15/05/23

INTEGER PROGRAMMING

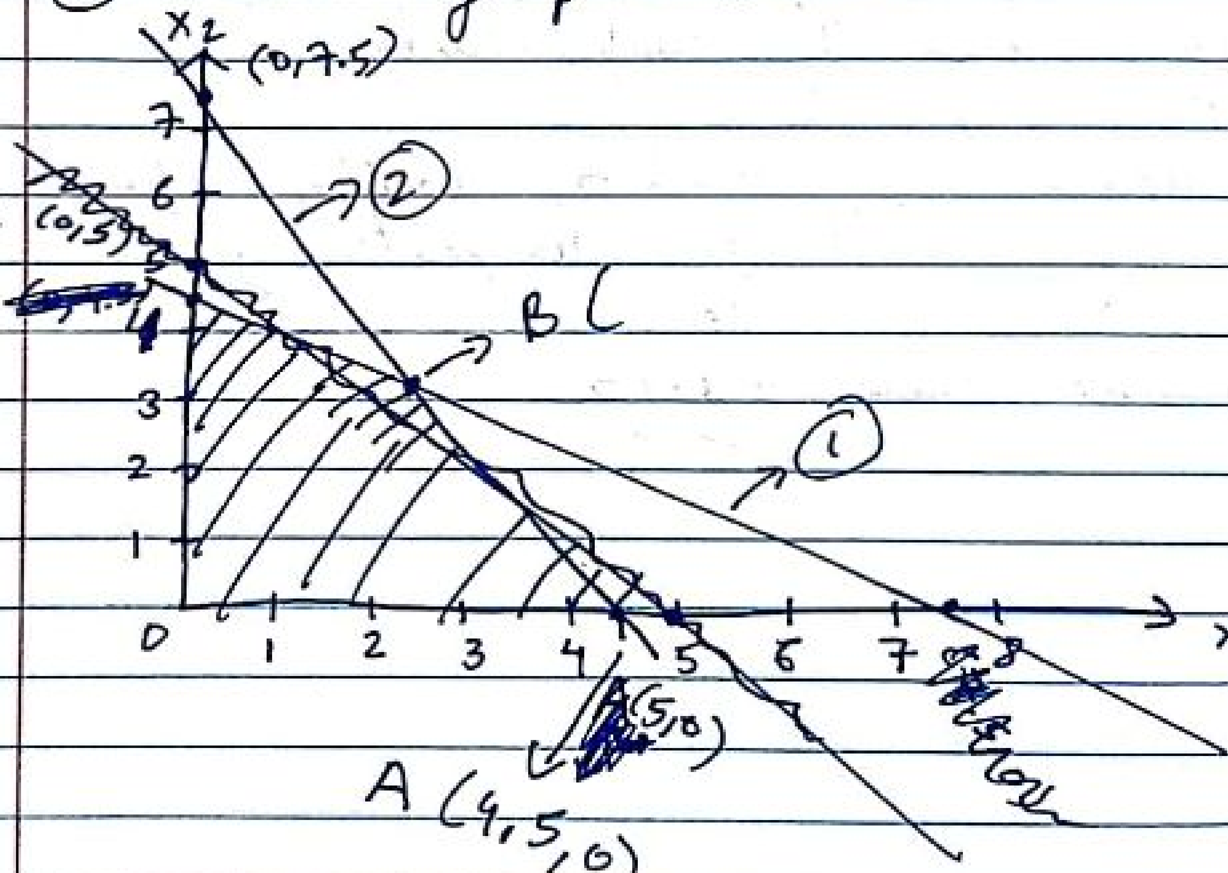
(Q) Max $z = 5x_1 + 4x_2$

subject to. $10x_1 + 6x_2 \leq 45$ — (2)

$x_1 + x_2 \leq 5$ — (1)

$x_1, x_2 \geq 0$ & integers

Ans) (i) Find the graphical solⁿ.



A : (4.5, 0) → $z = 22.5$

MAX ✓ B : (3.75, 1.25) → $z = 23.75$

C : (0, 5) → $z = 20$

$x_1 = 3.75$, $x_2 = 1.25$

$z = 23.75$

$x \le 3$ | $x_1 \ge 4$

(1 with 1)

$x_1 = 3$, $x_2 = 2$

$z = 23$

(1 with 2)

$x_1 = 4$, $x_2 = 0.83$

$z = 23.33$

Solut^n

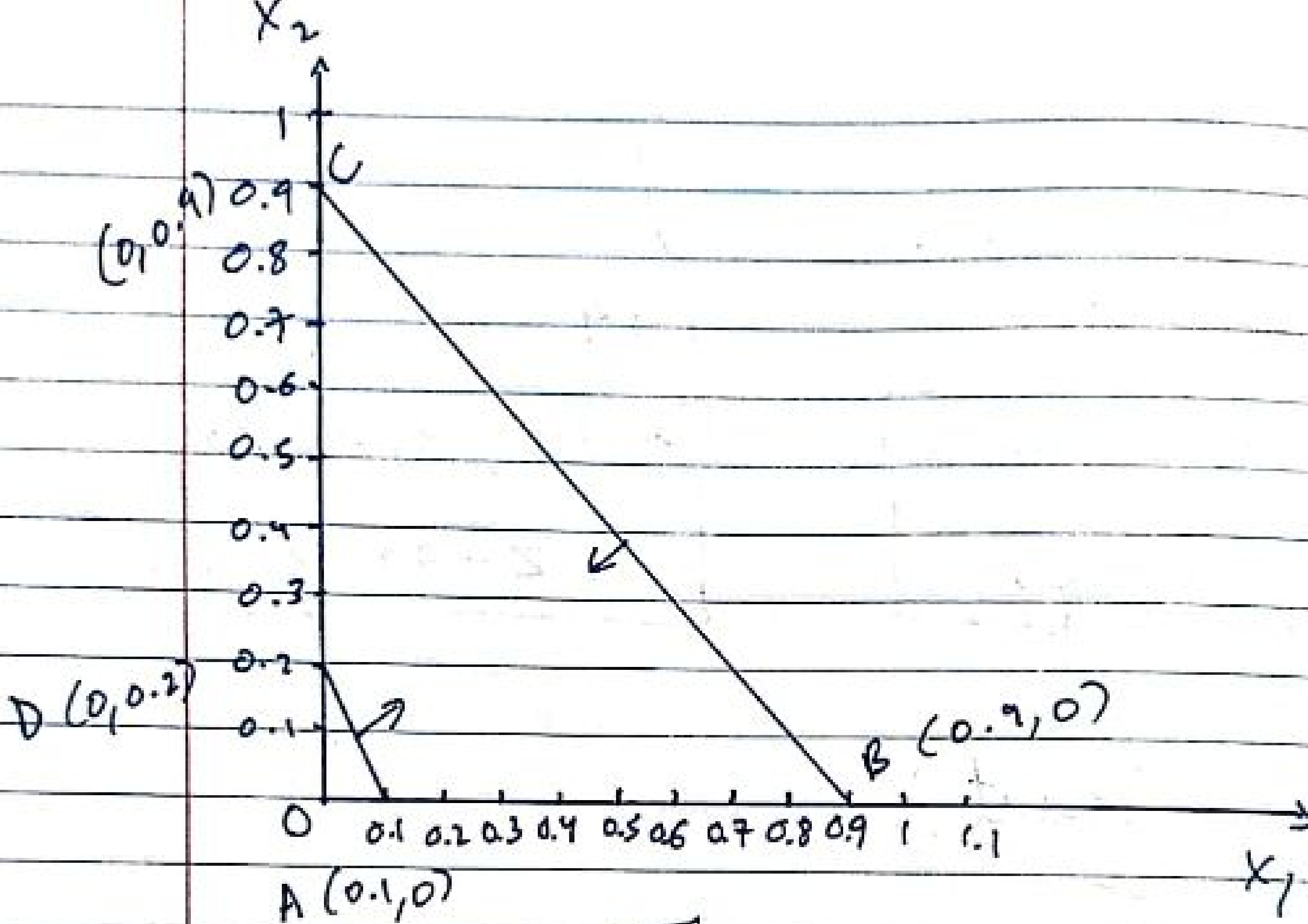
Q. Show graphically that the following integer linear programming has no feasible sol^n and then verify the result using branch and bound method.

$$\text{Max } z = 2x_1 + x_2$$

subject to $\quad 10x_1 + 10x_2 \le 9 \quad$ — (1)

$\quad\quad\quad\quad\quad 10x_1 + 5x_2 \ge 1 \quad$ — (2)

$x_1, x_2 \ge 0$ & integers



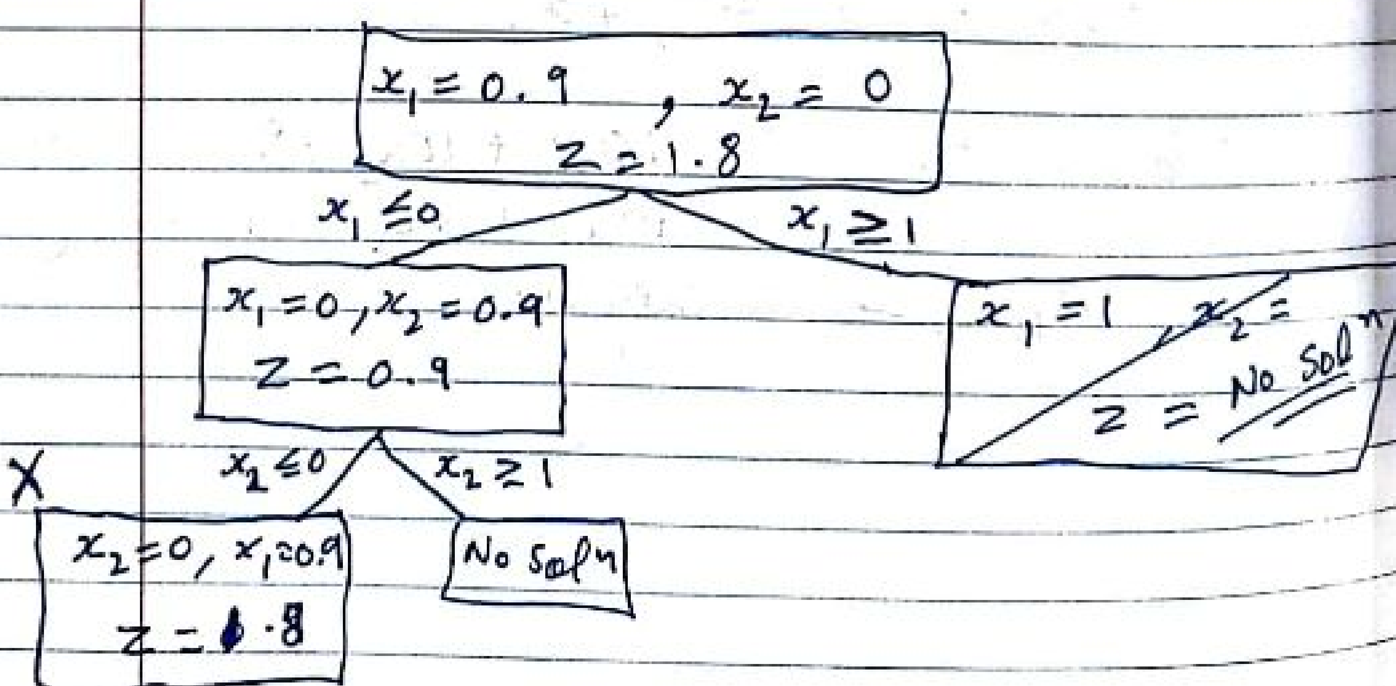
$\boxed{Z}$

Z for A → 0.2

B → 1.8 ✓ (Max)

C → 0.9

D → 0.2



∴ <u>No feasible sol^n exists in this case.</u>

17/05

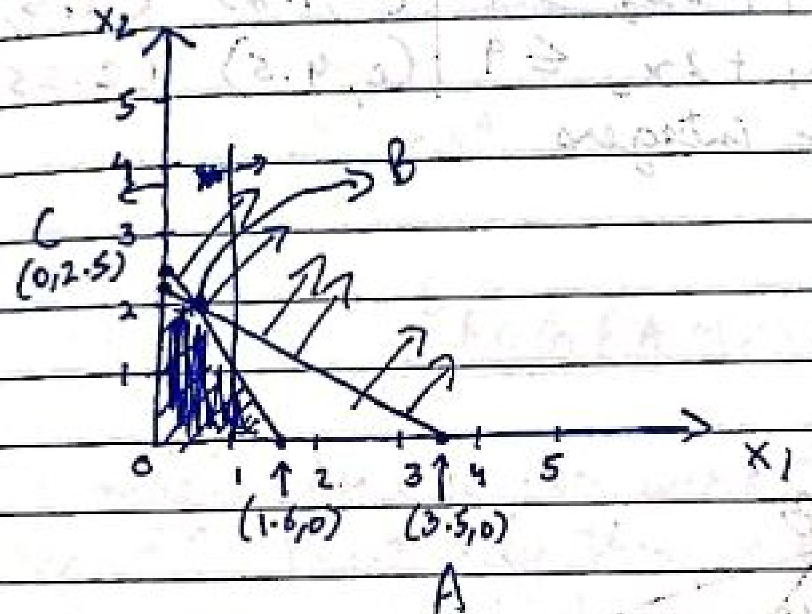
(Q.) Min $z = 5x_1 + 4x_2$

subject to $3x_1 + 2x_2 \geq 5$ $(0, 2.5)(1.6, 0)$

$2x_1 + 3x_2 \geq 7$ $(0, 2.3)(3.5, 0)$

$x_1, x_2 \geq 0$ & integers

Ans:)



C: $(0, 2.5) \rightarrow z = 10$

$\boxed{B : (0.2, 2.2) \rightarrow z = 9.8}$ → Min

A: $(3.5, 0) \rightarrow z = 17.5$

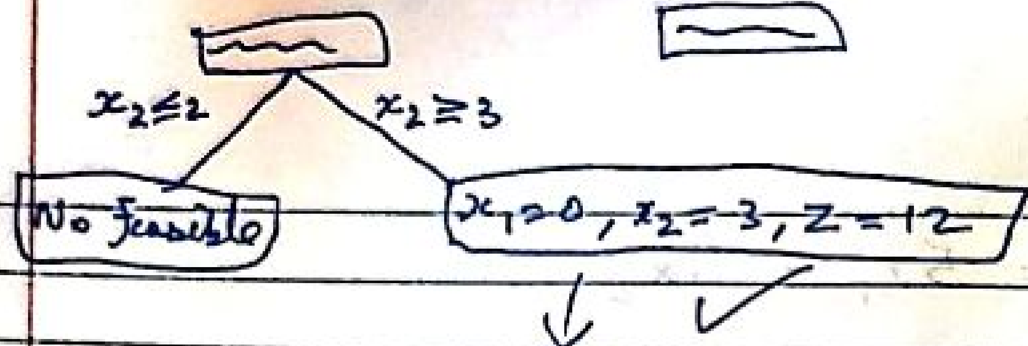
$\boxed{x_1 = 0.2, x_2 = 2.2, z = 9.8}$

$x_1 \leq 0$ $x_1 \geq 1$ ($\cap$ with 2^{nd} constraint)

$\boxed{x_1 = 0, x_2 = 2.5, z = 10}$ $\boxed{x_1 = 1, x_2 = 1.6, z = 11.4}$

fathom since its a min. problem and value of z is increasing ∴ we need not investigate it further as it will not give a better integer L.P solⁿ.

Continued



Int. sol^n

(Q.) Max $z = 3x_1 + 2x_2$

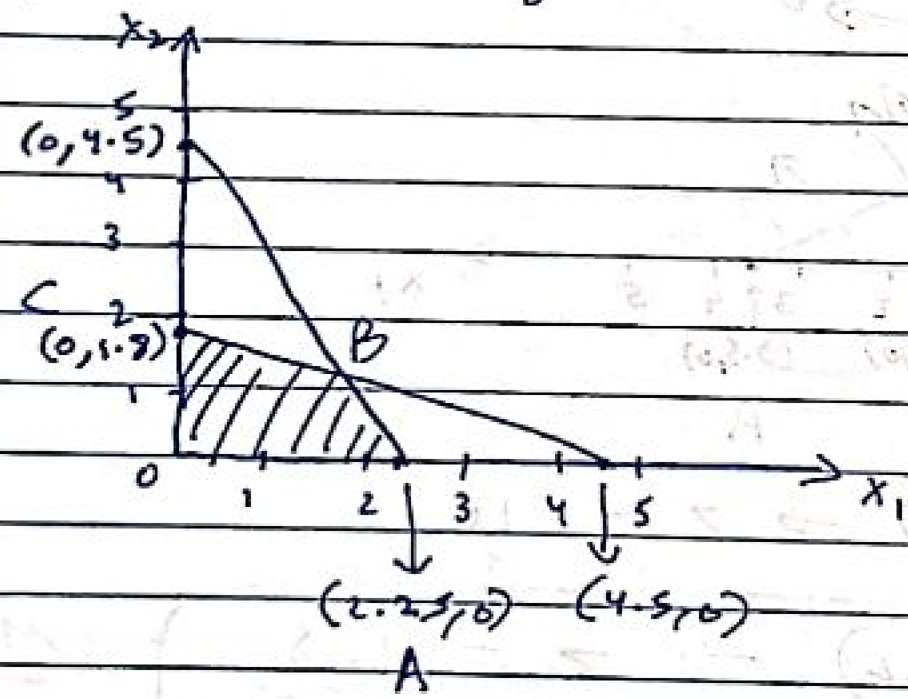
sub. to

$2x_1 + 5x_2 \leq 9$ | $(0, 1.8)$ $(4.5, 0)$

$4x_1 + 2x_2 \leq 9$ | $(0, 4.5)$ $(2.25, 0)$

$x_1, x_2 \geq 0$, & integers

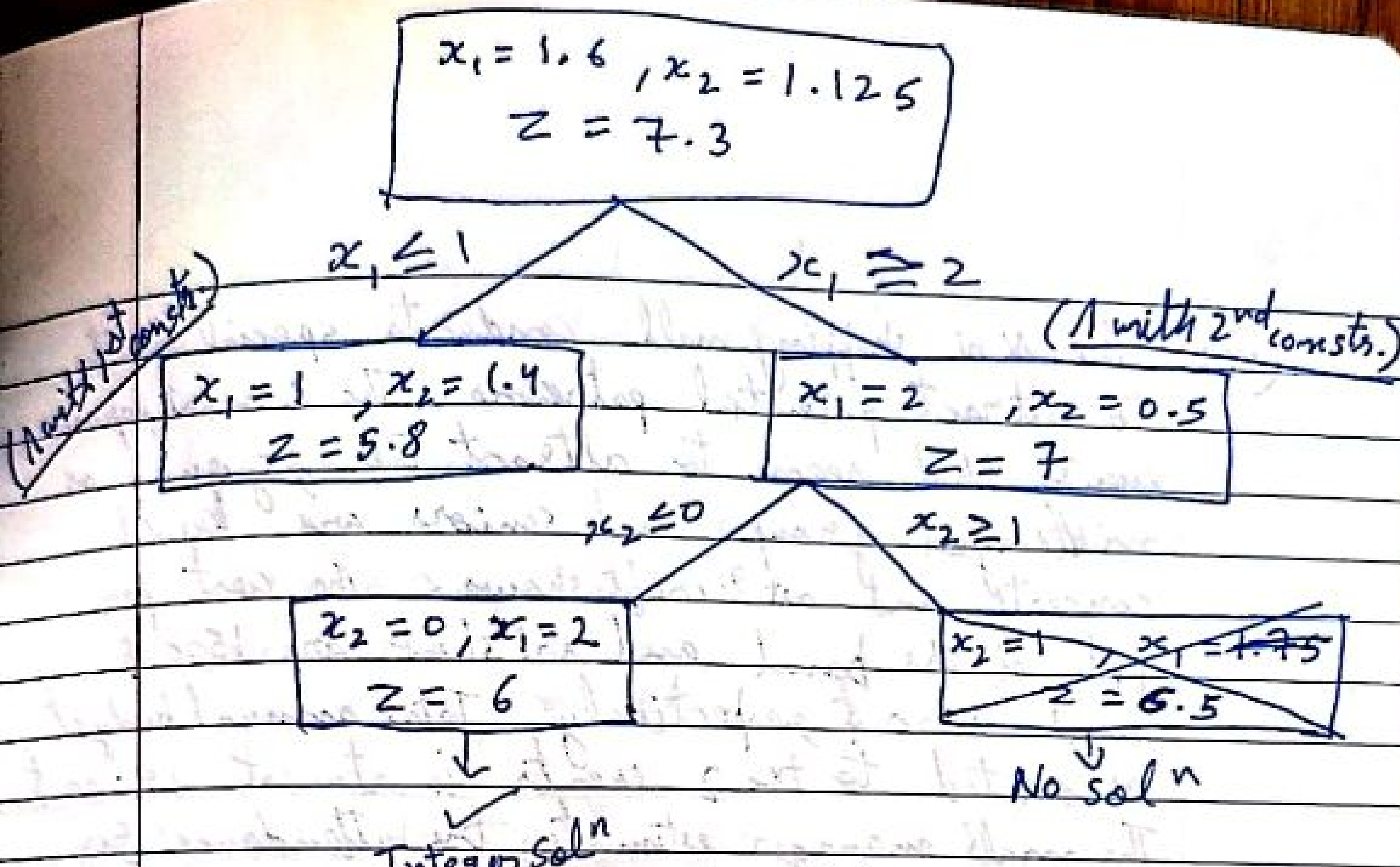
Ans)



$A \rightarrow (2.25, 0) \longrightarrow z = 6.75$

$B \rightarrow (1.6, 1.125) \longrightarrow z = 7.2$ ✓ (Max)

$C \rightarrow (0, 1.8) \longrightarrow z = 3.6$



18/03

GOAL PROGRAMMING

LP models done so far were to optimize a single obj. f^n. Practically there may be situat^n's where multiple objectives maybe more appropriate. Example:

↳ Politician may promise to reduce the national debt & at same time offer income tax relief. In such situations, a single sol^n that optimizes the conflicting objectives is not possible.

Compromise sol^n based on the relative importance of each objective is obtained. Efficient sol^n may not be the optimum sol^n.

(Q.) The N W shopping mall conducts special events to attract potential patreons. The 2 most popular events that seem to attract teens, young or middle-aged groups and seniors are band concerts and art & craft shows. The cost per ppt of the band and art show are 1500 \$ and 3000 \$ respectively. Total annual budget allocated to the 2 events is atmost 15000 \$. The mall manager estimates the attendance for events as follows :

Events	Teens	Y/M-aged grp	Seniors
Band Concert	200	100	0
Art & Crafts	0	400	250

The mgr. has set the minimum annual ~~potts~~ goals of 1000, 1200 and 800 for attendance of teenagers, Y/M-aged grp and seniors respectively

(i.) Formulate the prob. as a goal prog. model.

(ii.) Suppose that the goal of attracting young/middle-aged group is twice as impo. as the either of other 2 category, Formulate the goal.

Ans

$X_1 \to$ no. of band concerts per yr
$X_2 \to$ no. of art & crafts per yr

$$1500x_1 + 3000x_2 \leq 15000$$

$$200x_1 + 0x_2 \geq 1000$$
$$100x_1 + 400x_2 \geq 1200$$
$$0x_1 + 250x_2 \geq 800$$

Note

Introducing deviational variables
↳ represent the amount by which the goal will be violated.

$\boxed{s_i^+}$: deviatn above RHS of the constraint
$\boxed{s_i^-}$: deviatn below RHS of the constraint

s_i^+ & s_i^- are by defn dependent and hence cannot be taken as basic variables simultaneously. This means that in any simplex iteratn atmost one of the 2 deviational var. ~~will be~~ can assume +ve values

Ans continued

$$200x_1 + s_1^- - s_1^+ = 1000$$
$$100x_1 + 400x_2 + s_2^- - s_2^+ = 1200$$
$$250x_1 + s_3^- - s_3^+ = 800$$

Goal: Min $G_1 = s_1^-$

Min $G_2 = s_2^-$

Min $G_3 = s_3^-$

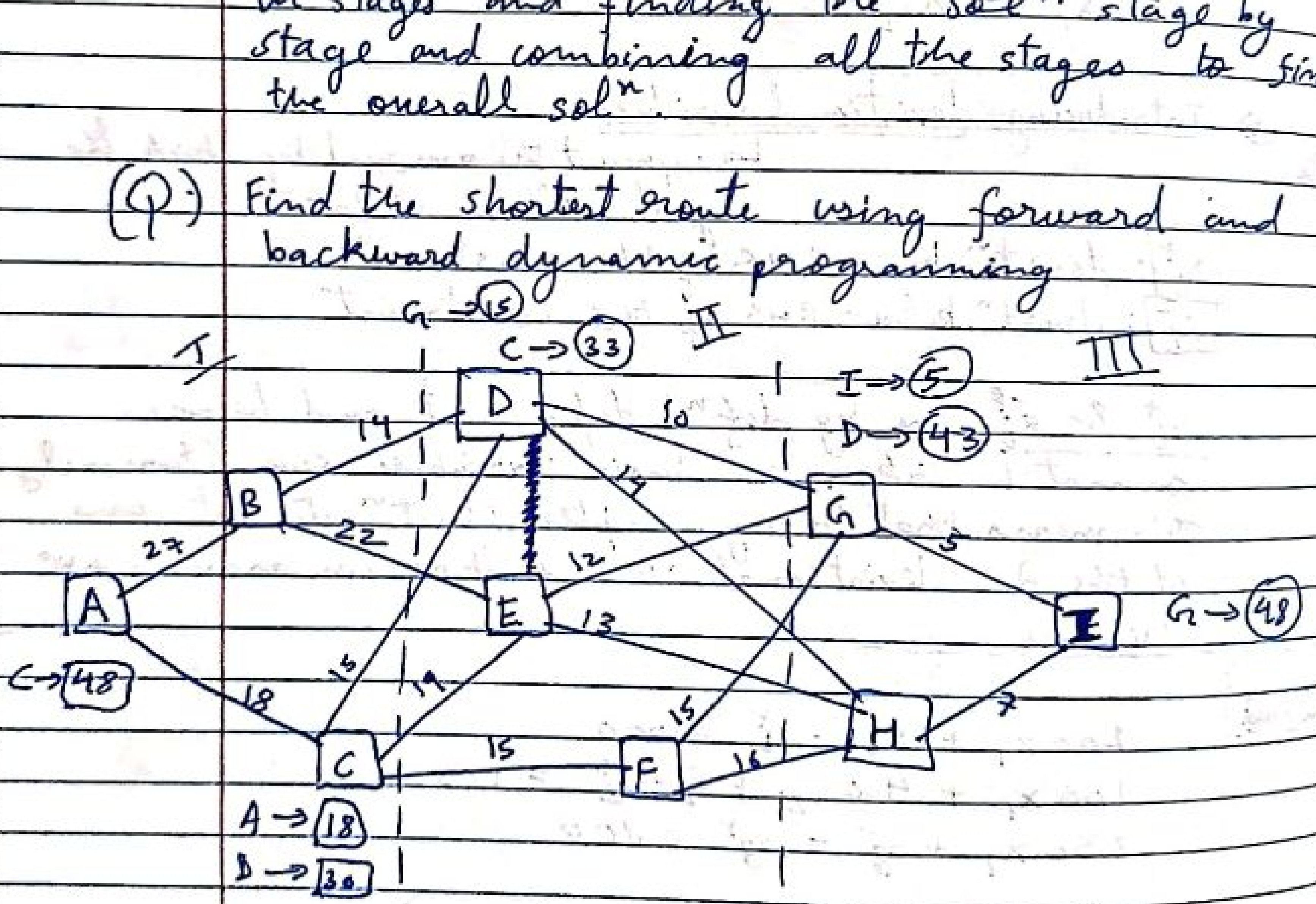
Goal as these be coz the dev. var. represent the amnt. by which goal will be violated

(ii) $\text{Min } G = s_1^- + 2s_2^- + s_3^-$

Dynamic Programming

This method works by dividing the prob^m in stages and finding the sol^n stage by stage and combining all the stages to find the overall sol^n.

(Q) Find the shortest route using forward and backward dynamic programming



Fwd: A → C → D → G → I

Total Distance = 48 units

Bwd: I → G → D → C → A

Total Durat^n = 48 units

~~(4)~~ Non-Linear Programming

(Q:) Find the extreme point using the necessary condⁿ.

$$f(x_1, x_2, x_3) = x_1 + 2x_3 + x_2 x_3 - x_1^2 - x_2^2 - x_3^2$$

Ans:)

$$\nabla f(x_0) = 0 \quad (\text{Necessary Condt}^n)$$

$$\frac{\delta f}{\delta x_1} = 0 \Rightarrow 1 - 2x_1 = 0 \Rightarrow x_1 = \frac{1}{2}$$

$$\left.\begin{array}{l} \dfrac{\delta f}{\delta x_2} = 0 \Rightarrow x_3 - 2x_2 = 0 \\ \dfrac{\delta f}{\delta x_3} = 0 \Rightarrow 2 + x_2 - 2x_3 = 0 \end{array}\right\} \rightarrow \begin{array}{l} x_2 = 2/3 \\ x_3 = 4/3 \end{array}$$