TSKS14

Multiple Antenna Communications

Lecture 1, 2020

Emil Björnson

LINKÖPING UNIVERSITY

Three main benefits of multiple antenna communications

1. Beamforming gain
2. Spatial diversity
3. Spatial multiplexing

The course will cover all of them:

- First historical perspective
- Then the theory behind them

Constructive and destructive interference

- One signal:

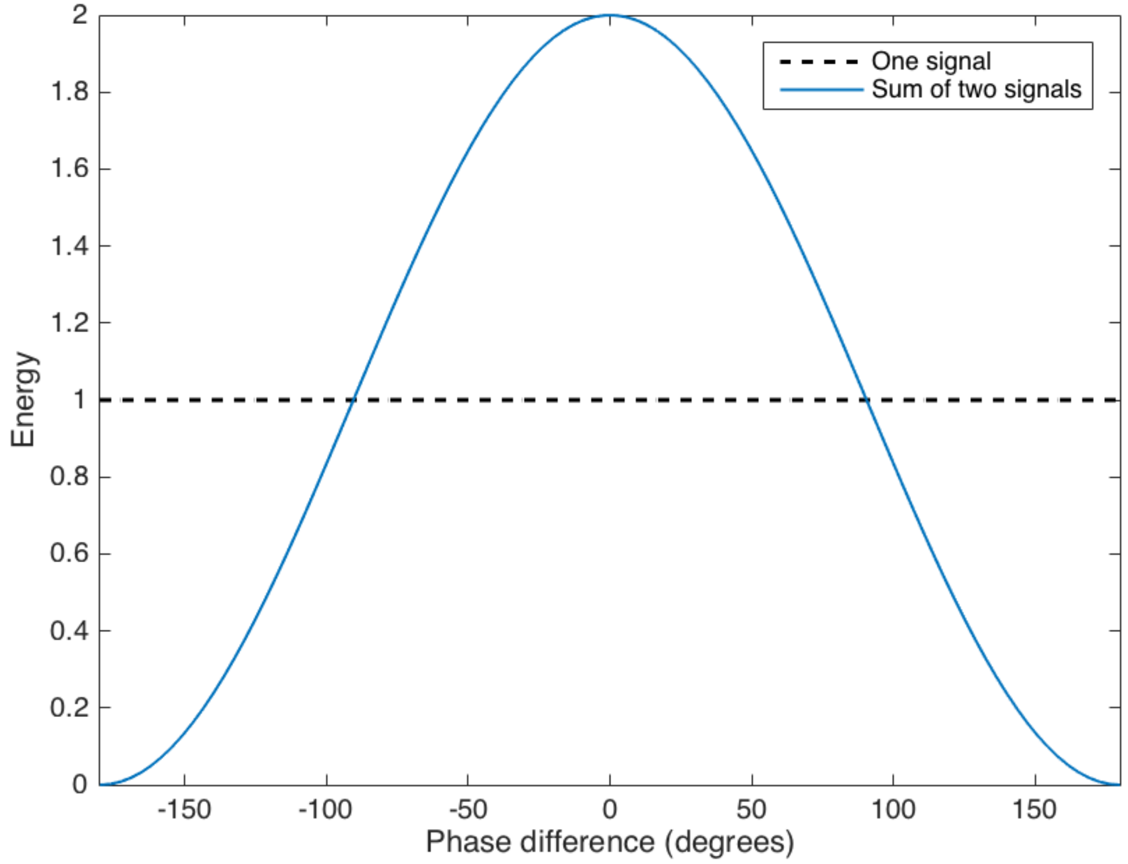
$$\sqrt{2}\sin(2\pi t)$$

(Normalization: $\int_0^1 \left(\sqrt{2}\sin(2\pi t)\right)^2 dt = 1$)

- Two signals:

$$\sin(2\pi t) + \sin(2\pi t - \theta)$$

Power depends on the phase difference θ (time delay)

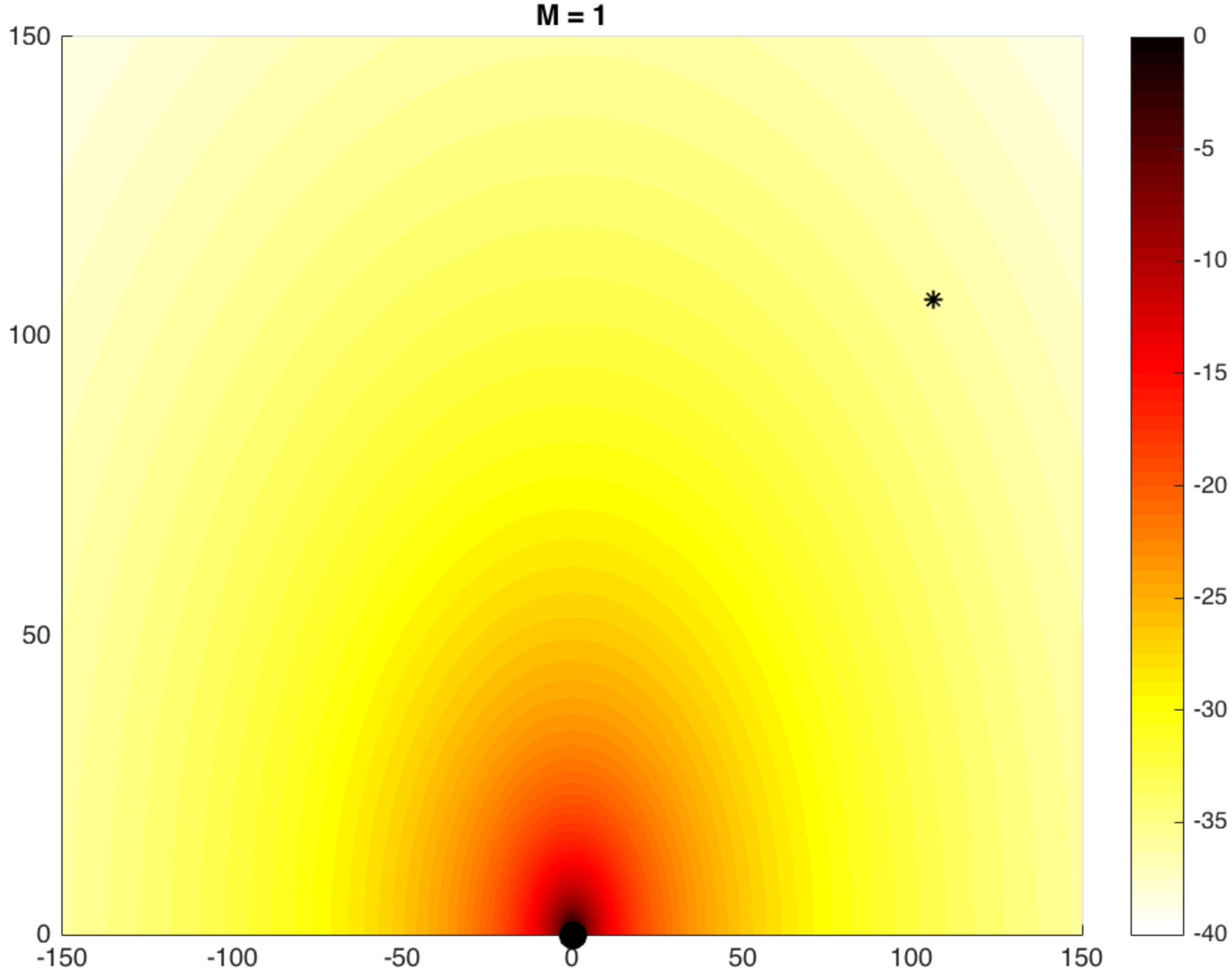


Omni-directional radiation

$M = 1$ antenna

Powers spread out as a sphere

- Received power decays as 1/distance^2



Directive radiation – Beamforming gain

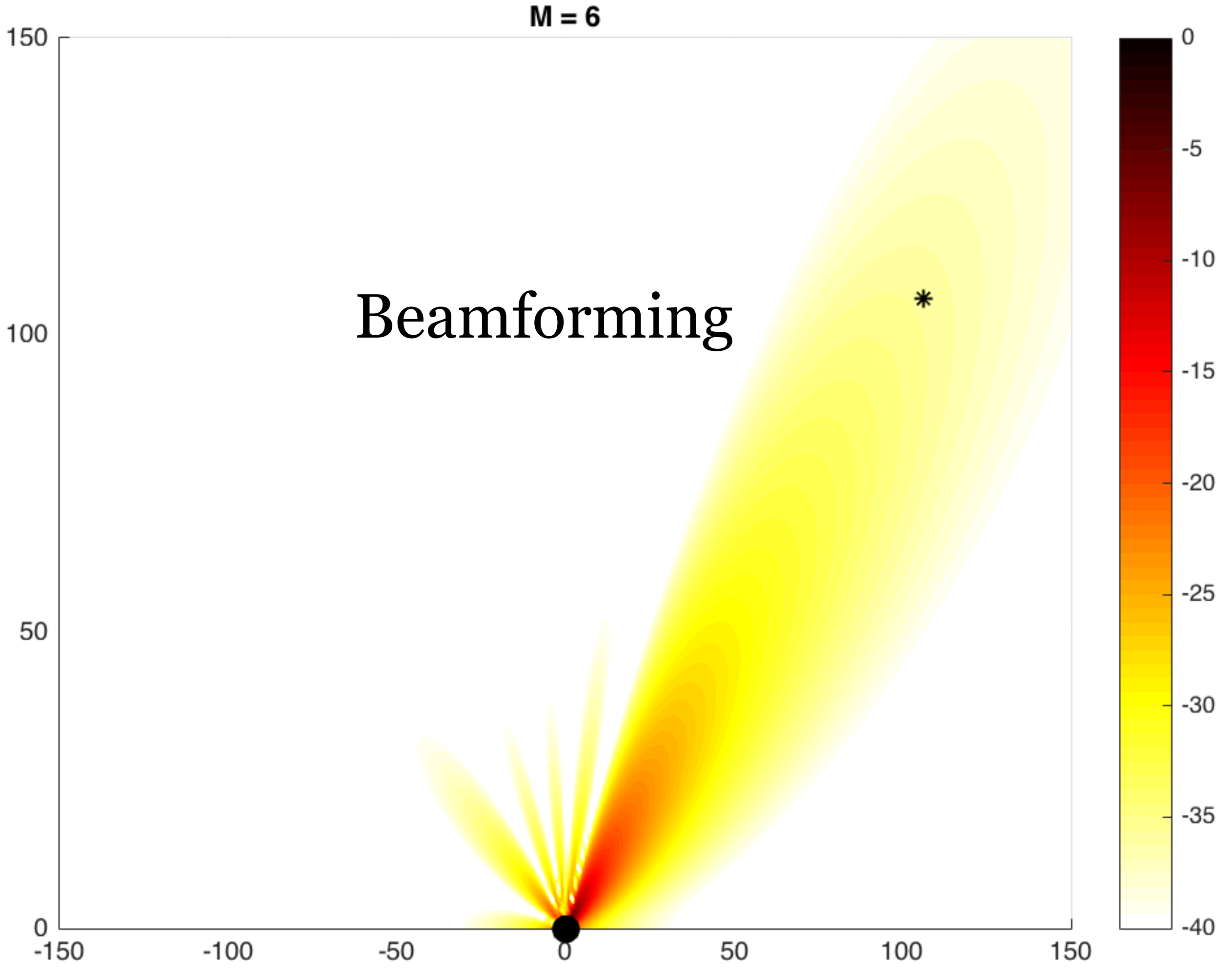
$M = 6$ antennas

Power directed towards receiver

- Constructive interference
- Send signals with time delays
- Needs to know "direction"

Two options:

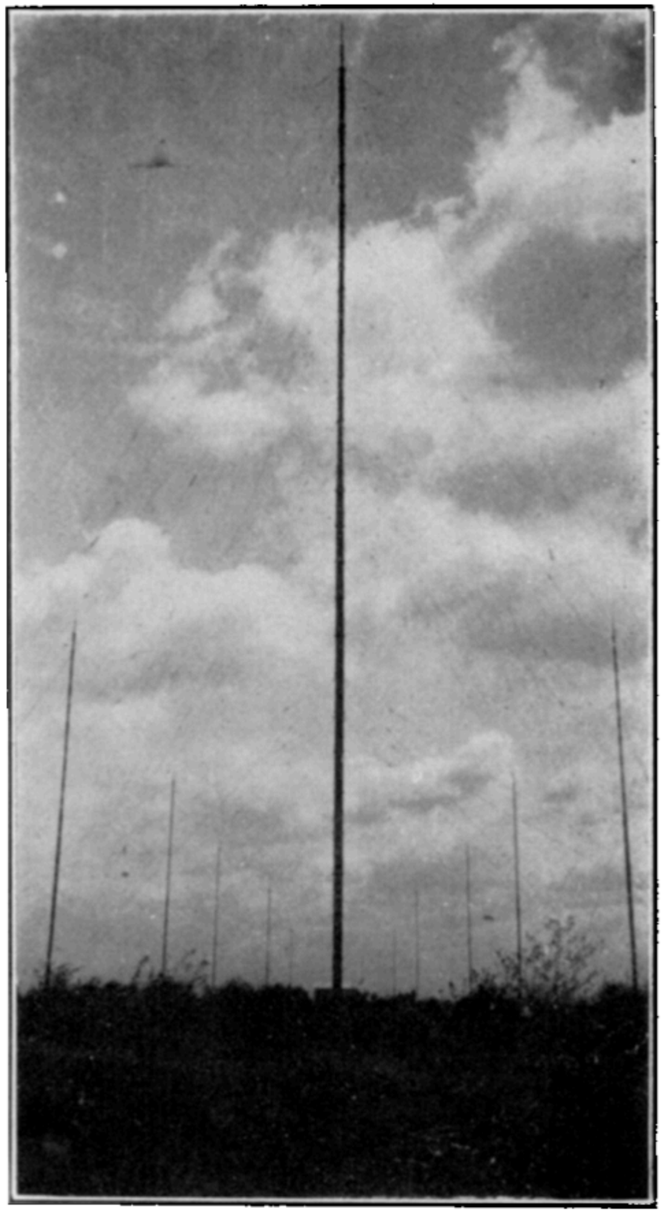
- 6 times larger received power
- Use 1/6 of power to achieve same received power



History: Beamforming

- Directional communication
  - E.F.W. Alexanderson, ”Transatlantic Radio Communication”, Trans. AIEE, 1919.
- Directive radiation pattern
  - Constructive/destructive interference patterns
  - Energy savings or better performance

Called *beamforming gain*



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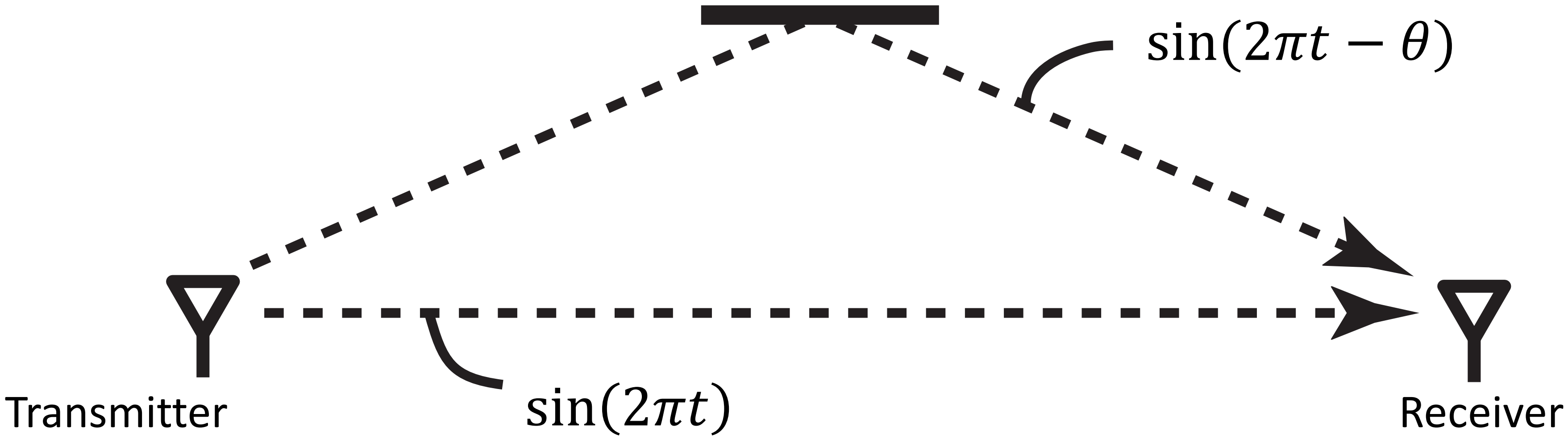
Multipath Propagation



Received signal: $\sin(2\pi t) + \sin(2\pi t - \theta)$

Multipath fading:
Add construtively or destructively depending on θ

History: Spatial diversity

- Diversity combining
  - D.G. Brennan, “Linear diversity combining techniques,” Proc. IRE, 1959

- Protect against fading/noise
  - Multiple antennas with independent observations
  - Small risk that multiple observations are all bad
  - Improve reliability

Called *diversity gain*

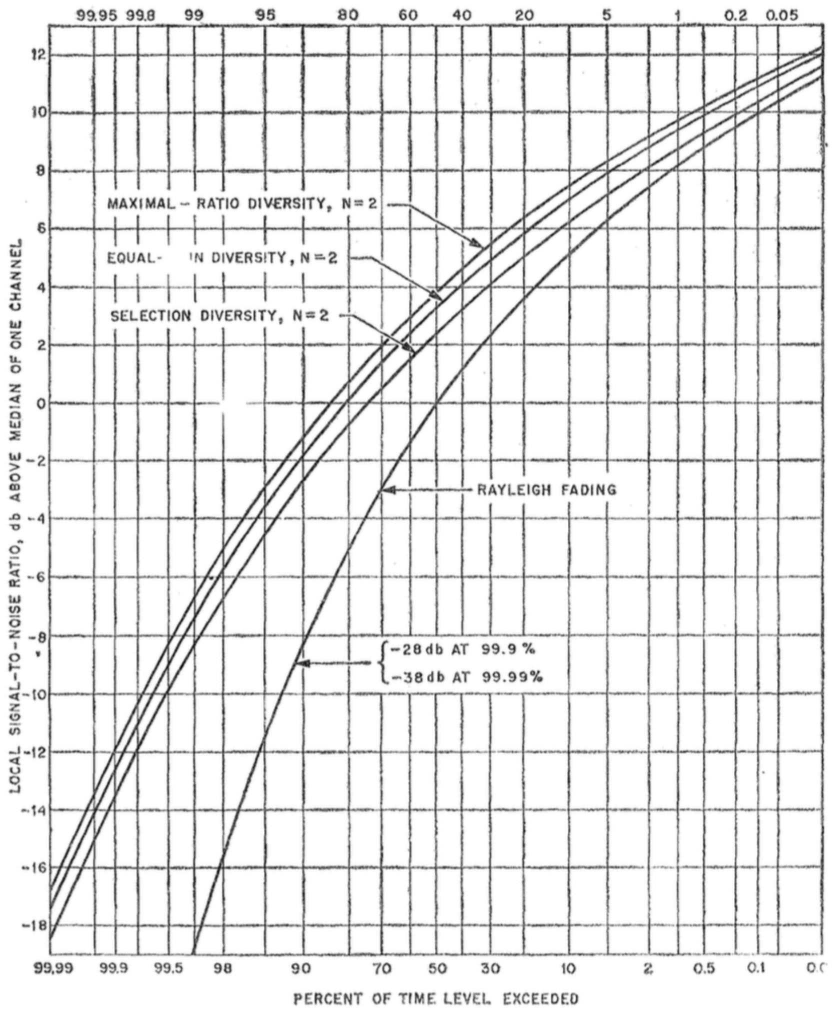
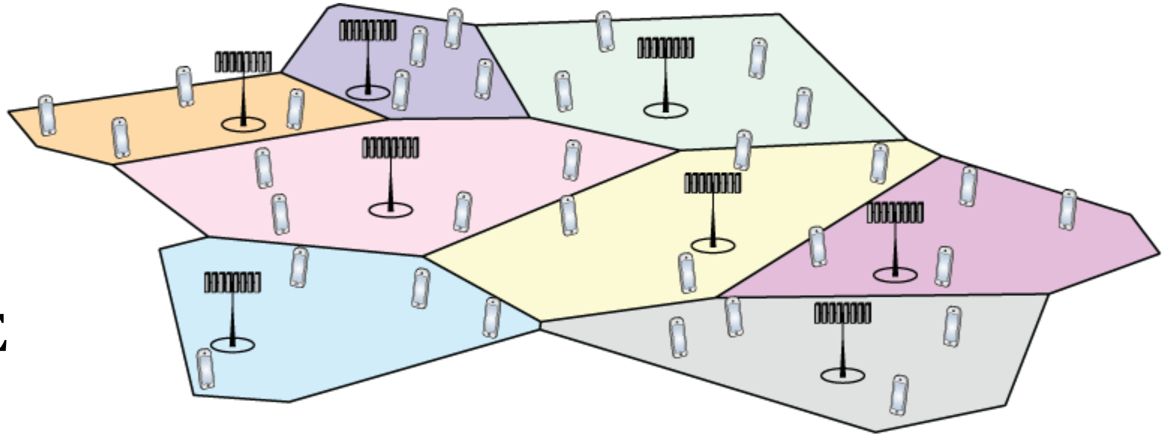


Fig. 9—Dual diversity distributions, conditions of Fig. 8.

Cellular networks

- Designed for mobile telephone systems
  - Bullington, K. (1953). “Frequency economy in mobile radio bands”. The Bell System Technical Journal.
  - Schulte, H. J. and W. A. Cornell (1960). “Multi-area mobile telephone system”. IEEE Trans. Veh. Technol.



- Reuse of spectrum in space:
  - Densify as usage increases
- Control interference by fractional spectrum reuse, power control

History: Space division multiple access (SDMA)

- Multiple user communication
  - S. C. Swales et al., “The Performance Enhancement of Multibeam Adaptive Base-Station Antennas for Cellular Land Mobile Radio Systems” Trans. on Vehicular Technology, 1990.
- Spatial multiplexing of users
  - Serve multiple users on same time and frequency
  - Handle more users per base station
  - Exists in 4G/5G and Wi-Fi

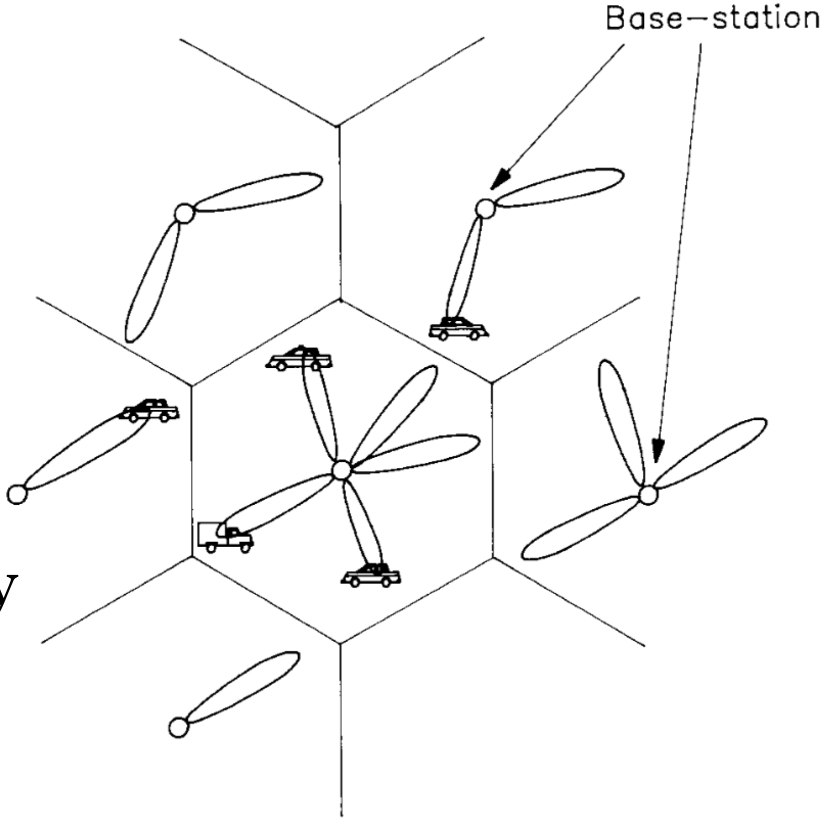


Fig. 2. Tracking of mobiles with multiple beams.

Called *multiplexing gain*

History: Point-to-point multi-antenna links

- Point-to-point multiple-input multiple-output (MIMO)
  - G. G. Raleigh and J. M. Cioffi, "*Spatio-temporal coding for wireless communications*," Globecom 1996.

- Multiple streams per user
  - Increase capacity (bit/s) per user
  - Requires multiple paths
  - Exists in 4G/5G and Wi-Fi

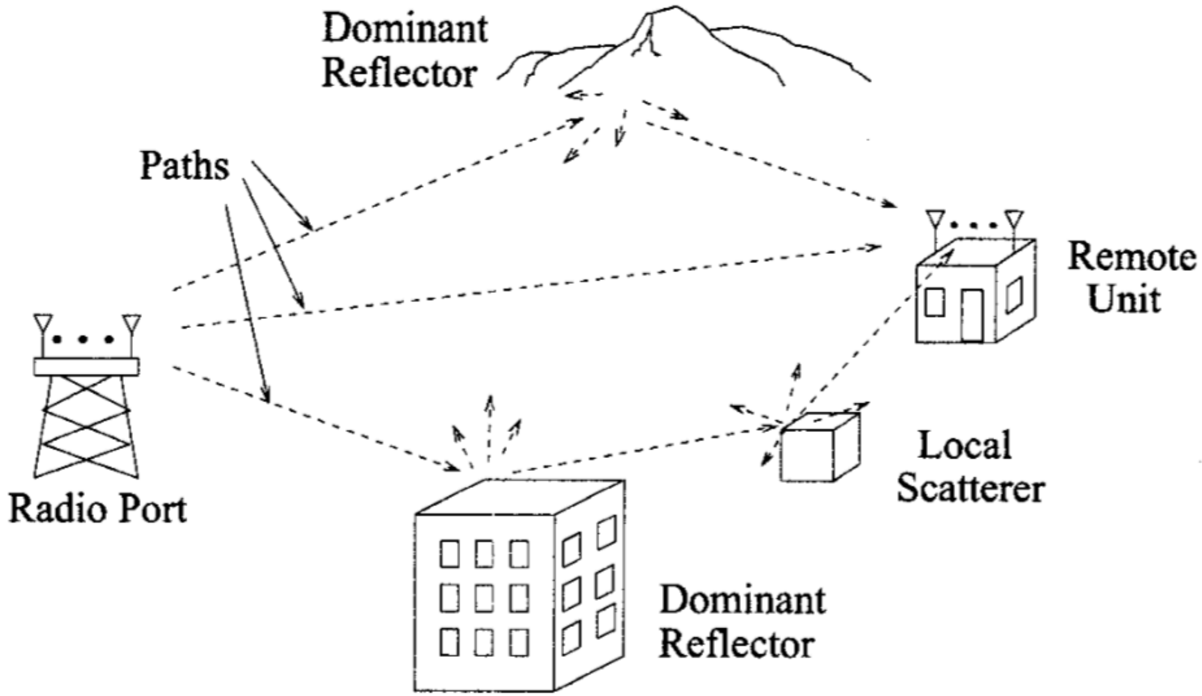


Figure 1: Illustration of the physical wireless channel.

Also called *multiplexing gain*

Outline of the course

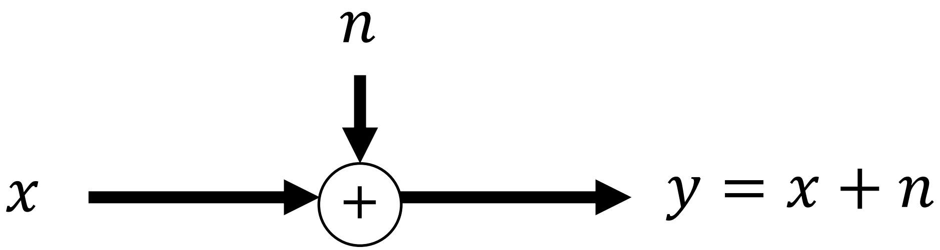
- Lecture 1:
  - History of multiple antenna communications
  - Channel capacity for single-antenna channels
- Basic multi–antenna channels (Lecture 2)
- Diversity and ergodic capacity (Lecture 3)
- Point-to-point MIMO (Lecture 4)

Based on lecture note document: “Introduction to Multiple Antenna Communications”

- Multi-user MIMO (Lectures 5-12)
  - Capacity bounds, Channel estimation, Power control, etc.

Based on the book “Fundamentals of Massive MIMO”

Example: Additive White Gaussian Noise (AWGN) Channel



- Noise: $n \sim N(0, N_0/2)$
- Energy per symbol: $E\{|x|^2\} \leq q$
- x, y, n real valued

How to measure performance?

- Data packet:

Information symbols

- Characterized by
  - How many symbols the packet contains
  - How many information bits these symbols represent (determined by the modulation and coding scheme)
  - Probability of incorrect decoding at the receiver

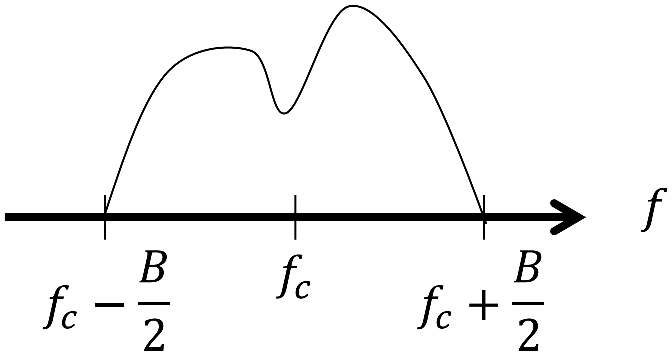
Channel capacity

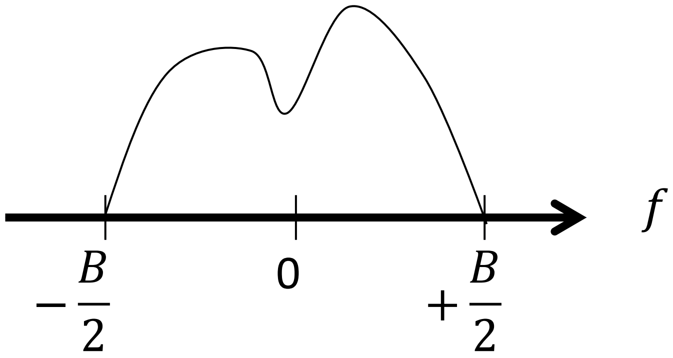
- Channel capacity
  - Random variables X and Y
  - Channel described by conditional distribution
    $$p_{Y|X}(y|x)$$

- **Channel coding theorem**
  - C [bit/channel use] is the capacity of the channel if:
    For any given $\delta > 0$ and $\gamma > 0$,
    there exist a channel coding codebook of a finite length N that has rate
    $R = C - \delta$ and offers an error probability $P(\text{error}) \leq \gamma$

Passband and baseband signals

- Bandwidth: Distance from smallest to largest frequency





Bandwidth (passband): B
Real-valued signal

Described by $2B$ real samples/s

Bandwidth (baseband): $\frac{B}{2}$
Complex-valued signal

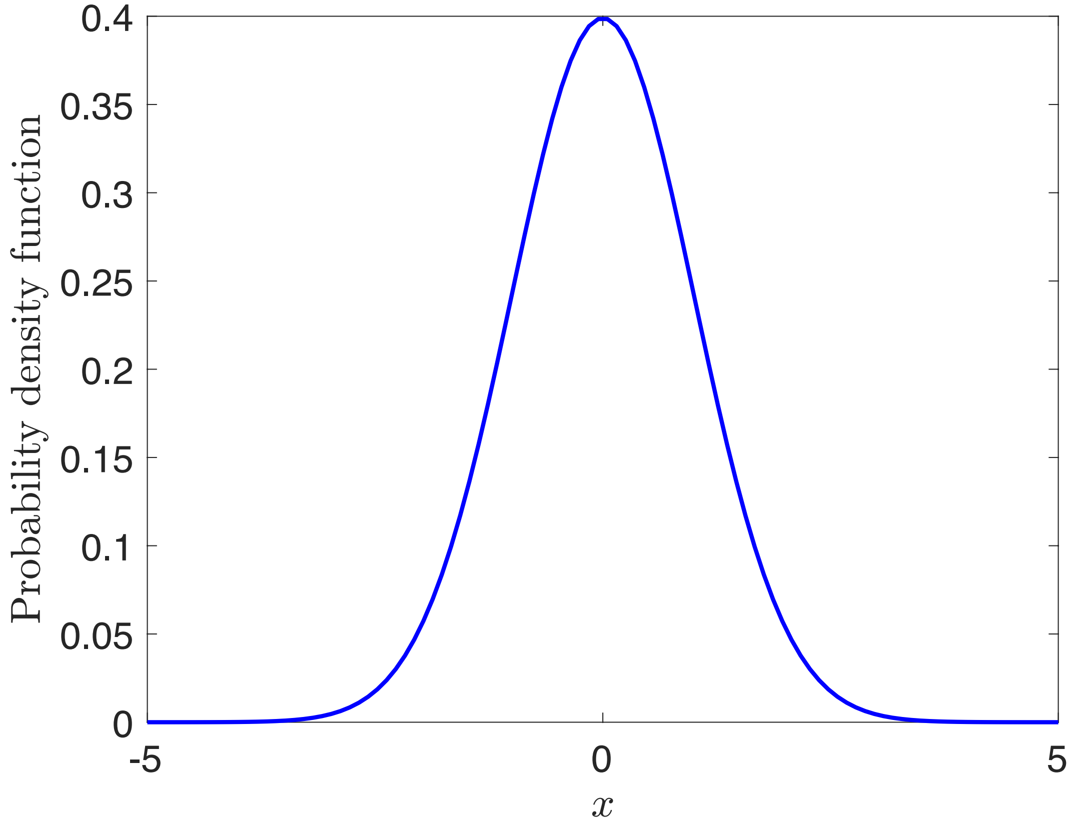
Described by B complex samples/s

These samples are used to convey information

Gaussian Distribution

- x is zero-mean Gaussian distributed, $x \sim N(0, \sigma^2)$
  - Probability density function (PDF):

$$p_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

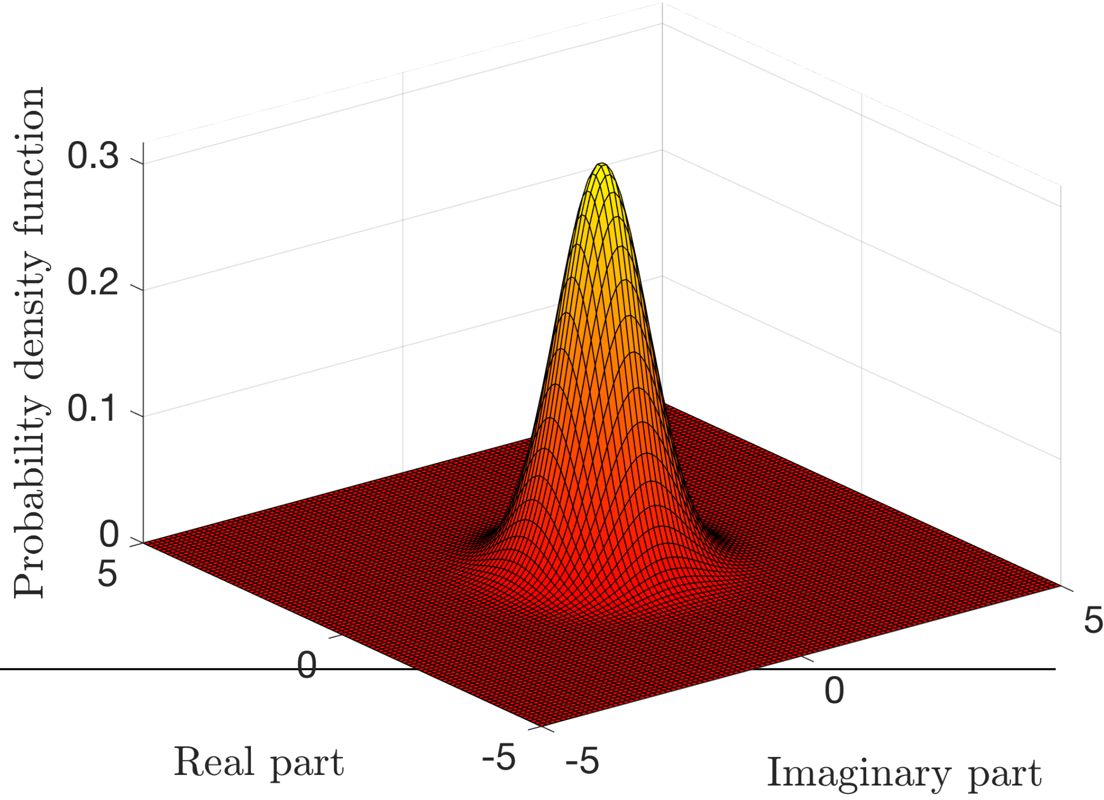


Properties: $E\{x\} = 0, E\{x^2\} = \sigma^2$

Complex Gaussian Distribution

- Consider independent $x_R, x_I \sim N(0, \sigma^2/2)$
  - $x = x_R + jx_I$ is circularly symmetric complex Gaussian distributed, $x \sim CN(0, \sigma^2)$, with

$$p_X(x) = \frac{1}{\sqrt{\pi\sigma^2}} e^{-\frac{x_R^2}{\sigma^2}} \frac{1}{\sqrt{\pi\sigma^2}} e^{-\frac{x_I^2}{\sigma^2}}$$
$$= \frac{1}{\pi\sigma^2} e^{-\frac{|x|^2}{\sigma^2}}$$



Properties: $E\{x\} = 0, E\{|x|^2\} = \sigma^2$

Capacity and mutual information

- Channel capacity

$$C = \max_{p_X(x)} I(x;y) \qquad x \longrightarrow \boxed{\text{Channel}} \longrightarrow y$$

- Mutual information: $I(x;y) = h(y) - h(y|x)$
- Differential entropy:

$$h(y) = -E\left\{\log_2\left(p_Y(y)\right)\right\} \leq \log_2(\pi e \mathrm{Var}\{y\})$$

- Conditional diff. entropy:

$$h(y|x) = -E\left\{\log_2\left(p_{Y|X}(y|x)\right)\right\}$$

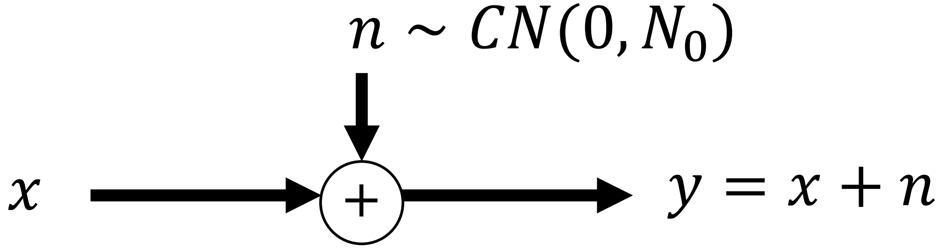
Equality if
Complex Gaussian

Differential entropy $h(x)$ of $x \sim CN(0, p)$

- Direct computation

$$
\begin{aligned}
h(x) &= -\int_{x\in\mathbb{C}} \frac{1}{\pi p} e^{-\frac{|x|^2}{p}} \log_2\left(\frac{1}{\pi p} e^{-\frac{|x|^2}{p}}\right) dx \\
&= \int_{x\in\mathbb{C}} \frac{1}{\pi p} e^{-\frac{|x|^2}{p}} \left(\log_2(\pi p) + \frac{|x|^2}{p}\log_2(e)\right) dx \\
&= \log_2(\pi p) \int_{x\in\mathbb{C}} \frac{1}{\pi p} e^{-\frac{|x|^2}{p}} dx + \frac{\log_2(e)}{p} \int_{x\in\mathbb{C}} \frac{|x|^2}{\pi p} e^{-\frac{|x|^2}{p}} dx \\
&= \log_2(\pi p) \cdot 1 + \frac{\log_2(e)}{p} E\{|x|^2\} = \log_2(\pi e p)
\end{aligned}
$$

Capacity of complex AWGN channel



- Recall: $I(x; y) = h(y) - h(y|x)$

$$h(y|x) = [y|x \sim CN(x, N_0)\ ] = \log_2(\pi e N_0)$$

- Mutual information maximized by $x \sim CN(0, q)$

$$h(y) \leq [y = x + n \sim CN(0, q + N_0)\ ] = \log_2(\pi e(q + N_0))$$

- Capacity:

$$C = h(y) - h(y|x) = \log_2\left(1 + \frac{q}{N_0}\right)$$

Point-to-point scalar channel

- Continuous-time AWGN channel with channel response $g \in \mathbb{C}$
  - Bandwidth B (Hz)
  - Power P (Watt)
  - Noise power spectral density N_0 (Watt/Hz)

- Sampling: $2B$ real samples/s, B complex samples/s:

$$y[l] = g \cdot x[l] + n[l], \qquad l = \text{sample index}$$

  - Energy per sample: $q = P/B$
  - $x[l] \sim CN(0, P/B)$
  - $n[l] \sim CN(0, N_0)$

Complex Gaussian signals are capacity achieving

Point-to-point scalar channel: Capacity

- Sampling: $2B$ real samples/s, B complex samples/s:

$$y[l] = g \cdot x[l] + n[l]$$

  - $x[l] \sim CN(0, P/B)$
  - $n[l] \sim CN(0, N_0)$

- Capacity:
  - Depends on P and B

$$\mathrm{C} = \log_2\left(1 + \frac{P|g|^2}{BN_0}\right) \text{ bits per complex sample}$$

$$C = B \cdot \log_2\left(1 + \frac{P|g|^2}{BN_0}\right) \text{ bits per second}$$

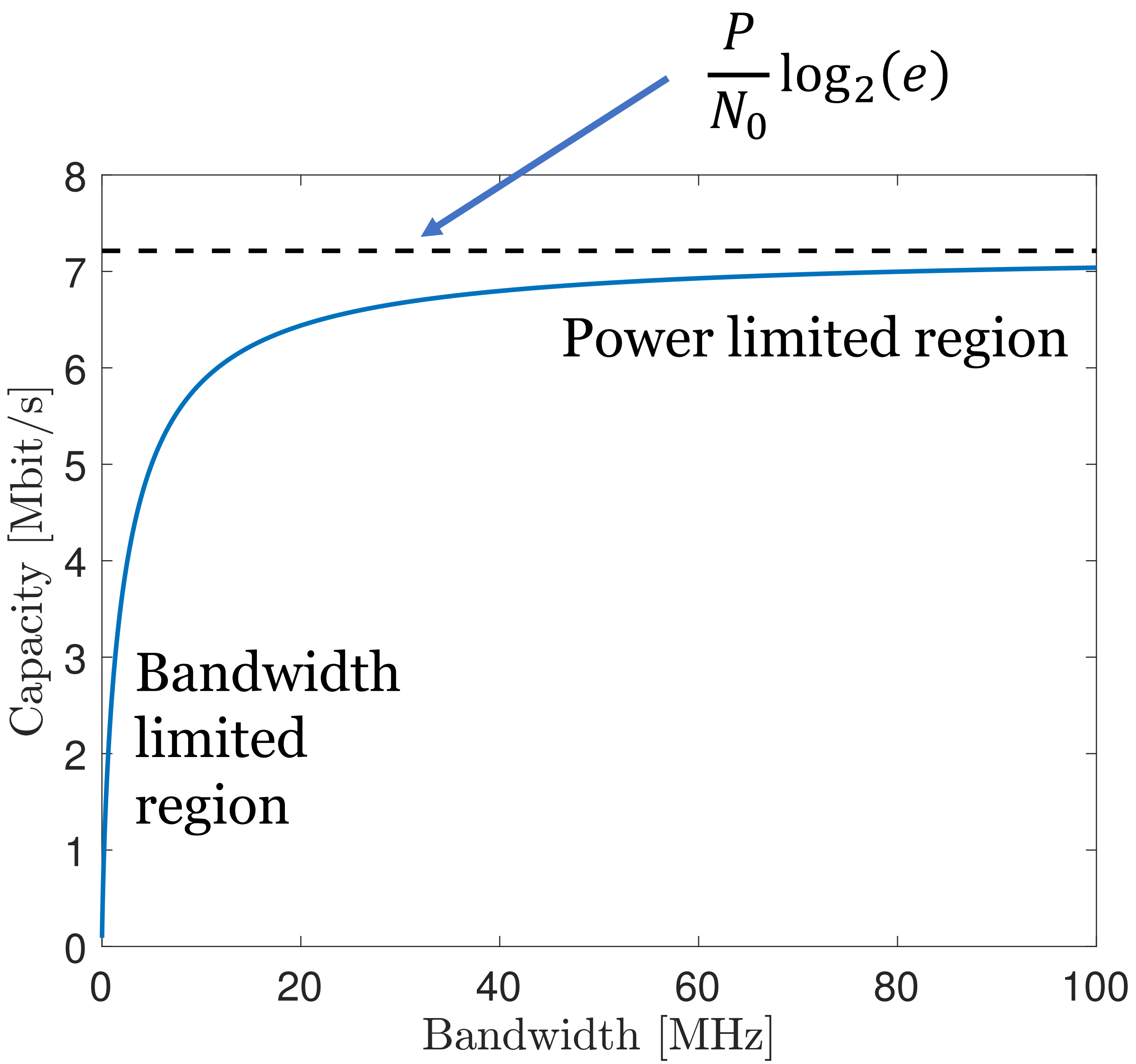
Capacity behaviors

- SNR:

$$\frac{P}{BN_0}$$

- $\log_2(1 + \mathrm{SNR})$
  - Linear for small SNR
  - Almost flat for large SNR

Assumption: $\frac{P|g|^2}{N_0} = 5 \cdot 10^6$ Hz



Summary

- Capacity of point-to-point channel:

$$C = B \cdot \log_2\left(1 + \frac{P}{BN_0}\right) \text{ bits per second}$$

- SNR per degree of freedom (symbol): $\frac{P}{BN_0}$
- Behaviors
  - Low SNR: C grows linearly with power P
  - High SNR: C grows linearly with bandwidth B

End of Lecture 1

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