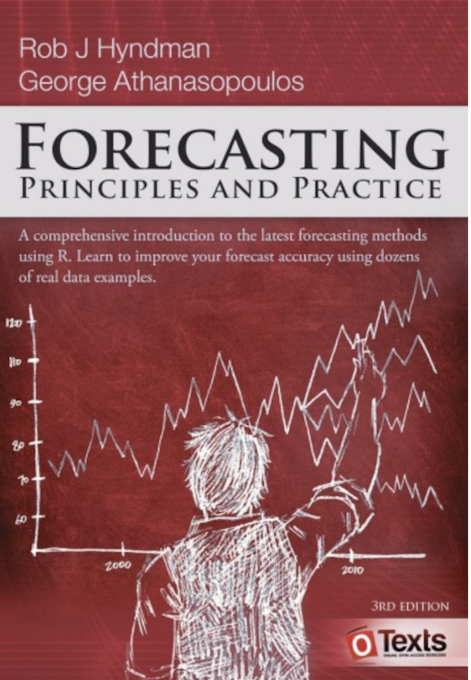



# 9. ARIMA models

## 9.3 Autoregressive models

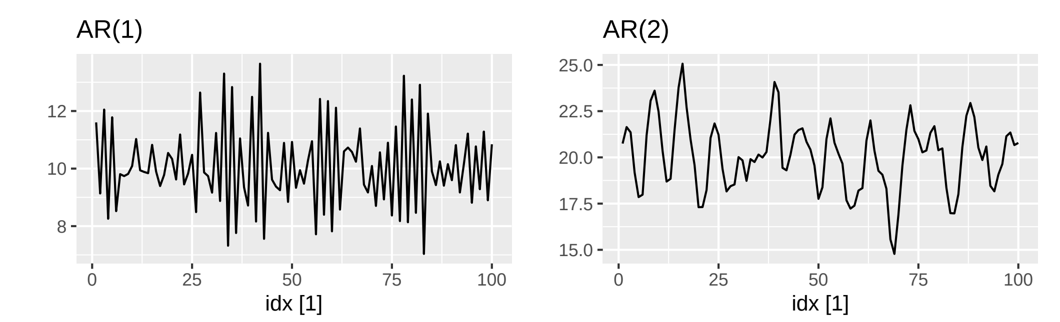
OTexts.org/fpp3/

# Autoregressive models

## Autoregressive model - AR($p$):

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t,$$

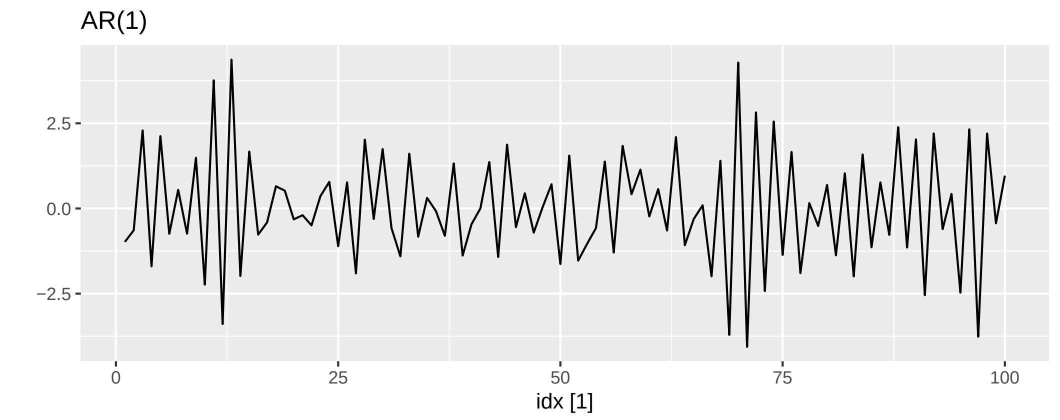
where $\varepsilon_t$ is white noise. This is a multiple regression with lagged values of $y_t$ as predictors.



# AR(1) model

$$y_t = -0.8y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$



# AR(1) model

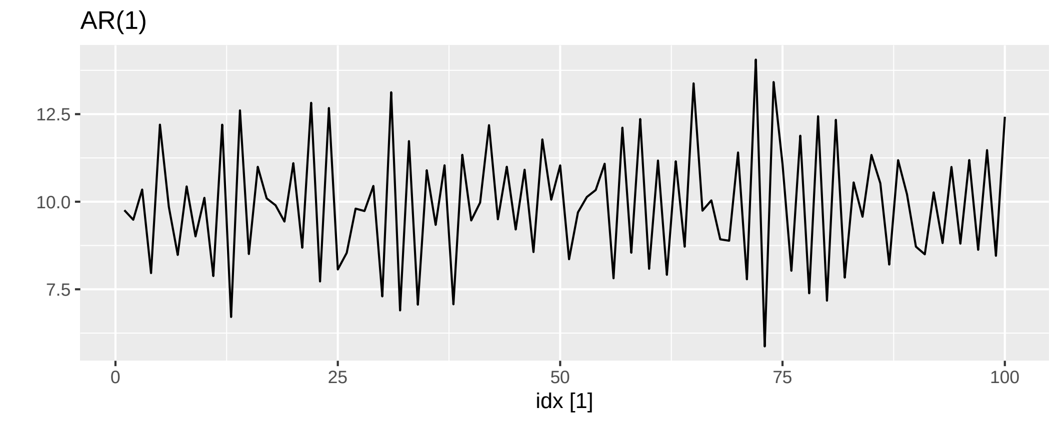
$$y_t = \phi_1 y_{t-1} + \varepsilon_t$$

- When $\phi_1 = 0$, $y_t$ is equivalent to a WN
- When $\phi_1 = 1$, $y_t$ is equivalent to a RW
- We require $|\phi_1| < 1$ for stationarity. The closer $\phi_1$ is to the bounds the more the process wanders above or below it's unconditional mean (zero in this case).
- When $\phi_1 < 0$, $y_t$ tends to oscillate between positive and negative values.

# AR(1) model including a constant

$$y_t = 18 - 0.8y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0,1), \quad T = 100.$$



# AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- When $\phi_1 = 0$ and $c = 0$, $y_t$ is equivalent to WN;
- When $\phi_1 = 1$ and $c = 0$, $y_t$ is equivalent to a RW;
- When $\phi_1 = 1$ and $c \neq 0$, $y_t$ is equivalent to a RW with drift;

# AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- $c$ is related to the mean of $y_t$.
- Let $E(y_t) = \mu$

# AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- $c$ is related to the mean of $y_t$.
- Let $E(y_t) = \mu$
- $\mu = c + \phi_1 \mu$

# AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- $c$ is related to the mean of $y_t$.
- Let $E(y_t) = \mu$
- $\mu = c + \phi_1 \mu$
- $\mu = \frac{c}{1-\phi_1}$

# AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- $c$ is related to the mean of $y_t$.
- Let $E(y_t) = \mu$
- $\mu = c + \phi_1 \mu$
- $\mu = \frac{c}{1-\phi_1}$
- `ARIMA()` takes care of whether you need a constant or not, or you can overide it.

# AR(1) model including a constant

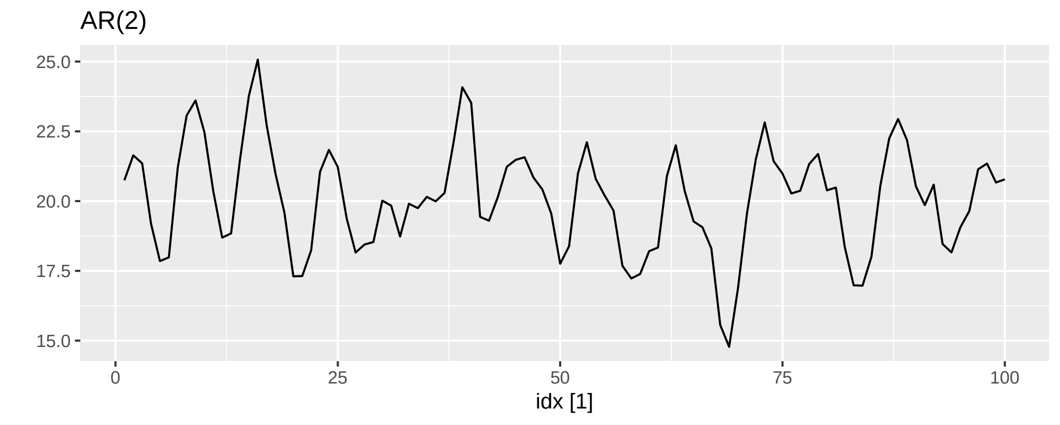
- If included estimated model returns `w/ mean`

```
Series: sim
Model: ARIMA(1,0,0) w/ mean

Coefficients:
          ar1  constant
      -0.8381   18.3527
s.e.   0.0540    0.1048

sigma^2 estimated as 1.11:  log likelihood=-146.7
AIC=299.4   AICc=299.7   BIC=307.2
```

# AR(2) model

$$y_t = 8 + 1.3y_{t-1} - 0.7y_{t-2} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \qquad T = 100.$$



# Stationarity conditions

We normally restrict autoregressive models to stationary data, and then some constraints on the values of the parameters are required.

**General condition for stationarity**

Complex roots of $1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_p z^p$ lie outside the unit circle on the complex plane.

# Stationarity conditions

We normally restrict autoregressive models to stationary data, and then some constraints on the values of the parameters are required.

**General condition for stationarity**

Complex roots of $1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_p z^p$ lie outside the unit circle on the complex plane.

- For $p = 1$: $-1 < \phi_1 < 1$.
- For $p = 2$:
  $-1 < \phi_2 < 1 \qquad \phi_2 + \phi_1 < 1 \qquad \phi_2 - \phi_1 < 1$.
- More complicated conditions hold for $p \geq 3$.
- Estimation software takes care of this.