16/03/23

POST OPTIMAL ANALYSIS

It deals with situation of finding new sol^n in efficient way when the parameters are changed.

Conditions after param. are changed.	Recommended Action
(1.) Sol^n is optimal & feasible.	No action recommen.
(2.) Sol^n becomes infeasible but stays optimal.	Use dual simplex.
(3.) Sol^n is feasible but not optimal.	Use simplex/primal simplex.
(4.) Sol^n is neither feasible nor optimal.	Generalized Simplex Method

I: Change in feasibility

It happens in 2 situations

(i.) RHS of the constraint eq^n changes

(ii.) Addition of a new constraint

(*) x_4, x_5, x_6 were the initial basic var. in the primal simplex
↳ like slack var.

(Q.) Max $z = 3x_1 + 2x_2 + 5x_3$

s.t $x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$

$x_1, x_2, x_3 \geq 0$

Ans) Associated primal optimal table

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol^n
z	4	0	0	1	2	0	1350
x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	100
x_3	$3/2$	0	1	0	$1/2$	0	230
x_6	2	0	0	-2	1	1	20

(i) Write the dual form.

Ans (i) Min $z = 430y_1 + 460y_2 + 420y_3$

s.t $y_1 + 3y_2 + y_3 \geq 3$

$2y_1 + 4y_3 \geq 2$

$y_1 + 2y_2 \geq 5$

$y_1, y_2, y_3 \geq 0$

(ii.) If RHS of the constraint eqⁿ changes to $\begin{pmatrix} 460 \\ 500 \\ 400 \end{pmatrix}$.

Use post optimal analysis to find optimal & feasible solⁿ.

Ans (ii). We use dual simplex method to check if the solⁿ continues to be feasible after the RHS of the constraint eqⁿ is changed.

$$\begin{pmatrix} x_2 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 460 \\ 500 \\ 400 \end{pmatrix} = \begin{pmatrix} 105 \\ 250 \\ -20 \end{pmatrix}$$

Since there is a -ve value in RHS of constraint, the solⁿ becomes infeasible.
To restore feasibility, we use dual simplex.

$z = 3(0) + 2(105) + 5(250) = \underline{1460}$

Basic	x_1	x_2	x_3	$x_4 \downarrow$	x_5	x_6	Sol^n
z	4	0	0	1	2	0	1460
x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	105
x_3	$3/2$	0	1	0	$1/2$	0	250
$\leftarrow x_6$	2	0	0	$\boxed{-2}$	1	1	-20
z	5	0	0	0	$5/2$	$1/2$	1450
x_2	$+1/4$	1	0	0	0	$1/4$	100
x_3	$3/2$	0	1	0	$1/2$	0	250
x_4	-1	0	0	1	$-1/2$	$-1/2$	10

$\therefore$ its now optimal & feasible.

$z = 1450, \; x_2 = 100, \; x_3 = 250, \; x_4 = 10$

$x_1, x_5, x_6 = 0$

(iii) If RHS of constraint changes to $\begin{pmatrix} 500 \\ 400 \\ 600 \end{pmatrix}$ in the initial primal optimal sol^n. Find the optimal & feasible sol^n.

Ans (iii)

$$\begin{pmatrix} x_2 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 500 \\ 400 \\ 600 \end{pmatrix} = \begin{pmatrix} 150 \\ 200 \\ 0 \end{pmatrix}$$

$\therefore$ feasible & optimal.

(opt. sol^n) $z = 3x_1 + 2x_2 + 5x_3 = 3(0) + 2(150) + 5(200)$

$= \underline{1300}$

Addition of a new constraint affects feasibility

(i) $3x_1 + x_2 + x_3 \leq 500$

$$3(0) + 100 + 230 \leq 500 \quad ✓$$

Since the constraint is satisfied by the optimal solⁿ it implies that the new constraint is redundant as it wont change the optimum solⁿ.

(ii) $3x_1 + 3x_2 + x_3 \leq 500$

$$3(0) + 3(100) + 230 \leq 500 \quad ✗$$

Since the additional constraint is not satified by the optimum solⁿ, it will be taken into consideration. So the simplex table after encorporating the constraint is. (next page)

(*) x_2 & x_3 are basic var. and are also a part of the additional constraint. Thus, this adjustment is made for the new x_7 row.

$$x_1 = \left(\frac{4}{9/4}\right) \quad ✗$$

$$\boxed{x_4 = \left(\frac{1}{-3/2}\right) = \frac{2}{3} \quad ✓} \quad ✓$$

$$x_5 = \left|\frac{2}{+1/4}\right| \neq \cancel{8} \quad ✗$$

(*) The $\boxed{\text{new } x_7 \text{ row} = x_7 \text{ row} - [3(x_2 \text{ row}) + x_3 \text{ row}]}$

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Sol^n
z	4	0	0	1	2	0	0	1350
x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	0	100
x_3	$3/2$	0	1	0	$1/2$	0	0	230
x_6	2	0	0	-2	1	1	0	20
x_7	3	3	1	0	0	0	1	500
z	[illegible]/4	0	0	$\downarrow$ 1	2	0	0	1350
x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	0	100
x_3	$3/2$	0	1	0	$1/2$	0	0	230
x_6	2	0	0	-2	1	1	0	20
$\leftarrow x_7$	$9/4$	0	0	$\boxed{-3/2}$	$+1/4$	0	1	$\boxed{-30}$
z	$11/2$	0	0	0	$13/6$	0	$2/3$	1330
x_2	$1/2$	1	0	0	$-1/6$	0	$1/3$	90
x_3	$3/2$	0	1	0	$1/2$	0	0	230
x_6	-1	0	0	0	$2/3$	1	$-4/3$	60
x_4	$-3/2$	0	0	1	$-1/6$	0	$-2/3$	20

$\therefore$ optimal & feasible.

$z = 1330, \quad x_2 = 90, \quad x_3 = 230, \quad x_4 = 20$

$x_5 = x_1 = 0 = x_7 \qquad x_6 = 60$

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II. Changes affecting optimality

(i.) changes in obj. f^n coeff.'s

We need to recompute the z-row coeff.'s.

Steps:

(i.) Find the dual values

(ii.) To use the new dual values to find non-basic var.'s

2 cases are likely to happen:

(i.)
↳ The new obj. f^n satisfies the optimality $cond^n$ then sol^n remains unchanged. In this case, optimum obj. values may change.

(ii.) Optimality $cond^n$ is not satisfied so we use primal simplex method to restore optimality.

(Q.) i.) If new obj. f^n is $\underline{Max\ z = 2x_1 + 3x_2 + 4x_3}$. check optimal.

Ans.) basic var. → $x_2 \ x_3 \ x_6$

coeff. of basic var. in new obj. f^n (taken in same order) are $(3 \ 4 \ 0)$.

$$(y_1, y_2, y_3) = (3 \ 4 \ 0)\begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} = \left(\frac{3}{2} \ \ \frac{5}{4} \ \ 0\right)$$

The coeff.'s of non-basic var.'s x_1, x_4, x_5 are obtained by subbing the values of y_1, y_2, y_3 in the constraints corresponding to non-basic var.'s.

$x_1 : y_1 + 3y_2 + y_3 - 2 = \left(\frac{3}{2} + \frac{15}{4} - 2\right) = \frac{13}{4}$ //

$x_4 : y_1 - 0 = \frac{3}{2}$ //

$x_5 : y_2 - 0 = \frac{5}{4}$ //

Since, coeff.'s of non-basic var.'s are +ve for the 'maximization problem', it implies that the optimality is not affected.

So, the optimal value of $z = 2x_1 + 3x_2 + 4x_3$

$z = 2(0) + 3(100) + 4(230)$

$z = \underline{\underline{1220}}$

(ii) If new obj. f^n is $\text{Max } z = 6x_1 + 3x_2 + 4x_3$
check if optimality is affected. If yes, find opt. sol^n.

Ans.) basic var. x_2, x_3, x_6.

$$(y_1, y_2, y_3) = (3 \quad 4 \quad 0) \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix}$$

$$= \left(\frac{3}{2} \quad \frac{5}{4} \quad 0 \right)$$

non basic var. $\rightarrow x_1, x_4, x_5$

$x_1 : y_1 + 3y_2 + y_3 - 6 = -3/4 \rightarrow$ -ve

$x_4 : y_1 - 0 = 3/2$

$x_5 : y_2 - 0 = 5/4$

Since one of the coeff. of non-basic var. is -ve.
it is not optimal sol^n.
Optimality will be restored using primal simplex.

Basic	x_1 ↓	x_2	x_3	x_4	x_5	x_6	sol^n
z	$-3/4$	0	0	$3/2$	$5/4$	0	1220
x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	100
x_3	$3/2$	0	1	0	$1/2$	0	230
← x_6	2	0	0	-2	1	1	20
z	0	0	0	$3/4$	$13/8$	$3/8$	$2455/2$
x_2							102.5
x_3							215
x_1	1	0	0	-1	$1/2$	$1/2$	10