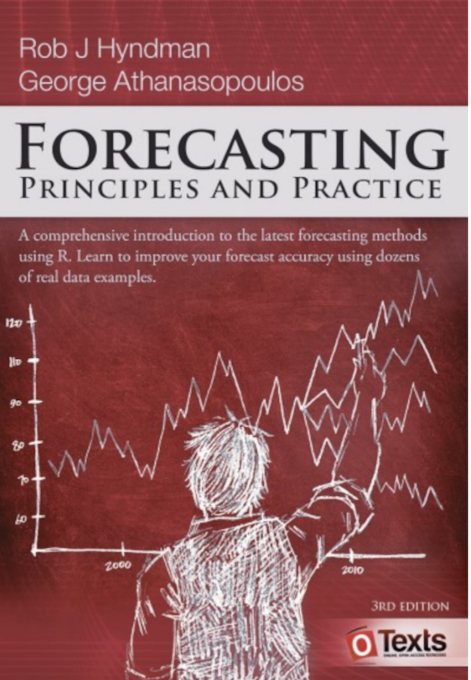

10. Dynamic regression models

10.6 Lagged predictors

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Lagged predictors

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- y_t = stream flow, x_t = rainfall.
- y_t = size of herd, x_t = breeding stock.

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- y_t = stream flow, x_t = rainfall.
- y_t = size of herd, x_t = breeding stock.

- These are dynamic systems with input (x_t) and output (y_t).
- x_t is often a leading indicator.
- There can be multiple predictors.

Lagged predictors

The model include present and past values of predictor:

$$y_t = a + \gamma_0 x_t + \gamma_1 x_{t-1} + \cdots + \gamma_k x_{t-k} + \eta_t$$

where η_t is an ARIMA process.

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where η_t is an ARIMA process.

Rewrite model as

$$\begin{aligned} y_t &= a + (\gamma_0 + \gamma_1 B + \gamma_2 B^2 + \cdots + \gamma_k B^k) x_t + \eta_t \\ &= a + \gamma(B) x_t + \eta_t. \end{aligned}$$

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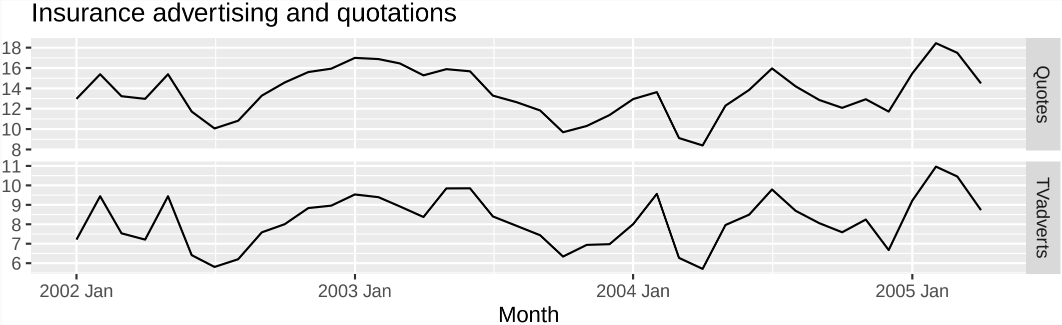
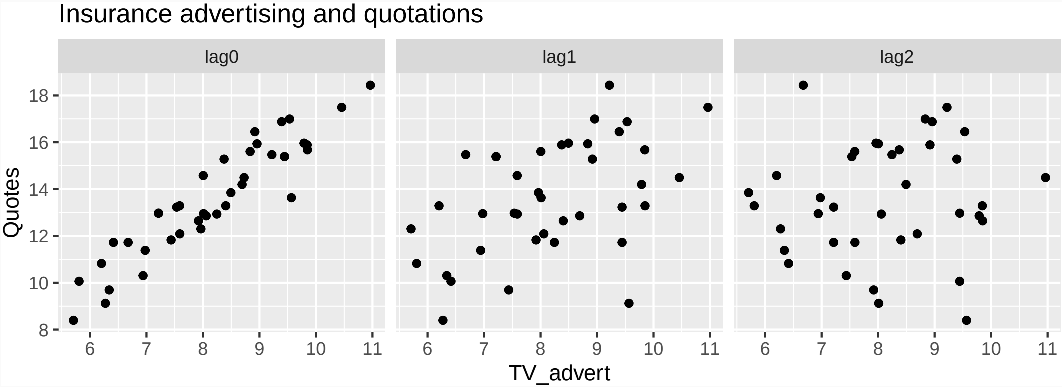
- $\gamma(B)$ is called a *transfer function* since it describes how change in x_t is transferred to y_t.
- x can influence y, but y is not allowed to influence x.

Example: Insurance quotes and TV adverts

```r
insurance |>
  pivot_longer(Quotes:TVadverts) |>
  ggplot(aes(x = Month, y = value)) +
  geom_line() +
  facet_grid(vars(name), scales = "free_y") +
  labs(y = NULL, title = "Insurance advertising and quotations")
```



Example: Insurance quotes and TV adverts



Example: Insurance quotes and TV adverts

```r
fit <- insurance |>
  # Restrict data so models use same fitting period
  mutate(Quotes = c(NA, NA, NA, Quotes[4:40])) |>
  # Estimate models
  model(
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts)),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts) +
      lag(TVadverts, 2)),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts) +
      lag(TVadverts, 2) + lag(TVadverts, 3))
  )
```

Example: Insurance quotes and TV adverts

```
glance(fit)
```

Lag order	sigma2	log_lik	AIC	AICc	BIC
0	0.265	-28.3	66.6	68.3	75.0
1	0.209	-24.0	58.1	59.9	66.5
2	0.215	-24.0	60.0	62.6	70.2
3	0.206	-22.2	60.3	65.0	73.8

Example: Insurance quotes and TV adverts

```r
fit_best <- insurance |>
  model(ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts)))
report(fit_best)
```

```
## Series: Quotes
## Model: LM w/ ARIMA(1,0,2) errors
##
## Coefficients:
##         ar1    ma1    ma2  TVadverts  lag(TVadverts)  intercept
##       0.512  0.917  0.459     1.2527          0.1464       2.16
## s.e.  0.185  0.205  0.190     0.0588          0.0531       0.86
##
## sigma^2 estimated as 0.2166:  log likelihood=-23.9
## AIC=61.9   AICc=65.4   BIC=73.7
```

Example: Insurance quotes and TV adverts

```r
fit_best <- insurance |>
  model(ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts)))
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##
## sigma^2 estimated as 0.2166:  log likelihood=-23.9
## AIC=61.9   AICc=65.4   BIC=73.7
```
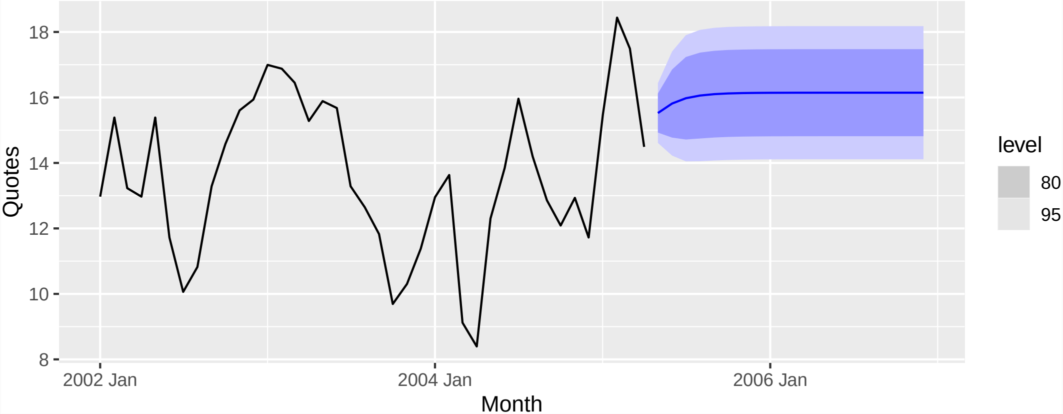

$$
\begin{aligned}
y_t &= 2.155 + 1.253x_t + 0.146x_{t-1} + \eta_t, \\
\eta_t &= 0.512\eta_{t-1} + \varepsilon_t + 0.917\varepsilon_{t-1} + 0.459\varepsilon_{t-2},
\end{aligned}
$$

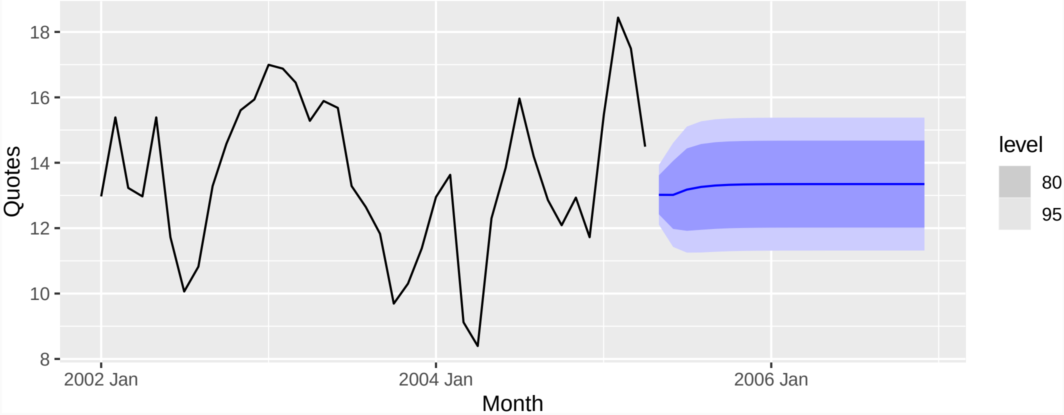
Example: Insurance quotes and TV adverts

```
advert_a <- new_data(insurance, 20) |>
  mutate(TVadverts = 10)
forecast(fit_best, advert_a) |> autoplot(insurance)
```



Example: Insurance quotes and TV adverts

```
advert_b <- new_data(insurance, 20) |>
  mutate(TVadverts = 8)
forecast(fit_best, advert_b) |> autoplot(insurance)
```



Example: Insurance quotes and TV adverts

```r
advert_c <- new_data(insurance, 20) |>
  mutate(TVadverts = 6)
forecast(fit_best, advert_c) |> autoplot(insurance)
```