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# FORECASTING

## PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**  
OPEN TEXTS FOR PRACTICE

## 10. Dynamic regression models

### 10.4 Stochastic and deterministic trends

[OTexts.org/fpp3/](http://OTexts.org/fpp3/)

# Stochastic & deterministic trends

## Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t \quad \text{where } \eta_t \sim \text{ARIMA}(p, 0, q)$$

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## Stochastic trend

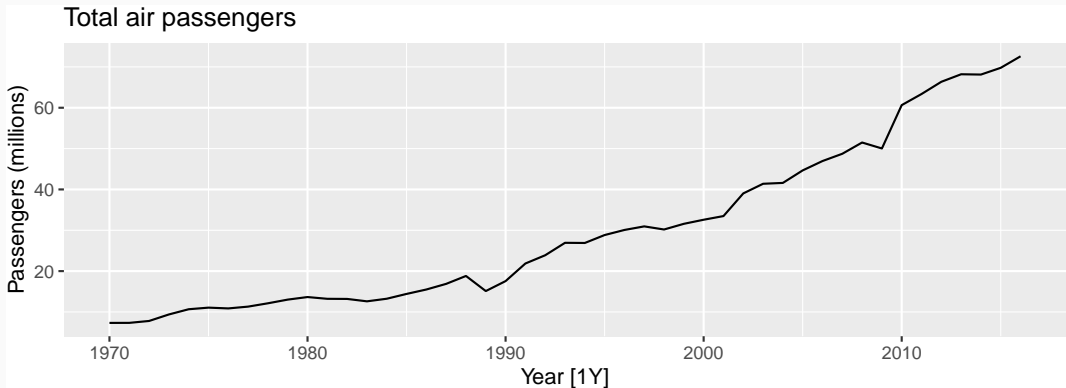
$$y_t = \beta_0 + \beta_1 t + \eta_t \quad \text{where } \eta_t \sim \text{ARIMA}(p, 1, q)$$

Difference both sides:

$$y_t = \beta_1 + \eta'_t \quad \text{where } \eta'_t \sim \text{ARIMA}(p, 0, q)$$

# Air transport passengers Australia

```
aus_airpassengers |>  
  autoplot(Passengers) +  
  labs(y = "Passengers (millions)", title = "Total air passengers")
```



# Air transport passengers Australia

## Deterministic trend

```
fit_deterministic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ 1 + trend() + pdq(d = 0)))
report(fit_deterministic)
```

```
## Series: Passengers
## Model: LM w/ ARIMA(1,0,0) errors
##
## Coefficients:
##           ar1  trend()  intercept
##       0.9564    1.415    0.901
## s.e.  0.0362    0.197    7.075
##
## sigma^2 estimated as 4.343:  log likelihood=-101
## AIC=210   AICc=211   BIC=217
```

# Air transport passengers Australia

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fit_deterministic <- aus_airpassengers |>  
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```

$$y_t = 0.901 + 1.415t + \eta_t$$
$$\eta_t = 0.956\eta_{t-1} + \varepsilon_t$$
$$\varepsilon_t \sim \text{NID}(0, 4.343).$$

# Air transport passengers Australia

## Stochastic trend

```
fit_stochastic <- aus_airpassengers |>  
  model(ARIMA(Passengers ~ 1 + pdq(d = 1)))  
report(fit_stochastic)
```

```
## Series: Passengers  
## Model: ARIMA(0,1,0) w/ drift  
##  
## Coefficients:  
##      constant  
##      1.419  
## s.e.      0.301  
##  
## sigma^2 estimated as 4.271: log likelihood=-98.2  
## AIC=200    AICc=201    BIC=204
```



# Air transport passengers Australia

## Stochastic trend

```
fit_stochastic <- aus_airpassengers |>  
  model(ARIMA(Passengers ~ 1 + pdq(d = 1)))  
report(fit_stochastic)
```

```
## Series: Passengers  
## Model: ARIMA(0,1,0) w/ drift  
##  
## Coefficients:  
##      constant  
##      1.419  
## s.e.      0.301  
##  
## sigma^2 estimated as 4.271: log likelihood=-98.2  
## AIC=200   AICc=201   BIC=204
```

$$y_t - y_{t-1} = 1.419 + \varepsilon_t,$$

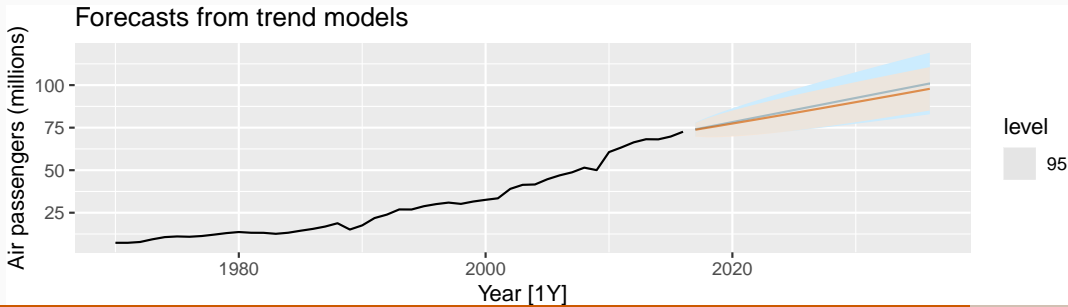
$$y_t = y_0 + 1.419t + \eta_t$$

$$\eta_t = \eta_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim \text{NID}(0, 4.271).$$

# Air transport passengers Australia

```
aus_airpassengers |>  
  autoplot(Passengers) +  
  autolayer(fit_stochastic |> forecast(h = 20),  
    colour = "#0072B2", level = 95) +  
  autolayer(fit_deterministic |> forecast(h = 20),  
    colour = "#D55E00", alpha = 0.65, level = 95) +  
  labs(y = "Air passengers (millions)", title = "Forecasts from trend models")
```



# Forecasting with trend

- Point forecasts are almost identical, but prediction intervals differ.
- Stochastic trends have much wider prediction intervals because the errors are non-stationary.
- Be careful of forecasting with deterministic trends too far ahead.