

Rob J Hyndman
George Athanasopoulos

FORECASTING

PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**
Open Texts Publishing

5. The forecaster's toolbox

5.5 Distributional forecasts

OTexts.org/fpp3/

Forecast distributions

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \dots, y_T$.
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

Forecast distributions

Assuming residuals are normal, uncorrelated, $\text{sd} = \hat{\sigma}$:

Mean: $y_{T+h|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$

Naïve: $y_{T+h|T} \sim N(y_T, h\hat{\sigma}^2)$

Seasonal naïve: $y_{T+h|T} \sim N(y_{T+h-m(k+1)}, (k+1)\hat{\sigma}^2)$

Drift: $y_{T+h|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where k is the integer part of $(h - 1)/m$.

Note that when $h = 1$ and T is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

Prediction intervals

- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where $\hat{\sigma}_h$ is the st dev of the h -step distribution.

- When $h = 1$, $\hat{\sigma}_h$ can be estimated from the residuals.

Prediction intervals

```
aus_production |>
  filter(!is.na(Bricks)) |>
  model(Seasonal_naive = SNAIVE(Bricks)) |>
  forecast(h = "5 years")
```

```
## # A tibble: 20 x 4 [1Q]
```

```
## # Key:           .model [1]
```

##	.model	Quarter	Bricks	.mean
##	<chr>	<qtr>	<dist>	<dbl>
##	1 Seasonal_naive	2005 Q3	N(428, 2336)	428
##	2 Seasonal_naive	2005 Q4	N(397, 2336)	397
##	3 Seasonal_naive	2006 Q1	N(355, 2336)	355
##	4 Seasonal_naive	2006 Q2	N(435, 2336)	435
##	5 Seasonal_naive	2006 Q3	N(428, 4672)	428
##	6 Seasonal_naive	2006 Q4	N(397, 4672)	397
##	7 Seasonal_naive	2007 Q1	N(355, 4672)	355

Prediction intervals

```
aus_production |>
  filter(!is.na(Bricks)) |>
  model(Seasonal_naive = SNAIVE(Bricks)) |>
  forecast(h = "5 years") |>
  hilo(level = 95)
```

```
## # A tsibble: 20 x 5 [1Q]
```

```
## # Key:           .model [1]
```

##	.model	Quarter	Bricks	.mean	`95%`
##	<chr>	<qtr>	<dist>	<dbl>	<hilo>
##	1 Seasonal_naive	2005 Q3	N(428, 2336)	428	[333, 523]95
##	2 Seasonal_naive	2005 Q4	N(397, 2336)	397	[302, 492]95
##	3 Seasonal_naive	2006 Q1	N(355, 2336)	355	[260, 450]95
##	4 Seasonal_naive	2006 Q2	N(435, 2336)	435	[340, 530]95
##	5 Seasonal_naive	2006 Q3	N(428, 4672)	428	[294, 562]95
##	6 Seasonal_naive	2006 Q4	N(397, 4672)	397	[263, 531]95

Prediction intervals

```
aus_production |>
  filter(!is.na(Bricks)) |>
  model(Seasonal_naive = SNAIVE(Bricks)) |>
  forecast(h = "5 years") |>
  hilo(level = 95) |>
  mutate(lower = `95%`$lower, upper = `95%`$upper)
```

```
## # A tsibble: 20 x 7 [1Q]
```

```
## # Key:           .model [1]
```

##	.model	Quarter	Bricks	.mean	`95%`	lower	upper
##	<chr>	<qtr>	<dist>	<dbl>	<hilo>	<dbl>	<dbl>
##	1 Seasonal_naive	2005 Q3	N(428, 2336)	428	[333, 523]95	333.	523.
##	2 Seasonal_naive	2005 Q4	N(397, 2336)	397	[302, 492]95	302.	492.
##	3 Seasonal_naive	2006 Q1	N(355, 2336)	355	[260, 450]95	260.	450.
##	4 Seasonal_naive	2006 Q2	N(435, 2336)	435	[340, 530]95	340.	530.
##	5 Seasonal_naive	2006 Q3	N(428, 4672)	428	[294, 562]95	294.	562.

Prediction intervals

- Point forecasts are often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them.
- Prediction intervals are usually too narrow due to unaccounted uncertainty.