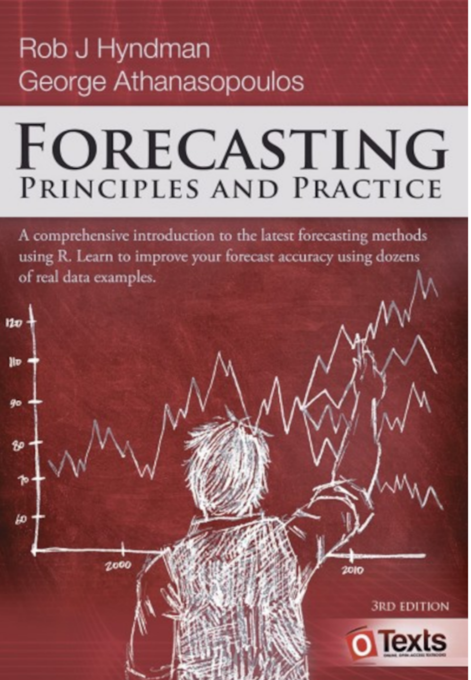

10. Dynamic regression models

10.1 Estimation

OTexts.org/fpp3/

Stochastic & deterministic trends

Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t \qquad \text{where } \eta_t \sim \text{ARIMA}(p, 0, q)$$

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Stochastic trend

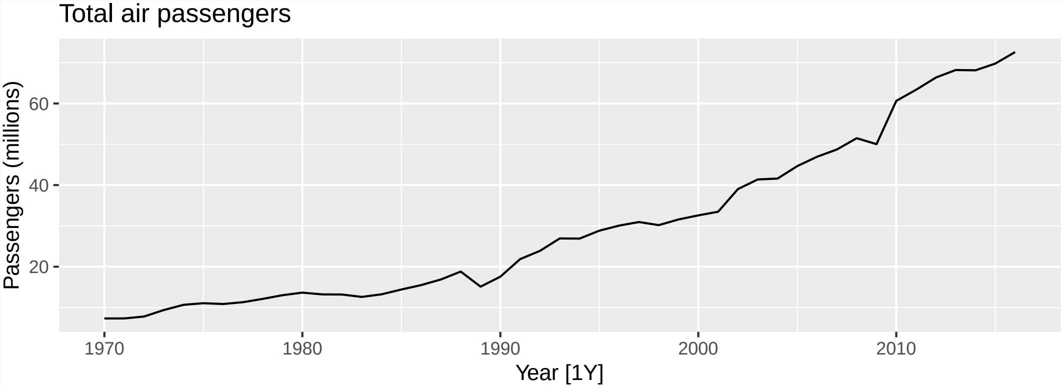
$$y_t = \beta_0 + \beta_1 t + \eta_t \qquad \text{where } \eta_t \sim \text{ARIMA}(p, 1, q)$$

Difference both sides:

$$y_t = \beta_1 + \eta'_t \qquad \text{where } \eta'_t \sim \text{ARIMA}(p, 0, q)$$

Air transport passengers Australia

```r
aus_airpassengers |>
  autoplot(Passengers) +
  labs(y = "Passengers (millions)", title = "Total air passengers")
```



Air transport passengers Australia

Deterministic trend

```r
fit_deterministic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ 1 + trend() + pdq(d = 0)))
report(fit_deterministic)
```

```
## Series: Passengers
## Model: LM w/ ARIMA(1,0,0) errors
##
## Coefficients:
##          ar1  trend()  intercept
##       0.9564    1.415      0.901
## s.e.  0.0362    0.197      7.075
##
## sigma^2 estimated as 4.343:  log likelihood=-101
## AIC=210   AICc=211   BIC=217
```

Air transport passengers Australia

Deterministic trend

```r
fit_deterministic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ 1 + trend() + pdq(d = 0)))
report(fit_deterministic)
```

```
## Series: Passengers
## Model: LM w/ ARIMA(1,0,0) errors
##
## Coefficients:
##          ar1  trend()  intercept
##       0.9564    1.415      0.901
## s.e.  0.0362    0.197      7.075
##
## sigma^2 estimated as 4.343:  log likelihood=-101
## AIC=210   AICc=211   BIC=217
```

$$y_t = 0.901 + 1.415t + \eta_t$$
$$\eta_t = 0.956\eta_{t-1} + \varepsilon_t$$
$$\varepsilon_t \sim \text{NID}(0, 4.343).$$

Air transport passengers Australia

Stochastic trend

```
fit_stochastic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ 1 + pdq(d = 1)))
report(fit_stochastic)
```

```
## Series: Passengers
## Model: ARIMA(0,1,0) w/ drift
##
## Coefficients:
##       constant
##          1.419
## s.e.     0.301
##
## sigma^2 estimated as 4.271:  log likelihood=-98.2
## AIC=200   AICc=201   BIC=204
```

Air transport passengers Australia

Stochastic trend

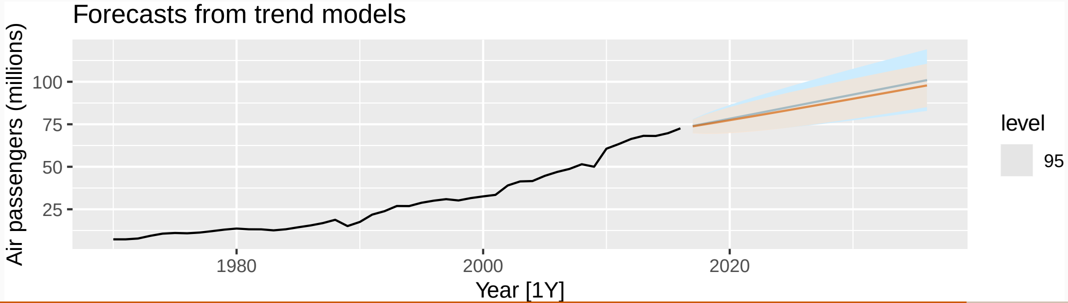
```
fit_stochastic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ 1 + pdq(d = 1)))
report(fit_stochastic)
```

```
## Series: Passengers
## Model: ARIMA(0,1,0) w/ drift
##
## Coefficients:
##       constant
##          1.419
## s.e.     0.301
##
## sigma^2 estimated as 4.271:  log likelihood=-98.2
## AIC=200   AICc=201   BIC=204
```

$$
\begin{aligned}
y_t - y_{t-1} &= 1.419 + \varepsilon_t, \\
y_t &= y_0 + 1.419t + \eta_t \\
\eta_t &= \eta_{t-1} + \varepsilon_t \\
\varepsilon_t &\sim \text{NID}(0, 4.271).
\end{aligned}
$$

Air transport passengers Australia

```
aus_airpassengers |>
  autoplot(Passengers) +
  autolayer(fit_stochastic |> forecast(h = 20),
    colour = "#0072B2", level = 95) +
  autolayer(fit_deterministic |> forecast(h = 20),
    colour = "#D55E00", alpha = 0.65, level = 95) +
  labs(y = "Air passengers (millions)", title = "Forecasts from trend models")
```



Forecasting with trend

- Point forecasts are almost identical, but prediction intervals differ.
- Stochastic trends have much wider prediction intervals because the errors are non-stationary.
- Be careful of forecasting with deterministic trends too far ahead.