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(1.) Primal Simplex

The problem starts at a basic feasible solⁿ. Successive iteration continue to be feasible until optimality is reached in the last iteration.

(2.) Dual Simplex

L.P starts at a better than optimal solⁿ which is infeasible. Successive iterations continue to be optimal and feasibility is restored at the last iteration.

Dual Simplex Algorithm

(i.) Feasibility:

The leaving var. x_r is the basic var. having the most -ve value (ties are broken arbitarily). If all the basic var. are non-ve, algorithm ends.

(ii.) Optimality condⁿ:

Given that x_r is the leaving var., let $\bar{c}_j$ be the reduced cost of non-basic var. x_j and α_{rj} is the constraint coeff. in x_r row and x_j column. The entering var. is the non-basic var. with α_{rj} less than 0.

$$\min_{\text{non-basic } x_j} \left\{ \left| \frac{\bar{c}_j}{\alpha_{rj}} \right| ; \alpha_{rj} < 0 \right\}$$

If $\alpha_{rj} \geq 0$ $\forall$ non-basic x_j, the problem has infeasible solⁿ.

The L.P model starts with 2 requirements:

(i.) obj. fⁿ must satisfy the optimality condⁿ of regular simplex method.

(ii.) All constraints should be $\leq$. If the constraints are of $\geq$ type then we multiply both sides by -1 to convert ~~it~~ to equality.

If constraint is an equality then it is converted to this as eg: $x_1 + x_2 = 1$

$$\hookrightarrow \begin{cases} x_1 + x_2 \leq 1 \\ x_1 + x_2 \geq 1 \rightarrow -x_2 - x_2 \leq -1 \end{cases}$$

Dual Simplex

(Q.) Min $z = 3x_1 + 2x_2 + x_3$

subject to
$$3x_1 + x_2 + x_3 \geq 3$$
$$-3x_1 + 3x_2 + x_3 \geq 6$$
$$x_1 + x_2 + x_3 \leq 3$$
$$x_1, x_2, x_3 \geq 0$$

Ans.)
$$-3x_1 - x_2 - x_3 \leq -3$$
$$3x_1 - 3x_2 - x_3 \leq -6$$
$$x_1 + x_2 + x_3 \leq 3$$

$x_1, x_2, x_3 \geq 0$

⊛ All constr. are $\leq$, ∴ starting basic var. will be slack var.

Basic	x_1	$x_2 \downarrow$	x_3	s_1	s_2	s_3	Sol^n	
Z	-3	-2	-1	0	0	0	0	
s_1	-3	-1	-1	1	0	0	-3	
$\leftarrow s_2$	3	$\boxed{-3}$	-1	0	1	0	-6	most -ve, so leaving var
s_3	1	1	1	0	0	1	3	
Z	-5	0	$-1/3 \downarrow$	0	$-2/3$	0	4	
$\leftarrow s_1$	-4	0	$\boxed{-2/3}$	1	$-1/3$	0	$\boxed{-1}$	→ -ve → ∴ enter. var.
x_2	-1	1	$1/3$	0	$-1/3$	0	2	
s_3	2	0	$2/3$	0	$1/3$	1	1	

⊛ Its a minimiz. prob. and obj. f^n coeff. are -ve which implies optimality.

$$\min_{\substack{\text{non-basic} \\ x_j}} \left\{ \frac{\bar{c}_j}{\alpha_{rj}} \; ; \; \alpha_{rj} < 0 \right\}$$

<u>Non-basic var.'s</u> (1st <u>iteration</u>)

$x_1 : \left| \dfrac{-3}{3} \right|$ $\times$ (as α_{rj} not < 0)

$x_2 : \left| \dfrac{-2}{-3} \right| = \dfrac{2}{3}$ $\rightarrow$ min. $\therefore$ it enters <u>(x_2)</u>

$x_3 : \left| \dfrac{-1}{-1} \right| = 1$

<u>Non-basic var.'s</u> (2nd <u>iteration</u>)

$x_1 = \left| \dfrac{-5}{-4} \right| = \dfrac{5}{4}$

$x_3 = \left| \dfrac{-1/3}{-2/3} \right| = \dfrac{1}{2}$

$s_2 = \left| \dfrac{-2/3}{-1/3} \right| = 2$

B	x_1	x_2	x_3	s_1	s_2	s_3	Sol^n
z	-3	0	0	-1/2	-1/2	0	9/2
x_3	6	0	1	-3/2	1/2	0	3/2
x_2	-3	1	0	1/2	-1/2	0	5/2
S_3	-2	0	0	1	0	1	0

$\therefore$ <u>optimal</u> & <u>feasible</u>.

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(Q7) $\text{Min } z = 5x_1 + 6x_2$

s.t $x_1 + x_2 \geq 2$

$4x_1 + x_2 \geq 4$

Use dual simplex.

Ans:) $-x_1 - x_2 \leq -2$

$-4x_1 - x_2 \leq -4$

Basic	x_1 ↓	x_2	s_1	s_2	Soln
z	-5	-6	0	0	0
s_1	-1	-1	1	0	-2
← s_2	$\boxed{-4}$	-1	0	1	-4
z	0	$-19/4$	0	$-5/4$ ↓	5
← s_1	0	$-3/4$	1	$-1/4$	-1
x_1	1	$1/4$	0	$-1/4$	1
z	0	-1	-5	0	10
s_2	0	3	-4	1	4
x_1	1	1	-1	0	2

$\therefore z = 10$

$x_1 = 2, \ x_2 = 0, \ s_1 = 0, \ s_2 = 4$

$\therefore$ optimal & feasible.

Iteration-1

$x_1 : \left|\frac{-5}{-4}\right| = \frac{5}{4}$ → entering var , $x_2 = \left|\frac{-6}{-1}\right| = 6$

Iteration-2

$x_2 = \left|\frac{-19/4}{-3/4}\right| = \frac{19}{3}$, $S_2 = \left|\frac{-5/4}{-1/4}\right| = 5$

Q.) Use dual simplex.

$$\text{Min } z = 4x_1 + 2x_2$$

s.t

$$x_1 + x_2 = 1$$
$$3x_1 - x_2 \geq 2$$

Ans:)

$$\boxed{x_1 + x_2 \leq 1}$$
$$+x_1 + x_2 \geq 1 \Rightarrow \boxed{-x_1 - x_2 \leq -1}$$

$$\boxed{-3x_1 + x_2 \leq -2}$$

Iteration-1 → $x_1 = \left|\frac{-4}{-3}\right| = \frac{4}{3}$, $x_2 = \left(\frac{-2}{1}\right) \rightarrow 2 > 0$ X

Iteration-2 → $s_3 = \left|\frac{-4/3}{-1/3}\right| = 4$, $x_2 = \left|\frac{-10/3}{-4/3}\right| = \frac{5}{2}$

Basic	x_1 ↓	x_2	s_1	s_2	s_3	sol^n
Z	-4	-2	0	0	0	0
s_1	1	1	1	0	0	1
s_2	-1	-1	0	1	0	-1
← s_3	$-\boxed{3}$	1	0	0	1	-2
Z	0	$-10/3$ ↓	0	0	$-4/3$	$8/3$
s_1	0	$4/3$	1	0	$+1/3$	$1/3$
← s_2	0	$\boxed{-4/3}$	0	1	$-1/3$	$-1/3$
x_1	1	$-1/3$	0	0	$-1/3$	$2/3$
Z	0	0	0	$-5/2$	$-1/2$	$7/2$
s_1	0	0	1	1	0	0
x_2	0	1	0	$-3/4$	$1/4$	$1/4$
x_1	1	0	0	$-1/4$	$-1/4$	$3/4$

$\therefore$ optimal & feasible

$\therefore Z = 7/2$

$s_2 = s_3 = s_1 = 0, \quad x_2 = 1/4, \quad x_1 = 3/4$