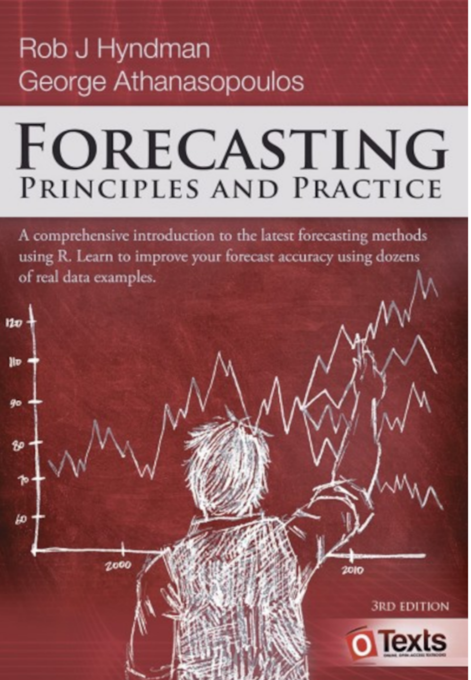
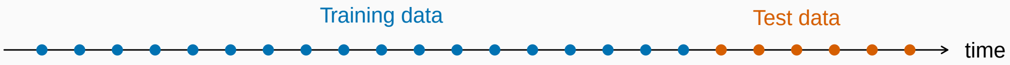


5. The forecaster’s toolbox

5.8 Evaluating point forecast accuracy

OTexts.org/fpp3/

Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Forecast errors

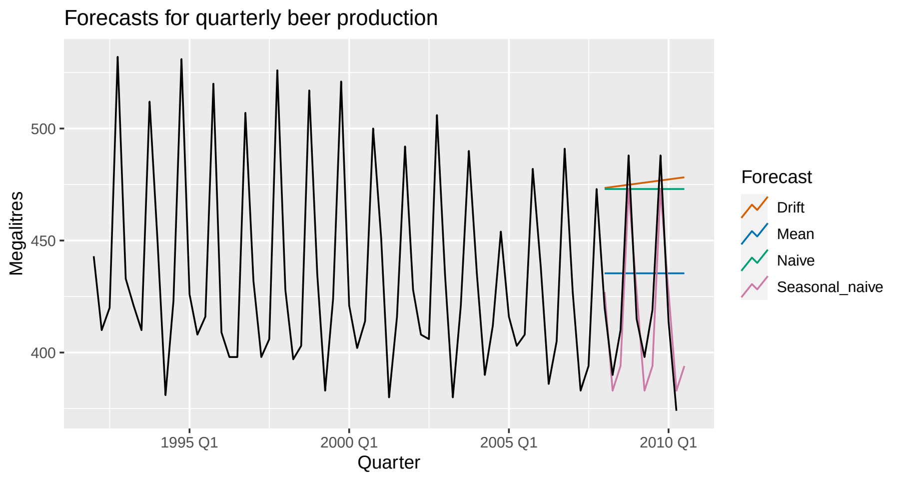
Forecast "error": the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing $\hat{y}_{T+h|T}$.

Measures of forecast accuracy



Measures of forecast accuracy

$$
\begin{aligned}
y_{T+h} &= (T+h)\text{th observation}, h = 1, \dots, H \\
\hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\
e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T}
\end{aligned}
$$

$$
\begin{aligned}
\text{MAE} &= \text{mean}(|e_{T+h}|) \\
\text{MSE} &= \text{mean}(e_{T+h}^2) \qquad \text{RMSE} = \sqrt{\text{mean}(e_{T+h}^2)} \\
\text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|)
\end{aligned}
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\end{aligned}
$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Scaled Errors

Proposed by Hyndman and Koehler (IJF, 2006).

- For non-seasonal time series, scale errors using naïve forecasts:

$$q_{T+h} = \frac{e_{T+h}}{\frac{1}{T-1}\sum_{t=2}^{T}|y_t - y_{t-1}|}.$$

Scaled Errors

Proposed by Hyndman and Koehler (IJF, 2006).

- For non-seasonal time series, scale errors using naïve forecasts:

$$q_{T+h} = \frac{e_{T+h}}{\frac{1}{T-1}\sum_{t=2}^{T}|y_t - y_{t-1}|}.$$

- For seasonal time series, scale forecast errors using seasonal naïve forecasts:

$$q_{T+h} = \frac{e_{T+h}}{\frac{1}{T-m}\sum_{t=m+1}^{T}|y_t - y_{t-m}|}.$$

Scaled errors

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|q_{T+h}|)$$

Scaled errors

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|q_{T+h}|)$$

Root Mean Squared Scaled Error

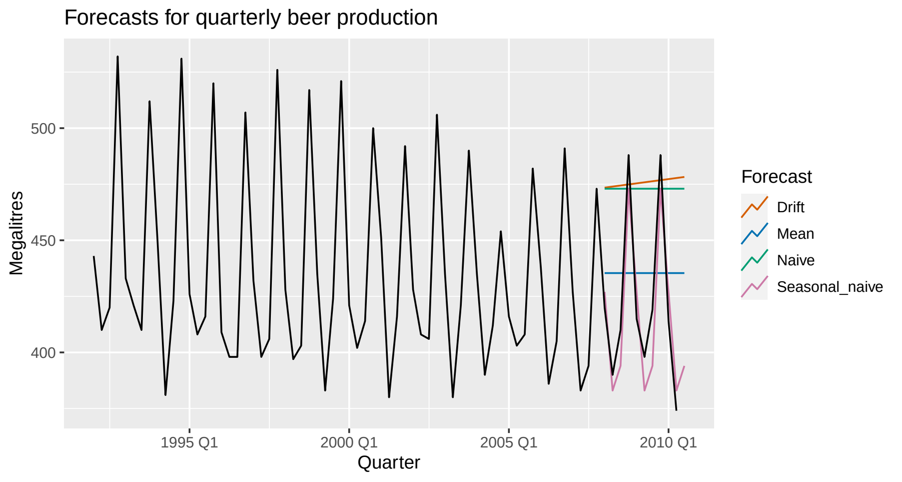
$$\text{RMSSE} = \sqrt{\text{mean}(q^2_{T+h})}$$

where

$$q^2_{T+h} = \frac{e^2_{T+h}}{\frac{1}{T-m}\sum_{t=m+1}^{T}(y_t - y_{t-m})^2},$$

and we set $m = 1$ for non-seasonal data.

Measures of forecast accuracy



Measures of forecast accuracy

```r
recent_production <- aus_production |>
  filter(year(Quarter) >= 1992)
train <- recent_production |>
  filter(year(Quarter) <= 2007)
beer_fit <- train |>
  model(
    Mean = MEAN(Beer),
    Naive = NAIVE(Beer),
    Seasonal_naive = SNAIVE(Beer),
    Drift = RW(Beer ~ drift())
  )
beer_fc <- beer_fit |>
  forecast(h = 10)
```

Measures of forecast accuracy

```
accuracy(beer_fit) |>
  arrange(.model) |>
  select(.model, .type, RMSE, MAE, MAPE, MASE, RMSSE)
```

```
## # A tibble: 4 x 7
##   .model          .type     RMSE   MAE  MAPE  MASE RMSSE
##   <chr>           <chr>    <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 Drift           Training  65.3  54.8 12.2   3.83  3.89
## 2 Mean            Training  43.6  35.2  7.89  2.46  2.60
## 3 Naive           Training  65.3  54.7 12.2   3.83  3.89
## 4 Seasonal_naive  Training  16.8  14.3  3.31  1     1
```

Measures of forecast accuracy

```
accuracy(beer_fc, recent_production) |>
  arrange(.model) |>
  select(.model, .type, RMSE, MAE, MAPE, MASE, RMSSE)
```

```
## # A tibble: 4 x 7
##   .model         .type  RMSE   MAE  MAPE  MASE RMSSE
##   <chr>          <chr> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 Drift          Test   64.9  58.9 14.6  4.12  3.87
## 2 Mean           Test   38.4  34.8  8.28 2.44  2.29
## 3 Naive          Test   62.7  57.4 14.2  4.01  3.74
## 4 Seasonal_naive Test   14.3  13.4  3.17 0.937 0.853
```