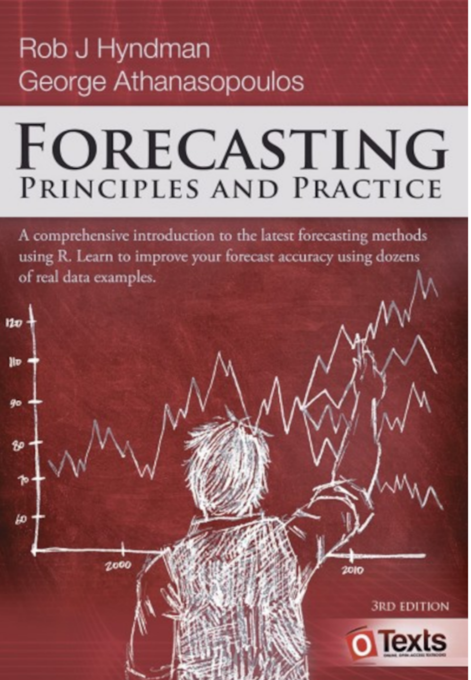

7. Time series regression models

7.7 Nonlinear regression

OTexts.org/fpp3/

Nonlinear regression

A log-log functional form

$$\log y = \beta_0 + \beta_1 \log x + \varepsilon$$

where β_1 is interpreted as an elasticity (the average percentage change in y resulting from a 1% increase in x).

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- alternative specifications: log-linear, linear-log.
- use $\log(x + 1)$ if required.

Piecewise linear and regression splines

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- In general, linear regression splines

$$x_1 = x \quad x_2 = (x - c_1)_+ \quad \ldots \quad x_k = (x - c_{k-1})_+$$

where $c_1, \ldots, c_{k-1}$ are knots.

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where $c_1, \ldots, c_{k-1}$ are knots.

- Need to select knots: can be difficult and arbitrary.
- Automatic knot selection algorithms very slow.
- Using piecewise cubics achieves a smoother result.

Warning: better fit but forecasting outside the range of the historical data is even more unreliable.

Nonlinear trends

Piecewise linear trend with bend at τ

$$x_{1,t} = t$$

$$x_{2,t} = \begin{cases} 0 & t < \tau \\ (t - \tau) & t \geq \tau \end{cases}$$

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Quadratic or higher order trend

$$x_{1,t} = t, \quad x_{2,t} = t^2, \quad \ldots$$

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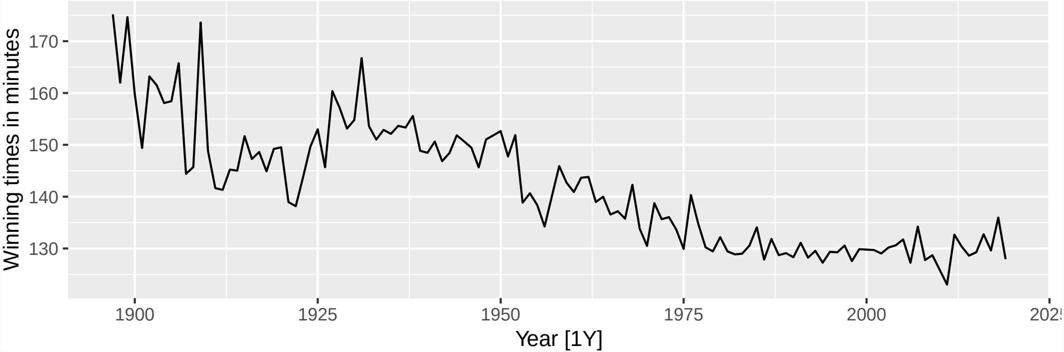
Quadratic or higher order trend

$$x_{1,t} = t, \quad x_{2,t} = t^2, \quad \dots$$

NOT RECOMMENDED!

Example: Boston marathon winning times

```r
marathon <- boston_marathon |>
  filter(Event == "Men's open division") |>
  select(-Event) |>
  mutate(Minutes = as.numeric(Time) / 60)
marathon |> autoplot(Minutes) + labs(y = "Winning times in minutes")
```



Example: Boston marathon winning times

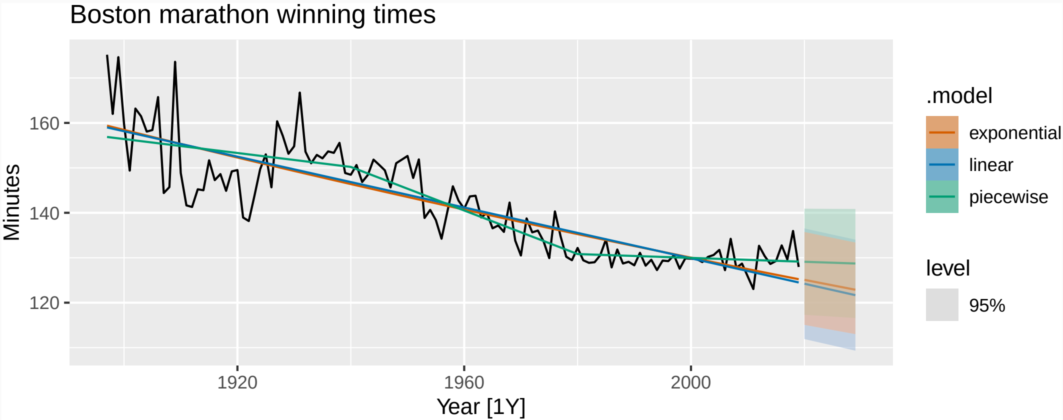
```
fit_trends <- marathon |>
  model(
    # Linear trend
    linear = TSLM(Minutes ~ trend()),
    # Exponential trend
    exponential = TSLM(log(Minutes) ~ trend()),
    # Piecewise linear trend
    piecewise = TSLM(Minutes ~ trend(knots = c(1940, 1980)))
  )
```

```
fit_trends
```

```
## # A mable: 1 x 3
##    linear exponential piecewise
##   <model>     <model>   <model>
## 1  <TSLM>      <TSLM>    <TSLM>
```

Example: Boston marathon winning times

```r
fit_trends |>
  forecast(h = 10) |>
  autoplot(marathon)
```



Example: Boston marathon winning times

```r
fit_trends |>
  select(piecewise) |>
  gg_tsresiduals()
```