

Rob J Hyndman  
George Athanasopoulos

# FORECASTING

## PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**  
OPEN TEXTS FOR PRACTICE

## 8. Exponential smoothing

### 8.2 Methods with trend

[OTexts.org/fpp3/](http://OTexts.org/fpp3/)

# Holt's linear trend

## Component form

Forecast

$$\hat{y}_{t+h|t} = \ell_t + hb_t$$

Level

$$\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + b_{t-1})$$

Trend

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1},$$

# Holt's linear trend

## Component form

Forecast	$\hat{y}_{t+h t} = \ell_t + hb_t$
Level	$\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + b_{t-1})$
Trend	$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1},$

- Two smoothing parameters  $\alpha$  and  $\beta^*$  ( $0 \leq \alpha, \beta^* \leq 1$ ).
- $\ell_t$  level: weighted average between  $y_t$  and one-step ahead forecast for time  $t$ , ( $\ell_{t-1} + b_{t-1} = \hat{y}_{t|t-1}$ )
- $b_t$  slope: weighted average of  $(\ell_t - \ell_{t-1})$  and  $b_{t-1}$ , current and previous estimate of slope.
- Choose  $\alpha, \beta^*, \ell_0, b_0$  to minimise SSE.

# Exponential smoothing: trend/slope

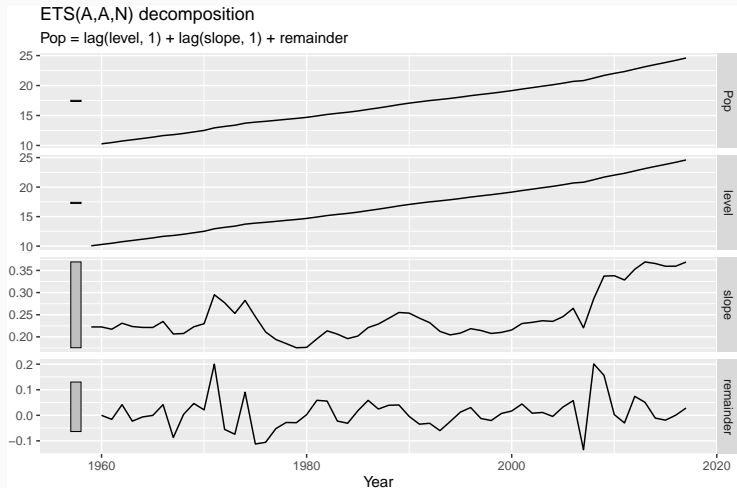
# Example: Australian population

```
aus_economy <- global_economy |>
  filter(Code == "AUS") |>
  mutate(Pop = Population / 1e6)
fit <- aus_economy |>
  model(AAN = ETS(Pop ~ error("A") + trend("A") + season("N")))
report(fit)
```

```
## Series: Pop
## Model: ETS(A,A,N)
## Smoothing parameters:
##   alpha = 1
##   beta  = 0.327
##
## Initial states:
## l[0]  b[0]
## 10.1  0.222
##
## sigma^2: 0.0041
##
## AIC  AICc  BIC
## -77.0 -75.8 -66.7
```

# Example: Australian population

```
components(fit) |> autoplot()
```



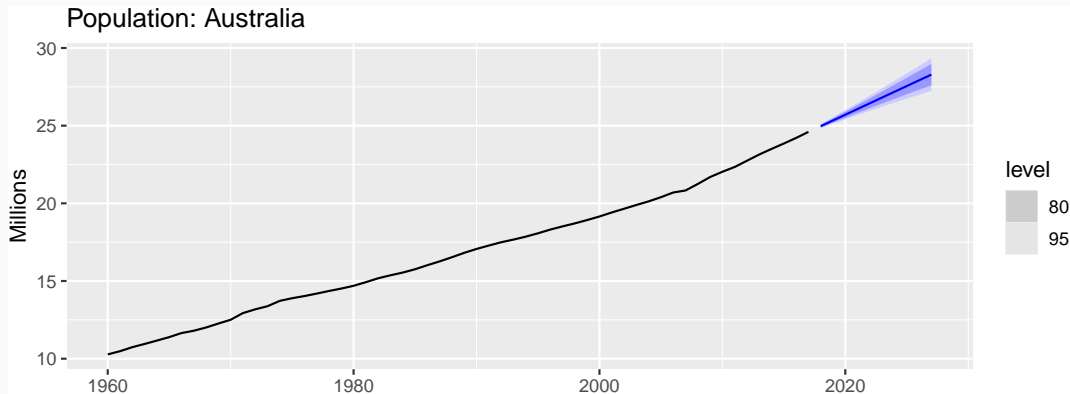
# Example: Australian population

```
components(fit) |>  
  left_join(fitted(fit), by = c("Country", ".model", "Year"))
```

```
## # A dable: 59 x 8 [1Y]  
## # Key:      Country, .model [1]  
## # :      Pop = lag(level, 1) + lag(slope, 1) + remainder  
##   Country   .model Year   Pop level slope remainder .fitted  
##   <fct>     <chr>  <dbl> <dbl> <dbl> <dbl>      <dbl>    <dbl>  
## 1 Australia AAN     1959  NA    10.1 0.222 NA        NA  
## 2 Australia AAN     1960  10.3  10.3 0.222 -0.000145  10.3  
## 3 Australia AAN     1961  10.5  10.5 0.217 -0.0159    10.5  
## 4 Australia AAN     1962  10.7  10.7 0.231  0.0418    10.7  
## 5 Australia AAN     1963  11.0  11.0 0.223 -0.0229    11.0  
## 6 Australia AAN     1964  11.2  11.2 0.221 -0.00641   11.2  
## 7 Australia AAN     1965  11.4  11.4 0.221 -0.000314  11.4  
## 8 Australia AAN     1966  11.7  11.7 0.235  0.0418    11.6  
## 9 Australia AAN     1967  11.8  11.8 0.206 -0.0869    11.9  
## 10 Australia AAN     1968  12.0  12.0 0.208  0.00350   12.0  
## # ... with 49 more rows
```

# Example: Australian population

```
fit |>  
  forecast(h = 10) |>  
  autoplot(aus_economy) +  
  labs(y = "Millions", title = "Population: Australia")
```





# Damped trend method

## Component form

$$\hat{y}_{t+h|t} = \ell_t + (\phi + \phi^2 + \dots + \phi^h)b_t$$

$$\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$$

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)\phi b_{t-1}.$$

# Damped trend method

## Component form

$$\hat{y}_{t+h|t} = \ell_t + (\phi + \phi^2 + \dots + \phi^h)b_t$$

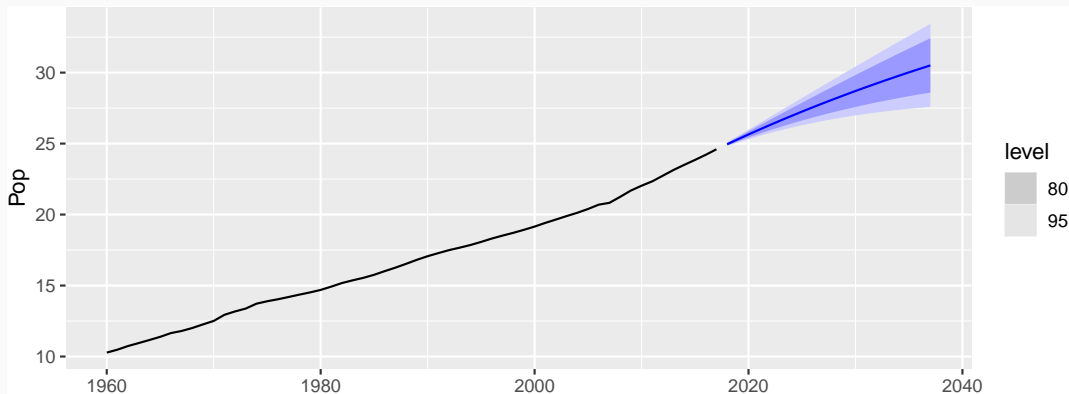
$$\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$$

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)\phi b_{t-1}.$$

- Damping parameter  $0 < \phi < 1$ .
- If  $\phi = 1$ , identical to Holt's linear trend.
- As  $h \rightarrow \infty$ ,  $\hat{y}_{T+h|T} \rightarrow \ell_T + \phi b_T / (1 - \phi)$ .
- Short-run forecasts trended, long-run forecasts constant.

# Example: Australian population

```
aus_economy |>  
  model(holt = ETS(Pop ~ error("A") + trend("Ad") + season("N"))) |>  
  forecast(h = 20) |>  
  autoplot(aus_economy)
```



# Example: Australian population

```
fit <- aus_economy |>
  filter(Year <= 2010) |>
  model(
    ses = ETS(Pop ~ error("A") + trend("N") + season("N")),
    holt = ETS(Pop ~ error("A") + trend("A") + season("N")),
    damped = ETS(Pop ~ error("A") + trend("Ad") + season("N"))
  )
```

```
tidy(fit)
accuracy(fit)
```

## Example: Australian population

	term	SES	Linear trend	Damped trend
	$\alpha$	1.00	1.00	1.00
	$\beta^*$		0.30	0.40
	$\phi$			0.98
	NA		0.22	0.25
	NA	10.28	10.05	10.04
Training RMSE		0.24	0.06	0.07
Test RMSE		1.63	0.15	0.21
Test MASE		6.18	0.55	0.75
Test MAPE		6.09	0.55	0.74
Test MAE		1.45	0.13	0.18