1 EKF in a nutshell

Extended Kalman Filter (EKF) is an modification of the well known Kalman Filter for the case of nonlinear dynamics. The Kalman Filter itself is an optimal state estimation algorithm for linear dynamics and measurement models which experience zero-mean Gaussian noise. In the EKF context however, neither the dynamical model nor the measurement model have to be linear, nonetheless, linearized dynamics and measurements models are still required within the filter. One dynamical system can be described as follows:

$$\dot{\mathbf{x}} = f(t, \mathbf{x}) + \mathbf{w} \tag{1}$$

in which the function $f : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ and $\mathbf{w}$ is a vector of n normally distributed random variables with zero mean and a covariance matrix $\mathbf{Q}$. The same system can be written in discrete time as:

$$\mathbf{x}_{k+1} = \mathbf{F}(t_k, t_{k+1}, \mathbf{x})\mathbf{x} + \mathbf{w}_k \tag{2}$$

where $\mathbf{F} : \mathbb{R} \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^{n \times n}$is the state transition matrix. A first order approximation of the state transition matrix is found below:

$$\mathbf{F}(t_k, t_{k+1}, \mathbf{x}) = \mathbf{I} + \frac{\partial f(t, \mathbf{x})}{\partial t} \cdot (t_k - t_{k+1}) \tag{3}$$

The measurements are modeled as follows:

$$\mathbf{z} = h(t, \mathbf{x}) + \mathbf{v} \tag{4}$$

in which the function $h : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^m$ and $\mathbf{v}$ is a vector of m normally distributed random variables with zero mean and a covariance matrix $\mathbf{R}$. One other important matrix that needs to be available for the EKF algorithm is the $H(t, \mathbf{x}) = \frac{\partial h(t,\mathbf{x})}{\partial t}$.

2 Example details

In the context of this example, the position and velocity of a vehicle moving along a line are to be estimated. The linear acceleration of the system is modeled follows:

$$\ddot{x} = -\sin(t) \tag{5}$$

letting $x_1 = x$ and $x_2 = \dot{x}$, the dynamical model can be written as:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} x_2 \\ -\sin(t) \end{bmatrix} \tag{6}$$

The state transition matrix of the system is written according to the approximation in (3) as:

$$\mathbf{F}(t_k, t_{k+1}, \mathbf{x}) = \begin{bmatrix} 1 & t_k - t_{k+1} \\ 0 & 1 \end{bmatrix} \tag{7}$$

The system is assumed to measure only the position of the vehicle, hence:

$$\mathbf{z} = x_1 \quad \Rightarrow \quad H(t, \mathbf{x}) = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The ground truth of the system is synthesized by analytically solving (5) and adding zero-mean Gaussian noise to the velocity (which in-turn affects the position), while the measurements where synthesized by adding zero-mean Gaussian noise to the position ground truth.