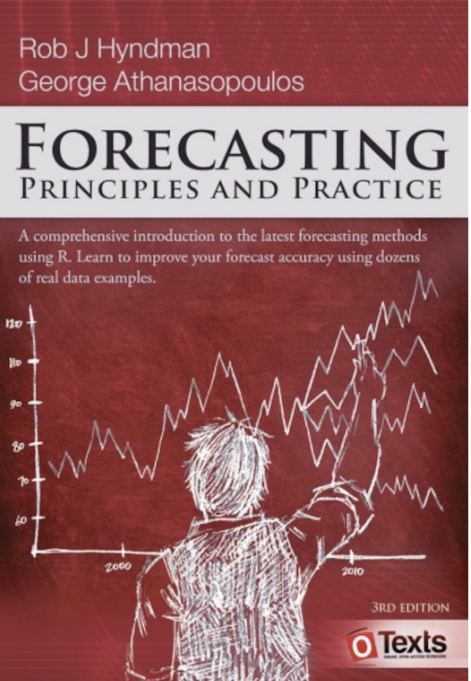

7. Time series regression models

7.1 The linear model

OTexts.org/fpp3/

Least squares estimation

- In practice we need to estimate the coefficients: $\beta_0, \beta_1, \ldots, \beta_k$.

$$\sum_{t=1}^{T} \varepsilon_t^2 = \sum_{t=1}^{T} (y_t - \beta_0 - \beta_1 x_{1,t} - \beta_2 x_{2,t} - \cdots - \beta_k x_{k,t})^2$$

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```
model(TSLM(y ~ x_1 + x_2 + ... + x_k))
```

- Estimated coefficients: $\hat{\beta}_0, \ldots, \hat{\beta}_k$

Example: US consumption expenditure

```
fit_consMR <- us_change |>
  model(lm = TSLM(Consumption ~ Income + Production + Unemployment + Savings))
report(fit_consMR)
```

```
## Series: Consumption
## Model: TSLM
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -0.906 -0.158 -0.036  0.136  1.155
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.25311    0.03447    7.34  5.7e-12 ***
## Income        0.74058    0.04012   18.46  < 2e-16 ***
## Production    0.04717    0.02314    2.04    0.043 *
## Unemployment -0.17469    0.09551   -1.83    0.069 .
## Savings      -0.05289    0.00292  -18.09  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.31 on 193 degrees of freedom
## Multiple R-squared: 0.768,   Adjusted R-squared: 0.763
## F-statistic:  160 on 4 and 193 DF, p-value: <2e-16
```

Fitted values

$$\hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1 x_{1,t} + \hat{\beta}_2 x_{2,t} + \cdots + \hat{\beta}_k x_{k,t}$$

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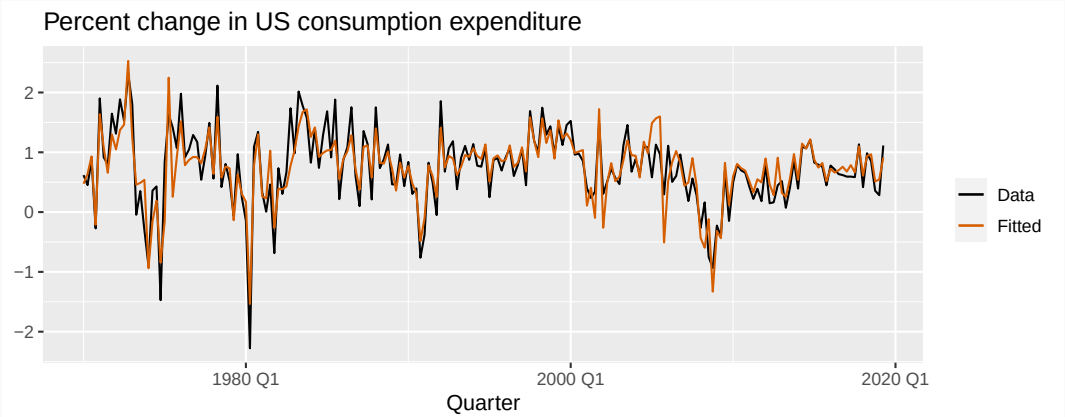
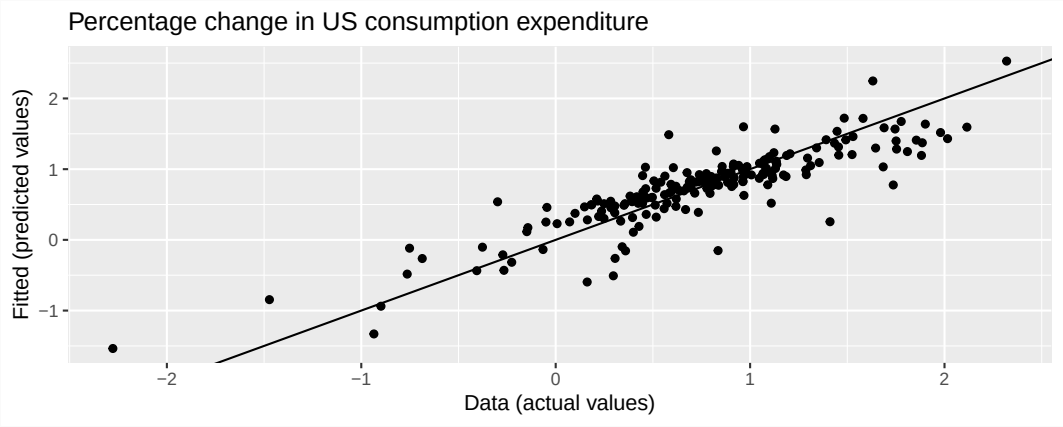


Example: US consumption expenditure



Goodness of fit

Coefficient of determination

$$R^2 = \frac{\sum(\hat{y}_t - \bar{y})^2}{\sum(y_t - \bar{y})^2}$$

Standard error of the regression

$$\hat{\sigma}_e = \sqrt{\frac{1}{T-k-1}\sum_{t=1}^{T} e_t^2}$$

where k is the number of predictors in the model.