TSKS14

Multiple Antenna Communications

Lecture 10, 2020

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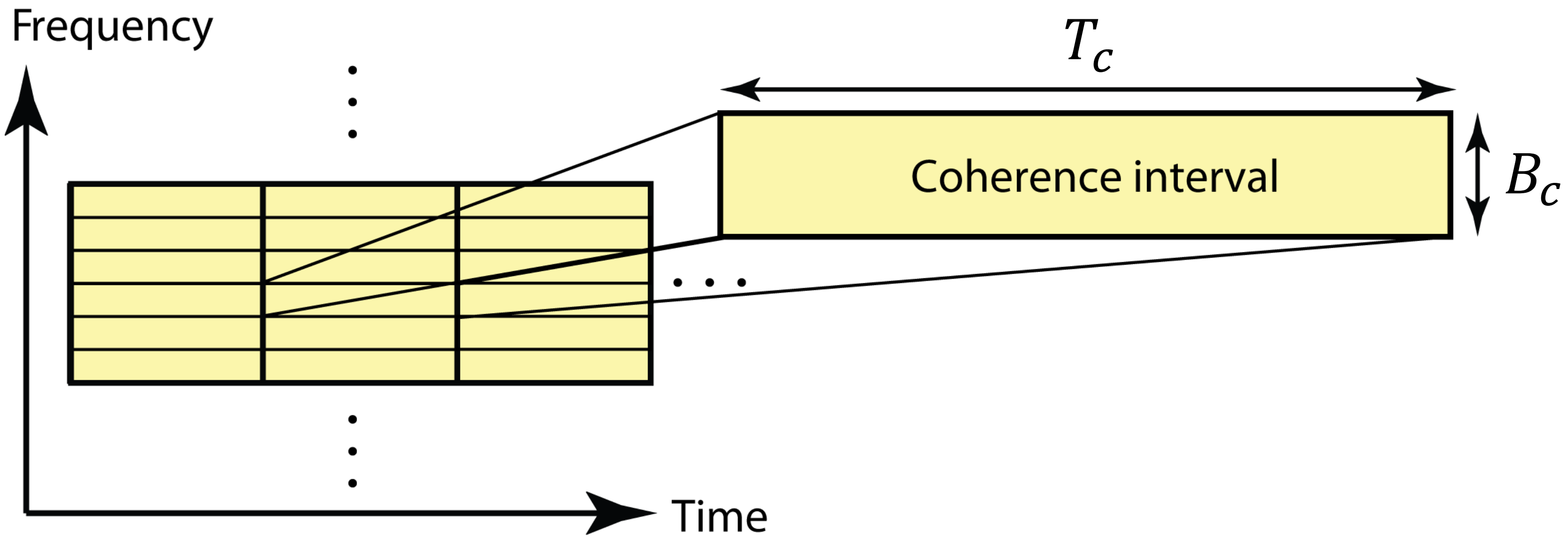
Outline of this lecture

- Cellular networks
  - Basic structure
- Channel estimation
  - Pilot contamination
- Spectral efficiency expressions with MR
  - Uplink
  - Downlink

Recall: Coherence interval



- Divide bandwidth and time into coherence intervals
  - According to sampling theorem:
    $\tau_c = B_c T_c$ complex samples
  - Channel time-invariant and described by a scalar

Net spectral efficiency

- Recall: Capacity bounds

Uplink:

$$\log_2\left(1+\frac{M\rho_{ul}\eta_k\gamma_k}{\sum_{i=1}^{K}\rho_{ul}\eta_i\beta_i+1}\right)$$

Downlink:

$$\log_2\left(1+\frac{M\rho_{dl}\eta_k\gamma_k}{\beta_k\sum_{i=1}^{K}\rho_{dl}\eta_i+1}\right)$$

  - When we use the channel for data
  - Pilot overhead: τ_p samples per coherence interval

Net spectral efficiency:

Uplink:

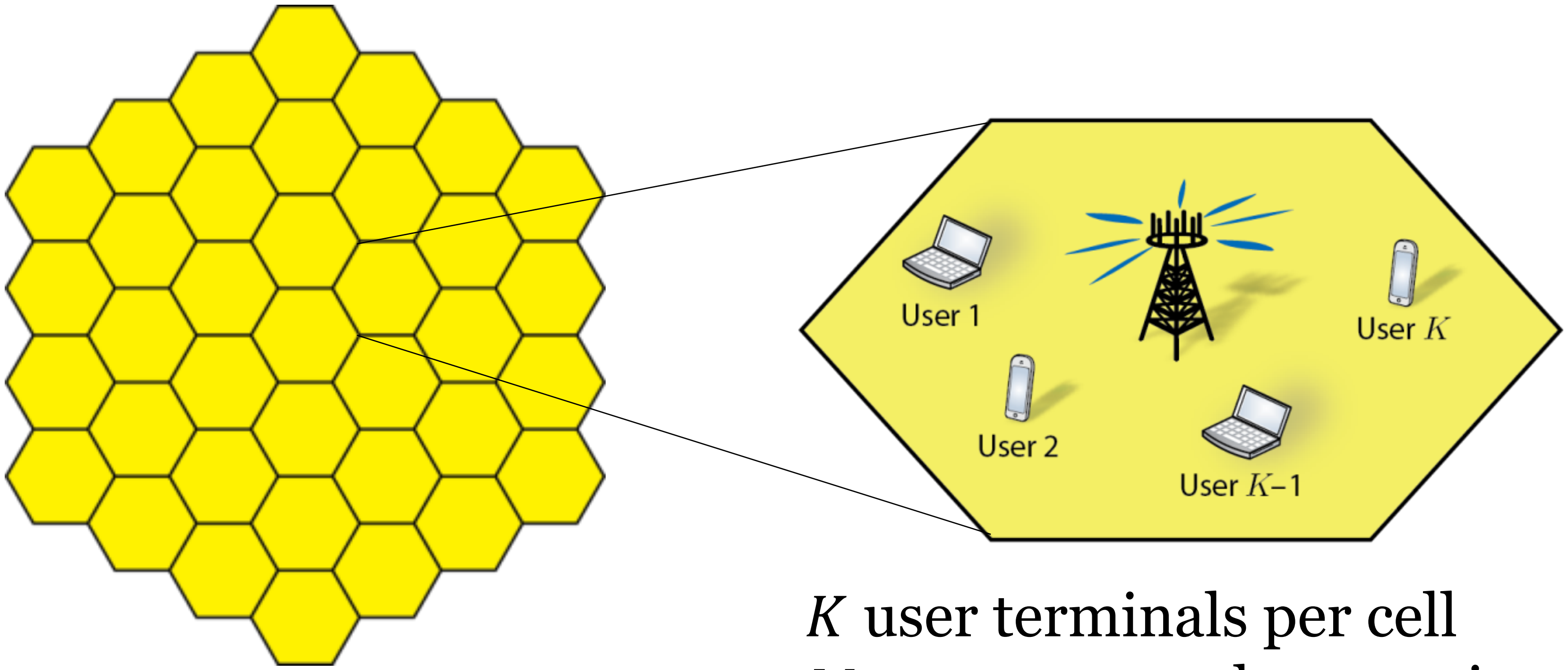
$$\left(1-\frac{\tau_p}{\tau_c}\right)\log_2\left(1+\frac{M\rho_{ul}\eta_k\gamma_k}{\sum_{i=1}^{K}\rho_{ul}\eta_i\beta_i+1}\right)$$

Downlink:

$$\left(1-\frac{\tau_p}{\tau_c}\right)\log_2\left(1+\frac{M\rho_{dl}\eta_k\gamma_k}{\beta_k\sum_{i=1}^{K}\rho_{dl}\eta_i+1}\right)$$

Cellular networks

- Canonical hexagonal model:



K user terminals per cell
M antennas per base station

Multi-cell propagation model

Notation:

$$g^{\text{BS index,BS antenna}}_{\text{User cell,user index}}$$

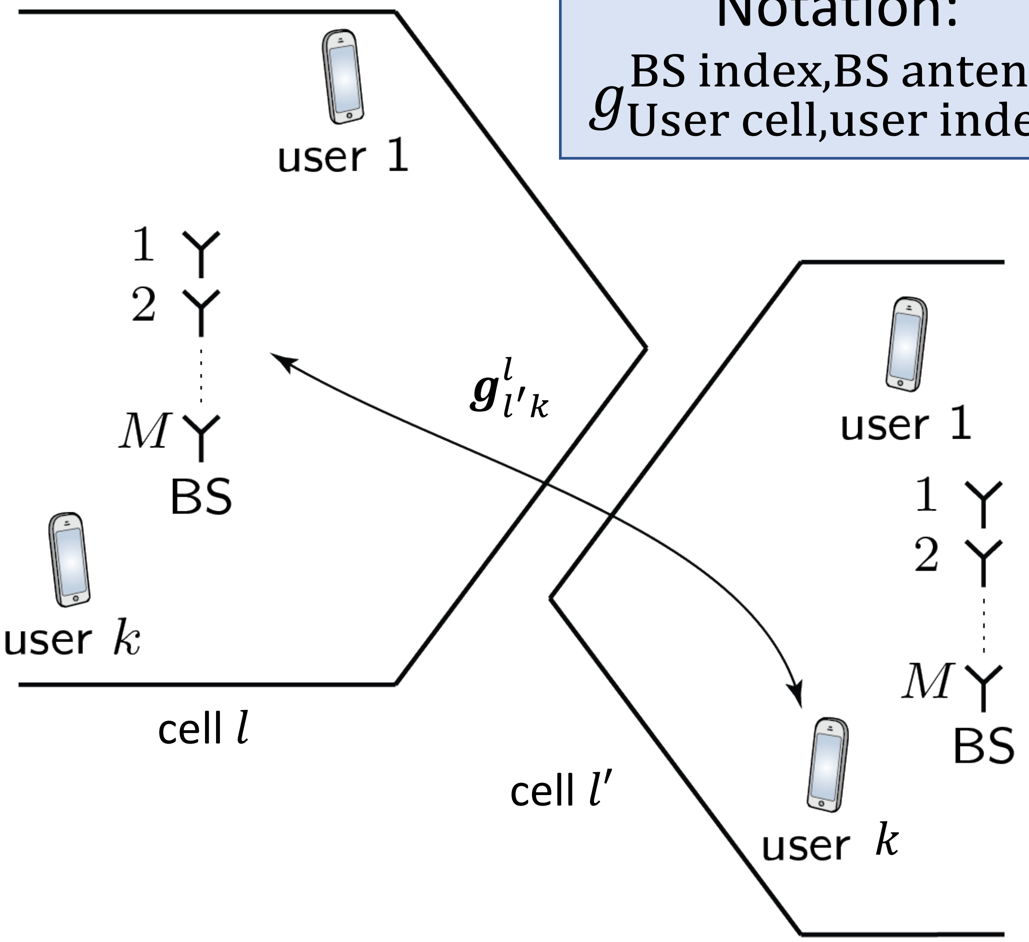
- L cells

- Channel from BS l to user k in cell l':

$$\boldsymbol{g}^{l}_{l'k} = \left[g^{l1}_{l'k} \quad \dots \quad g^{lM}_{l'k}\right]^T$$

- Rayleigh fading:

$$\boldsymbol{g}^{l}_{l'k} \sim CN\left(\boldsymbol{0}, \beta^{l}_{l'k}\boldsymbol{I}_M\right)$$



Uplink multi-cell MIMO model

- Received signal at the BS in cell l:

$$\boldsymbol{y}_l = \sqrt{\rho_{ul}}\boldsymbol{G}_l^l \boldsymbol{x}_l + \sqrt{\rho_{ul}} \sum_{l'=1, l' \neq l}^{L} \boldsymbol{G}_{l'}^l \boldsymbol{x}_{l'} + \boldsymbol{w}_l$$

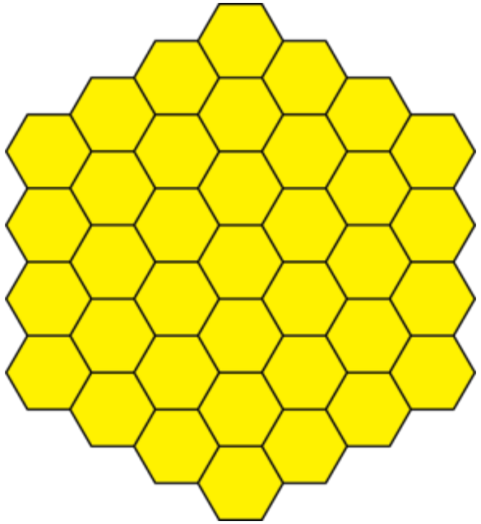
where

$$\boldsymbol{y}_l = \begin{pmatrix} y_{l1} \\ \vdots \\ y_{lM} \end{pmatrix}, \qquad \boldsymbol{G}_{l'}^l = \begin{pmatrix} g_{l'1}^{l1} & \cdots & g_{l'K}^{l1} \\ \vdots & \ddots & \vdots \\ g_{l'1}^{lM} & \cdots & g_{l'K}^{lM} \end{pmatrix}, \qquad \boldsymbol{x}_l = \begin{pmatrix} x_{l1} \\ \vdots \\ x_{lK} \end{pmatrix}, \qquad \boldsymbol{w}_l = \begin{pmatrix} w_{l1} \\ \vdots \\ w_{lM} \end{pmatrix}$$

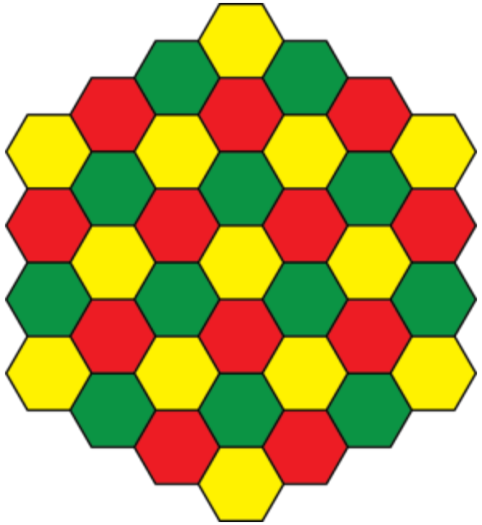
- Each user's signal is power-limited as $\mathbb{E}\{|x_{lk}|^2\} \leq 1$
- Normalized noise: $\boldsymbol{w}_l \sim CN(\boldsymbol{0}, \boldsymbol{I}_M)$

Examples of pilot reuse

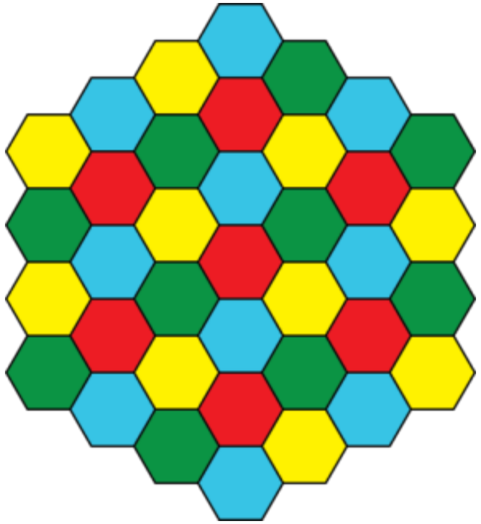
- Divide cells into n_{reuse} clusters
  - Use $K \cdot n_{reuse}$ pilots, same K in each cell of a cluster



Pilot reuse
$n_{reuse} = 1$



Pilot reuse
$n_{reuse} = 3$

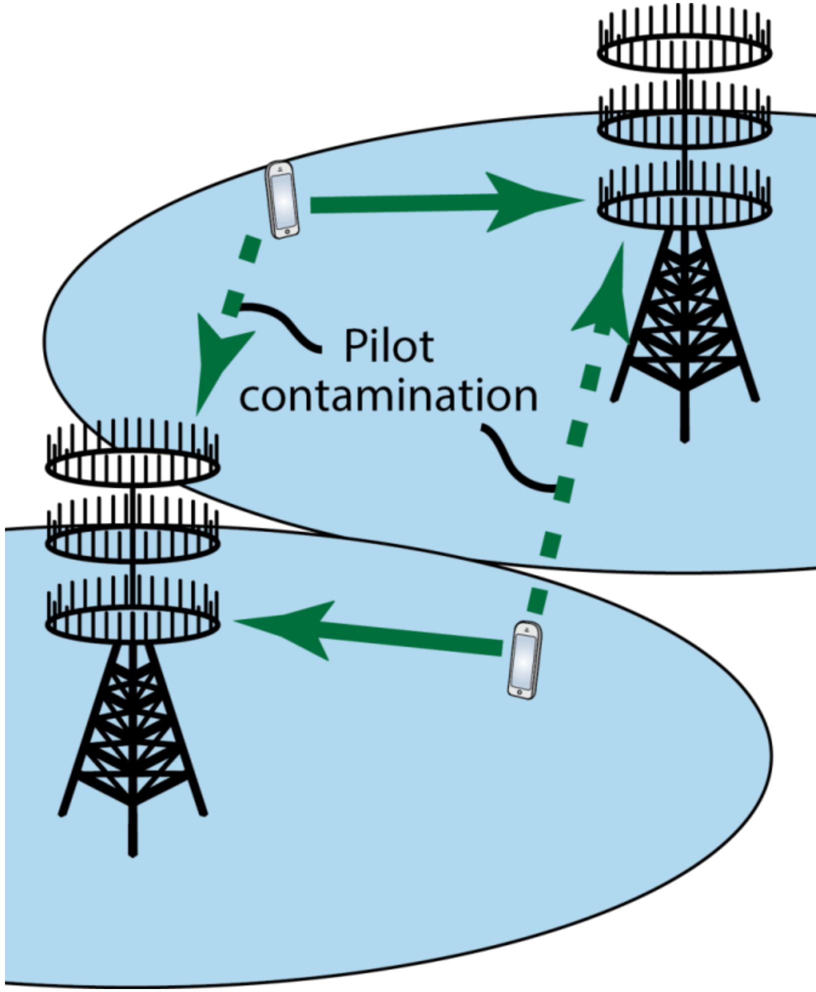


Pilot reuse
$n_{reuse} = 4$

Impact of pilot reuse

- Same pilot sequence sent by multiple users
  - Creates interference
  - Called: “pilot contamination”

- Contaminating cells
  - $\mathcal{P}_l$: Set of cell using same pilots as cell l (including itself)



Estimating Gaussian variable in noise

- Consider $y = \sqrt{p}g + w$ where
  - p is a constant, $g \sim CN(0, \beta)$, $w \sim CN(0, \sigma^2)$

Mean squared error:
$E\{|\hat{g} - g|^2\}$

Minimum mean squared error (MMSE) estimator:
$$\hat{g} = E\{g|y\} = \frac{\sqrt{p}\beta}{\sigma^2 + p\beta} y$$

Estimation error: $\tilde{g} = \hat{g} - g \sim CN\left(0, \beta - \frac{p\beta^2}{\sigma^2 + p\beta}\right)$

Estimate: $\hat{g} \sim CN\left(0, \frac{p\beta^2}{\sigma^2 + p\beta}\right)$

Independent random variables

MMSE estimates of channels in cellular networks

- Received pilot signal (after despreading):

$$\boldsymbol{Y}'_{pl} = \sqrt{\tau_p \rho_{ul}} \sum_{l'' \in \mathcal{P}_l} \boldsymbol{G}^l_{l''} + \boldsymbol{W}'_{pl}$$

- Estimate of $g^{lm}_{l'k}$ from user k in cell l' to antenna m at BS l
    - Estimate:

$$\hat{g}^{lm}_{l'k} = E\{g^{lm}_{l'k} | \boldsymbol{Y}'_{pl}\} = \frac{\sqrt{\tau_p \rho_{ul}} \beta^l_{l'k}}{1 + \tau_p \rho_{ul} \sum_{l'' \in \mathcal{P}_l} \beta^l_{l''k}} \left[\boldsymbol{Y}'_{pl}\right]_{mk} \sim CN(0, \gamma^l_{l'k})$$

with $\tilde{g}^{lm}_{l'k} = \hat{g}^{lm}_{l'k} - g^{lm}_{l'k} \sim CN\left(0, \beta^l_{l'k} - \gamma^l_{l'k}\right)$ and

$$\gamma^l_{l'k} = \frac{\tau_p \rho_{ul} \left(\beta^l_{l'k}\right)^2}{1 + \tau_p \rho_{ul} \sum_{l'' \in \mathcal{P}_l} \beta^l_{l''k}}$$

Vector notation:

$$\hat{\boldsymbol{g}}^l_{l'k} = \begin{bmatrix} \hat{g}^{l1}_{l'k} \\ \vdots \\ \hat{g}^{lM}_{l'k} \end{bmatrix}$$

Pilot contamination

- Two consequences:
  - Lower estimation quality:

$$\gamma_{l'k}^{l} = \frac{\tau_p \rho_{ul} \left(\beta_{l'k}^{l}\right)^2}{1+\tau_p \rho_{ul} \sum_{l'' \in \mathcal{P}_l} \beta_{l''k}^{l}} < \frac{\tau_p \rho_{ul} \left(\beta_{l'k}^{l}\right)^2}{1+\tau_p \rho_{ul} \beta_{l'k}^{l}}$$

Without inter-cell interference

  - Correlated channel estimates
    $\hat{g}_{lk}^{lm}$ correlated with $g_{l'k}^{lm}$ for all $l' \in \mathcal{P}_l$

$$\sum_{m=1}^{M} E\left\{\hat{g}_{lk}^{lm}\left(g_{l'k}^{lm}\right)^*\right\} = \begin{cases} 0 \text{ if } l' \notin \mathcal{P}_l \\ \text{Proportional to } M \text{ if } l' \in \mathcal{P}_l \end{cases}$$

Linear receiver processing

- Received signal at the BS in cell l:

$$\boldsymbol{y}_l = \sqrt{\rho_{ul}}\boldsymbol{G}_l^l\boldsymbol{x}_l + \sqrt{\rho_{ul}} \sum_{l'=1, l'\neq l}^{L} \boldsymbol{G}_{l'}^l \boldsymbol{x}_{l'} + \boldsymbol{w}_l$$

where $\boldsymbol{x}_l = \boldsymbol{D}_{\boldsymbol{\eta}_l}^{1/2}\boldsymbol{q}_l$

$$\boldsymbol{D}_{\boldsymbol{\eta}_l} = \begin{pmatrix} \eta_{l1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \eta_{lK} \end{pmatrix} \quad \boldsymbol{q}_l = \begin{pmatrix} q_{l1} \\ \vdots \\ q_{lK} \end{pmatrix}$$

Data signals

- Assign receiver filter $\boldsymbol{a}_{lk}$ for user k in cell l
  - Select it to make $\boldsymbol{a}_{lk}^H\boldsymbol{y}_l \approx q_{lk}$

MR processing:
$\boldsymbol{a}_{lk} = \widehat{\boldsymbol{g}}_{lk}^l$

Uplink capacity lower bound with MR

$$\log_2\left(1 + \frac{\overbrace{M\rho_{ul}\eta_{lk}\gamma_{lk}^{l}}^{\text{Desired signal (coherent)}}}{\underbrace{\rho_{ul}\sum_{l'=1}^{L}\sum_{k'=1}^{K}\eta_{l'k'}\beta_{l'k'}^{l}}_{\text{Non-coherent interference from all users}} + \underbrace{M\rho_{ul}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\eta_{l'k}\gamma_{l'k}^{l}}_{\text{Coherent interference from pilot-sharing users}} + \underbrace{1}_{\text{Noise}}}\right)$$

- Comments
  - Derived in same way as in single-cell case
  - **New term**: Coherent interference

Downlink multi-cell MIMO model

- Received signal at users in cell l:

$$\boldsymbol{y}_l = \sqrt{\rho_{dl}}\left(\boldsymbol{G}_l^l\right)^T \boldsymbol{x}_l + \sqrt{\rho_{dl}} \sum_{l'=1, l' \neq l}^{L} \left(\boldsymbol{G}_l^{l'}\right)^T \boldsymbol{x}_{l'} + \boldsymbol{w}_l$$

where

$$\boldsymbol{y}_l = \begin{pmatrix} y_{l1} \\ \vdots \\ y_{lK} \end{pmatrix} \quad \boldsymbol{G}_{l'}^{l} = \begin{pmatrix} g_{l'1}^{l1} & \cdots & g_{l'K}^{l1} \\ \vdots & \ddots & \vdots \\ g_{l'1}^{lM} & \cdots & g_{l'K}^{lM} \end{pmatrix} \quad \boldsymbol{x}_l = \begin{pmatrix} x_{l1} \\ \vdots \\ x_{lM} \end{pmatrix} \quad \boldsymbol{w}_l = \begin{pmatrix} w_{l1} \\ \vdots \\ w_{lK} \end{pmatrix}$$

- Maximum power is ρ_{dl}, $\mathrm{E}\{\|\boldsymbol{x}_l\|^2\} \leq 1$
- Normalized noise: $\boldsymbol{w}_l \sim CN(\boldsymbol{0}, \boldsymbol{I}_K)$

MR precoding:

$$\boldsymbol{x}_l = \sum_{k=1}^{K} \frac{\widehat{\boldsymbol{g}}_{lk}^l}{\sqrt{E\left\{\left\|\widehat{\boldsymbol{g}}_{lk}^l\right\|^2\right\}}} \sqrt{\eta_{lk}} q_{lk}$$

Downlink capacity lower bound with MR

$$\log_2\left(1+\frac{\overbrace{M\rho_{dl}\eta_{lk}\gamma_{lk}^{l}}^{\text{Desired signal (coherent)}}}{\underbrace{\rho_{dl}\sum_{l'=1}^{L}\beta_{lk}^{l'}\sum_{k'=1}^{K}\eta_{l'k'}}_{\substack{\text{Non-coherent interference}\\ \text{from all users}}}+\underbrace{M\rho_{dl}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\eta_{l'k}\gamma_{lk}^{l'}}_{\substack{\text{Coherent interference}\\ \text{from pilot-sharing users}}}+\underset{\text{Noise}}{1}}\right)$$

- Comments
  - Derived in same way as in single-cell case
  - **New term**: Coherent interference

Comparing uplink and downlink

- Non-coherent interference
  - Uplink: $\rho_{ul} \sum_{l'=1}^{L} \sum_{k'=1}^{K} \eta_{l'k'} \beta_{l'k'}^{l}$
  - Downlink: $\rho_{dl} \sum_{l'=1}^{L} \beta_{lk}^{l'} \sum_{k'=1}^{K} \eta_{l'k'}$

  - Differences:

$$\rho_{ul} \leftrightarrow \rho_{dl}$$
$$\beta_{l'k'}^{l} \leftrightarrow \beta_{lk}^{l'}$$

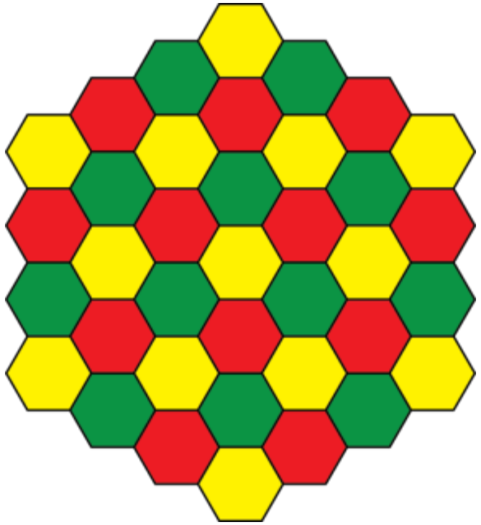
Uplink: Interference comes from each user
Downlink: Interference comes from each base station

Uplink asymptotic limit

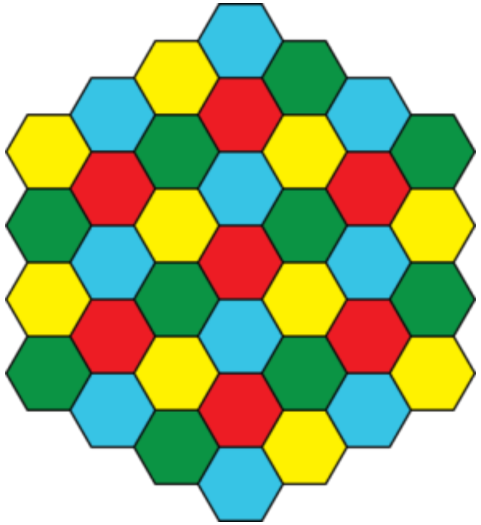
- As $M \to \infty$: Capacity lower bound has a finite limit:

$$\log_2\left(1+\frac{M\rho_{ul}\eta_{lk}\gamma_{lk}^{l}}{\rho_{ul}\sum_{l'=1}^{L}\sum_{k'=1}^{K}\eta_{l'k'}\beta_{l'k'}^{l}+M\rho_{ul}\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\eta_{l'k}\gamma_{l'k}^{l}+1}\right)\to\log_2\left(1+\frac{\eta_{lk}\left(\beta_{lk}^{l}\right)^2}{\sum_{l'\in\mathcal{P}_l\setminus\{l\}}\eta_{l'k}\left(\beta_{l'k}^{l}\right)^2}\right)$$

Larger if contaminating cells are further away: when n_{reuse} is larger



$n_{reuse} = 3$



$n_{reuse} = 4$

Summary

- Massive MIMO in cellular networks
  - Similar capacity bounds as in a single cell, but more complicated notation

- New phenomenon: Pilot contamination
  - Reduces estimation quality
  - Causes coherent interference

End of Lecture 10

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