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# FORECASTING

## PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**  
OPEN TEXTS FOR PRACTICE

## 8. Exponential smoothing

### 8.1 Simple exponential smoothing

[OTexts.org/fpp3/](https://OTexts.org/fpp3/)

# Simple methods

Time series  $y_1, y_2, \dots, y_T$ .

## Random walk forecasts

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- Want something in between these methods.
- Most recent data should have more weight.

# Simple Exponential Smoothing

## Forecast equation

$$\hat{y}_{T+1|T} = \alpha y_T + \alpha(1 - \alpha)y_{T-1} + \alpha(1 - \alpha)^2 y_{T-2} + \cdots,$$

where  $0 \leq \alpha \leq 1$ .

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Observation	Weights assigned to observations for:			
	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$
$y_T$	0.2	0.4	0.6	0.8
$y_{T-1}$	0.16	0.24	0.24	0.16
$y_{T-2}$	0.128	0.144	0.096	0.032
$y_{T-3}$	0.1024	0.0864	0.0384	0.0064
$y_{T-4}$	$(0.2)(0.8)^4$	$(0.4)(0.6)^4$	$(0.6)(0.4)^4$	$(0.8)(0.2)^4$
$y_{T-5}$	$(0.2)(0.8)^5$	$(0.4)(0.6)^5$	$(0.6)(0.4)^5$	$(0.8)(0.2)^5$

# Simple Exponential Smoothing

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## Component form

Forecast equation

$$\hat{y}_{t+h|t} = \ell_t$$

Smoothing equation

$$\ell_t = \alpha y_t + (1 - \alpha)\ell_{t-1}$$

- $\ell_t$  is the level (or the smoothed value) of the series at time  $t$ .
- $\hat{y}_{t+1|t} = \alpha y_t + (1 - \alpha)\hat{y}_{t|t-1}$



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Iterate to get exponentially weighted moving average form.

## Weighted average form

$$\hat{y}_{T+1|T} = \sum_{j=0}^{T-1} \alpha(1 - \alpha)^j y_{T-j} + (1 - \alpha)^T \ell_0$$

# Optimising smoothing parameters

- Need to choose best values for  $\alpha$  and  $\ell_0$ .
- Similarly to regression, choose optimal parameters by minimising SSE:

$$\text{SSE} = \sum_{t=1}^T (y_t - \hat{y}_{t|t-1})^2.$$

- Unlike regression there is no closed form solution — use numerical optimization.

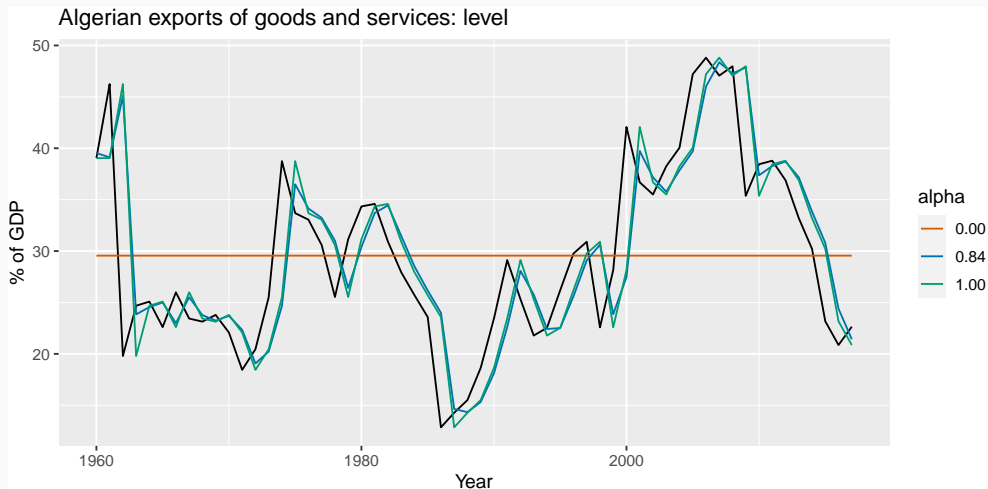
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- For Algerian Exports example:
  - ▶  $\hat{\alpha} = 0.8400$
  - ▶  $\hat{\ell}_0 = 39.54$

# Simple Exponential Smoothing



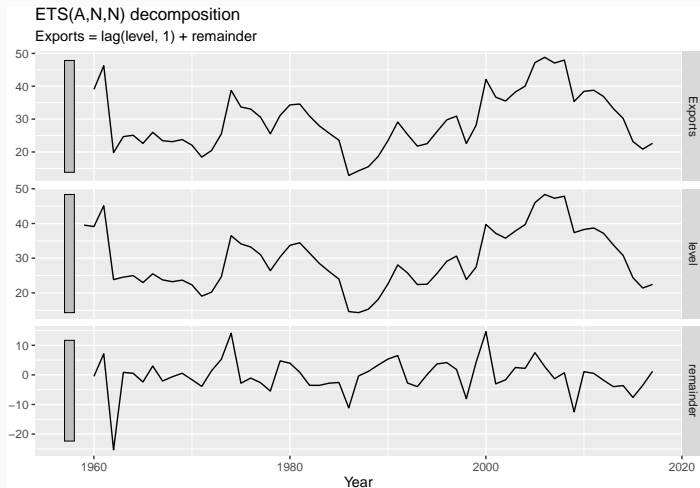
# Example: Algerian Exports

```
algeria_economy <- global_economy |>
  filter(Country == "Algeria")
fit <- algeria_economy |>
  model(ANN = ETS(Exports ~ error("A") + trend("N") + season("N")))
report(fit)
```

```
## Series: Exports
## Model: ETS(A,N,N)
##   Smoothing parameters:
##     alpha = 0.84
##
##   Initial states:
##   l[0]
##   39.5
##
##   sigma^2: 35.6
##
##   AIC AICc BIC
##   447 447 453
```

# Example: Algerian Exports

```
components(fit) |> autoplot()
```



# Example: Algerian Exports

```
components(fit) |>  
  left_join(fitted(fit), by = c("Country", ".model", "Year"))
```

```
## # A dable: 59 x 7 [1Y]  
## # Key:      Country, .model [1]  
## # :        Exports = lag(level, 1) + remainder  
##   Country .model Year Exports level remainder .fitted  
##   <fct>   <chr>  <dbl>   <dbl> <dbl>      <dbl>   <dbl>  
## 1 Algeria ANN    1959    NA    39.5      NA      NA  
## 2 Algeria ANN    1960   39.0   39.1    -0.496   39.5  
## 3 Algeria ANN    1961   46.2   45.1     7.12    39.1  
## 4 Algeria ANN    1962   19.8   23.8   -25.3    45.1  
## 5 Algeria ANN    1963   24.7   24.6     0.841   23.8  
## 6 Algeria ANN    1964   25.1   25.0     0.534   24.6  
## 7 Algeria ANN    1965   22.6   23.0    -2.39    25.0  
## 8 Algeria ANN    1966   26.0   25.5     3.00    23.0  
## 9 Algeria ANN    1967   23.4   23.8    -2.07    25.5  
## 10 Algeria ANN   1968   23.1   23.2    -0.630   23.8  
## # ... with 49 more rows
```

# Example: Algerian Exports

```
fit |>  
  forecast(h = 5) |>  
  autoplot(algeria_economy) +  
  labs(y = "% of GDP", title = "Exports: Algeria")
```

