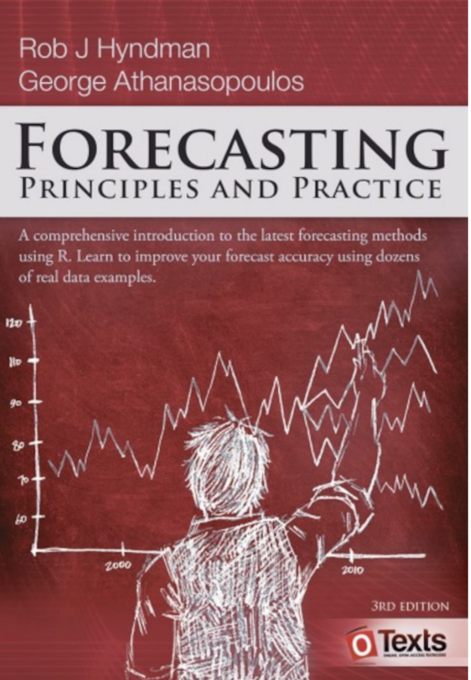


10. Dynamic regression models

10.5 Dynamic harmonic regression

OTexts.org/fpp3/

Dynamic harmonic regression

Combine Fourier terms with ARIMA errors

Advantages

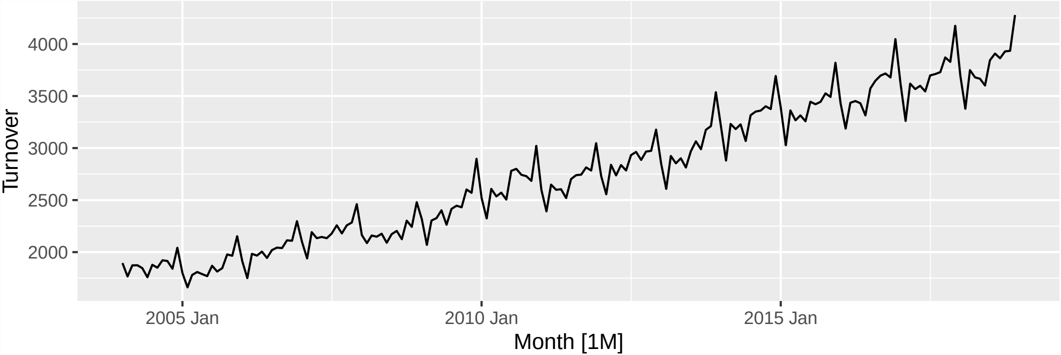
- it allows any length seasonality;
- for data with more than one seasonal period, you can include Fourier terms of different frequencies;
- the seasonal pattern is smooth for small values of K (but more wiggly seasonality can be handled by increasing K);
- the short-term dynamics are easily handled with a simple ARMA error.

Disadvantages

- seasonality is assumed to be fixed

Eating-out expenditure

```r
aus_cafe <- aus_retail |>
  filter(Industry == "Cafes, restaurants and takeaway food services",
         year(Month) %in% 2004:2018) |>
  summarise(Turnover = sum(Turnover))
aus_cafe |> autoplot(Turnover)
```

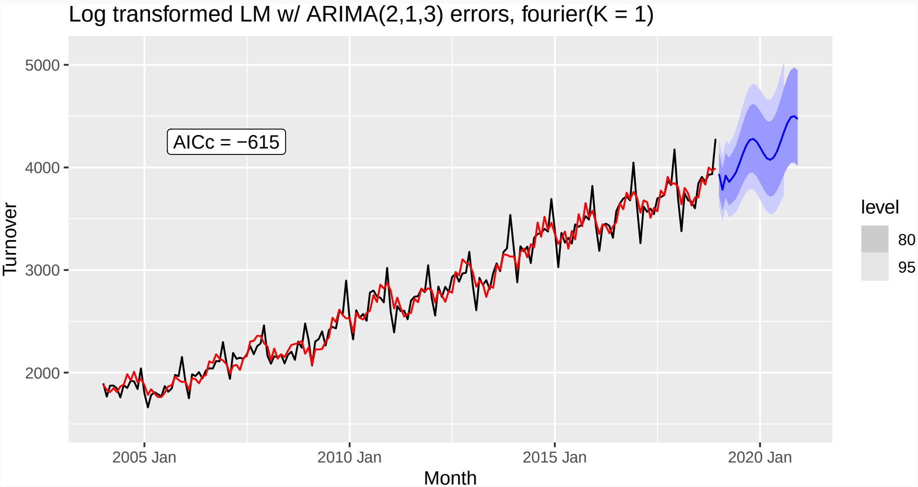


Eating-out expenditure

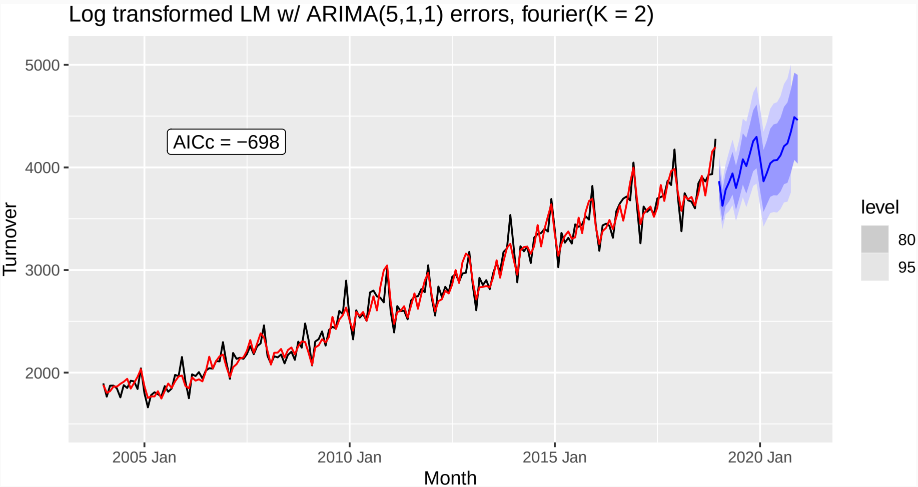
```
fit <- aus_cafe |> model(
  `K = 1` = ARIMA(log(Turnover) ~ fourier(K = 1) + PDQ(0, 0, 0)),
  `K = 2` = ARIMA(log(Turnover) ~ fourier(K = 2) + PDQ(0, 0, 0)),
  `K = 3` = ARIMA(log(Turnover) ~ fourier(K = 3) + PDQ(0, 0, 0)),
  `K = 4` = ARIMA(log(Turnover) ~ fourier(K = 4) + PDQ(0, 0, 0)),
  `K = 5` = ARIMA(log(Turnover) ~ fourier(K = 5) + PDQ(0, 0, 0)),
  `K = 6` = ARIMA(log(Turnover) ~ fourier(K = 6) + PDQ(0, 0, 0)))
glance(fit)
```

.model	sigma2	log_lik	AIC	AICc	BIC
K = 1	0.002	317	-616	-615	-588
K = 2	0.001	362	-700	-698	-661
K = 3	0.001	394	-763	-761	-725
K = 4	0.001	427	-822	-818	-771
K = 5	0.000	474	-919	-917	-875
K = 6	0.000	474	-920	-918	-875

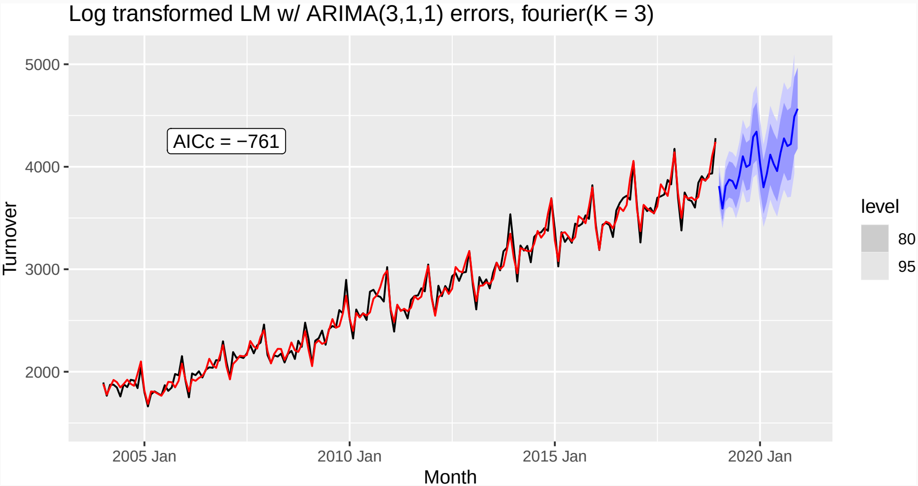
Eating-out expenditure



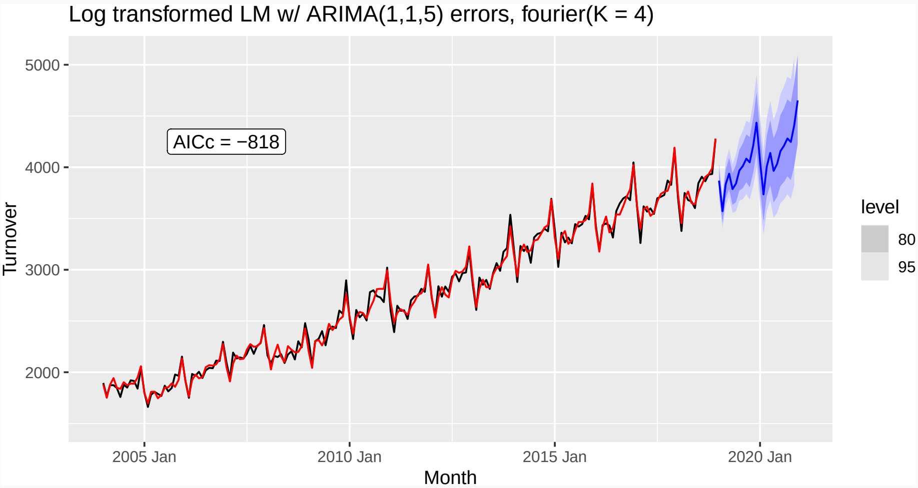
Eating-out expenditure



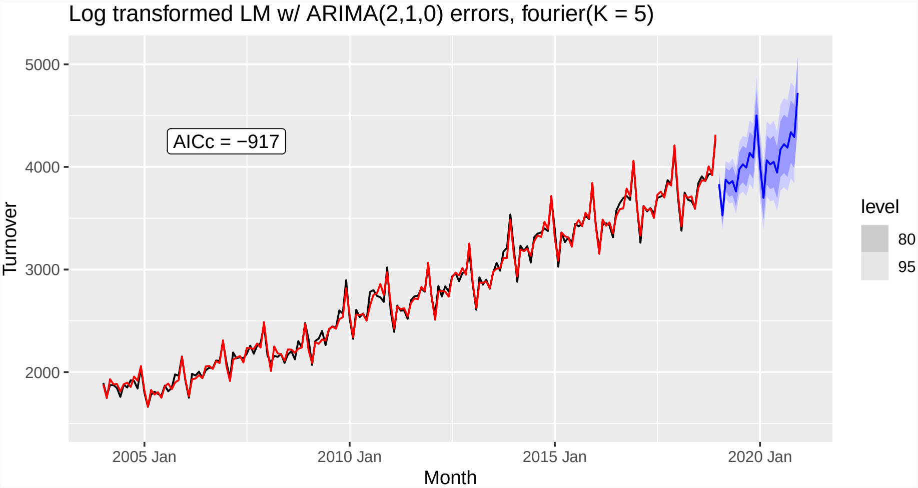
Eating-out expenditure



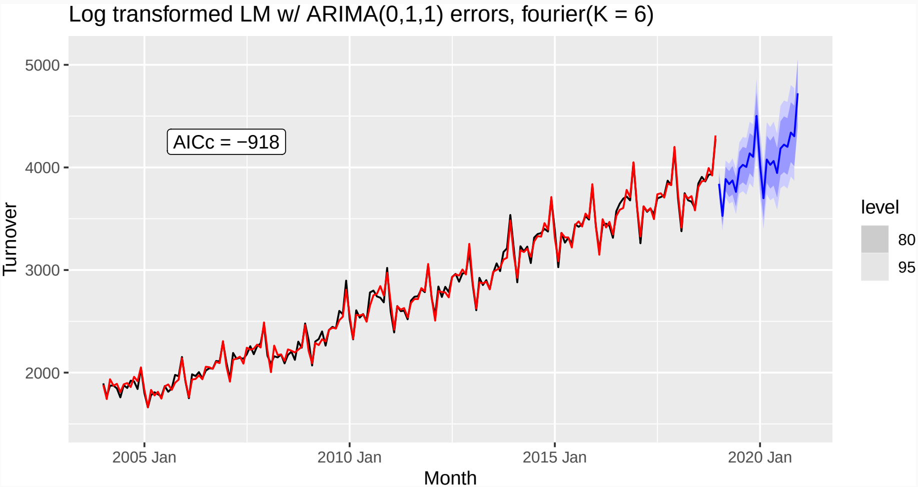
Eating-out expenditure



Eating-out expenditure



Eating-out expenditure



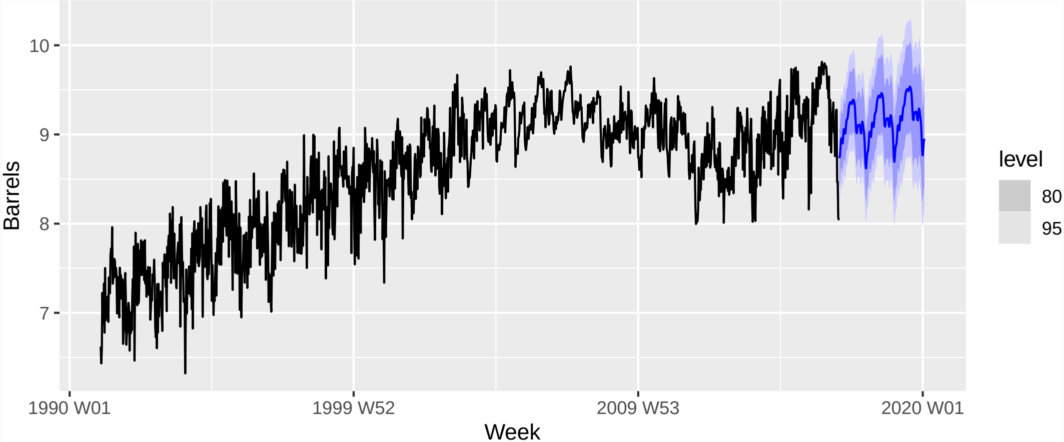
Example: weekly gasoline products

```r
fit <- us_gasoline |>
  model(ARIMA(Barrels ~ fourier(K = 13) + PDQ(0, 0, 0)))
report(fit)
```

```
## Series: Barrels
## Model: LM w/ ARIMA(0,1,1) errors
##
## Coefficients:
##           ma1  fourier(K = 13)C1_52  fourier(K = 13)S1_52
##       -0.8934               -0.1121               -0.2300
## s.e.   0.0132                0.0123                0.0122
##       fourier(K = 13)C2_52  fourier(K = 13)S2_52
##                     0.0420                0.0317
## s.e.                0.0099                0.0099
##       fourier(K = 13)C3_52  fourier(K = 13)S3_52
##                     0.0832                0.0346
## s.e.                0.0094                0.0094
##       fourier(K = 13)C4_52  fourier(K = 13)S4_52
##                     0.0185                0.0398
## s.e.                0.0092                0.0092
##       fourier(K = 13)C5_52  fourier(K = 13)S5_52
```

Example: weekly gasoline products

```r
forecast(fit, h = "3 years") |>
  autoplot(us_gasoline)
```



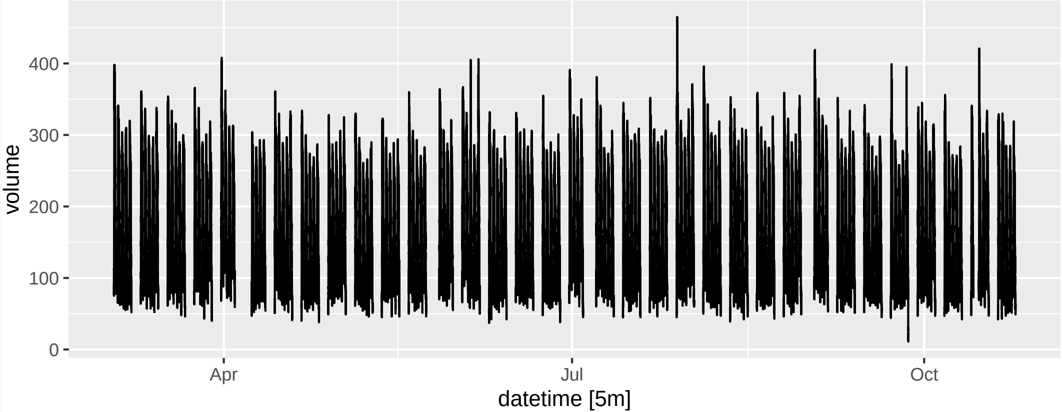
5-minute call centre volume

```
(calls <- readr::read_tsv("http://robjhyndman.com/data/callcenter.txt") |>
  rename(time = `...1`) |>
  pivot_longer(-time, names_to = "date", values_to = "volume") |>
  mutate(date = as.Date(date, format = "%d/%m/%Y"),
         datetime = as_datetime(date) + time) |>
  as_tsibble(index = datetime))
```

```
## # A tsibble: 27,716 x 4 [5m] <UTC>
##    time   date       volume datetime
##    <time> <date>      <dbl> <dttm>
##  1 07:00  2003-03-03    111 2003-03-03 07:00:00
##  2 07:05  2003-03-03    113 2003-03-03 07:05:00
##  3 07:10  2003-03-03     76 2003-03-03 07:10:00
##  4 07:15  2003-03-03     82 2003-03-03 07:15:00
##  5 07:20  2003-03-03     91 2003-03-03 07:20:00
##  6 07:25  2003-03-03     87 2003-03-03 07:25:00
##  7 07:30  2003-03-03     75 2003-03-03 07:30:00
##  8 07:35  2003-03-03     89 2003-03-03 07:35:00
```

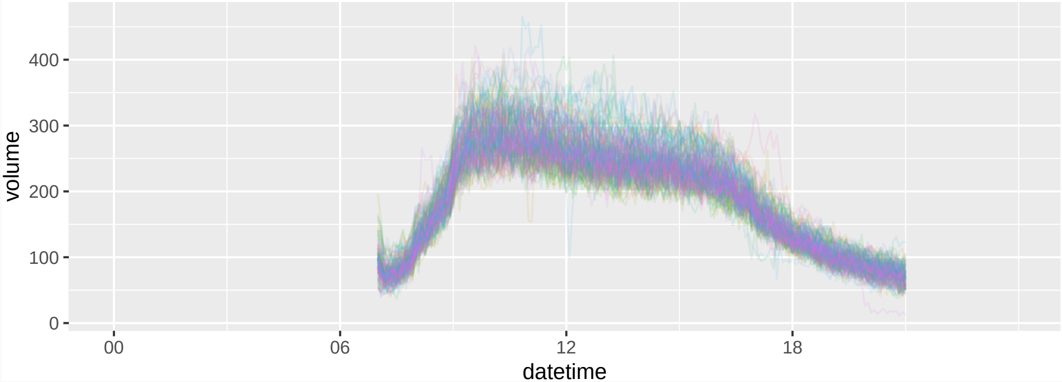
5-minute call centre volume

```
calls |>
  fill_gaps() |>
  autoplot(volume)
```



5-minute call centre volume

```r
calls |>
  fill_gaps() |>
  gg_season(volume, period = "day", alpha = 0.1) +
  guides(colour = FALSE)
```



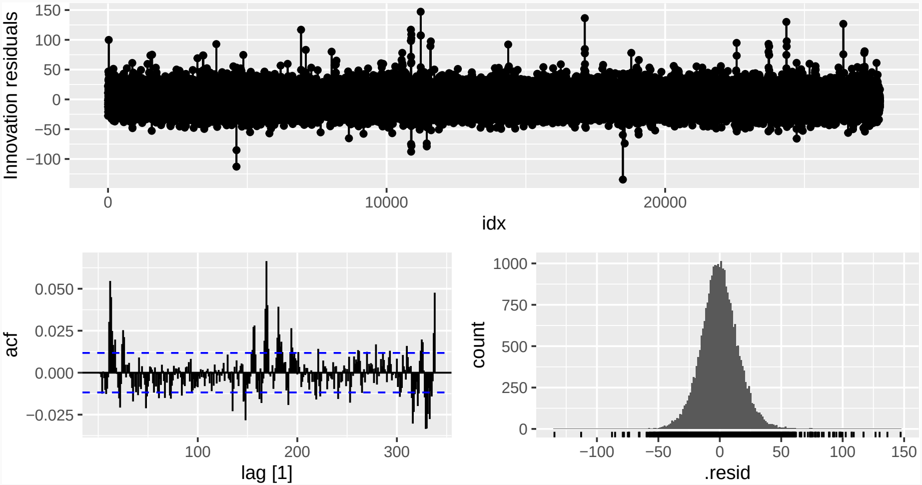
5-minute call centre volume

```
calls_mdl <- calls |>
  mutate(idx = row_number()) |>
  update_tsibble(index = idx)
fit <- calls_mdl |>
  model(ARIMA(volume ~ fourier(169, K = 10) + pdq(d = 0) + PDQ(0, 0, 0)))
report(fit)
```

```
## Series: volume
## Model: LM w/ ARIMA(1,0,3) errors
##
## Coefficients:
##          ar1      ma1      ma2      ma3  fourier(169, K = 10)C1_169
##        0.989  -0.7383  -0.0333  -0.0282                       -79.1
## s.e.   0.001   0.0061   0.0075   0.0060                         0.7
##        fourier(169, K = 10)S1_169  fourier(169, K = 10)C2_169
##                            55.298                     -32.361
## s.e.                        0.701                       0.378
##        fourier(169, K = 10)S2_169  fourier(169, K = 10)C3_169
##                            13.742                      -9.318
## s.e.                        0.379                       0.273
```

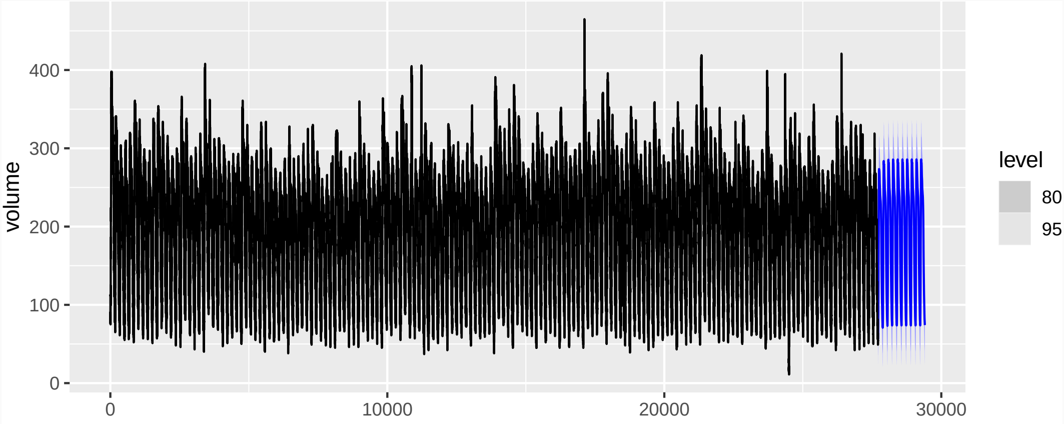
5-minute call centre volume

```r
gg_tsresiduals(fit, lag = 338)
```



5-minute call centre volume

```r
fit |>
  forecast(h = 1690) |>
  autoplot(calls_mdl)
```



5-minute call centre volume

```
fit |>
  forecast(h = 1690) |>
  autoplot(filter(calls_mdl, idx > 25000))
```