

Rob J Hyndman  
George Athanasopoulos

# FORECASTING

## PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**  
OPEN TEXTS FOR PRACTICE

## 8. Exponential smoothing

### 8.3 Methods with seasonality

[OTexts.org/fpp3/](http://OTexts.org/fpp3/)

# Holt-Winters additive method

Holt and Winters extended Holt's method to capture seasonality.

## Component form

$$\hat{y}_{t+h|t} = \ell_t + hb_t + s_{t+h-m(k+1)}$$

$$\ell_t = \alpha(y_t - s_{t-m}) + (1 - \alpha)(\ell_{t-1} + b_{t-1})$$

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$$

$$s_t = \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1 - \gamma)s_{t-m}$$

- $k = \text{integer part of } (h - 1)/m$ . Ensures estimates from the final year are used for forecasting.
- Parameters:  $0 \leq \alpha \leq 1$ ,  $0 \leq \beta^* \leq 1$ ,  $0 \leq \gamma \leq 1 - \alpha$  and  $m = \text{period of seasonality}$  (e.g.  $m = 4$  for quarterly data).

# Holt-Winters additive method

- Seasonal component is usually expressed as

$$s_t = \gamma^*(y_t - \ell_t) + (1 - \gamma^*)s_{t-m}.$$

- Substitute in for  $\ell_t$ :

$$s_t = \gamma^*(1 - \alpha)(y_t - \ell_{t-1} - b_{t-1}) + [1 - \gamma^*(1 - \alpha)]s_{t-m}$$

- We set  $\gamma = \gamma^*(1 - \alpha)$ .
- The usual parameter restriction is  $0 \leq \gamma^* \leq 1$ , which translates to  $0 \leq \gamma \leq (1 - \alpha)$ .

# Exponential smoothing: seasonality

# Holt-Winters multiplicative method

Seasonal variations change in proportion to the level of the series.

## Component form

$$\hat{y}_{t+h|t} = (\ell_t + hb_t)s_{t+h-m(k+1)}$$

$$\ell_t = \alpha \frac{y_t}{s_{t-m}} + (1 - \alpha)(\ell_{t-1} + b_{t-1})$$

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$$

$$s_t = \gamma \frac{y_t}{(\ell_{t-1} + b_{t-1})} + (1 - \gamma)s_{t-m}$$

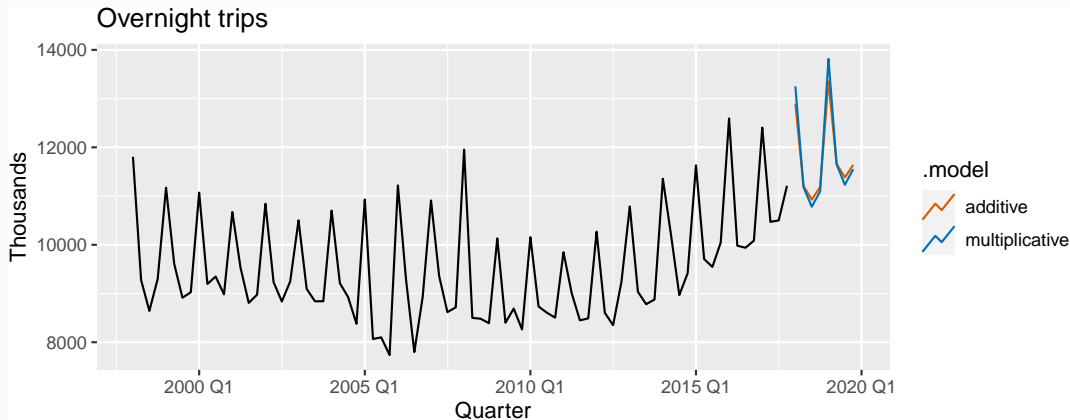
- $k$  is integer part of  $(h - 1)/m$ .
- Additive method:  $s_t$  in absolute terms — within each year  $\sum_i s_i \approx 0$ .
- Multiplicative method:  $s_t$  in relative terms — within each year  $\sum_i s_i \approx m$ .

# Example: Australian holiday tourism

```
aus_holidays <- tourism |>
  filter(Purpose == "Holiday") |>
  summarise(Trips = sum(Trips))
fit <- aus_holidays |>
  model(
    additive = ETS(Trips ~ error("A") + trend("A") + season("A")),
    multiplicative = ETS(Trips ~ error("M") + trend("A") + season("M"))
  )
fc <- fit |> forecast()
```

# Example: Australian holiday tourism

```
fc |>  
  autoplot(aus_holidays, level = NULL) +  
  labs(y = "Thousands", title = "Overnight trips")
```



# Estimated components

```
components(fit)
```

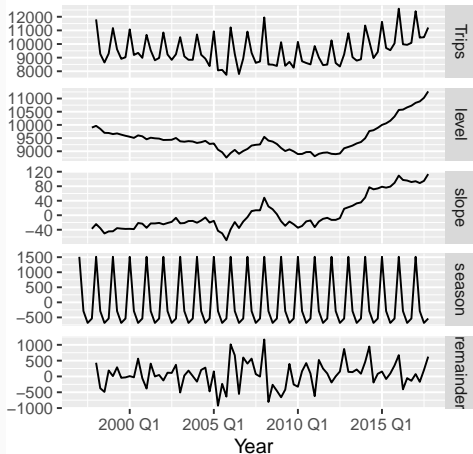
```
## # A dable: 168 x 7 [1Q]
## # Key:      .model [2]
## # :      Trips = lag(level, 1) + lag(slope, 1) + lag(season, 4) +
## # remainder
```

##	.model	Quarter	Trips	level	slope	season	remainder
##	<chr>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
##	1 additive	1997 Q1	NA	NA	NA	1512.	NA
##	2 additive	1997 Q2	NA	NA	NA	-290.	NA
##	3 additive	1997 Q3	NA	NA	NA	-684.	NA
##	4 additive	1997 Q4	NA	9899.	-37.4	-538.	NA
##	5 additive	1998 Q1	11806.	9964.	-24.5	1512.	433.
##	6 additive	1998 Q2	9276.	9851.	-35.6	-290.	-374.
##	7 additive	1998 Q3	8642.	9700.	-50.2	-684.	-489.
##	8 additive	1998 Q4	9300.	9694.	-44.6	-538.	188.

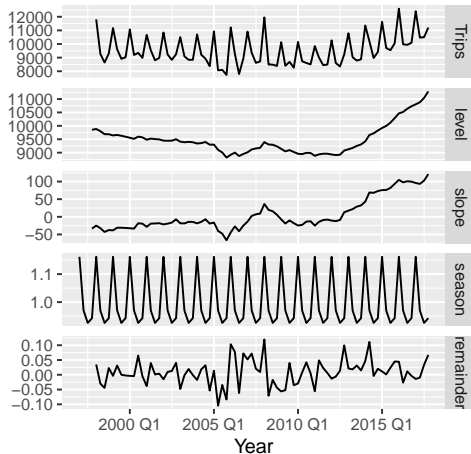


# Estimated components

## Additive states



## Multiplicative states



# Holt-Winters damped method

Often the single most accurate forecasting method for seasonal data:

$$\hat{y}_{t+h|t} = [\ell_t + (\phi + \phi^2 + \dots + \phi^h)b_t]s_{t+h-m(k+1)}$$

$$\ell_t = \alpha(y_t/s_{t-m}) + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$$

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)\phi b_{t-1}$$

$$s_t = \gamma \frac{y_t}{(\ell_{t-1} + \phi b_{t-1})} + (1 - \gamma)s_{t-m}$$

# Holt-Winters with daily data

```
sth_cross_ped <- pedestrian |>
  filter(
    Date >= "2016-07-01",
    Sensor == "Southern Cross Station"
  ) |>
  index_by(Date) |>
  summarise(Count = sum(Count) / 1000)
sth_cross_ped |>
  filter(Date <= "2016-07-31") |>
  model(hw = ETS(Count ~ error("M") + trend("Ad") + season("M"))) |>
  forecast(h = "2 weeks") |>
  autoplot(sth_cross_ped |> filter(Date <= "2016-08-14")) +
  labs(
    title = "Daily traffic: Southern Cross",
    y = "Pedestrians ('000)"
  )
```

# Holt-Winters with daily data

