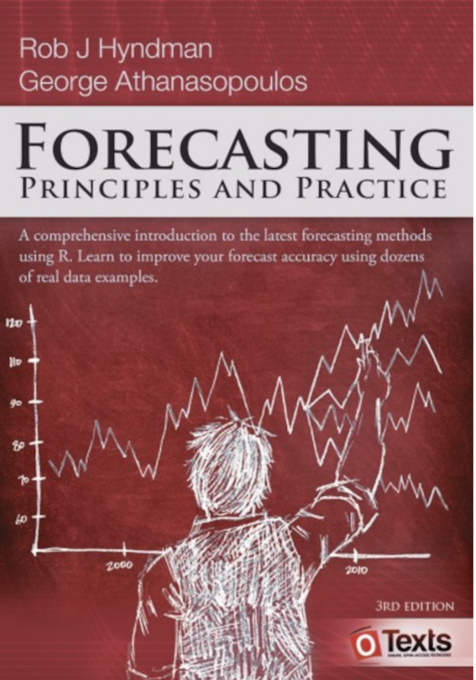

9. ARIMA models

9.2 Backshift notation

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For monthly data, if we wish to shift attention to "the same month last year", then B^{12} is used, and the notation is $B^{12}y_t = y_{t-12}$.

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Similarly, if second-order differences (i.e., first differences of first differences) have to be computed, then:

$$y''_t = y_t - 2y_{t-1} + y_{t-2} = (1 - B)^2 y_t$$

Backshift notation

- Second-order difference is denoted $(1 - B)^2$.
- *Second-order difference* is not the same as a *second difference*, which would be denoted $1 - B^2$;
- In general, a dth-order difference can be written as

$$(1 - B)^d y_t$$

- A seasonal difference followed by a first difference can be written as

$$(1 - B)(1 - B^m) y_t$$

Backshift notation

The “backshift” notation is convenient because the terms can be multiplied together to see the combined effect.

$$
\begin{aligned}
(1-B)(1-B^m)y_t &= (1-B-B^m+B^{m+1})y_t \\
&= y_t - y_{t-1} - y_{t-m} + y_{t-m-1}.
\end{aligned}
$$

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For monthly data, $m = 12$ and we obtain the same result as earlier.