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# FORECASTING

## PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**  
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## 9. ARIMA models

### 9.3 Autoregressive models

[OTexts.org/fpp3/](http://OTexts.org/fpp3/)

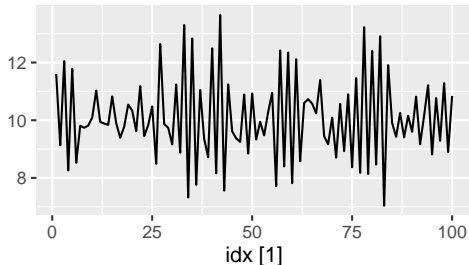
# Autoregressive models

## Autoregressive model - AR( $p$ ):

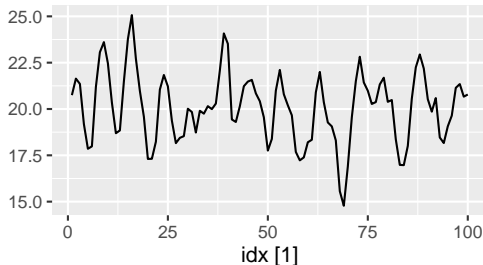
$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t,$$

where  $\varepsilon_t$  is white noise. This is a multiple regression with **lagged values** of  $y_t$  as predictors.

AR(1)



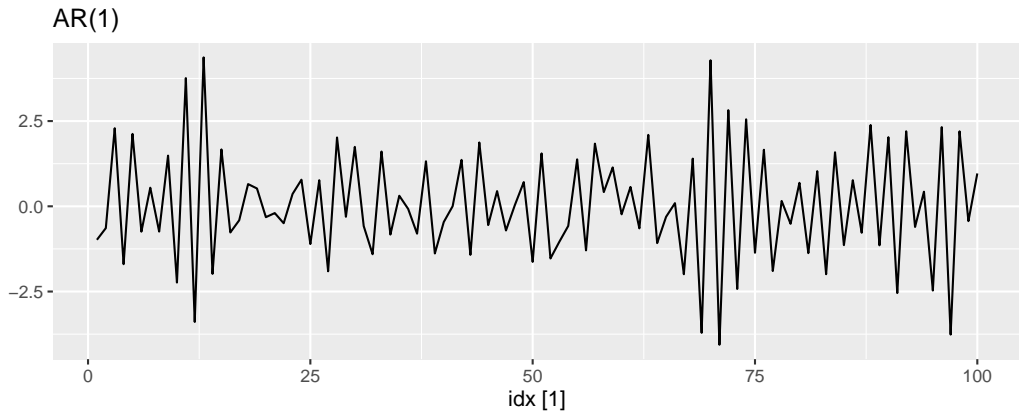
AR(2)



# AR(1) model

$$y_t = -0.8y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$



# AR(1) model

$$y_t = \phi_1 y_{t-1} + \varepsilon_t$$

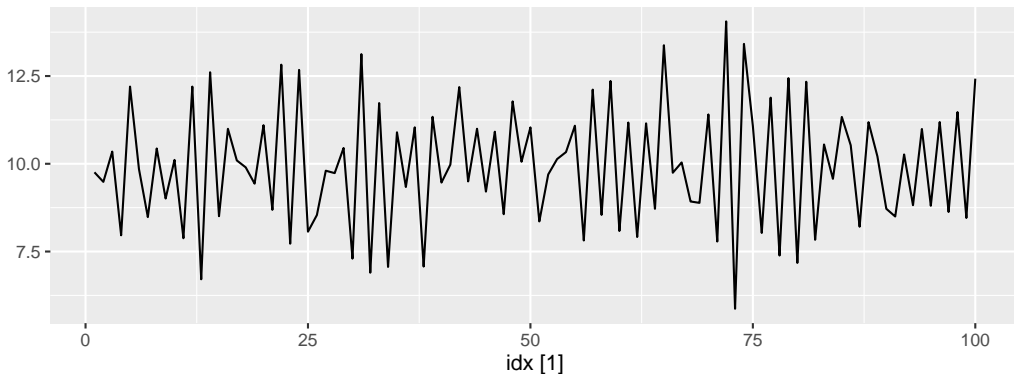
- When  $\phi_1 = 0$ ,  $y_t$  is equivalent to a WN
- When  $\phi_1 = 1$ ,  $y_t$  is equivalent to a RW
- We require  $|\phi_1| < 1$  for stationarity. The closer  $\phi_1$  is to the bounds the more the process wanders above or below it's unconditional mean (zero in this case).
- When  $\phi_1 < 0$ ,  $y_t$  tends to oscillate between positive and negative values.

# AR(1) model including a constant

$$y_t = 18 - 0.8y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$

AR(1)



# AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- When  $\phi_1 = 0$  and  $c = 0$ ,  $y_t$  is equivalent to WN;
- When  $\phi_1 = 1$  and  $c = 0$ ,  $y_t$  is equivalent to a RW;
- When  $\phi_1 = 1$  and  $c \neq 0$ ,  $y_t$  is equivalent to a RW with drift;

# AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- $c$  is related to the mean of  $y_t$ .
  - Let  $E(y_t) = \mu$  then  $c = \mu \times (1 - \phi_1)$ .
- 
- `ARIMA()` takes care of whether you need a constant or not, or you can override it.

# AR(1) model including a constant

- If included estimated model returns w/ mean

Series: sim

Model: ARIMA(1,0,0) w/ mean

Coefficients:

	ar1	constant
	-0.8381	18.3527
s.e.	0.0540	0.1048

sigma<sup>2</sup> estimated as 1.11: log likelihood=-146.7

AIC=299.4 AICc=299.7 BIC=307.2

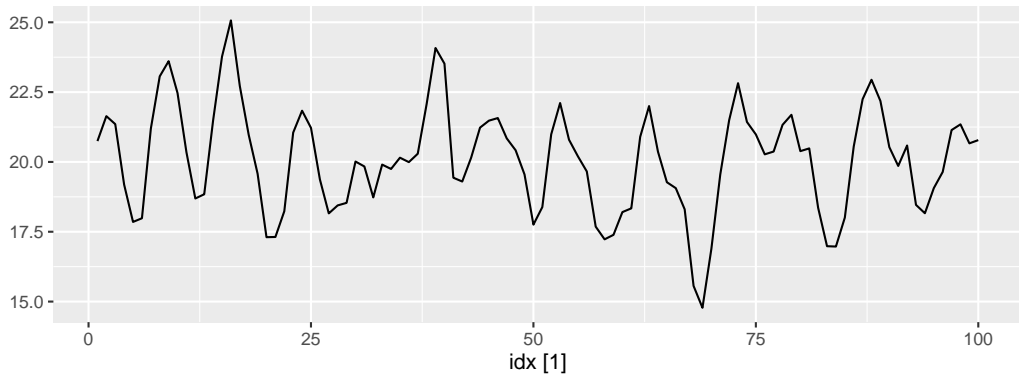


# AR(2) model

$$y_t = 8 + 1.3y_{t-1} - 0.7y_{t-2} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$

AR(2)



# Stationarity conditions

We normally restrict autoregressive models to stationary data, and then some constraints on the values of the parameters are required.

## General condition for stationarity

Complex roots of  $1 - \phi_1 z - \phi_2 z^2 - \dots - \phi_p z^p$  lie outside the unit circle on the complex plane.

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We normally restrict autoregressive models to stationary data, and then some constraints on the values of the parameters are required.

## General condition for stationarity

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- For  $p = 1$ :  $-1 < \phi_1 < 1$ .
- For  $p = 2$ :  
 $-1 < \phi_2 < 1 \quad \phi_2 + \phi_1 < 1 \quad \phi_2 - \phi_1 < 1$ .
- More complicated conditions hold for  $p \geq 3$ .
- Estimation software takes care of this.