Optimization Algorithms

Sheet — Spring 21

Problem Modeling

Disclaimer: All variables used in all questions $\in \mathbb{Z}^+$ unless otherwise is stated explicitly.

1. **Variables definition:**

   (a) Input variables

   - n: Total number of users.
   - dl: distance limit to match users.
   - c_i: Car Capacity for driver i.
   - $dist_{i,j}$: distance between user i and user j.
   - pay_i: the max money passenger i can pay.
   - $fare_i$: money charged by the driver i per passenger
   - $minc_i$: the min number of passengers to travel with the driver i.

   (b) Decision variables

   - $x_{i,j}$: Binary value indicates whether passenger j is matched with driver i.
   - d_i: Binary value indicates whether user i is a driver.
   - p_i: Binary value indicates whether user i is a passenger.

   $1 \leq i \leq n$, $1 \leq j \leq n$.

2. **Objective:**

   Maximize the number of matches, and hence, the number of travellers

$$max(\sum_{i=1}^{n}\sum_{j=1}^{n} x_{i,j}) \tag{1}$$

3. **subject to:**

   (a) If a user is a driver, he/she is matched with at most c_i passengers.

$$\forall i \in [1,n], \sum_{j=1}^{n} x_{i,j} \leq c_i \times d_i \tag{2}$$

   (b) If a user is a passenger, he/she is matched with at most 1 driver

$$\forall j \in [1,n], \sum_{i=1}^{n} x_{i,j} \leq p_j \tag{3}$$

   (c) Every user is either a driver or a passenger

$$\forall i \in [1,n], d_i + p_i \leq 1 \tag{4}$$

(d) Any matched driver and passenger should be within a distance of distance_limit from each other

$$\forall i, j \in [1, n], x_{i,j} \times dist_{i,j} \leq dl \tag{5}$$

(e) Passenger can pay the fare that is charged by the matched driver

$$\forall i, j \in [1, n], p_j \times pay_j \geq x_{i,j} \times fare_i \tag{6}$$

(f) $driver_i$ has at least $minc_i$ matched passengers

$$\forall i \in [1, n], \sum_{j=1}^{n} x_{i,j} \geq minc_i \times d_i \tag{7}$$

Note: $minc_i$ is used in this constraint to ensure that all passengers with the same driver pay the same cost.