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FORECASTING

PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**
OPEN TEXTS FOR PRACTICE

9. ARIMA models

9.4 Moving average models

OTexts.org/fpp3/

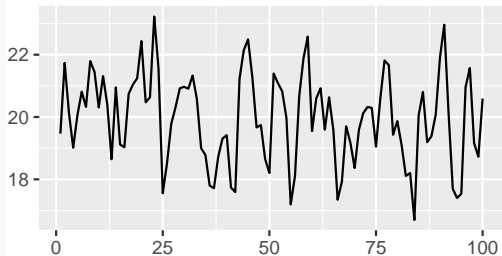
Moving Average (MA) models

Moving Average (MA) models:

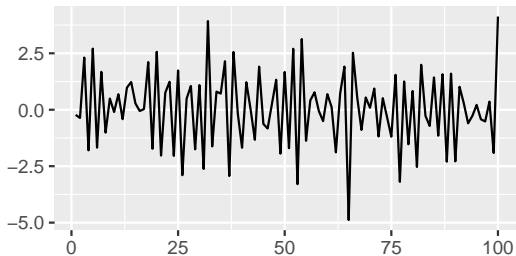
$$y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q},$$

where ε_t is white noise. This is a multiple regression with **past errors** as predictors. *Don't confuse this with moving average smoothing!*

MA(1)



MA(2)

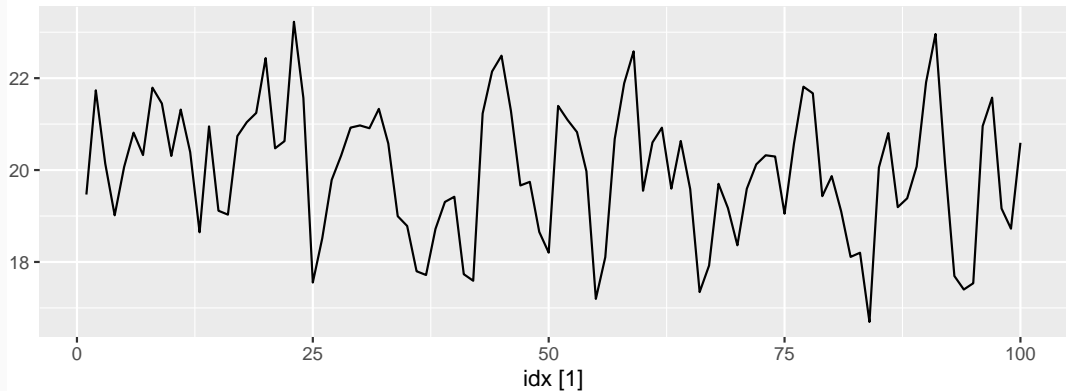


MA(1) model

$$y_t = 20 + \varepsilon_t + 0.8\varepsilon_{t-1}$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$

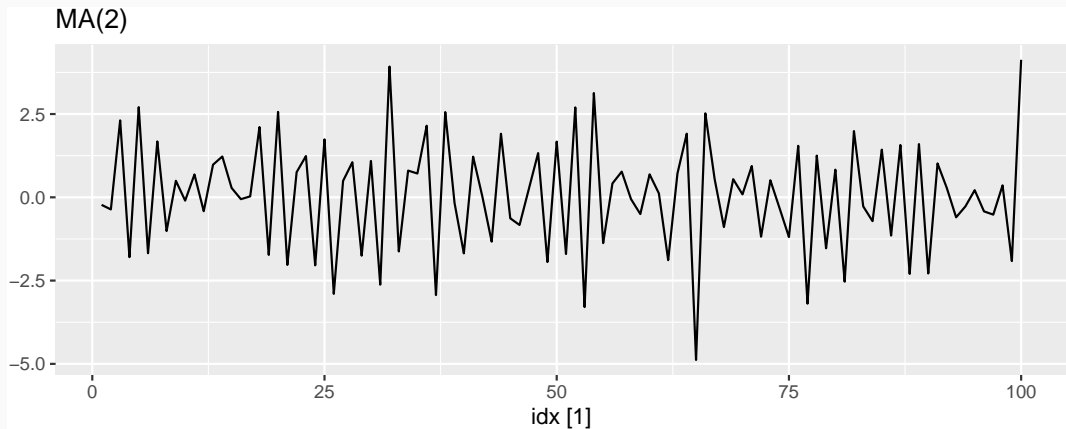
MA(1)



MA(2) model

$$y_t = \varepsilon_t - \varepsilon_{t-1} + 0.8\varepsilon_{t-2}$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$



MA(∞) models

It is possible to write any stationary AR(p) process as an MA(∞) process.

Example: AR(1)

$$\begin{aligned}y_t &= \phi_1 y_{t-1} + \varepsilon_t \\&= \phi_1(\phi_1 y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t \\&= \phi_1^2 y_{t-2} + \phi_1 \varepsilon_{t-1} + \varepsilon_t \\&= \phi_1^3 y_{t-3} + \phi_1^2 \varepsilon_{t-2} + \phi_1 \varepsilon_{t-1} + \varepsilon_t \\&\dots\end{aligned}$$

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Provided $-1 < \phi_1 < 1$:

Invertibility

- Any $MA(q)$ process can be written as an $AR(\infty)$ process if we impose some constraints on the MA parameters.
- Then the MA model is called “invertible”.
- Invertible models have some mathematical properties that make them easier to use in practice.
- Invertibility of an ARIMA model is equivalent to forecastability of an ETS model.

Invertibility

General condition for invertibility

Complex roots of $1 + \theta_1 z + \theta_2 z^2 + \dots + \theta_q z^q$ lie outside the unit circle on the complex plane.

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Complex roots of $1 + \theta_1 z + \theta_2 z^2 + \dots + \theta_q z^q$ lie outside the unit circle on the complex plane.

- For $q = 1$: $-1 < \theta_1 < 1$.
- For $q = 2$: $-1 < \theta_2 < 1$ $\theta_2 + \theta_1 > -1$ $\theta_1 - \theta_2 < 1$.
- More complicated conditions hold for $q \geq 3$.
- Estimation software takes care of this.