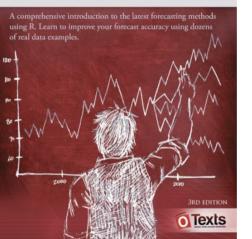
Rob J Hyndman George Athanasopoulos

FORECASTING PRINCIPLES AND PRACTICE



5. The forecaster's toolbox

5.5 Distributional forecasts

OTexts.org/fpp3/

Forecast distributions

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \dots, y_T$.
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

Forecast distributions

Assuming residuals are normal, uncorrelated, sd = $\hat{\sigma}$:

Mean:
$$y_{T+h|T} \sim N(\bar{y}, (1+1/T)\hat{\sigma}^2)$$

Naïve:
$$y_{T+h|T} \sim N(y_T, h\hat{\sigma}^2)$$

Seasonal naïve:
$$y_{T+h|T} \sim N(y_{T+h-m(k+1)}, (k+1)\hat{\sigma}^2)$$

Drift:
$$y_{T+h|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h^{\frac{T+h}{T}}\hat{\sigma}^2)$$

where k is the integer part of (h-1)/m.

Note that when h = 1 and T is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{\mathbf{y}}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where $\hat{\sigma}_h$ is the st dev of the *h*-step distribution.

■ When h = 1, $\hat{\sigma}_h$ can be estimated from the residuals.

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```
aus production |>
 filter(!is.na(Bricks)) |>
 model(Seasonal_naive = SNAIVE(Bricks)) |>
 forecast(h = "5 years")
## # A fable: 20 x 4 [10]
  # Key: .model [1]
##
      .model
                                    Bricks .mean
##
                     Ouarter
##
     <chr>
                       <atr>
                                   <dist> <dbl>
   1 Seasonal_naive 2005 Q3 N(428, 2336)
##
                                             428
   2 Seasonal_naive 2005 Q4
                              N(397, 2336)
##
                                             397
##
    3 Seasonal naive 2006 01
                              N(355, 2336)
                                             355
    4 Seasonal naive 2006 02
                              N(435, 2336)
                                             435
##
##
    5 Seasonal_naive 2006 Q3 N(428, 4672)
                                             428
##
   6 Seasonal naive 2006 04 N(397, 4672)
                                             397
```

```
aus_production |>
  filter(!is.na(Bricks)) |>
  model(Seasonal_naive = SNAIVE(Bricks)) |>
  forecast(h = "5 years") |>
  hilo(level = 95)

## # A tsibble: 20 x 5 [10]
```

```
## # Key: .model [1]
     .model
                  Ouarter
                               Bricks .mean
                                                 `95%`
##
                  <qtr> <dist> <dbl> <hilo>
##
     <chr>
##
   1 Seasonal_naive 2005 Q3 N(428, 2336) 428 [333, 523]95
   2 Seasonal_naive 2005 Q4 N(397, 2336) 397 [302, 492]95
##
##
   3 Seasonal_naive 2006 Q1 N(355, 2336) 355 [260, 450]95
##
   4 Seasonal_naive 2006 Q2 N(435, 2336)
                                       435 [340, 530]95
##
   5 Seasonal_naive 2006 Q3 N(428, 4672)
                                        428 [294, 562]95
## 6 Seasonal naive 2006 04 N(397 4672)
                                        397 [263 531]95
```

```
aus_production |>
  filter(!is.na(Bricks)) |>
  model(Seasonal_naive = SNAIVE(Bricks)) |>
  forecast(h = "5 years") |>
  hilo(level = 95) |>
  mutate(lower = `95%`$lower, upper=`95%`$upper)
```

```
## # A tsibble: 20 x 7 [1Q]
## # Kev: .model [1]
##
     .model
                   Ouarter
                               Bricks .mean
                                                   `95%` lower upper
##
     <chr>
                     <qtr>
                              <dist> <dbl>
                                                   <hilo> <dbl> <dbl>
   1 Seasonal_naive 2005 Q3 N(428, 2336) 428 [333, 523]95 333.
##
                                                               523.
   2 Seasonal_naive 2005 Q4 N(397, 2336)
                                         397 [302, 492]95 302.
                                                               492.
##
##
   3 Seasonal_naive 2006 Q1 N(355, 2336)
                                         355 [260, 450]95 260.
                                                                450.
   4 Seasonal naive 2006 02 N(435, 2336)
                                         435 [340, 530]95 340.
                                                               530.
##
   5 Seasonal naive 2006 Q3
                           N(428, 4672)
                                          428 [294, 562]95 294. 562.
##
```

- Point forecasts are often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use level argument to control coverage.
- Check residual assumptions before believing them.
- Prediction intervals are usually too narrow due to unaccounted uncertainty.