Antenna Azimuth Position Control System

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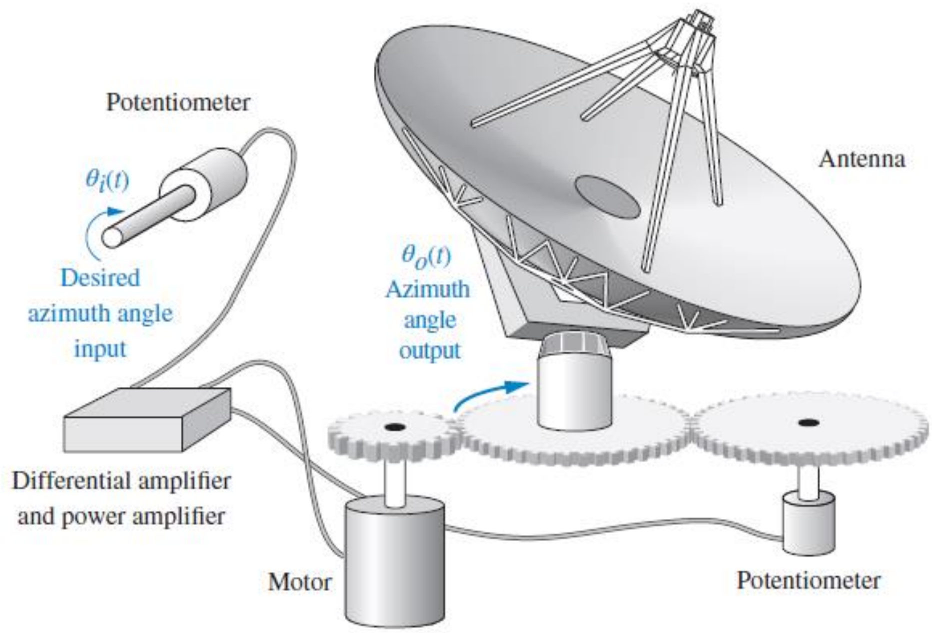
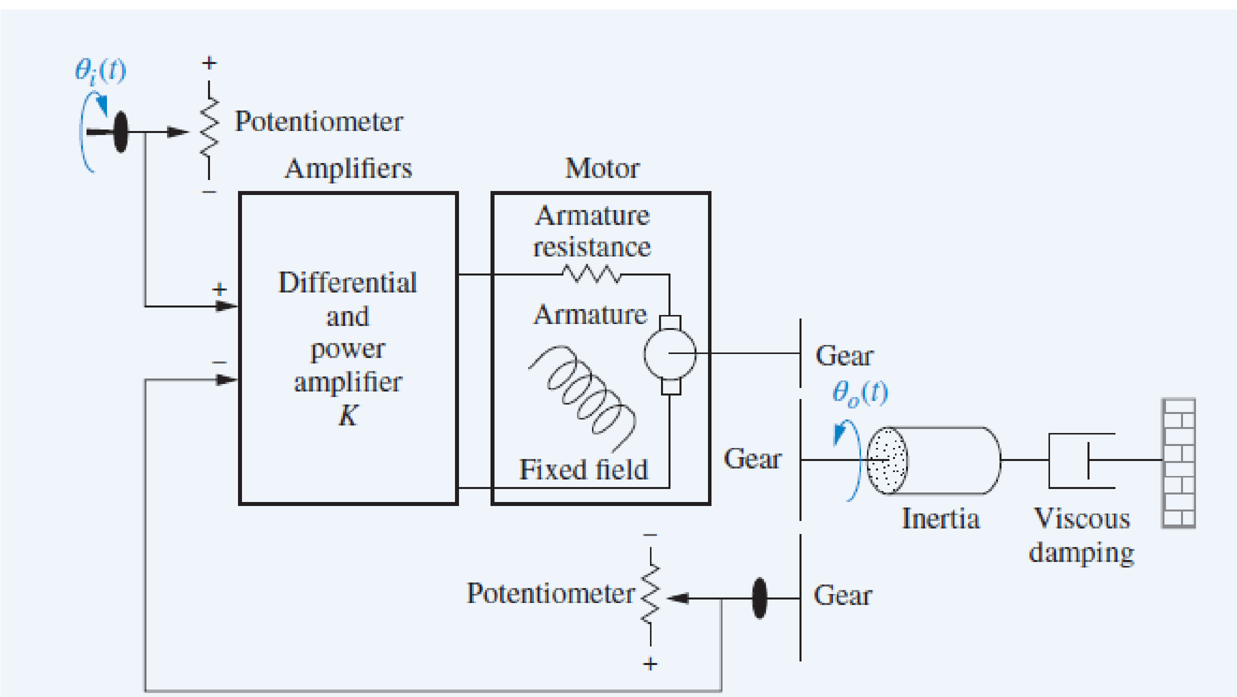
Layout of the System:



Schematic:



Transfer functions of subsystems

Input and Output Potentiometer

- The input azimuthal angle is fed manually to the potentiometer which in turn transforms the angular displacement into voltage.
- The input and output potentiometers are identical so they will have the same transfer functions.

Evaluation of the transfer function:

Assumptions:

1. Let us assume an n-turns potentiometer that is maintained at a voltage $+V$ and $-V$ at its two terminals.
2. $\frac{n}{2}$ turns to the right or the left is equivalent to a voltage change V.

From the previous assumptions

$$\frac{V(s)}{\theta(s)} = \frac{V}{\frac{n}{2}\cdot 2\pi} = \frac{V}{n\pi}$$

where $V(s)$ is the voltage on the potentiometer and $\theta(s)$ is the Azimuthal angle.

Preamplifier

- The preamplifier is composed of two stages, the first stage is a differential amplifier and the second stage is a power amplifier.
- The differential amplifier acts as a summing junction with an additional gain.
- The purpose of the power amplifier is to limit power dissipation ensuring high power efficiency.

Evaluation of the transfer function:

Assumptions:

1. Under ideal conditions, the differential amplifier is a gain stage for the differential signal that is frequency invariant.
2. There is a wide range of power amplifiers, but for simplicity we will assume a single finite pole and no finite zeros power amplifiers.

$$G_1(s) = K$$

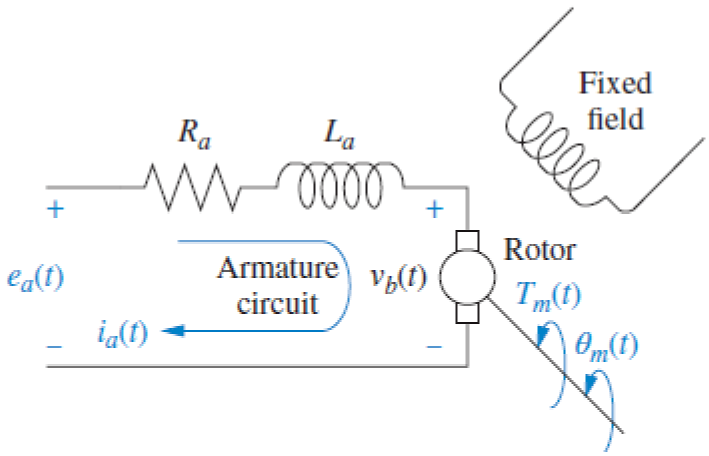
$$G_2(s) = \frac{K_1}{s+a}$$

$$G(s) = G_1(s)\cdot G_2(s) = \frac{KK_1}{s+a}$$

Armature Controlled DC Motor and Load

The motor is an electromechanical device which converts the input voltage into mechanical displacement.

The current $i_a(t)$ results in the rotation of the rotor.



schematic for the circuit model of the motor

Evaluation of the transfer function:

The electromotive force generated at the terminals of conductor moving normal to a constant magnetic field is given by

$$EMF = Blv \quad \textbf{(1)}$$

where B is the magnetic field intensity, l is the length of the conductor and v is the velocity of the conductor.

From equation (1)

$$v_b(t) = k_b \frac{d\theta_m(t)}{dt}$$

Using the transformed circuit and Kirchhoff's Loop rule

$$K_b s\theta_m(s) + I_a(s)R + I_a(s)sL = E_a(s) \quad (\mathbf{2})$$

The torque developed on the rotor is proportional to the armature current

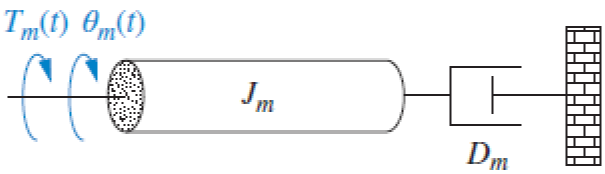
$$T_m(t) = K_t i_a(t) \quad (\mathbf{3})$$

k_t is a proportionality constant that relates depends on the motor and the magnetic field characteristics.

By substituting the Laplace transform of equation **(3)** into equation **(2)**

$$K_b s\theta_m(s) + \frac{T_m(s)}{k_t}R_a + \frac{T_m(s)}{k_t}sL_a = E_a(s) \quad (\mathbf{4})$$

Next, we will relate $T_m(t)$ and $\theta_m(t)$



Equivalent inertia and viscous damping of the armature and the load

where J_m includes both the armature and load inertia and D_m includes both the armature and load viscous damping.

$$\frac{T_m(s)}{\theta_m(s)} = J_m s^2 + D_m s \quad (\mathbf{5})$$

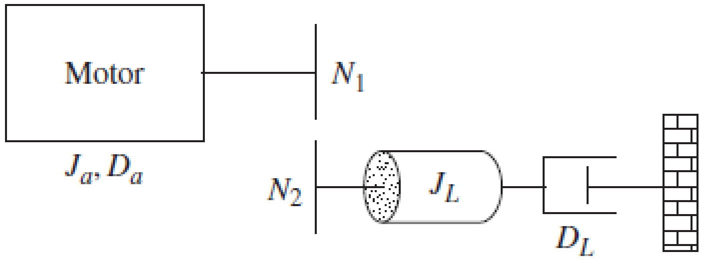
By substituting the Laplace transform of equation **(5)** into equation **(4)**

$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{\frac{1}{K_t}(sL_a + R_a)(J_m s^2 + D_m s) + K_b s}$$

By assuming $L \ll R$ and simplifying

$$\frac{\theta_m(s)}{E_a(s)} = \frac{{K_t}/{J_m R_a}}{s(s + \frac{1}{J_m}\left(D + \frac{K_b K_t}{R}\right))} = \frac{K}{s(s + a_m)}$$

J_m and D_m are calculated as follows



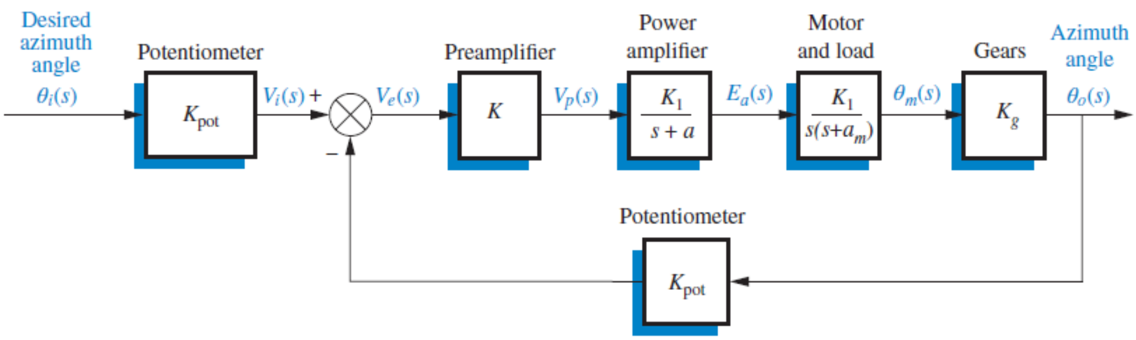
Motor and load

From Fig (e)

$$J_m = J_a + J_L(\frac{N_1}{N_2})^2$$

$$D_m = D_a + D_L(\frac{N_1}{N_2})^2$$

Block Diagram of the system



By pushing the input potentiometer past the summing junction, the system becomes a unity feedback system whose open loop transfer function is:

$$G(s) = \frac{K_{pot}KK_1K_mK_g}{s(s+a)(s+a_m)}$$

Then the closed loop transfer function is:

$$T(s) = \frac{K_{pot}KK_1K_mK_g}{s(s+a)(s+a_m) + K_{pot}KK_1K_mK_g}$$

System Configurations

Parameter	Configuration 1	Configuration 2	Configuration 3
V	10	10	10
n	10	1	1
K	-	-	-
K_1	100	150	100
a	100	150	100
R_a	8	5	5
J_a	0.02	0.05	0.05
D_a	0.01	0.01	0.01
K_b	0.5	1	1
K_t	0.5	1	1
N_1	25	50	50
N_2	250	250	250
N_3	250	250	250

J_L	1	5	5
D_L	1	3	3

Block Diagram Parameters:

Parameter	Configuration 1	Configuration 2	Configuration 3
K_{pot}	0.318	3.18	3.18
K	-	-	-
K_1	100	150	100
a	100	150	100
K_m	2.083	0.8	0.8
a_m	1.708	1.32	1.32
K_g	0.1	0.2	0.2

This table is filled using the relations derived earlier.

Configuration 1:

$$G(s) = \frac{6.624K}{s(s+100)(s+1.708)}$$

$$T(s) = \frac{6.624K}{s(s+100)(s+1.708)+6.624K}$$

Configuration 2:

$$G(s) = \frac{76.32K}{s(s+150)(s+1.32)}$$

$$T(s) = \frac{76.32K}{s(s+150)(s+1.32)+76.32K}$$

Configuration 3:

$$G(s) = \frac{50.88K}{s(s+100)(s+1.32)}$$

$$T(s) = \frac{50.88K}{s(s+100)(s+1.32)+50.88K}$$

Steady state error

$$e(\infty) = \frac{sR(s)}{1+G(s)}$$

For a step input:

$$\lim_{s \to 0} G(s) = \infty$$

$$e_{step}(\infty) = 0$$

For a ramp input:

$$\lim_{s \to 0} sG(s) = \frac{K_{pot} K K_1 K_m K_g}{a \cdot a_m}$$

$$e_{ramp}(\infty) = \frac{a \cdot a_m}{K_{pot} K K_1 K_m K_g}$$

For a parabolic input:

$$\lim_{s \to 0} s^2 G(s) = 0$$

$$e_{parabolic}(\infty) = \infty$$

The system is type 1 as it has only one pure integration. Consequently, the steady state error for a step input will always be zero, the steady state error for a ramp input will be finite and the steady state error for a parabolic input will always be infinite.

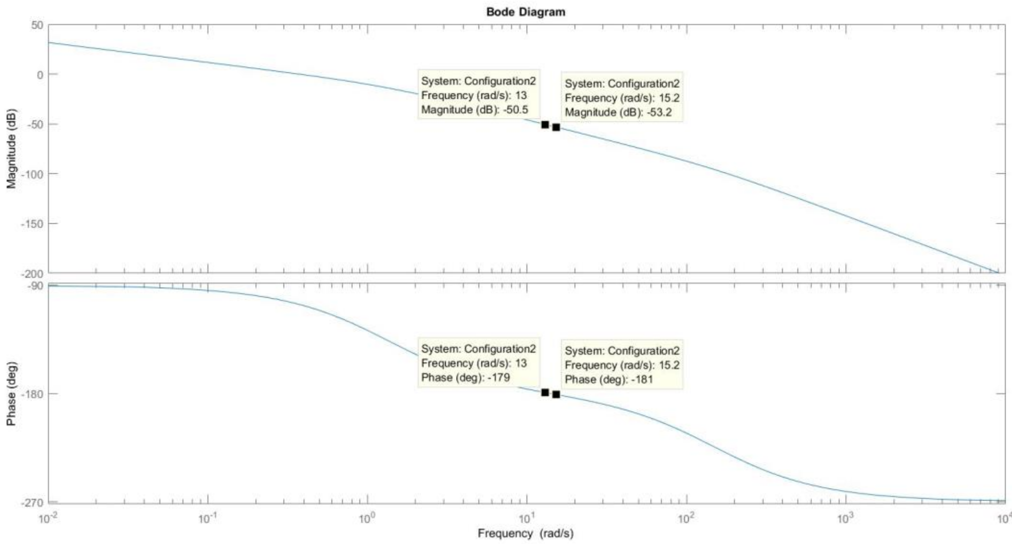
Steady state error in case of a ramp input for each configuration:

$$e_{ramp-1}(\infty) = \frac{25.79}{K}$$

$$e_{ramp-2}(\infty) = \frac{2.59}{K}$$

$$e_{ramp-3}(\infty) = \frac{2.59}{K}$$

Range of stability for Configuration 2



Bode plot of the open loop transfer function

Using the bode plot of the system, At $Phase = -180°$

$$Gain\ margin \approx -51.85dB$$

Then the range of stability is approximately given by:

$$0 < K < 10^{51.85/20}$$

$$0 < K < 391.29$$

Using Routh Table:

s^3	1	198
s^2	151.32	76.32K
s^1	198-0.504 K	0
s^0	76.32 K	0

To ensure stability:

$$198 - 0.504\ K > 0$$

$$\therefore 0 < K < 392.85$$

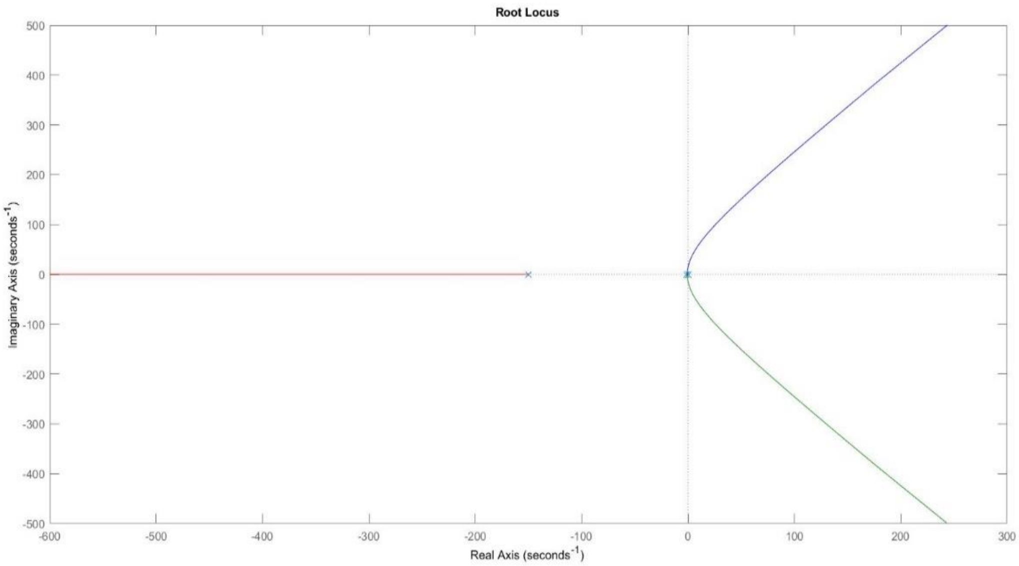
Following the case study in Norman Nise textbook

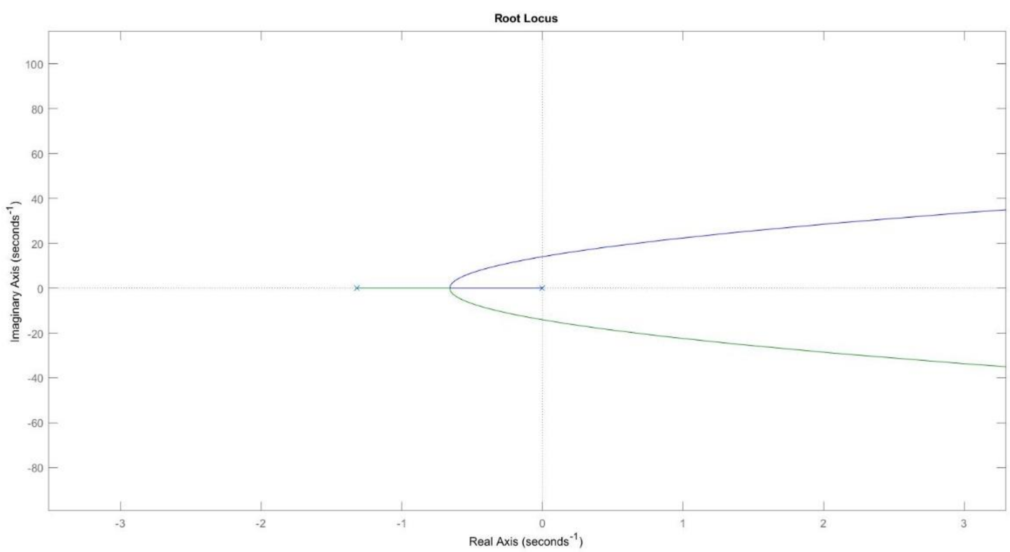
Objective 1: We will find the preamplifier gain at 8 seconds settling time for Configuration 2

$$T_s = \frac{4}{\sigma_d} = 8$$

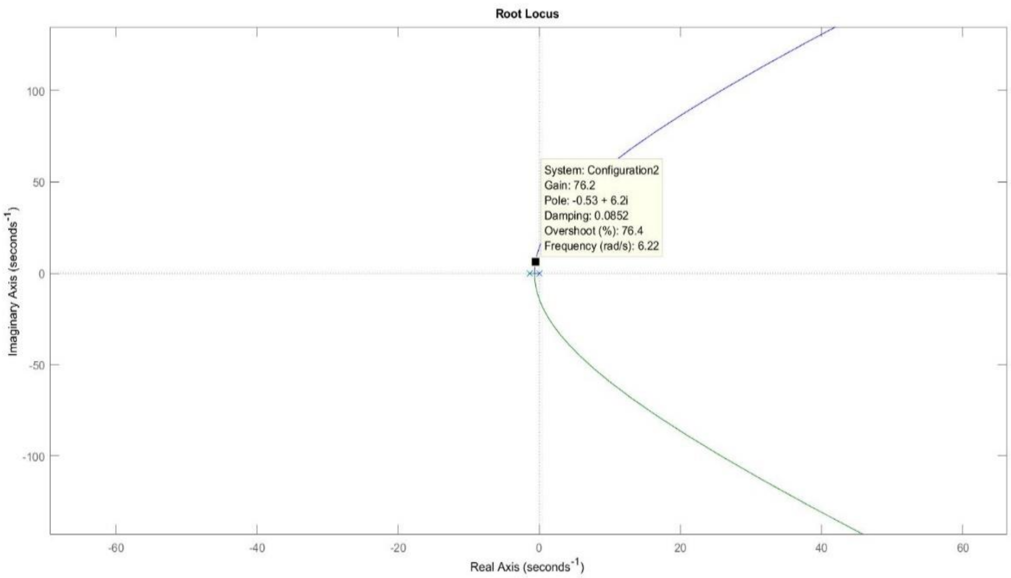
$$\therefore \sigma_d = \frac{1}{2}$$

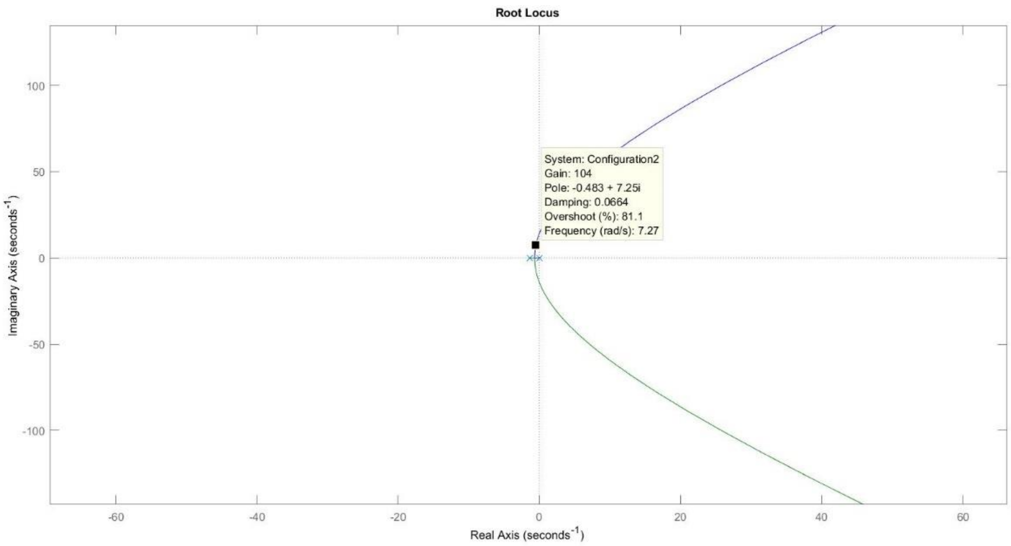
By using MATLAB to plot the root locus:





Due to the limited precision, we managed to find the gain at $\sigma_d = 0.483$ and $\sigma_d = 0.53$





Using linear interpolation to approximate the margin between the two points

$$\omega_d = \frac{1050}{47}\sigma_d + \frac{8479}{470}$$

At $\sigma_d = -0.5$, $\omega_d = 6.87$

$$p = -0.5 + 6.87j$$

From the open loop transfer function

$$G(s) = \frac{76.32K}{s(s+150)(s+1.32)}$$

$$K = \left|\frac{-s(s+150)(s+1.32)}{76.32}\right| \approx 93.45$$

Objective 2: We will find the percentage overshoot and velocity constant of the system

Percent overshoot:

To calculate the percent overshoot, we have to calculate the damping factor.

$$\because \sigma_d = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\therefore \frac{\sigma_d}{\omega_d} = \frac{\zeta}{\sqrt{1-\zeta^2}}$$

Using the values -we found- for σ_d and ω_d then solving for ζ,

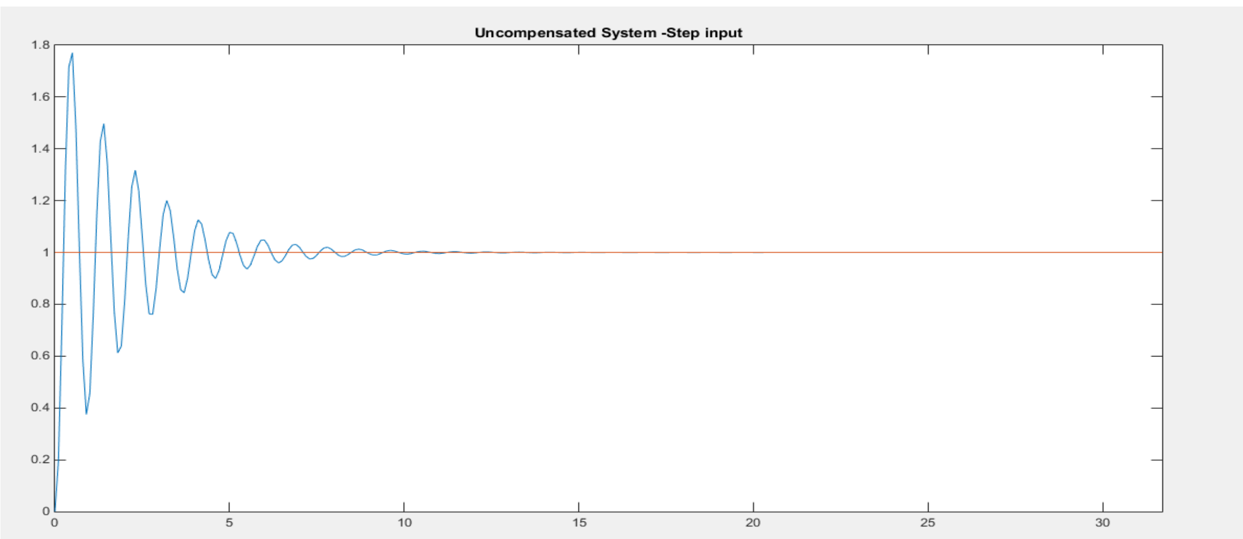
$$\zeta = \sqrt{\frac{50^2}{50^2+687^2}} \approx 0.0726$$

$$\%OS = e^{\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100 \approx 79.56\% \;\; (too\ high)$$

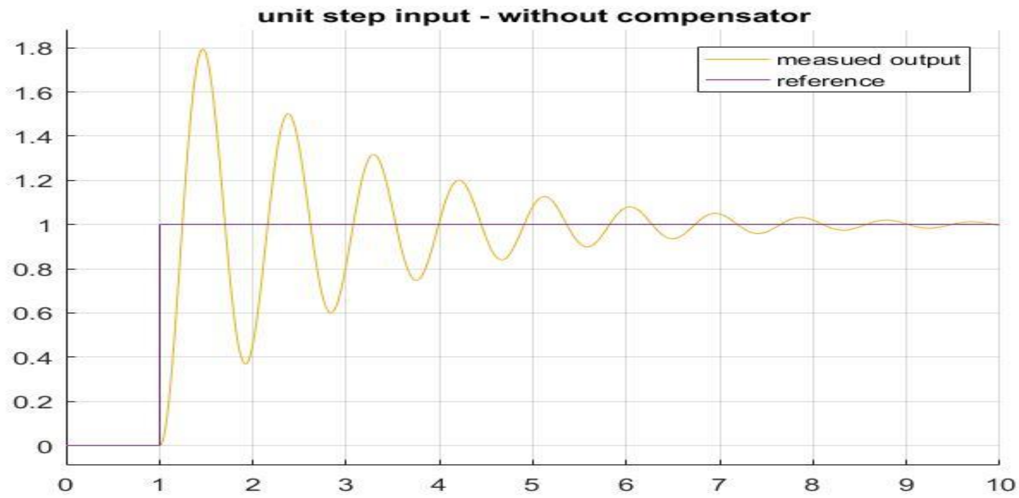
Velocity constant:

$$K_v = \frac{K}{2.59} = \frac{93.45}{2.59} \approx 36.08$$

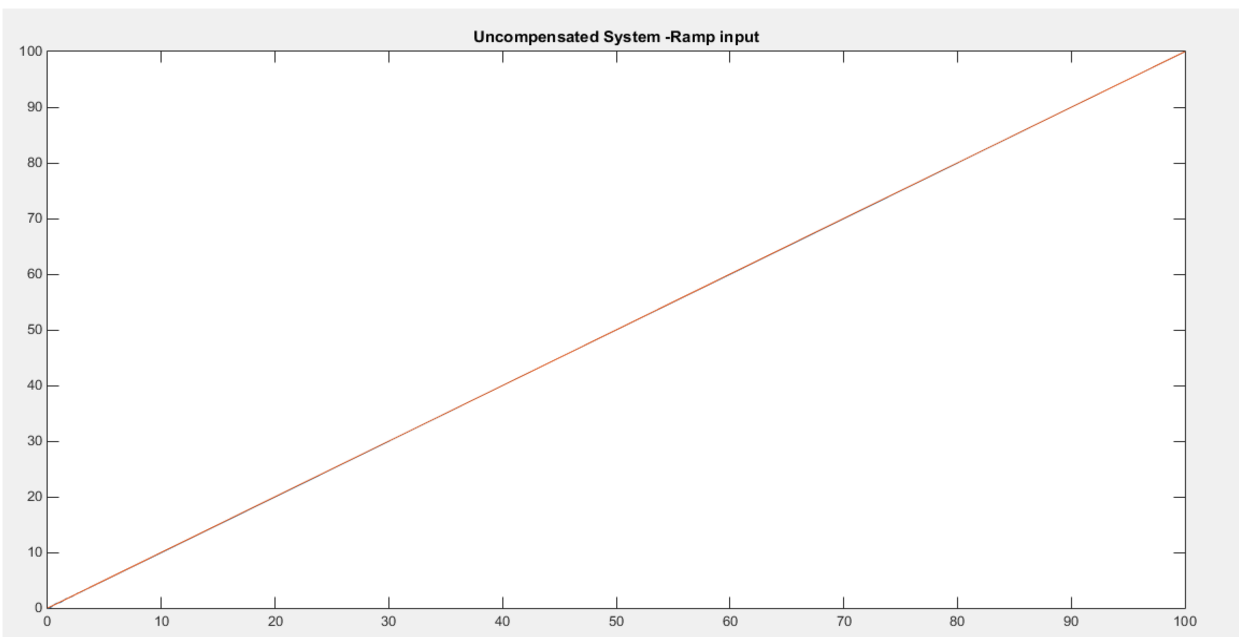
Time response of the system for different inputs



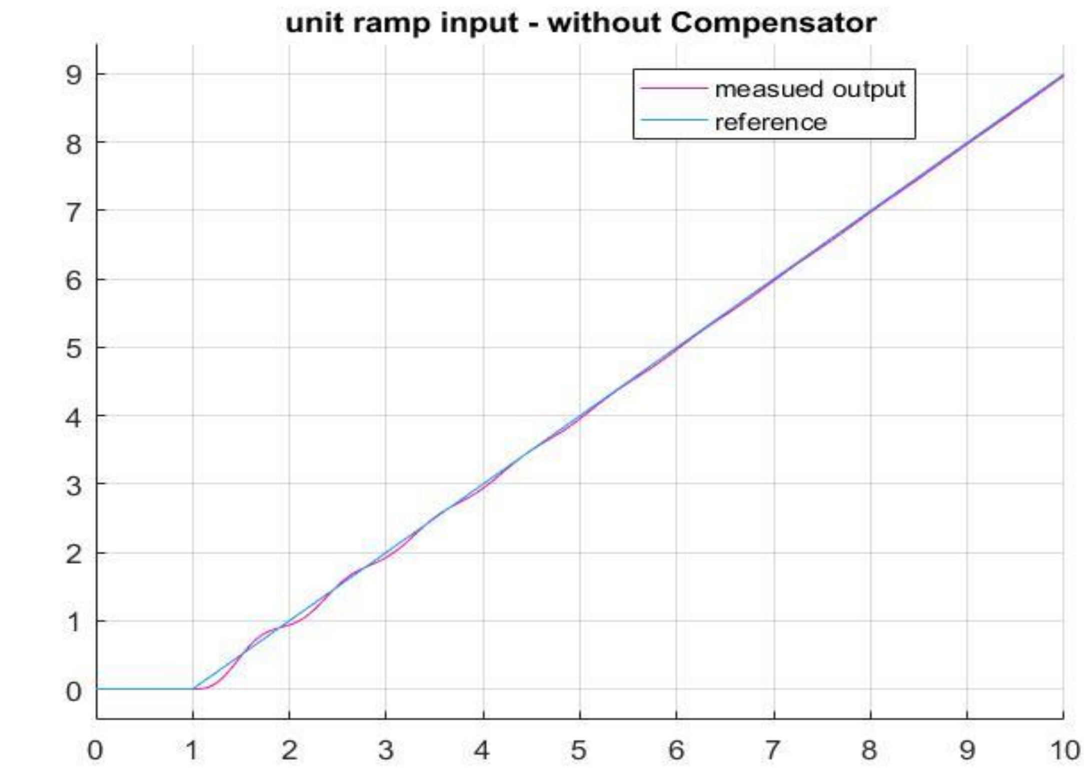
Time response for a step input using MATLAB



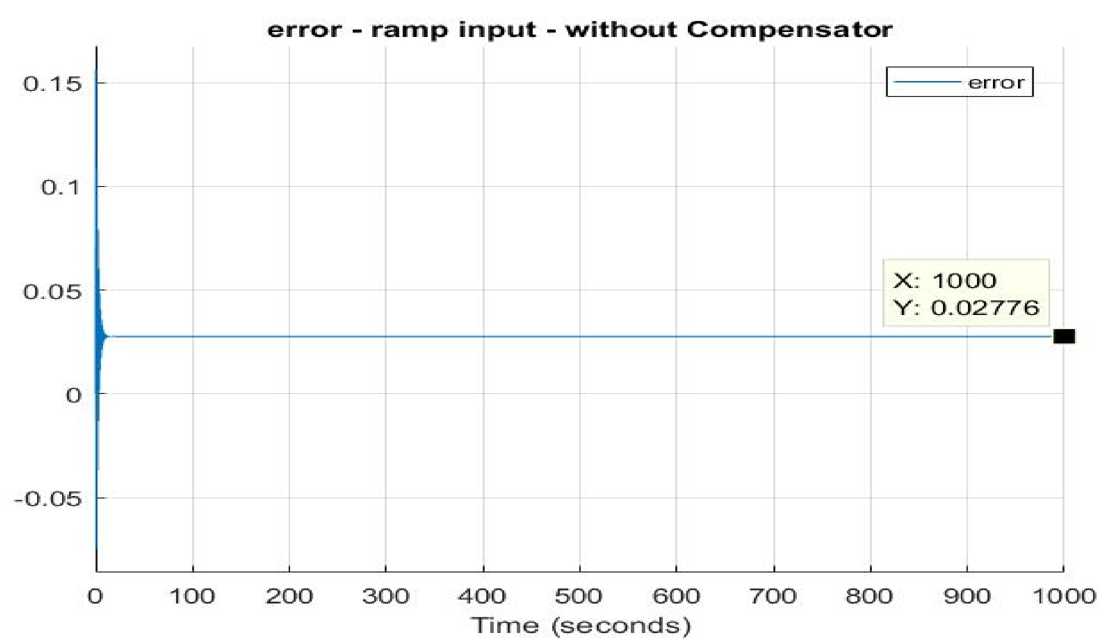
Time response for a step input using SIMULINK



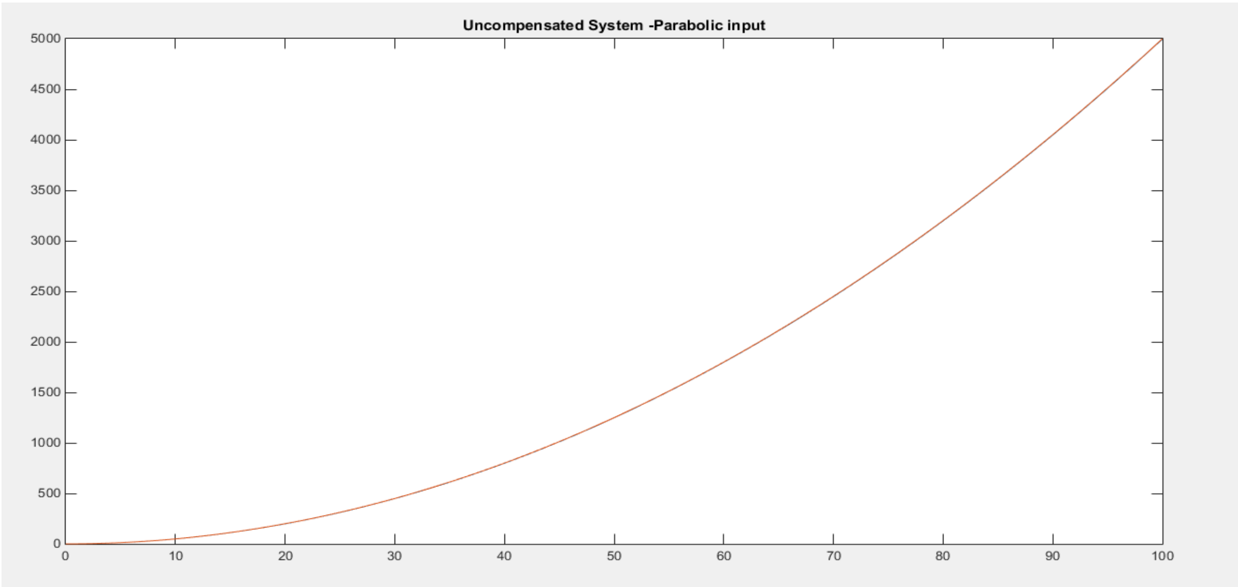
Time response for a ramp input using MATLAB



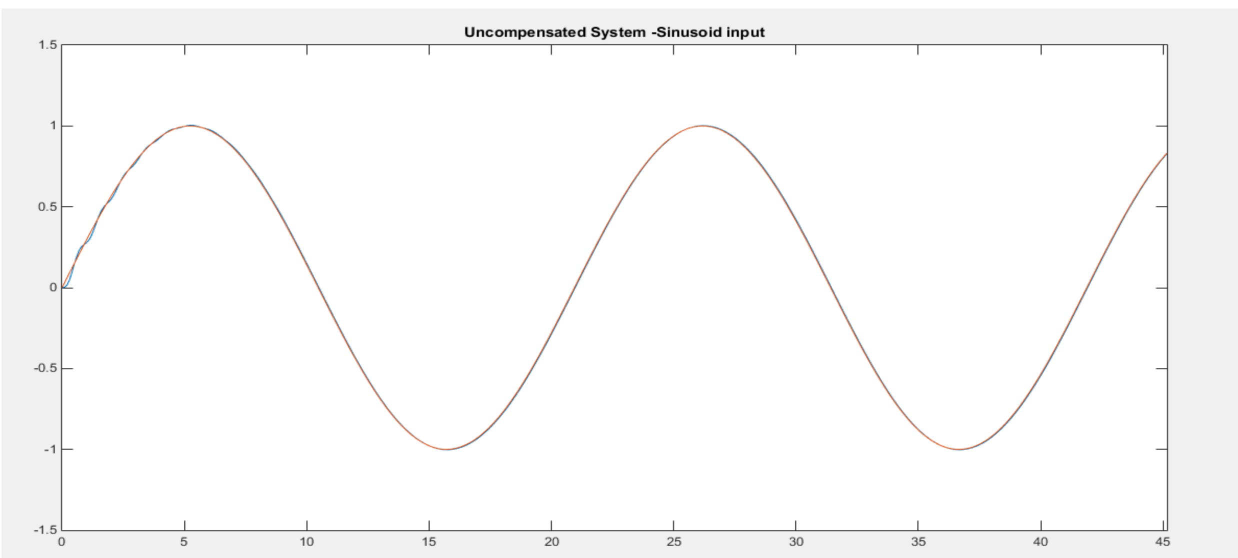
Time response for a ramp input using SIMULINK



Steady state error for a ramp input using SIMULINK



Time response for a parabolic input using MATLAB



Time response for a sinusoidal input using MATLAB

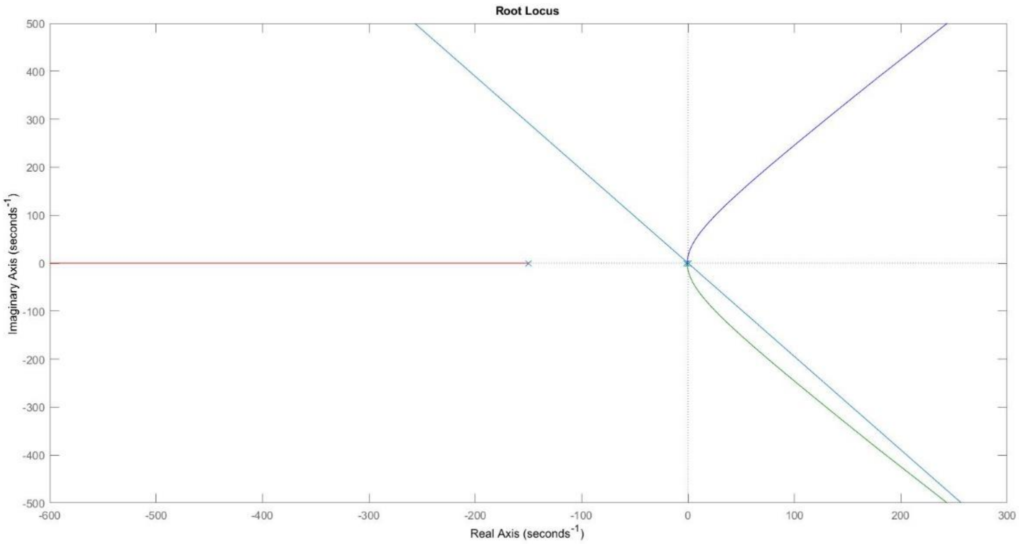
Objective 3: We will design a Lag Lead Compensator to reduce the percent overshoot by a factor of 4 and the settling time by a factor of 2 and improve the velocity constant by a factor of 10 after lead compensation.

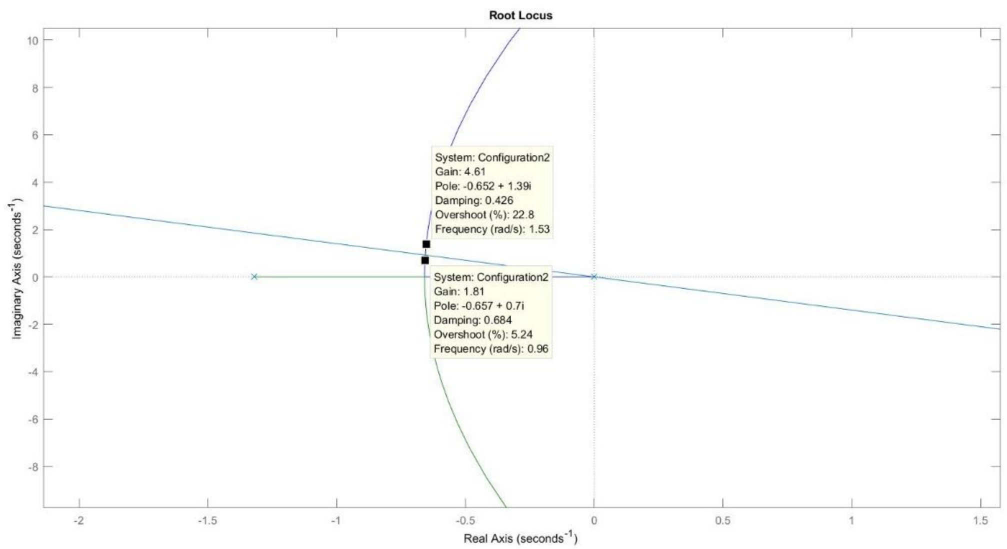
Lead Compensator

At $\%OS = \frac{79.56\%}{4} = 19.89\%$

$$\zeta_{new} = -\frac{\ln(\%OS/100)}{\sqrt{\pi^2 + (\ln(\%OS/100))^2}} \approx 0.457$$

$$\theta = 180 - cos^{-1}(0.457) \approx 117.19°$$





$$p \approx -0.653 + 1.271j$$

It is required to reduce T_s by a factor of two, then σ_d will be doubled. Thus, p will be doubled

$$p_{new} = -1.306 + 2.542j$$

Assuming a compensator zero at -2

$$G_{Lead}(s) = \frac{s+2}{s+a}$$

$$G_{Lead\ Compensated}(s) = \frac{76.32(s+2)}{s(s+150)(s+1.32)(s+a)}$$

By assuming that the phase of $G_{Lead\ Compensated}(s)$ at p_{new} is $-180°$, we find that

$$a \approx 3.708$$

$$G_{Lead\ Compensated}(s) \approx \frac{76.32(s+2)}{s(s+150)(s+1.32)(s+3.708)}$$

$$K \approx \frac{-p_{new}(p_{new}+150)(p_{new}+1.32)(p_{new}+3.708)}{76.32(p_{new}+2)} \approx 18.79$$

$$kG_{Lead-Compensated}(s) \approx \frac{1434.053(s+2)}{s(s+150)(s+1.32)(s+3.708)}$$

Lag Compensator

$$G_{Lag}(s) = \frac{s+z_1}{s+p_1}$$

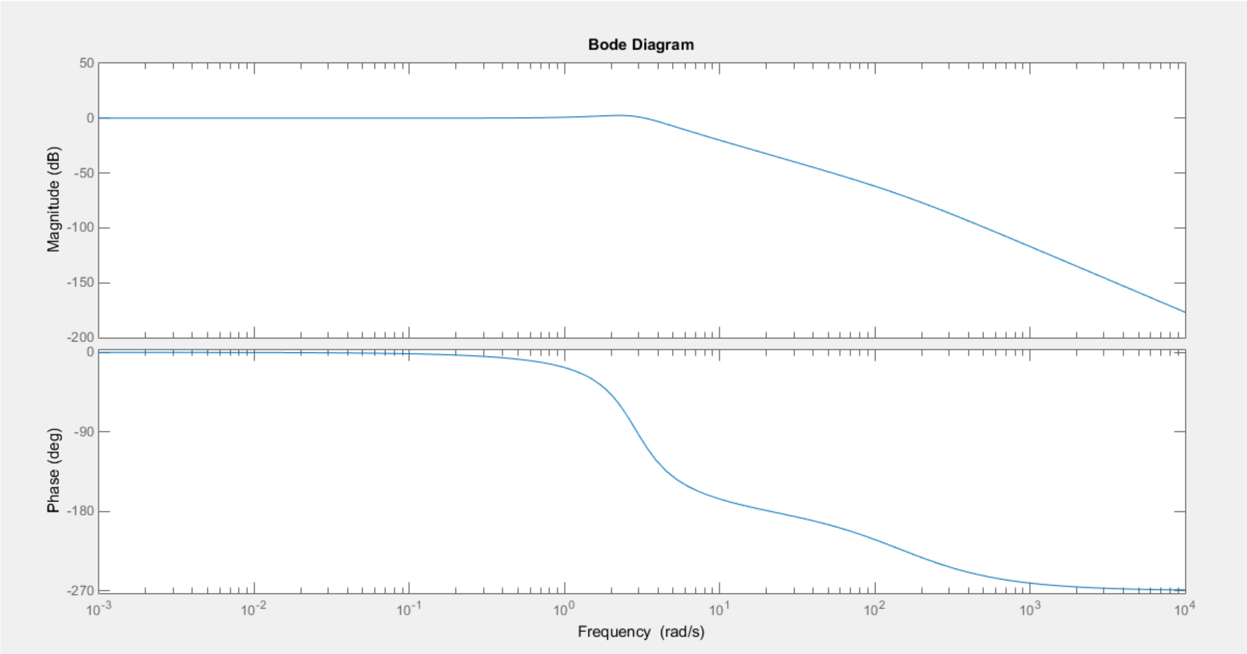
$$\lim_{s\to 0} G_{Lag}(s) = \frac{z_1}{p_1} = 10$$

Let $z_1 = 0.01$ and $p_1 = 0.001$ such that their phase contribution is approximately zero to maintain the transient response.

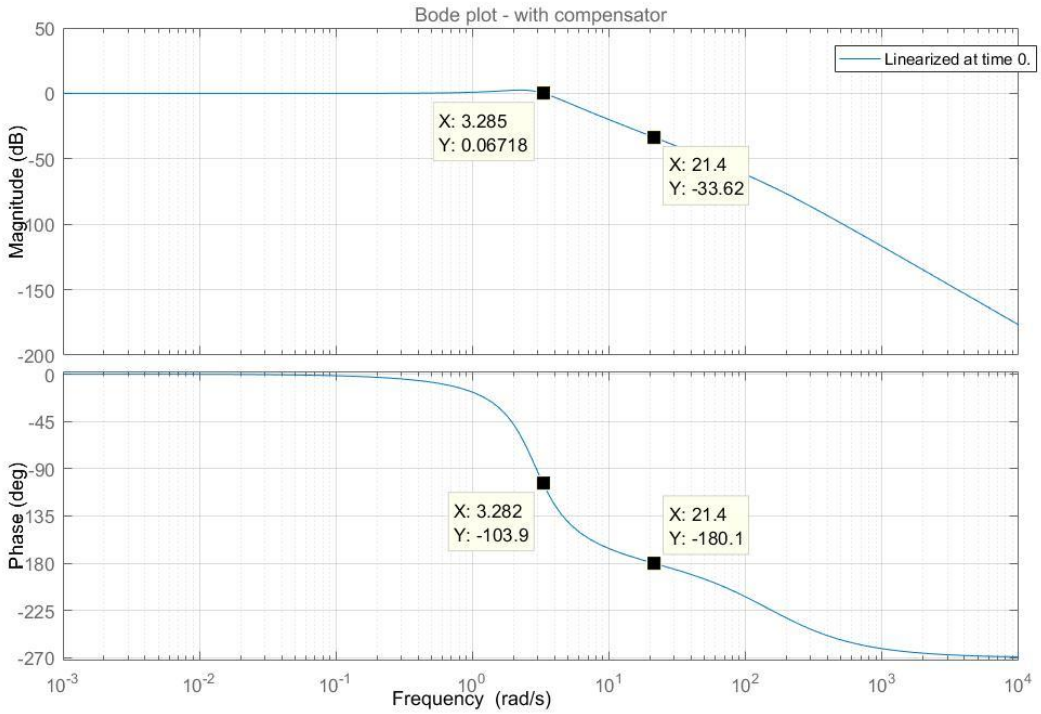
$$G_{Lead\ Lag\ Compensated}(s) = \frac{1434.053(s+2)(s+0.01)}{s(s+150)(s+1.32)(s+3.708)(s+0.001)}$$

$$e_{ramp}(\infty) \approx 0.0256$$

Bode Plot of the compensated system

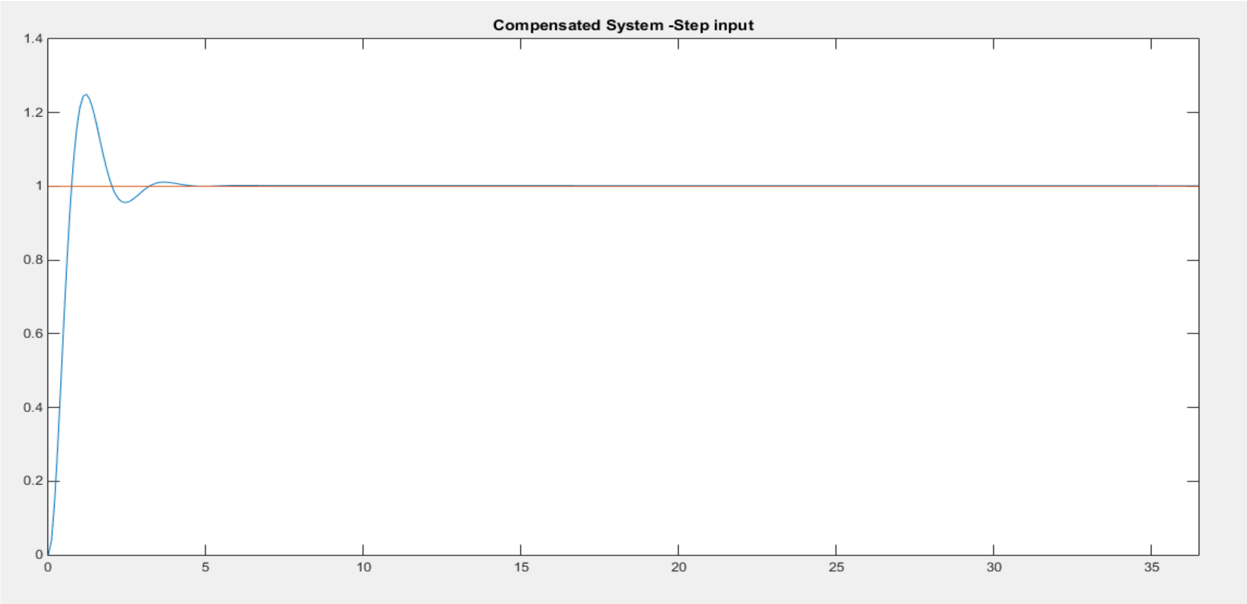


Bode plot of the compensated system using MATLAB

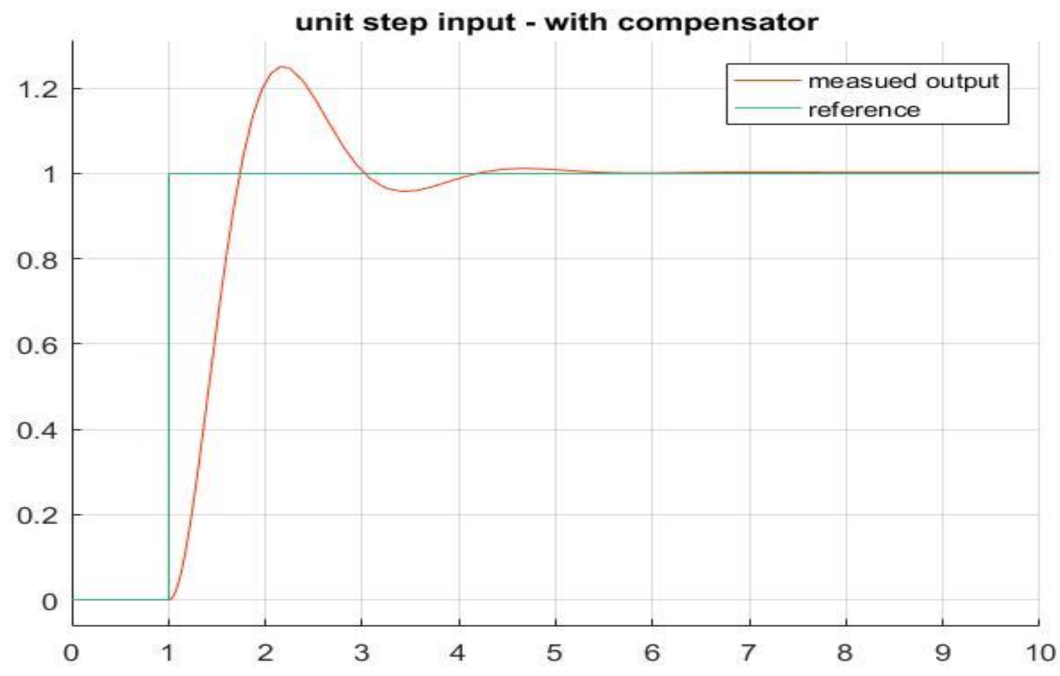


Bode plot of the compensated system using SIMULINK

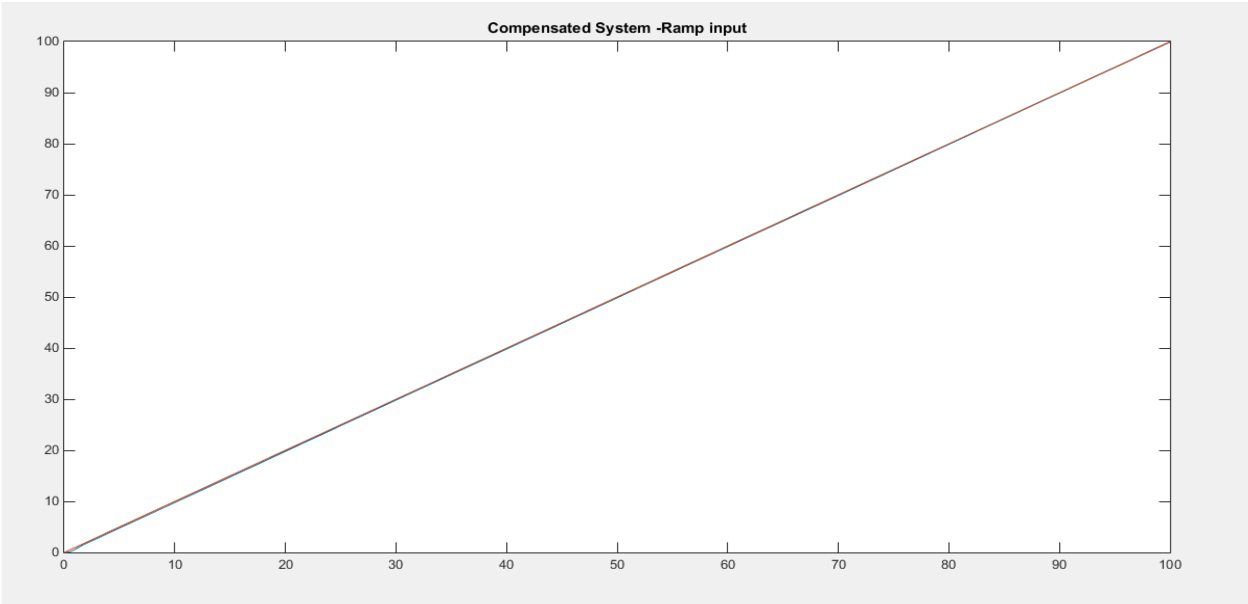
Time response of the compensated system for different inputs



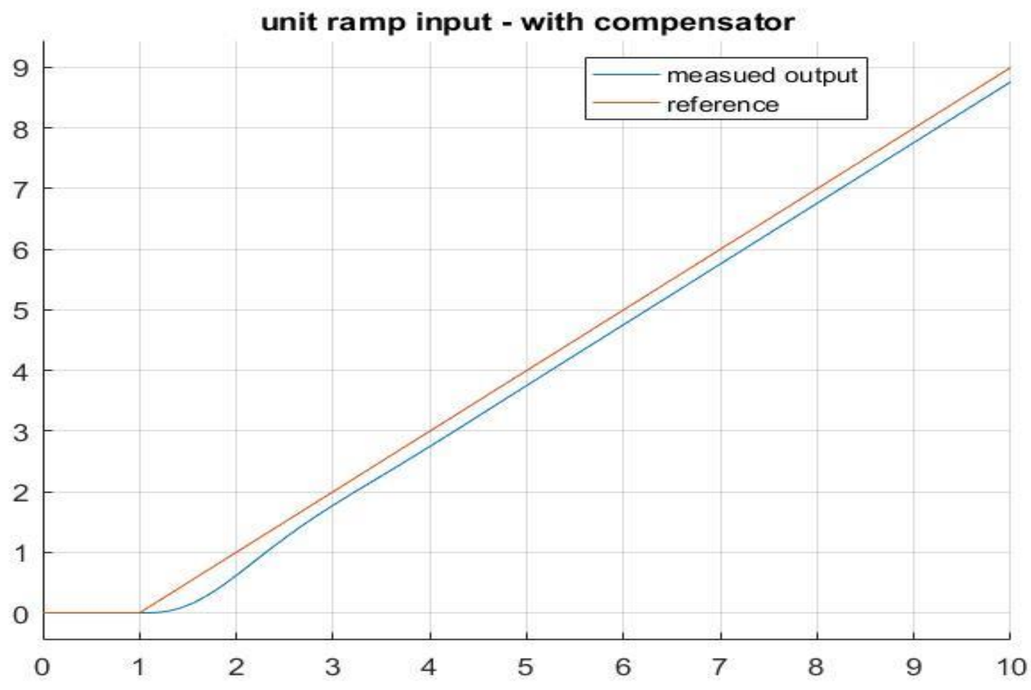
Time response of the compensated system for a step input using MATLAB



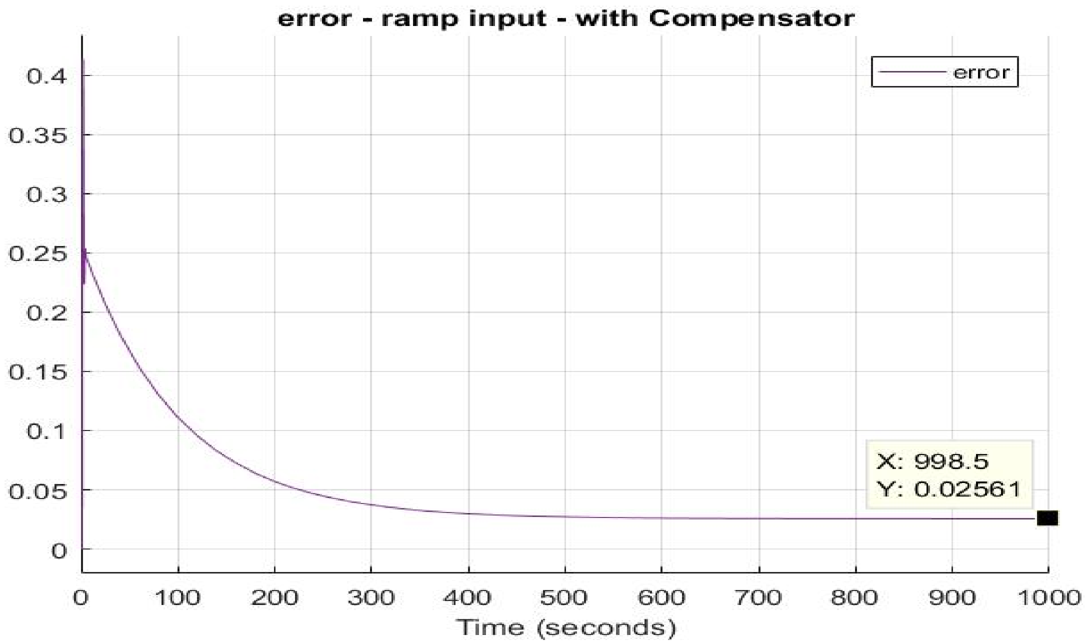
Time response of the compensated system for a step input using SIMULINK



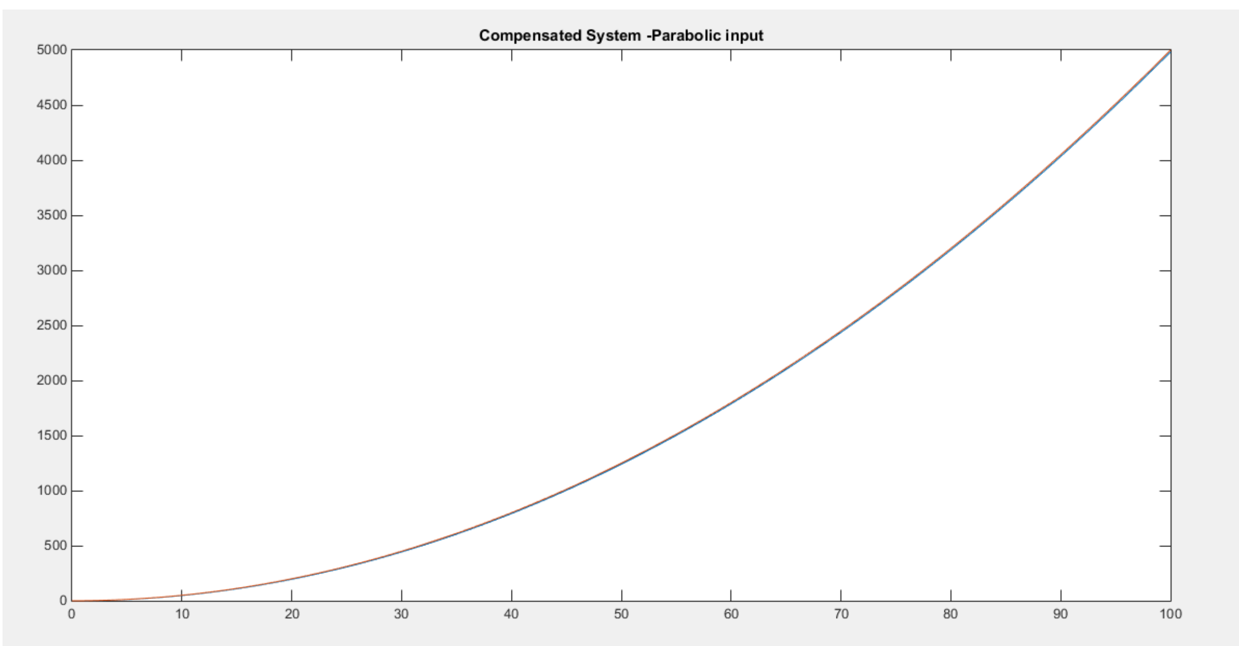
Time response of the compensated system for a ramp input using MATLAB



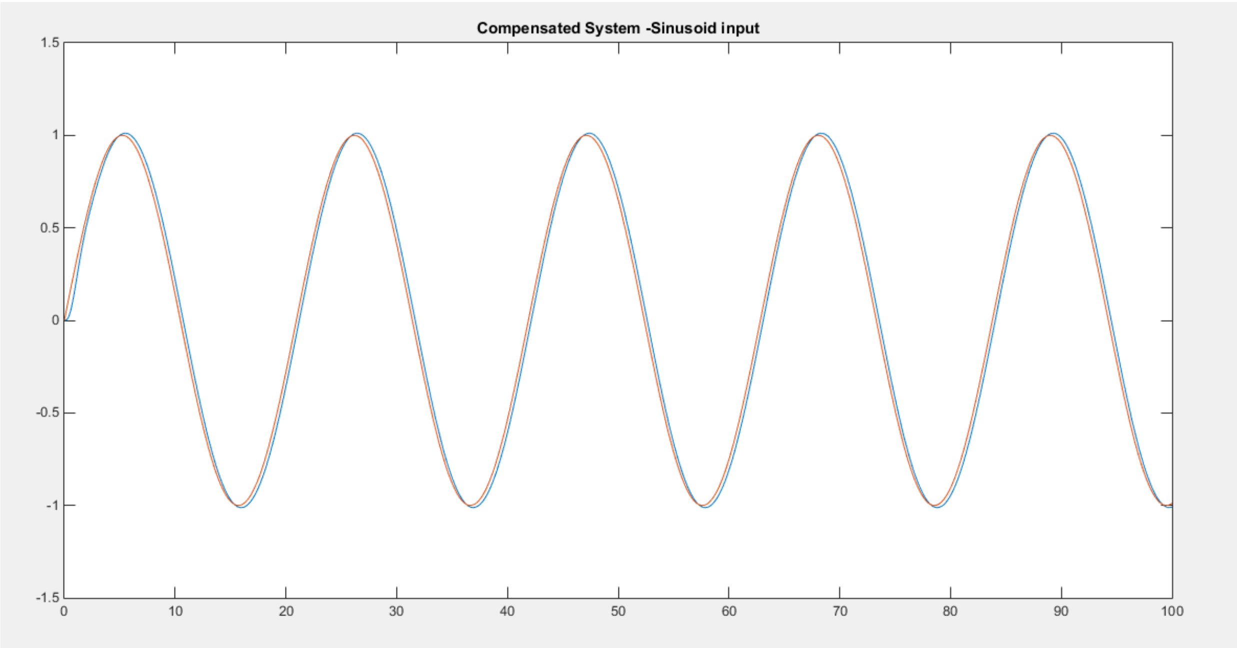
Time response of the compensated system for a ramp input using SIMULINK



Steady state error for a ramp input using SIMULINK



Time response of the compensated system for a parabolic input



Time response of the compensated system for a sinusoidal input

References

[1] N. Nise, *Control systems engineering*. Hoboken, NJ: Wiley, 2015.

[2] A. Sedra and K. Smith, *Microelectronic circuits*. Oxford: Oxford University Press, 2016.