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FORECASTING PRINCIPLES AND PRACTICE



9. ARIMA models

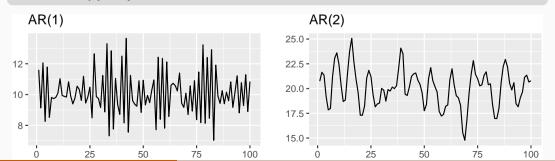
9.3 Autoregressive modelsOTexts.org/fpp3/

Autoregressive models

Autoregressive (AR) models:

$$\mathbf{y}_t = \mathbf{c} + \phi_1 \mathbf{y}_{t-1} + \phi_2 \mathbf{y}_{t-2} + \cdots + \phi_p \mathbf{y}_{t-p} + \varepsilon_t,$$

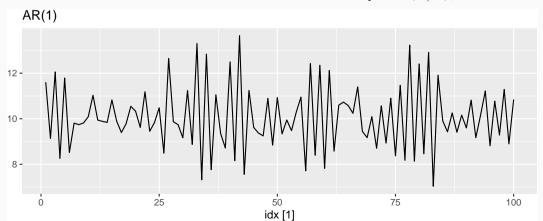
where ε_t is white noise. This is a multiple regression with **lagged** values of y_t as predictors.



AR(1) model

$$y_t = 18 - 0.8y_{t-1} + \varepsilon_t$$

 $\varepsilon_t \sim N(0, 1), \quad T = 100.$



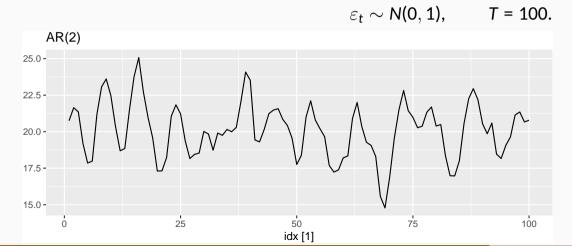
AR(1) model

$$\mathbf{y}_t = \mathbf{c} + \phi_1 \mathbf{y}_{t-1} + \varepsilon_t$$

- When ϕ_1 = 0, y_t is equivalent to WN
- When ϕ_1 = 1 and c = 0, y_t is equivalent to a RW
- When ϕ_1 = 1 and $c \neq 0$, y_t is **equivalent to a RW with drift**
- When ϕ_1 < 0, y_t tends to oscillate between positive and negative values.

AR(2) model

$$y_t = 8 + 1.3y_{t-1} - 0.7y_{t-2} + \varepsilon_t$$



Stationarity conditions

We normally restrict autoregressive models to stationary data, and then some constraints on the values of the parameters are required.

General condition for stationarity

Complex roots of $1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_p z^p$ lie outside the unit circle on the complex plane.

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Complex roots of $1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_p z^p$ lie outside the unit circle on the complex plane.

- For p = 1: $-1 < \phi_1 < 1$.
- For p = 2: $-1 < \phi_2 < 1$ $\phi_2 + \phi_1 < 1$ $\phi_2 \phi_1 < 1$.
- More complicated conditions hold for $p \ge 3$.
- Estimation software takes care of this.