TSKS14

# Multiple Antenna Communications

Lecture 6, 2020

Emil Björnson

LINKÖPING UNIVERSITY

# Outline of this lecture

- Channel coherence intervals
    - Coherence time and coherence bandwidth
- Massive MIMO
    - Motivation and basic properties
    - Duplexing modes
    - Uplink system model

# Linear time-invariant channels

$$x(t) \longrightarrow \boxed{\text{Channel}} \longrightarrow y(\mathrm{t})$$

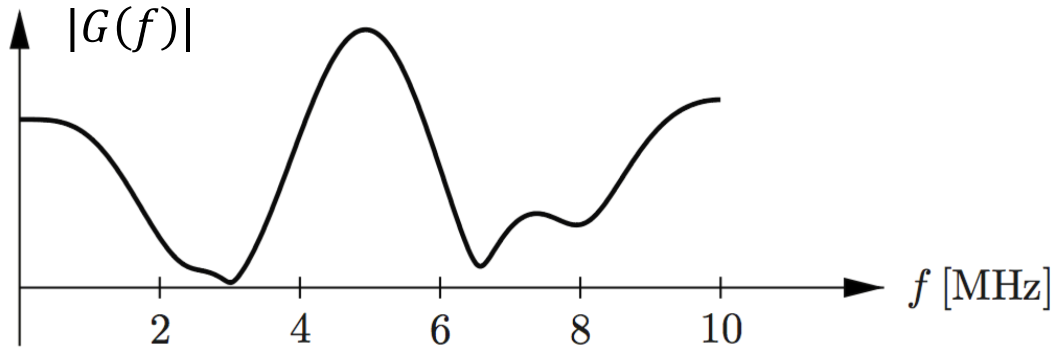
- Is the channel a linear time-invariant (LTI) filter?
- Linearity due to Maxwell's equations

- Coherence time $T_c$
  - Time that channel is *approximately* time-invariant
  - Simple model: $T_c = \lambda/(2v)$ or $T_c = \lambda/(4v)$ seconds

Proportional to wavelength, inversely to speed $v$

# Channel dispersion

$$x(t) \longrightarrow \boxed{\text{Channel}} \longrightarrow y(t)$$




- Is the channel dispersive?

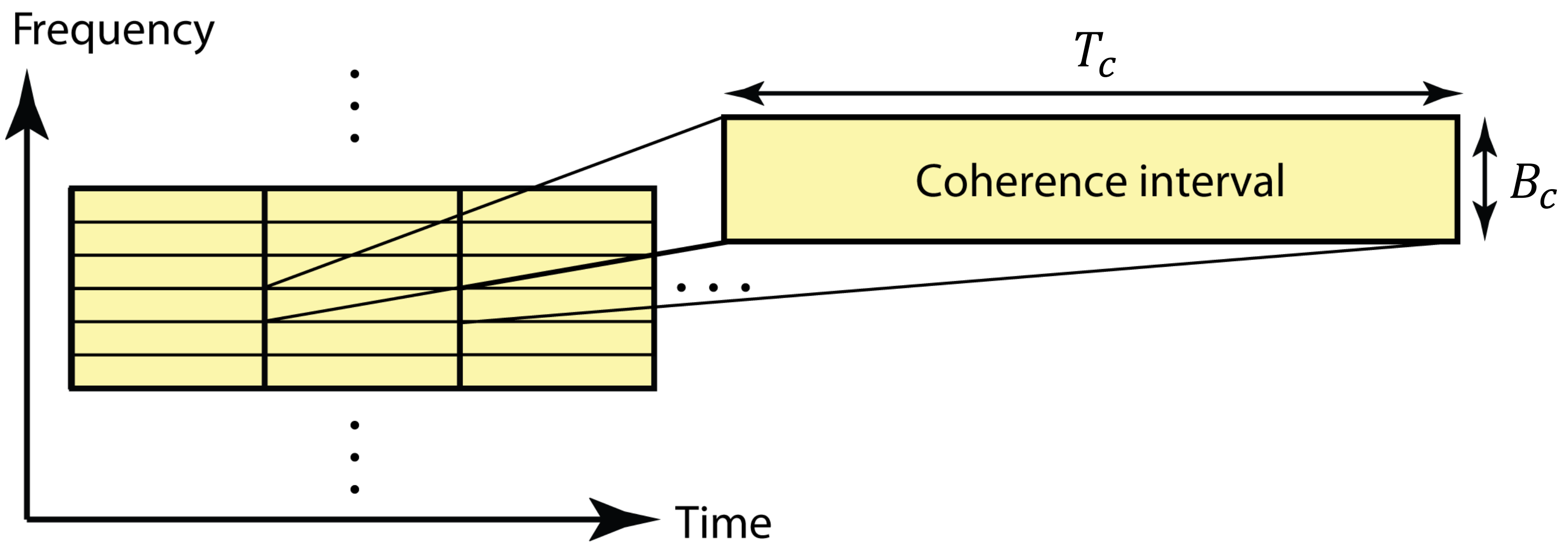
- Coherence bandwidth $B_c$
  - Bandwidth over which frequency response $G(f) \approx g$ is almost constant:

$$g(t) = g \cdot \delta(t)$$

  - Simple model: $B_c = c/|d_{\max} - d_{\min}|$ or $B_c = c/(2|d_{\max} - d_{\min}|)$ Hz

Inversely proportional to path length difference

# Coherence interval (block fading)




- Divide bandwidth and time resources into coherence intervals
  - According to sampling theorem:
    $$\tau_c = B_c T_c \text{ complex samples}$$
  - Channel time-invariant and described by a scalar

This is an example of fast fading

# How large is a coherence interval?

- **Example:** Support vehicular speed in suburban area:

$$v = 30\text{ m/s} = 108\text{ km/h}$$
$$|d_{\max} - d_{\min}| = 1000\text{ m}$$

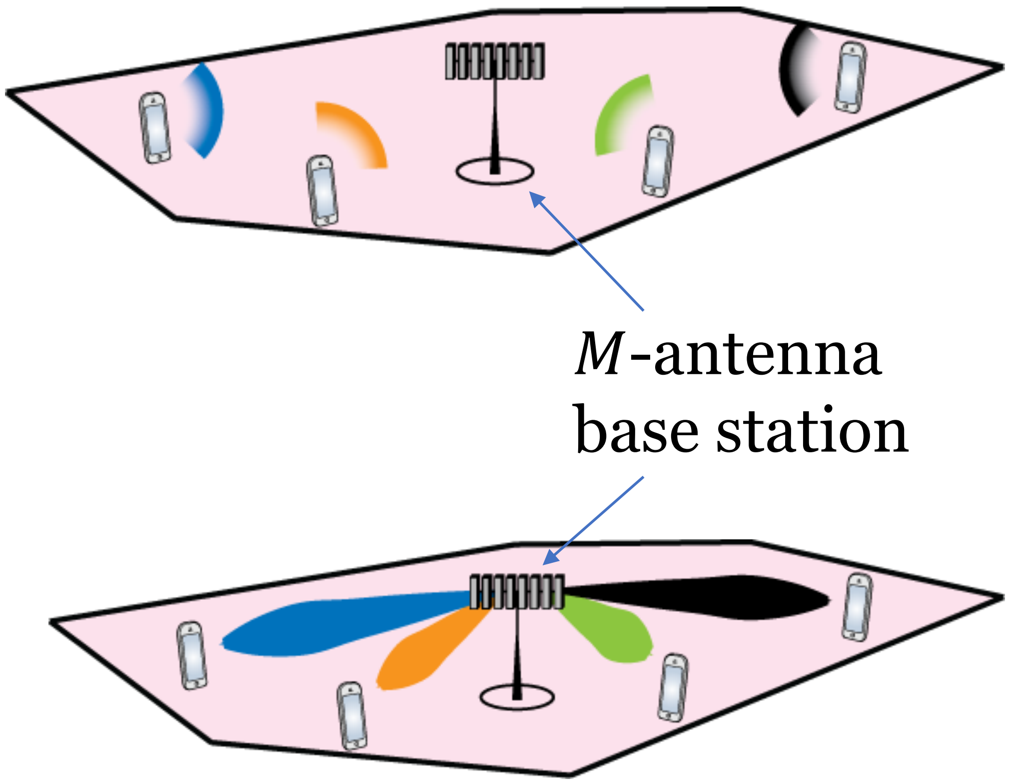
- Typical carrier frequency: $f = 2\text{ GHz}, \lambda = 15\text{ cm}$
  - Coherence time: $T_c = \frac{\lambda}{2v} = \frac{0.15}{2 \cdot 30} = 2.5\text{ ms}$
  - Coherence bandwidth: $B_c = \frac{c}{|d_{\max} - d_{\min}|} = \frac{3 \cdot 10^8}{1000} = 300\text{ kHz}$

**Coherence interval:**
$\tau_c = 750$ complex samples

# Recall: Multiuser MIMO Communication

- Uplink
  - From $K$ users to base station
  - Multipoint-to-point MIMO

- Downlink
  - From base station to $K$ users
  - Point-to-multipoint MIMO



# Multiuser MIMO vs. Massive MIMO

- Conventional multiuser MIMO
  - $M \leq 8, \ K \leq 4$
  - Used in LTE and WiFi
  - Seldom reaches the $\min(M, K) = K$ capacity gain

- Massive MIMO
  - $M \approx 100, \ K \approx 10$ (or more)
  - More directive signals:
    Less randomness, larger beamforming gain, less interference

# Motivation for Massive MIMO

- Recall: Sum Capacity with $K = 2$:

$$R_1 + R_2 = \log_2(\det(\boldsymbol{I}_M + \rho_{ul}\boldsymbol{G}\boldsymbol{G}^H)) = \log_2(\det(\boldsymbol{I}_2 + \rho_{ul}\boldsymbol{G}^H\boldsymbol{G}))$$

- If $\boldsymbol{G} = [\boldsymbol{g}_1\ \boldsymbol{g}_2]$: $\boldsymbol{G}^H\boldsymbol{G} = \begin{bmatrix} \|\boldsymbol{g}_1\|^2 & \boldsymbol{g}_1^H\boldsymbol{g}_2 \\ \boldsymbol{g}_2^H\boldsymbol{g}_1 & \|\boldsymbol{g}_2\|^2 \end{bmatrix}$

- Expanding sum capacity:

$$\log_2(\det(\boldsymbol{I}_2 + \rho_{ul}\boldsymbol{G}^H\boldsymbol{G})) = \log_2\left((1 + \rho_{ul}\|\boldsymbol{g}_1\|^2)(1 + \rho_{ul}\|\boldsymbol{g}_2\|^2) - \rho_{ul}^2\left|\boldsymbol{g}_1^H\boldsymbol{g}_2\right|^2\right)$$

$$\leq \log_2(1 + \rho_{ul}\|\boldsymbol{g}_1\|^2) + \log_2(1 + \rho_{ul}\|\boldsymbol{g}_2\|^2)$$

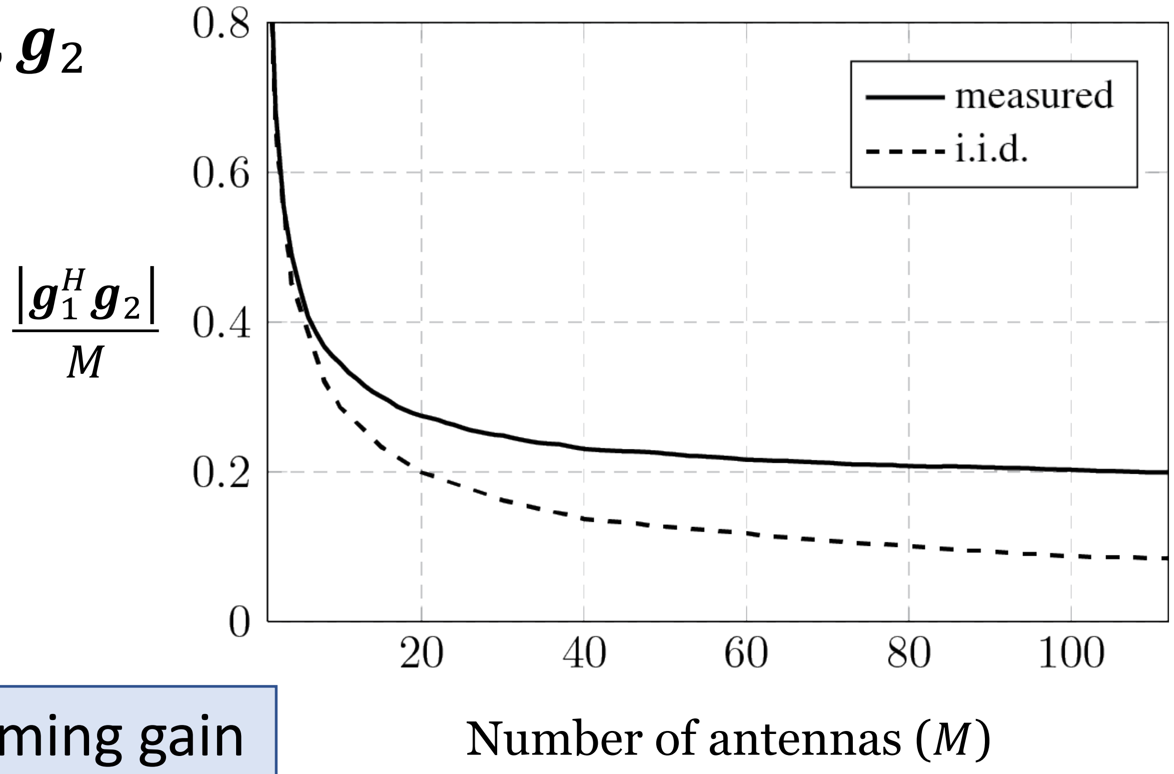
Equality if and only if $\boldsymbol{g}_1^H\boldsymbol{g}_2 = 0$

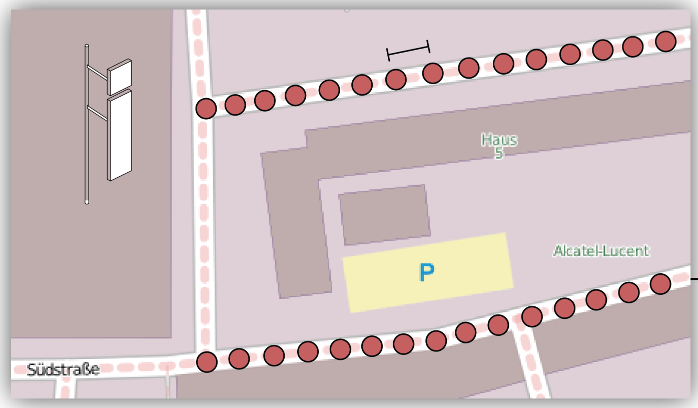
# Motivation for Massive MIMO: Favorable propagation

- Consider two $M$-antenna channels $\boldsymbol{g}_1, \boldsymbol{g}_2$

Inner product $\frac{|\boldsymbol{g}_1^H \boldsymbol{g}_2|}{M}$ converges to zero as $M \to \infty$

**Less interference**




Related to beamforming gain and beamwidth

Reference: J. Hoydis, C. Hoek, T. Wild, and S. ten Brink, “Channel Measurements for Large Antenna Arrays,” ISWCS 2012

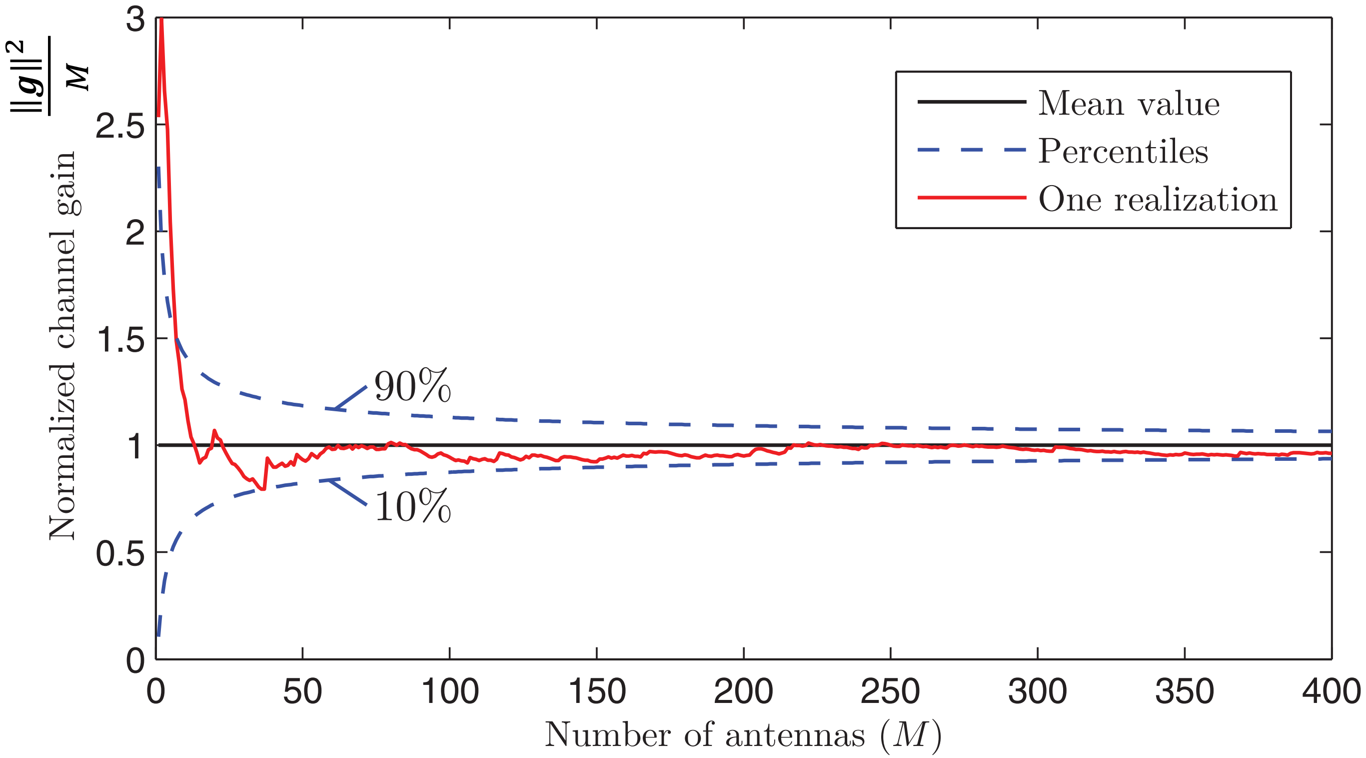
# Motivation for Massive MIMO: Channel hardening

- Consider an $M$-antenna channel $\boldsymbol{g} \sim CN(\mathbf{0}, \boldsymbol{I}_M)$

$$\frac{1}{M}\|\boldsymbol{g}\|^2 \text{ has } \begin{cases} \text{Mean: } 1 \\ \text{Variance: } 1/M \end{cases}$$

Consequence of spatial diversity:
$\|\boldsymbol{g}\|^2 \approx \mathrm{E}\{\|\boldsymbol{g}\|^2\}$
when $M$ is large

Consequence of beamforming gain:
$\|\boldsymbol{g}\|^2 \approx M$
when $M$ is large



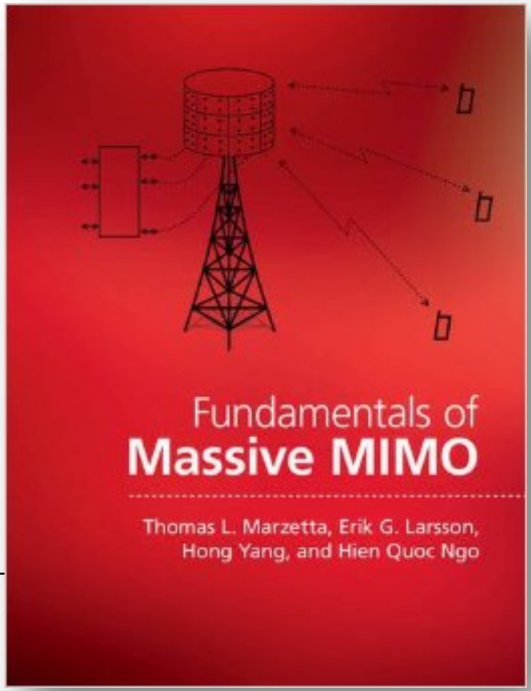
# Background of Massive MIMO

- Proposed by
  Thomas L. Marzetta
  - Awarded *honorary doctor*
    at Linköping University, 2015

- First paper:

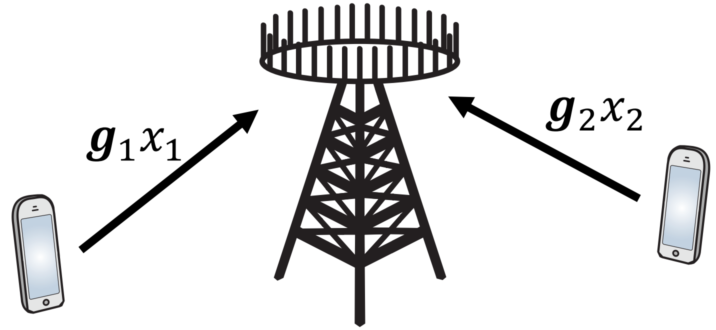
  "*Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas*," IEEE Trans. Wireless Communications, 2010.

Now a main component of 5G



# Marzetta’s asymptotic motivation

- Example: Uplink with i.i.d. Rayleigh fading
  - Two users, send signals $x_k$ for $k = 1,2$
  - Channels: $\boldsymbol{g}_k = [g_k^1 \dots g_k^M]^T \sim CN(\mathbf{0}, \boldsymbol{I}_M)$
  - Noise: $\boldsymbol{w} \sim CN(\mathbf{0}, \boldsymbol{I}_M)$
  - Received: $\mathbf{y} = \boldsymbol{g}_1 x_1 + \boldsymbol{g}_2 x_2 + \boldsymbol{w}$

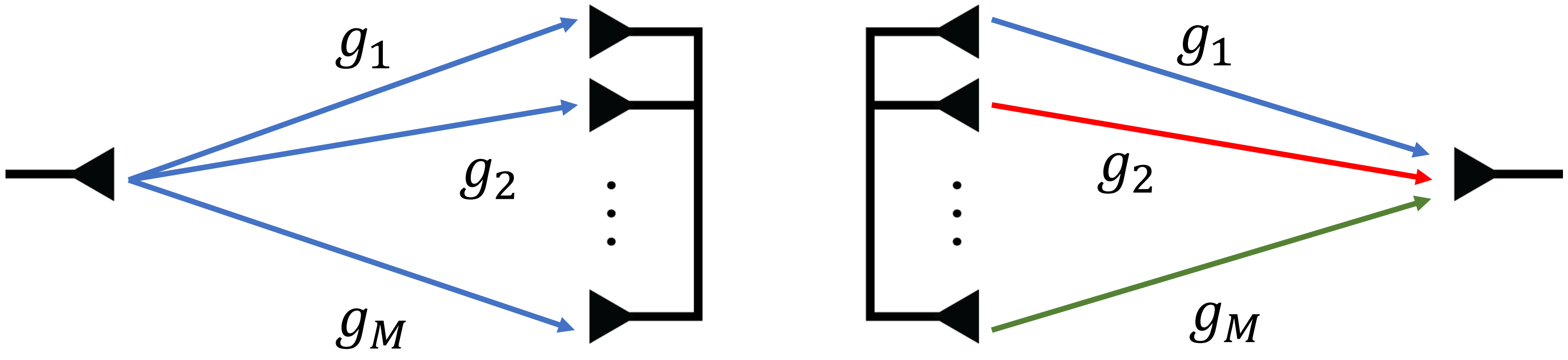



- Linear detector $\boldsymbol{a}_1 = \frac{1}{M}\boldsymbol{g}_1$ for User 1: $\tilde{y}_1 = \boldsymbol{a}_1^H \mathbf{y} = \boxed{\boldsymbol{a}_1^H \boldsymbol{g}_1} x_1 + \boxed{\boldsymbol{a}_1^H \boldsymbol{g}_2} x_2 + \boxed{\boldsymbol{a}_1^H \boldsymbol{w}}$
  - Signal remains: $\boldsymbol{a}_1^H \boldsymbol{g}_1 = \frac{1}{M}\|\boldsymbol{g}_1\|^2 \xrightarrow{M\to\infty} \mathrm{E}[|g_1^1|^2] = 1$
  - Interference vanishes: $\boldsymbol{a}_1^H \boldsymbol{g}_2 = \frac{1}{M}\boldsymbol{g}_1^H \boldsymbol{g}_2 \xrightarrow{M\to\infty} \mathrm{E}[g_1^{1*} g_2^1] = 0$
  - Noise vanishes: $\boldsymbol{a}_1^H \boldsymbol{w} = \frac{1}{M}\boldsymbol{g}_1^H \boldsymbol{w} \xrightarrow{M\to\infty} \mathrm{E}[g_1^{1*} w_1] = 0$

Asymptotically noise/interference-free communication: $\tilde{y}_1 \xrightarrow{M\to\infty} x_1$

# Estimating the channel responses

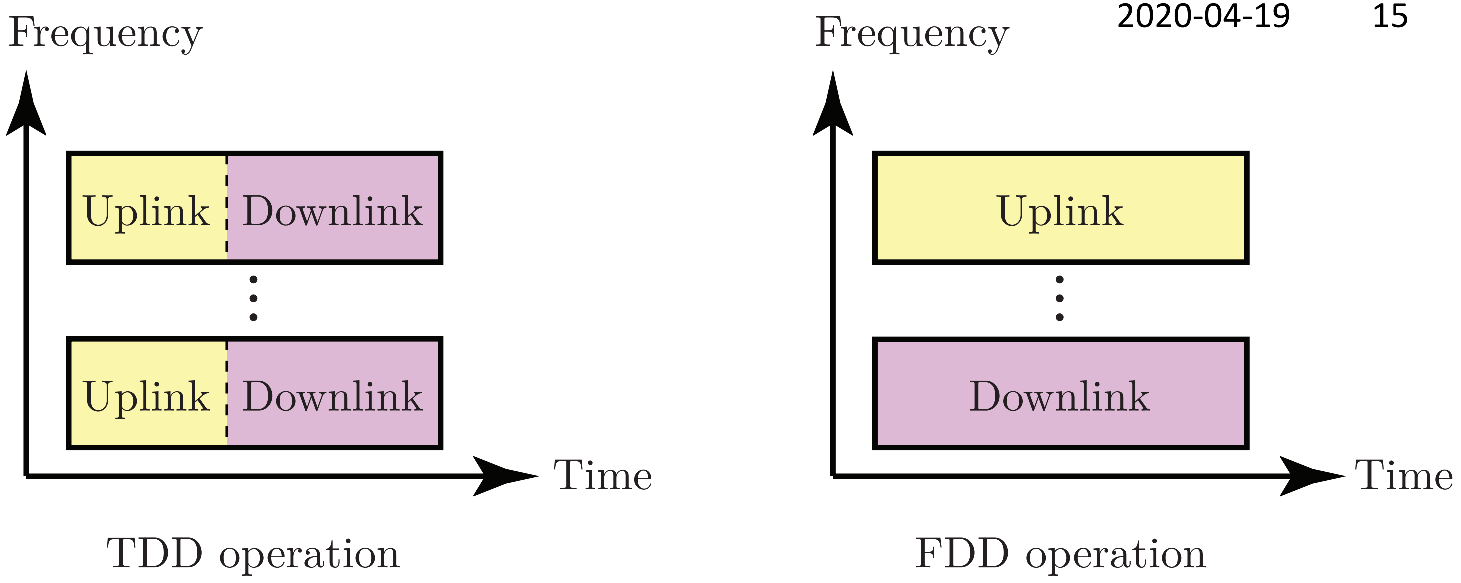
- Channels: $K$ users with $M$-length channels
  - Estimate $MK$ coefficients in each coherence interval
- Basic principle: Send known *pilot* signal




One pilot: Estimate all coefficients

$M$ pilots to estimate all coefficients

# Duplexing




- Time-division duplex (TDD): Separate uplink and downlink in time
  - $K$ pilots are needed
- Frequency-division duplex (FDD): Separate uplink and downlink in frequency
  - $M$ pilots are needed

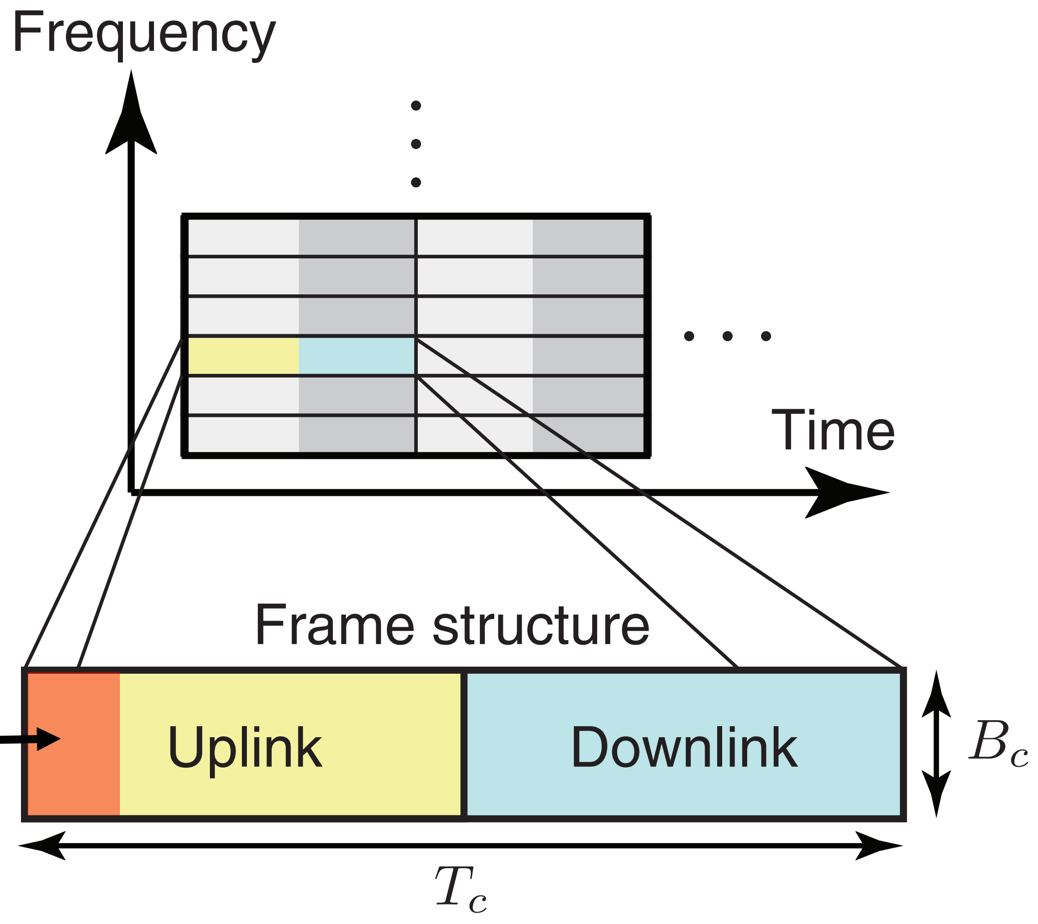
- Example: $M = 100, K = 10$, $\tau_c = 200$

**TDD operation is key!**

# Operating a TDD Massive MIMO System

- Frames matched to coherence intervals
  - Fixed single-tap channel responses
  - Coherence time: $T_c$ s
  - Coherence bandwidth: $B_c$ Hz
  - Interval contains $\tau_c = T_c B_c$ symbols

**Uplink pilots**:
Enable channel estimation



# Uplink Massive MIMO system model

- Received signal:

$$\boldsymbol{y} = \sqrt{\rho_{ul}}\boldsymbol{G}\boldsymbol{x} + \boldsymbol{w}$$

where

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_M \end{pmatrix} \quad \boldsymbol{G} = \begin{pmatrix} g_1^1 & \cdots & g_K^1 \\ \vdots & \ddots & \vdots \\ g_1^M & \cdots & g_K^M \end{pmatrix} \quad \boldsymbol{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_K \end{pmatrix} \quad \boldsymbol{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_M \end{pmatrix}$$

  - Parameters are normalized: Maximum power is $\rho_{ul}$
    $x_1, \dots, x_K$ has power $\leq 1$
  - Channel of user $k$: $g_k^1, \dots, g_k^M \sim CN(0, \beta_k)$
  - Normalized noise: $\boldsymbol{w} \sim CN(\boldsymbol{0}, \boldsymbol{I}_M)$

Large-scale fading coefficient

# Modeling of power and large-scale fading coefficients

- Maximum SNR of user $k$ is $\rho_{ul}\beta_k$

- How to model $\rho_{ul}$?

$$\rho_{ul} = \frac{\text{Uplink radiated power} \cdot \text{Antenna gains}}{BN_0}$$

$N_0 = 10^{-17}$

- How to model $\beta_k$?
  - Example: 3GPP-type model at distance $d_k$:

$$\beta_k = 10^{-1.53}\left(\frac{d_k}{1\text{ m}}\right)^{-3.76} \quad \text{for } d_k \geq 35\text{ m}$$

**Typical values:**
$B = 10$ MHz
Radiated power: 100 mW
Antenna gains = 0 dBi

**Typical values:**
$d_k = 35$ m: $\beta_k = -73$ dB
$d_k = 1$ km: $\beta_k = -128$ dB

# Summary

- Massive MIMO is multi-user MIMO with many antennas and users
  - No strict definition exists

- TDD operation
  - Divide time-frequency resources into frames
  - Match frame size to coherence intervals
  - Send pilots in uplink for channel estimation
  - Switch between uplink/downlink data in same coherence interval

End of Lecture 6

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