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FORECASTING

PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**
Open Texts Publishing

3. Time series decomposition

3.4 Classical decomposition

OTexts.org/fpp3/

Trend-cycle

Additive decomposition: $y_t = T_t + S_t + R_t = \hat{T}_t + \hat{S}_t + \hat{R}_t$

Multiplicative decomposition: $y_t = T_t \times S_t \times R_t = \hat{T}_t \times \hat{S}_t \times \hat{R}_t$

Trend-cycle

Additive decomposition: $y_t = T_t + S_t + R_t = \hat{T}_t + \hat{S}_t + \hat{R}_t$

Multiplicative decomposition: $y_t = T_t \times S_t \times R_t = \hat{T}_t \times \hat{S}_t \times \hat{R}_t$

- Estimate \hat{T} using $(2 \times m)$ -MA if m is even. Otherwise, estimate \hat{T} using m -MA

Compute de-trended series

- Additive decomposition: $y_t - \hat{T}_t$
- Multiplicative decomposition: y_t / \hat{T}_t

De-trending

Remove smoothed series \hat{T}_t from y_t to leave S_t and R_t .

- Additive model: $y_t - \hat{T}_t = (\hat{T}_t + \hat{S}_t + \hat{R}_t) - \hat{T}_t = \hat{S}_t + \hat{R}_t$
- Multiplicative model: $\frac{y_t}{\hat{T}_t} = \frac{\hat{T}_t \times \hat{S}_t \times \hat{R}_t}{\hat{T}_t} = \hat{S}_t \times \hat{R}_t$

Estimating seasonal component

- Seasonal index for each season is estimated as an **average** of the detrended series for that season of successive years.
- E.g., take averages across all Januaries to get $S^{(1)}$ if your data is monthly.
- If necessary, adjust the seasonal indices so that:
 - ▶ for additive: $S^{(1)} + S^{(2)} + \dots + S^{(12)} = 0$
 - ▶ for multiplicative: $S^{(1)} + S^{(2)} + \dots + S^{(12)} = m$
- The seasonal component \hat{S}_t simply consists of replications of the seasonal indices.

Remainder component

Additive decomposition: $\hat{R}_t = y_t - \hat{T}_t - \hat{S}_t$

Multiplicative decomposition: $\hat{R}_t = y_t / (\hat{T}_t \hat{S}_t)$

Remainder component

Additive decomposition: $\hat{R}_t = y_t - \hat{T}_t - \hat{S}_t$

Multiplicative decomposition: $\hat{R}_t = y_t / (\hat{T}_t \hat{S}_t)$

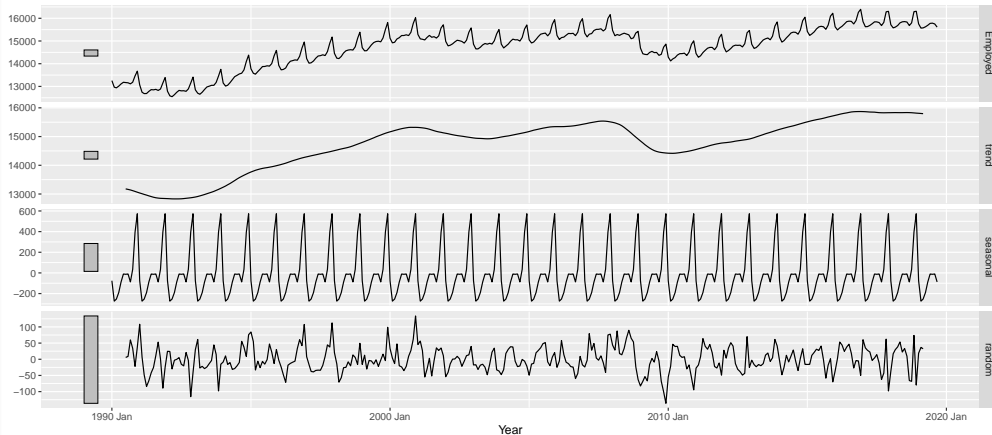
Classical decomposition

- Choose additive or multiplicative depending on which gives the most stable components.
- For multiplicative model, this method of estimation is known as **ratio-to-moving-average method**.

US Retail Employment

Classical additive decomposition of total US retail employment

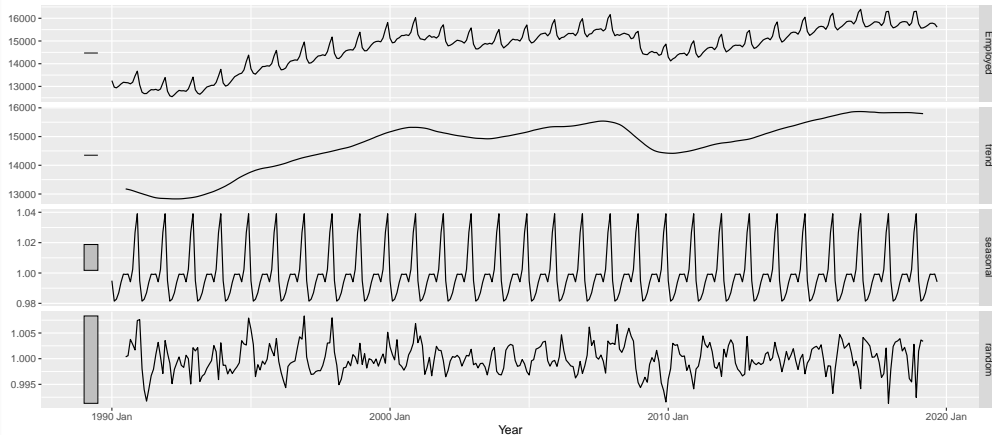
Employed = trend + seasonal + random



US Retail Employment

Classical multiplicative decomposition of total US retail employment

$\text{Employed} = \text{trend} * \text{seasonal} * \text{random}$



US Retail Employment

```
us_retail_employment |>
  model(classical_decomposition(Employed, type = "additive")) |>
  components() |>
  autoplot() + xlab("Year") +
  ggtitle("Classical additive decomposition of total
          US retail employment")
```

Comments on classical decomposition

- Estimate of trend is **unavailable** for first few and last few observations.
- **Seasonal component repeats** from year to year. May not be realistic.
- **Not robust to outliers.**
- Newer methods designed to overcome these problems.