opticut: likelihood based optimal partitioning for indicator species analysis

Peter Solymos and Ermias T. Azeria November 15, 2016

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Introduction

Identifying and monitoring indicator species has long been considered a cost-effective way of tracking environmental change or the status of the biota. Examples include the characterization of vegetation types (Chytry et al. 2002), degradation of ecosystems (McGeoch & Chown 1988), or signalling cryptic or rare species (Halme et al. 2009). Throughout these examples, a key attribute of indicator species (also referred to as character or differential species) is that they have strong associations with the environmental variables that they are supposed to indicate.

Approaches to quantify the degree of environmental associations for species (indicator value) traditionally falls into three major types of approaches:

- 1. contingency table based measures (De Caceres & Legendre 2009);
- 2. analysis of variance (ANOVA; Wildi & Feldmeyer-Christe 2013); and
- 3. the widely used non-parametric IndVal method (Dufrene & Legendre 1997).

While the different approaches have strong appeal and applications, they do not always meet the challenges presented by ecological data.

Ecological data come in different forms: binary, ordinal, count, abundance, or presence only data. Some of these data types are suitable for a particular approach, while some formats need 'tweaking'. For example, binarizing abundance or count data for contingency tables leads to information loss. ANOVA, on the other hand, implies normality and homoscedastic errors, which might not always be satisfied by 0/1, ordinal, skewed, or percent cover data. Finally, randomization test for the IndVal approach requires count data, which renders hypothesis testing difficult if not impossible for continuous or ordinal data.

Another staple of observation field studies is the presence of modifying or confounding variables, or the presence of systematic biases (variable sampling effort, imperfect detectability, sample selection bias). Ignoring these effects can lead to erroneous indicator species analysis (Zettler et al. 2013). Controlling for these effects can improve the assessment of species-environment relationships, thus lead to better evaluation of indicator species.

To address these limitations, Kemencei et al. (2014) proposed a model-based indicator species analysis that accounted for the effects of modifying variables, and non-independence in the data due to paired sampling design. This model-based approach has been generalized and made available in the opticut R extension package. The opticut package offers computationally efficient and extensible algorithms for finding indicator species, tools for exploring and visualizing the results, and quantifying uncertainties. This manual showcases the functionality of the package.

Install

The opticut R package can be installed from the Comprehensive R Archive Network (CRAN) as:

```
install.packages("opticut")
```

Install development version from GitHub:

```
library(devtools)
install_github("psolymos/opticut")
```

User visible changes in the package are listed in the NEWS file.

Report a problem

Use the issue tracker to report a problem.

License

GPL-2

Loading the package

library(opticut)

To get started, open R and load tha **opticut** package as:

```
## Loading required package: pbapply
## opticut 0.0-9 2016-10-30
```

Partitioning for single species

Optimal partitioning (optimal cut, or in short: opticut) is found for each species independent of each other. We make observations (y_i) of possibly multiple species at i = 1, ..., n sites. Now let us consider a discrete site descriptor (g_i) with K levels or strata (K > 2). This stratification might come from remotely sensed or other geospatial information, field measurements, or from multivariate clustering. We can use g_i to create m = 1, ..., M possible binary partitions based on coding one or more levels as 1s and the rest with 0s. We denote any such possible partition as $z^{(m)}$. The total number of binary partitions is $M = 2^{K-1} - 1$, not counting cases when 0s or 1s are completely missing (which is the null model). The opticut method, as opposed to for example IndVal method is invariant to the coding of 0s and 1s in $z^{(m)}$. This

means that complementary cases, such as $z^{(m)}$ and $1-z^{(m)}$, are treated as interchangeable. The opticut package provides utility functions to create and check binary partitions from multi-level vectors (kComb, allComb, checkComb).

Manipulating partitions

Finding all combinations does not require a model or observed responses. It only takes a classification vector with K > 1 strata. The kComb function returns a 'contrast' matrix corresponding to all possible binary partitions of the factor with K levels:

```
kComb(k = 2)
##
         [,1]
## [1,]
             1
## [2,]
            0
kComb(k = 3)
         [,1] [,2] [,3]
##
## [1,]
            1
                  0
## [2,]
            0
                   1
                        0
## [3,]
                  0
kComb(k = 4)
         [,1] [,2] [,3] [,4] [,5] [,6] [,7]
##
## [1,]
                  0
                        0
                              0
                                    1
## [2,]
            0
                   1
                        0
                              0
                                    1
                                          0
                                                0
## [3,]
            0
                  0
                        1
                              0
                                    0
                                          1
                                                0
                              1
                  0
                        0
                                          0
                                                1
## [4,]
                                    0
```

allComb takes a classification vector with at least 2 levels and returns a model matrix with binary partitions. checkComb checks if combinations are unique and non-complementary (misfits are returned as attributes):

```
(f <- rep(LETTERS[1:4], each=2))</pre>
## [1] "A" "A" "B" "B" "C" "C" "D" "D"
(mc <- allComb(f, collapse = " "))</pre>
     A B C D A_B A_C A_D
## A 1 0 0 0
                         1
## A 1 0 0 0
                         1
                1
## B O 1 O O
                1
                         0
## B O 1 O O
                1
                         0
## C O O 1 O
                0
                     1
                         0
## C O O 1 O
                0
                     1
                         0
```

```
## D O O O 1
                        1
## D O O O 1
                0
                        1
## attr(,"collapse")
## [1] " "
## attr(,"comb")
## [1] "all"
checkComb(mc)
## [1] TRUE
## attr(,"comp")
        i j
##
## attr(,"same")
        i j
##
mc2 \leftarrow cbind(z = 1 - mc[,1], mc[,c(1:ncol(mc), 1)])
colnames(mc2) <- 1:ncol(mc2)</pre>
mc2
##
     1 2 3 4 5 6 7 8 9
## A O 1 O O O 1 1 1 1
## A O 1 O O O 1 1 1 1
## B 1 O 1 O 0 1 O 0
## B 1 O 1 O 0 1 O 0 O
## C 1 O O 1 O O 1 O O
## C 1 0 0 1 0 0 1 0 0
## D 1 0 0 0 1 0 0 1 0
## D 1 0 0 0 1 0 0 1 0
checkComb(mc2)
## [1] FALSE
## attr(,"comp")
##
        i j
## [1,] 1 2
## [2,] 1 9
## attr(,"same")
##
        i j
## [1,] 9 2
```

Choosing a parametric model

A suitable parametric (or semi-parametric) model can be chosen to describe the relationship between the observations for a single species and the site descriptors. The choice of the parametric model depends on the nature of the observations and the goals of the study. The systematic component of the model (also called the linear predictor), $f(\mu_i) = \beta_0^{(m)} + \beta_0^{(m)}$

 $\beta_1^{(m)} z_i^{(m)} + \sum_{j=1}^p \alpha_j^{(m)} x_{ij}$ x_ij, is linked to the random component of the model through the link function f. The expected value is given by the inverse link function: $E[Y_i] = \mu_i = f^{-1}(\beta_0^{(m)} + \beta_1^{(m)} z_i^{(m)} + \sum_{j=1}^p \alpha_j^{(m)} x_{ij})$. Expected values can then be estimated for each partition. The symbol x_{ij} denotes other site descriptors (j = 1, ..., p; number of predictors besides g_i) that can take discrete or continuous values. These variables might describe variation in the observations not fully explained by the partitions, e.g. due to spatially uneven distribution, differences in sampling effort, or environmental variables interfering with the observation process.

We can estimate the parameters in the linear predictor (and possibly other "nuisance" factors such as variance components in mixed effects models) and calculate expected values. The probability density function for the model $P(Y_i = y_i \mid z_i^{(m)}, x_{ij}, \theta)$ is used to find the maximum likelihood estimates (MLE) of the model parameters $\hat{\theta}^{(m)} = (\hat{\beta}_0^{(m)}, \hat{\beta}_1^{(m)}, \hat{\alpha}_1^{(m)}, \dots, \hat{\alpha}_p^{(m)})$ that jointly maximize the log-likelihood function. The log-likelihood function evaluated at the MLE is $l(\hat{\theta}^{(m)}; y)$.

The opticut package has several built-in distributions that can be specified though the dist argument. Currently available distributions:

- "gaussian": real valued continuous observations, e.g. biomass;
- "poisson": Poisson count data;
- "binomial": presence-absence type data;
- "negbin": overdispersed Negative Binomial count data;
- "beta": continuous response in the unit interval, e.g. percent cover;
- "zip", "zip2": zero-inflated Poisson counts (partitioning in count model: "zip", or in zero model: "zip2");
- "zinb", "zinb2": zero-inflated Negative Binomial counts (partitioning in count model: "zinb", or in zero model: "zinb2");
- "ordered": response measured on ordinal scale, e.g. ordinal vegetation cover (only available for single species because ordinal levels often do not match across different species thus leading to different intercept terms);
- "rsf", "rspf": presence-only data using resource selection and resource selection probability functions (only available for single species because used distribution is unique for all species thus multiple species cannot be combined in a single input matrix).

Other distributions can be specified by user-defined functions as explained later in the manual.

All combinations

Fitting the model to all the M candidate binary partitions leads to a set of log-likelihood values. One can compare the log-likelihood values $l(\hat{\theta}^{(m)}; y)$ to the log-likelihood value based on the null model $l(\hat{\theta}^{(0)}; y)$. We define the null model the same way as the other M models but without the binary partition: $\beta_1^{(m)} = 0$. The log of the likelihood ratio between the M candidate models and the null model can be calculated as $l(\hat{\theta}^{(m)}; y) - l(\hat{\theta}^{(0)}; y)$.

The best-supported model m' and the corresponding binary partition $z^{(m')}$ is the model with the highest log-likelihood ratio value $l(\hat{\theta}^{(m')};y)$. Model weights are calculated as $w_m = exp\{l(\hat{\theta}^{(m)};y) - l(\hat{\theta}^{(m')};y)\}/\sum_{m=1}^{M} exp\{l(\hat{\theta}^{(m)};y) - l(\hat{\theta}^{(m')};y)\}$. These weights sum to 1 and indicate asymptotic probabilities of finding the same best partition when the sampling is replicated. The concentration of asymptotic probabilities among the models can be expressed through the Simpson index $H = \sum_{m=1}^{M} w_m^2$. High values of H indicate high concentration of the model weights for one or few model out of the total number of models compared (Kemencei et al. 2014).

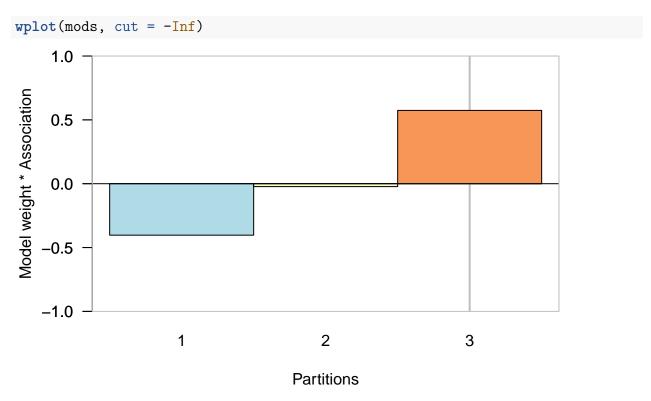
The opticut package provides the opticut1 function to fit a chosen parametric model to a set of binary partitions and the summary returns the model output for each candidate partition with log likelihood ratios and model weights.

```
## stratification
g \leftarrow c(1,1,1,1,2,2,2,2,3,3,3,3)
## abundance
y \leftarrow c(0,0,3,0,2,3,0,5,5,6,3,4)
mods <- opticut1(Y = y, Z = allComb(g), dist = "gaussian")</pre>
mods
## Univariate opticut results, comb = all, dist = gaussian
## I = 0.8932; w = 0.5742; H = 0.4926; logL null = -25.93
## Best supported models with logLR >= 2:
##
     assoc
                 Ι
                     mu0 mu1 logLR
## 3
        ++ 0.8932 1.625 4.50 3.233 0.5742
        -- 0.8798 3.500 0.75 2.879 0.4031
## 3 binary splits (1 model not shown)
```

Not all the models are printed, only the ones where the log likelihood ratio (logLR) value is ≥ 2 by default. We can change this by explicitly defining the cut argument in the print method (or by setting the cut global option via ocoptions, as explained later):

```
print(mods, cut = -Inf)
```

Model support across the partitions can be visualized by the model weight plot (wplot):



The opticut function can take matrices or a model formula as its input, and repeats the procedure done by opticut1 for multiple species in a community matrix. The summary shows the best-supported model for each species. It is the preferred way of specifying the model for single species as well. Single species results are part of the \$species element of the output object:

```
(oc <- opticut(y, strata = g, comb = "all", dist = "gaussian"))</pre>
## Multivariate opticut results, comb = all, dist = gaussian
##
## Call:
## opticut.default(Y = y, strata = g, dist = "gaussian", comb = "all")
## 1 species, 3 binary splits
summary(oc)
## Multivariate opticut results, comb = all, dist = gaussian
##
## Call:
## opticut.default(Y = y, strata = g, dist = "gaussian", comb = "all")
##
## Best supported model with logLR >= 2:
        split assoc
                             mu0 mu1 logLR
                         Ι
            3
                 ++ 0.8932 1.625 4.5 3.233 0.5742
## Sp 1
## 3 binary splits
```

oc\$species

The use of the opticut1 function is generally discouraged: some of the internal checks are not guaranteed to flag issues when the formula-to-model-matrix translation is side-stepped (this is what is happening when the modifier variables are supplied as X argument in opticut1). Use the opticut with a single species instead, as shown above.

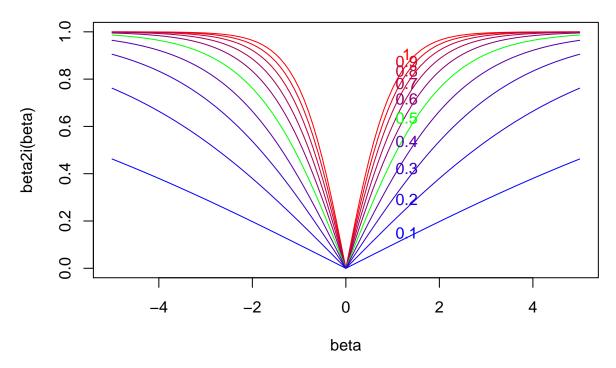
Indicator value

Once a model is fit a to a given binary partition, we can quantify the indicator value for the species. The indicator value denoted by $I^{(m)}$ describes the contrast between the two subset of the data represented by the binary partition $z^{(m)}$. We define indicator value as a scaled version of the $\beta_1^{(m)}$ coefficient estimate: $I^{(m)} = |\tanh(c\beta_1^{(m)})|$, where c = 0.5. The hyperbolic tangent function is used as an inverse Fisher transformation to scale real valued coefficients into the [-1,1] range. The absolute value then results in a [0, 1] range for the indicator value.

The c is a scaling constant that modifies the shape of the function. We chose 0.5 as the default value that allows the indicator value to change more gradually according to our experience with real-world data sets. The c = 0.5 setting is also identical to an inverse logistic function transformed into the [-1, 1] range.

The beta2i function is used internally to calculate the indicator value. Here we show the effect of the scaling constant c on the shape of the function, the default c = 0.5 is in green:

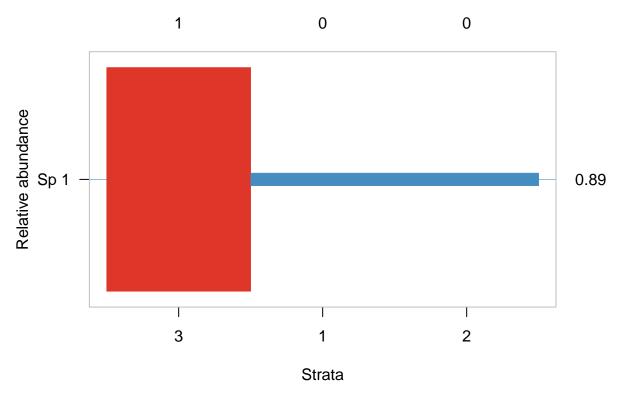
```
beta <- seq(-5, 5, 0.1)
Col <- occolors(c("red", "blue"))(10)
Col[6] <- "#00FF00"
plot(beta, beta2i(beta), type = "n")
s <- seq(1, 0.1, -0.1)
for (i in 1:10) {
    lines(beta, beta2i(beta, scale = s[i]), col = Col[i])
    text(1.5 - 0.2, beta2i(1.5, scale = s[i]), s[i], col = Col[i])
}</pre>
```



An alternative way to define the indicator value would be taking the relative difference between the expected values for the 0 and 1 stratum in $z^{(m)}$. This, however, depends on the response scale and the baseline values chosen for possible modifying effects.

Our definition of indicator value is on the linear predictor scale and is more readily compared across species without respect to their relative abundance and values of other modifying factors. Note however, that the meaning of the indicator value might be quite different for studies using different parametric models: it is a difference in the Gaussian case, multiplier in log-linear models, change in log-odds in logistic or ordinal regression. The indicator value given a binary partition is returned by the model summaries, and used in visualization:

plot(oc)



Use the ocoptions function to change the default scale (c) setting.

User defined combinations

It is possible to

```
comb <- cbind(</pre>
    A = c(rep(1, 4), rep(0, 8)),
    B = c(rep(0, 4), rep(1, 4), rep(0, 4)))
comb
##
          {\tt A} {\tt B}
##
     [1,] 1 0
    [2,] 1 0
##
    [3,] 1 0
##
##
     [4,] 1 0
     [5,] 0 1
##
    [6,] 0 1
##
    [7,] 0 1
##
##
    [8,] 0 1
    [9,] 0 0
##
## [10,] 0 0
## [11,] 0 0
## [12,] 0 0
```

```
print(opticut1(Y = y, Z = comb, dist = "gaussian"), cut = -Inf)
## Univariate opticut results, comb = NA, dist = gaussian
## I = 0.8798; w = 0.9466; H = 0.8988; logL null = -25.93
##
## Best supported models with logLR >= -Inf:
                  Ι
                      mu0 mu1
##
     assoc
                                   logLR
        -- 0.87983 3.500 0.75 2.878892 0.94655
## A
## B
         - 0.06242 2.625 2.50 0.004726 0.05345
## 2 binary splits
If the user happen to define complementary partitions, an error message is thrown:
comb <- cbind(comb, 1-comb)</pre>
colnames(comb) <- LETTERS[1:4]</pre>
comb
         ABCD
##
##
    [1,] 1 0 0 1
    [2,] 1 0 0 1
##
##
    [3,] 1 0 0 1
    [4,] 1 0 0 1
##
    [5,] 0 1 1 0
##
    [6,] 0 1 1 0
##
    [7,] 0 1 1 0
##
    [8,] 0 1 1 0
##
## [9,] 0 0 1 1
## [10,] 0 0 1 1
## [11,] 0 0 1 1
## [12,] 0 0 1 1
try(opticut1(Y = y, Z = comb, dist = "gaussian"))
checkComb(comb)
## [1] FALSE
## attr(,"comp")
##
        i j
## [1,] 1 3
## [2,] 2 4
## attr(,"same")
##
        iј
The global option check comb can be set to override this default behaviour, although there
```

The global option check_comb can be set to override this default behaviour, although there is no real point in duplicating reparametrized but otherwised identical models:

```
op <- ocoptions(check_comb = FALSE, cut = -Inf)
opticut1(Y = y, Z = comb, dist = "gaussian")</pre>
```

```
## Univariate opticut results, comb = NA, dist = gaussian
## I = 0.8798; w = 0.4733; H = 0.4494; logL null = -25.93
##
## Best supported models with logLR >= -Inf:
##
     assoc
                 Ι
                     mu0
                           mu1
                                   logLR
## A
        -- 0.87983 3.500 0.750 2.878892 0.47328
## C
        ++ 0.87983 0.750 3.500 2.878892 0.47328
         - 0.06242 2.625 2.500 0.004726 0.02672
## B
## D
         + 0.06242 2.500 2.625 0.004726 0.02672
## 4 binary splits
ocoptions(op)
```

Rank based combinations

The IndVal method requires the algorithm to evaluate $2^K - 1$ binary partitions. Our opticut approach is parametrization invariant with respect to coding the levels in the binary partitions (it affects the intercept term but not the contrast or the log likelihood ratio). This effectively halves the number of partitions we need to compare $(2^{K-1}-1, comb = "all" in opticut)$. Still, the number of partitions increases according to powers of 2. Here we propose an approach that increases linearly with K. This algorithm is based on sorting all the K partitions in g according to increasing order of the linear predictor estimates for K coefficients (as opposed to estimating 2 coefficients for a binary partition). The logic follows from the fact that the optimal binary partitioning tries to find the best split in terms of likelihood ratio with lower estimates on one side, and higher estimates on the other side of the split. As a consequence, we only need to try K-1 binary partitions to find the optimal $z^{(m')}$. This algorithm is implemented in the rankComb function that is called by opticut with the argument comb = "rank".

The function rankComb evaluates a model with a K-level factor and returns a corresponding partitioning matrix. Attibutes hold the estimates for the K levels:

```
rankComb(Y = y, Z = as.factor(g), dist = "gaussian", collapse = " ")
     3 2_3
##
## 1 0
         0
## 1 0
         0
## 1 0
         0
## 1 0
         0
## 2 0
         1
## 2 0
         1
## 2 0
         1
## 2 0
         1
## 3 1
         1
## 3 1
```

The collapse argument can be important to make partitions more distinguishable. The global option is " " that can be modified via the ocoptions function. It can cause problems when the the collapse character is part of the factor levels used for g. The fix_levels function comes handy for fixing these levels. The function replaces the collapse character to something else:

```
getOption("ocoptions")$collapse

## [1] " "

fix_levels(as.factor(c("A b", "C d")), sep=":")

## [1] A:b C:d

## Levels: A:b C:d

fix_levels(as.factor(c("A b", "C d")), sep="")

## [1] Ab Cd

## Levels: Ab Cd
```

There is an overhead of fitting the model to calculate the ranking first. But computing efficiencies can be still high compared to all partitions. As a consequence of the ranking process, we do not have summaries for all the possible binary partitions, only for the top candidates. Moreover, the partitions produced for each species might not be identical. Therefore the model weights (w) and Simpson index (H) have different interpretation and cannot be that easily compared across species, unless the model weights are highly concentrated for the top models. In this case, the sum of weights for the missing models becomes negligible. Another consequence of the ranking process is that $\beta_1^{(m)}$ estimates are always positive. In the case of comparing all the partitions, the full set of partitions is fixed for all species, and some respond positively while others respond negatively to the same binary variable in terms of the $\beta_1^{(m)}$ values. Thus it is required to store the sign of the relationship as part of the summary.

The comb = "rank" is the defult setting in opticut. It is clear that the 2 approaches lead to identical best partitions. Only the model weights (w) are different:

```
summary(opticut(y, strata = g, comb = "all", dist = "gaussian"))
## Multivariate opticut results, comb = all, dist = gaussian
##
```

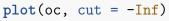
```
## Call:
## opticut.default(Y = y, strata = g, dist = "gaussian", comb = "all")
##
## Best supported model with logLR >= 2:
        split assoc
                         Ι
                             mu0 mu1 logLR
            3
                 ++ 0.8932 1.625 4.5 3.233 0.5742
## 3 binary splits
summary(opticut(y, strata = g, comb = "rank", dist = "gaussian"))
## Multivariate opticut results, comb = rank, dist = gaussian
##
## Call:
## opticut.default(Y = y, strata = g, dist = "gaussian", comb = "rank")
## Best supported model with logLR >= 2:
        split assoc
                             mu0 mu1 logLR
##
                         Ι
                 ++ 0.8932 1.625 4.5 3.233 0.5875
## Sp 1
## 2 binary splits
```

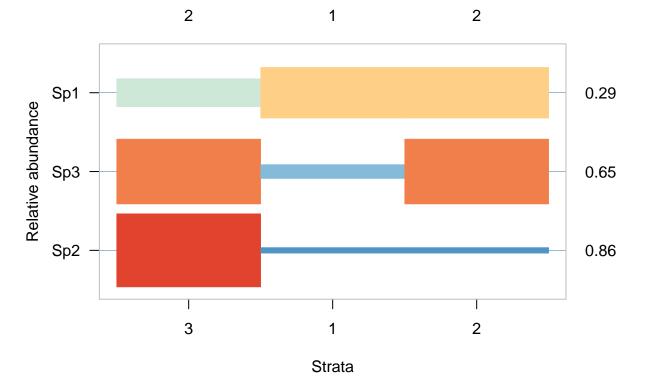
Partitioning for multiple species

The opticut function can take matrices or a model formula as its input, and repeats the procedure done by opticut1 for multiple species in a community matrix. The summary shows the best-supported model for each species, in this case based on a Poisson count model (dist = "poisson"). The plot method uses the indicator value (I in the summary) to represent the contrast between the two strata of the best supported binary partition:

```
## stratification
        c(1,1,1,1, 2,2,2,2, 3,3,3,3)
g <-
## community matrix
y <- cbind(
    Sp1=c(4,6,3,5,5,6,3,4,4,1,3,2),
    Sp2=c(0,0,0,0,1,0,0,1,4,2,3,4),
    Sp3=c(0,0,3,0,2,3,0,5,5,6,3,4))
oc <- opticut(formula = y ~ 1, strata = g, dist = "poisson", comb = "all")
summary(oc)
## Multivariate opticut results, comb = all, dist = poisson
##
## Call:
## opticut.formula(formula = y ~ 1, strata = g, dist = "poisson",
##
       comb = "all")
```

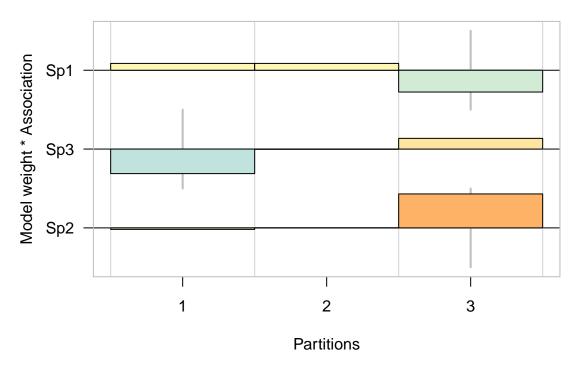
```
##
## Best supported models with logLR >= 2:
##
      split assoc
                        I muO mu1 logLR
                -- 0.6471 3.50 0.75 4.793 0.6922
## Sp3
           1
              +++ 0.8571 0.25 3.25 9.203 0.9573
## Sp2
## 3 binary splits
## 1 species not shown
```





Model support across the partitions can be visualized by the model weight plot (wplot).

```
wplot(oc, cut = -Inf)
```



Note that the wplot uses 'splits' (binary partitions) whereas plot uses the K levels. Therefore, wplot does not work for multiple species when comb = "rank". In this case the splits can be different across the species, whereas comb = "all" uses the same pre-defined partitions across all species.

The bestmodel method returns the best-supported model for further model diagnostics and prediction, the getMLE prints out the estimated coefficients and the variance-covariance matrix:

```
mods <- bestmodel(oc)</pre>
mods
## $Sp1
##
          stats::glm(formula = Y ~ . - 1, family = Family, data = XX)
##
## Coefficients:
##
        V1
                 Z1
    1.5041 -0.5878
##
##
## Degrees of Freedom: 12 Total (i.e. Null); 10 Residual
## Null Deviance:
                         63.01
## Residual Deviance: 4.39 AIC: 46.05
##
## $Sp2
##
          stats::glm(formula = Y ~ . - 1, family = Family, data = XX)
## Call:
##
```

```
## Coefficients:
      V1
               Z1
##
## -1.386
            2.565
##
## Degrees of Freedom: 12 Total (i.e. Null); 10 Residual
## Null Deviance:
                        25.54
## Residual Deviance: 6.445
                            AIC: 26.58
##
## $Sp3
##
## Call: stats::glm(formula = Y ~ . - 1, family = Family, data = XX)
##
## Coefficients:
##
      V1
  1.253 - 1.540
##
## Degrees of Freedom: 12 Total (i.e. Null); 10 Residual
## Null Deviance:
                        49.33
## Residual Deviance: 18.9 AIC: 48.37
## explore further
str(predict(mods[[1]]))
## Named num [1:12] 1.5 1.5 1.5 1.5 1.5 ...
## - attr(*, "names")= chr [1:12] "1" "2" "3" "4" ...
confint(mods[[1]])
## Waiting for profiling to be done...
          2.5 %
##
                    97.5 %
## V1 1.158610 1.81387284
## Z1 -1.343213 0.07397014
## MLE and variance-covariance matrix (species 1)
getMLE(oc, which = 1)
## $coef
##
           V1
                      Z1
   1.5040774 -0.5877867
##
##
## $vcov
               V1
                           Z1
## V1 0.02777777 -0.02777777
## Z1 -0.02777777 0.12777667
##
## $dist
```

[1] "poisson"

The bestpart method returns the binary partition for the best-supported model:

bestpart(oc)

```
Sp1 Sp2 Sp3
##
## 1
          0
## 1
      0
          0
              1
## 1
      0
          0
              1
## 1
      0
          0
              1
## 2
      0
              0
## 2
      0 0
              0
## 2
      0 0
              0
## 2
      0
          0
              0
## 3
      1 1
              0
## 3
      1 1
              0
## 3
      1
          1
              0
## 3
      1
          1
              0
```

Compare the "all" and "rank" based combinations: same best partitions putting aside complementaryity

```
op <- ocoptions(cut = -Inf)</pre>
summary(opticut(formula = y ~ 1, strata = g, dist = "poisson", comb = "all"))
## Multivariate opticut results, comb = all, dist = poisson
##
## Call:
## opticut.formula(formula = y ~ 1, strata = g, dist = "poisson",
       comb = "all")
##
## Best supported models with logLR >= -Inf:
##
       split assoc
                        I muO mu1 logLR
           3
## Sp1
                 - 0.2857 4.50 2.50 1.498 0.6144
                -- 0.6471 3.50 0.75 4.793 0.6922
## Sp3
           1
               +++ 0.8571 0.25 3.25 9.203 0.9573
           3
## Sp2
## 3 binary splits
summary(opticut(formula = y ~ 1, strata = g, dist = "poisson", comb = "rank"))
## Multivariate opticut results, comb = rank, dist = poisson
##
## Call:
## opticut.formula(formula = y ~ 1, strata = g, dist = "poisson",
       comb = "rank")
##
## Best supported models with logLR >= -Inf:
```

```
## split assoc I mu0 mu1 logLR w
## Sp1 1 2 + 0.2857 2.50 4.50 1.498 0.7611
## Sp3 2 3 ++ 0.6471 0.75 3.50 4.793 0.6962
## Sp2 3 +++ 0.8571 0.25 3.25 9.203 0.9577
## 2 binary splits
ocoptions(op)
```

Quantifying uncertainty

Uncertainty in the estimated coefficients and uncertainty in the derived indicator value for the best-supported model $(I^{(m')})$ is quantified based on the estimate of the Hessian matrix assuming asymptotic normality of the MLE. The distribution of $I^{(m')}$ in this case is based on parametric bootstrap. This approach (type = "asymp" in uncertainty) is suitable when asymptotic normality assumption is reasonable, i.e. sample size is large. For small sample situations, a parametric bootstrap algorithm is implemented (type = "boot") to estimate uncertainty in $I^{(m')}$. The summary contains lower and upper confidence limits (for a given error rate) representing the asymptotic or bootstrap distribution (based on B number of iterations) given the fixed partition for the best model, $z^{(m')}$.

The output is summarized, the **\$uncertainty** component contains individual species results:

```
c(1,1,1,1, 2,2,2,2, 3,3,3,3)
y <- cbind(
    Sp1=c(4,6,3,5,5,6,3,4,4,1,3,2),
    Sp2=c(0,0,0,0,1,1,0,0,1,4,2,3,4),
    Sp3=c(0,0,3,0,2,3,0,5,5,6,3,4))
oc <- opticut(formula = y ~ 1, strata = g, dist = "poisson")
uc <- uncertainty(oc, type = "asymp", B = 999)
summary(uc)
## Multivariate opticut uncertainty results
## type = asymp, B = 999, level = 0.95
##
##
       split R
                        Lower Upper
                    Ι
        1 2 1 0.2906 0.03057 0.5656
## Sp1
## Sp3
         2 3 1 0.6195 0.17788 0.8697
           3 1 0.8201 0.48827 0.9664
## Sp2
uc$uncertainty
```

\$Sp1

```
## Univariate opticut uncertainty results, type = asymp, B = 999
##
##
                      Ι
     best
                                           mu0
                                                             mu1
    1 2:1000
                Min.
                       :0.0000873
                                             :0.7993
                                                               :2.557
##
                                     Min.
                                                        Min.
##
                1st Qu.:0.1831074
                                     1st Qu.:2.0171
                                                        1st Qu.:3.980
                Median :0.2900418
                                     Median :2.4432
##
                                                        Median :4.466
                                     Mean
##
                Mean
                       :0.2906122
                                             :2.5914
                                                        Mean
                                                               :4.541
                3rd Qu.:0.3961506
                                     3rd Qu.:3.0070
                                                        3rd Qu.:5.025
##
                       :0.6995527
                                             :8.7078
                                                               :8.356
##
                Max.
                                     Max.
                                                        Max.
##
## $Sp2
## Univariate opticut uncertainty results, type = asymp, B = 999
##
##
    best
                    Ι
                                     mu0
                                                         mu1
                     :0.3064
##
    3:1000
             Min.
                                Min.
                                        :0.02919
                                                   Min.
                                                           :1.411
              1st Qu.:0.7631
                                1st Qu.:0.16521
                                                   1st Qu.:2.701
##
                                                   Median :3.278
##
             Median : 0.8541
                                Median :0.26081
##
             Mean
                     :0.8201
                                Mean
                                        :0.33242
                                                   Mean
                                                           :3.409
              3rd Qu.:0.9067
##
                                3rd Qu.:0.41959
                                                   3rd Qu.:3.941
##
                     :0.9873
                                        :1.88247
                                                           :7.852
             Max.
                                Max.
                                                   Max.
##
## $Sp3
## Univariate opticut uncertainty results, type = asymp, B = 999
##
##
     best
                      Ι
                                         mu0
                                                            mu1
                       :0.006351
##
    2 3:1000
                Min.
                                    Min.
                                            :0.1531
                                                      Min.
                                                              :1.946
                                    1st Qu.:0.5000
                                                       1st Qu.:3.106
##
                1st Qu.:0.514800
                Median : 0.653531
                                    Median :0.7280
                                                      Median :3.492
##
##
                Mean
                       :0.619532
                                            :0.8695
                                                              :3.556
                                    Mean
                                                       Mean
                                    3rd Qu.:1.0993
                                                       3rd Qu.:3.948
##
                3rd Qu.:0.750730
##
                       :0.922248
                                            :3.8991
                                                              :6.079
                Max.
                                    Max.
                                                       Max.
```

The bootstrap can be difficult for small sample sizes, strata can go completely missing:

```
try(uc <- uncertainty(oc, type = "boot", B = 99))</pre>
```

A general requirement for the bootstrap approach ("boot" and "multi") is that the bootstrap samples contain observations from each stratum. We recommend having at least 5 observations per strata. Possible problems with missing partitions in small data sets can be remedied by supplying pre-defined indices for resampling, for example, based on jackknife (leave-one-out) approach. The resampling scheme can be customized for such needs. Use the check_strata function:

```
B <- sapply(1:length(g), function(i) which((1:length(g)) != i))
check_strata(oc, B) # check representation</pre>
```

```
## attr(,"nx")
## [1] 3
## attr(,"nmat")
   [1] 3 3 3 3 3 3 3 3 3 3 3 3
summary(uncertainty(oc, type = "boot", B = B))
## Multivariate opticut uncertainty results
## type = boot, B = 12, level = 0.95
##
##
       split R
                    I Lower Upper
## Sp1
         1 2 1 0.2866 0.2167 0.3644
         2 3 1 0.6523 0.5556 0.9053
## Sp3
## Sp2
           3 1 0.8572 0.8384 0.9158
```

The reliability of the best partition can also be assessed using the setting type = "multi" (as in multiple models). In this case, the model partitions are re-evaluated for each bootstrap sample. Model uncertainty is assessed as the number of times a partition is supported out of the B bootstrap runs. The reliability (R) metric in the summary is the proportion for the most frequently supported partition. The corresponding indicator value and confidence interval is conditional on this most commonly supported partition.

```
## Multivariate opticut uncertainty results
## type = multi, B = 12, level = 0.95
##
## split R I Lower Upper
## Sp1     1 2 1.0000 0.2866 0.2167 0.3644
## Sp3     2 3 0.6923 0.6872 0.6174 0.9368
## Sp2     3 1.0000 0.8572 0.8384 0.9158
```

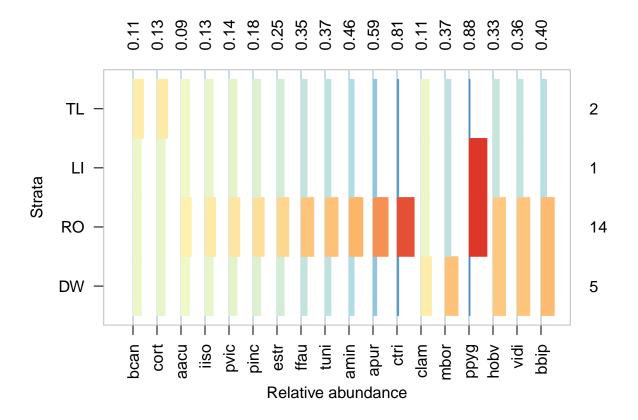
Distributions

For most distributions, we will use the dolina data set that is part of the **opticut** package. It is a comprehensive and micro-scale land snail data set from 16 dolines of the Aggtelek Karst Area, Hungary. Data set containing land snail counts as described in Kemecei et al. 2014.

```
data(dolina)
## stratum as ordinal
dolina$samp$stratum <- as.integer(dolina$samp$stratum)
## filter species to speed up things a bit
Y <- dolina$xtab[,colSums(dolina$xtab > 0) >= 20]
```

Gaussian

```
dol <- opticut(Y ~ stratum + lmoist + method, data=dolina$samp,</pre>
    strata=dolina$samp$mhab, dist="gaussian")
summary(dol)
## Multivariate opticut results, comb = rank, dist = gaussian
##
## Call:
## opticut.formula(formula = Y ~ stratum + lmoist + method, data = dolina$samp,
       strata = dolina$samp$mhab, dist = "gaussian")
##
##
## Best supported models with logLR >= 2:
        split assoc
                         Ι
                                mu0
                                       mu1
                                            logLR
## dper DW RO
                +++ 0.5988
                            3.03383 4.4165
                                            9.238 0.6600
## bbip DW RO
                +++ 0.4028
                            0.11875 0.9727 26.906 0.5892
## vidi DW RO
                +++ 0.3552  0.64966  1.3925  10.360  0.9866
## hobv DW RO
                +++ 0.3297 -0.07760 0.6074 23.017 0.6582
## ppyg LI RO
                ++ 0.8822 4.47006 7.2414 4.533 0.8833
## mbor
                +++ 0.3662 -0.17269 0.5952 25.555 1.0000
## clam
           DW
               ++ 0.1078  0.12200  0.3385  5.527  0.8665
## ctri
           RO
                +++ 0.8063 -0.37944 1.8531 9.223 0.9954
## apur
          RO
                +++ 0.5912 1.82422 3.1831 9.096 0.9754
                +++ 0.4560 2.17491 3.1595 9.546 0.5919
## amin
           RO
## tuni
           RO
                +++ 0.3683 1.01426 1.7872 21.063 1.0000
## ffau
          RO
                +++ 0.3480 0.02962 0.7560 30.899 1.0000
## estr
          RO
                +++ 0.2462 0.23962 0.7423 21.892 1.0000
## pinc
           RO
                +++ 0.1836  0.26944  0.6409  20.890  0.9924
## pvic
           RO
                ++ 0.1431  0.36807  0.6562  7.107  0.7323
## iiso
           RO
                +++ 0.1274  0.06840  0.3245  25.005  1.0000
## cort
           TL
                +++ 0.1321 -0.03773 0.2280 21.583 0.9921
## bcan
           TL
                +++ 0.1086  0.11177  0.3298  12.425  0.9988
## 3 binary splits
## 1 species not shown
## vertical plot orientation
plot(dol, horizontal=FALSE, pos=1, upper=0.8)
```

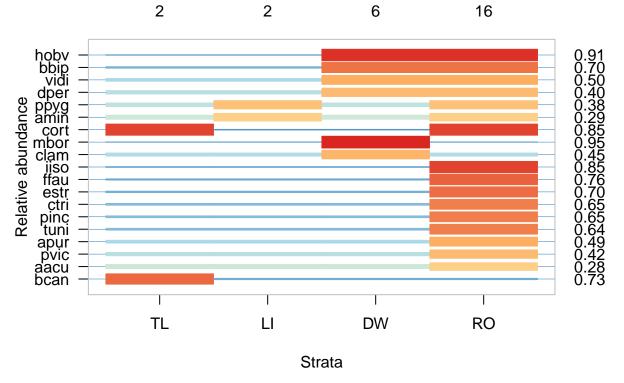


Poisson and Negative Binomial

```
dol <- opticut(Y ~ stratum + lmoist + method, data=dolina$samp,</pre>
    strata=dolina$samp$mhab, dist="poisson")
summary(dol)
## Multivariate opticut results, comb = rank, dist = poisson
##
## Call:
## opticut.formula(formula = Y ~ stratum + lmoist + method, data = dolina$samp,
##
       strata = dolina$samp$mhab, dist = "poisson")
##
## Best supported models with logLR >= 2:
##
        split assoc
                         Ι
                                 mu0
                                          mu1
                                                logLR
## hobv DW RO
                +++ 0.9083 0.0384393 0.80014
                                               75.633 1.0000
## bbip DW RO
                +++ 0.6986 0.1843710 1.03897
                                               63.376 1.0000
## vidi DW RO
                +++ 0.4955 0.7994519
                                      2.36967
                                               38.141 1.0000
## dper DW RO
                +++ 0.4029 4.5082800 10.59304
                                               54.686 0.8816
                +++ 0.3758 3.0963391 6.82425 100.632 1.0000
## ppyg LI RO
## amin LI RO
               +++ 0.2884 2.1715347 3.93142 22.048 0.7260
## cort TL RO
                +++ 0.8506 0.0212887 0.26361 22.532 0.4901
                +++ 0.9489 0.0004436 0.01693 80.850 1.0000
## mbor
           DW
```

```
++ 0.4461 0.1125752
## clam
           DW
                                       0.29391
                                                  7.122 0.6114
                 +++ 0.8545 0.0504103
                                                 21.695 1.0000
## iiso
           RO
                                        0.64254
## ffau
           RO
                 +++ 0.7557 0.1043986
                                        0.75035
                                                 49.352 1.0000
## estr
           RO
                +++ 0.7050 0.1853057
                                        1.07085
                                                 29.952 1.0000
## ctri
           RO
                 +++ 0.6495 0.2931486
                                        1.37962 124.868 1.0000
                 +++ 0.6476 0.2513583
## pinc
           RO
                                        1.17509
                                                 19.970 0.8020
## tuni
           RO
                 +++ 0.6418 2.3448978 10.74959
                                                 42.793 1.0000
## apur
                 +++ 0.4897 2.6442605
                                        7.72006
                                                 53.488 1.0000
           RO
## pvic
           RO
                +++ 0.4150 0.4325312
                                        1.04624
                                                  9.007 0.6502
## aacu
           RO
                 ++ 0.2821 0.6989512
                                                  3.741 0.4915
                                        1.24819
                +++ 0.7292 0.1594665
## bcan
           TL
                                        1.01806
                                                 15.246 0.9958
## 3 binary splits
```

```
## horizontal plot orientation
plot(dol)
```



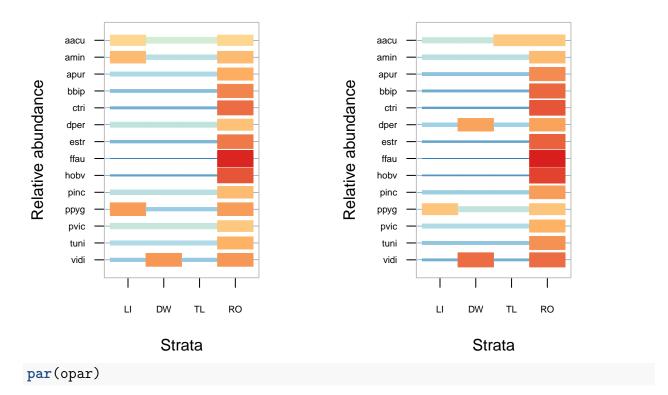
Because opticut uses the stats::glm function to fit the Poisson model, it accepts other arguments, e.g. offsets. Let's subset the dolina data set for the litter sampling method ("Q"). Pool the abundances in each of the 16 dolines by microhabitat types. By doing this, we make sampling effort uneven. Litter microhabitat was sampled along a North-South transect (7 locations), whereas the other three strata (rock, live trees, dead wood) were sampled at 3 random locations in each dolina.

```
DQ <- dolina$samp[dolina$samp$method == "Q",]
DQ$dol_mhab <- paste0(DQ$dolina, "_", DQ$mhab)
head(DQ)</pre>
```

```
aspect stratum lmoist lthick
         sample dolina microhab mhab method
##
           10A1
## 10A1Q
                     10
                                    LI
                                             Q southern
                                                                     1.0
                                                                            2.0
                           litter
## 10A2Q
           10A2
                     10
                                    LI
                                             Q southern
                                                               3
                                                                     1.0
                                                                            2.5
                           litter
## 10A3Q
           10A3
                                    LI
                                             Q southern
                                                               2
                                                                     1.0
                                                                            3.0
                     10
                          litter
## 10A4Q
           10A4
                     10
                          litter
                                    LI
                                             Q
                                                    flat
                                                               1
                                                                     1.5
                                                                            0.5
                                                               2
## 10A5Q
           10A5
                                    LI
                                             Q northern
                                                                     1.0
                                                                            1.5
                     10
                          litter
## 10A6Q
           10A6
                                             Q northern
                                                               3
                                                                     1.0
                                                                            3.0
                     10
                          litter
                                    LI
##
         dol mhab
## 10A1Q
             10 LI
## 10A2Q
             10 LI
## 10A3Q
             10_LI
## 10A4Q
             10 LI
## 10A5Q
             10 LI
## 10A6Q
             10_LI
YQ <- dolina$xtab[dolina$samp$method == "Q",]
YQ \leftarrow YQ[,colSums(YQ > 0) >= 20]
YQ <- mefa4::groupSums(YQ, 1, DQ$dol mhab)
DQ <- mefa4::nonDuplicated(DQ, dol mhab, TRUE)
```

Let's compare the results of ignoring sampling effort differences with a casewhen we use sampling effort as offset. Offsets are defined as log(7) or log(3) representing the sampling volume differences (more debris searched leads to more smails found). Using offsets results in less species that is associated with litter:

```
op <- ocoptions(collapse="_", sort=FALSE, cut=-Inf)</pre>
dol0 <- opticut(YQ, strata=DQ$mhab, dist="poisson")</pre>
off <- ifelse(DQ$mhab == "LI", log(7), log(3))
dol1 <- opticut(YQ, strata=DQ$mhab, dist="poisson", offset=off)</pre>
table(wo offset=summary(dol0)$summary$split,
    with offset=summary(dol1)$summary$split)
##
            with_offset
## wo offset DW RO LI RO RO TL RO
##
       DW RO
                  1
                        0
                           0
##
       LI RO
                  0
                            1
                                  1
                         1
##
       RO
                  1
                                  0
ocoptions(op)
opar \leftarrow par(mfrow=c(1,2))
plot(dol0, show I=FALSE, show S=FALSE, sort=FALSE, cex.axis=0.6)
plot(dol1, show_I=FALSE, show_S=FALSE, sort=FALSE, cex.axis=0.6)
```



The Negative Binomial model can be quite picky, as it gives an error somewhere in the middle of the process:

```
try(dol <- opticut(Y ~ stratum + lmoist + method, data=dolina$samp,
    strata=dolina$samp$mhab, dist="negbin"))</pre>
```

Warning: step size truncated due to divergence

Changing the try_error global option will allow the process to go on after catching the error:

```
op <- ocoptions(try_error = TRUE)
dol <- opticut(Y ~ stratum + lmoist + method, data=dolina$samp,
    strata=dolina$samp$mhab, dist="negbin")</pre>
```

Warning: step size truncated due to divergence

Warning in opticut.default(Y = Y, X = X, strata = strata, dist = dist, comb ## = comb, : Bad news: opticut failed for 1 out of 19 species.

dol\$failed

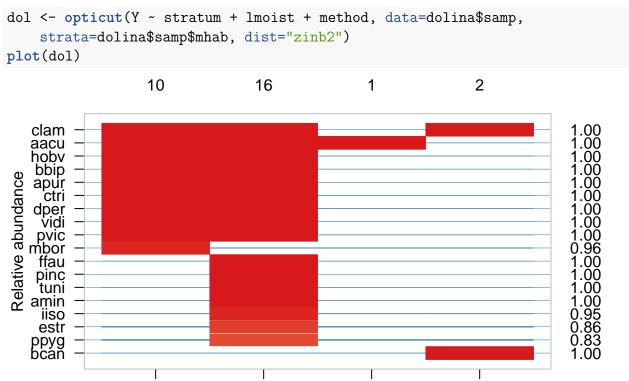
```
## [1] "mbor"
ocoptions(op)
```

It failed for the "mbor" (Macrogastra borealis) species, which is now excluded from the output.

Zero-inflated distributions

The Zero-inflated Negative Binomial implementation in the **pscl** package seems more robust, no error messages. We use the dist = "zinb2" option so that we test for optimal partitioning in the zero-inflation (ZI) component. Using a mixture distribution can be important if 0s can occur no only as a result of the ZI component, but due to low abundance in the count distribution (Poisson or Negative Binomial). Differentiating among these different types of zeros is not possible by binarizing the data and using logistic regression.

In the case of "zip2" and "zinb2" distributions, the coefficients refer to the probability of non-zero, so that positive and negative effects are properly identified (as opposed to "zip" and "zinb" where the ZI coefficients refer to probability of zero):



Binomial, Beta distribution and ordinal data

RO

Stratigraphy example using Beta distribution:

DW

```
library(rioja)
```

Strata

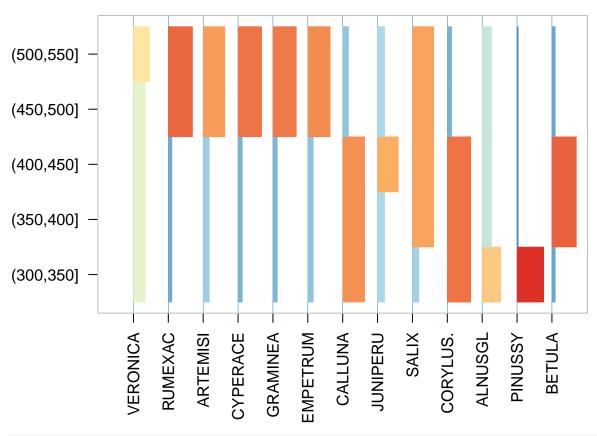
LI

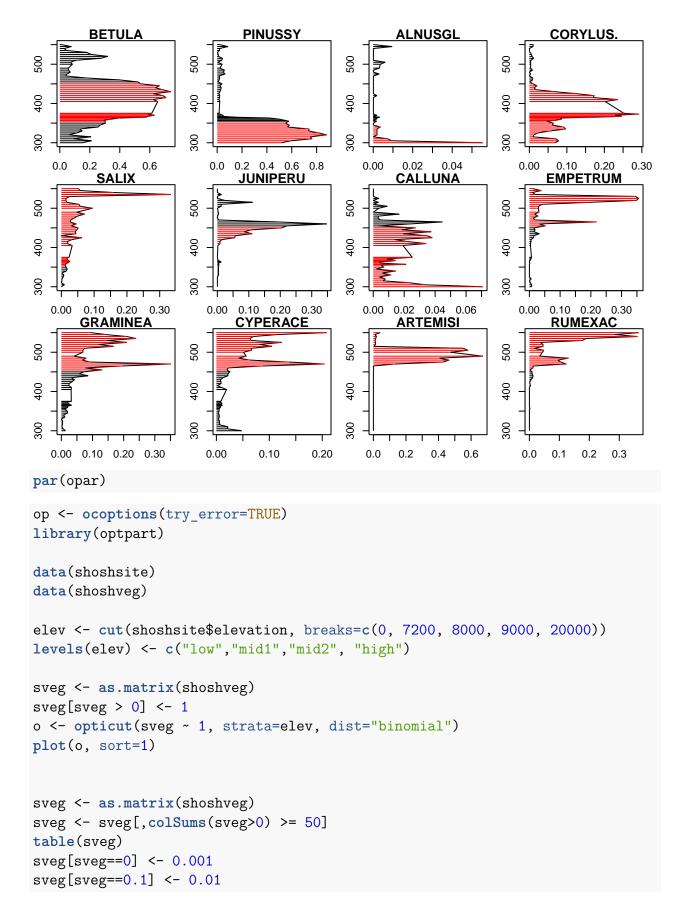
TL

This is rioja 0.9-9

```
data(aber)
strat.plot(aber$spec, aber$ages$Depth, scale.percent=TRUE, y.rev=TRUE)
                                   GRAMINE
300
350
400
450
500
550
     z <- as.factor(cut(aber$ages$Depth, 5))</pre>
ab <- as.matrix(aber$spec) / 100
ab[ab == 0] <- 0.0001
ab \leftarrow ab[,apply(ab, 2, max) > 0.05]
a <- opticut(ab, strata=z, comb="rank", dist="beta")</pre>
summary(a)
## Multivariate opticut results, comb = rank, dist = beta
##
## Call:
## opticut.default(Y = ab, strata = z, dist = "beta", comb = "rank")
## Best supported models with logLR >= 2:
##
                                                             Ι
                                            split assoc
                                                                   mu0
           (350,400] (400,450] (450,500] (500,550]
## SALIX
                                                    +++ 0.5335 0.014015
                     (300,350] (350,400] (400,450]
## CORYLUS.
                                                    +++ 0.6883 0.019366
## CALLUNA
                     (300,350] (350,400] (400,450]
                                                    +++ 0.5802 0.005234
                               (350,400] (400,450]
## BETULA
                                                    +++ 0.7576 0.153662
```

```
(450,500] (500,550]
## RUMEXAC
                                                    +++ 0.7376 0.015648
## CYPERACE
                                (450,500] (500,550]
                                                     +++ 0.6909 0.015826
## GRAMINEA
                                (450,500] (500,550]
                                                      +++ 0.6798 0.028003
## EMPETRUM
                                (450,500] (500,550]
                                                     +++ 0.6048 0.020863
## ARTEMISI
                                (450,500] (500,550]
                                                      ++ 0.5586 0.046111
## PINUSSY
                                          (300,350]
                                                    +++ 0.9142 0.083196
## ALNUSGL
                                          (300, 350]
                                                      ++ 0.3289 0.002089
                                          (400, 450]
                                                      ++ 0.4933 0.025317
## JUNIPERU
##
                 mu1 logLR
## SALIX
            0.044639 8.393 0.5344
## CORYLUS. 0.096616 17.133 0.9940
## CALLUNA 0.019421 12.719 0.5430
## BETULA
            0.568254 27.575 1.0000
## RUMEXAC 0.095248 15.892 0.9888
## CYPERACE 0.080857 24.351 1.0000
## GRAMINEA 0.131311 21.569 0.9914
## EMPETRUM 0.079641 10.171 0.6500
## ARTEMISI 0.145790 7.025 0.9708
## PINUSSY 0.669499 26.667 1.0000
## ALNUSGL 0.004128 2.081 0.4201
## JUNIPERU 0.071110 4.017 0.6459
## 4 binary splits
## 1 species not shown
plot(a, sort=FALSE, horizontal=FALSE, pos=1, upper=0.8,
    show_I=FALSE, show_S=FALSE, mar=c(6,6,1,1), xlab="", ylab="")
```





```
sveg[sveg==0.5] < -0.05
sveg[sveg==1] \leftarrow 0.15
sveg[sveg==2] \leftarrow 0.25
sveg[sveg==3] \leftarrow 0.35
sveg[sveg==4] \leftarrow 0.45
sveg[sveg==5] \leftarrow 0.55
sveg[sveg==6] <- 0.65
sveg[sveg==7] \leftarrow 0.75
sveg[sveg==8] <- 0.8
table(sveg)
o2 <- opticut(sveg ~ 1, strata=elev, dist="beta")
plot(o2, sort=1)
## ZI-Beta
zi_beta_fun <- function(Y, X, linkinv, ...) {</pre>
    kx \leftarrow ncol(X)
    id1 <- Y > 0
    id0 <- !id1
    nll_ZIB_ML <- function(parms) {</pre>
         mu <- plogis(X %*% parms[1:kx])</pre>
         gamma <- exp(parms[kx + 1]) # precision</pre>
         phi <- plogis(parms[kx+2])</pre>
         alpha <- mu * gamma
         beta <- (1 - mu) * gamma
         loglik0 <- log(phi)</pre>
         loglik1 <- log(1 - phi) + suppressWarnings(dbeta(Y,</pre>
             alpha, beta, log = TRUE))
         loglik <- sum(loglik0[id0]) + sum(loglik1[id1])</pre>
         if (!is.finite(loglik) || is.na(loglik))
             loglik <- -.Machine$double.xmax^(1/3)</pre>
         -loglik
    }
    Yv <- Y
    Yv[Y \le 0.001] < 0.001
    ini <- c(coef(betareg::betareg(Yv ~ .-1, data=X)), zi=-5)</pre>
    X <- as.matrix(X)</pre>
    res <- optim(ini, nll ZIB ML, ...)
    list(coef=res$par,
         logLik=-res$value,
         linkinv=binomial("logit")$linkinv)
}
sveg <- as.matrix(shoshveg)</pre>
sveg <- sveg[,colSums(sveg>0) >= 50]
table(sveg)
```

```
#sveg[sveg==0] <- 0.001
sveg[sveg==0.1] <- 0.01
sveg[sveg==0.5] <- 0.05
sveg[sveg==1] <- 0.15
sveg[sveg==2] \leftarrow 0.25
sveg[sveg==3] \leftarrow 0.35
sveg[sveg==4] \leftarrow 0.45
sveg[sveg==5] \leftarrow 0.55
sveg[sveg==6] \leftarrow 0.65
sveg[sveg==7] \leftarrow 0.75
sveg[sveg==8] <- 0.8
table(sveg)
o2 <- opticut(sveg ~ 1, strata=elev, dist=zi_beta_fun)
plot(o2, sort=1)
y <- as.matrix(varespec / 100)
range(y[y>0])
y \leftarrow y[,apply(y, 2, max) > 0.05]
zi_beta_fun(y[,3], data.frame(matrix(1, nrow(y), 1)))
opticut1(y[,1], matrix(1, nrow(y), 1), varechem$grazing, dist=zi beta fun)
sveg2 <- as.matrix(shoshveg)</pre>
sveg2[sveg2 > 0] <- 1
sveg2 <- sveg2[,colnames(sveg)]</pre>
o3 <- opticut(sveg2 ~ 1, strata=elev, dist="binomial")
plot(o3, sort=1)
sveg2 <- as.matrix(shoshveg)</pre>
sveg2[sveg2 > 0] <- 1
sveg2 <- sveg2[,colnames(sveg)]</pre>
Y <- shoshveg$ASTMIS
table(Y)
o4 <- opticut(Y ~ 1, strata=elev, dist="ordered")
o4$species
## check for modifier variable
ocoptions(op)
```

Presence-only (use-availability) data

```
library(ResourceSelection)
data(goats)
slp <- cut(goats$SLOPE,c(-1, 20, 30, 40, 50, 90))
table(slp, goats$STATUS)
o <- opticut(STATUS ~ ELEVATION, data=goats, strata=slp, dist="rsf")
o <- opticut(goats$STATUS, strata=slp, dist="rsf")
o$species
plot(o, sort=FALSE)</pre>
```

Mixed-effects models (LMM, GLMM)

Here is an example using mixed models and the package **lme4**:

```
library(lme4)
```

Loading required package: Matrix

```
set.seed(1234)
n <- 200
x0 \leftarrow sample(1:4, n, TRUE)
x1 <- ifelse(x0 %in% 1:2, 1, 0)
x2 <- rnorm(n, 0.5, 1)
ee <- rnorm(n/5)
g \leftarrow rep(1:5, each=n/5)
lam1 \leftarrow exp(0.5 + 0.5*x1 + -0.2*x2 + ee[g])
Y1 <- rpois(n, lam1)
X <- model.matrix(~x2)</pre>
Z \leftarrow allComb(x0)
lmefun <- function(Y, X, linkinv, gr, ...) {</pre>
    X <- as.matrix(X)</pre>
    m <- glmer(Y ~ X-1 + (1|gr), family=poisson("log"), ...)
    list(coef=fixef(m),
         logLik=logLik(m),
         linkinv=family(m)$linkinv)
lmefun(Y1, X, gr=g)
```

```
##
## $logLik
## 'log Lik.' -345.1799 (df=3)
##
## $linkinv
## function (eta)
## pmax(exp(eta), .Machine$double.eps)
## <environment: namespace:stats>
opticut1(Y1, X, Z, dist=lmefun, gr=g)
## Univariate opticut results, comb = all, dist = lmefun
## I = 0.2518; w = 0.9985; H = 0.997; logL null = -345.2
##
## Best supported models with logLR >= 2:
      assoc
                  Ι
                      mu0
                            mu1
                                logLR
## 1 2
       +++ 0.2518 1.480 2.476 13.582 0.9984969
## 1
         ++ 0.2120 1.745 2.683 6.864 0.0012072
         -- 0.1792 2.168 1.509 4.739 0.0001441
## 4
         -- 0.1805 2.134 1.482 4.620 0.0001279
## 3
         ++ 0.1242 1.858 2.386 2.813 0.0000210
## 7 binary splits (2 models not shown)
```

Generalized additive models (GAM)

```
library(mgcv)
## Loading required package: nlme
##
## Attaching package: 'nlme'
## The following object is masked from 'package:lme4':
##
##
       lmList.
## This is mgcv 1.8-15. For overview type 'help("mgcv-package")'.
library(detect)
## Loading required package: Formula
## Loading required package: stats4
## detect 0.4-0
                     2016-03-02
data(oven)
oven$veg <- factor(NA, c("agr", "open", "decid", "conif", "mix"))</pre>
```

```
oven$veg[oven$pforest < 0.5] <- "open"</pre>
oven$veg[oven$pagri > 0.5 & oven$pforest < 0.5] <- "agr"
oven$veg[oven$pforest >= 0.5] <- "mix"</pre>
oven$veg[oven$pforest >= 0.5 & oven$pdecid >= 0.8] <- "decid"</pre>
oven$veg[oven$pforest >= 0.5 & oven$pdecid < 0.2] <- "conif"</pre>
table(oven$veg, useNA="always")
##
##
          open decid conif
                              mix <NA>
##
     530
            33
                   78
                         30
                              220
oven$xlat <- scale(oven$lat)</pre>
oven$xlong <- scale(oven$long)</pre>
gamfun <- function(Y, X, linkinv, Data, ...) {</pre>
    X <- as.matrix(X)</pre>
    m \leftarrow mgcv::gam(Y \sim X-1 + s(xlat) + s(xlong), Data, ...)
    list(coef=coef(m),
        logLik=logLik(m),
        linkinv=family(m)$linkinv)
}
x <- ifelse(oven$veg=="agr",1,0)
X <- model.matrix(~x)</pre>
gamfun(oven$count, X, Data=oven, family=poisson)
## $coef
## X(Intercept)
                           Хx
                                 s(xlat).1
                                               s(xlat).2
                                                             s(xlat).3
## -0.306661304 -1.382449156 -0.555089095 -0.890381109 0.306380012
##
      s(xlat).4
                   s(xlat).5
                                 s(xlat).6
                                               s(xlat).7
                                                             s(xlat).8
   1.122776013 0.132117780 -0.052077672 0.036702449 -3.041886993
##
                                              s(xlong).3
      s(xlat).9
                  s(xlong).1
                                s(xlong).2
                                                            s(xlong).4
##
  0.955525315 -1.090347031 1.922368963 -0.004724731 -0.428722334
##
##
     s(xlong).5
                  s(xlong).6
                                s(xlong).7
                                              s(xlong).8
                                                            s(xlong).9
##
   0.369410432 0.241421901 -0.228281375 -0.857581486 0.995878919
##
## $logLik
## 'log Lik.' -730.8404 (df=16.34444)
##
## $linkinv
## function (eta)
## pmax(exp(eta), .Machine$double.eps)
## <environment: namespace:stats>
print(opticut1(oven$count, X=X[,1,drop=FALSE], oven$veg, dist=gamfun,
    Data=oven, family=poisson), cut=-Inf)
```

Univariate opticut results, comb = rank, dist = gamfun

```
## I = 0.5988; w = 0.9985; H = 0.997; logL null = -777.7
##
## Best supported models with logLR >= -Inf:
                        assoc
                                   Ι
                                        mu0
                                               mu1 logLR
                          +++ 0.5988 0.1847 0.7359 46.906 9.985e-01
## open decid conif mix
## decid conif mix
                          +++ 0.5801 0.2049 0.7712 40.373 1.451e-03
## decid mix
                          +++ 0.5211 0.2298 0.7298 37.182 5.973e-05
                           ++ 0.2455 0.3462 0.5714 6.942 4.396e-18
## decid
## 4 binary splits
o <- opticut(count ~ 1, oven, strata=veg, dist=gamfun, Data=oven, family=poisson)
summary(o)
## Multivariate opticut results, comb = rank, dist = gamfun
##
## Call:
## opticut.formula(formula = count ~ 1, data = oven, strata = veg,
##
       dist = gamfun, Data = oven, family = poisson)
##
## Best supported model with logLR >= 2:
##
                       split assoc
                                        Ι
                                             mu0
                                                    mu1 logLR
## Sp 1 open decid conif mix
                               +++ 0.5988 0.1847 0.7359 46.91 0.9985
## 4 binary splits
o <- opticut(count ~ 1, oven, strata=veg, dist="poisson")
summary(o)
## Multivariate opticut results, comb = rank, dist = poisson
##
## Call:
## opticut.formula(formula = count ~ 1, data = oven, strata = veg,
##
       dist = "poisson")
##
## Best supported model with logLR >= 2:
##
                       split assoc
                                        Ι
                                                   mu1 logLR
                                             muO
                               +++ 0.6903 0.1868 1.019 142.7 0.9998
## Sp 1 open decid conif mix
## 4 binary splits
oven$pa <- ifelse(oven$count > 0, 1, 0)
o <- opticut(pa ~ 1, oven, strata=veg, dist=gamfun, Data=oven, family=binomial)
summary(o)
## Multivariate opticut results, comb = rank, dist = gamfun
##
## Call:
## opticut.formula(formula = pa ~ 1, data = oven, strata = veg,
       dist = gamfun, Data = oven, family = binomial)
##
```

```
##
## Best supported model with logLR >= 2:
##
                       split assoc
                                        Ι
                                             mu0
                                                    mu1 logLR
## Sp 1 open decid conif mix
                               +++ 0.6982 0.1421 0.4824 28.84 0.919
## 4 binary splits
o <- opticut(pa ~ 1, oven, strata=veg, dist="binomial")
summary(o)
## Multivariate opticut results, comb = rank, dist = binomial
##
## Call:
## opticut.formula(formula = pa ~ 1, data = oven, strata = veg,
       dist = "binomial")
##
##
## Best supported model with logLR >= 2:
##
                       split assoc
                                        Ι
                                             mu0
                                                    mu1 logLR
## Sp 1 open decid conif mix +++ 0.7582 0.1377 0.5374 82.37 0.6569
## 4 binary splits
```

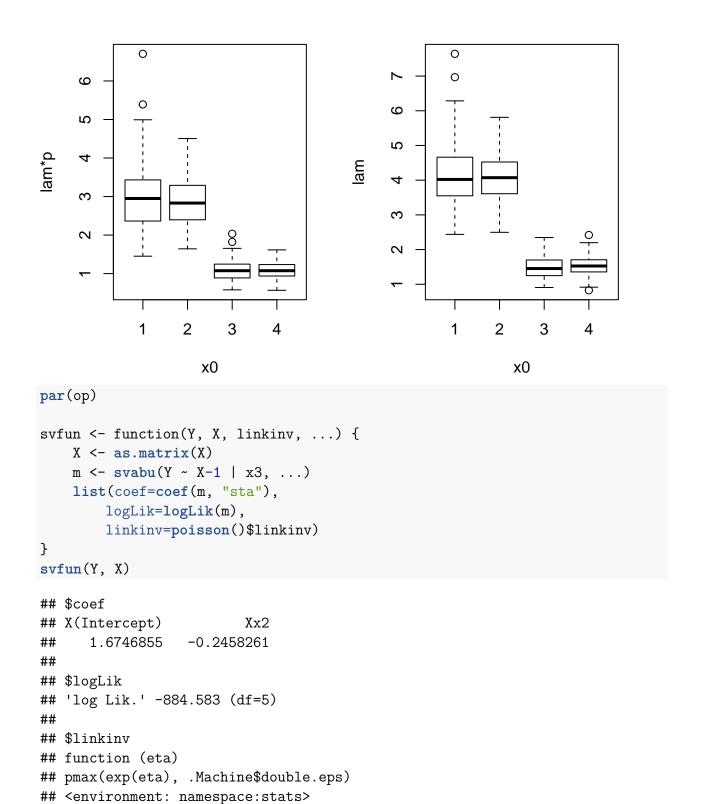
N-mixture models

A single-visit based N-mixture is an example where detection error is estimated. Let us compare results based on naive GLM and N-mixture:

```
library(detect)
set.seed(2345)
n <- 500
x0 <- sample(1:4, n, TRUE)
x1 <- ifelse(x0 %in% 1:2, 1, 0)
x2 <- rnorm(n, 0.5, 1)
x3 <- runif(n, 0, 1)
lam <- exp(0.5 + 1*x1 + -0.2*x2)
p <- plogis(2 + -2*x3)
Y <- rpois(n, lam*p)

X <- model.matrix(~x2)

op <- par(mfrow=c(1,2))
boxplot((lam*p) ~ x0, ylab="lam*p", xlab="x0")
boxplot(lam ~ x0, ylab="lam", xlab="x0")</pre>
```



```
## Univariate opticut results, comb = rank, dist = poisson ## I = 0.474; w = 1; H = 1; logL_null = -930
```

print(opticut1(Y, X, as.factor(x0), dist="poisson"), cut=-Inf)

naive GLM

```
##
## Best supported models with logLR >= -Inf:
##
                    Ι
                       mu0
                             mu1 logLR
        assoc
## 1 2
          +++ 0.4740 1.139 3.193 115.16 1.000e+00
## 1 2 4 +++ 0.4082 1.078 2.566 52.38 5.383e-28
          +++ 0.2842 1.808 3.243 38.04 3.211e-34
## 3 binary splits
## N-mixture
print(opticut1(Y, X, as.factor(x0), dist=svfun), cut=-Inf)
## Univariate opticut results, comb = rank, dist = svfun
## I = 0.4873; w = 1; H = 0.9999; logL null = -884.6
## Best supported models with logLR >= -Inf:
       assoc
                   Ι
                       muO
                             mu1 logLR
## 1 2
          +++ 0.4873 2.409 6.989 26.239 1.000e+00
## 1 2 3 +++ 0.4303 1.936 4.859 15.787 2.888e-05
           ++ 0.2475 4.142 6.866 3.284 1.073e-10
## 3 binary splits
```

Package options

High performance computing

```
data(dolina)
dolina$samp$stratum <- as.integer(dolina$samp$stratum)</pre>
Y <- ifelse(dolina$xtab > 0, 1, 0)
dol <- opticut(Y ~ stratum + lmoist + method, data=dolina$samp,</pre>
    strata=dolina$samp$mhab, dist="binomial")
## parallel computing comparisons
library(parallel)
cl <- makeCluster(2)</pre>
## sequential, all combinations (2^(K-1) - 1)
system.time(opticut(Y ~ stratum + lmoist + method, data=dolina$samp,
    strata=dolina$samp$mhab, dist="binomial", comb="all", cl=NULL))
##
      user system elapsed
     2.275
            0.047
                     2.376
##
## sequential, rank based combinations (K - 1)
system.time(opticut(Y ~ stratum + lmoist + method, data=dolina$samp,
```

```
strata=dolina$samp$mhab, dist="binomial", comb="rank", cl=NULL))
##
           system elapsed
      user
            0.040
##
     1.665
                     1.746
## parallel, all combinations (2^(K-1) - 1)
system.time(opticut(Y ~ stratum + lmoist + method, data=dolina$samp,
    strata=dolina$samp$mhab, dist="binomial", comb="all", cl=cl))
##
      user system elapsed
##
     0.016
             0.003
                     2.806
## parallel, rank based combinations (K - 1)
system.time(opticut(Y ~ stratum + lmoist + method, data=dolina$samp,
    strata=dolina$samp$mhab, dist="binomial", comb="rank", cl=cl))
##
           system elapsed
     user
##
     0.012
            0.003
                     0.859
## compare timings for uncertainty calculations
system.time(uncertainty(dol, type="asymp", B=999))
##
      user system elapsed
     0.742
            0.014
                     0.770
##
system.time(uncertainty(dol, type="asymp", B=999, cl=cl))
##
           system elapsed
      user
##
     0.029
            0.005
                     0.622
system.time(uncertainty(dol, type="boot", B=4))
##
           system elapsed
     user
##
     2.596
             0.063
                     2.709
system.time(uncertainty(dol, type="boot", B=4, cl=cl))
##
           system elapsed
      user
     0.027
             0.002
                     1.471
##
system.time(uncertainty(dol, type="multi", B=4))
##
      user
            system elapsed
##
     6.505
             0.106
                     6.623
system.time(uncertainty(dol, type="multi", B=4, cl=cl))
##
      user
           system elapsed
##
     0.022
           0.002
                     3.895
```

Global options

The ocoptions function provides a convenient way of handling options related to the **opticut** package. The function takes arguments in <tag> = <value> form, or a list of tagged values. The tags must come from the parameters described below. When parameters are set by ocoptions, their former values are returned in an invisible named list. Such a list can be passed as an argument to ocoptions to restore the parameter values. Tags are the following:

- collapse: character value to be used when merging factor levels, the default is " ".
- cut: log likelihood ratio value, model/species with lower values are excluded from summaries and plots, the default is 2.
- sort: logical value indicating if species/partitions should be meaningfully sorted, the default is TRUE. It can take numeric value when only species (1) or partitions (2) are to be sorted (1:2 is equivalent to TRUE, wile any other numeric value is equivalent to FALSE).
- theme: the color theme to be used based on occolors, the default is "br".
- check_comb: check the design matrices for complementary partitions using checkComb, the default is TRUE.
- try_error: if opticut should try to exclude species where the models failed (TRUE), the default is to stop when an error is encountered (FALSE).
- scale: the scaling factor used to calculate indicator value (I) based on the estimated coefficient (b): I = abs(tanh(b*scale)), the default is 0.5.

```
## List of 7
## $ collapse : chr " "
## $ cut : num 2
## $ sort : logi TRUE
## $ theme : chr "br"
## $ check_comb: logi TRUE
```

```
## $ try error : logi FALSE
## $ scale
              : num 0.5
str(getOption("ocoptions"))
## List of 7
## $ collapse : chr " "
## $ cut
              : num 2
## $ sort
               : logi TRUE
## $ theme
              : chr "br"
## $ check_comb: logi TRUE
## $ try error : logi FALSE
## $ scale
              : num 0.5
summary(ocall <- opticut(spp ~ 1, strata=gr, dist="gaussian", comb="all"))</pre>
## Multivariate opticut results, comb = all, dist = gaussian
##
## Call:
## opticut.formula(formula = spp ~ 1, strata = gr, dist = "gaussian",
      comb = "all")
##
## Best supported models with logLR >= 2:
           split assoc
                            I muO mu1 logLR
                   +++ 0.9866 2.0 7.0 14.82 0.4995
## Species2 X1 X3
## Species1 X2 X3
                   +++ 0.8483 3.0 5.5 17.33 0.4999
                   +++ 1.0000 0.5 18.0 55.19 1.0000
## Species3
              X1
## 15 binary splits
## resetting phoptions and checking new settings
ocop <- ocoptions(collapse="+", sort=FALSE)</pre>
str(ocoptions())
## List of 7
## $ collapse : chr "+"
## $ cut
              : num 2
## $ sort
              : logi FALSE
## $ theme
              : chr "br"
## $ check comb: logi TRUE
## $ try error : logi FALSE
## $ scale
              : num 0.5
## running again with new settings
summary(ocall <- opticut(spp ~ 1, strata=gr, dist="gaussian", comb="all"))</pre>
## Multivariate opticut results, comb = all, dist = gaussian
##
## Call:
```

```
## opticut.formula(formula = spp ~ 1, strata = gr, dist = "gaussian",
       comb = "all")
##
##
## Best supported models with logLR >= 2:
            split assoc
                             I muO mu1 logLR
## Species1 X2+X3
                    +++ 0.8483 3.0 5.5 17.33 0.4999
## Species2 X1+X3
                    +++ 0.9866 2.0 7.0 14.82 0.4995
## Species3
                    +++ 1.0000 0.5 18.0 55.19 1.0000
## 15 binary splits
## resetting original
ocoptions(ocop)
str(ocoptions())
## List of 7
## $ collapse : chr " "
## $ cut
               : num 2
## $ sort
               : logi TRUE
## $ theme
               : chr "br"
## $ check_comb: logi TRUE
## $ try error : logi FALSE
## $ scale
               : num 0.5
```

Color themes

The **opticut** package uses color themes for plotting and provides a convenient way of setting color palettes via the **occolors** function. The function takes a single theme argument and returns a function as in **colorRampPalette**.

The theme argument can be a character value, character vector, or a function used to interpolate the colors. The built-in values are "br" (blue-red divergent palette, colorblind safe), "gr" (green-red divergent palette), "bw" (black and white). Hexadecimal values for the built-in palettes are taken from http://colorbrewer2.org.

```
plot(1:100, rep(2, 100), pch = 15,
    ylim = c(0, 21), axes = FALSE, ann = FALSE,
    col = occolors()(100)) # default 'bg'

text(50, 1, "theme = 'br'")
points(1:100, rep(5, 100), pch = 15,
    col=occolors("gr")(100))

text(50, 4, "theme = 'gr'")
points(1:100, rep(8, 100), pch = 15,
    col=occolors("bw")(100))

text(50, 7, "theme = 'bw'")
points(1:100, rep(11, 100), pch = 15,
```

```
col=occolors(terrain.colors)(100))
text(50, 10, "theme = terrain.colors")
points(1:100, rep(14, 100), pch = 15,
    col=occolors(c("purple", "pink", "orange"))(100))
text(50, 13, "theme = c('purple', 'pink', 'orange')")
points(1:100, rep(17, 100), pch = 15,
    col=occolors(c("#a6611a", "#ffffbf", "#018571"))(100))
text(50, 16, "theme = c('#a6611a', '#ffffbf', '#018571')")
points(1:100, rep(20, 100), pch = 15,
    col=occolors(c("#7b3294", "#ffffbf", "#008837"))(100))
text(50, 19, "theme = c('#7b3294', '#ffffbf', '#008837')")
            theme = c('#7b3294', '#ffffbf', '#008837')
            theme = c('#a6611a', '#ffffbf', '#018571')
              theme = c('purple', 'pink', 'orange')
                    theme = terrain.colors
                        theme = 'bw'
                        theme = 'gr'
```

Progress bar

The expected completion time of extensive calculations and the progress is shown by the progress bar via the **pbapply** package. Default options with **opticut** are:

theme = 'br'

```
str(pboptions())
```

```
## List of 10
   $ type
              : chr "none"
## $ char
              : chr "[=-]"
## $ txt.width: num 50
## $ gui.width: num 300
## $ style
              : num 6
## $ initial : num 0
## $ title
              : chr "R progress bar"
              : chr ""
## $ label
## $ nout
              : int 100
## $ min_time : num 2
```

See ?pboptions for a description of these options. Use pboptions (type = "none") to turn

off the probress bar in interactive R sessions. The progress bar is automatically turned off during non-interactive sessions.

Dynamic documents

opticut object summaries come with an as.data.frame method that can be used to turn the summary into a data frame, which is what for example the kable function from knitr package expects. This way, formatting the output isw much facilitated, and the user does not have to dig into the structure of the summary object.

The GitHub repository has a minimal Rmarkdown example do demonstrate how to format opticut objects for best effects: Rmd source, knitted PDF.

```
##
##
          split
                                 Ι
                                      mu0
                                                     logLR
                  assoc
                                              mu1
## ----
## Sp3
          2 3
                             0.647
                                     0.75
                                             3.50
                                                     4.793
                                                              0.696
          3
                                     0.25
                                             3.25
## Sp2
                  +++
                            0.857
                                                     9.203
                                                              0.958
print(kable(oc$species[[1]][,c(1,2,4,5,8,9,10)], digits=3))
```

```
##
##
##
                         Ι
                             mu0
                                    mu1
                                              logL
                                                      logLR
           assoc
## 1
                1
                    0.125
                             3.5
                                    4.5
                                           -22.185
                                                      0.339
                                                               0.239
## 1 2
                1
                    0.286
                             2.5
                                    4.5
                                           -21.026
                                                      1.498
                                                               0.761
print(kable(as.data.frame(uc), digits=3))
```

```
##
##
split R I Lower Upper
```

```
## Sp1
          1 2
                          0.277
                                   0.024
                                            0.564
                     1
## Sp3
          2 3
                          0.622
                                   0.204
                                            0.880
                     1
## Sp2
          3
                     1
                          0.824
                                   0.496
                                            0.962
```

The kable output is rendered as nice tables (without the print part):

kable(as.data.frame(oc), digits=3)

	split	assoc	Ι	mu0	mu1	logLR	W
•						4.793 9.203	

kable(oc\$species[[1]][,c(1,2,4,5,8,9,10)], digits=3)

	assoc	I	mu0	mu1	logL	logLR	W
1	1	0.125	3.5	4.5	-22.185	0.339	0.239
1 2	1	0.286	2.5	4.5	-21.026	1.498	0.761

kable(as.data.frame(uc), digits=3)

	split	R	I	Lower	Upper
Sp1	1 2	1	0.277	0.024	0.564
Sp3	2 3	1	0.622	0.204	0.880
Sp2	3	1	0.824	0.496	0.962

Summary

The likelihood-based optimal partitioning framework implemented in the opticut R package provides a compelling alternative to other available R packages (**indicspecies**, De Caceres & Legendre 2009; **labdsv** and **optpart**, Roberts 2016a, 2016b; **vegan**, Oksanen et al. 2016), especially when the type of data or the presence of modifying effects call for an alternative approach.

The package comes with many parametric models included for binary, count, abundance, percent cover, ordinal, and presence-only data, and the approach can be extended to more complex situations, such as mixed models, additive models. The opticut package leverages other R packages (MASS, Venables & Ripley 2002; betareg, Cribari-Neto & Zeileis 2010; pscl, Zeileis et al. 2008; ResourceSelection, Lele et al. 2016) for fitting parametric models. The approach can be extended to linear and generalized linear mixed models (LMM, GLMM; see Kemencei et al. 2014), generalized additive models, N-mixture models.

Computing times are shortened by the application of efficient algorithms and through high performance computing options. The **opticut** package provides progress bars with estimated remaining time for long-running evaluations (through the **pbapply** package, Solymos & Zawadzki 2016), natively supports several parallel back-ends (multicore machines, computing clusters, forking; through the **parallel** package, R Core Team 2016) to speed calculations up, and provides options and methods for dynamic report generation (coercion methods and color themes). Functions in the package can be customized and extended to meet the needs under a wide range of real-world situations.

Please cite the **opticut** package in scholarly publications as:

Peter Solymos and Ermias T. Azeria (2016). opticut: Likelihood Based Optimal Partitioning for Indicator Species Analysis. R package version . https://github.com/psolymos/opticut

Use citation("opticut") for an up-to-date citation.

References

McGeoch MA and Chown SL (1998) Scaling up the value of bioindicators. Trends in Ecology and Evolution 13: 46–47.

Dufrene M and Legendre P (1997) Species assemblages and indicator species: The need for a flexible asymmetrical approach. Ecological Monographs 67: 345–366.

De Caceres M and Legendre P (2009) Associations between species and groups of sites: Indices and statistical inference. Ecology 90: 3566–3574.

O. Wildi and E. Feldmeyer-Christe 2013. Indicator values (IndVal) mimic ranking by F-ratio in real-world vegetation data. COMMUNITY ECOLOGY 14(2): 139–143.

Chytry, Milan; Tichy, Lubomir; Holt, Jason & Botta-Dukat, Zoltan 2002. Determination of diagnostic species with statistical fidelity measures. Journal of Vegetation Science 13: 79–90.

Halme, P., Monkkonen, M., Kotiaho, J. S, Ylisirniö, A-L. 2009. Quantifying the indicator power of an indicator species. Conservation Biology 23: 1008–1016.

Zettler ML, Proffitt CE, Darr A, et al. On the Myths of Indicator Species: Issues and Further Consideration in the Use of Static Concepts for Ecological Applications. Thompson F, ed. PLoS ONE. 2013;8(10):e78219. doi:10.1371/journal.pone.0078219.

Kemencei, Z., Farkas, R., Pall-Gergely, B., Vilisics, F., Nagy, A., Hornung, E. & Solymos, P. (2014): Microhabitat associations of land snails in forested dolinas: implications for coarse filter conservation. Community Ecology 15:180–186.

Jari Oksanen, F. Guillaume Blanchet, Michael Friendly, Roeland Kindt, Pierre Legendre, Dan McGlinn, Peter R. Minchin, R. B. O'Hara, Gavin L. Simpson, Peter Solymos, M. Henry H. Stevens, Eduard Szoecs and Helene Wagner (2016). vegan: Community Ecology Package. R package version 2.4-1. https://CRAN.R-project.org/package=vegan

Achim Zeileis, Christian Kleiber, Simon Jackman (2008). Regression Models for Count Data in R. Journal of Statistical Software 27(8). URL http://www.jstatsoft.org/v27/i08/

Francisco Cribari-Neto, Achim Zeileis (2010). Beta Regression in R. Journal of Statistical Software 34(2), 1-24. URL http://www.jstatsoft.org/v34/i02/.

Venables, W. N. & Ripley, B. D. (2002) Modern Applied Statistics with S. Fourth Edition. Springer, New York. ISBN 0-387-95457-0

David W. Roberts (2016a). labdsv: Ordination and Multivariate Analysis for Ecology. R package version 1.8-0. https://CRAN.R-project.org/package=labdsv

David W. Roberts (2016b). optpart: Optimal Partitioning of Similarity Relations. R package version 2.3-0. https://CRAN.R-project.org/package=optpart

Subhash R. Lele, Jonah L. Keim and Peter Solymos (2016). ResourceSelection: Resource Selection (Probability) Functions for Use-Availability Data. R package version 0.3-0. https://CRAN.R-project.org/package=ResourceSelection

R Core Team (2016). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

Peter Solymos and Zygmunt Zawadzki (2016). pbapply: Adding Progress Bar to '*apply' Functions. R package version 1.3-2. https://github.com/psolymos/pbapply