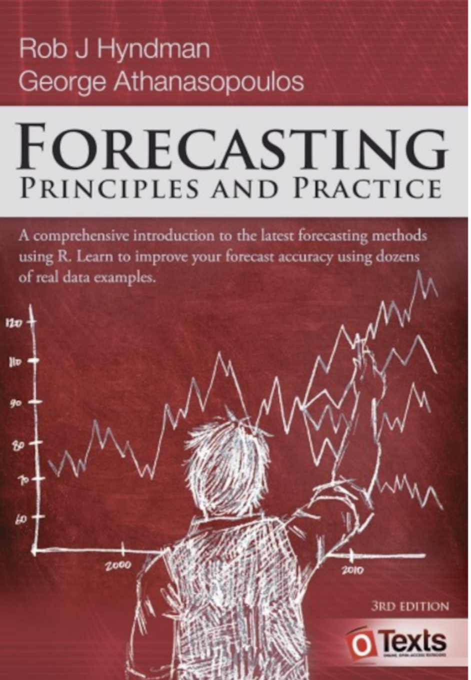


5. The forecaster’s toolbox

5.6 Forecasting using transformations

OTexts.org/fpp3/

Mathematical transformations

If the data show different variation at different levels of the series, then a transformation can be useful.

Denote original observations as $y_1, \dots, y_n$ and transformed observations as $w_1, \dots, w_n$.

Box-Cox transformations

$$w_t = \begin{cases} \log(y_t), & \lambda = 0; \\ (\text{sign}(y_t)|y_t|^\lambda - 1)/\lambda, & \lambda \neq 0. \end{cases}$$

- Natural logarithm, particularly useful because they are more interpretable: changes in a log value are relative (percent) changes on the original scale.

Back-transformation

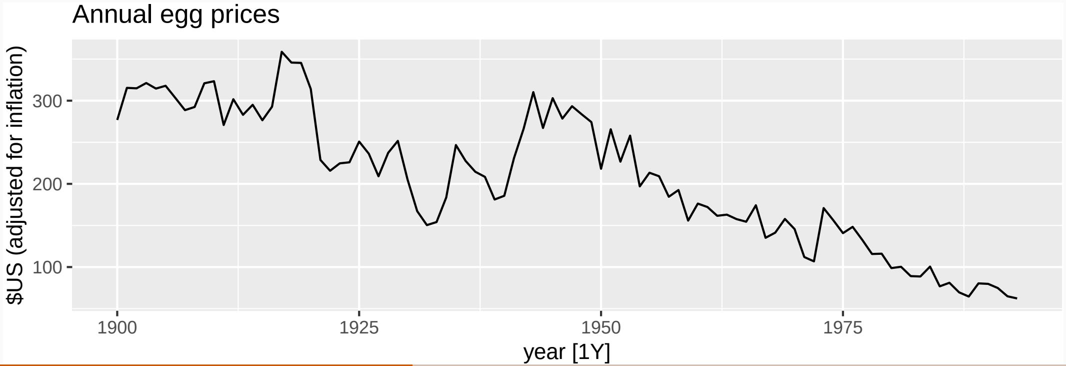
We must reverse the transformation or back-transform to obtain forecasts on the original scale. The reverse Box-Cox transformations are given by

Box-Cox back-transformations

$$y_t = \begin{cases} \exp(w_t), & \lambda = 0; \\ \text{sign}(\lambda w_t + 1)|\lambda w_t + 1|^{1/\lambda}, & \lambda \neq 0. \end{cases}$$

Modelling with transformations

```r
eggs <- prices |>
  filter(!is.na(eggs)) |>
  select(eggs)
eggs |> autoplot() +
  labs(title = "Annual egg prices", y = "$US (adjusted for inflation)")
```



Modelling with transformations

Transformations used in the left of the formula will be automatically back-transformed. To model log-transformed egg prices, you could use:

```
fit <- eggs |>
  model(RW(log(eggs) ~ drift()))
fit
```

```
## # A mable: 1 x 1
##   `RW(log(eggs) ~ drift())`
##                     <model>
## 1             <RW w/ drift>
```

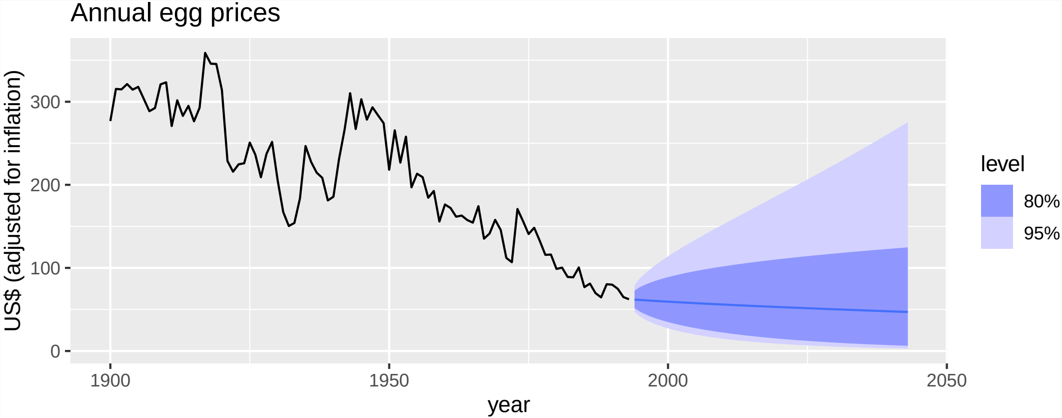
Forecasting with transformations

```r
fc <- fit |>
  forecast(h = 50)
fc
```

```
## # A fable: 50 x 4 [1Y]
## # Key:     .model [1]
##    .model                       year              eggs .mean
##    <chr>                       <dbl>            <dist> <dbl>
##  1 RW(log(eggs) ~ drift())  1994 t(N(4.1, 0.018))  61.8
##  2 RW(log(eggs) ~ drift())  1995 t(N(4.1, 0.036))  61.4
##  3 RW(log(eggs) ~ drift())  1996 t(N(4.1, 0.055))  61.0
##  4 RW(log(eggs) ~ drift())  1997 t(N(4.1, 0.074))  60.6
##  5 RW(log(eggs) ~ drift())  1998 t(N(4.1, 0.093))  60.2
##  6 RW(log(eggs) ~ drift())  1999    t(N(4, 0.11))  59.8
##  7 RW(log(eggs) ~ drift())  2000    t(N(4, 0.13))  59.4
##  8 RW(log(eggs) ~ drift())  2001    t(N(4, 0.15))  59.0
```

Forecasting with transformations

```r
fc |> autoplot(eggs) +
  labs(title = "Annual egg prices",
       y = "US$ (adjusted for inflation)")
```



Bias adjustment

- Back-transformed point forecasts are medians.
- Back-transformed PI have the correct coverage.

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Back-transformed means

Let X have mean μ and variance σ^2.

Let $f(x)$ be back-transformation function, and $Y = f(X)$.

Taylor series expansion about μ:

$$Y = f(X) \approx f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2 f''(\mu).$$

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$$\text{E}[Y] = \text{E}[f(X)] \approx f(\mu) + \tfrac{1}{2}\sigma^2 f''(\mu)$$

Bias adjustment

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1 - \lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

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$$\mathrm{E}[Y] \approx \begin{cases} e^{\mu}\left[1 + \frac{\sigma^2}{2}\right] & \lambda = 0; \\ (\lambda\mu + 1)^{1/\lambda}\left[1 + \frac{\sigma^2(1-\lambda)}{2(\lambda\mu+1)^2}\right] & \lambda \neq 0. \end{cases}$$

Bias adjustment

```
fc |>
  autoplot(eggs, level = 80, point_forecast = lst(mean, median)) +
  labs(title = "Annual egg prices",
       y = "US$ (adjusted for inflation)")
```