27/04/2023

CPM & PERT

CPM → Critical Path Method

PERT → Project Evaluation Review Technique

These are network based methods designed to assist in planning, scheduling and control of projects.

CPM assumes deterministic durations for the activity whereas PERT assumes probabilistic durations.

Objectives:

(1.) To determine the min. possible completion time for the project.

(2.) To determine a range of start and finish time for each activity so that the project can be completed in min. time.

Activities:

These are classified as critical & non-critical.

Critical activities → have no leeway in determining its start and finish times.
Non-critical activities → allows some scheduling slack so that start time of activity can be delayed within limits

without affecting the total completion time of the project.

Notations:

(i.) → ES_j = $\square_j$

↳ Earliest occurence time of the event j.

(ii.) → LC_j = Δ_j

↳ Latest completion time of the event j.

(iii.) → D_{ij} : (i, j)

↳ Duration of the activity b/w i & j.

Critical path calculations involve 2 passes:

(1.) The forward pass determines the earliest occurence times of the events.

(2.) The backward pass determines the latest completion times of the events.

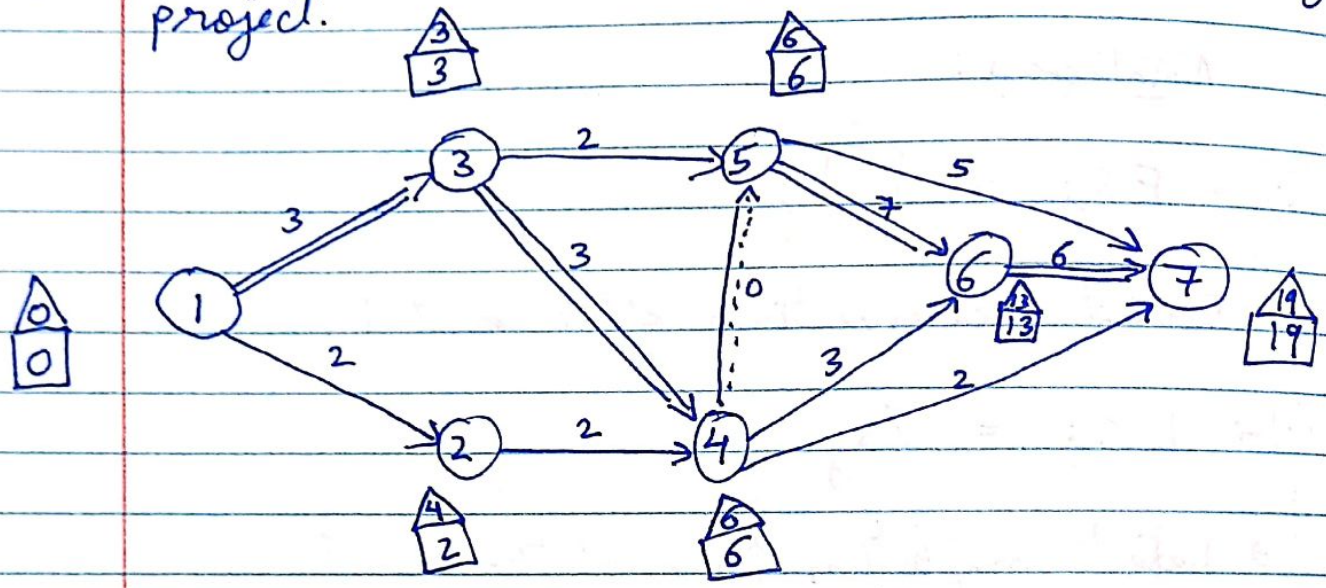
(*) Critical activity is defined if the following 3 cond^{n}'s are satisfied

(i.) $\square_i = \Delta_i$

(ii.) $\square_j = \Delta_j$

(iii.) $\square_j - \square_i = \Delta_j - \Delta_i = D_{ij}$

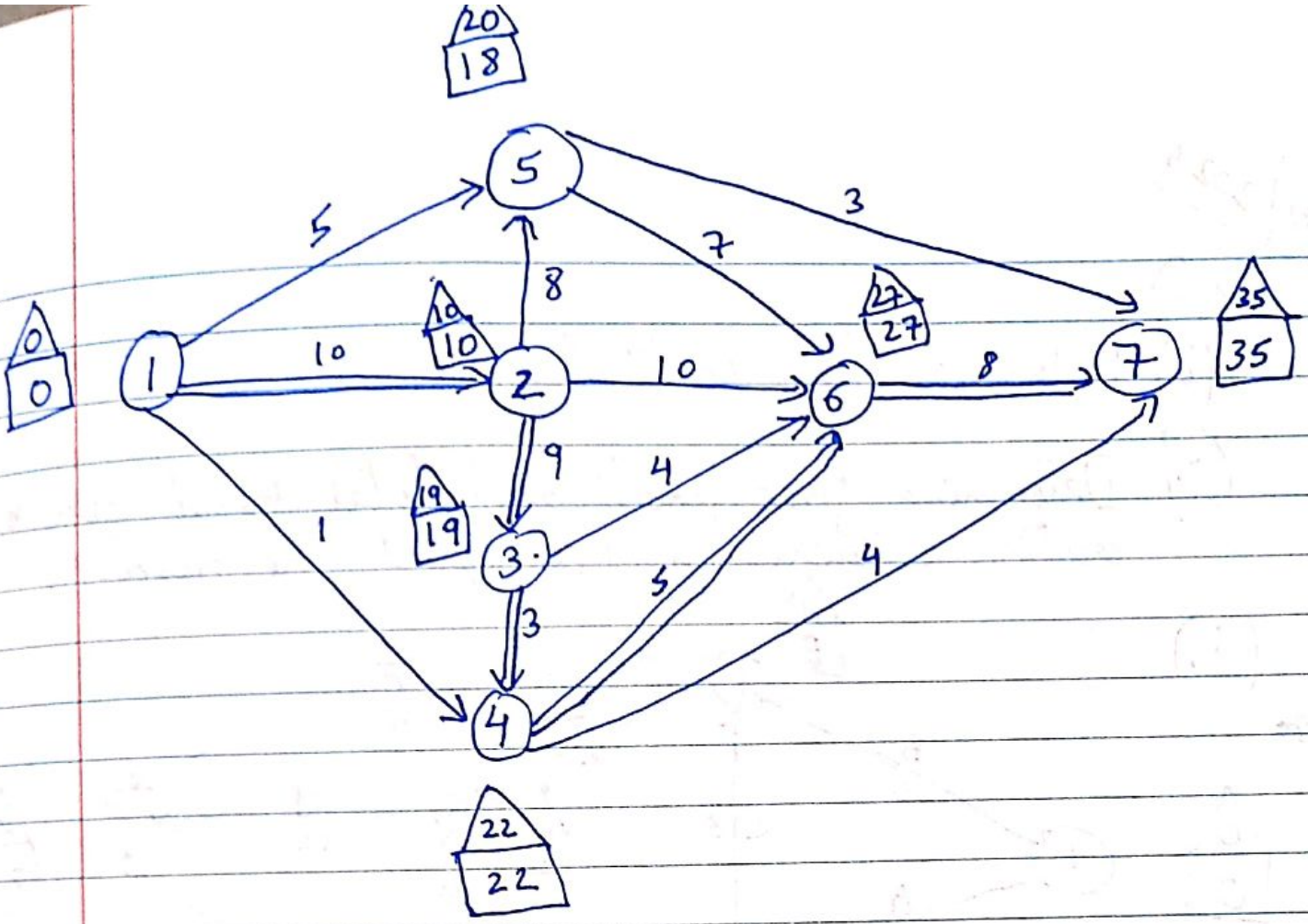
(Q.) Determine the critical path for the following project.



Critical path : 1 – 3 – 4 – 5 – 6 – 7

(Q.) Draw the network & find the critical path.

Nodes	Dij
1-2	10
1-4	1
1-5	5
2-3	9
2-5	8
2-6	10
3-4	3
3-6	4
4-6	5
4-7	4
5-6	7
5-7	3
6-7	8



Critical Path : 1 - 2 - 3 - 4 - 6 - 7

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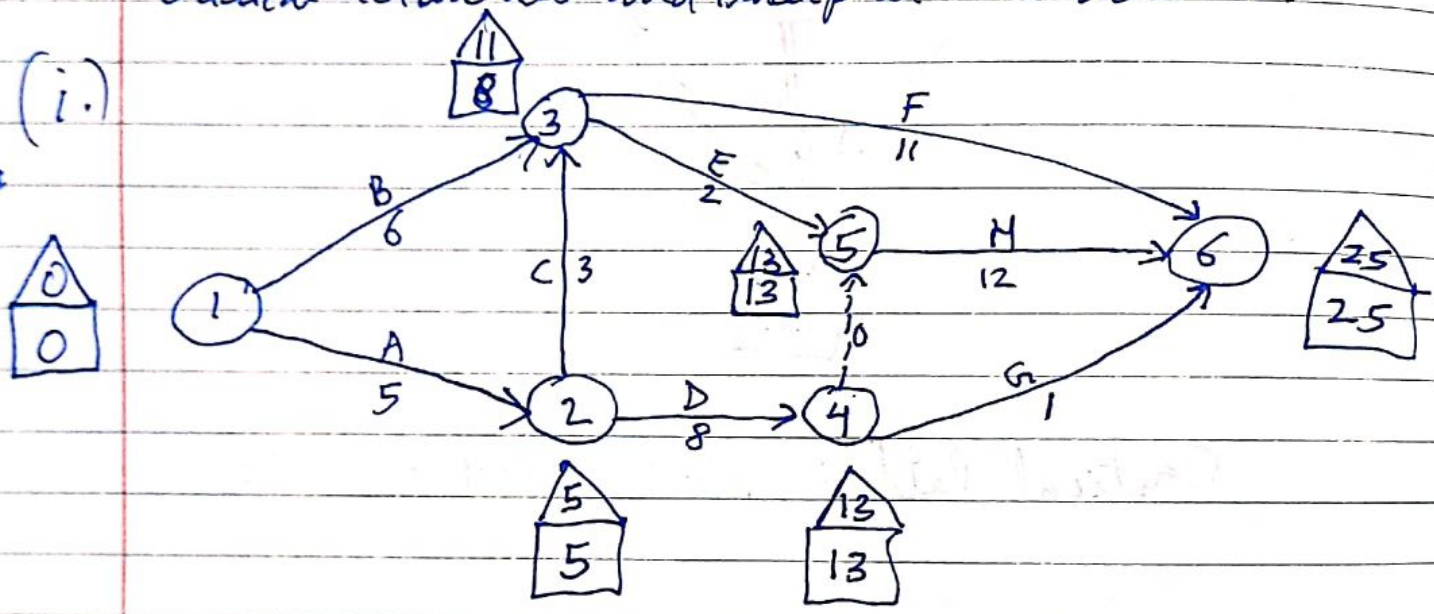
(Q.)

(i.) Determine the critical path..

(ii.) Determine free float and total float for non-critical activities and interpret the result.

(i.)



Critical path = 1 - 2 - 4 - 5 - 6

Critical activities = (A, D, H)

Non-critical activities = (B, C, E, F, G)

(ii.)

Non-crit. activities	Dij	TF = $\triangle_j - \square_i - D_{ij}$	FF = $\square_j - \square_i - D_{ij}$
B	6	11-0-6 = 5	8-0-6 = 2
C	3	11-5-3 = 3	8-5-3 = 0
E	2	13-8-2 = 3	13-8-2 = 3
F	11	25-8-11 = 6	25-8-11 = 6
G	1	25-13-1 = 11	25-13-1 = 11

Interpretation

(*) if FF < TF:

For B: B can start anytime b/w 0 to 2 units w/o delaying the successive activities. If however B starts at anytime 2 + d (< 5) then the start time of the consecutive activity will be pushed forward.

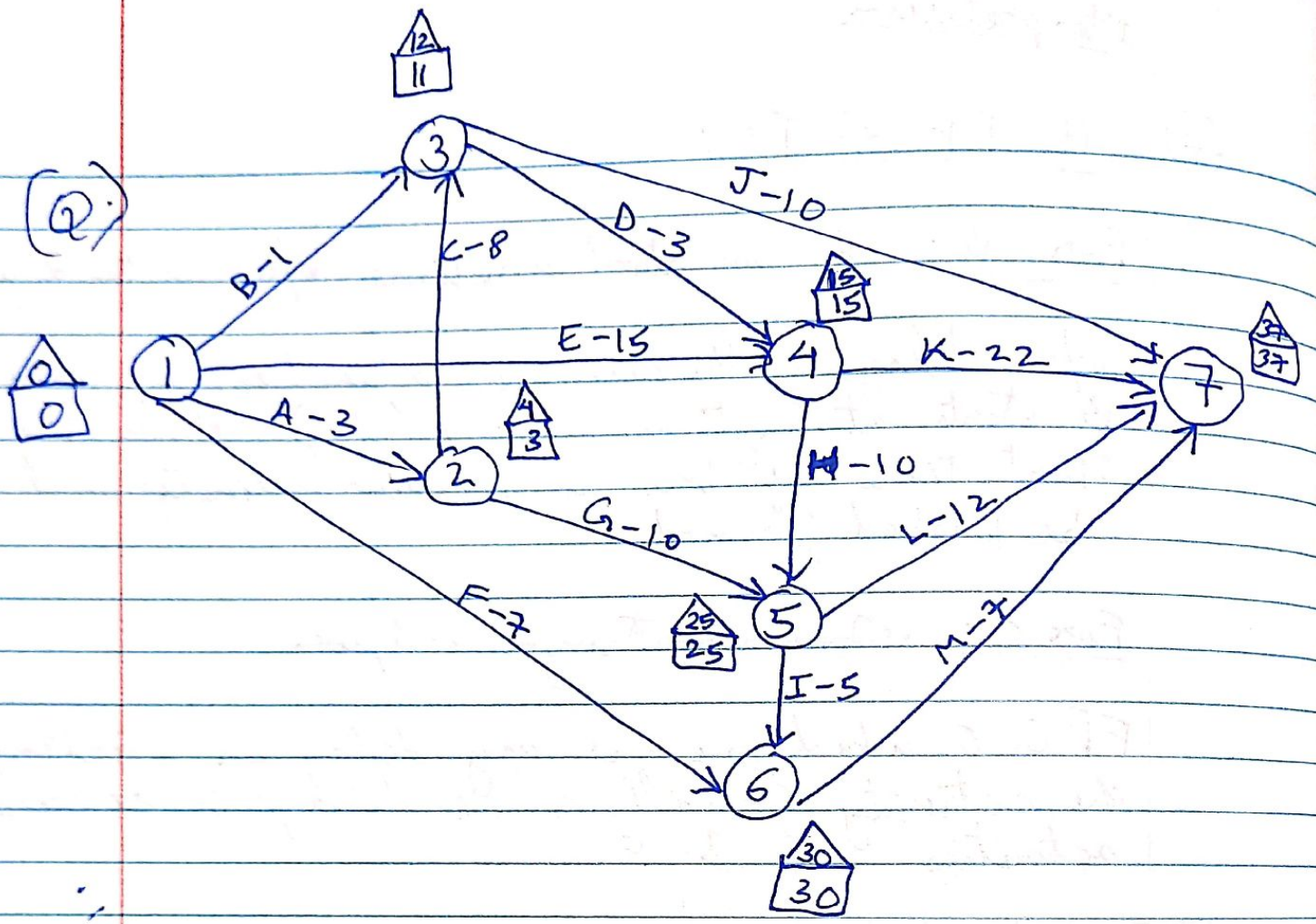
For C: ~~C can start anytime b/w~~

FF is 0 which means any delay in starting the activity C will cause delays in successive activities E & F.

(*) if FF = TF:

Activities E, F & G may be scheduled anywhere b/w the earliest start time and the latest completion time w/o delaying the project.

(Q)



C.P = 1-4-7
1-4-5-7
1-4-5-6-7

Non-Critical Act.	Dij	T F	F F	
A	3	1	0	→ FF = 0
B	1	11	10	→ FF < TF
C	8	1	0	→ FF = 0
D	3	1	1	FF = TF
F	7	23	23	
G	10	12	12	
J	10	16	16	

Write the explanation as per prev. qⁿ.

3/05/23

PERT [Program Evaluation and Review Technique]

The durat^n of the activity is not fixed rather it is decided on the basis of most optimistic time - 'a', most likely time - 'm', and pessimistic time - 'b'.

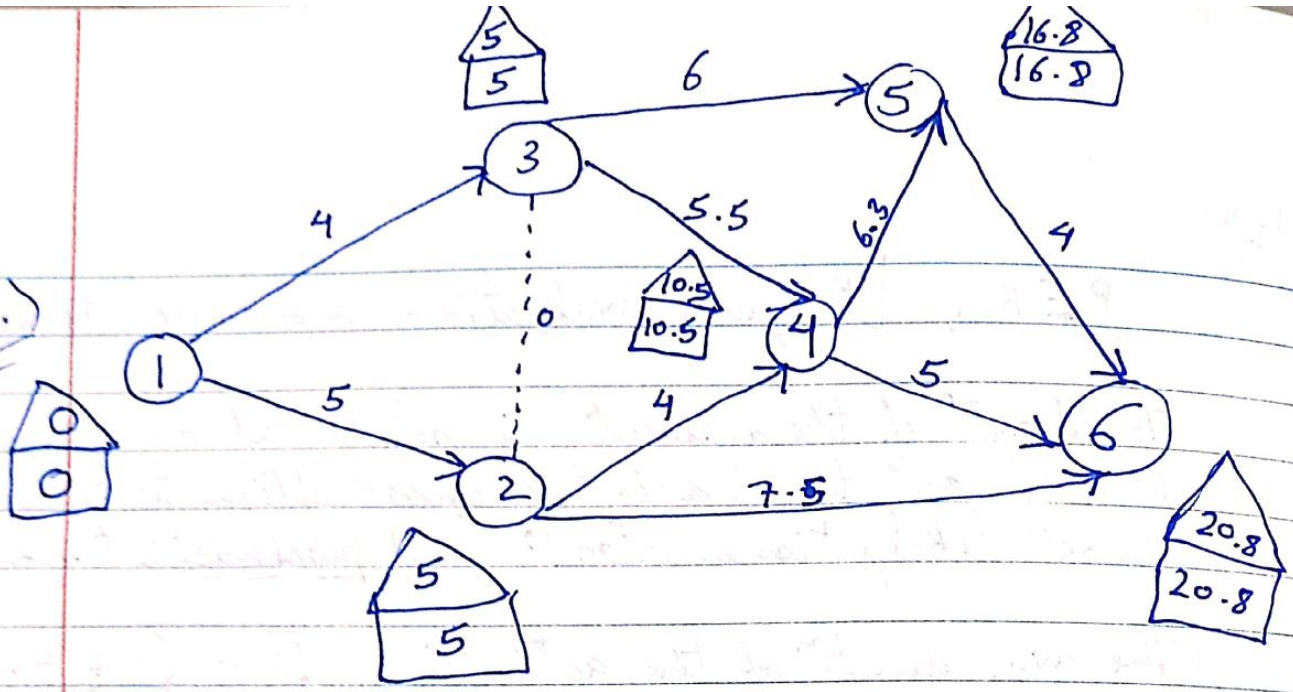
The avg. durat^n of the activity : $\bar{D} = \frac{(a + 4m + b)}{6}$

$$\text{Variance} = \left(\frac{b-a}{6}\right)^2$$

(Q.) (i) Draw the network
(ii) Determine critical path
(iii) Find the probability that the earliest occurence time of the 4th event is atmost 12 days.
(iv) Find the prob. that the entire project will be completed within 23 days.

Activity	a	b	m	$\bar{D}$	Variance
1-2	2	8	5	5	1
1-3	1	7	4	4	1
2-3	0	0	0	0	0
2-4	2	6	4	4	4/9 → 0.44
2-6	5	12	7	15/2	1.36
3-4	3	10	5	11/2	1.36
3-5	3	9	6	6	1
4-5	4	10	6	19/3 → 6.33	1
4-6	2	8	5	5	1
5-6	2	6	4	4	0.44

Ans:)



C.P : 1 - 2 - 3 - 4 - 5 - 6

⊛ If 2 or more paths lead to the event i then we consider the one with largest durat^n.

⊛ If there is a tie amongst the durat^n's then we select the one which corresponds to the larger variance thereby reflecting more uncertainty.

Critical Activities	E(Di)	Var.	
1	0	0	
2	5	1	(1→2) : 0+1 = 1
3	5	1	(2→3) : 1+0 = 1
4	10.5	2.36	(3→4) : 1+1.36 = 2.36
5	16.8	3.36	(4→5) : 2.36+1 = 3.36
6	20.8	3.8	(5→6) : 3.36+0.44 = 3.8

$X \rightarrow Z$

$$Z = \frac{X - \mu}{\sigma}$$

(iii) $P(X_4 \leq 12)$

$$\Downarrow$$

$$P\left(Z \leq \frac{12 - 10.5}{\sqrt{2.36}}\right) \Rightarrow P(Z \leq 0.98)$$

$$P(Z \leq 0.98) = 0.8365 \approx 83.65\%$$

Probability

(iv) $P(X_6 \leq 23)$

$$\Downarrow$$

$$P\left(Z \leq \frac{23 - 20.8}{\sqrt{3.8}}\right)$$

$$\Downarrow$$

$$P(Z \leq 1.13) = 0.8708 \approx 87.08\%$$

or

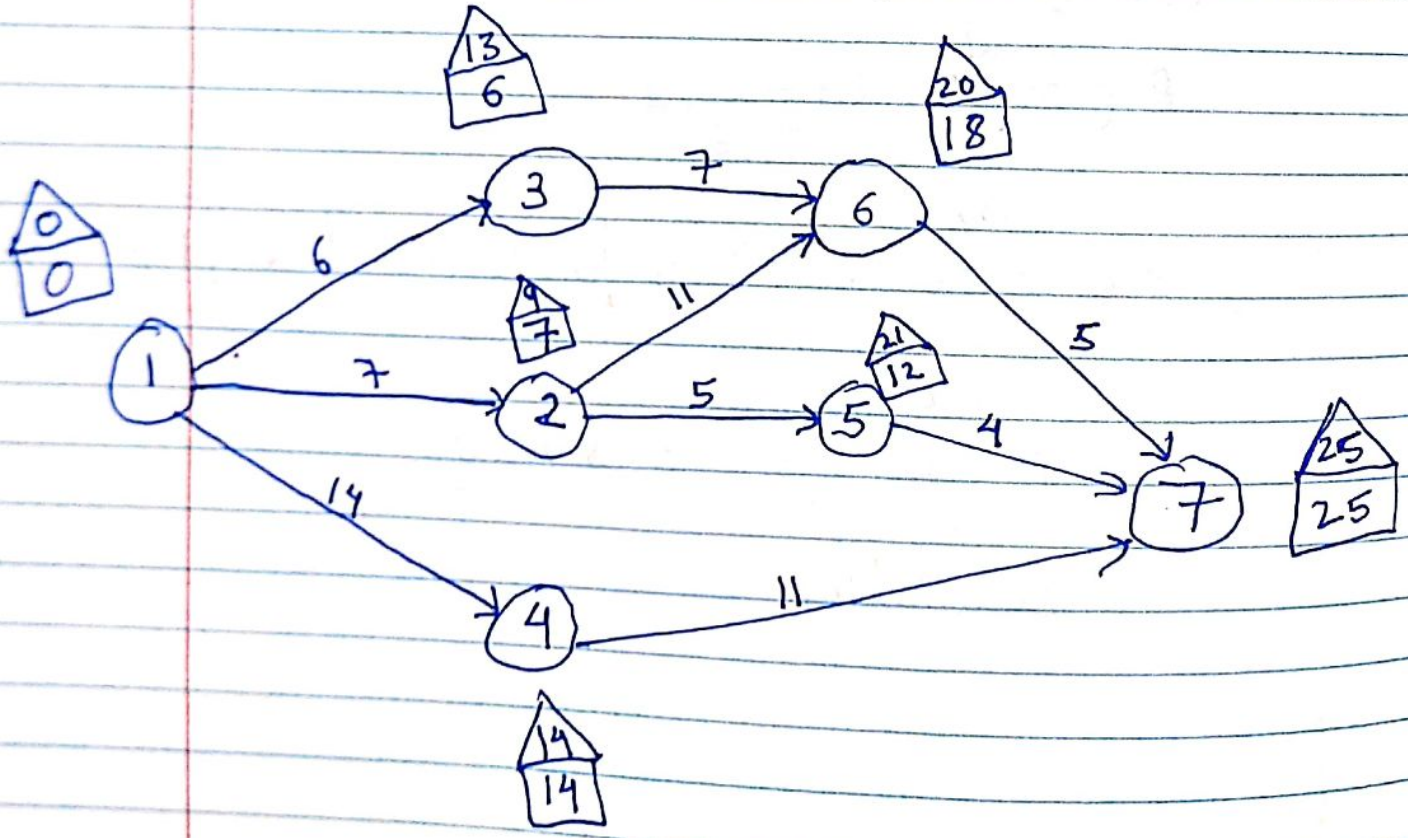
$$F(1.13) = 0.8708$$

cumulative

09/05/23

(Q.)

Activity	a	b	m	$\bar{D}$	Variance
1-2	3	15	6	7	4
1-3	2	14	5	6	4
1-4	6	30	12	14	16
2-5	2	8	5	5	1
2-6	5	17	11	11	4
3-6	3	15	6	7	4
4-7	3	27	9	11	16
5-7	1	7	4	4	1
6-7	2	8	5	5	1



C.P = 1 - 4 - 7

(Grit)

Activity	E(Di)	Var.	
1	0	0	
4	14	16	(1→4 : 0+16=16)
7	25	32	(4→7 : 16+16=32)

(i.) Find Prob. that it will take atleast 23 days to complete project.

Ans.) $P(X_7 \geq 23)$

$$P\left(Z \geq \frac{23-25}{\sqrt{32}}\right)$$

$$P(Z \geq -0.35)$$

$$= 1 - F(-0.35)$$

$$= 1 - 0.3632$$

$$= 0.6368$$

(ii.) Find $P(X_7)$ —(≤ 5 days prior to the expected duration) → within (4)

Ans.) $P(X_7 < 20) \rightarrow P\left(Z < \frac{20-25}{\sqrt{32}}\right)$

$\rightarrow P(Z < -0.88) \rightarrow F(-0.88)$

$= 0.1894$