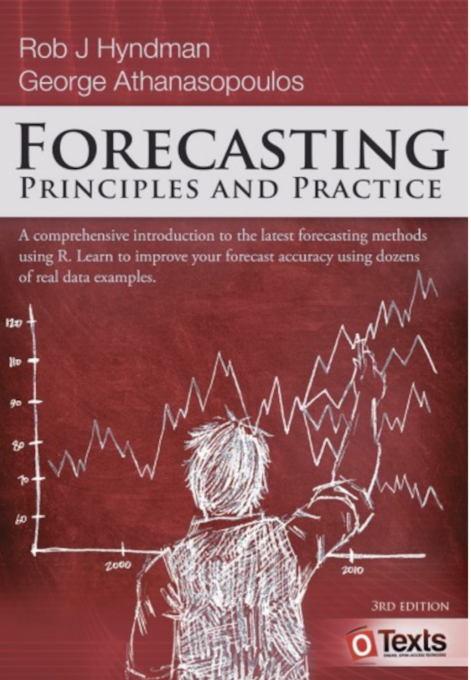


3. Time series decomposition

3.4 Classical decomposition

OTexts.org/fpp3/

Trend-cycle

Additive decomposition: $y_t = T_t + S_t + R_t = \hat{T}_t + \hat{S}_t + \hat{R}_t$

Multiplicative decomposition: $y_t = T_t \times S_t \times R_t = \hat{T}_t \times \hat{S}_t \times \hat{R}_t$

Trend-cycle

Additive decomposition: $y_t = T_t + S_t + R_t = \hat{T}_t + \hat{S}_t + \hat{R}_t$
Multiplicative decomposition: $y_t = T_t \times S_t \times R_t = \hat{T}_t \times \hat{S}_t \times \hat{R}_t$

- Estimate $\hat{T}$ using $(2 \times m)$-MA if m is even. Otherwise, estimate $\hat{T}$ using m-MA

Compute de-trended series

- Additive decomposition: $y_t - \hat{T}_t$
- Multiplicative decomposition: $y_t / \hat{T}_t$

De-trending

Remove smoothed series $\hat{T}_t$ from y_t to leave S_t and R_t.

- Additive model: $y_t - \hat{T}_t = (\hat{T}_t + \hat{S}_t + \hat{R}_t) - \hat{T}_t = \hat{S}_t + \hat{R}_t$
- Multiplicative model: $\frac{y_t}{\hat{T}_t} = \frac{\hat{T}_t \times \hat{S}_t \times \hat{R}_t}{\hat{T}_t} = \hat{S}_t \times \hat{R}_t$

Estimating seasonal component

- Seasonal index for each season is estimated as an average of the detrended series for that season of successive years.
- E.g., take averages across all Januaries to get $S^{(1)}$ if your data is monthly.
- If necessary, adjust the seasonal indices so that:
  - for additive: $S^{(1)} + S^{(2)} + \ldots + S^{(12)} = 0$
  - for multiplicative: $S^{(1)} + S^{(2)} + \ldots + S^{(12)} = m$
- The seasonal component $\hat{S}_t$ simply consists of replications of the seasonal indices.

Remainder component

Additive decomposition: $\hat{R}_t = y_t - \hat{T}_t - \hat{S}_t$

Multiplicative decomposition: $\hat{R}_t = y_t / (\hat{T}_t \hat{S}_t)$

Remainder component

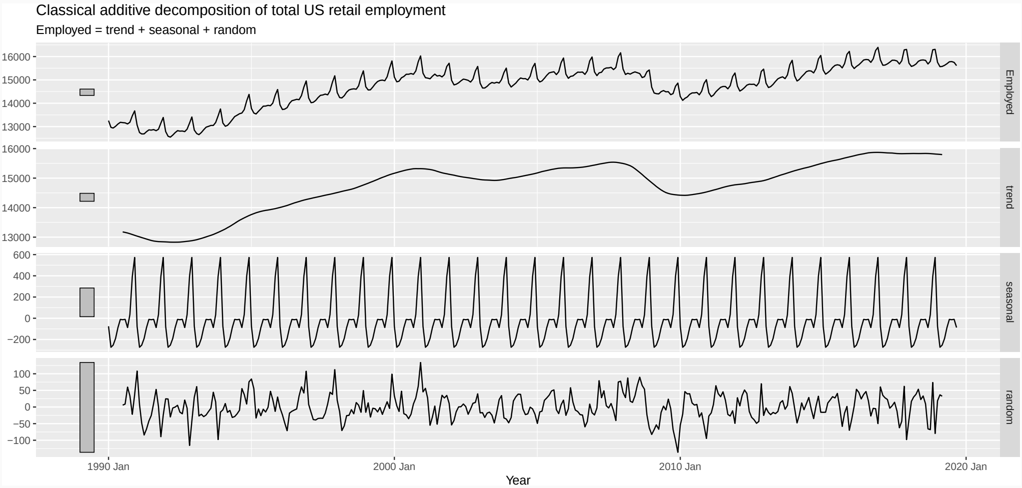
Additive decomposition: $\hat{R}_t = y_t - \hat{T}_t - \hat{S}_t$

Multiplicative decomposition: $\hat{R}_t = y_t / (\hat{T}_t \hat{S}_t)$

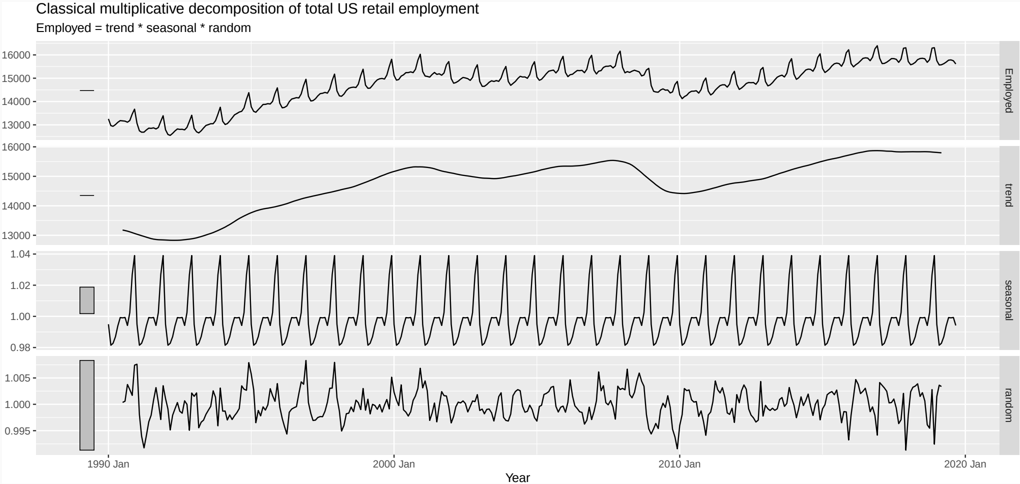
Classical decomposition

- Choose additive or multiplicative depending on which gives the most stable components.
- For multiplicative model, this method of estimation is known as ratio-to-moving-average method.

US Retail Employment



US Retail Employment



US Retail Employment

```r
us_retail_employment |>
  model(classical_decomposition(Employed, type = "additive")) |>
  components() |>
  autoplot() + xlab("Year") +
  ggtitle("Classical additive decomposition of total
          US retail employment")
```

Comments on classical decomposition

- Estimate of trend is unavailable for first few and last few observations.
- Seasonal component repeats from year to year. May not be realistic.
- Not robust to outliers.
- Newer methods designed to overcome these problems.