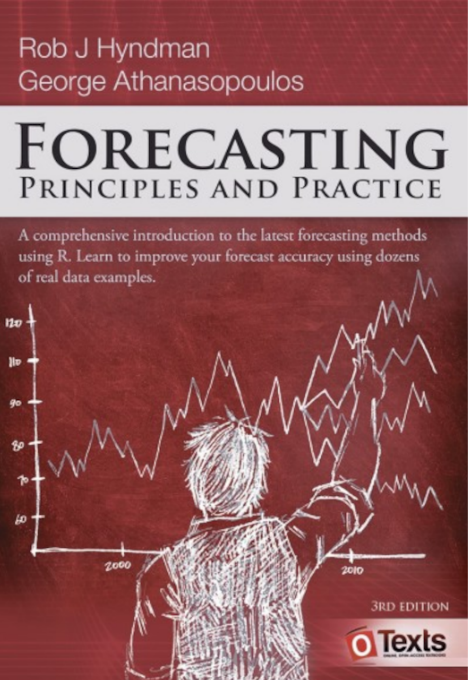


9. ARIMA models

9.3 Autoregressive models

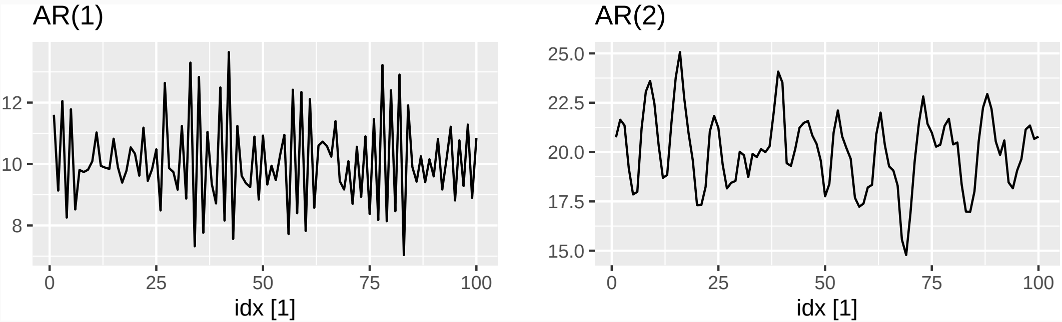
OTexts.org/fpp3/

Autoregressive models

Autoregressive model - AR(p):

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t,$$

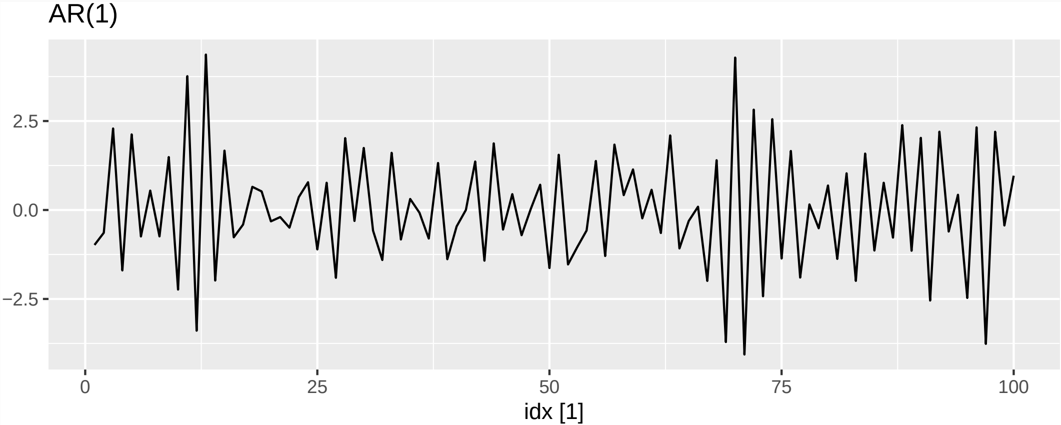
where ε_t is white noise. This is a multiple regression with lagged values of y_t as predictors.



AR(1) model

$$y_t = -0.8y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$



AR(1) model

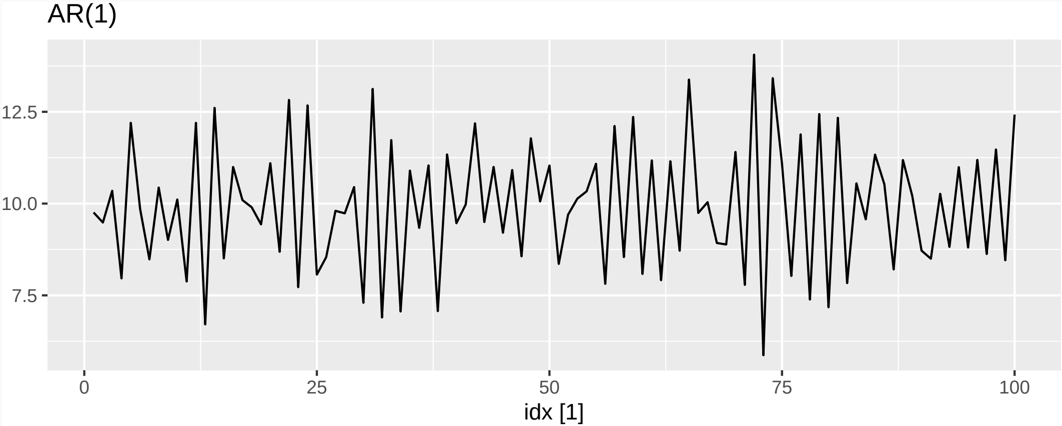
$$y_t = \phi_1 y_{t-1} + \varepsilon_t$$

- When $\phi_1 = 0$, y_t is equivalent to a WN
- When $\phi_1 = 1$, y_t is equivalent to a RW
- We require $|\phi_1| < 1$ for stationarity. The closer ϕ_1 is to the bounds the more the process wanders above or below it's unconditional mean (zero in this case).
- When $\phi_1 < 0$, y_t tends to oscillate between positive and negative values.

AR(1) model including a constant

$$y_t = 18 - 0.8y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$



AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- When $\phi_1 = 0$ and $c = 0$, y_t is equivalent to WN;
- When $\phi_1 = 1$ and $c = 0$, y_t is equivalent to a RW;
- When $\phi_1 = 1$ and $c \neq 0$, y_t is equivalent to a RW with drift;

AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- c is related to the mean of y_t.
- Let $E(y_t) = \mu$

AR(1) model including a constant

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- c is related to the mean of y_t.
- Let $E(y_t) = \mu$
- $\mu = c + \phi_1 \mu$

AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- c is related to the mean of y_t.
- Let $E(y_t) = \mu$
- $\mu = c + \phi_1 \mu$
- $\mu = \frac{c}{1-\phi_1}$

AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- c is related to the mean of y_t.
- Let $E(y_t) = \mu$
- $\mu = c + \phi_1 \mu$
- $\mu = \frac{c}{1-\phi_1}$
- `ARIMA()` takes care of whether you need a constant or not, or you can overide it.

AR(1) model including a constant

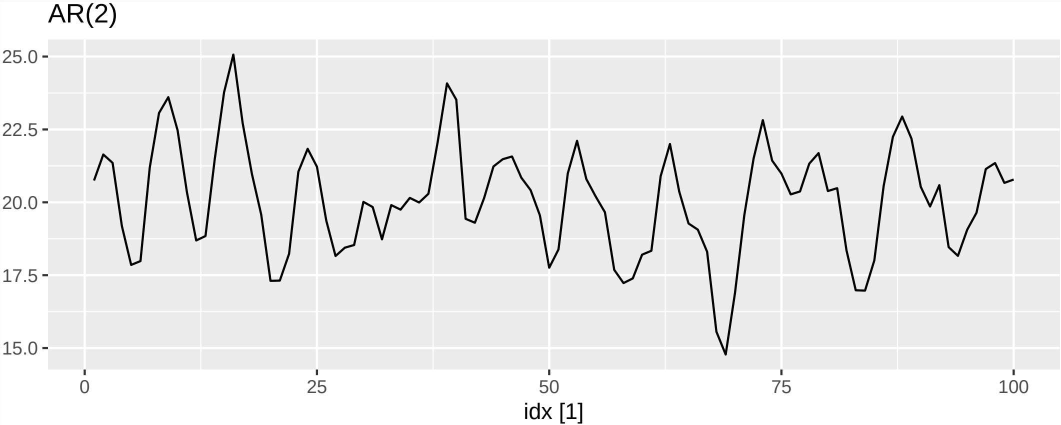
- If included estimated model returns `w/ mean`

```
Series: sim
Model: ARIMA(1,0,0) w/ mean

Coefficients:
          ar1  constant
      -0.8381   18.3527
s.e.   0.0540    0.1048

sigma^2 estimated as 1.11:  log likelihood=-146.7
AIC=299.4   AICc=299.7   BIC=307.2
```

AR(2) model

$$y_t = 8 + 1.3y_{t-1} - 0.7y_{t-2} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \qquad T = 100.$$



Stationarity conditions

We normally restrict autoregressive models to stationary data, and then some constraints on the values of the parameters are required.

General condition for stationarity

Complex roots of $1 - \phi_1 z - \phi_2 z^2 - \dots - \phi_p z^p$ lie outside the unit circle on the complex plane.

Stationarity conditions

We normally restrict autoregressive models to stationary data, and then some constraints on the values of the parameters are required.

General condition for stationarity

Complex roots of $1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_p z^p$ lie outside the unit circle on the complex plane.

- For $p = 1$: $-1 < \phi_1 < 1$.
- For $p = 2$:
  $-1 < \phi_2 < 1 \qquad \phi_2 + \phi_1 < 1 \qquad \phi_2 - \phi_1 < 1.$
- More complicated conditions hold for $p \geq 3$.
- Estimation software takes care of this.