

Rob J Hyndman  
George Athanasopoulos

# FORECASTING

## PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**  
Open Texts Publishing

## 9. ARIMA models

### 9.1 Random walk model

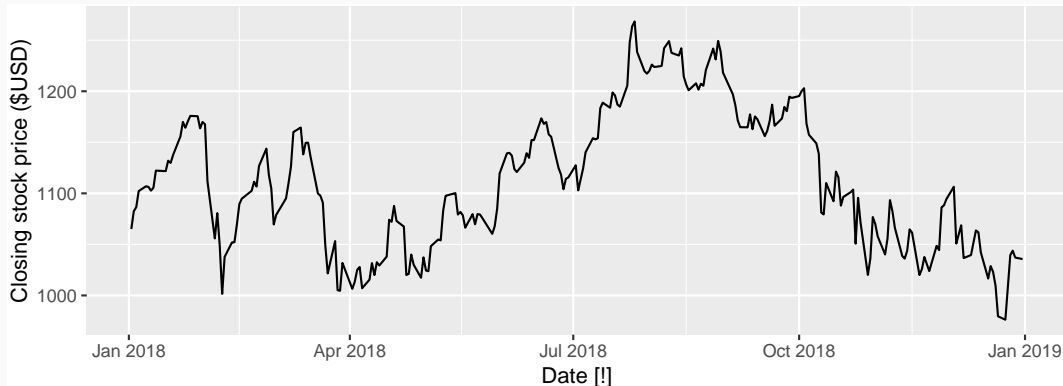
[OTexts.org/fpp3/](http://OTexts.org/fpp3/)

# Example: Google stock price

```
google_2018 <- gafa_stock |>  
  filter(Symbol == "GOOG", year(Date) == 2018)
```

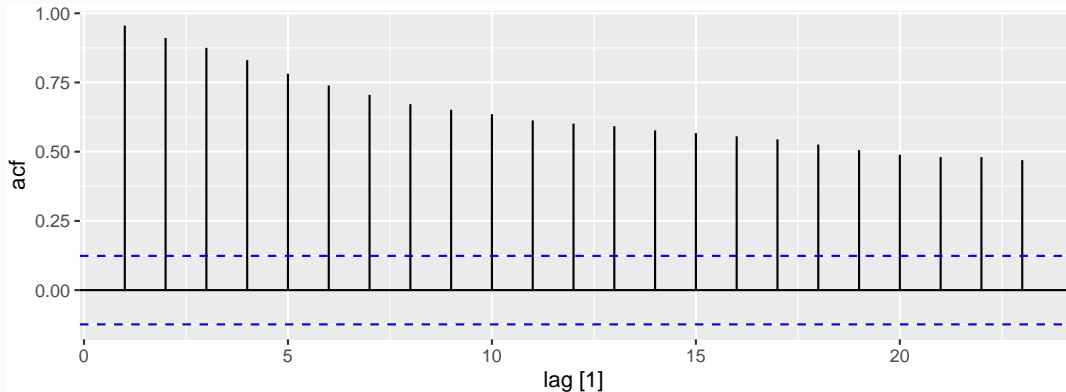
# Example: Google stock price

```
google_2018 |>  
  autoplot(Close) +  
  labs(y = "Closing stock price ($USD)")
```



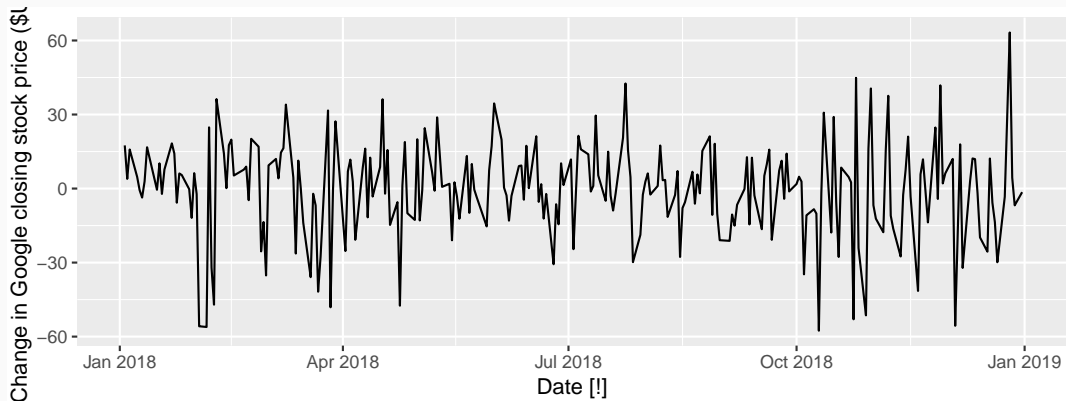
# Example: Google stock price

```
google_2018 |>  
  ACF(Close) |>  
  autoplot()
```



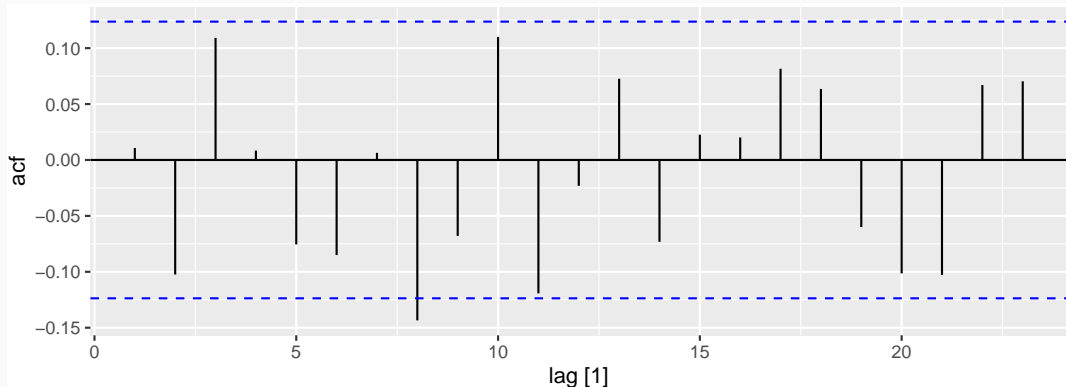
# Example: Google stock price

```
google_2018 |>  
  autoplot(difference(Close)) +  
  labs(y = "Change in Google closing stock price ($USD)")
```



# Example: Google stock price

```
google_2018 |>  
  ACF(difference(Close)) |>  
  autoplot()
```



## Example: Google stock price

- The differences are the **day-to-day** changes.
- Now the series looks just like a white noise series:
  - ▶ No autocorrelations outside the 95% limits.
- **Conclusion:** The daily change in the Google stock price is essentially a random amount uncorrelated with previous days.

# Random walk model

If differenced series is white noise with zero mean:

$$y_t - y_{t-1} = \varepsilon_t \quad \text{or} \quad y_t = y_{t-1} + \varepsilon_t$$

where  $\varepsilon_t \sim NID(0, \sigma^2)$ .

- Very widely used for non-stationary data.
- This is the model behind the **naïve method**.



# Random walk model

If differenced series is white noise with zero mean:

$$y_t - y_{t-1} = \varepsilon_t \quad \text{or} \quad y_t = y_{t-1} + \varepsilon_t$$

where  $\varepsilon_t \sim NID(0, \sigma^2)$ .

- Very widely used for non-stationary data.
- This is the model behind the **naïve method**.
- Random walks typically have:
  - ▶ long periods of apparent trends up or down
  - ▶ Sudden/unpredictable changes in direction

# Random walk model

If differenced series is white noise with zero mean:

$$y_t - y_{t-1} = \varepsilon_t \quad \text{or} \quad y_t = y_{t-1} + \varepsilon_t$$

where  $\varepsilon_t \sim NID(0, \sigma^2)$ .

- Very widely used for non-stationary data.
- This is the model behind the **naïve method**.
- Random walks typically have:
  - ▶ long periods of apparent trends up or down
  - ▶ Sudden/unpredictable changes in direction
- Forecast are equal to the last observation (naïve)
  - ▶ future movements up or down are equally likely.

# Random walk with drift model

If differenced series is white noise with non-zero mean:

$$y_t - y_{t-1} = c + \varepsilon_t \quad \text{or} \quad y_t = c + y_{t-1} + \varepsilon_t$$

where  $\varepsilon_t \sim NID(0, \sigma^2)$ .

- $c$  is the **average change** between consecutive observations.
- If  $c > 0$ ,  $y_t$  will tend to drift upwards and vice versa.
- This is the model behind the **drift method**.

# Seasonal differencing

If seasonally differenced data is white noise it implies:

$$y_t - y_{t-m} = \varepsilon_t \quad \text{or} \quad y_t = y_{t-m} + \varepsilon_t$$

where  $\varepsilon_t \sim NID(0, \sigma^2)$ .

- The model behind the **seasonal naïve** method.