Forward Kinematics Assignment

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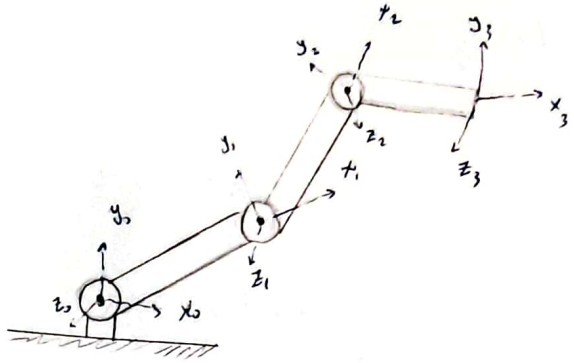
3.4 Problems:

Q(2)



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	0	0	θ_3^*

a_i : Link length

α_i : Link twist

d_i : Link offset

θ_i : joint angle

$$A_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & a_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & a_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$* H = A_1 A_2 A_3 = {}^0T_1\ {}^1T_2\ {}^2T_3$$

$$A_1A_2 = \begin{bmatrix} (\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) & (-\cos\theta_2\sin\theta_2 - \sin\theta_1\cos\theta_2) & 0 & a_2\cos\theta_1\cos\theta_2 + a_2\sin\theta_1\sin\theta_2 + a_1\cos\theta_1 \\ (\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2) & (-\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2) & 0 & a_2\sin\theta_1\sin\theta_2 + a_2\cos\theta_1\cos\theta_2 + a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

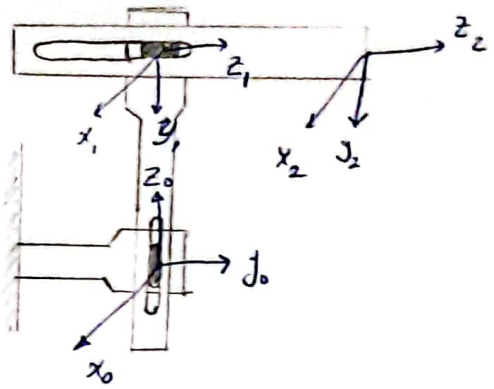
$$A_1A_2 = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & -\cos(\theta_1+\theta_2) & 0 & a_2\sin\theta_2 + a_2\cos(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = A_1 A_2 A_3$$

$$= \begin{bmatrix} c_1 c_2 c_3 - s_1 s_2 - s_3 c_1 c_2 & c_1 c_2 c_3 - s_1 s_2 - s_3 c_1 c_2 & 0 & q_3 c_1 c_2 c_3 \\ \quad + c_1 s_1 & \quad + c_2 s_1 & & - q_2 s_1 s_2 \\ & & & + q_2 c_1 c_2 \\ & & & - s_1 s_2 \\ & & & - q_3 s_3 c_1 c_2 \\ c_3 c_1 s_2 + c_2 s_1 + s_3 c_1 c_2 & c_1 c_2 c_3 - s_1 s_2 - s_3 c_1 s_2 & 0 & a_1 s_1 + a_2 c_1 s_2 \\ \quad - s_1 s_2 & \quad + s_1 c_2 & & + a_2 s_1 c_2 + \\ & & & a_3 s_3 c_1 c_2 + s_1 c_2 \\ 0 & 0 & 1 & + a_3 - c_1 c_2 \\ 0 & & & 0 \\ & 0 & 0 & 0 \end{bmatrix}$$

(3)

Link	a_i	α_i	d_i	θ_i
1	0	-90	d_1 ①	0
2	0	0	d_2 ②	0

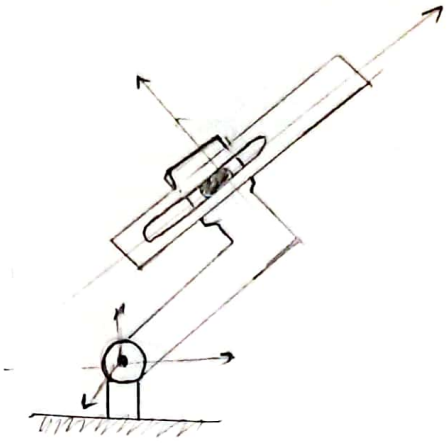


$$A_1 = {}^0T_1 = \begin{bmatrix} C\theta_1 & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1 = A_1 A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q(4)



Link	a	α	d	θ
1	0	-90	0	θ_1
2	0	0	d_2	0

$$ {}^{0}T_1 = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

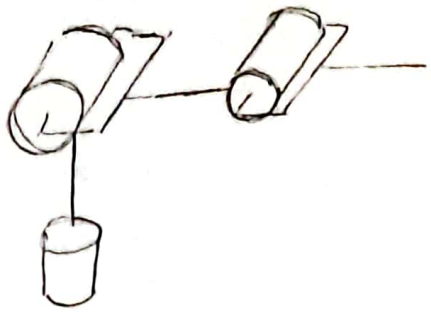
$$ {}^{0}T_1 = \begin{bmatrix} C\theta_1 & 0 & -\sin\theta_1 & a_1 C\theta_1 \\ S\theta_1 & 0 & \cos\theta_1 & a_2 \sin\theta_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ T = {}^{0}T_1 * {}^{2}T_1 = \begin{bmatrix} C_1 & 0 & S_1 & d_2 S_1 + q_1 C_1 \\ S_1 & 0 & -C_1 & -d_2 C_1 + q_1 S_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Q(6)

Link	θ	d	a	α	
1	θ_1	d_1	0	90	R
2	θ_2	0	a_2	0	R
3	θ_3	0	a_3	0	R



$$^0T_1 = \begin{bmatrix} C\theta_1 & -0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

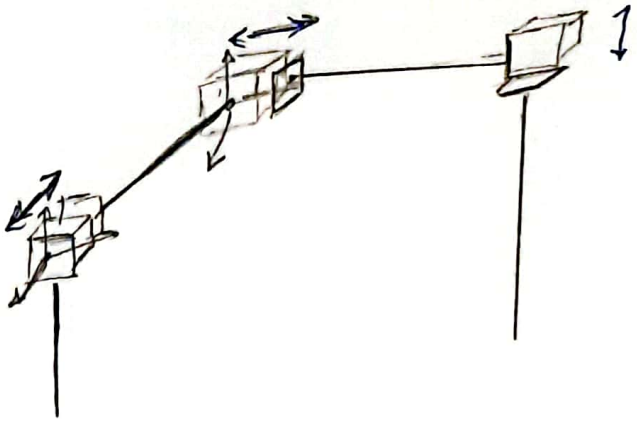
$$^1T_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2 C\theta_1 \\ 0 & C\theta_2 & 0 & a_2 S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3 C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3 S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = {}^0T_1\ {}^1T_2\ {}^2T_3 = \begin{bmatrix} c_1c_2c_3 - c_1s_2s_3 & -s_3c_1c_2 - c_1c_2c_3 & s_1 & a_3c_1c_2c_3 + a_3c_1c_2s_3 + a_2c_1c_2 \\ s_1c_2c_3 - s_1s_2s_3 & -s_1s_3c_2 - s_1s_2c_3 & -c_1 & a_3c_3s_1c_2 - a_3c_3s_1c_2 - a_2s_1s_2 + a_2c_2s_1 \\ s_2c_3 + c_2s_3 & -s_2c_3 + c_2s_3 & 0 & a_3c_3s_2 + a_3s_2s_3 + a_2s_1a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q(7)

Link	a_i	d_i	d_i	θ_i
L_1	0	d_1	$-g_0$	0
L_2	0	d_2	g_0	0
L_3	0	d_3	0	0



$$^0T_1 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_2 = {}^0T_1 * {}^1T_2 = \begin{bmatrix} 0 & 1 & 0 & -d_2 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$^0T_3 = {}^0T_2 * {}^3T_2 = \begin{bmatrix} 0 & 1 & 0 & -d_2 \\ 0 & 0 & 1 & d_3 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$