27/03/23

TRANSPORTATION MODELS

Its a Special case of L.P. It deals with shipping of commodities from m sources to n nodes.
The basic var.s will be $(m+n-1)$ and the independent constraints will be $(m+n-1)$.

Since transportation model deals with the shipping cost of the commodities,

∴ it will always be a minimization prob.

Transportation models are solved in 2 phases:

(i.) To find initial basic feasible sol^n.

(ii.) To find the optimal sol^n.

Initial basic feasible sol^n is determined by 3 methods:

(i.) North West Corner Method

(ii.) Least cost Method

(iii.) Vogel's Approximation Method

Any basic feasible sol^n will have $(m+n-1)$ basic vari's that assume non-zero +ve values.

And the non-basic var's will have zero values.

The cells corresponding to non-basic var. will be empty.

→ Basic feas. Sol^n

DEGENERATE BFS

If in a cell we find zero mentioned, it means that the cell corresponds to a basic var. that has assumed 0 value. It implies degenerate basic feasible sol^n.

BALANCED TRANSPORTATION

Σ supply = Σ demand

(Q.) Find the initial BFS for the following prob.:

				Demand
	1	2	6	7
	0	4	2	12
	3	1	5	11
Supply	10	10	10	

(a) North West corner Method

				Demand
	1 7 ↓	2	6	7 ✓
	0 3 →	4 9 ↓	2	12 ✓
	3	1 1 →	5 10	11 ✓
Supply	10 ✓	10 ✓	10 ✓	

∴ Cost = (7×1) + (3 ×0) + (9×4) + (1×1) +(10×5

= $94

(b.) Least Cost Method

			Demand
1	2	6 7	~~7~~
0 10	4	2 2	~~12~~
3	1 10	5 1	~~11~~
Supply ~~10~~	~~10~~	10 ✓	

$$\text{Cost} = (10\times0) + (10 * 1) + (1\times5) + (2\times2) + (7\times6)$$
$$= \$61$$

(c.) Vogel's Approximation Method: (VAM)

			Demand	P_1	P_2	P_3
1 7	2	6	7	1	1	1
0 2	4	2 10	12	2	④	-
3 1	1 10	5	11	2	2	2
Supply → 10	10	10				
P_1 → 1	1	③				
P_2 → 1	1	=				
P_3 2	1	=				

Cost = $40

㊉ The column with max penalty is selected to make the allocation then in that column we select that particular block and make max. allocation where cost is least.

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(Q.)

			Demand
5	1	8	12
2	4	0	14
3	6	7	4
9	10	11	

Supply

(i) NW corner method

5 9 →	1 3 ↓	8	12
2	4 7 →	0 7 ↓	14
3	6	7 4	4
9	10	11	

cost = 45 + 3 + 28 + 0 + 28 = \$ 104

(ii) Least Cost Method

5 2	1 10	8	12
2 3	4	0 11	14
3 4	6	7	4
9	10	11	

Cost = 10 + 10 + 6
+ 0 + 12
= 38 \$

(iii.) Vogel's Approx. Menthod (VAM)

					P_1 ↓	P_2 ↓
	2^{5}	10^{1}	^{8}	12	4	④
	3^{2}	^{4}	11^{0}	14	2	2
	4^{3}	^{6}	^{7}	4	3	3
	9	10	11			
P_1 →	1	3	⑦			
P_2 →	1	3	—			

Cost = 10 + 10 + 6 + 0 + 12
= 38 $

(Q.) A transport company ships truckloads of grain from 3 farms to 4 mills. The supply and the demand (in truckloads) together with the unit transportation cost per truckload on different routes are summarized in the model. The unit transportation cost C_{ij} (shown in NE corner of each box) are in 100's of \$'s. Use the 3 methods to find the starting feasible solⁿ.

Mill

Farm 1	10	2	20	11	15
2	12	7	9	20	25
3	4	14	16	18	10
	5	15	15	1.5	

(i) <u>NW corner Method</u>

Cost = \$ 520

10 5	2 10	20	11	15
12	7 5	9 15	20 5	25
4	14	16	18 10	10
5	15	15	15	

Least Cost Method

(ii)

10	2 15	20	11	15
12	7	9 15	20 10	25
4 5	14	16	18 5	10
5	15	15	15	

Cost = $ 475

(iii) VAM

					P_1	P_2	P_3
10	2 15	20	11	15	8	⑨	–
12	7	9 15	20 10	25	2	2	⑪
4 5	14	16	18 5	10	⑩	2	2
5	15	15	15				
$P_1 \rightarrow$ 6	5	7	7				
$P_2 \rightarrow$ –	5	7	7				
$P_3 \rightarrow$ –	–	7	2				

Cost = $475

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(Q.)

0	2	1	6
2	1	5	9
2	4	3	5
5	5	10	

(i.) Find starting BFS using N-W method.

(ii.) Find the optimal sol^n.

Ans-(i.)

0 5 →	2 1 ↓	1	6
2	1 4 →	5 5 ↓	9
2	4	3 5	5
5	5	10	

∴ Cost = 0 + 2 + 4 + 25 + 15 = \$ 46

⊛ To find the optimal sol^n, we use the method of multipliers. 2 conditions have to be satisfied:

(a.) supply limits & demand requirements remain satisfied.

(b.) shipments through all the routes must be non--ve.

→

⊛ non-basic = $u_i + v_j - c_{ij}$

Ans-ii)

	$v_1 = 0$	$v_2 = 2$	$v_3 = 6$	
$u_1 = 0$	0 5 →	2 $1-\theta$ ↓	1 θ $\boxed{5}$	6
$u_2 = -1$	2 $\boxed{-3}$	1 4 → $+\theta$	5 $5-\theta$ ↓	9
$u_3 = -3$	2 $\boxed{-5}$	4 $\boxed{-5}$	3 5	5
	5	5	10	

⊛ '5' is the entering var. as it is the most +ve coeff. ~~of~~ for the non-basic var's.

max value that θ ~~x~~ can assume such that both conditions are satisfied then $\theta = 1$

$$\left.\begin{array}{l} 1-\theta \geq 0 \\ 5-\theta \geq 0 \end{array}\right\} \begin{array}{c} \text{max} \\ \theta = 1 \end{array}$$

∴ leaving var. is $\textcircled{1-\theta}$.

	$v_1 = 0$	$v_2 = -3$	$v_3 = 1$	
$u_1 = 0$	0 $5-\theta$	2 $\boxed{-5}$	1 $1+\theta$	6
$u_2 = 4$	2 θ $\boxed{2}$	1 5	5 $4-\theta$	9
$u_3 = 2$	2 $\boxed{0}$	4 $\boxed{-5}$	3 5	5
	5	5	10	

$$\therefore \quad \left.\begin{array}{l} (5-\theta) \geq 0 \\ (4-\theta) \geq 0 \end{array}\right\} \begin{array}{c} \text{max} \\ \underline{\underline{\theta = 4}} \end{array}$$

	$v_1 = 0$	$v_2 = -1$	$v_3 = 1$
$u_1 = 0$	0 1	2 $\boxed{-3}$	1 5
$u_2 = 2$	2 4	1 5	5 $\boxed{-2}$
$u_3 = 2$	2 $\boxed{0}$	4 $\boxed{-3}$	3 5

∴ Since all non-basic are -ve, ∴ optimal.

∴ Optimal cost = 0 + 8 + 5 + 15 + 5
= $33

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(Q.)

5	1	7	10
6	4	6	80
3	2	5	15
75	20	50	

Σ Supply $\neq$ Σ demand

105 $\neq$ 145

$\therefore$ unbalanced transportat.

(i.) VAM for BFS

Ans -i.)

				P_1	P_2	P_3
5	1 10	7	10	4	$\boxed{4}$	—
6 60	4 10	6 10	80	2	2	2
3 15	2	5	15	1	1	1
0	0	0 40	40	0	–	–
75	20	50				

P_1 →	3	1	$\boxed{5}$
P_2 →	2	1	1
P_3 →	$\boxed{3}$	2	1

Cost = 10 + 40 + 60 + 360 + 45 + 0

= 515 $

(ii.) Optimal Sol^n

	$v_1 = 3$	$v_2 = 1$	$v_3 = 3$
$u_1 = 0$	5 $\boxed{-2}$	1 10	7 $\boxed{-4}$
$u_2 = 3$	6 60 $-\theta$	4 10	6 10 $+\theta$
$u_3 = 0$	3 15	2 $\boxed{-1}$	5 $\boxed{-2}$
$u_4 = -3$	0 θ $\boxed{0}$	0 $\boxed{-2}$	0 40 $-\theta$

$$\left.\begin{array}{l} 60 - \theta \geq 0 \\ 40 - \theta \geq 0 \end{array}\right\} \text{Cond}^{n}\text{'s}$$

$$\therefore \theta = 40$$

	$v_1 = 3$	$v_2 = 1$	$v_3 = 3$
$u_1 = 0$	5 $\boxed{-2}$	1 10	7 $\boxed{-4}$
$u_2 = 3$	6 20	4 10	6 50
$u_3 = 0$	3 15	2 $\boxed{-1}$	5 $\boxed{-2}$
$u_4 = -3$	0 40	0 $\boxed{-2}$	0 $\boxed{0}$

Cost = 10 + 120 + 40 + 300 + 45 + 0
= 515 $

Since 0 appears in non-basic var. and there is no other +ve value in non-basic that can be taken as the entering var., this means optimality has reached and future

17/04/23

UNBALANCED TRANSPORTATION with PENALTIES

(Q.) In an unbalanced trasp. prob., sometimes there are penalties for unsatified demand to reflect the failure of supplier to meet the required demand. The matrix is as follows:

	D1	D2	D3	
	1	5	6	90
	3	2	3	10
	2	6	1	20
	60	50	50	

Let the penalty cost per unit of unsatisfied demand be 6, 4 and 2 for destinations D_1, D_2, D_3.
Find (i) BFS using VAM.
(ii) check for optimality.

(i)

	D1	D2	D3		P_1	P_2	P_3
	1 60	5 30	6	90	$\boxed{4}$	1	1
	3	2 10	3	10	1	1	1
	2	6	1 20	20	1	$\boxed{5}$	—
	6	4 10	2 30	40	2	2	$\boxed{2}$
	60	50	50				

$P_1 \rightarrow$ 1 2 1 | $P_3 \rightarrow$ — 2 1

$P_2 \rightarrow$ — 2 1

$$Cost = 60 + 150 + 20 + 20 + 60 + 40$$
$$= \underline{\underline{350}} \$$$

(ii)

	$v_1 = 1$	$v_2 = 5$	$v_3 = 3$
$u_1 = 0$	1 60	5 30	6 $\boxed{-3}$
$u_2 = -3$	3 $\boxed{-5}$	2 10	3 $\boxed{-3}$
$u_3 = -2$	2 $\boxed{-3}$	6 $\boxed{-3}$	1 20
$u_4 = -1$	6 $\boxed{-6}$	4 10	2 30

∴ optimal as all non-basic var's are −ve.

∴ Cost = $\underline{\underline{350}}$ /-

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MAXIMIZATION $Prob^m$ in $Transportat^n$

(Q.) There are 4 $\xrightarrow{}$ (D_1, D_2, D_3, D_4) areas which are affected due to floods, food grain is to be dropped in these areas by 3 aircrafts (S_1, S_2, S_3). The following matrix is given:

	D_1	D_2	D_3	D_4	a_i
S_1	10	8	6	3	60
S_2	8	10	5	3	40
S_3	2	5	9	10	50
b_i	30	50	60	40	

where a_i denotes total no. of trips that aircraft S_i can make in one day.

b_i → No. of trips required to the area D_j in one day

C_{ij} ⇒ denotes amount of food grain that aircraft S_i can carry to the area D_j in one trip.

Find # of trips that aircraft S_i should make to the area D_j so that the total quantity of food dropped is maximized.

⊛ C_{ij} is converted as a −ve value for transportation model to be treated as a minimization problem.

$\therefore$ Max^n $prob^m \rightarrow C_{ij} \rightarrow -ve$

Using VAM

	D1	D2	D3	D4		P1	P2	P3	P4
S1	−10 30	−8 10	−6 20	−3	60	2	2	②	2
S2	−8	−10 40	−5	−3	40	2	2	2	⑤
S3	−2	−5	−9 10	−10 40	50	1	$\boxed{4}$	—	—
S4	0	0	0 30	0	30	0	0	0	0
	30	50	60	40					

					P5
P1 →	2	2	3	$\boxed{7}$	2
P2 →	2	2	3	—	—
P3 →	2	2	1	—	—
P4 →	—	2	1	—	0
P5 →	—	⑧	6	—	

$\therefore$ Cost = −300 − 80 − 120 − 400 − 90 − 400
= 1390/−

	$v_1 = -10$	$v_2 = -8$	$v_3 = -6$	$v_4 = -7$
$u_1 = 0$	30 −10	10 −8	20 −6	−3 $\boxed{-4}$
$u_2 = -2$	−8 $\boxed{-4}$	40 −10	−5 $\boxed{-3}$	−3 $\boxed{-6}$
$u_3 = -3$	−2 $\boxed{-11}$	−5 $\boxed{-6}$	10 −9	40 −10
$u_4 = 6$	0 $\boxed{-4}$	0 $\boxed{-2}$	30 0	0 $\boxed{-1}$

$\therefore$ optimal as all −ve non-basic var.

$\therefore$ No. of trips
= 1390