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FORECASTING

PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**
OPEN TEXTS FOR PRACTICE

5. The forecaster's toolbox

5.8 Evaluating point forecast accuracy

OTexts.org/fpp3/

Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Forecast errors

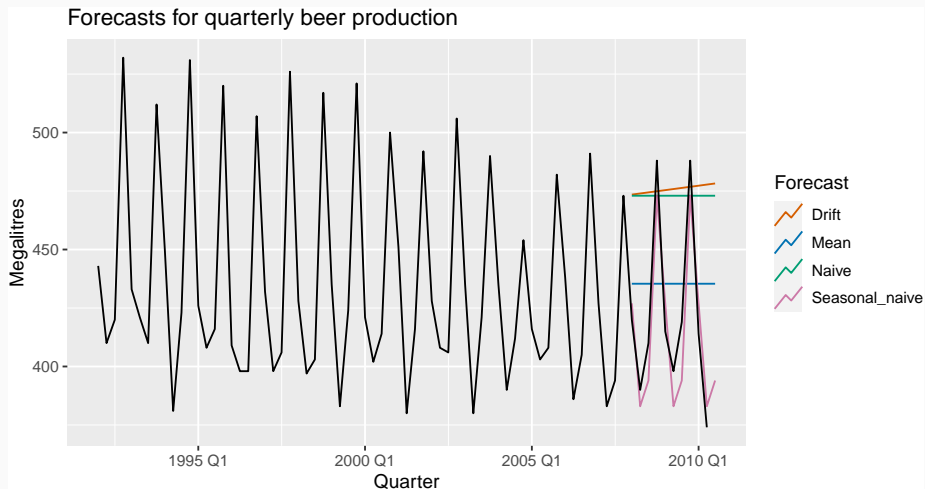
Forecast “error”: the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing $\hat{y}_{T+h|T}$.

Measures of forecast accuracy



Measures of forecast accuracy

y_{T+h} = $(T + h)$ th observation, $h = 1, \dots, H$

$\hat{y}_{T+h|T}$ = its forecast based on data up to time T .

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T}$$

$$\text{MAE} = \text{mean}(|e_{T+h}|)$$

$$\text{MSE} = \text{mean}(e_{T+h}^2)$$

$$\text{MAPE} = 100\text{mean}(|e_{T+h}|/|y_{T+h}|)$$

$$\text{RMSE} = \sqrt{\text{mean}(e_{T+h}^2)}$$

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- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t , and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = (T - 1)^{-1} \sum_{t=2}^T |y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve

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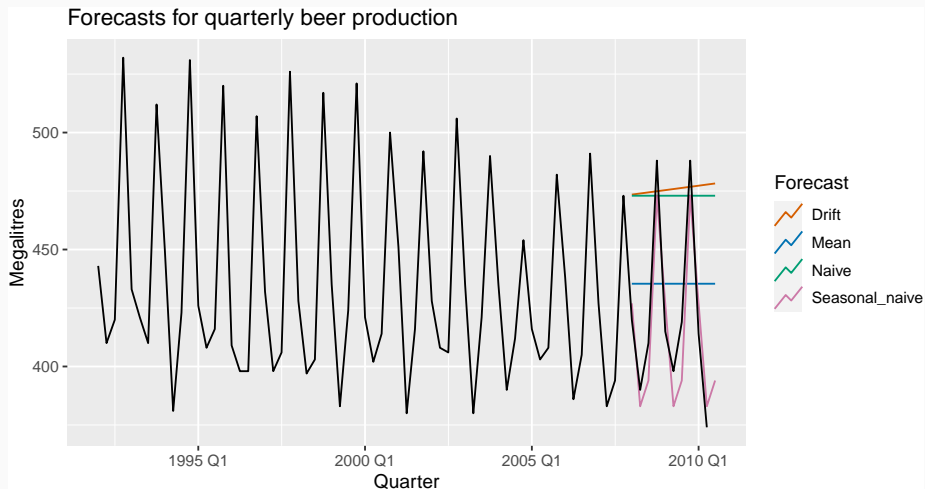
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For seasonal time series,

$$Q = (T - m)^{-1} \sum_{t=m+1}^T |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal

Measures of forecast accuracy



Measures of forecast accuracy

```
recent_production <- aus_production |>
  filter(year(Quarter) >= 1992)
train <- recent_production |>
  filter(year(Quarter) <= 2007)
beer_fit <- train |>
  model(
    Mean = MEAN(Beer),
    Naive = NAIVE(Beer),
    Seasonal_naive = SNAIVE(Beer),
    Drift = RW(Beer ~ drift())
  )
beer_fc <- beer_fit |>
  forecast(h = 10)
```

Measures of forecast accuracy

```
accuracy(beer_fit)
```

```
## # A tibble: 4 x 6
##   .model      .type    RMSE    MAE  MAPE  MASE
##   <chr>      <chr>    <dbl> <dbl> <dbl> <dbl>
## 1 Drift      Training  65.3   54.8  12.2   3.83
## 2 Mean      Training  43.6   35.2   7.89   2.46
## 3 Naive     Training  65.3   54.7  12.2   3.83
## 4 Seasonal_naive Training  16.8   14.3   3.31   1
```

```
accuracy(beer_fc, recent_production)
```

```
## # A tibble: 4 x 6
##   .model      .type    RMSE    MAE  MAPE  MASE
##   <chr>      <chr>    <dbl> <dbl> <dbl> <dbl>
## 1 Drift      Test     64.9   58.9  14.6   4.12
## 2 Mean      Test     38.4   34.8   8.28   2.44
## 3 Naive     Test     62.7   57.4  14.2   4.01
## 4 Seasonal_naive Test     14.3   13.4   3.17  0.937
```