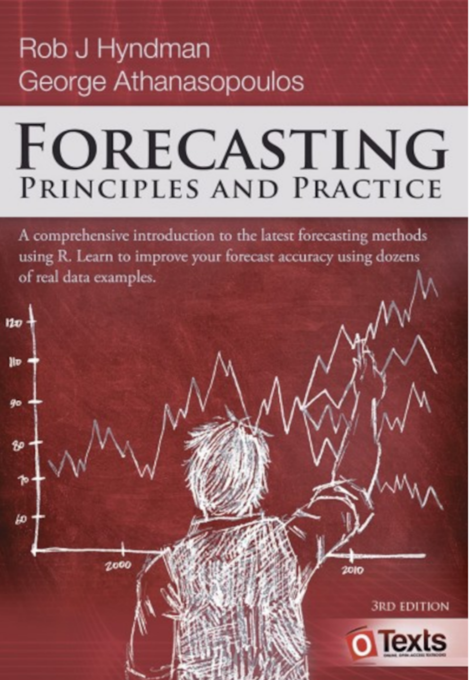
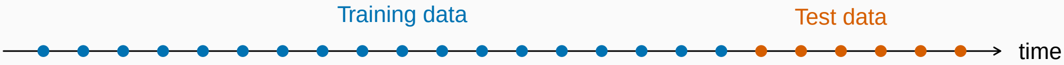


5. The forecaster's toolbox

5.8 Evaluating point forecast accuracy

OTexts.org/fpp3/

Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Forecast errors

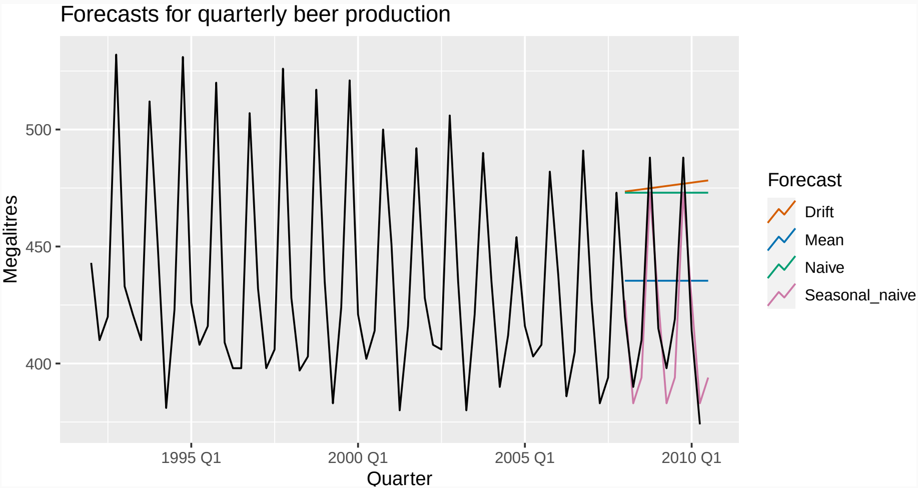
Forecast "error": the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing $\hat{y}_{T+h|T}$.

Measures of forecast accuracy



Measures of forecast accuracy

$$
\begin{aligned}
y_{T+h} &= \text{(}T+h\text{)th observation, } h = 1, \dots, H \\
\hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\
e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T}
\end{aligned}
$$

$$
\begin{aligned}
\text{MAE} &= \text{mean}(|e_{T+h}|) \\
\text{MSE} &= \text{mean}(e_{T+h}^2) \qquad \text{RMSE} = \sqrt{\text{mean}(e_{T+h}^2)} \\
\text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|)
\end{aligned}
$$

Measures of forecast accuracy

$$
\begin{aligned}
y_{T+h} &= (T+h)\text{th observation, } h = 1, \dots, H \\
\hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\
e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T}
\end{aligned}
$$

$$
\begin{aligned}
\text{MAE} &= \text{mean}(|e_{T+h}|) \\
\text{MSE} &= \text{mean}(e_{T+h}^2) \qquad \text{RMSE} = \sqrt{\text{mean}(e_{T+h}^2)} \\
\text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|)
\end{aligned}
$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = (T-1)^{-1}\sum_{t=2}^{T}|y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

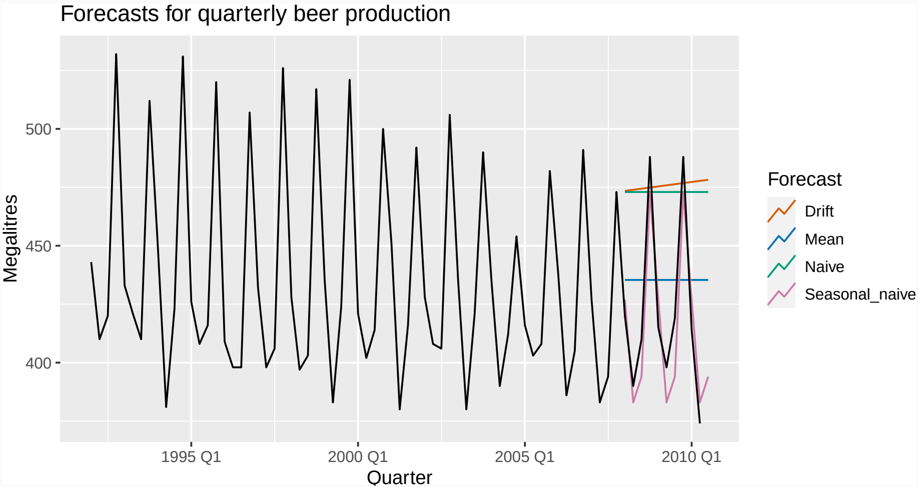
Proposed by Hyndman and Koehler (IJF, 2006).

For seasonal time series,

$$Q = (T-m)^{-1} \sum_{t=m+1}^{T} |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal

Measures of forecast accuracy



Measures of forecast accuracy

```
recent_production <- aus_production |>
  filter(year(Quarter) >= 1992)
train <- recent_production |>
  filter(year(Quarter) <= 2007)
beer_fit <- train |>
  model(
    Mean = MEAN(Beer),
    Naive = NAIVE(Beer),
    Seasonal_naive = SNAIVE(Beer),
    Drift = RW(Beer ~ drift())
  )
beer_fc <- beer_fit |>
  forecast(h = 10)
```

Measures of forecast accuracy

```
accuracy(beer_fit)
```

```
## # A tibble: 4 x 6
##   .model         .type     RMSE   MAE  MAPE  MASE
##   <chr>          <chr>    <dbl> <dbl> <dbl> <dbl>
## 1 Drift          Training  65.3  54.8 12.2   3.83
## 2 Mean           Training  43.6  35.2  7.89  2.46
## 3 Naive          Training  65.3  54.7 12.2   3.83
## 4 Seasonal_naive Training  16.8  14.3  3.31  1
```

```
accuracy(beer_fc, recent_production)
```

```
## # A tibble: 4 x 6
##   .model         .type  RMSE   MAE  MAPE  MASE
##   <chr>          <chr> <dbl> <dbl> <dbl> <dbl>
## 1 Drift          Test   64.9  58.9 14.6  4.12
## 2 Mean           Test   38.4  34.8  8.28 2.44
## 3 Naive          Test   62.7  57.4 14.2  4.01
## 4 Seasonal_naive Test   14.3  13.4  3.17 0.937
```