left(\log_2\left(1+\frac{P}{BN_0}\right), \log_2\left(1+\frac{P}{P+BN_0}\right)\right)$

Time sharing:
We can achieve all points in between



Uplink in Multiuser MIMO



- Notation:
  - K single-antenna users, M base station antennas
  - Channel response g_i^j from user i to antenna j
  - Data signals $x_1, \dots, x_K$, received signals $y_1, \dots, y_M$

Uplink Multiuser MIMO: System model

- Received signal:

$$\boldsymbol{y} = \sqrt{\rho_{ul}}\boldsymbol{G}\boldsymbol{x} + \boldsymbol{w}$$

where

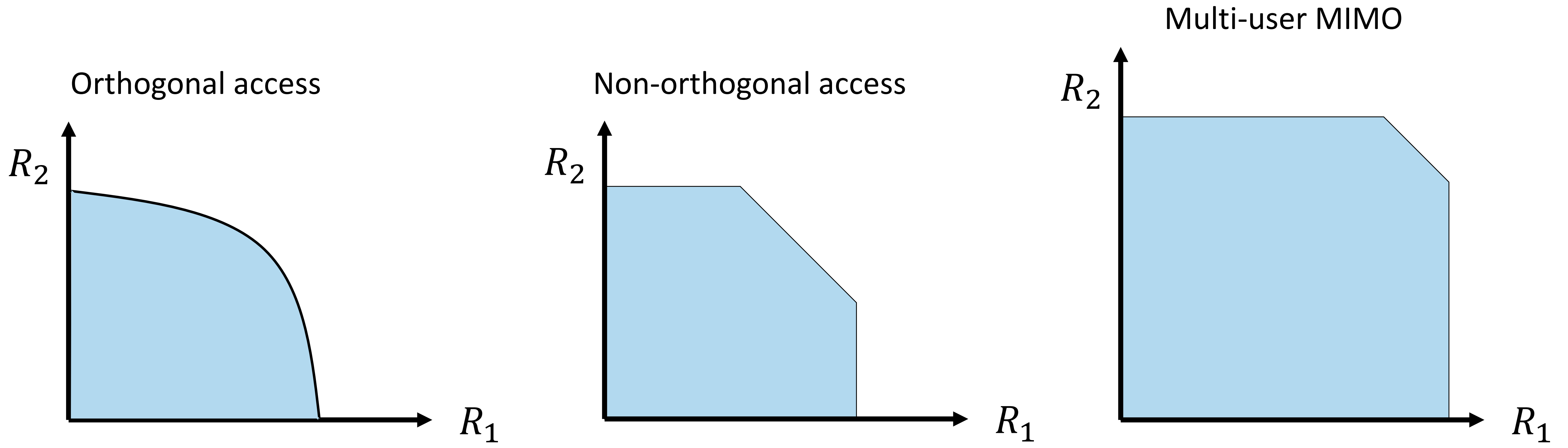
$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_M \end{pmatrix} \quad \boldsymbol{G} = \begin{pmatrix} g_1^1 & \cdots & g_K^1 \\ \vdots & \ddots & \vdots \\ g_1^M & \cdots & g_K^M \end{pmatrix} \quad \boldsymbol{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_K \end{pmatrix} \quad \boldsymbol{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_M \end{pmatrix}$$

- Parameters are normalized: SNR is ρ_{ul}
- Each users signal is power-limited as $\mathbb{E}\{|x_k|^2\} \leq 1$
- Normalized noise: $\boldsymbol{w} \sim CN(\boldsymbol{0}, \boldsymbol{I}_M)$

What is the difference from point-to-point MIMO?

- **Difference 1**: Users do not cooperate
  - $x_1, \ldots, x_K$ are independent signals
- **Difference 2**: Each user cares about its own performance
  - K user capacities instead of one capacity
- **Difference 3**: Each user has its own power budget
  - Power constraint: $E\{|x_k|^2\} \leq 1$
- **Difference 4**: The channel matrix $\boldsymbol{G}$ is modeled differently
  - Each column can be modeled as a SIMO channel

Motivating example

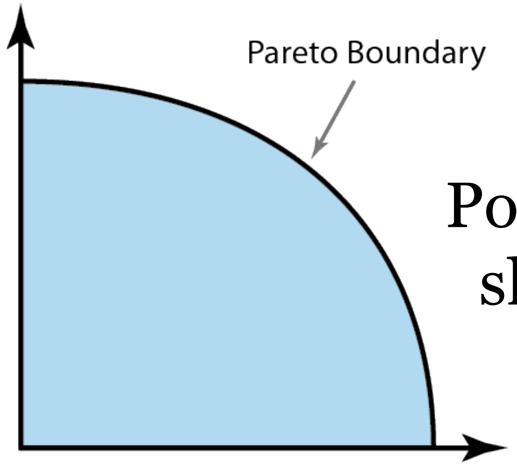


Capacity or rate region?
Capacity region is the largest possible rate region

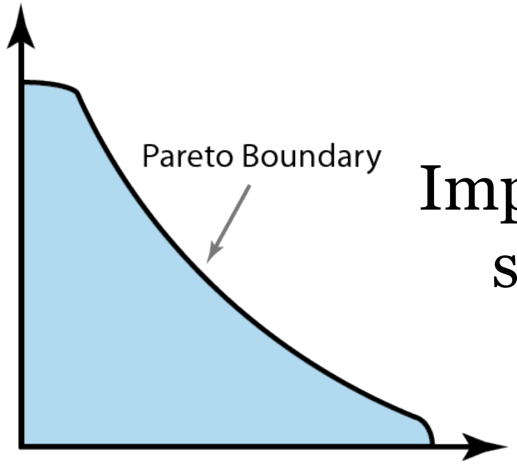
Two benefits:
Beamforming gain for every user
Smaller interference loss

Shape of capacity region

- One can pick two points and use them fractions of the time
  - Similar to time-sharing
  - Hence: Line between any two points are in the region
  - Conclusion: Region must be a *convex set*



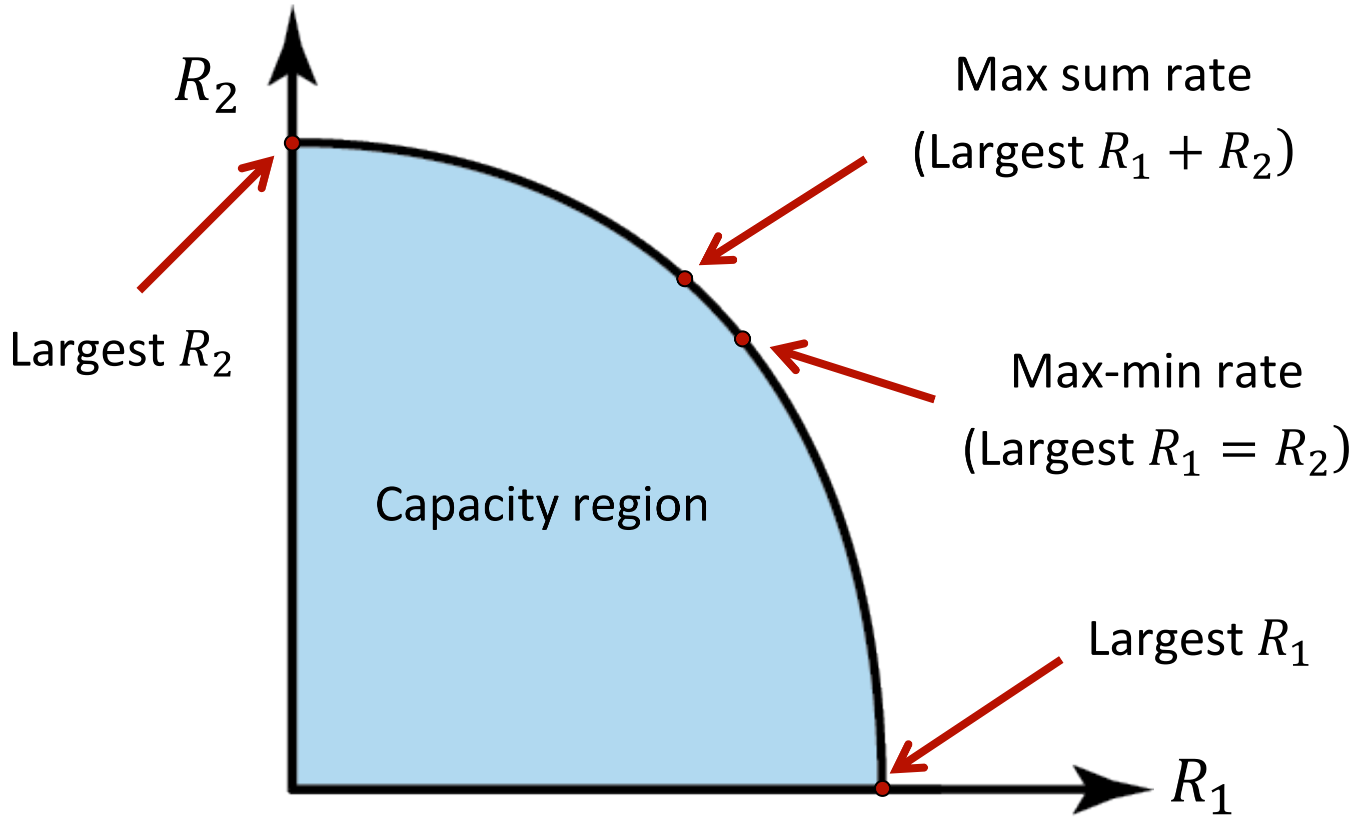
Possible shape



Impossible shape

Points in the capacity region

- Combinations (R_1, R_2) of rates that can be simultaneously achieved



Many choices!

Only rule:
Always pick something on the outer boundary (Pareto boundary)!

Fairness scale:

Max sum rate

Max-min fairness

Sum Capacity of Uplink Multiuser MIMO

- Recall: Received signal:

$$\boldsymbol{y} = \sqrt{\rho_{ul}}\boldsymbol{G}\boldsymbol{x} + \boldsymbol{w}$$

- Assume a deterministic $\boldsymbol{G}$
    - Let all users transmit with full power: $\boldsymbol{x} \sim CN(\boldsymbol{0}, \boldsymbol{I}_K)$

Like a point-to-point MIMO channel
But with a “suboptimal” signal covariance matrix $\boldsymbol{Q} = \boldsymbol{I}_M$

- Sum rate: $R_1 + \cdots + R_K = \log_2(\det(\boldsymbol{I}_M + \rho_{ul}\boldsymbol{G}\boldsymbol{G}^H))$

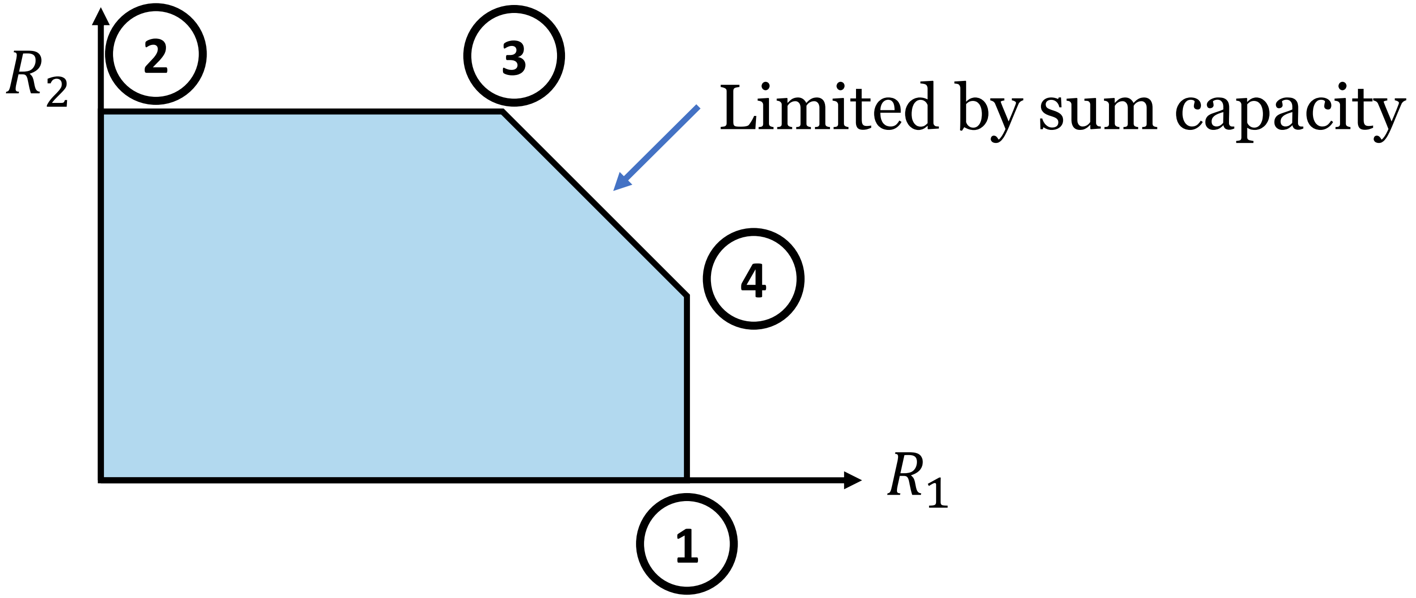
This is the sum capacity!
Achieved by successive interference cancellation
Decoding order determines who gets which share

Uplink Capacity Region with $K = 2$

- Region contains all (R_1, R_2) satisfying:

$$R_1 \leq \log_2(1 + \rho_{ul}\|\boldsymbol{g}_1\|^2)$$
$$R_2 \leq \log_2(1 + \rho_{ul}\|\boldsymbol{g}_2\|^2)$$
$$R_1 + R_2 \leq \log_2(\det(\boldsymbol{I}_M + \rho_{ul}\boldsymbol{G}\boldsymbol{G}^H))$$

$$\boldsymbol{G} = [\boldsymbol{g}_1 \ \boldsymbol{g}_2]$$



Summary

- Point-to-point MIMO channels
  - Large multiplexing gains are hard to achieve in practice

- Multi-user MIMO channels
  - Similar system model
  - Key differences: Independent users, different power, different performance
  - Capacity and rate regions
  - Orthogonal and non-orthogonal access

End of Lecture 5

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