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FORECASTING
PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.

3RD EDITION

OTexts

8. Exponential smoothing

8.1 Simple exponential smoothing

OTexts.org/fpp3/

Simple methods

Time series $y_1, y_2, \ldots, y_T$.

Random walk forecasts

$$\hat{y}_{T+h|T} = y_T$$

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- Want something in between these methods.
- Most recent data should have more weight.

Simple Exponential Smoothing

Forecast equation

$$\hat{y}_{T+1|T} = \alpha y_T + \alpha(1-\alpha)y_{T-1} + \alpha(1-\alpha)^2 y_{T-2} + \cdots,$$
$$\text{where } 0 \le \alpha \le 1.$$

Simple Exponential Smoothing

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| | Weights assigned to observations for: | | | |
Observation	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$
y_T	0.2	0.4	0.6	0.8
y_{T-1}	0.16	0.24	0.24	0.16
y_{T-2}	0.128	0.144	0.096	0.032
y_{T-3}	0.1024	0.0864	0.0384	0.0064
y_{T-4}	$(0.2)(0.8)^4$	$(0.4)(0.6)^4$	$(0.6)(0.4)^4$	$(0.8)(0.2)^4$
y_{T-5}	$(0.2)(0.8)^5$	$(0.4)(0.6)^5$	$(0.6)(0.4)^5$	$(0.8)(0.2)^5$

Simple Exponential Smoothing

Simple Exponential Smoothing

Component form

Forecast equation	$\hat{y}_{t+h\|t} = \ell_t$
Smoothing equation	$\ell_t = \alpha y_t + (1 - \alpha)\ell_{t-1}$

- ℓ_t is the level (or the smoothed value) of the series at time t.
- $\hat{y}_{t+1\|t} = \alpha y_t + (1 - \alpha)\hat{y}_{t\|t-1}$

Simple Exponential Smoothing

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Iterate to get exponentially weighted moving average form.

Weighted average form

$$\hat{y}_{T+1|T} = \sum_{j=0}^{T-1} \alpha(1 - \alpha)^j y_{T-j} + (1 - \alpha)^T \ell_0$$

Optimising smoothing parameters

- Need to choose best values for α and ℓ_0.
- Similarly to regression, choose optimal parameters by minimising SSE:

$$\text{SSE} = \sum_{t=1}^{T}(y_t - \hat{y}_{t|t-1})^2.$$

- Unlike regression there is no closed form solution — use numerical optimization.

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- For Algerian Exports example:
  - $\hat{\alpha}$ = 0.8400
  - $\hat{\ell}_0$ = 39.54

Simple Exponential Smoothing

Algerian exports of goods and services: level
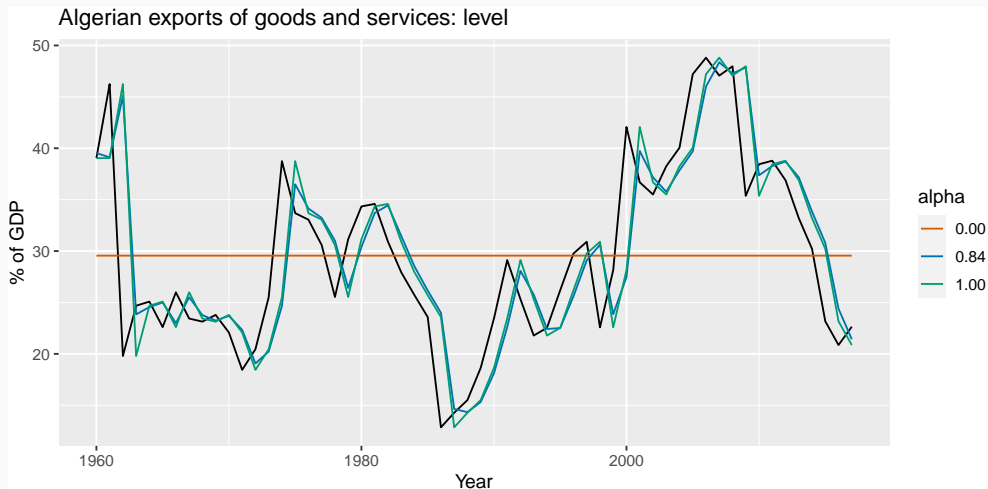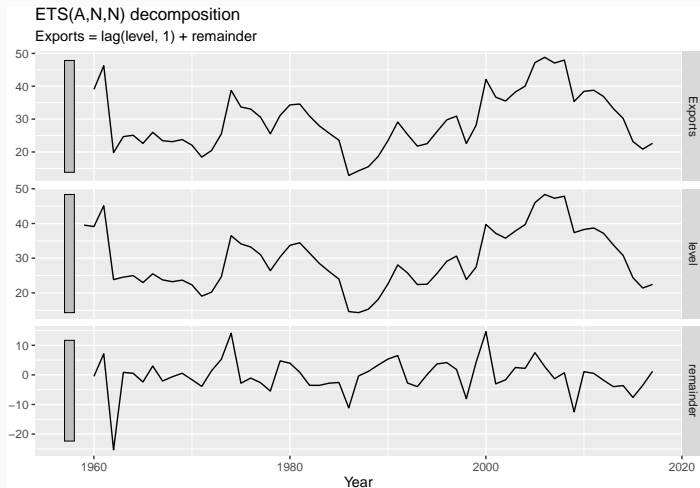
Example: Algerian Exports

```
algeria_economy <- global_economy |>
  filter(Country == "Algeria")
fit <- algeria_economy |>
  model(ANN = ETS(Exports ~ error("A") + trend("N") + season("N")))
report(fit)
```

```
## Series: Exports
## Model: ETS(A,N,N)
##   Smoothing parameters:
##     alpha = 0.84
##
##   Initial states:
##  l[0]
##  39.5
##
##   sigma^2:  35.6
##
##  AIC AICc  BIC
##  447  447  453
```

Example: Algerian Exports

```
components(fit) |> autoplot()
```

Example: Algerian Exports

```
components(fit) |>
  left_join(fitted(fit), by = c("Country", ".model", "Year"))
```

```
## # A dable: 59 x 7 [1Y]
## # Key:      Country, .model [1]
## # :         Exports = lag(level, 1) + remainder
##    Country .model  Year Exports level remainder .fitted
##    <fct>   <chr>  <dbl>   <dbl> <dbl>     <dbl>   <dbl>
##  1 Algeria ANN     1959      NA  39.5       NA      NA
##  2 Algeria ANN     1960    39.0  39.1   -0.496    39.5
##  3 Algeria ANN     1961    46.2  45.1    7.12     39.1
##  4 Algeria ANN     1962    19.8  23.8  -25.3      45.1
##  5 Algeria ANN     1963    24.7  24.6    0.841    23.8
##  6 Algeria ANN     1964    25.1  25.0    0.534    24.6
##  7 Algeria ANN     1965    22.6  23.0   -2.39     25.0
##  8 Algeria ANN     1966    26.0  25.5    3.00     23.0
##  9 Algeria ANN     1967    23.4  23.8   -2.07     25.5
## 10 Algeria ANN     1968    23.1  23.2   -0.630    23.8
## # ... with 49 more rows
```

Example: Algerian Exports

```
fit |>
  forecast(h = 5) |>
  autoplot(algeria_economy) +
  labs(y = "% of GDP", title = "Exports: Algeria")
```