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# FORECASTING

## PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**  
Open Texts Publishing

## 10. Dynamic regression models

### 10.1 Estimation

[OTexts.org/fpp3/](http://OTexts.org/fpp3/)

# Regression with ARIMA errors

## Regression models

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t,$$

- $y_t$  modeled as function of  $k$  explanatory variables  $x_{1,t}, \dots, x_{k,t}$ .
- In regression, we assume that  $\varepsilon_t$  is WN.
- Now we want to allow  $\varepsilon_t$  to be autocorrelated.

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## Example: ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

where  $\varepsilon_t$  is white noise.

# Residuals and errors

**Example:**  $\eta_t = \text{ARIMA}(1,1,1)$

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- Be careful in distinguishing  $\eta_t$  from  $\varepsilon_t$ .
- Only the errors  $\varepsilon_t$  are assumed to be white noise.
- In ordinary regression,  $\eta_t$  is assumed to be white noise and so  $\eta_t = \varepsilon_t$ .

# Estimation

If we minimize  $\sum \eta_t^2$  (by using ordinary regression):

- 1 Estimated coefficients  $\hat{\beta}_0, \dots, \hat{\beta}_k$  are no longer optimal as some information ignored;
- 2 Statistical tests associated with the model (e.g., t-tests on the coefficients) are incorrect.
- 3  $p$ -values for coefficients usually too small (“spurious regression”).
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- 3  $p$ -values for coefficients usually too small (“spurious regression”).
- 4 AIC of fitted models misleading.
  - Minimizing  $\sum \varepsilon_t^2$  avoids these problems.
  - Maximizing likelihood similar to minimizing  $\sum \varepsilon_t^2$ .

# Regression with ARIMA errors

## Model with ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
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## Equivalent to model with ARIMA(1,0,1) errors

$$y'_t = \beta_1 x'_{1,t} + \cdots + \beta_k x'_{k,t} + \eta'_t,$$
$$(1 - \phi_1 B)\eta'_t = (1 + \theta_1 B)\varepsilon_t,$$

where  $y'_t = y_t - y_{t-1}$ ,  $x'_{t,i} = x_{t,i} - x_{t-1,i}$  and  $\eta'_t = \eta_t - \eta_{t-1}$ .

## Regression with ARIMA errors

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$$\text{where } \phi(B)(1 - B)^d \eta_t = \theta(B)\varepsilon_t$$

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## After differencing all variables

$$y'_t = \beta_1 x'_{1,t} + \cdots + \beta_k x'_{k,t} + \eta'_t.$$

where

$$\phi(B)\eta'_t = \theta(B)\varepsilon_t, \quad y'_t = (1-B)^d y_t, \quad x'_{i,t} = (1-B)^d x_{i,t}, \quad \eta'_t = (1-B)^d \eta_t$$