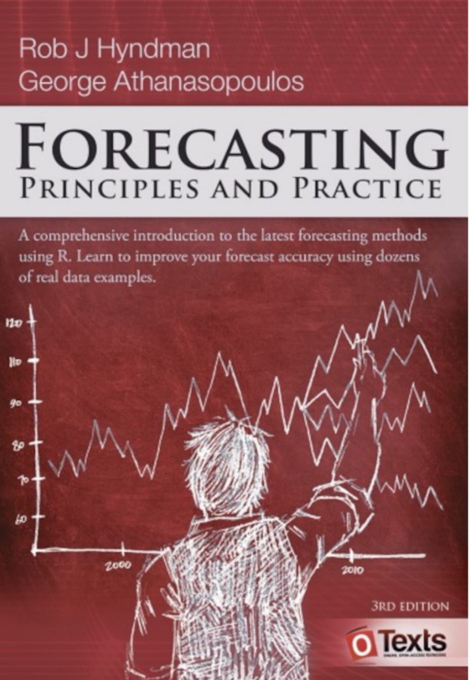


3. Time series decomposition

3.3 Moving averages

OTexts.org/fpp3/

Moving averages

The simplest estimate of the trend-cycle uses **moving averages**.

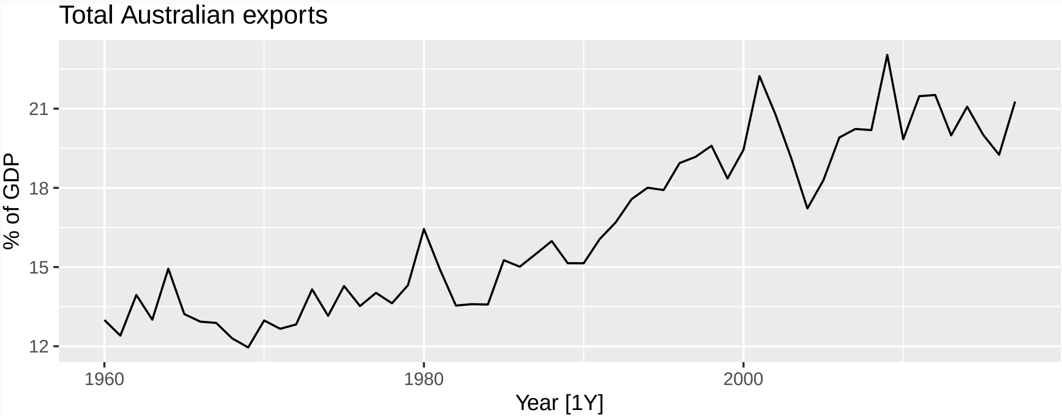
m-MA

$$\hat{T}_t = \frac{1}{m} \sum_{j=-k}^{k} y_{t+j}$$

where $k = \frac{m-1}{2}$.

Moving averages

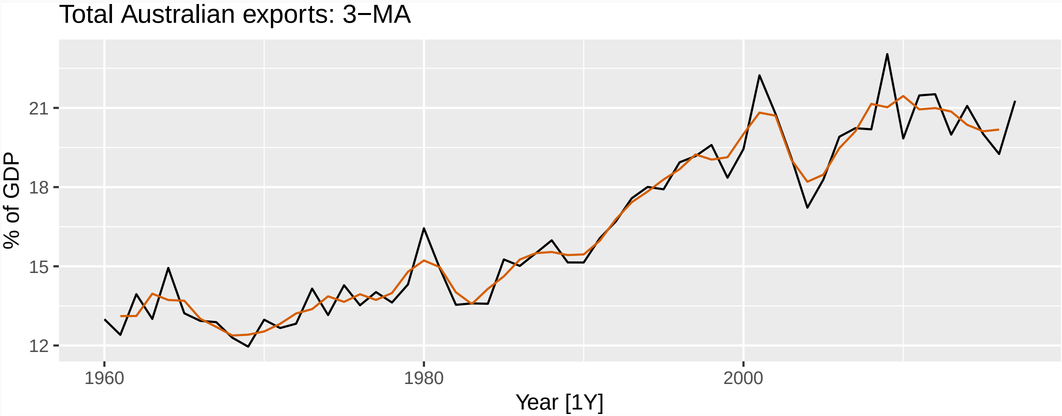
```r
global_economy |> filter(Country == "Australia") |>
  autoplot(Exports) +
  labs(y="% of GDP", title= "Total Australian exports")
```



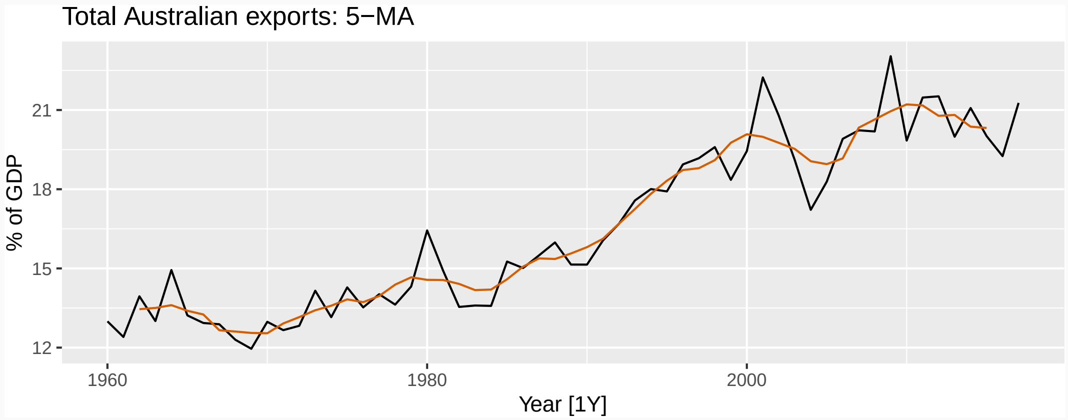
Moving average smoothing

Year	Exports	5-MA
1960.00	12.99	
1961.00	12.40	
1962.00	13.94	13.46
1963.00	13.01	13.50
1964.00	14.94	13.61
...	...	...
2012.00	21.52	20.78
2013.00	19.99	20.81
2014.00	21.08	20.37
2015.00	20.01	20.32
2016.00	19.25	
2017.00	21.27	

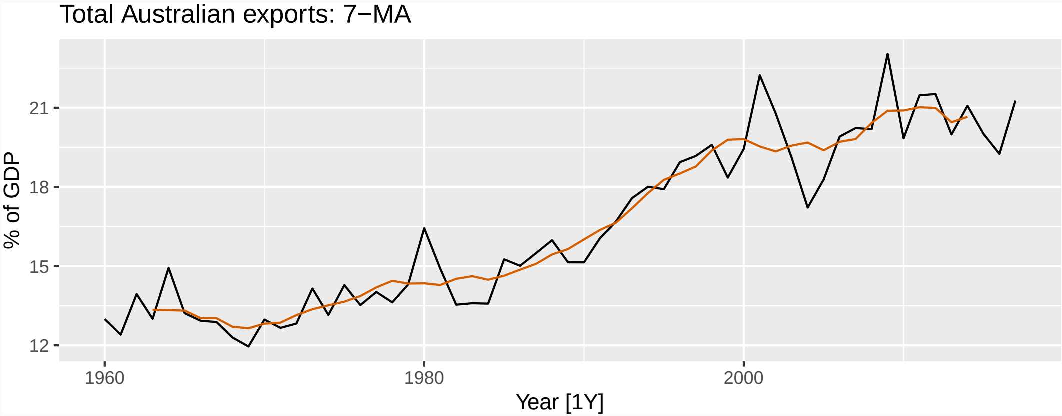
Moving average smoothing



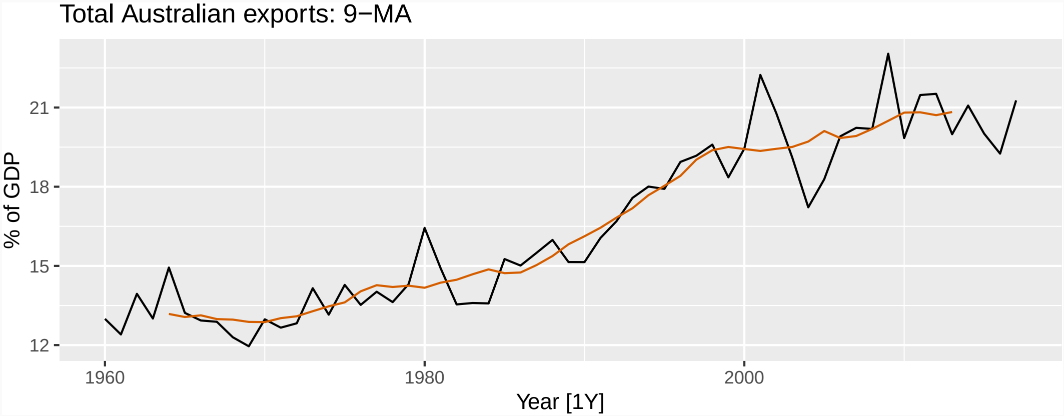
Moving average smoothing



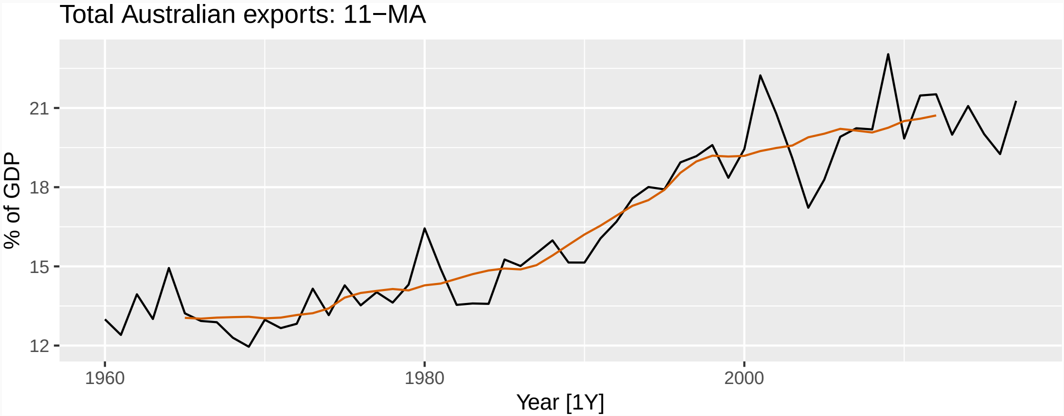
Moving average smoothing



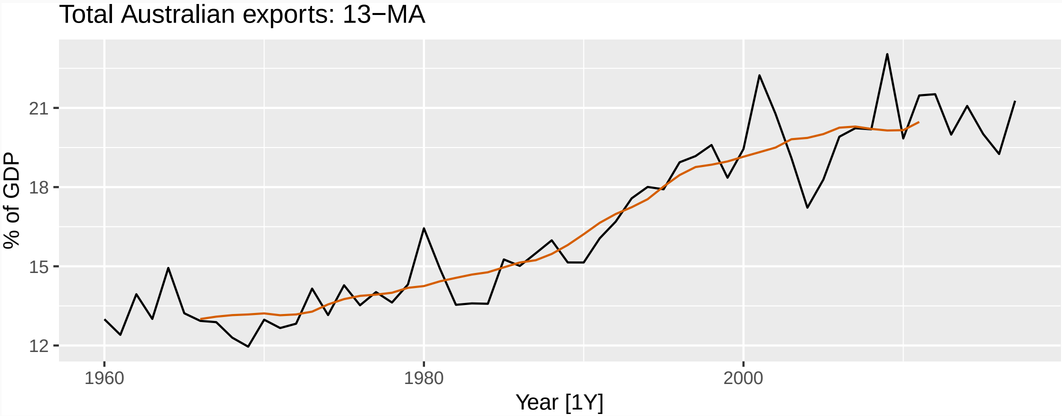
Moving average smoothing



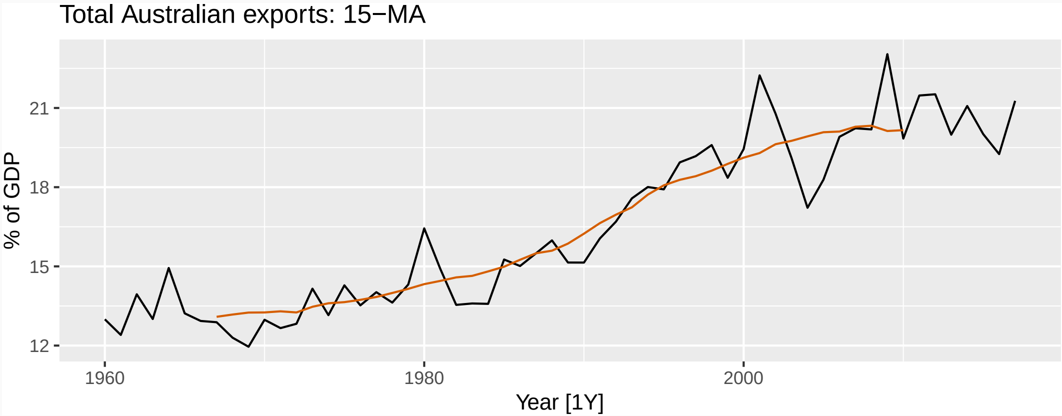
Moving average smoothing



Moving average smoothing



Moving average smoothing



Moving average smoothing

So a moving average is an average of nearby points

- observations nearby in time are also likely to be close in value.
- average eliminates some randomness in the data, leaving a smooth trend-cycle component.

$$\text{3-MA:} \quad \hat{T}_t = (y_{t-1} + y_t + y_{t+1})/3$$

$$\text{5-MA:} \quad \hat{T}_t = (y_{t-2} + y_{t-1} + y_t + y_{t+1} + y_{t+2})/5$$

- each average computed by dropping oldest observation and including next observation.
- averaging moves through time series until trend-cycle computed at each observation possible.

Endpoints

Why is there no estimate at ends?

- For a 3 MA, there cannot be estimates at time 1 or time T because the observations at time 0 and $T + 1$ are not available.
- Generally: there cannot be estimates at times near the endpoints.

The order of the MA

- larger order means smoother, flatter curve
- larger order means more points lost at ends
- order = length of season *or cycle* removes pattern
- But so far odd orders?

Centered MA

4 MA:

$$\frac{1}{4}(y_{t-2} + y_{t-1} + y_t + y_{t+1})$$

or

$$\frac{1}{4}(y_{t-1} + y_t + y_{t+1} + y_{t+2})$$

Centered MA

4 MA:

$$\frac{1}{4}(y_{t-2} + y_{t-1} + y_t + y_{t+1})$$

or

$$\frac{1}{4}(y_{t-1} + y_t + y_{t+1} + y_{t+2})$$

Solution: take a further 2-MA to "centre" result.

$$\begin{aligned} T_t &= \frac{1}{2}\Big\{\frac{1}{4}(y_{t-2} + y_{t-1} + y_t + y_{t+1}) \\ &\qquad\qquad + \frac{1}{4}(y_{t-1} + y_t + y_{t+1} + y_{t+2})\Big\} \\ &= \frac{1}{8}y_{t-2} + \frac{1}{4}y_{t-1} + \frac{1}{4}y_t + \frac{1}{4}y_{t+1} + \frac{1}{8}y_{t+2} \end{aligned}$$

Centered MA

Year	Data	4-MA	$2 \times$ 4-MA
1992 Q1	443.00		
1992 Q2	410.00		
		451.25	
1992 Q3	420.00		450.00
		448.75	
1992 Q4	532.00		450.12
		451.50	
1993 Q1	433.00		450.25
		449.00	
1993 Q2	421.00		446.50
		444.00	
1993 Q3	410.00		446.00
		448.00	
1993 Q4	512.00		443.00
		438.00	
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Moving average trend-cycle

A moving average of the same length as the season removes the seasonal pattern.

- For quarterly data: use a 2×4 MA
- For monthly data: use a 2×12 MA

$$\hat{T}_t = \frac{1}{24}y_{t-6} + \frac{1}{12}y_{t-5} + \cdots + \frac{1}{12}y_{t+5} + \frac{1}{24}y_{t+6}$$

Moving average trend-cycle

```r
us_retail_employment_ma <- us_retail_employment |>
  mutate(
    `12-MA` = slider::slide_dbl(Employed, mean,
                .before = 5, .after = 6, .complete = TRUE),
    `2x12-MA` = slider::slide_dbl(`12-MA`, mean,
                .before = 1, .after = 0, .complete = TRUE)
  )

us_retail_employment_ma |>
  autoplot(Employed, color = "gray") +
  autolayer(us_retail_employment_ma, vars(`2x12-MA`),
            color = "#D55E00") +
  labs(y = "Persons (thousands)",
       title = "Total employment in US retail")
```

Moving average trend-cycle