Discrete Game
Homework Challenge (2 Extra Points)

In the random utility framework, we model individual choice as individual i choosing among K choices by maximizing her utility $u(i, k)$, with $u(i, k)$ being a function of observed individual and alternative characteristics $x(i, k)$.

Now imagine you are studying a problem in which several individuals make desions simultaneously and each individual's utility depends not only on $x(i, k)$, but also on what other individuals will do.

Exhibit 1

Movie Release Date. When a film studio is deciding on the release date of a movie, it has to consider not only consumer demand (e.g., demand is higher in the summer or around big holidays), but also what its competitors will do (e.g., I want to release my movie in April, but what if Disney drops its Avengers in April as well?). For an analysis of this problem, see Einav (2010).

Exhibit 2

Firm Entry. When a firm decides whether to enter a particular market, it has to consider not only the characteristics of the market, but also whether its competitors will enter the market too.

- When Walmart decides whether to build a store in a small town, it has to consider whether discount stores like K-mart will enter as well. For an analysis of this problem, see Jia (2008).
- When a shopping mall decides whether to enter a market, it has to consider the decisions of other shopping malls and how they may affect its profits depending on whether they are high-end malls or low-end malls. See Vitorino (2012).

In these cases, individual choices involve **strategic interactions**, which we can model using **game theory**. Each individual's choice can be considered the result of an **equilibrium strategy**. The resulting model is called a **discrete game** model.

Model

Consider a simple game with two players: A and B. A and B are simultaneously choosing among a binary choice set $\mathbb{C} = \{0, 1\}$. The utility function of player i, $i \in \{A, B\}$, is:

$$\begin{aligned} u_i(0) &= \epsilon_i^0 \\ u_i(1) &= \pi_i(a_j) + \epsilon_i^1 \end{aligned}$$

, where π_i is the profit function of i and depends on the choice $a_j \in \{0, 1\}$ of the other player j[1].

Assume that players A and B simultaneously make their decisions without knowing what the other player will do. Such a game is called a **game of incomplete information**. Because players do not know what others will do, their choices will depend on their *beliefs* (expectations) about other players' *choice probabilities*:

$$\begin{aligned} a_i &= \arg\max \{u_i(0), \mathbb{E}_i[u_i(1)]\} \\ &= \arg\max \left\{\epsilon_i^0, \pi_i(0)\, p_i(a_j = 0) + \pi_i(1)\, p_i(a_j = 1) + \epsilon_i^1\right\} \end{aligned}$$

, where $p_i(a_j)$ denote i's *belief* about j's *probability* of choosing a_j[2].

A solution concept for games of incomplete expectation is the **Nash Bayesian equilibrium**, which states that in equilibrium, each player has the *correct* belief about the choice probabilities of other players, i.e., $p_i(a_j) = p(a_j)$ for all (i, j). Assuming $(\epsilon_i^0, \epsilon_i^1) \sim$ type I extreme value[3], this implies:

$$p(a_i = 1) = \frac{\exp(\pi_i(0)\, p(a_j = 0) + \pi_i(1)\, p(a_j = 1))}{1 + \exp(\pi_i(0)\, p(a_j = 0) + \pi_i(1)\, p(a_j = 1))}, \quad \forall i, j$$

[1] A reasonable assumption is that $\pi_i(0) > \pi_i(1)$ if the two playes are competitors, and $\pi_i(0) < \pi_i(1)$ if the two players complement each other or have a symbiosis relationship.

[2] Similarly, $\mathbb{E}_i$ denotes i's subjective expectation.

[3] i.e., Gumbel$(0, 1)$.

Task

- Write an introduction to discrete game models and their applications in economic analyses.
- Simulate data from a discrete game model and estimate player utility parameters from your simulated data. Are you able to obtain correct estimates of the underlying utility model?

Reference

- Ellickson, P. B. and S. Misra. (2011). "Estimating Discrete Games," *Marketing Science*, 30(6). [paper, slides]