2023

Duality

is a L.P.P defined ~~of~~ directly or systematically from the primal or original L.P model.

The 2 problems are so closely related that the optimal solⁿ of one problem automatically provides the optimal solⁿ of the other.

Need or purpose of using Dual form

In L.P numerical work increases more with the increase in the no. of constraints whereas in dual problem it gets interchanged, thus, it is easy to calculate.

Dual problem is constructed from primal

- Dual var. is defined for each primal constraint
- Dual constraint is defined for each primal var.
- The constraint (column) coeff. of a primal var. defines the LHS coeff. of the dual constraint and its obj. coeff. is defined by the RHS.
- Obj. coeff. of the dual equals the RHS of the primal constraint eqⁿ.

The sense of optimization in the dual is always the opposite of that of primal. If dual obj. f^n is minimization, the constraints are $\geq$ or $=$ ($\geq$) type.

And vice-versa for maximization. ($\leq$)

Starting point in all constraints are eqn's with non-ve RHS and all var. are also -ve.

Q.) write the dual for each of the primal problems:

1. Max $z = -5x_1 + 2x_2$

s.t $-x_1 + x_2 \leq -2$

$2x_1 + 3x_2 \leq 5$ $\quad x_1, x_2 \geq 0$

Ans.) $x_1 - x_2 + s_1 = 2$

$2x_1 + 3x_2 + s_2 = 5$ $\quad x_1, x_2, s_1, s_2 \geq 0$

Dual form

Min $z = 2y_1 + 5y_2$

s.t $\quad y_1 + 2y_2 \geq -5$

$-y_1 + 3y_2 \geq 2$

$y_1 \geq 0 \quad , \quad y_2 \geq 0$

2. Min $z = 6x_1 + 3x_2$

s.t. $6x_1 - 3x_2 + x_3 \geq 2$

$3x_1 + 4x_2 + x_3 \geq 5$

$x_1, x_2, x_3 \geq 0$

Ans:

$6x_1 - 3x_2 + x_3 - x_4 = 2$

$3x_1 + 4x_2 + x_3 - x_4 = 5$

$x_1 \ldots x_4 \geq 0$

Dual form

Max $z = 2y_1 + 5y_2$

s.t

$6y_1 + 3y_2 \leq 6$

$-3y_1 + 4y_2 \leq 3$

$y_1 + y_2 \leq 0$

$-y_1 \leq 0$

$-y_2 \leq 0$

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Simplex Table Computations

1. <u>Constraint column computations:</u> (<u>Feasibility</u>)

$$\begin{pmatrix}\text{constraint} \\ \text{column in} \\ \text{iteration 'i'}\end{pmatrix} = \begin{pmatrix}\text{Inverse in} \\ \text{iteration} \\ \text{'i'}\end{pmatrix}\begin{pmatrix}\text{original} \\ \text{constraint} \\ \text{column}\end{pmatrix}$$

2. <u>Objective Z-row computations:</u> (Optimality)

$$\begin{pmatrix}\text{Primal } z\text{-eq}^n \\ \text{coeff. of var. } x_j\end{pmatrix} = \begin{pmatrix}\text{LHS of corresp.} \\ \text{dual constraint}\end{pmatrix} - \begin{pmatrix}\text{RHS of corresponding} \\ \text{dual constraint}\end{pmatrix}$$

(Q) Max $z = 4x_1 + 14x_2$

s.t. $2x_1 + 7x_2 + x_3 = 21$

$7x_1 + 2x_2 + x_4 = 21$ $\quad x_1, x_2, x_3, x_4 \geq 0.$

(i) write the dual form

Ans-(i) $\text{Min } z = 21y_1 + 21y_2$

s.t $2y_1 + 7y_2 \geq 4$

$7y_1 + 2y_2 \geq 14$

$y_1 \geq 0, \; y_2 \geq 0$

(ii) Check feasibility & optimality of each of the following:

(a) basic var. $= (x_2, x_4)$

inverse $= \begin{pmatrix} 1/7 & 0 \\ -2/7 & 1 \end{pmatrix}$

Ans) (a) Feasibility:

$$\begin{pmatrix} x_2 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1/7 & 0 \\ -2/7 & 1 \end{pmatrix} \begin{pmatrix} 21 \\ 21 \end{pmatrix} = \begin{pmatrix} 3 \\ 15 \end{pmatrix} \begin{matrix} \rightarrow x_2 \\ \rightarrow x_4 \end{matrix}$$

Since both values are +ve, it implies feasibility.

Optimality:

$$(y_1, y_2) = (14 \;\; 0) \begin{pmatrix} 1/7 & 0 \\ -2/7 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \end{pmatrix}$$

$\uparrow y_1 \quad \uparrow y_2$

Non-basic var.

$x_1 : 2y_1 + 7y_2 - 4$

$2(2) + 7(0) - 4 = 0$

$x_3 : y_1 - 0 = 2$

Its a maximization prob. and coeff. of non-basic var. are +ve which implies optimality.

ii. b. basic var. $= (x_2, x_3)$

inverse $= \begin{pmatrix} 0 & 1/2 \\ 1 & -7/2 \end{pmatrix}$

Feasibility

$$\begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 & 1/2 \\ 1 & -7/2 \end{pmatrix} \begin{pmatrix} 21 \\ 21 \end{pmatrix} = \begin{pmatrix} 21/2 \\ \frac{-105}{2} \end{pmatrix}$$

∴ Not feasible (as -ve)

Optimality

$$(y_1, y_2) = (14 \quad 0) \begin{pmatrix} 0 & 1/2 \\ 1 & -7/2 \end{pmatrix} = \begin{array}{cc} x_2 & \\ \uparrow & \\ 0 & 7 \end{array}$$

$(\cancel{0} \quad 7)$

Non-basic var.

x_1: $2y_1 + 7y_2 - 4 = (2(0) + 7(7) - 4) = \underline{\underline{45}}$

x_4: $y_2 - 0 = 7 - 0 = \underline{\underline{7}}$

$\therefore$ optimal.

(Q.) $\text{Max } z = 3x_1 + 2x_2 + 5x_3$

s.t
$$x_1 + 2x_2 + x_3 + x_4 = 30$$
$$3x_1 + 2x_3 + x_5 = 60$$
$$x_1 + 4x_2 + x_6 = 20$$

$$x_1 \ldots x_6 \geq 0$$

Ans.) Dual form

$$\text{Min } z = 30y_1 + 60y_2 + 20y_3$$

$$y_1 + 3y_2 + y_3 \geq 3$$

$$\cancel{2y_1 + 2y_2}\; 2y_1 + 4y_3 \geq 2$$

$$y_1 + 2y_2 \geq 5$$

$$y_1 \geq 0, \quad y_2 \geq 0, \quad y_3 \geq 0$$

Check feasi. and optim. for following

basic var. = $(x_4 \quad x_3 \quad x_6)$

inverse = $\begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Feasibility:

$$\begin{pmatrix} x_4 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 30 \\ 60 \\ 20 \end{pmatrix} = \begin{pmatrix} 0 \\ 30 \\ 20 \end{pmatrix}$$

$\therefore$ feasible

Optimality

$$(y_1, y_2, y_3) = (0 \quad \cancel{30}\ 5 \quad 0) \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= (0 \quad 5/2 \quad 0)$$

~~$\therefore$ Optimal~~

Non-basic:

$x_1 : y_1 + 3y_2 + y_3 - 3 = 9/2$

$x_2 : 2y_1 + 4y_3 - 2 = -2 \rightarrow$ not optimal

$x_5 : y_2 - 0 = 5/2$

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(ii)

~~Max z = $3x_1$~~

Basic = $(x_2 \quad x_3 \quad x_6)$

$$\text{Inverse} = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix}$$

<u>Feasibility</u>

$$\begin{pmatrix} x_2 \\ x_3 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 30 \\ 60 \\ 20 \end{pmatrix} = \begin{pmatrix} 0 \\ 30 \\ 20 \end{pmatrix}$$

$\therefore$ feasible

<u>Optimality</u>

$$(y_1, y_2, y_3) = (2 \quad 5 \quad 0) \begin{pmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{pmatrix}$$

$$= (1 \quad 2 \quad 0)$$

<u>Non-basic</u> :

x_1 : $y_1 + 3y_2 + y_3 - 3 = 4$

x_4 : $y_1 - 0 = 1$

x_5 : $y_2 - 0 = 2$ $\qquad \therefore$ Optimal