TSKS14

Multiple Antenna Communications

Lecture 2, 2020

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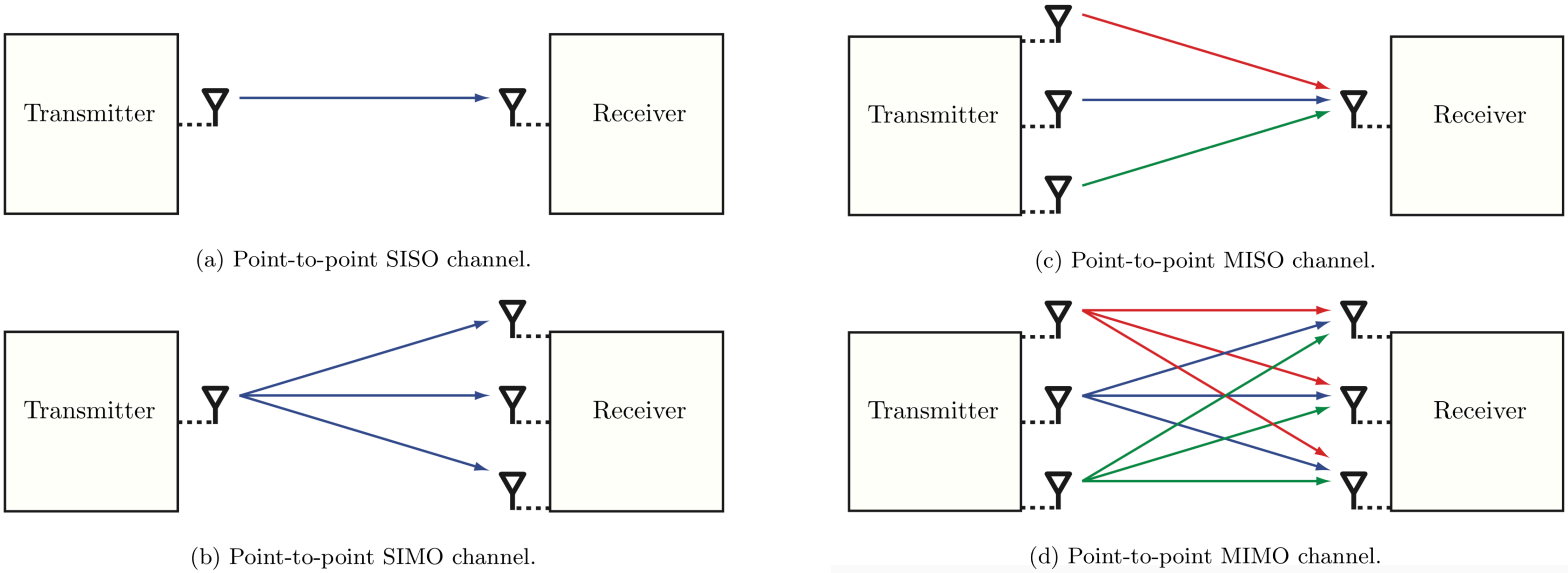
Taxonomy: Point-to-point channels



Single-input single-output (SISO)
Single-input multiple-output (SIMO)

Multiple-input single-output (MISO)
Multiple-input multiple-output (MIMO)

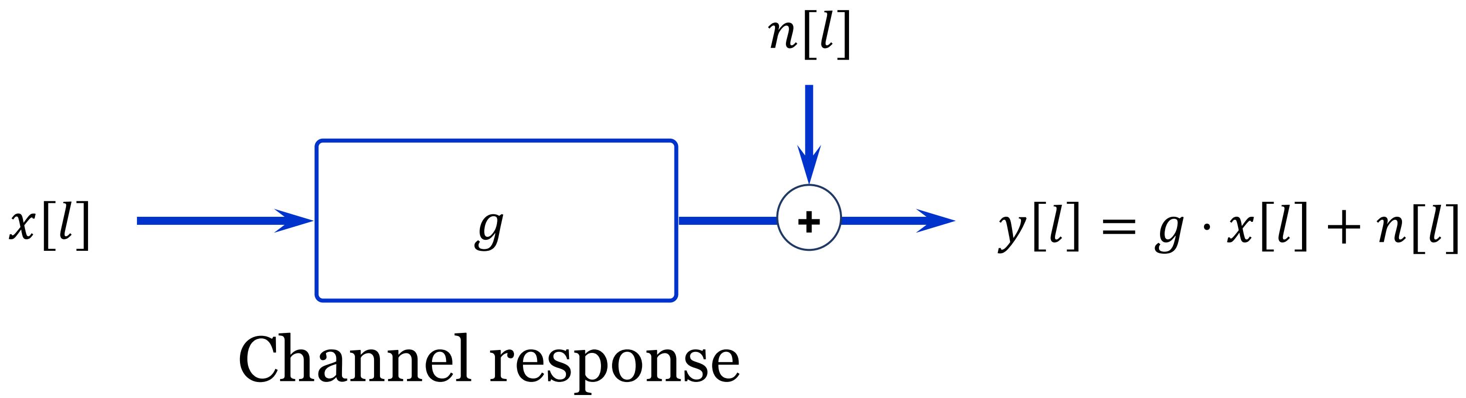
Outline of this lecture

- Recap of SISO channel capacity
- Capacity of SIMO and MISO channels
- Channel models for line-of-sight channels
- Beamwidth with uniform linear arrays

Not all transmit power is received

- Wireless signals spread out as a sphere
  - Fraction of received signal at distance d: $\dfrac{\lambda^2}{(4\pi d)^2}$
- Cellular communication: 70 dB to 130 dB is lost!
  - Channel gain $|g|^2$ is around -70 dB to -130 dB

Also known as *pathloss*



Capacity of memoryless SISO channel

- AWGN channel with a complex channel gain g:
$$y[l] = g \cdot x[l] + n[l]$$
  - $x[l] \sim CN(0, q)$, energy per sample: $q = P/B$
  - $n[l] \sim CN(0, N_0)$

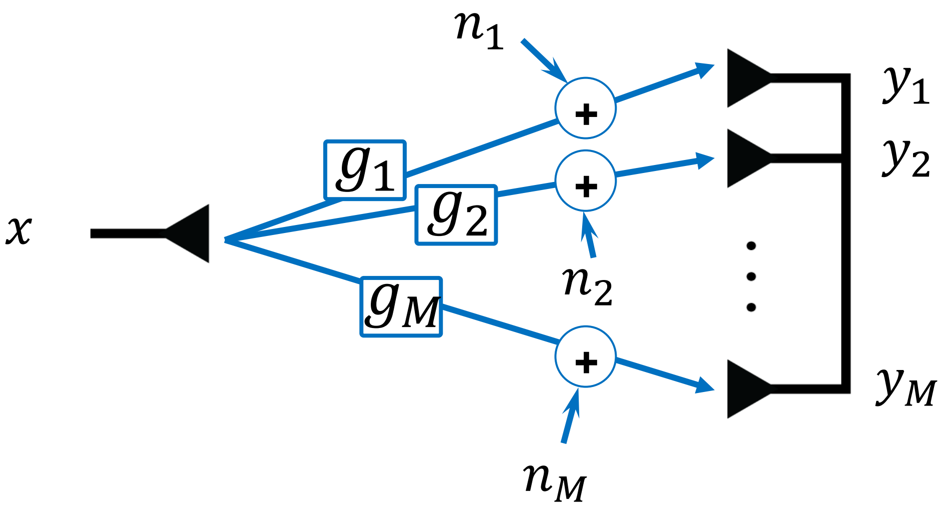
Memoryless channel: Drop dependence on l

- Capacity computation: Treat $g \cdot x \sim CN(0, q|g|^2)$ as the signal!
$$\mathrm{C} = \log_2\left(1 + \frac{q|g|^2}{N_0}\right) \text{bits per complex sample}$$

Received SNR

How can we increase the SNR?

Single-input multiple-output (SIMO) channel



Noise: $n_m \sim CN(0, N_0)$

- Received vector signal:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} g_1 \\ \vdots \\ g_M \end{bmatrix} x + \begin{bmatrix} n_1 \\ \vdots \\ n_M \end{bmatrix}$$

  - Short form:

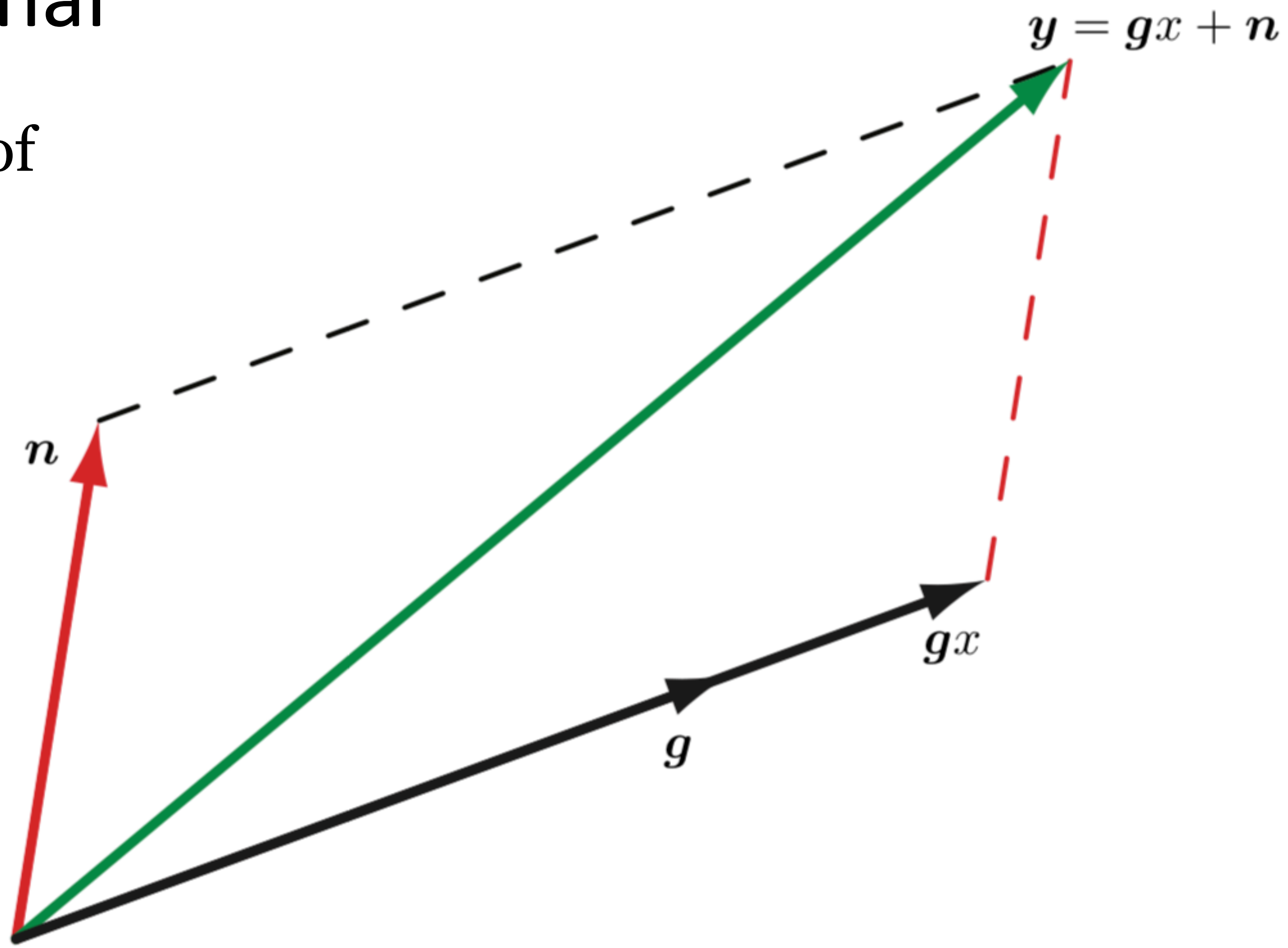
$$\boldsymbol{y} = \boldsymbol{g} x + \boldsymbol{n}$$

Geometry of received signal

- Received vector $\boldsymbol{y}$ is summation of
  - Desired signal $\boldsymbol{g}x$
  - Noise vector $\boldsymbol{n}$

- Want to turn $\boldsymbol{y}$ into an estimate of x

Only part of $\boldsymbol{y}$ *parallel to* $\boldsymbol{g}$ *contains desired signal*



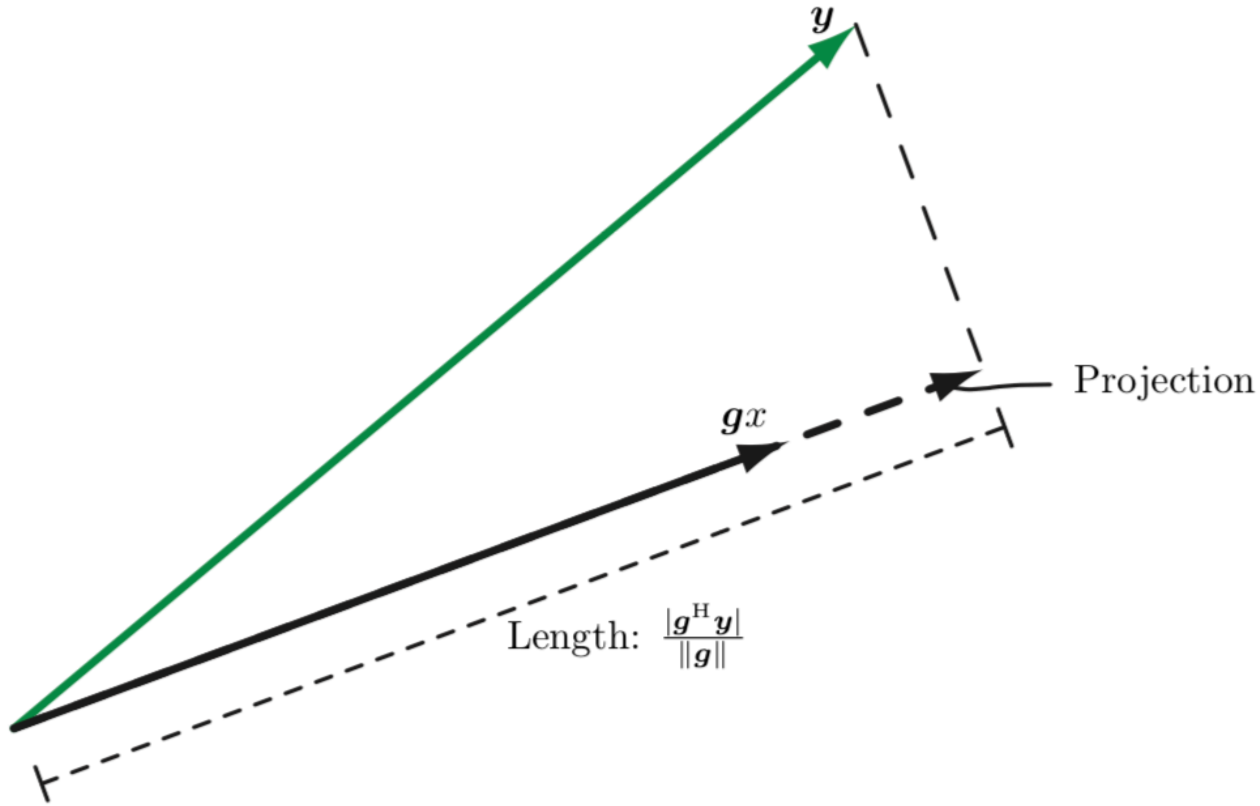
Capacity of SIMO channel

- Maximum ratio combining (MRC):

$$\frac{\boldsymbol{g}^H}{\|\boldsymbol{g}\|}\boldsymbol{y} = \|\boldsymbol{g}\|x + \frac{\boldsymbol{g}^H\boldsymbol{n}}{\|\boldsymbol{g}\|}$$

Combining vector

Scalar

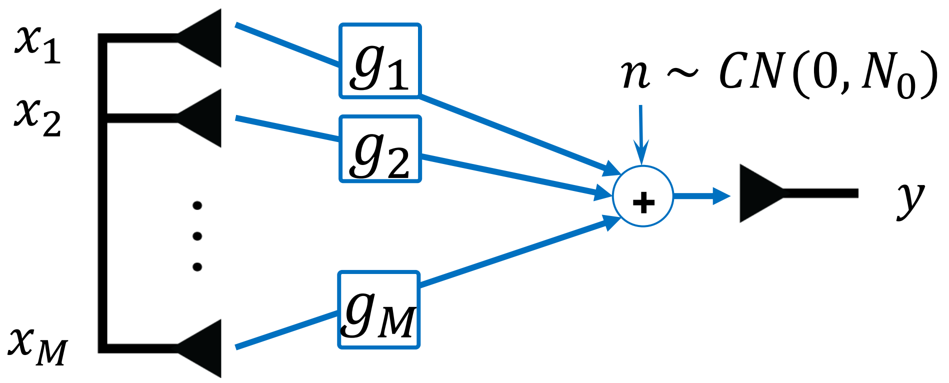


> Like a SISO channel with $g = \|\boldsymbol{g}\|$

- Capacity:

$$\mathrm{C} = \log_2\left(1 + \frac{q\|\boldsymbol{g}\|^2}{N_0}\right) \text{ bits per complex sample}$$

Multiple-input single-output (MISO) channel



- Received vector signal:

$$y = [g_1 \quad \cdots \quad g_M] \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} + n = \boldsymbol{g}^T \boldsymbol{x} + n$$

- Precoding: $\boldsymbol{x} = \boldsymbol{w}\tilde{x}$

Information signal: $\tilde{x} \sim CN(0, q)$

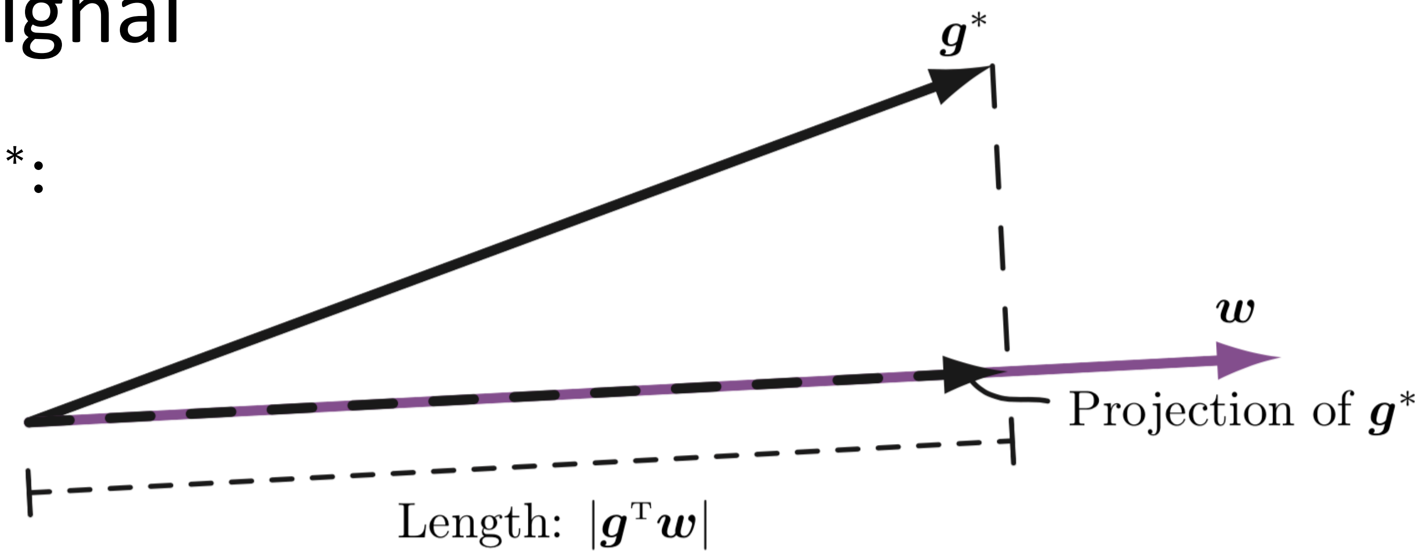
Unit-norm precoding vector

Geometry of received signal

- Channel projects signal onto $\boldsymbol{g}^*$:

Maximum ratio transmission (MRT):

$$\boldsymbol{w} = \frac{\boldsymbol{g}^*}{\|\boldsymbol{g}\|}$$



$$y = \boldsymbol{g}^T \frac{\boldsymbol{g}^*}{\|\boldsymbol{g}\|} \tilde{x} + n = \|\boldsymbol{g}\| \tilde{x} + n$$

- Capacity:

$$\mathrm{C} = \log_2\left(1 + \frac{q\|\boldsymbol{g}\|^2}{N_0}\right) \text{ bits per complex sample}$$

Receive and transmit beamforming

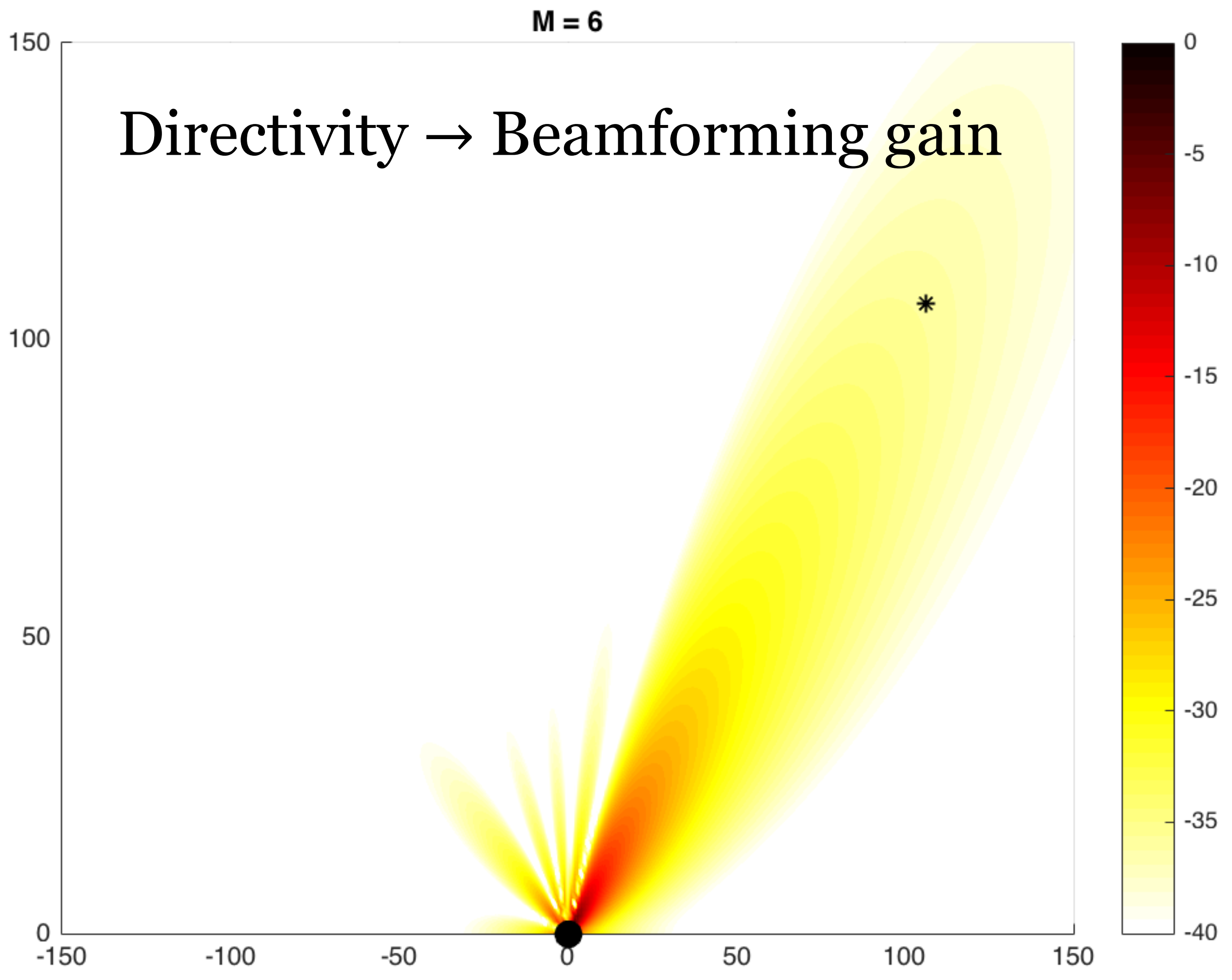
- Same capacity of SIMO and MISO channel

$$\mathrm{C} = \log_2\left(1 + \frac{q\|\boldsymbol{g}\|^2}{N_0}\right) \text{ bits per complex sample}$$

  - Achieved by “beamforming” along channel vector $\boldsymbol{g}$
  - Beamforming gain of $\|\boldsymbol{g}\|^2$ (often proportional to M)

- Several different names
  - Maximum ratio combining/transmission (MRC, MRT)
  - Conjugate beamforming, matched filtering

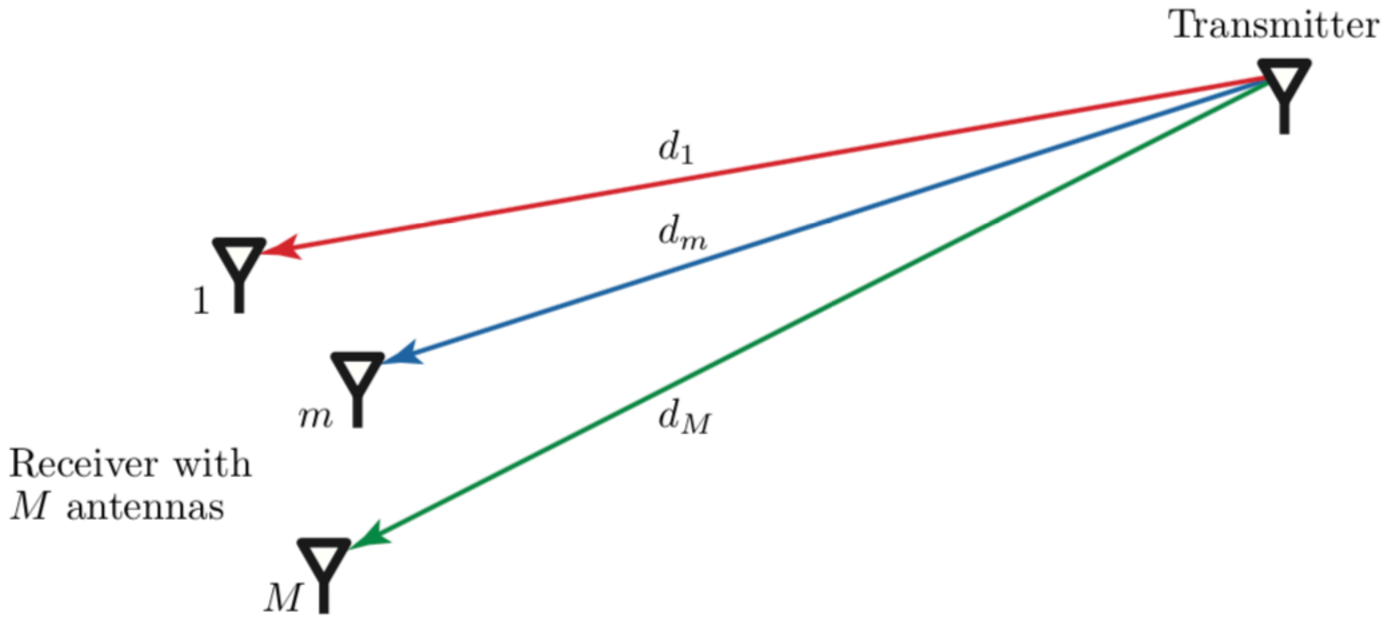
Transmit beamforming



- Other names:
  - Array gain
  - Power gain

Free-space line-of-sight commmunication

- Direct path between transmitter and receiver
  - No other paths
- We will derive a channel model



Complex baseband representation

- Real-valued baseband signals (bandwidth $B/2$):

$$s_I(t) \text{ and } s_Q(t)$$

- Complex baseband representation:

$$s_b(t) = s_I(t) + j s_Q(t)$$

- Real-valued passband signal

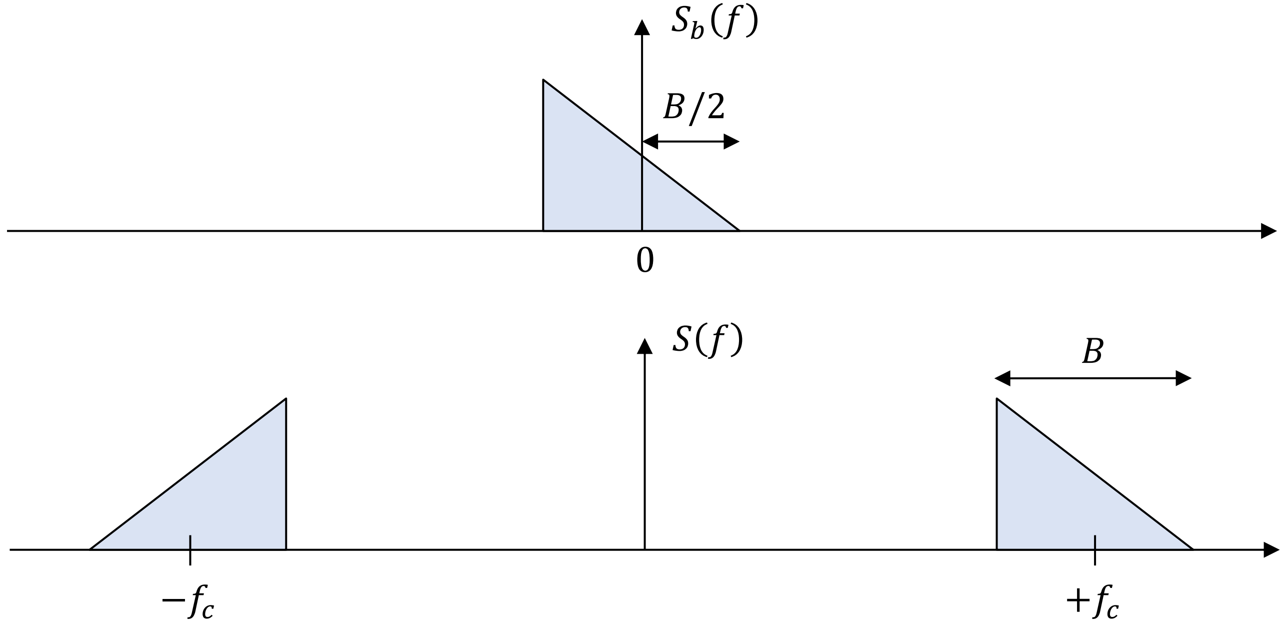
$$s(t) = s_I(t)\sqrt{2}\cos(2\pi f_c t) - s_Q(t)\sqrt{2}\sin(2\pi f_c t)$$
$$= Re\left\{\sqrt{2} s_b(t) e^{j2\pi f_c t}\right\}$$

  - Carrier frequency: $f_c \gg B$
  - Wavelength: $\lambda = c/f_c$

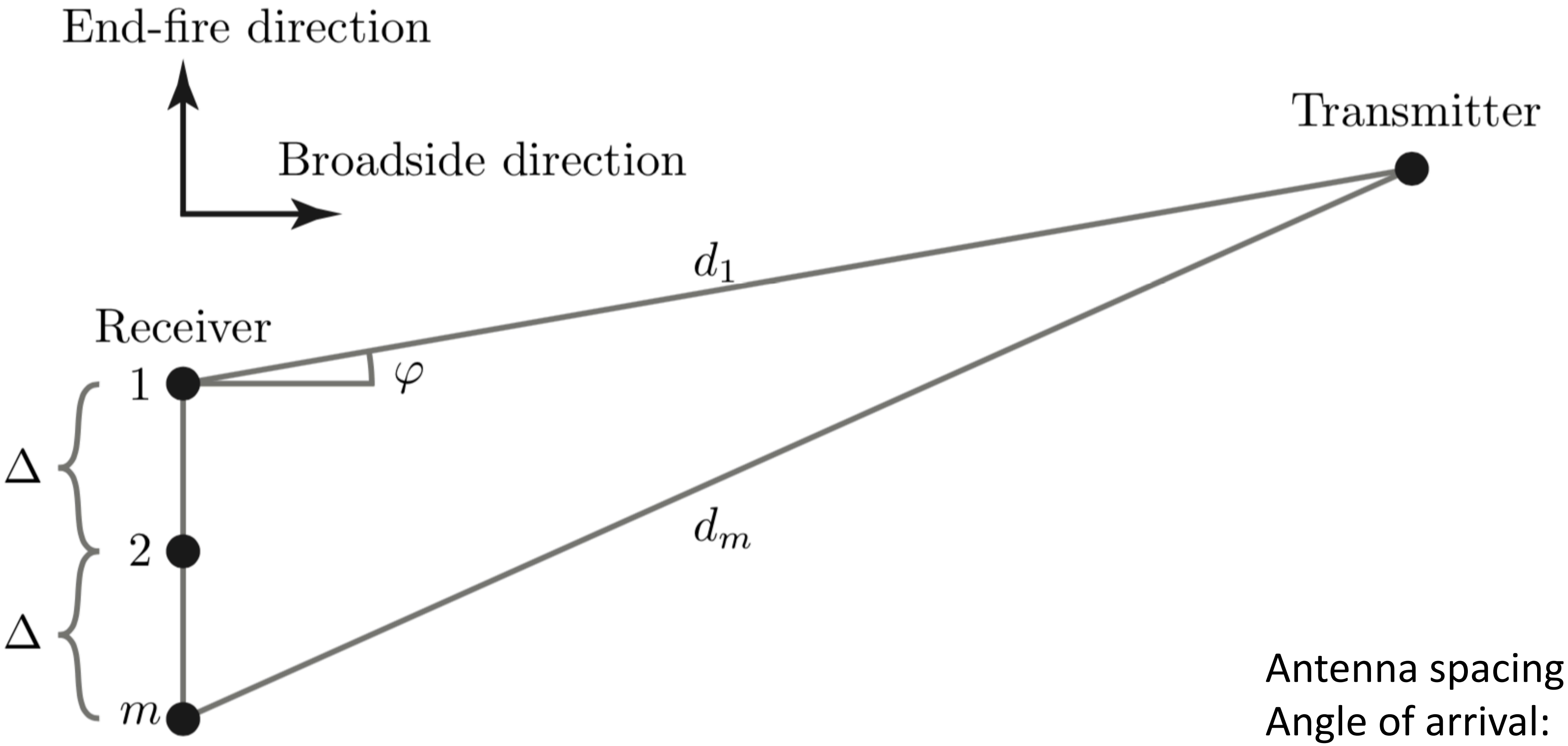
Complex baseband representation (2)

Fourier transforms: $S_b(f) = \mathcal{F}\{s_b(t)\}$

$$S(f) = \frac{1}{\sqrt{2}}\left(S_b(f - f_c) + S_b^*(-f - f_c)\right)$$



Uniform linear array (ULA)



Antenna spacing: Δ
Angle of arrival: φ

Impact of channel in complex baseband

- Transmitted signal:

$$s(t) = Re\{\sqrt{2}s_b(t)e^{j2\pi f_c t}\}$$

- Received signal at distance d_m:

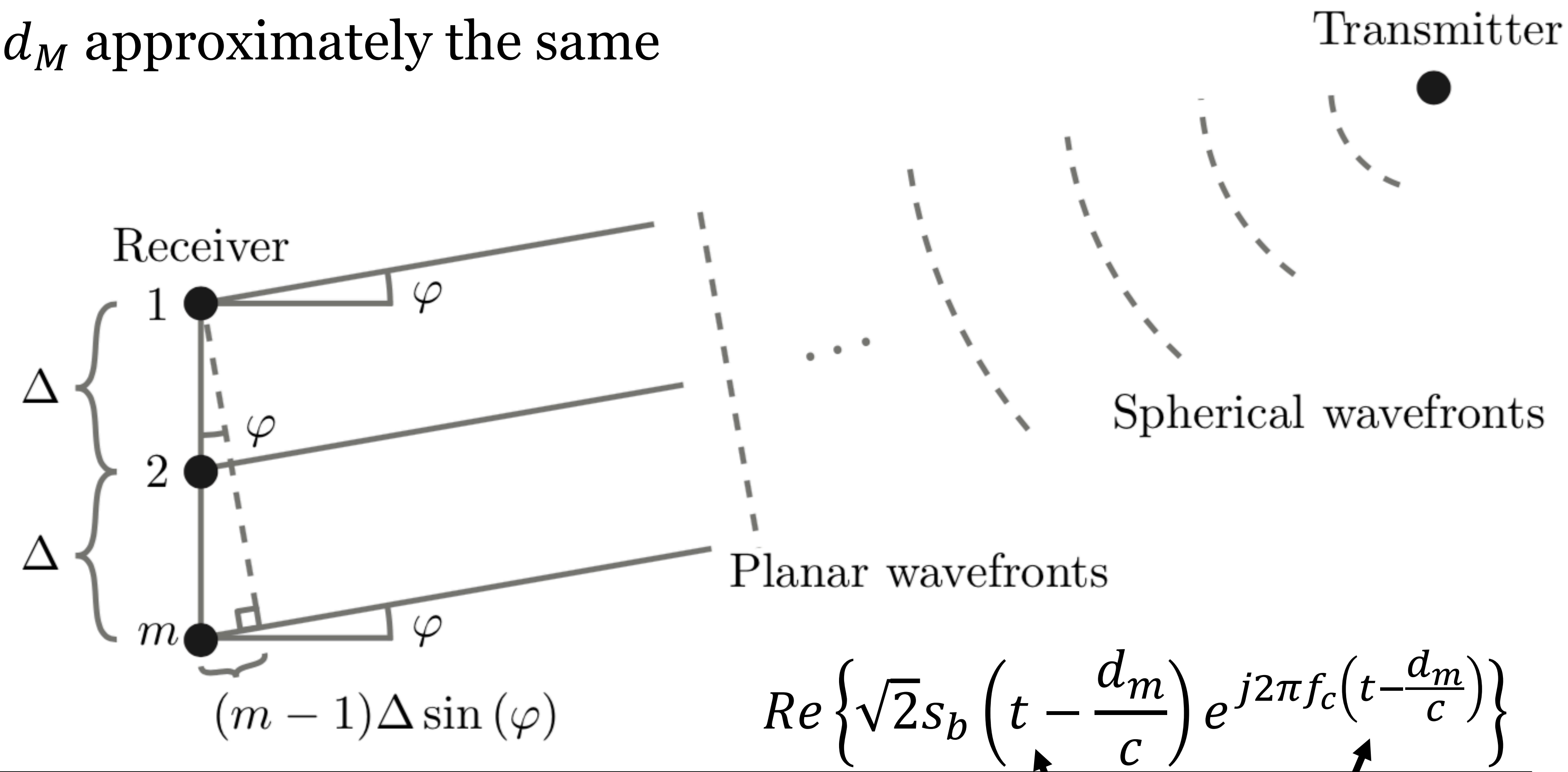
$$\mu_m(t) = \frac{\lambda}{4\pi d_m} s\left(t - \frac{d_m}{c}\right) = \frac{\lambda}{4\pi d_m} Re\left\{\sqrt{2}s_b\left(t - \frac{d_m}{c}\right)e^{j2\pi f_c\left(t - \frac{d_m}{c}\right)}\right\}$$

- Receiver can compensate for time delay by selecting sampling time
  - Different d_m: Only one antenna has perfect sampling

Far-field approximation

- Distances $d_1, \dots, d_M$ approximately the same

Approximately same channel gain:
$$\frac{\lambda^2}{(4\pi d_m)^2} \approx \frac{\lambda^2}{(4\pi d)^2}$$



$$Re\left\{\sqrt{2}s_b\left(t-\frac{d_m}{c}\right)e^{j2\pi f_c\left(t-\frac{d_m}{c}\right)}\right\}$$

Sample at $t = \frac{d_1}{c} = \frac{d}{c}$

ULA channel model

- M antennas in receiver array
  - Angle of arrival φ
  - Wavelength λ
  - Distance Δ between antennas
  - Distance d to first antenna
- Channel vector:

$$\boldsymbol{g} = \frac{\lambda}{4\pi d} \begin{bmatrix} 1 \\ e^{-j2\pi \frac{\Delta \sin(\varphi)}{\lambda}} \\ \vdots \\ e^{-j2\pi \frac{(M-1)\Delta \sin(\varphi)}{\lambda}} \end{bmatrix}$$

Applies to both SIMO and MISO channels

Channel capacity with line-of-sight channel

- Recall:

$$C = \log_2\left(1 + \frac{q\|\boldsymbol{g}\|^2}{N_0}\right) \text{ bits per complex sample}$$

- We have $\|\boldsymbol{g}\|^2 = \beta M$ where $\beta = \frac{\lambda^2}{(4\pi d)^2}$:

$$C = \log_2\left(1 + \frac{q\beta M}{N_0}\right)$$

Beamforming gain: M

Beamwidth

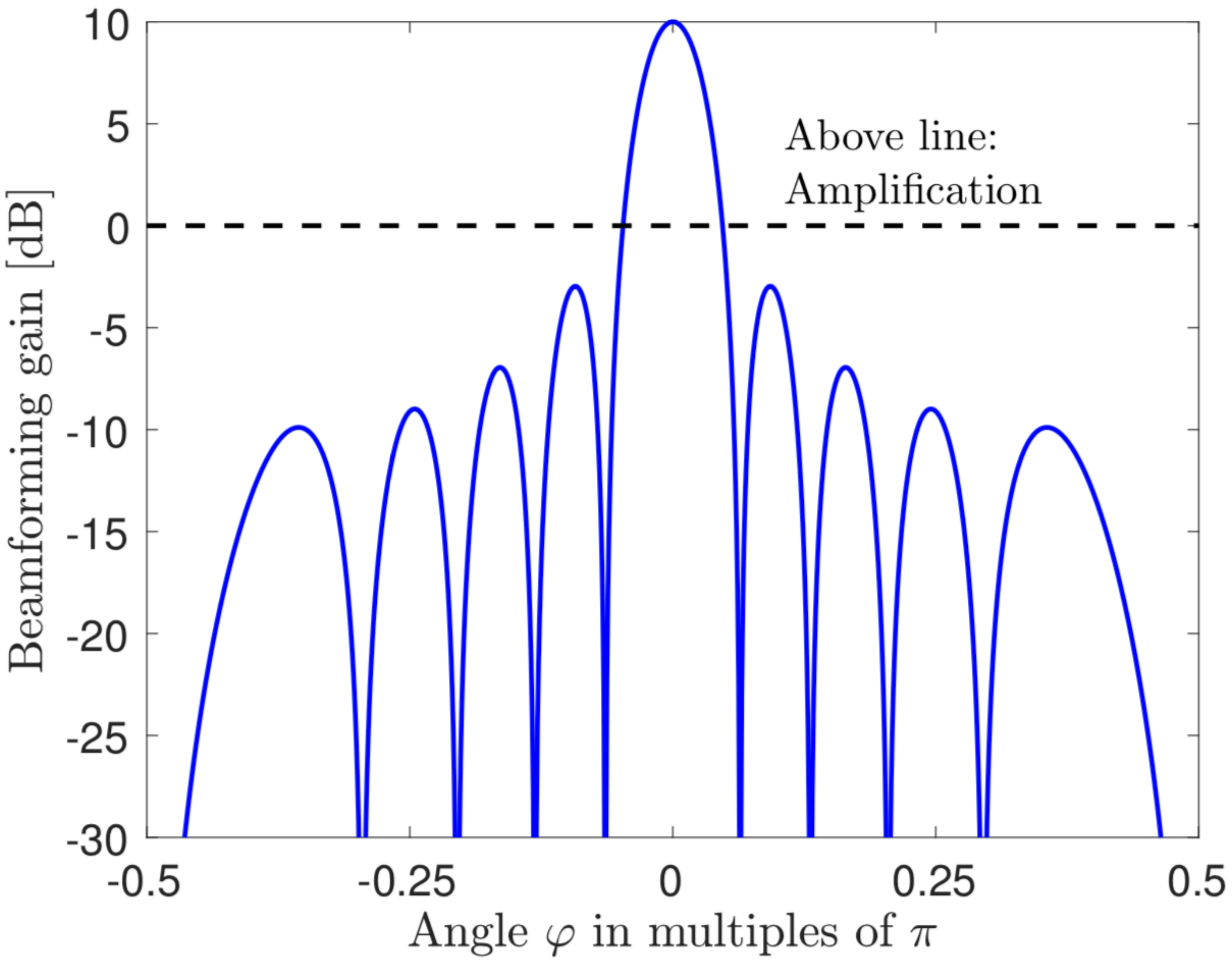
- Beamforming gain in more than one direction

- Example: $M = 10, \Delta = \lambda/2$

- First-null beamwidth:

$$\approx \frac{4}{M}$$

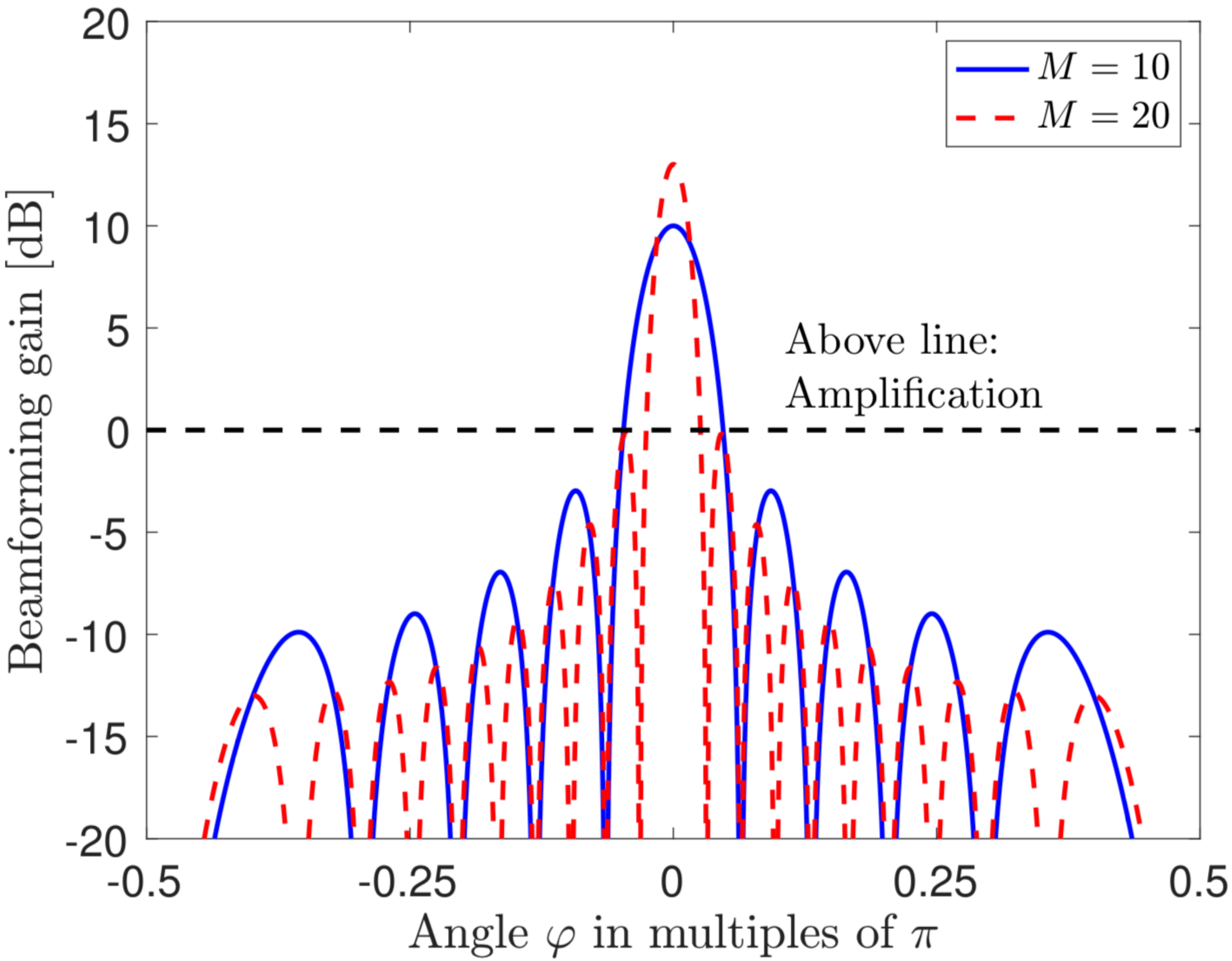
More antennas: Narrower beams



Two benefits of beamforming

- Beamforming gain
    - Stronger signal at desired location

- Narrower beam
    - Less interference at undesired locations



End of Lecture 2

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