24/04/2023

# ASSIGNMENT MODEL

It is a special case of transportation model where no. of supply nodes = no. of demand nodes. It is an $n \times n$ square design where n workers are assigned to n jobs.

Objective → to ~~minimize~~ minimize the total time to complete a set of jobs or to maximize the skill ratings or total satisfaction of customers or minimize the cost of assignments.

Assumptions:

* Each machine or worker is assigned no more than one job. And each job is assigned to exactly one worker/machine.

(Q.)

| | $J_1$ | $J_2$ | $J_3$ |
|---|---|---|---|
| $A_1$ | 15 | 10 | 9 |
| $A_2$ | 9 | 15 | 10 |
| $A_3$ | 10 | 12 | 8 |

1st step

| |
|---|
| 9 |
| 9 |
| 8 |

2nd step

| | | |
|---|---|---|
| 6 | 1 | 0 |
| 0 | 6 | 1 |
| 2 | 4 | 0 |

0 1 0

| | $J_1$ | $J_2$ | $J_3$ |
|---|---|---|---|
| $A_1$ | 6 | (0) | 0 |
| $A_2$ | (0) | 5 | 1 |
| $A_3$ | 2 | 3 | (0) |

$A_1 \rightarrow J_2$
$A_2 \rightarrow J_1$
$A_3 \rightarrow J_3$

$\therefore$ Cost = 9 + 10 + 8 = 27

(Q.)

| | $J_1$ | $J_2$ | $J_3$ | $J_4$ | |
|---|---|---|---|---|---|
| $A_1$ | 8 | (6) | 5 | 7 | 5 |
| $A_2$ | 6 | 5 | 3 | (4) | 3 |
| $A_3$ | 7 | 8 | (4) | 6 | 4 |
| $A_4$ | (6) | 7 | 5 | 6 | 5 |

| | $J_1$ | $J_2$ | $J_3$ | $J_4$ |
|---|---|---|---|---|
| $A_1$ | 3 | 1 | 0 | 2 |
| $A_2$ | 3 | 2 | 0 | 1 |
| $A_3$ | 3 | 4 | 0 | 2 |
| $A_4$ | 1 | 2 | 0 | 1 |
| | 1 | 1 | 0 | 1 |

| | $J_1$ | $J_2$ | $J_3$ | $J_4$ |
|---|---|---|---|---|
| $A_1$ | 2 | [0] | 0 | 1 |
| $A_2$ → | 2 | 1 | 0 | [0] |
| $A_3$ | 2 | 3 | [0] | 1 |
| $A_4$ | [0] | 1 | 0 | 0 |

$A_1 \rightarrow J_2$
$A_2 \rightarrow J_4$
$A_3 \rightarrow J_3$
$A_4 \rightarrow J_1$

Cost = 6 + 6 + 4 + 4 = 20

Q.

| | | | | | |
|---|---|---|---|---|---|
| 3 | 8 | 2 | 10 | (3) | 2 |
| 8 | 7 | (2) | 9 | 7 | 2 |
| 6 | (4) | 2 | 7 | 5 | 2 |
| 8 | 4 | 2 | (3) | 5 | 2 |
| (9) | 10 | 6 | 9 | 10 | 6 |

| | | | | |
|---|---|---|---|---|
| 1 | 6 | 0 | 8 | 1 |
| 6 | 5 | 0 | 7 | 5 |
| 4 | 2 | 0 | 5 | 3 |
| 6 | 2 | 0 | 1 | 3 |
| 3 | 4 | 0 | 3 | 4 |

1 2 0 1 1

| | | | | |
|---|---|---|---|---|
| 0 | (0) | (0) | (7) | 0 |
| 5 | 3 | [0] | 6 | 4 |
| 3 | 0 | 0 | 4 | 2 |
| 5 | (0) | (0) | 0 | 2 |
| 2 | 2 | [0] | 2 | 3 |

(2)

(*) If no feasible sol$^n$ with all 0 entries are obtained. Then draw min no. of horizontal or vertical lines in the last reduced matrix such that it covers all 0 entries.

Select smallest uncovered element and subtract it from every uncovered element and add it to every element at the intersection. If no feasible sol$^n$ is found then repeat the process else determine the optimal sol$^n$.

| | | | | |
|---|---|---|---|---|
| 0 | 6 | 2 | 7 | [0] |
| 3 | 3 | [0] | 4 | 2 |
| 1 | [0] | 0 | 2 | 0 |
| 5 | 2 | 2 | [0] | 2 |
| [0] | 2 | 0 | 0 | 1 |

Cost = 9+4+2+3+3

= 21

Q4/4

# Unbalanced Assignment Problem

(Q.)

| | | | | |
|---|---|---|---|---|
| 7 | 5 | 8 | (4) | 4 |
| (5) | 6 | 7 | 4 | 4 |
| 8 | (7) | 9 | 8 | 7 |
| 0 | 0 | (0) | 0 | 0 |

added → to make square design

| | | | | |
|---|---|---|---|---|
| 3 | 1 | 4 | 0 | |
| 1 | 2 | 3 | 0 | |
| 1 | 0 | 2 | (1) | (1) → minimum |
| 0 | 0 | 0 | (0) | |
| 0 | 0 | 0 | 0 | |

| | | | |
|---|---|---|---|
| 2 | 0 | 3 | [0] |
| [0] | 1 | 2 | 0 |
| 1 | [0] | 2 | 2 |
| 0 | 0 | [0] | 1 |

Cost = 5 + 7 + 4 = 16 //

26/04/

(8.) A company has 4 territories open & 4 salesmen available for the assignment. The territories are not equally rich in their sales potential. It is estimated that a typical salesman operating in each territory would bring in the following annual sales.

| Territory: | I | II | III | IV |
|---|---|---|---|---|
| Annual sales (AED): | 60,000 | 50,000 | 40,000 | 30,000 |

4 salesmen are considered to defer in their ability. It is estimated that working under the same cond$^{n}$'s their yearly sales would be proportional as follows:

| Salesmen: | A | B | C | D |
|---|---|---|---|---|
| proper$^{n}$: | 7 | 5 | 4 | 4 |

The criteria is to maximize the expected total sales using the assignment model.

Territory (Sales in AED 10000)

Ans.)

| | I (6) | II (5) | III (4) | IV (3) | |
|---|---|---|---|---|---|
| A (7) | -42 | -35 | -28 | -21 | -42 |
| B (6) | -30 | -25 | -20 | -15 | -30 |
| C (5) | -30 | -25 | -20 | -15 | -30 |
| D (4) | -24 | -20 | -16 | -12 | -24 |

Values are taken as -ve to convert max to min prob.

| | | | |
|---|---|---|---|
| 0 | 7 | 14 | 21 |
| 0 | 5 | 10 | 15 |
| 0 | 5 | 10 | 15 |
| 0 | 4 | 8 | 12 |

0 4 8 12

| | | | |
|---|---|---|---|
| 0 | 3 | 6 | 9 |
| 0 | 1 | 2 | 3 |
| 0 | 1 | 2 | 3 |
| 0 | 0 | 0 | 0 |

1 min

| | | | |
|---|---|---|---|
| 0 | 2 | 5 | 8 |
| 0 | 0 | 1 | 2 |
| 0 | 0 | 1 | 2 |
| 1 | 0 | 0 | 0 |

min
1

| | | | |
|---|---|---|---|
| 0 | 2 | 4 | 7 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 |
| 2 | 1 | 0 | 0 |

A - I
B - II
C - III
D - IV

Cost => Max expected annual sales
= 42 + 25 + 20 + 12
= 99,000 AED

(Q.) 3 work centers are required to manufacture, assemble and package a product.
4 locations are available in the plant.
The materials handling cost at each location for the work centers is given by the following cost matrix.
Determine the Locat$^{n}$s of work centers that minimize the total material handling cost.

| | Locat$^{n}$s 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | 18 | 15 | 16 | 13 |
| B | 16 | 11 | – | 15 |
| C | 9 | 10 | 12 | 18 |

Ans.)

| | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| A | 18 | 15 | 16 | 13 | 13 |
| B | 16 | 11 | M | 15 | 11 |
| C | 9 | 10 | 12 | 18 | 9 |
| D | 0 | 0 | 0 | 0 | 0 |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | ~~5~~ | 2 | 3 | [0] |
| B | 5 | [0] | M-11 | 4 |
| C | [0] | 1 | 3 | 9 |
| D | 0 | 0 | [0] | 0 |
| | 0 | 0 | 0 | 0 |

(same remains)

A → 4
B → 2
C → 1
D → 3

Cost = 13 + 11 + 9 + 0
= 33