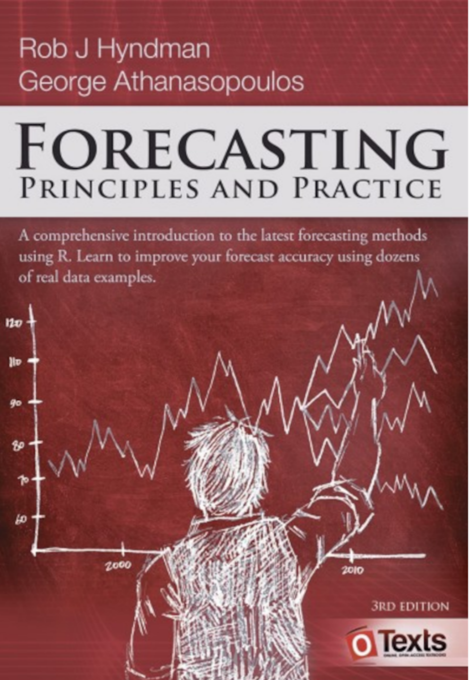

7. Time series regression models

7.5 Selecting predictors

OTexts.org/fpp3/

Comparing regression models

Computer output for regression will always give the R^2 value. This is a useful summary of the model.

- It is equal to the square of the correlation between y and $\hat{y}$.
- It is often called the "coefficient of determination".
- It can also be calculated as follows:

$$R^2 = \frac{\sum(\hat{y}_t - \bar{y})^2}{\sum(y_t - \bar{y})^2}$$

- It is the proportion of variance accounted for (explained) by the predictors.

Comparing regression models

However ...

- R^2 does not allow for “degrees of freedom”.
- Adding *any* variable tends to increase the value of R^2, even if that variable is irrelevant.

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where k = no. predictors and T = no. observations.

Maximizing $\bar{R}^2$ is equivalent to minimizing $\hat{\sigma}^2$.

$$\hat{\sigma}^2 = \frac{1}{T - k - 1}\sum_{t=1}^{T}\varepsilon_t^2$$

Akaike's Information Criterion

$$\text{AIC} = -2\log(L) + 2(k+2)$$

where L is the likelihood and k is the number of predictors in the model.

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- AIC penalizes terms more heavily than $\bar{R}^2$.
- Minimizing the AIC is asymptotically equivalent to minimizing MSE via **leave-one-out cross-validation** (for any linear regression).

Corrected AIC

For small values of T, the AIC tends to select too many predictors, and so a bias-corrected version of the AIC has been developed.

$$\text{AIC}_\text{C} = \text{AIC} + \frac{2(k+2)(k+3)}{T-k-3}$$

As with the AIC, the AIC_C should be minimized.

Bayesian Information Criterion

$$\mathrm{BIC} = -2\log(L) + (k+2)\log(T)$$

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where L is the likelihood and k is the number of predictors in the model.

- BIC penalizes terms more heavily than AIC
- Also called SBIC and SC.
- Minimizing BIC is asymptotically equivalent to leave-v-out cross-validation when $v = T[1 - 1/(log(T) - 1)]$.

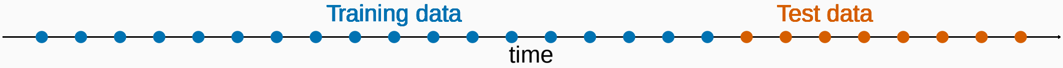
Leave-one-out cross-validation

For regression, leave-one-out cross-validation is faster and more efficient than time-series cross-validation.

- Select one observation for test set, and use *remaining* observations in training set. Compute error on test observation.
- Repeat using each possible observation as the test set.
- Compute accuracy measure over all errors.

Cross-validation

Traditional evaluation



Cross-validation

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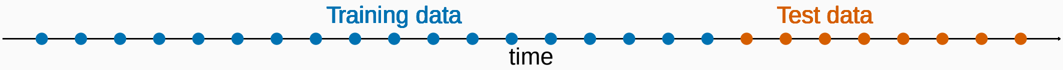
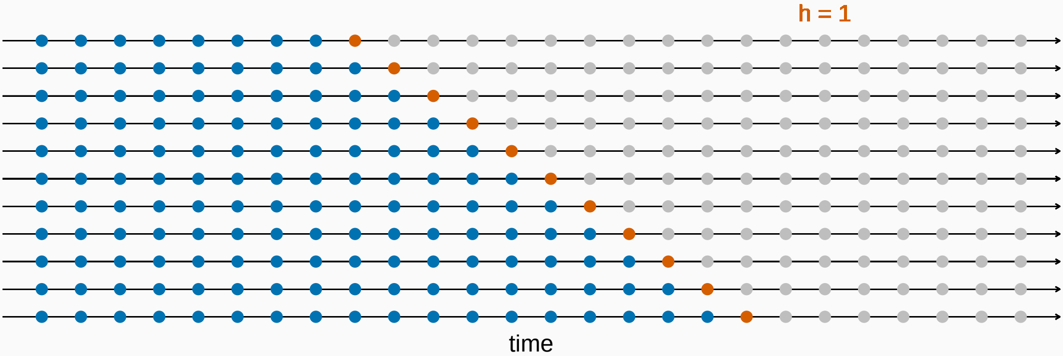


Time series cross-validation



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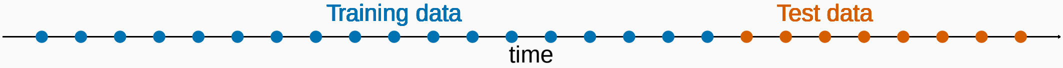
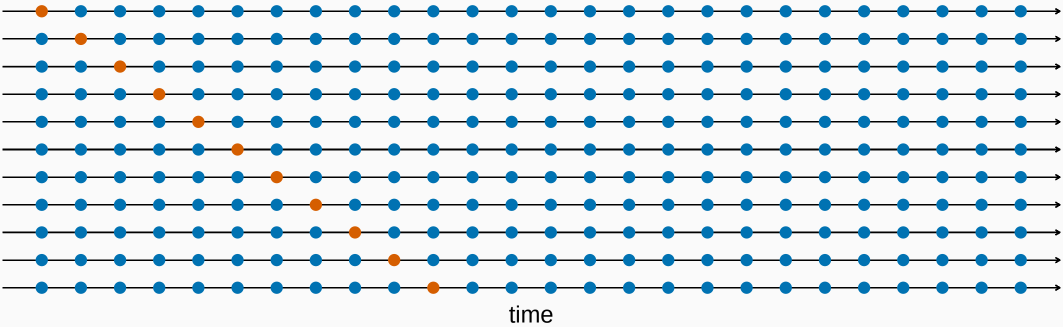


Leave-one-out cross-validation



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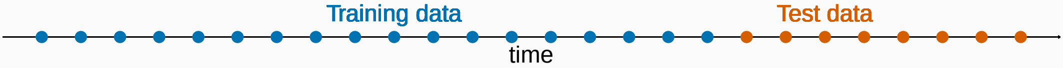
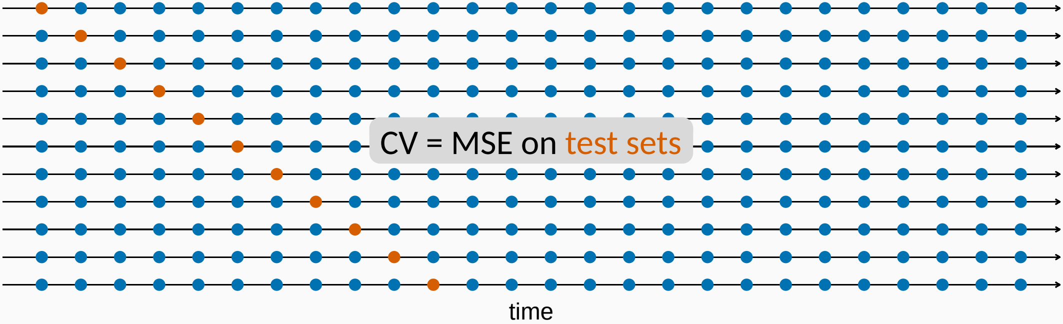


Leave-one-out cross-validation



Choosing regression variables

Best subsets regression

- Fit all possible regression models using one or more of the predictors.
- Choose the best model based on one of the measures of predictive ability (CV, AIC, AICc).

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Warning!

- If there are a large number of predictors, this is not possible.
- For example, 50 predictors leads to over 1 quadrillion possible models!

Choosing regression variables

Backwards stepwise regression

- Start with a model containing all variables.
- Try subtracting one variable at a time. Keep the model if it has lower CV or AICc.
- Iterate until no further improvement.

Choosing regression variables

Backwards stepwise regression

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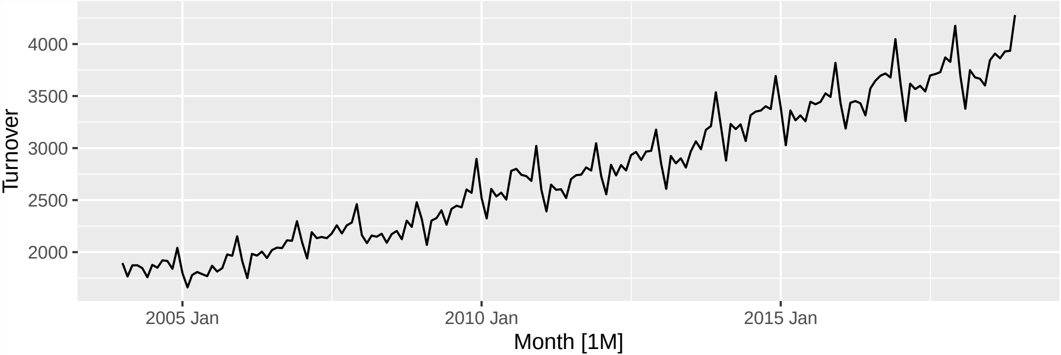
Notes

- Stepwise regression is not guaranteed to lead to the best possible model.
- Inference on coefficients of final model will be wrong.

Harmonic regression: eating-out expenditure

```r
aus_cafe <- aus_retail |>
  filter(Industry == "Cafes, restaurants and takeaway food services",
         year(Month) %in% 2004:2018) |>
  summarise(Turnover = sum(Turnover))
aus_cafe |> autoplot(Turnover)
```



Harmonic regression: eating-out expenditure

```r
fit <- aus_cafe |>
  model(
    K1 = TSLM(log(Turnover) ~ trend() + fourier(K = 1)),
    K2 = TSLM(log(Turnover) ~ trend() + fourier(K = 2)),
    K3 = TSLM(log(Turnover) ~ trend() + fourier(K = 3)),
    K4 = TSLM(log(Turnover) ~ trend() + fourier(K = 4)),
    K5 = TSLM(log(Turnover) ~ trend() + fourier(K = 5)),
    K6 = TSLM(log(Turnover) ~ trend() + fourier(K = 6))
  )
glance(fit) |> select(.model, r_squared, adj_r_squared, CV, AICc)
```

```
## # A tibble: 6 x 5
##   .model r_squared adj_r_squared      CV   AICc
##   <chr>      <dbl>         <dbl>   <dbl>  <dbl>
## 1 K1         0.962         0.962 0.00238 -1085.
## 2 K2         0.966         0.965 0.00220 -1099.
## 3 K3         0.976         0.975 0.00157 -1160.
## 4 K4         0.980         0.979 0.00138 -1183.
## 5 K5         0.985         0.984 0.00104 -1234.
## 6 K6         0.985         0.984 0.00105 -1232.
```

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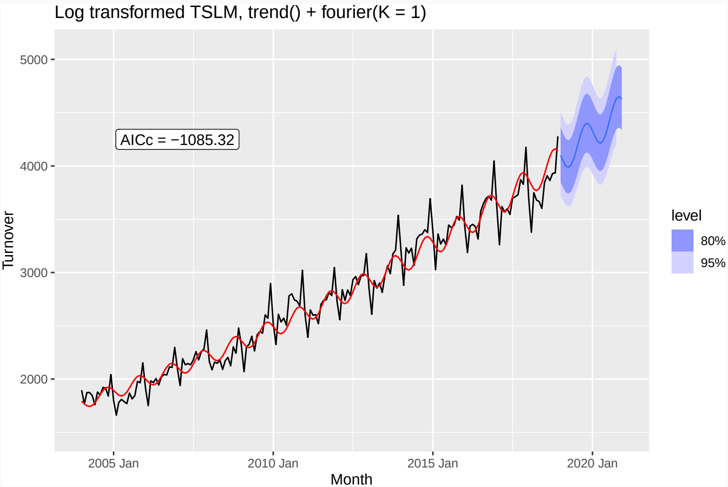

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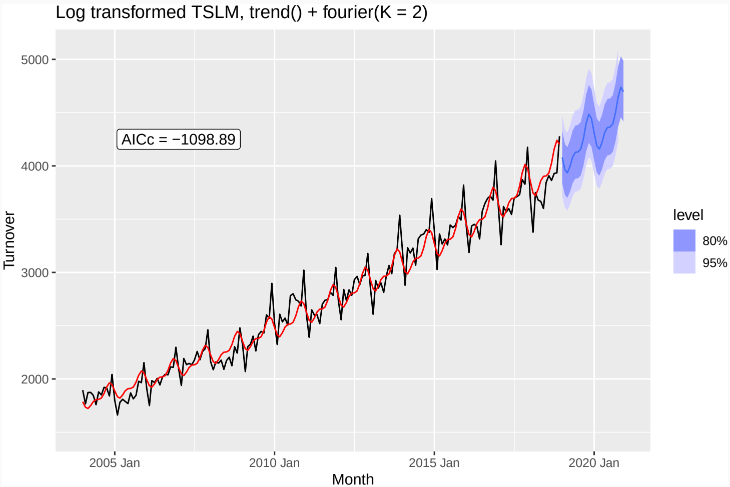

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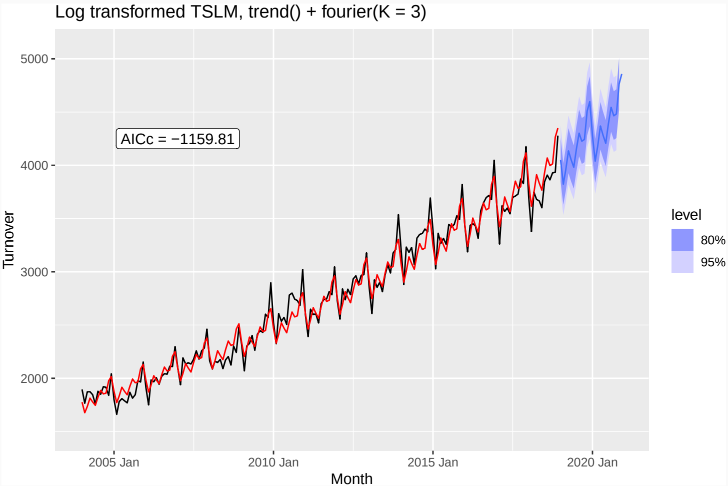
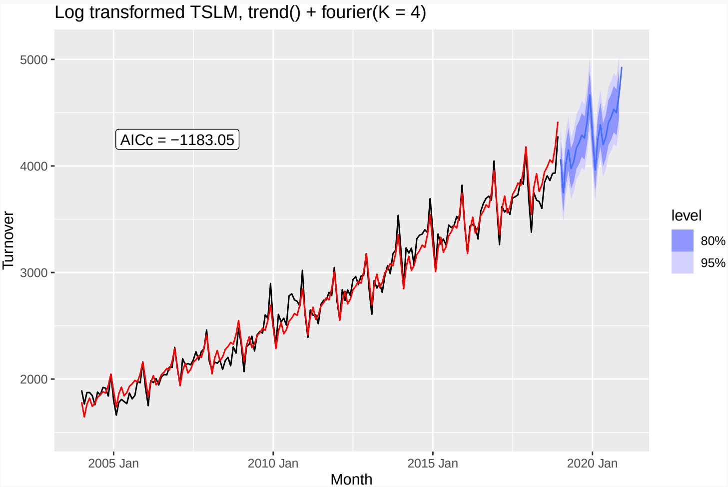


Harmonic regression: eating-out expenditure



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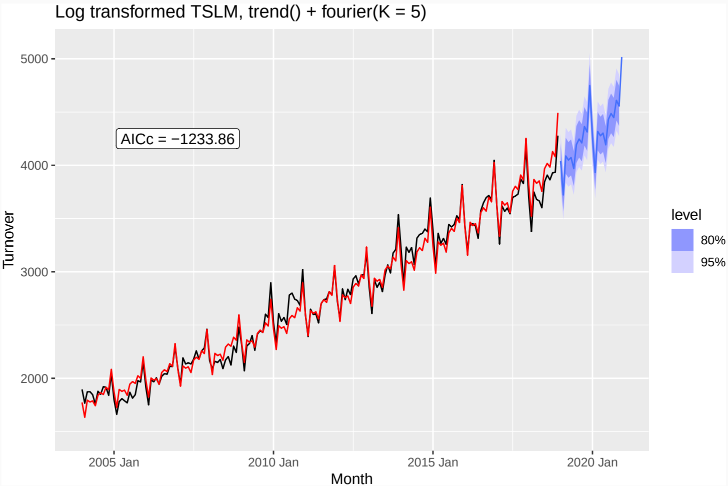

Harmonic regression: eating-out expenditure