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# FORECASTING

## PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**  
Oxford Texts in Finance and Probability

## 9. ARIMA models

### 9.3 Autoregressive models

[OTexts.org/fpp3/](http://OTexts.org/fpp3/)

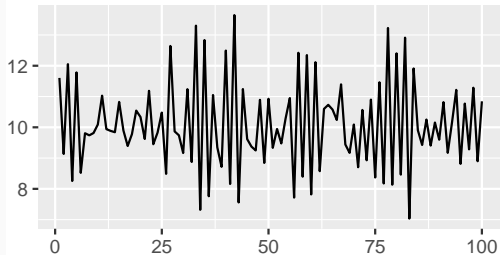
# Autoregressive models

## Autoregressive (AR) models:

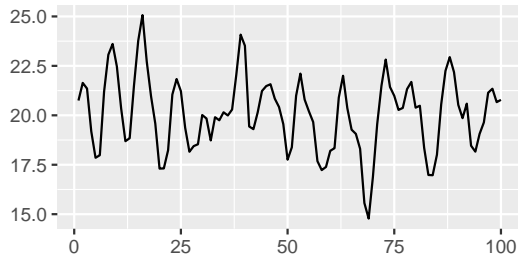
$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t,$$

where  $\varepsilon_t$  is white noise. This is a multiple regression with **lagged values** of  $y_t$  as predictors.

AR(1)



AR(2)

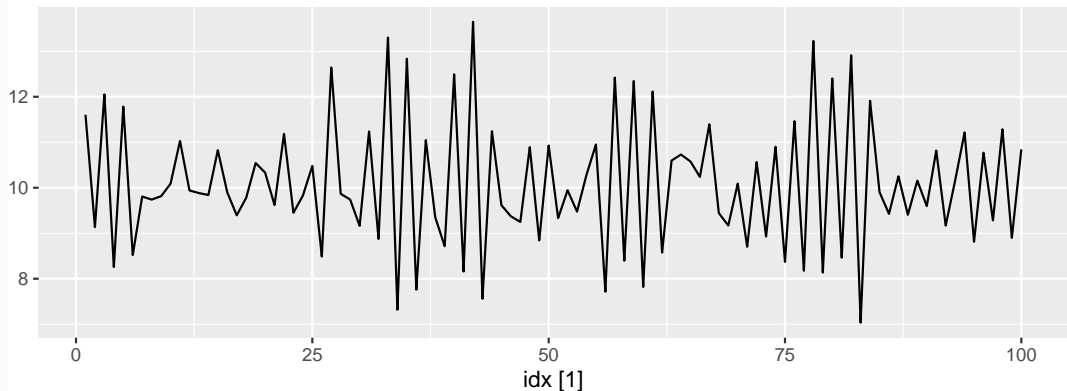


# AR(1) model

$$y_t = 18 - 0.8y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$

AR(1)



# AR(1) model

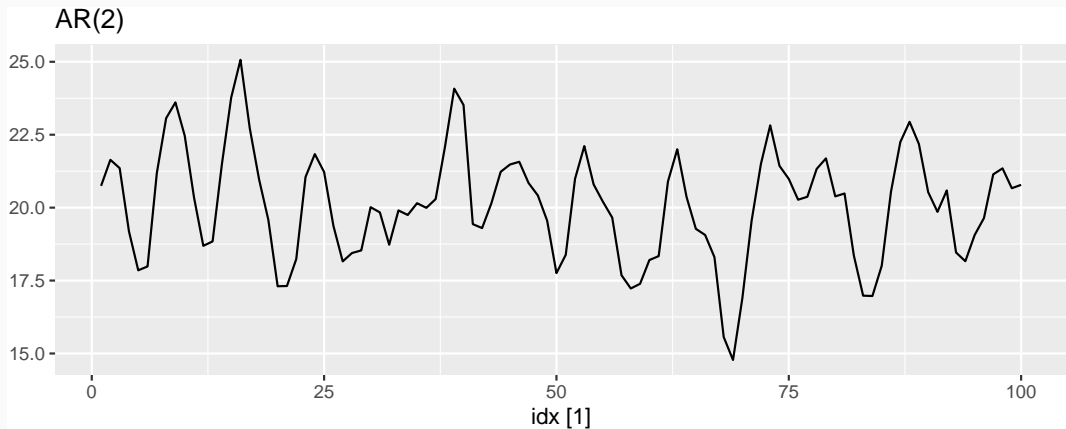
$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- When  $\phi_1 = 0$ ,  $y_t$  is **equivalent to WN**
- When  $\phi_1 = 1$  and  $c = 0$ ,  $y_t$  is **equivalent to a RW**
- When  $\phi_1 = 1$  and  $c \neq 0$ ,  $y_t$  is **equivalent to a RW with drift**
- When  $\phi_1 < 0$ ,  $y_t$  tends to **oscillate between positive and negative values.**

# AR(2) model

$$y_t = 8 + 1.3y_{t-1} - 0.7y_{t-2} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$



# Stationarity conditions

We normally restrict autoregressive models to stationary data, and then some constraints on the values of the parameters are required.

## General condition for stationarity

Complex roots of  $1 - \phi_1 z - \phi_2 z^2 - \dots - \phi_p z^p$  lie outside the unit circle on the complex plane.

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- For  $p = 1$ :  $-1 < \phi_1 < 1$ .
- For  $p = 2$ :  $-1 < \phi_2 < 1$        $\phi_2 + \phi_1 < 1$        $\phi_2 - \phi_1 < 1$ .
- More complicated conditions hold for  $p \geq 3$ .
- Estimation software takes care of this.