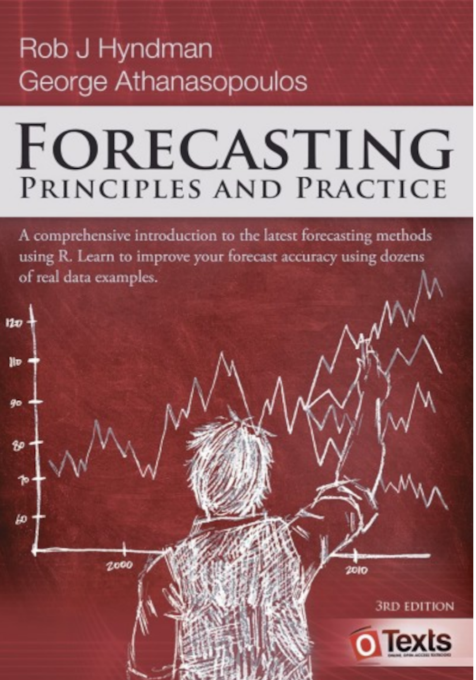

9. ARIMA models

9.8 Forecasting

OTexts.org/fpp3/

Point forecasts

1. Rearrange ARIMA equation so y_t is on LHS.
2. Rewrite equation by replacing t by $T + h$.
3. On RHS, replace future observations by their forecasts, future errors by zero, and past errors by corresponding residuals.

Start with $h = 1$. Repeat for $h = 2, 3, \ldots$.

Point forecasts

ARIMA(3,1,1) forecasts: Step 1

$$(1 - \hat{\phi}_1 B - \hat{\phi}_2 B^2 - \hat{\phi}_3 B^3)(1 - B)y_t = (1 + \hat{\theta}_1 B)\varepsilon_t,$$

Point forecasts

ARIMA(3,1,1) forecasts: Step 1

$$(1 - \hat{\phi}_1 B - \hat{\phi}_2 B^2 - \hat{\phi}_3 B^3)(1 - B) y_t = (1 + \hat{\theta}_1 B)\varepsilon_t,$$

$$\left[1 - \hat{\phi}_1 B - \hat{\phi}_2 B^2 - \hat{\phi}_3 B^3 - B + \hat{\phi}_1 B^2 + \hat{\phi}_2 B^3 + \hat{\phi}_3 B^4\right] y_t = (1 + \hat{\theta}_1 B)\varepsilon_t$$

Point forecasts

ARIMA(3,1,1) forecasts: Step 1

$$(1 - \hat{\phi}_1 B - \hat{\phi}_2 B^2 - \hat{\phi}_3 B^3)(1 - B) y_t = (1 + \hat{\theta}_1 B)\varepsilon_t,$$

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$$\left[1 - (1 + \hat{\phi}_1) B + (\hat{\phi}_1 - \hat{\phi}_2) B^2 + (\hat{\phi}_2 - \hat{\phi}_3) B^3 + \hat{\phi}_3 B^4\right] y_t = (1 + \hat{\theta}_1 B)\varepsilon_t$$

Point forecasts

ARIMA(3,1,1) forecasts: Step 1

$$(1 - \hat{\phi}_1 B - \hat{\phi}_2 B^2 - \hat{\phi}_3 B^3)(1 - B) y_t = (1 + \hat{\theta}_1 B)\varepsilon_t,$$

$$\left[1 - \hat{\phi}_1 B - \hat{\phi}_2 B^2 - \hat{\phi}_3 B^3 - B + \hat{\phi}_1 B^2 + \hat{\phi}_2 B^3 + \hat{\phi}_3 B^4\right] y_t = (1 + \hat{\theta}_1 B)\varepsilon_t$$

$$\left[1 - (1 + \hat{\phi}_1) B + (\hat{\phi}_1 - \hat{\phi}_2) B^2 + (\hat{\phi}_2 - \hat{\phi}_3) B^3 + \hat{\phi}_3 B^4\right] y_t = (1 + \hat{\theta}_1 B)\varepsilon_t$$

$$y_t - (1 + \hat{\phi}_1) y_{t-1} + (\hat{\phi}_1 - \hat{\phi}_2) y_{t-2} + (\hat{\phi}_2 - \hat{\phi}_3) y_{t-3} + \hat{\phi}_3 y_{t-4} = \varepsilon_t + \hat{\theta}_1 \varepsilon_{t-1}$$

Point forecasts

ARIMA(3,1,1) forecasts: Step 1

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$$y_t - (1 + \hat{\phi}_1) y_{t-1} + (\hat{\phi}_1 - \hat{\phi}_2) y_{t-2} + (\hat{\phi}_2 - \hat{\phi}_3) y_{t-3} + \hat{\phi}_3 y_{t-4} = \varepsilon_t + \hat{\theta}_1 \varepsilon_{t-1}$$

$$y_t = (1 + \hat{\phi}_1) y_{t-1} - (\hat{\phi}_1 - \hat{\phi}_2) y_{t-2} - (\hat{\phi}_2 - \hat{\phi}_3) y_{t-3} - \hat{\phi}_3 y_{t-4} + \varepsilon_t + \hat{\theta}_1 \varepsilon_{t-1}$$

Point forecasts (h=1)

$$y_t = (1 + \hat{\phi}_1)y_{t-1} - (\hat{\phi}_1 - \hat{\phi}_2)y_{t-2} - (\hat{\phi}_2 - \hat{\phi}_3)y_{t-3} - \hat{\phi}_3 y_{t-4} + \varepsilon_t + \hat{\theta}_1\varepsilon_{t-1}$$

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ARIMA(3,1,1) forecasts: Step 2

$$y_{T+1} = (1 + \hat{\phi}_1)y_T - (\hat{\phi}_1 - \hat{\phi}_2)y_{T-1} - (\hat{\phi}_2 - \hat{\phi}_3)y_{T-2} - \hat{\phi}_3 y_{T-3} + \varepsilon_{T+1} + \hat{\theta}_1\varepsilon_T$$

Point forecasts (h=1)

$$y_t = (1 + \hat{\phi}_1)y_{t-1} - (\hat{\phi}_1 - \hat{\phi}_2)y_{t-2} - (\hat{\phi}_2 - \hat{\phi}_3)y_{t-3} - \hat{\phi}_3 y_{t-4} + \varepsilon_t + \hat{\theta}_1\varepsilon_{t-1}$$

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ARIMA(3,1,1) forecasts: Step 3

$$\hat{y}_{T+1|T} = (1 + \hat{\phi}_1)y_T - (\hat{\phi}_1 - \hat{\phi}_2)y_{T-1} - (\hat{\phi}_2 - \hat{\phi}_3)y_{T-2} - \hat{\phi}_3 y_{T-3} + \hat{\theta}_1 e_T$$

Point forecasts (h=2)

$$y_t = (1 + \hat{\phi}_1)y_{t-1} - (\hat{\phi}_1 - \hat{\phi}_2)y_{t-2} - (\hat{\phi}_2 - \hat{\phi}_3)y_{t-3} - \hat{\phi}_3 y_{t-4} + \varepsilon_t + \hat{\theta}_1\varepsilon_{t-1}$$

Point forecasts (h=2)

$$y_t = (1 + \hat{\phi}_1)y_{t-1} - (\hat{\phi}_1 - \hat{\phi}_2)y_{t-2} - (\hat{\phi}_2 - \hat{\phi}_3)y_{t-3} - \hat{\phi}_3 y_{t-4} + \varepsilon_t + \hat{\theta}_1\varepsilon_{t-1}$$

ARIMA(3,1,1) forecasts: Step 2

$$y_{T+2} = (1 + \hat{\phi}_1)y_{T+1} - (\hat{\phi}_1 - \hat{\phi}_2)y_T - (\hat{\phi}_2 - \hat{\phi}_3)y_{T-1} - \hat{\phi}_3 y_{T-2} + \varepsilon_{T+2} + \hat{\theta}_1\varepsilon_{T+1}.$$

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ARIMA(3,1,1) forecasts: Step 3

$$\hat{y}_{T+2|T} = (1 + \hat{\phi}_1)\hat{y}_{T+1|T} - (\hat{\phi}_1 - \hat{\phi}_2)y_T - (\hat{\phi}_2 - \hat{\phi}_3)y_{T-1} - \hat{\phi}_3 y_{T-2}.$$

Prediction intervals

Assuming $\varepsilon_t \sim N(0, \sigma^2)$

95% prediction interval

$$\hat{y}_{T+h|T} \pm 1.96\sqrt{v_{T+h|T}}$$

where $v_{T+h|T}$ is estimated forecast variance.

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where $v_{T+h|T}$ is estimated forecast variance.

- $v_{T+1|T} = \hat{\sigma}^2$ for all ARIMA models
- Multi-step prediction intervals for ARIMA(0,0,q):

$$y_t = \varepsilon_t + \sum_{i=1}^{q} \hat{\theta}_i \varepsilon_{t-i}.$$

$$v_{T|T+h} = \hat{\sigma}^2 \left[1 + \sum_{i=1}^{h-1} \hat{\theta}_i^2\right], \qquad \text{for } h = 2, 3, \ldots.$$

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$$v_{T|T+h} = \hat{\sigma}^2 \left[1 + \sum_{i=1}^{h-1} \hat{\theta}_i^2\right], \qquad \text{for } h = 2, 3, \dots .$$

- Other models beyond scope of this book.

Prediction intervals

- Prediction intervals increase in size with forecast horizon.
- Prediction intervals can be difficult to calculate by hand
- Calculations assume residuals are uncorrelated and normally distributed.
- Prediction intervals tend to be too narrow.
  - the uncertainty in the parameter estimates has not been accounted for.
  - model uncertainty has not been accounted for.
  - the ARIMA model assumes historical patterns will not change during the forecast period.
  - the ARIMA model assumes uncorrelated future errors.