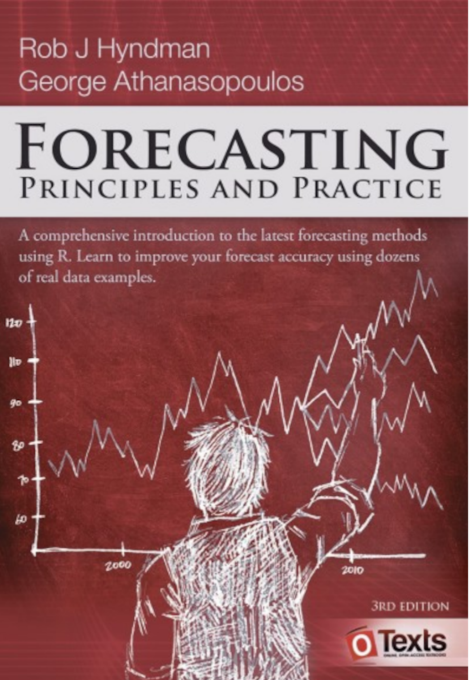


8. Exponential smoothing

8.7 Forecasting with ETS models

OTexts.org/fpp3/

Forecasting with ETS models

Traditional point forecasts: iterate the equations for $t = T + 1, T + 2, \dots, T + h$ and set all $\varepsilon_t = 0$ for $t > T$.

Forecasting with ETS models

Traditional point forecasts: iterate the equations for $t = T+1, T+2, \dots, T+h$ and set all $\varepsilon_t = 0$ for $t > T$.

- Not the same as $\text{E}(y_{t+h}|\boldsymbol{x}_t)$ unless seasonality is additive.
- `fable` uses $\text{E}(y_{t+h}|\boldsymbol{x}_t)$.
- Point forecasts for ETS(A,$*$,$*$) are identical to ETS(M,$*$,$*$) if the parameters are the same.

Example: ETS(A,A,N)

$$
\begin{aligned}
y_{T+1} &= \ell_T + b_T + \varepsilon_{T+1} \\
\hat{y}_{T+1|T} &= \ell_T + b_T \\
y_{T+2} &= \ell_{T+1} + b_{T+1} + \varepsilon_{T+2} \\
&= (\ell_T + b_T + \alpha\varepsilon_{T+1}) + (b_T + \beta\varepsilon_{T+1}) + \varepsilon_{T+2} \\
\hat{y}_{T+2|T} &= \ell_T + 2b_T
\end{aligned}
$$

etc.

Example: ETS(M,A,N)

$$
\begin{aligned}
y_{T+1} &= (\ell_T + b_T)(1 + \varepsilon_{T+1}) \\
\hat{y}_{T+1|T} &= \ell_T + b_T. \\
y_{T+2} &= (\ell_{T+1} + b_{T+1})(1 + \varepsilon_{T+2}) \\
&= \left\{(\ell_T + b_T)(1 + \alpha\varepsilon_{T+1}) + [b_T + \beta(\ell_T + b_T)\varepsilon_{T+1}]\right\}(1 + \varepsilon_{T+2}) \\
\hat{y}_{T+2|T} &= \ell_T + 2b_T
\end{aligned}
$$

etc.

Forecasting with ETS models

Prediction intervals: can only be generated using the models.

- The prediction intervals will differ between models with additive and multiplicative errors.
- Exact formulae for some models.
- More general to simulate future sample paths, conditional on the last estimate of the states, and to obtain prediction intervals from the percentiles of these simulated future paths.

Prediction intervals

PI for most ETS models: $\hat{y}_{T+h|T} \pm c\sigma_h$, where c depends on coverage probability and σ_h is forecast standard deviation.

(A,N,N) $\quad \sigma_h = \sigma^2\Big[1 + \alpha^2(h-1)\Big]$

(A,A,N) $\quad \sigma_h = \sigma^2\Big[1 + (h-1)\{\alpha^2 + \alpha\beta h + \frac{1}{6}\beta^2 h(2h-1)\}\Big]$

(A,A$_d$,N) $\quad \sigma_h = \sigma^2\Big[1 + \alpha^2(h-1) + \frac{\beta\phi h}{(1-\phi)^2}\left\{2\alpha(1-\phi) + \beta\phi\right\} - \frac{\beta\phi(1-\phi^h)}{(1-\phi)^2(1-\phi^2)}\left\{2\alpha(1-\phi^2) + \beta\phi(1+2\phi-\phi^h)\right\}\Big]$

(A,N,A) $\quad \sigma_h = \sigma^2\Big[1 + \alpha^2(h-1) + \gamma k(2\alpha + \gamma)\Big]$

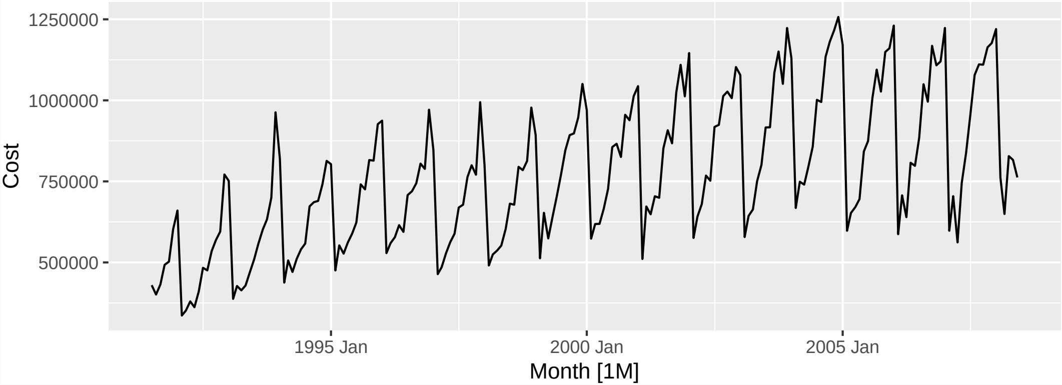
(A,A,A) $\quad \sigma_h = \sigma^2\Big[1 + (h-1)\{\alpha^2 + \alpha\beta h + \frac{1}{6}\beta^2 h(2h-1)\} + \gamma k\{2\alpha + \gamma + \beta m(k+1)\}\Big]$

(A,A$_d$,A) $\quad \sigma_h = \sigma^2\Big[1 + \alpha^2(h-1) + \frac{\beta\phi h}{(1-\phi)^2}\left\{2\alpha(1-\phi) + \beta\phi\right\} - \frac{\beta\phi(1-\phi^h)}{(1-\phi)^2(1-\phi^2)}\left\{2\alpha(1-\phi^2) + \beta\phi(1+2\phi-\phi^h)\right\}$

$\qquad + \gamma k(2\alpha + \gamma) + \frac{2\beta\gamma\phi}{(1-\phi)(1-\phi^m)}\left\{k(1-\phi^m) - \phi^m(1-\phi^{mk})\right\}\Big]$

Example: Corticosteroid drug sales

```r
h02 <- PBS |>
  filter(ATC2 == "H02") |>
  summarise(Cost = sum(Cost))
h02 |> autoplot(Cost)
```



Example: Corticosteroid drug sales

```r
h02 |>
  model(ETS(Cost)) |>
  report()
```

```
## Series: Cost
## Model: ETS(M,Ad,M)
##   Smoothing parameters:
##     alpha = 0.307
##     beta  = 0.000101
##     gamma = 0.000101
##     phi   = 0.978
##
##   Initial states:
##    l[0] b[0]  s[0] s[-1] s[-2] s[-3] s[-4] s[-5] s[-6] s[-7] s[-8] s[-9]
##  417269 8206 0.872 0.826 0.756 0.773 0.687  1.28  1.32  1.18  1.16   1.1
##  s[-10] s[-11]
##    1.05  0.981
##
##   sigma^2:  0.0046
##
##  AIC AICc  BIC
## 5515 5519 5575
```

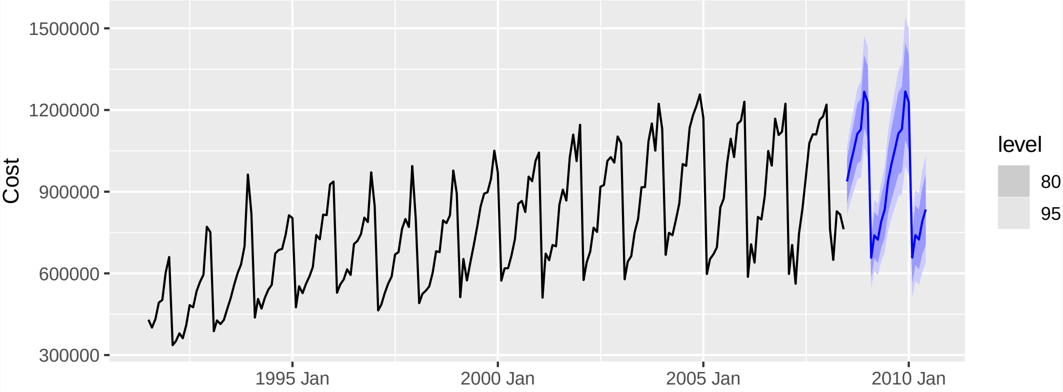
Example: Corticosteroid drug sales

```r
h02 |>
  model(ETS(Cost ~ error("A") + trend("A") + season("A"))) |>
  report()
```

```
## Series: Cost
## Model: ETS(A,A,A)
##   Smoothing parameters:
##     alpha = 0.17
##     beta  = 0.00631
##     gamma = 0.455
##
##   Initial states:
##    l[0] b[0]   s[0]   s[-1]   s[-2]   s[-3]   s[-4]  s[-5]  s[-6]  s[-7]
##   409706 9097 -99075 -136602 -191496 -174531 -241437 210644 244644 145368
##   s[-8] s[-9] s[-10] s[-11]
##   130570 84458  39132 -11674
##
##   sigma^2:  3.5e+09
##
##   AIC AICc  BIC
## 5585 5589 5642
```

Example: Corticosteroid drug sales

```r
h02 |>
  model(ETS(Cost)) |>
  forecast() |>
  autoplot(h02)
```



Example: Corticosteroid drug sales

```r
h02 |>
  model(
    auto = ETS(Cost),
    AAA = ETS(Cost ~ error("A") + trend("A") + season("A"))
  ) |>
  accuracy()
```

```
## # A tibble: 2 x 10
##   .model .type         ME   RMSE    MAE     MPE  MAPE  MASE RMSSE    ACF1
##   <chr>  <chr>      <dbl>  <dbl>  <dbl>   <dbl> <dbl> <dbl> <dbl>   <dbl>
## 1 auto   Training  2461. 51102. 38649. -0.0127  4.99 0.638 0.689 -0.0958
## 2 AAA    Training -5780. 56784. 43378. -1.30    6.05 0.716 0.766  0.0258
```