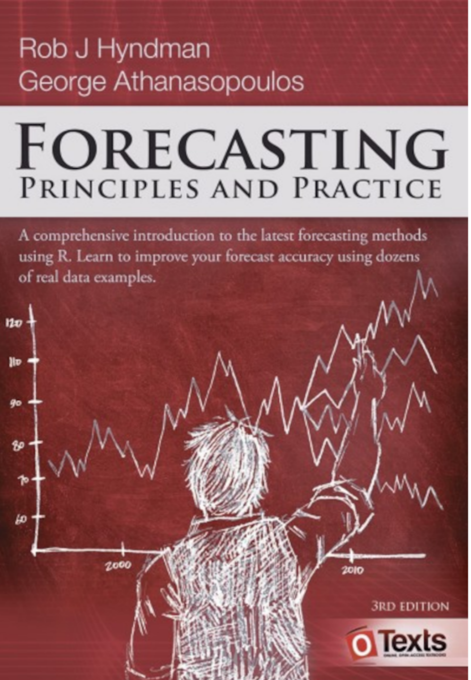


10. Dynamic regression models

10.5 Dynamic harmonic regression

OTexts.org/fpp3/

Dynamic harmonic regression

Combine Fourier terms with ARIMA errors

Advantages

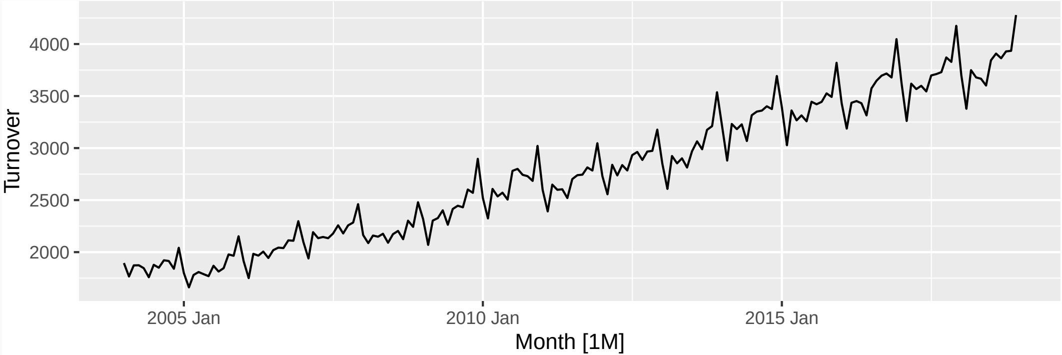
- it allows any length seasonality;
- for data with more than one seasonal period, you can include Fourier terms of different frequencies;
- the seasonal pattern is smooth for small values of K (but more wiggly seasonality can be handled by increasing K);
- the short-term dynamics are easily handled with a simple ARMA error.

Disadvantages

- seasonality is assumed to be fixed

Eating-out expenditure

```
aus_cafe <- aus_retail |>
  filter(Industry == "Cafes, restaurants and takeaway food services",
         year(Month) %in% 2004:2018) |>
  summarise(Turnover = sum(Turnover))
aus_cafe |> autoplot(Turnover)
```

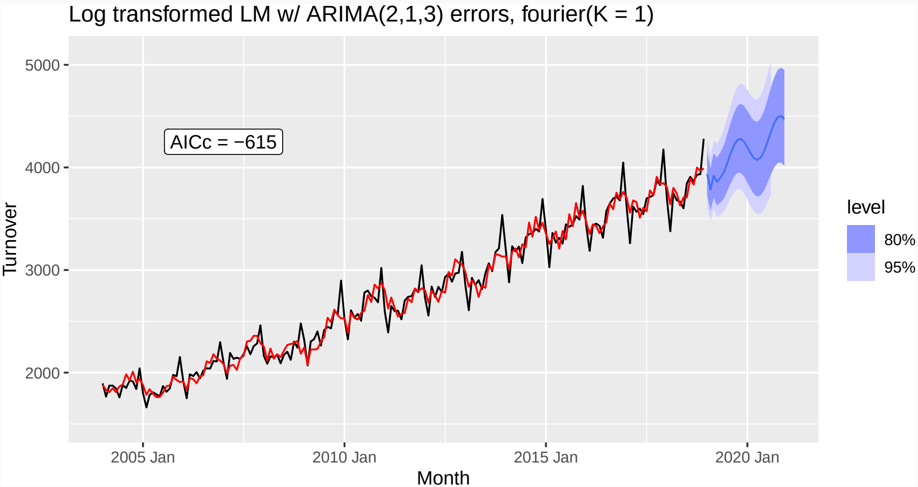


Eating-out expenditure

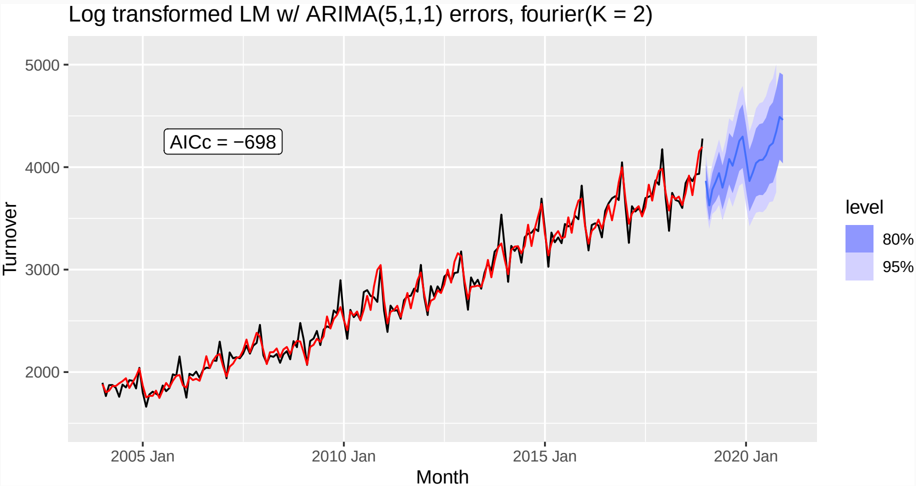
```r
fit <- aus_cafe |> model(
  `K = 1` = ARIMA(log(Turnover) ~ fourier(K = 1) + PDQ(0, 0, 0)),
  `K = 2` = ARIMA(log(Turnover) ~ fourier(K = 2) + PDQ(0, 0, 0)),
  `K = 3` = ARIMA(log(Turnover) ~ fourier(K = 3) + PDQ(0, 0, 0)),
  `K = 4` = ARIMA(log(Turnover) ~ fourier(K = 4) + PDQ(0, 0, 0)),
  `K = 5` = ARIMA(log(Turnover) ~ fourier(K = 5) + PDQ(0, 0, 0)),
  `K = 6` = ARIMA(log(Turnover) ~ fourier(K = 6) + PDQ(0, 0, 0)))
glance(fit)
```

.model	sigma2	log_lik	AIC	AICc	BIC
K = 1	0.002	317	-616	-615	-588
K = 2	0.001	362	-700	-698	-661
K = 3	0.001	394	-763	-761	-725
K = 4	0.001	427	-822	-818	-771
K = 5	0.000	474	-919	-917	-875
K = 6	0.000	474	-920	-918	-875

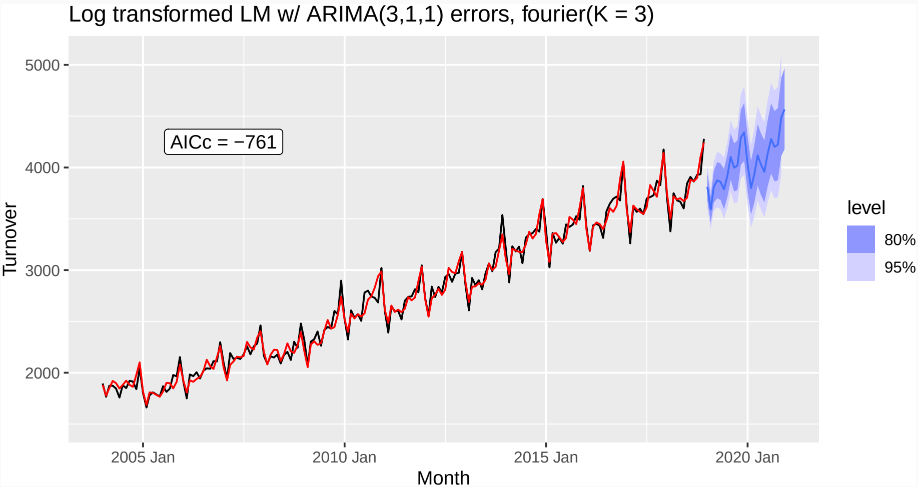
Eating-out expenditure



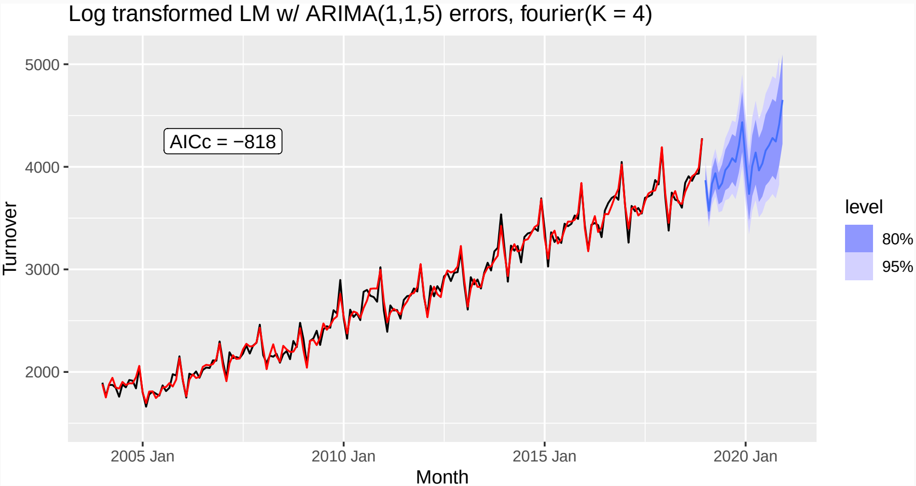
Eating-out expenditure



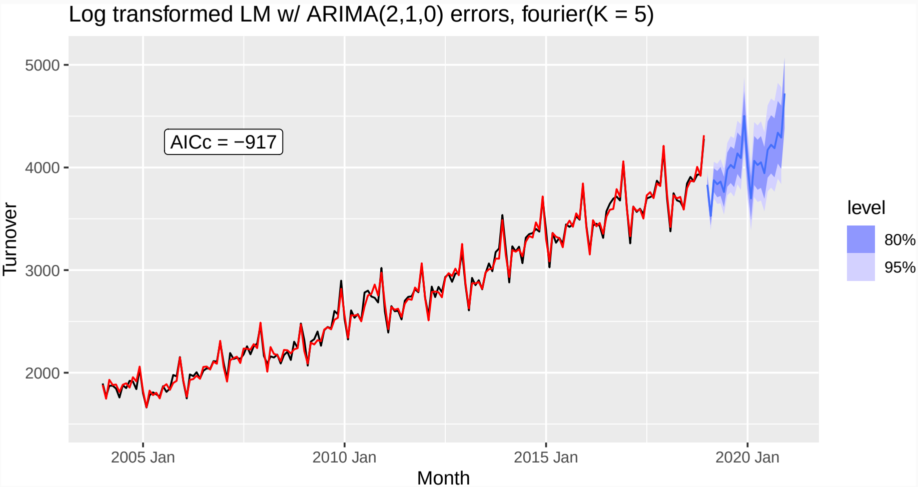
Eating-out expenditure



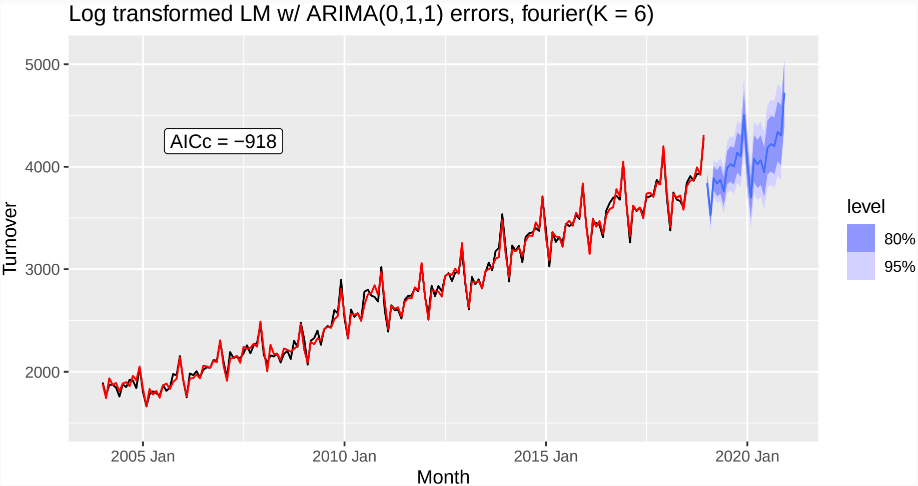
Eating-out expenditure



Eating-out expenditure



Eating-out expenditure



Example: weekly gasoline products

```r
fit <- us_gasoline |>
  model(K06 = ARIMA(Barrels ~ fourier(K = 6) + PDQ(0, 0, 0)))
report(fit)
```

```
## Series: Barrels
## Model: LM w/ ARIMA(0,1,1) errors
##
## Coefficients:
##          ma1  fourier(K = 6)C1_52  fourier(K = 6)S1_52  fourier(K = 6)C2_52  fourier(K = 6)S2_52
##       -0.896              -0.1122              -0.2300               0.0419               0.0316
## s.e.   0.013               0.0123               0.0122               0.0099               0.0099
##       fourier(K = 6)C3_52  fourier(K = 6)S3_52  fourier(K = 6)C4_52  fourier(K = 6)S4_52
##                    0.0832               0.0345               0.0186               0.0397
## s.e.               0.0094               0.0094               0.0093               0.0092
##       fourier(K = 6)C5_52  fourier(K = 6)S5_52  fourier(K = 6)C6_52  fourier(K = 6)S6_52  intercept
##                   -0.0314               0.0010              -0.0522               0.0002     0.0014
## s.e.               0.0092               0.0092               0.0091               0.0091     0.0007
##
## sigma^2 estimated as 0.06205:  log likelihood=-33.1
## AIC=96.2   AICc=96.6   BIC=174
```

Example: weekly gasoline products

```r
forecast(fit, h = "3 years") |>
  autoplot(us_gasoline)
```