Chapter 72

ECONOMETRIC EVALUATION OF SOCIAL PROGRAMS, PART III: DISTRIBUTIONAL TREATMENT EFFECTS, DYNAMIC TREATMENT EFFECTS, DYNAMIC DISCRETE CHOICE, AND GENERAL EQUILIBRIUM POLICY EVALUATION*

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Abstract

This chapter develops three topics. (1) Identification of the distributions of treatment effects and the distributions of agent subjective evaluations of treatment effects. Methods for identifying *ex ante* and *ex post* distributions are presented and empirical examples are given. (2) Identification of dynamic treatment effects. The relationship between the statistical literature on dynamic causal inference based on sequential-randomization and the dynamic discrete-choice literature is exposited. The value of well posed economic choice models for decision making under uncertainty in analyzing and interpreting dynamic intervention studies is developed. A survey of the dynamic discrete-choice literature is presented. (3) The key ideas and papers in the recent literature on general equilibrium evaluations of social programs are summarized.

Keywords

distributions of treatment effects, dynamic treatment effects, dynamic discrete choice, general equilibrium policy evaluation

JEL classification: C10, C23, C41, C50

1. Introduction

Part I of this Handbook contribution by Heckman and Vytlacil (Chapter 70) presents a general framework for policy evaluation. Three distinct policy problems are analyzed: P-1 (Internal Validity)—evaluating the effects of a policy in place; P-2 (External Validity)—forecasting the effect of a policy in place in a new environment, and P-3—forecasting the effect of new policies never previously implemented. Among other topics, Part I considers the analysis of distributions of treatment effects and distinguishes private (subjective) valuations of programs from objective valuations. It also discusses the dynamic revelation of information and the uncertainty facing agents. It makes a distinction between *ex ante* expectations of subjective and objective treatment effects and *ex post* realizations of subjective and objective treatment effects. It presents a framework for defining the option value of participating in social programs. The analysis there is largely microeconomic in focus and does not consider the full general equilibrium impacts of policies.

Part II by Heckman and Vytlacil (Chapter 71) focuses primarily on methods for conducting *ex post* evaluations of policies in place (problem P-1), organizing our discussion around the marginal treatment effect (MTE). Mean treatment effect parameters receive the most attention. The methods exposited there can be used to identify marginal impact distributions for Y_0 and Y_1 separately. We show how to use the marginal treatment effect to solve problems P-2 and P-3 in constructing *ex post* evaluations but we do not consider general equilibrium policy analysis.

This chapter presents methods that implement the most innovative aspects of Part I. It is organized in three sections. The first section analyzes methods for the identification of distributions of treatment effects ($Y_1 - Y_0$) and not just the distribution of marginal outcome distributions (or their means) for Y_0 and Y_1 separately. We first analyze *ex post* realized distributions. A different way to say this is that we initially ignore uncertainty. We then present methods for identifying *ex ante* distributions of treatment effects and the information that agents act on when they make their treatment choices prior to the realization of outcomes. Agent *ex ante* expectations are one form of subjective valuation. We present empirical examples based on the research of Carneiro, Hansen and Heckman (2001, 2003), Cunha, Heckman and Navarro (2005, 2006) and Cunha and Heckman (2007b, 2007c, 2008). This part of the chapter helps move the evaluation literature out of statistics and into economics. It presents methods for developing subjective and objective distributions of outcomes.

In the second portion of this contribution, we build on the analysis in the first portion to consider dynamic treatment effects, where sequential revelation of information plays a prominent role. We consider dynamic matching models introduced by Robins (1997), Gill and Robins (2001) and Lok (2007), and applied in economics by Lechner and Miquel (2002) and Fitzenberger, Osikominu and Völter (2006). We then consider more economically motivated models based on continuous-time duration analysis [see Abbring and Van den Berg (2003b)] and dynamic generalizations of the Roy model [Heckman and Navarro (2007)]. We consider identification of mean treatment effects and joint distributions of both objective and subjective outcomes. In the third section of the paper, we briefly consider general equilibrium policy evaluation for distributions of outcomes. We now turn to identification of the distributions of treatment effects.

2. Identifying distributions of treatment effects

The fundamental problem of policy evaluation is that we cannot observe agents in more than one possible state. Chapter 71 focused on various methods for identifying mean outcomes and marginal distributions. Methods useful for identifying means apply in a straightforward way to identification of quantiles of marginal distributions as well as the full marginal distributions. In a two potential outcome world, we can identify $\Pr(Y_1 \leqslant y \mid X) = E(\mathbf{1}[Y_1 \leqslant y] \mid X)$ and $\Pr(Y_0 \leqslant y \mid X) = E(\mathbf{1}[Y_0 \leqslant y] \mid X)$ using the variety of methods summarized in that chapter. One can compare outcomes at one quantile of Y_1 with outcomes at a quantile of Y_0. See, e.g., Heckman, Smith and Clements (1997) or Abadie, Angrist and Imbens (2002). However, these methods do not in general identify the quantiles of the distribution of $Y_1 - Y_0$.

The research reported here is based on work by Aakvik, Heckman and Vytlacil (2005), Heckman and Smith (1998), Heckman, Smith and Clements (1997), Carneiro, Hansen and Heckman (2001, 2003), Cunha, Heckman and Navarro (2005, 2006), and Cunha and Heckman (2007b, 2007c, 2008). It moves beyond means as descriptions of policy outcomes and considers joint counterfactual distributions of outcomes (for example, $F(y_1, y_0)$, gains, $F(y_1 - y_0)$ or outcomes for participants $F(y_1, y_0 \mid D = 1)$). These are the *ex post* distributions realized after treatment is received. We also analyze *ex ante* distributions, inferring the information available to agents when they make their choices. From knowledge of the *ex post* joint distributions of counterfactual outcomes, it is possible to determine the proportion of people who benefit or lose from treatment, and hence *ex post* regret, the origin and destination outcomes of those who change status because of treatment and the amount of gain (or loss) from various policies targeted to persons at different deciles of an initial pre-policy income distribution.[1] Using the joint distribution of counterfactuals, it is possible to develop a more nuanced understanding of the distributional impacts of public policies, and to move beyond comparisons of aggregate distributions induced by different policies to consider how people in different portions of an initial distribution are affected by public policy.

Except in special cases, which we discuss in this portion of the chapter, the methods discussed in Chapter 71 do not solve the fundamental problem of identifying the distribution of treatment effects, i.e., constructing the joint distribution of (Y_0, Y_1) and of the treatment effects $Y_1 - Y_0$. This part of the Handbook reviews methods for constructing or bounding these distributions. We now state precisely the problem analyzed in this section.

[1] It is also possible to generate all mean, median or other quantile gains to treatment, to identify all pairwise treatment effects in a multi-outcome setting, and to determine how much of the variability in returns across persons comes from variability in the distributions of the outcome selected and how much comes from variability in distributions for alternative opportunities.

2.1. The problem

Consider a two-outcome model. The methods surveyed apply in a straightforward way to models with more than two outcomes, as we demonstrate after analyzing the two-outcome case. For expositional convenience, we focus on scalar outcomes unless explicitly stated otherwise. We do not usually observe (Y_0, Y_1) as a pair, but rather only one coordinate and that subject to a selection bias. Thus the problem of recovering joint distributions from cross-section data has two aspects. The first is the selection problem. From data on outcomes, $F_1(y_1 \mid D = 1, X)$, $F_0(y_0 \mid D = 0, X)$, under what conditions can one recover $F_1(y_1 \mid X)$ and $F_0(y_0 \mid X)$, respectively? The second problem is how to construct the joint distribution of $F(y_0, y_1 \mid X)$ from the two marginal distributions. We assume in this section that one of the methods for dealing with the selection problem discussed in Chapters 70 and 71 has been applied and the analyst knows $\Pr(Y_0 \leqslant y_0 \mid X) = F_0(y_0 \mid X)$ and $\Pr(Y_1 \leqslant y_1 \mid X) = F_1(y_1 \mid X)$. The problem is to construct $\Pr(Y_0 \leqslant y_0, Y_1 \leqslant y_1 \mid X) = F(y_0, y_1 \mid X)$. A related problem is how to construct the joint distribution of (Y_0, Y_1, D): $F(y_0, y_1, d \mid X)$. We also consider methods for bounding joint distributions. But first we answer the question, "Why bother"?

2.2. Why bother identifying joint distributions?

Given the intrinsic difficulty in identifying joint distributions of counterfactual outcomes, it is natural to ask, "why not settle for the marginals $F_0(y_0 \mid X)$ and $F_1(y_1 \mid X)$?" The methods surveyed in Chapter 71 afford identification of the marginal distributions. Any method that can identify means or quantiles of distributions can be modified to identify marginal distributions since $E[\mathbf{1}(Y_j \leqslant y_j) \mid X] = F_j(y_j \mid X)$, $j = 0, 1$.[2]

The literature on the measurement of economic inequality as surveyed by Foster and Sen (1997) focuses on marginal distributions across different policy states. Invoking the anonymity postulate, it does not keep track of individual fortunes across different policy states. It does not decompose overall outcomes in each policy state, $Y = DY_1 + (1 - D)Y_0$, into their component parts Y_1, Y_0, attributable to treatment, and D due to choice or assignment mechanisms. Thus, in comparing policies p and $p' \in \mathcal{P}$, it compares the marginal distributions of

$$Y^p = D^p Y_1^p + \left(1 - D^p\right) Y_0^p,$$

where D^p is the treatment choice indicator under policy p, and

$$Y^{p'} = D^{p'} Y_1^{p'} + \left(1 - D^{p'}\right) Y_0^{p'}$$

without seeking information on the subjective valuations of the policy change or the components of the treatment distributions under each policy (Y_0^p and Y_1^p; $Y_0^{p'}$ and $Y_1^{p'}$).

[2] Quantile methods [Chesher (2003), Koenker and Xiao (2002), Koenker (2005)] and many of the methods surveyed in Chapter 73 (Matzkin) of this Handbook also recover these marginal distributions under appropriate assumptions.

It compares $F_{Y^p}(y^p \mid X)$ and $F_{Y^{p'}}(y^{p'} \mid X)$ in making comparisons of welfare and does not worry about the component distributions, subjective valuations of agents, or any issues of self-selection.

Distinguishing the contributions of the component outcome distributions $F_0(y_0)$ and $F_1(y_1)$ and the choice mechanisms is essential for understanding the channels through which different policies operate [Carneiro, Hansen and Heckman (2001, 2003), and Cunha and Heckman (2008)]. Throughout this section, we assume policy invariance for outcomes, (PI-1) and (PI-2), in the notation of Chapter 70, unless otherwise noted.

Heckman, Smith and Clements (1997) and Heckman (1998) apply the concepts of first and second order stochastic dominance used in the conventional inequality measurement literature to compare outcome distributions across treatment states within a policy regime.[3] The same methods can be used to compare treatment outcome distributions across policy states.[4]

Some economists appeal to classical welfare economics and classical decision theory to argue that marginal distributions of treatment outcomes are all that is required to implement the criteria used by these approaches. The argument is that under expected utility maximization with information set $\mathcal{I}$, the agent should be assigned to (choose) treatment 1 if

$$E\bigl(\Upsilon(Y_1) - \Upsilon(Y_0) \mid \mathcal{I}\bigr) \geqslant 0,$$

where Υ is the preference function and $\mathcal{I}$ is the appropriate information set (that of the social planner or the agent). To compute this expectation it is only necessary to know $F_1(y_1 \mid \mathcal{I})$ and $F_0(y_0 \mid \mathcal{I})$, and not the full joint distribution $F(y_0, y_1 \mid \mathcal{I})$. For many other criteria used in classical decision theory, marginal distributions are all that is required.

As noted in Section 2.5 of Chapter 70, if one seeks to know the proportion of people who benefit in terms of income from the program in gross terms ($\Pr(Y_1 \geqslant Y_0 \mid \mathcal{I})$), one needs to know the joint distribution of (Y_0, Y_1) given the appropriate information set. Thus if one seeks to determine the proportion of agents who benefit from 1 compared to 0, it is necessary to determine the joint distribution of (Y_0, Y_1) unless information set $\mathcal{I}$ is known to the econometrician and the agent uses the Roy model to make choices. For the Roy model,

$$D = \mathbf{1}[Y_1 \geqslant Y_0],$$

the probability of selecting treatment given the econometrician's information set $\mathcal{I}_E$ is

$$\Pr(D = 1 \mid \mathcal{I}_E) = \Pr(Y_1 \geqslant Y_0 \mid \mathcal{I}_E).$$

[3] Abadie (2002) develops standard errors for this method and presents additional references.

[4] Under the conventional microeconometric partial equilibrium approach to policy evaluation surveyed in Chapter 70, the marginal distributions of (Y_0, Y_1) are invariant to the choice of the policy regime. This assumption is relaxed in our analysis of general equilibrium effects in Section 4.

If there are no direct costs of participation, and agents participate in the program based on self-selection under perfect certainty, and $\mathcal{I}_E$ is both the econometrician's and the agent's information set, then data on choices identify this proportion without need for any further econometric analysis. See the discussion in Section 2.6 below. More generally, agents may use the generalized Roy model presented in Chapter 70, or some other model, to make decisions, but the analyst seeks to know the proportion who gain *ex post*, conditioning on a different information set. Individual choice data will not reveal this probability if (a) agents do not use a Roy model formulated in *ex post* outcomes, (b) they use a more general decision rule, or (c) the information set of the agent is different from that of the econometrician. In these cases, further econometric analysis is required to identify $\Pr(Y_1 \geqslant Y_0 \mid \mathcal{I})$ for any particular information set.

Clearly the joint distribution of (Y_0, Y_1) given $\mathcal{I}$ is required to compute the gain in gross outcomes in general terms. In analyzing the option values of social programs and the distribution of returns to schooling (e.g., $(Y_1 - Y_0)$), in identifying dynamic discrete-choice models (reviewed in Section 3), and in determining *ex post* regret, knowledge of the full joint distribution of outcomes is required.

Section 2.10 presents examples of the richer, more nuanced, approach to policy evaluation that is possible when the analyst has access to the joint distribution of outcomes across counterfactual states. We show how the tools presented in this section allow economists to move beyond the limitations of the anonymity postulate to consider who benefits and who loses from policy reforms. We present estimates of the proportion of people who have *ex post* regret about their schooling choices and estimates of the *ex ante* and *ex post* distributions of returns to schooling ($\frac{Y_1 - Y_0}{Y_0}$) which inherently require knowledge of the joint distribution of outcomes across states. We now turn to methods for identifying or bounding joint distributions.

2.3. Solutions

There are two basic approaches in the literature to solving the problem of identifying $F(y_0, y_1 \mid X)$: (A) solutions that postulate assumptions about dependence between Y_0 and Y_1 and (B) solutions based on information from agent participation rules and additional data on choice. Recently developed methods build on these two basic approaches and combine choice theory with supplementary data and assumptions about the structure of dependence among model unobservables. We survey all of these methods. In addition to methods for exact identification, Fréchet bounds can be placed on the joint distributions from knowledge of the marginals [see, e.g., Heckman and Smith (1993, 1995), Heckman, Smith and Clements (1997), Manski (1997)]. We first consider these bounds.

2.4. Bounds from classical probability inequalities

The problem of bounding an unknown joint distribution from known marginal distributions is a classical problem in mathematical statistics. Hoeffding (1940) and Fréchet

(1951) demonstrate that the joint distribution is bounded by two functions of the marginal distributions. Their inequalities state that

$$\max\left[F_0(y_0 \mid X) + F_1(y_1 \mid X) - 1, 0\right] \leqslant F(y_0, y_1 \mid X) \leqslant \min\left[F_0(y_0 \mid X), F_1(y_1 \mid X)\right].$$[5]

To simplify the notation, we keep conditioning on X implicit in the remainder of this section. Rüschendorf (1981) establishes that these bounds are tight.[6] Mardia (1970) establishes that both the lower bound and the upper bound are proper probability distributions. At the upper bound, $Y_1 = F_1^{-1}(F_0(Y_0))$ is a non-decreasing deterministic function of Y_0. At the lower bound, Y_1 is a non-increasing deterministic function of Y_0: $Y_1 = F_1^{-1}(1 - F_0(Y_0))$.

By a theorem of Cambanis, Simons and Stout (1976), if $k(y_1, y_0)$ is superadditive (or subadditive), then extreme values of $E(k(Y_1, Y_0))$ are obtained from the upper and lower bounding distributions.[7,8] Since $k(y_1, y_0) = (y_1 - E(Y_1))(y_0 - E(Y_0))$ is superadditive, the maximum attainable product-moment correlation $r_{Y_0 Y_1}$ is obtained from the upper bound distribution while the minimum attainable product moment correlation is obtained at the lower bound distribution. Let $\Delta = Y_1 - Y_0$. It is possible to bound $\mathrm{Var}(\Delta) = (\mathrm{Var}(Y_1) + \mathrm{Var}(Y_0) - 2r_{Y_0 Y_1}[\mathrm{Var}(Y_1)\,\mathrm{Var}(Y_0)]^{1/2})$ with the minimum obtained from the Fréchet–Hoeffding upper bound.[9] Checking whether the lower bound of $\mathrm{Var}(\Delta)$ is statistically significantly different from zero provides a test of whether or not the data are consistent with the common effect model. For example, if $Y_1 - Y_0 = \beta$, a constant, $\mathrm{Var}(\Delta) = 0$.

Tchen (1980) establishes that Kendall's τ and Spearman's ρ also attain their extreme values at the bounding distributions. The upper and lower bounding distributions produce the cases of perfect positive dependence and perfect negative dependence, respectively. Often the bounds on the quantiles of the Δ distribution obtained from the Fréchet–Hoeffding bounds are very wide.[10] Table 1 presents the range of values of $r_{Y_0 Y_1}$,

[5] King (1997) applies these inequalities to solve the problem of ecological correlation. These inequalities are used in the missing data literature for contingency tables [see, e.g., Bishop, Fienberg and Holland (1975)].

[6] An upper bound is "tight" if it is the smallest possible upper bound. A lower bound is tight if it is the largest lower bound.

[7] k is assumed to be Borel-measurable and right-continuous. k is strictly superadditive if $y_1 > y_1'$ and $y_0 > y_0'$ imply that $k(y_1, y_0) + k(y_1', y_0') > k(y_1, y_0') + k(y_1', y_0)$. k is strictly subadditive if the final inequality is reversed.

[8] An interesting application of the analysis of Cambanis, Simons and Stout (1976) is to the assignment problem studied by Koopmans and Beckmann (1957) and Becker (1974). If total output of a match $k(y_0, y_1)$ is superadditive, as it is in the Cobb–Douglas model ($k(y_0, y_1) = y_0 y_1$), then the optimal sorting rule is obtained by the upper bound of the Fréchet distribution.

[9] Note that the maximum value of $r_{Y_0 Y_1}$ is obtained at the upper bound and that all other components of the variance of Δ are obtained from the marginal distributions. Thus the minimum variance of Δ is obtained from the Fréchet–Hoeffding upper bound distribution.

[10] See the examples in Heckman and Smith (1993).

Table 1
Characteristics of the distribution of impacts on earnings in the 18 months after random assignment at the Fréchet–Hoeffding bounds (National JTPA Study 18 month impact sample: adult females)

Statistic	From lower bound distribution	From upper bound distribution
Impact standard deviation	14968.76	674.50
	(211.08)	(137.53)
Outcome correlation	−0.760	0.998
	(0.013)	(0.001)
Spearman's ρ	−0.9776	0.9867
	(0.0016)	(0.0013)

Notes: 1. These estimates were obtained using the empirical c.d.f.s calculated at 100 dollar earnings intervals rather than using the percentiles of the two c.d.f.s.
2. Bootstrap standard errors in parentheses.
Source: Heckman, Smith and Clements (1997).

Spearman's ρ and $[\mathrm{Var}(\Delta)]^{1/2}$ for the Job Training Partnership Act (JTPA) data analyzed in Heckman, Smith and Clements (1997).[11] The ranges are rather wide, but it is interesting to observe that the bounds rule out the common effect model, as $\mathrm{Var}(\Delta)$ is bounded away from zero.

The Fréchet–Hoeffding bounds apply to all joint distributions.[12] The outcome variables may be discrete, continuous or both discrete and continuous. It is fruitful to consider the bounds for this model with binary outcomes to establish the variability in the distribution of impacts for a discrete variable such as employment. For specificity, we analyze the employment data from the JTPA experiment reported in Heckman, Smith and Clements (1997). The data are multinomial.[13] Let (E, E) denote the event "employed with treatment" and "employed without treatment" and let (E, N) be the event "employed with treatment, not employed without treatment." Similarly, (N, E) and (N, N) refer respectively to cases where a person would not be employed if treated but would be employed if not treated, and where a person would not be employed in either case. The probabilities associated with these events are P_{EE}, P_{EN}, P_{NE} and P_{NN}, respectively. This model can be written in the form of a contingency table. The columns refer to employment and nonemployment in the untreated state. The rows refer to employment and nonemployment in the treated state.

[11] Heckman, Smith and Clements (1997) discuss the properties of the estimates of the standard errors reported in Table 1. JTPA was a job training program in place in the US in the 1980s and 1990s.
[12] Formulae for multivariate bounds are given in Tchen (1980) and Rüschendorf (1981).
[13] The following formulation owes a lot to the missing cell literature in contingency table analysis. See, e.g., Bishop, Fienberg and Holland (1975).

		Untreated		
		E	N	
Treated	E	P_{EE}	P_{EN}	$P_{E\cdot}$
	N	P_{NE}	P_{NN}	$P_{N\cdot}$
		$P_{\cdot E}$	$P_{\cdot N}$	

If we observed the same person in both the treated and untreated states, we could fill in the table and estimate the full distribution. With experimental data or data corrected for selection using the methods discussed in Chapter 71, one can estimate the marginals of the table parameters:

$$P_{E\cdot} = P_{EE} + P_{EN} \quad \text{(employment proportion among the treated)}, \tag{2.1a}$$

$$P_{\cdot E} = P_{EE} + P_{NE} \quad \text{(employment proportion among the untreated)}. \tag{2.1b}$$

The treatment effect is usually defined as

$$\Delta = P_{EN} - P_{NE}. \tag{2.2}$$

This is the proportion of people who would switch from being nonemployed to being employed as a result of treatment minus the proportion of persons who would switch from being employed to not being employed as a result of treatment. Using (2.1a) and (2.1b), we obtain the treatment effect as

$$\Delta = P_{E\cdot} - P_{\cdot E}, \tag{2.3}$$

so that Δ is identified by subtracting the proportion employed in the control group ($\hat{P}_{\cdot E}$) from the proportion employed in the treatment group ($\hat{P}_{E\cdot}$).

If we wish to decompose Δ into its two components, experimental data or selection-corrected data do not in general give an exact answer. In terms of the contingency table presented above, we know the row and column marginals but not the individual elements in the table. The case in the 2×2 table corresponding to the common effect model for continuous outcomes restricts the effect of the program on employment to be always positive or always negative, so that either P_{EN} or $P_{NE} = 0$, respectively. Under such assumptions, the model is fully identified. This is analogous to the continuous case in which the common effect assumption, or more generally, an assumption of perfect positive dependence, identifies the joint distribution of outcomes.

More generally, the Fréchet–Hoeffding bounds restrict the range of admissible values for the cell probabilities. Their application in this case produces:

$$\max[P_{E\cdot} + P_{\cdot E} - 1, 0] \leqslant P_{EE} \leqslant \min[P_{E\cdot}, P_{\cdot E}],$$
$$\max[P_{E\cdot} - P_{\cdot E}, 0] \leqslant P_{EN} \leqslant \min[P_{E\cdot}, 1 - P_{\cdot E}],$$
$$\max[-P_{E\cdot} + P_{\cdot E}, 0] \leqslant P_{NE} \leqslant \min[1 - P_{E\cdot}, P_{\cdot E}],$$
$$\max[1 - P_{E\cdot} - P_{\cdot E}, 0] \leqslant P_{NN} \leqslant \min[1 - P_{E\cdot}, 1 - P_{\cdot E}].$$

Table 2
Fraction employed in the 16th, 17th or 18th month after random assignment and Fréchet–Hoeffding bounds on the probabilities P_{NE} and P_{EN} (National JTPA study 18 month impact sample: adult females)

Parameter	Estimate
Fraction employed in the treatment group	0.64 (0.01)
Fraction employed in the control group	0.61 (0.01)
Bounds on P_{EN}	[0.03, 0.39] (0.01), (0.01)
Bounds on P_{NE}	[0.00, 0.36] (0.00), (0.01)

Notes: 1. Employment percentages are based on self-reported employment in months 16, 17 and 18 after random assignment. A person is coded as employed if the sum of their self-reported earnings over these three months is positive.
2. P_{ij} is the probability of having employment status i in the treated state and employment status j in the untreated state, where i and j take on the values E for employed and N for not employed. The Fréchet–Hoeffding bounds are given in the text.
3. Standard errors are discussed in Heckman, Smith and Clements (1997).
Source: Heckman, Smith and Clements (1997).

Table 2, taken from the analysis of Heckman, Smith and Clements (1997), presents the Fréchet–Hoeffding bounds for P_{NE} and P_{EN} from the national JTPA experiment—the source of data for Table 1. The outcome variable is whether or not a person is employed in the 16th, 17th or 18th month after random assignment. The bounds are very wide. Even without taking into account sampling error, the experimental evidence for adult females is consistent with a value of P_{NE} ranging from 0.00 to 0.36. The range for P_{EN} is equally large. Thus as many as 39% and as few as 3% of adult females may have had their employment status improved by participating in the training program. As many as 36% and as few as 0% may have had their employment status harmed by participating in the program. From (2.2), we know that the net difference $P_{EN} - P_{NE} = \Delta$, so that high values of P_{EN} are associated with high values of P_{NE}. As few as 25% $[(0.64 - 0.39) \times 100\%]$ and as many as 61% of the women would have worked whether or not they entered the program ($P_{EE} \in [0.25, 0.61]$).

From the evidence presented in Table 2, one cannot distinguish two different stories. The first story is that the JTPA program benefits many people by facilitating their employment but it also harms many people who would have worked if they had not participated in the program. The second story is that the program benefits and harms few people.[14] Heckman, Smith and Clements (1997) and Manski (1997, 2003) develop these bounds further. We next consider methods to point identify the joint distributions of outcomes. All entail using some auxiliary information.

2.5. Solutions based on dependence assumptions

A variety of approaches solve the problem of identifying the joint distribution of potential outcomes by making dependence assumptions connecting Y_0 and Y_1. We review some of the major approaches.

2.5.1. Solutions based on conditional independence or matching

An approach based on matching postulates access to variables Q that have the property that conditional on Q, $F_0(y_0 \mid D = 0, X, Q) = F_0(y_0 \mid X, Q)$ and $F_1(y_1 \mid D = 1, X, Q) = F_1(y_1 \mid X, Q)$. As discussed in Section 9 of Chapter 71, matching assumes that conditional on observed variables, Q, there is no selection problem: $(Y_0 \perp\!\!\!\perp D \mid X, Q)$ and $(Y_1 \perp\!\!\!\perp D \mid X, Q)$. If it is further assumed that all of the dependence between (Y_0, Y_1) given X comes through Q, it follows that

$$F(y_1, y_0 \mid X, Q) = F_1(y_1 \mid X, Q) F_0(y_0 \mid X, Q).$$

Using these results, it is possible to identify the joint distribution $F(y_0, y_1 \mid X)$ because

$$F(y_0, y_1 \mid X) = \int F_0(y_0 \mid X, Q) F_1(y_1 \mid X, Q)\, d\mu(Q \mid X),$$

where $\mu(Q \mid X)$ is the conditional distribution of Q given X. Under the assumption that we observe X and Q, this conditional distribution can be constructed from data. We obtain $F_0(y_0 \mid X, Q)$, $F_1(y_1 \mid X, Q)$ by matching. Thus we can construct the right-hand side of the preceding expression. As noted in Chapter 71, matching makes the strong assumption that conditional on (Q, X) the marginal return to treatment is the same as the average return, although returns may differ by the level of Q and X.

2.5.2. The common coefficient approach

The traditional approach in economics to identifying joint distributions is to assume that the joint distribution $F(y_0, y_1 \mid X)$ is a degenerate, one dimensional distribution. Conditional on X, Y_0 and Y_1 are assumed to be deterministically related:

$$Y_1 - Y_0 = \Delta, \tag{2.4}$$

[14] Heckman, Smith and Clements (1997) show that conditioning on other background variables does not reduce the intrinsic uncertainty in the data. Thus in both the discrete and continuous cases, the data from the JTPA experiment are consistent with a wide variety of impact distributions.

where Δ is a constant given X. It is the difference in means between Y_1 and Y_0 for the selection corrected distribution.[15] This approach assumes that treatment has the same effect on everyone (with the same X), and that the effect is Δ. Because (2.4) implies a perfect ranking across quantiles of the outcome distributions Y_0 and Y_1, Δ can be identified from the difference in the quantiles between Y_0 and Y_1 for any quantile. Even if the means do not exist, one can still identify Δ. From knowledge of $F_0(y_0 \mid X)$ and $F_1(y_1 \mid X)$, one can identify the means and quantiles. Hence one can identify Δ.

2.5.3. *More general dependence assumptions*

Heckman, Smith and Clements (1997) and Heckman and Smith (1998) relax the common coefficient assumption by postulating perfect ranking in the positions of individuals in the $F_1(y_1 \mid X)$ and $F_0(y_0 \mid X)$ distributions. The best in one distribution is the best in the other. Assuming continuous and strictly increasing marginal distributions, they postulate that quantiles are perfectly ranked so $Y_1 = F_1^{-1}(F_0(Y_0))$. This is the tight upper bound of the Fréchet bounds. An alternative assumption is that people are perfectly inversely ranked so the best in one distribution is the worst in the other:

$$Y_1 = F_1^{-1}\bigl(1 - F_0(Y_0)\bigr).$$

This is the tight Fréchet lower bound.

One can associate quantiles across the marginal distributions more generally. Heckman, Smith and Clements (1997) use Markov transition kernels that stochastically map quantiles of one distribution into quantiles of another. They define a pair of Markov kernels $M(y_1, y_0 \mid X)$ and $\tilde{M}(y_0, y_1 \mid X)$ with the property that they map marginals into marginals:

$$F_1(y_1 \mid X) = \int M(y_1, y_0 \mid X)\, dF_0(y_0 \mid X),$$

$$F_0(y_0 \mid X) = \int \tilde{M}(y_0, y_1 \mid X)\, dF_1(y_1 \mid X).$$

Allowing these kernels to be degenerate produces a variety of deterministic transformations, including the two previously presented, as special cases of a general mapping. Different $(M, \tilde{M})$ pairs produce different joint distributions. These transformations supply the missing information needed to construct the joint distributions.[16]

[15] Δ may be a function of X.

[16] For given marginal distributions F_0 and F_1, we cannot independently pick M and $\tilde{M}$. Consistency requires that

$$\int_{-\infty}^{y_0} M(y_1, y \mid X)\, dF_0(y \mid X) = \int_{-\infty}^{y_1} \tilde{M}(y_0, y \mid X)\, dF_1(y \mid X),$$

for all y_0, y_1.

A perfect ranking (or perfect inverse ranking) assumption generalizes the perfect ranking, constant-shift assumptions implicit in the conventional literature. It allows analysts to apply conditional quantile methods to estimate the distributions of gains.[17] However, it imposes a strong and arbitrary dependence across distributions. Lehmann and D'Abrera (1975), Robins (1989, 1997), Koenker and Xiao (2002), and many others maintain this assumption under the rubric of "rank invariance" in order to identify the distribution of treatment effects.

Table 3 shows the percentiles of the earnings impact distribution ($F_\Delta(y_1 - y_0)$) for females in the National JTPA experiment under various assumptions about dependence

Table 3
Percentiles of the impact distribution as ranking across distributions (τ) varies based on random samples of 50 permutations with each value of τ (National JTPA study 18 month impact sample: adult females)

Measure of rank correlation τ	Minimum	5th percentile	25th percentile	50th percentile	75th percentile	95th percentile	Maximum
1.00	0.00	0.00	572.00	864.00	966.00	2003.00	18550.00
	(703.64)	(47.50)	(232.90)	(269.26)	(305.74)	(543.03)	(5280.67)
0.95	−14504.00	0.00	125.50	616.00	867.00	1415.50	48543.50
	(1150.01)	(360.18)	(124.60)	(280.19)	(272.60)	(391.51)	(8836.49)
0.90	−18817.00	−1168.00	0.00	487.00	876.50	2319.50	49262.00
	(1454.74)	(577.84)	(29.00)	(265.71)	(282.77)	(410.27)	(6227.38)
0.70	−25255.00	−8089.50	−136.00	236.50	982.50	12158.50	55169.50
	(1279.50)	(818.25)	(260.00)	(227.38)	(255.78)	(614.45)	(5819.28)
0.50	−28641.50	−12037.00	−1635.50	0.00	1362.50	16530.00	58472.00
	(1149.22)	(650.31)	(314.39)	(83.16)	(249.29)	(329.44)	(5538.14)
0.30	−32621.00	−14855.50	−3172.50	0.00	4215.50	16889.00	54381.00
	(1843.48)	(548.48)	(304.62)	(37.96)	(244.67)	(423.05)	(5592.86)
0.00	−44175.00	−18098.50	−6043.00	0.00	7388.50	19413.25	60599.00
	(2372.05)	(630.73)	(300.47)	(163.17)	(263.25)	(423.63)	(5401.02)
−0.30	−48606.00	−20566.00	−8918.50	779.50	9735.50	21093.25	65675.00
	(1281.80)	(545.99)	(286.92)	(268.02)	(300.59)	(462.13)	(5381.91)
−0.50	−48606.00	−21348.00	−9757.50	859.00	10550.50	22268.00	67156.00
	(1059.06)	(632.55)	(351.55)	(315.37)	(255.28)	(435.78)	(5309.90)
−0.70	−48606.00	−22350.00	−10625.00	581.50	11804.50	23351.00	67156.00
	(1059.06)	(550.00)	(371.38)	(309.84)	(246.58)	(520.93)	(5309.90)
−0.90	−48606.00	−22350.00	−11381.00	580.00	12545.00	23351.00	67156.00
	(1059.06)	(547.17)	(403.30)	(346.12)	(251.07)	(341.41)	(5309.90)
−0.95	−48606.00	−22350.00	−11559.00	580.00	12682.00	23351.00	67156.00
	(1059.06)	(547.17)	(404.67)	(366.37)	(255.97)	(341.41)	(5309.90)
−1.00	−48606.00	−22350.00	−11755.00	580.00	12791.00	23351.00	67156.00
	(1059.06)	(547.17)	(411.83)	(389.51)	(253.18)	(341.41)	(5309.90)

(*continued on next page*)

[17] See, e.g., Heckman, Smith and Clements (1997).

Table 3
(*continued*)

Notes: 1. This table shows selected percentiles of the empirical distribution of $Y_1 - Y_0$ under different assumptions about the dependence of Y_1 and Y_0. The empirical distribution of $Y_1 - Y_0$ for each indicated value of Kendall's rank correlation τ is constructed by pairing the percentiles of the empirical distributions of Y_0 and Y_1 in a way consistent with the value of τ. There are $J!$ ways of pairing the $J = 100$ percentiles of both marginal distributions, each corresponding to lining up the Y_0 percentiles to one of the $J!$ permutations of the Y_1 percentiles. First, consider the two extreme cases, $\tau = 1$ and $\tau = -1$. If the percentiles of Y_0 are assigned to the corresponding percentile of Y_1, then the rank correlation τ between the percentiles among the resulting J pairs equals 1. The difference between the percentile of Y_1 and the associated percentile of Y_0 in each pair is the impact for that pair. Taken together, the J pairs' impacts form the distribution of impacts for $\tau = 1$. It is the minimum, maximum and percentiles of this impact distribution that are reported in the first row of the table. If the percentile comparisons are based on pairing the biggest in one distribution with the smallest in the other distribution, then $\tau = -1$. Computations for $\tau = -1$ are reported in the table's last row.

Intermediate values of τ are obtained by considering pairings of percentiles with a specified number of inversions in the ranks. An inversion is said to arise if, among two pairs of quantiles, a lower Y_0 quantile is matched with a higher Y_1 quantile. For a given pairing of percentiles (permutation of the Y_1 percentiles) the total number of inversions is

$$\eta = \sum_j \sum_{i<j} h_{ij}, \quad h_{ij} = \begin{cases} 1, & Y_1^{(i)} > Y_1^{(j)}, \\ 0 \end{cases}$$

where $Y_1^{(j)}$ is the percentile of Y_1 associated with the jth percentile of Y_0. The value of η ranges from 0 (corresponding to perfect positive rank correlation) to $\frac{1}{2}J(J-1)$ (perfect negative rank correlation). Kendall's rank correlation measure τ is

$$\tau = 1 - \frac{4\eta}{J(J-1)}, \quad \text{where } \tau \in [-1, 1].$$

There are multiple pairings of percentiles consistent with each intermediate value of τ (number of inversions η), unlike in the cases of $\tau = 1$ and $\tau = -1$. Therefore, for intermediate values of τ the table reports the mean of the indicated parameters of the impact distribution over a random sample of 50 pairings having the indicated value of τ.

2. Bootstrap standard errors in parentheses.

Source: Heckman, Smith and Clements (1997).

between Y_1 and Y_0. The experiment identified $F_1(y_1)$ and $F_0(y_0)$ separately. The table reports selected percentiles of the estimated impact distributions for different assumed levels of dependence, τ (Kendall's rank correlation). As shown in the first footnote to the table, $\tau = 1$ corresponds to the Fréchet upper bound. $\tau = -1$ corresponds to the Fréchet lower bound. Since without further information in hand, the joint distribution is not identified, the data are consistent with all values of τ and so each row of the table is a possible outcome distribution. Notice that the medians (50th percentile) are reasonable, but many percentiles are not. Heckman, Smith and Clements (1997) suggest that prior information about plausible outcomes, possibly formalized by a Bayesian analysis, can be used to pick reasonable values of τ. We next consider alternative deconvolution assumptions that can be used to point identify the joint distributions.

2.5.4. Constructing distributions from assuming independence of the gain from the base

An alternative assumption about the dependence across outcomes is that $Y_1 = Y_0 + \Delta$, where Δ, the treatment effect, is a random variable stochastically independent of Y_0 given X, i.e.,

(CON-1) $Y_0 \perp\!\!\!\perp \Delta \mid X$.

This assumption states that the gain from participating in the program is independent of the base Y_0. If we assume

(M-1) $(Y_0, Y_1) \perp\!\!\!\perp D \mid X$,

and (CON-1), we can identify $F(y_0, y_1 \mid X)$ from the cross-section outcome distributions of participants and nonparticipants and estimate the joint distribution by using deconvolution.[18] Methods for using this information are presented in Appendix A.

Horowitz and Markatou (1996) develop the asymptotic properties of convolution estimators with regression building on the work of Stefanski and Carroll (1991). Heckman, Smith and Clements (1997) and Heckman and Smith (1998) use deconvolution to analyze the distribution of gains from the JTPA data. Neither (CON-1) nor (M-1) is an attractive assumption from the point of view of economic choice models. (M-1) implies that marginal entrants into a social program have the same return as average participants. The assumption (CON-1) is not a prediction of general choice models.

2.5.5. Random coefficient regression approaches

In a regression setting in which means and variances are assumed to capture all of the relevant information about the distributions of outcomes and treatment effects, the convolution approach discussed in the preceding section is equivalent to the traditional normal random coefficient model. Letting

$$Y_1 = \mu_1(X) + U_1, \quad E(U_1 \mid X) = 0,$$
$$Y_0 = \mu_0(X) + U_0, \quad E(U_0 \mid X) = 0,$$

this version of the model may be written as

$$Y = \mu_0(X) + \underbrace{\big(\mu_1(X) - \mu_0(X) + U_1 - U_0\big)}_{\beta(X)} D + U_0$$

[18] Barros (1987) uses this assumption in the context of an analysis of selection bias.

$$= \mu_0(X) + \big(\mu_1(X) - \mu_0(X)\big)D + (U_1 - U_0)D + U_0$$
$$= \mu_0(X) + \bar{\beta}(X)D + \upsilon D + U_0, \tag{2.5}$$

where in the notation of Chapter 71, $\beta(X)$ is the treatment effect ($= \Delta$), $\bar{\beta}(X) = \mu_1(X) - \mu_0(X)$, and $\upsilon = U_1 - U_0$. From (M-1), $(U_0, U_1) \perp\!\!\!\perp D \mid X$.

Nonparametric regression methods may be used to recover $\mu_0(X)$ and $\mu_1(X) - \mu_0(X)$ or one may use ordinary parametric regression methods if one assumes that $\mu_1(X) = X\beta_1$ and $\mu_0(X) = X\beta_0$. Equation (2.5) is a components-of-variance model and a test of (CON-1) given (M-1) is that

$$\begin{aligned}\operatorname{Var}(Y \mid D = 1, X) &= \operatorname{Var}(Y_0 + \Delta \mid D = 1, X)\\ &= \operatorname{Var}(Y_0 \mid X) + \operatorname{Var}(\Delta \mid X)\\ &\geqslant \operatorname{Var}(Y \mid D = 0, X) = \operatorname{Var}(Y_0 \mid X).\end{aligned}$$

Under standard conditions, each component of variance is identified and estimable from the residuals obtained from the nonparametric regression of Y on D and X. Thus one can jointly test a prediction of (CON-1) and (M-1) by checking these inequalities.

2.6. *Information from revealed preference*

An alternative approach, rooted more deeply in economics, uses information on agent choices to recover the joint population distribution of potential outcomes.[19] Unlike the method of matching or the methods based on particular assumptions about dependence between Y_0 and Y_1, the method based on revealed preference capitalizes on a close relationship between (Y_0, Y_1) and decisions about program participation. Participation includes voluntary entry into a program or attrition from it.

The prototypical framework is the Roy (1951) model extensively utilized in Chapters 70 and 71. In that setup, as previously noted in Section 2.2,

$$D = \mathbf{1}[Y_1 \geqslant Y_0]. \tag{2.6}$$

If we postulate that the outcome equations can be written in a separable form, so that

$$Y_1 = \mu_1(X) + U_1, \quad E(U_1 \mid X) = 0,$$
$$Y_0 = \mu_0(X) + U_0, \quad E(U_0 \mid X) = 0,$$

then $\Pr(D = 1 \mid X) = \Pr(Y_1 - Y_0 \geqslant 0 \mid X) = \Pr(U_1 - U_0 \geqslant -(\mu_1(X) - \mu_0(X)))$. Heckman and Honoré (1990) demonstrate that if $X \perp\!\!\!\perp (U_0, U_1)$, $\operatorname{Var}(U_0) < \infty$ and $\operatorname{Var}(U_1) < \infty$, and (U_0, U_1) are normal, the full model $F(y_0, y_1, D \mid X)$ is identified even if we only observe Y_0 or Y_1 for any person and there are no regressors and

[19] Heckman (1974a, 1974b) demonstrates how access to censored samples on hours of work, wages for workers, and employment choices identifies the joint distribution of the value of nonmarket time and potential market wages under a normality assumption. Heckman and Honoré (1990) consider nonparametric versions of this model without labor supply.

no exclusion restrictions. If instead of assuming normality, it is assumed that the support of $(\mu_0(X), \mu_1(X))$ contains the support of (U_0, U_1), $(\mu_0(X), \mu_1(X))$ and the joint distribution of (U_0, U_1) are nonparametrically identified up to location normalizations. The proof of this theorem due to Heckman and Honoré (1990) is a special case of the general theorem proved in Appendix B of Chapter 70.

A crucial feature of the Roy model is that the decision to participate in the program is made solely in terms of potential outcomes. No new unobserved variables enter the model that do not also appear in the outcome equations (Y_0, Y_1). We could augment decision rule (2.6) to be $D = \mathbf{1}[Y_1 - Y_0 - \mu_C(Z) \geqslant 0]$, where $\mu_C(Z)$ is the cost of participation in the program and Z is observed, and still preserve the identifiability of the Roy model. Provided that we measure Z and condition on it, and provided that $(U_0, U_1) \perp\!\!\!\perp (X, Z)$, the model remains nonparametrically identified. The crucial property of the identification result is that no new unobservable enters the model through the participation equation. However, if we add components of cost based on observables, subjective valuations of gain $(Y_1 - Y_0 - \mu_C(Z))$ no longer equal "objective" measures $(Y_1 - Y_0)$. This is the distinction between the generalized Roy model and the extended Roy model extensively discussed in Chapter 71.

In the case of the Roy model, information about who participates in the program also informs the analyst about the distribution of the value of the program to participants $F_\Delta(y_1 - y_0 \mid Y_1 \geqslant Y_0, X)$. Thus, we acquire the distribution of implicit values of the program for participants. In the Roy model, "objective" and "subjective" outcomes coincide and agent's choices are informative on the outcome not chosen.

For more general decision rules with additional sources of unobservables apart from those arising from (Y_0, Y_1), it is not generally possible to identify $F(y_0, y_1)$ from information on (Y, D, X, Z) without invoking additional assumptions. For the generalized Roy model,

$$D = \mathbf{1}[Y_1 - Y_0 - C \geqslant 0],$$

where, for example,

$$C = \mu_C(Z) + U_C.$$

Let $U_I = U_1 - U_0 - U_C$, $I = Y_1 - Y_0 - C$ and $\mu_I(X, Z) = \mu_1(X) - \mu_0(X) - \mu_C(Z)$. Define $P(X, Z) = \Pr(D = 1 \mid X, Z)$. If U_C is not perfectly predicted by (U_0, U_1), then we cannot, in general, estimate the joint distribution of (Y_0, Y_1, C) given (X, Z) or the distribution of (U_0, U_1, U_C) from data on Y, D, X and Z.

However, under the conditions in Appendix B of Chapter 70, we can identify up to an unknown scale for I, $F_{Y_0,I}(y_0, i \mid X, Z)$ and $F_{Y_1,I}(y_1, i \mid X, Z)$.[20] The following intuition motivates the conditions under which $F_{Y_0,I}(y_0, i \mid X, Z)$ is identified. A parallel argument holds for $F_{Y_1,I}(y_1, i \mid X, Z)$. First, under the conditions given in Cosslett (1983), Manski (1988), Matzkin (1992) and Appendix B of Chapter 70, we can identify

[20] In our application of that theorem, there are only two choices so $\bar{S} = 2$ in the notation of that theorem.

$\frac{\mu_I(X,Z)}{\sigma_{U_I}}$ from $\Pr(D = 1 \mid X, Z) = \Pr(\mu_I(X, Z) + U_I \geqslant 0 \mid X, Z)$. $\sigma^2_{U_I}$ is the variance of U_I. We can also identify the distribution of $\frac{U_I}{\sigma_{U_I}}$. Second, from this information and $F_0(y_0 \mid D = 0, X, Z) = \Pr(Y_0 \leqslant y_0 \mid \mu_I(X, Z) + U_I < 0, X, Z)$, we can form

$$F_0(y_0 \mid D = 0, X, Z) \Pr(D = 0 \mid X, Z) = \Pr(Y_0 \leqslant y_0, I < 0 \mid X, Z).$$

The left-hand side of this expression is known (we observe Y_0 when $D = 0$ and we know the probability that $D = 0$ given X, Z). The right-hand side can be written as

$$\Pr\left(Y_0 \leqslant y_0, \frac{U_I}{\sigma_{U_I}} < -\frac{\mu_I(X, Z)}{\sigma_{U_I}} \;\middle|\; X, Z\right).$$

In particular if $\mu_I(X, Z)$ can be made arbitrarily small ($\mu_I(X, Z) \to -\infty$), for a given X, we can recover the marginal distribution Y_0 from which we can recover $\mu_0(X)$, and hence the distribution of U_0.

From the definition of Y_0, $U_0 = Y_0 - \mu_0(X)$. We may write the preceding probability as

$$\Pr\left(U_0 \leqslant y_0 - \mu_0(X), \frac{U_I}{\sigma_{U_I}} < \frac{-\mu_I(X, Z)}{\sigma_{U_I}} \;\middle|\; X, Z\right).$$

Note that the X and Z can be varied and y_0 is a number. Thus, by varying the known y_0 and $\frac{\mu_I(X,Z)}{\sigma_{U_I}}$, we can trace out the joint distribution of $(U_0, \frac{U_I}{\sigma_{U_I}})$. Thus we can recover the joint distribution of

$$(Y_0, I) = \left(\mu_0(X) + U_0, \frac{\mu_I(X, Z) + U_I}{\sigma_{U_I}}\right).$$

Notice the three key ingredients required to recover the joint distribution:

(a) The independence between (U_0, U_I) and (X, Z).
(b) The assumption that we can make $\frac{\mu_I(X,Z)}{\sigma_{U_I}}$ arbitrarily small for a given X (so we get the marginal distribution of Y_0 and hence $\mu_0(X)$). As noted in Chapter 71, this type of identification-at-infinity assumption plays a key role in the entire selection and evaluation literature for identifying many important evaluation parameters, such as the average treatment effect and treatment on the treated.
(c) The assumption that $\frac{\mu_I(X,Z)}{\sigma_{U_I}}$ can be varied independently of $\mu_0(X)$. This enables us to trace out the joint distribution of $(U_0, \frac{U_I}{\sigma_{U_I}})$.[21]

[21] Another way to see how identification works is to note that from Cosslett (1983), Manski (1988), Matzkin (1992) and ingredients (a) and (b), we can express

$$F_0(y_0 \mid D = 0, X, Z) \Pr(D = 0 \mid X, Z)$$

as a function of $\mu_0(X)$ and $\frac{\mu_I(X,Z)}{\sigma_{U_I}}$. The dependence on X and Z operating only through the indices $\mu_0(X)$ and $\frac{\mu_I(X,Z)}{\sigma_{U_I}}$ is called index sufficiency. Varying the $\mu_0(X)$ and $\frac{\mu_I(X,Z)}{\sigma_{U_I}}$ traces out the distribution of $(U_0, \frac{U_I}{\sigma_{U_I}})$.

A parallel argument establishes identification of the distribution of (Y_1, I) given X and Z.

Identification of the Roy model follows from this analysis. Recall that the model assumes that $U_I = U_1 - U_0$ so $\sigma_{U_I}^2 = \mathrm{Var}(U_1 - U_0)$. From the distributions of (Y_0, I) and (Y_1, I), given X and Z, we can recover the joint distributions of $(U_0, \frac{U_1-U_0}{\sigma_{U_I}})$ and $(U_1, \frac{U_1-U_0}{\sigma_{U_I}})$ and hence the joint distribution of (U_0, U_1). We can recover the joint distribution of $U_1 - U_0$ even if $\mu_I(X, Z) \neq \mu_1(X) - \mu_0(X)$ as long as $U_C \equiv 0$.

2.7. *Using additional information*

We have established that data from social experiments or observational data corrected for selection do not in general identify joint distributions of potential outcomes. In the special case of the Roy model, choice data supplemented with outcome data will identify the joint distribution. But this result is fragile. For more general choice criteria, we cannot without further assumptions identify the joint distribution of potential outcomes. Recent approaches build on these results to supplement choice models with dependence assumptions to identify the joint distribution of (U_0, U_1).

Aakvik, Heckman and Vytlacil (2005), Carneiro, Hansen and Heckman (2001, 2003), Cunha, Heckman and Navarro (2005, 2006), and Cunha and Heckman (2007b, 2008) use factor models to capture the dependence across the unobservables (U_0, U_1, U_I) and to supplement the information used in order to construct the joint distribution of counterfactuals. Their approach is a version of the proxy/replacement function approach developed in Heckman and Robb (1985, 1986) that is discussed in Section 10 of Chapter 71 and in Chapter 73 (Matzkin) of this Handbook. It extends factor models developed by Jöreskog and Goldberger (1975) and Jöreskog (1977) to restrict the dependence among the (U_0, U_1, U_I). A low dimensional set of random variables generates the dependence across the outcome unobservables. Such dimension reduction coupled with the use of choice data and additional measurements that proxy or replace the factors can provide enough information to identify the joint distributions of (Y_0, Y_1) and (Y_0, Y_1, D).

The factor models are built around a conditional-independence assumption. Conditional on the factors, outcomes and choice equations are independent. Thus the factor models have a close affinity with matching except that they do not assume that the analyst observes the factors and must instead integrate them out and identify their distribution.

To demonstrate how this approach works, assume separability between observables and unobservables:

$$Y_1 = \mu_1(X) + U_1,$$
$$Y_0 = \mu_0(X) + U_0.$$

Denote I as the latent variable generating treatment choices:

$$I = \mu_I(Z) + U_I,$$

$$D = \mathbf{1}[I \geqslant 0].$$

Allow any X to be in Z so the notation is general.

To understand this approach, it is convenient but not essential to assume that (U_0, U_1, U_I) is normally distributed with mean zero and covariance matrix Σ. Normality plays no essential role in the analysis of this section. The key role is played by the factor structure assumption introduced below. Assume access to data on (Y, D, X, Z). We can identify $F_0(y_0 \mid D = 0, X, Z)$, $F_1(y_1 \mid D = 1, X, Z)$ and $\Pr(D = 1 \mid X, Z)$. Under certain conditions presented in Appendix B, Chapter 70 and the preceding section, we can identify the distributions of $(U_0, \frac{U_I}{\sigma_{U_I}})$ and $(U_1, \frac{U_I}{\sigma_{U_I}})$ nonparametrically. We can sometimes identify the scale on U_I.

To restrict the dependence across the unobservables, we adopt a factor structure model for the U_0, U_1, U_I. Other restrictions across the unobservables are possible. Models for a single factor are extensively developed by Jöreskog and Goldberger (1975). Aakvik, Heckman and Vytlacil (2005) and Carneiro, Hansen and Heckman (2001, 2003) extend their analysis to generate distributions of counterfactuals.

Initially assume a one-factor model where θ is a scalar factor (say unmeasured ability) that generates dependence across the unobservables assumed to be independent of (X, Z):

$$\begin{aligned}
&U_0 = \alpha_0\theta + \varepsilon_0,\\
&U_1 = \alpha_1\theta + \varepsilon_1,\\
&U_I = \alpha_{U_I}\theta + \varepsilon_{U_I},\\
&\theta \perp\!\!\!\perp (\varepsilon_0, \varepsilon_1, \varepsilon_{U_I}), \qquad (\varepsilon_0, \varepsilon_1, \varepsilon_{U_I}) \text{ are mutually independent.}
\end{aligned}$$

We discuss methods for multiple factors in the next section. Assume that $E(U_0) = 0$, $E(U_1) = 0$ and $E(U_I) = 0$. In addition, $E(\theta) = 0$. Thus $E(\varepsilon_0) = 0$, $E(\varepsilon_1) = 0$ and $E(\varepsilon_{U_I}) = 0$. To set the scale of the unobserved factor, normalize one "loading" (coefficient on θ) to 1. Note that all the dependence in the unobservables across equations arises from θ.

From the joint distributions of $(U_0, \frac{U_I}{\sigma_{U_I}})$ and $(U_1, \frac{U_I}{\sigma_{U_I}})$ we can identify

$$\begin{aligned}
\operatorname{Cov}\left(U_0, \frac{U_I}{\sigma_{U_I}}\right) &= \frac{\alpha_0\alpha_{U_I}}{\sigma_{U_I}}\sigma_\theta^2,\\
\operatorname{Cov}\left(U_1, \frac{U_I}{\sigma_{U_I}}\right) &= \frac{\alpha_1\alpha_{U_I}}{\sigma_{U_I}}\sigma_\theta^2,
\end{aligned}$$

assuming that the covariances on the left-hand side exist. From the ratio of the second covariance to the first we obtain $\frac{\alpha_1}{\alpha_0}$. Thus we obtain the sign of the dependence between U_0, U_1 because

$$\operatorname{Cov}(U_0, U_1) = \alpha_0\alpha_1\sigma_\theta^2.$$

From the ratio, we obtain α_1 if we normalize $\alpha_0 = 1$. Without further information, we cannot identify the variance of U_I, $\sigma_{U_I}^2$. We normalize it to 1. (Alternatively, we could normalize the variance of ε_{U_I} to 1.) Below, we present a condition that sets the scale of U_I.

With additional information, one can identify the full joint distribution of (U_0, U_1, U_I) and hence can construct the joint distribution of potential outcomes. In this section, we show this by a series of examples for a normal model. In a normal model, the joint distribution of (Y_0, Y_1) is determined (given X) if one can identify the variances of Y_0 and Y_1 and their covariance. We then show that normality plays no essential role in this analysis. We first consider what can be identified from access to a proxy M for θ (e.g., a test score).

2.7.1. Some examples

EXAMPLE 1 *(Access to a single proxy measure (e.g., a test score)).* Assume access to data on Y_0 given $D = 0$, X, Z; to data on Y_1 given $D = 1$, X, Z; and to data on D given X, Z. Suppose that the analyst also has access to a proxy for θ. Denote the proxy measure by M. In a schooling example, it could be a test score:

$$M = \mu_M(X) + U_M,$$

where

$$U_M = \alpha_M\theta + \varepsilon_M,$$

so

$$M = \mu_M(X) + \alpha_M\theta + \varepsilon_M,$$

where ε_M is independent of ε_0, ε_1, ε_{U_I} and θ, as well as (X, Z) ($\varepsilon_M \perp\!\!\!\perp (\varepsilon_0, \varepsilon_1, \varepsilon_{U_I}, \theta, X, Z)$). We can identify the mean $\mu_M(X)$ from observations on M and X. From this additional information, we acquire three additional covariance terms, conditional on X, Z, where we keep the conditioning implicit and define I as normalized by σ_{U_I}:

$$\begin{aligned}
\operatorname{Cov}(Y_1, M) &= \alpha_1\alpha_M\sigma_\theta^2,\\
\operatorname{Cov}(Y_0, M) &= \alpha_0\alpha_M\sigma_\theta^2,\\
\operatorname{Cov}(I, M) &= \frac{\alpha_{U_I}}{\sigma_{U_I}}\alpha_M\sigma_\theta^2.
\end{aligned}$$ [22]

Suppose that we normalize the loading on the proxy (or test score) to one ($\alpha_M = 1$). It is no longer necessary to normalize $\alpha_0 = 1$ as in the preceding section. From the ratio of the covariance of Y_1 with I with the covariance of I with M, we obtain the right-hand

[22] Conditioning on X, Z, we can remove the dependence of Y_1, Y_0, M and I on these variables and effectively work with the residuals $Y_0 - \mu_0(X) = U_0$, $Y_1 - \mu_1(X) = U_1$, $M - \mu_M(X) = U_M$, $I - \mu_I(Z) = U_I$, where we keep the scale on I implicit.

side of

$$\frac{\mathrm{Cov}(Y_1, I)}{\mathrm{Cov}(I, M)} = \frac{\alpha_1 \alpha_{U_I} \sigma_\theta^2}{\alpha_{U_I} \alpha_M \sigma_\theta^2} = \alpha_1,$$

because $\alpha_M = 1$ (normalization). From the discussion in the preceding section where no proxy is assumed, we obtain α_0 since

$$\frac{\mathrm{Cov}(Y_1, I)}{\mathrm{Cov}(Y_0, I)} = \frac{\alpha_1 \alpha_{U_I} \sigma_\theta^2}{\alpha_0 \alpha_{U_I} \sigma_\theta^2} = \frac{\alpha_1}{\alpha_0}.$$

From knowledge of α_1 and α_0 and the normalization for α_M, we obtain σ_θ^2 from $\mathrm{Cov}(Y_1, M)$ or $\mathrm{Cov}(Y_0, M)$. We obtain α_{U_I} (up to scale σ_{U_I}) from $\mathrm{Cov}(I, M) = \frac{\alpha_{U_I} \alpha_M \sigma_\theta^2}{\sigma_{U_I}}$ since we know α_M $(= 1)$ and σ_θ^2. The model is overidentified. We can identify the scale of σ_{U_I} by a standard argument from the discrete-choice literature. We review this argument below.

Observe that if we write out the decision rule in terms of costs, we can characterize the latent variable determining choices as:

$$I = Y_1 - Y_0 - C,$$

where $C = \mu_C(Z) + U_C$ and $U_C = \alpha_C \theta + \varepsilon_C$, where ε_C is independent of θ and the other ε's. $E(U_C) = 0$ and U_C is independent of (X, Z). Then, $U_I = U_1 - U_0 - U_C$ and

$$\begin{aligned} \alpha_{U_I} &= \alpha_1 - \alpha_0 - \alpha_C, \\ \varepsilon_{U_I} &= \varepsilon_1 - \varepsilon_0 - \varepsilon_C, \\ \mathrm{Var}(\varepsilon_{U_I}) &= \mathrm{Var}(\varepsilon_1) + \mathrm{Var}(\varepsilon_0) + \mathrm{Var}(\varepsilon_C). \end{aligned}$$

Identification of α_0, α_1 and α_{U_I} implies identification of α_C. Identification of the variance of ε_{U_I} implies identification of the variance of ε_C since the variances of ε_1 and ε_0 are known.

Observe further that the scale σ_{U_I} is identified if there are variables in X but not in Z [see Heckman (1976, 1979), Heckman and Robb (1985, 1986), Willis and Rosen (1979)].[23] From the variance of M given X, we obtain $\mathrm{Var}(\varepsilon_M)$ since we know $\mathrm{Var}(M)$ (conditional on X) and we know $\alpha_M^2 \sigma_\theta^2$:

$$\mathrm{Var}(M) - \alpha_M^2 \sigma_\theta^2 = \sigma_{\varepsilon_M}^2.$$

(Recall that we keep the conditioning on X implicit.) By similar reasoning, it is possible to identify $\mathrm{Var}(\varepsilon_0)$, $\mathrm{Var}(\varepsilon_1)$ and the fraction of $\mathrm{Var}(U_I)$ due to ε_{U_I}. We can thus

[23] The easiest case to understand is one where $\mu_C(Z) = Z\gamma$, $\mu_1(X) = X\beta_1$, $\mu_0(X) = X\beta_0$ and $\mu_I(Z, X) = X(\beta_1 - \beta_0) - Z\gamma$. We identify the coefficients of the index $\mu_I(Z, X)$ up to scale σ_{U_I}, but we know $\beta_1 - \beta_0$ from the earnings functions. Thus if one X is not in Z and its associated coefficient is not zero, we can identify σ_{U_I}. See, e.g., Heckman (1976).

construct the joint distribution of (Y_0, Y_1, C) and hence the joint distribution of (Y_0, Y_1) since we identified $\mu_C(Z)$ and all of the factor loadings. Thus we can identify the objective outcome distribution for (Y_0, Y_1) and the subjective distribution for C as well as their joint distribution (Y_0, Y_1, C).

We have assumed normality because it is convenient to do so. Carneiro, Hansen and Heckman (2003), Cunha, Heckman and Navarro (2005, 2006) and Cunha and Heckman (2008) show that it is possible to nonparametrically identify the distributions of θ, ε_0, ε_1, ε_{U_I} and ε_M so our results do not hinge on arbitrary distributional assumptions as we establish in the next section.

We next show by way of example that choice data are not strictly required to secure identification of the joint distributions of counterfactuals. It is the extra information joined with the factor restriction on the dependence that allows us to identify the joint distribution of outcomes.

EXAMPLE 2 *(Identification without choice data).* This example builds on Example 1. Let M be two dimensional so $M = (M_1, M_2)$, and M_1, M_2 are indicators that depend on θ and assume that they are both observed. In place of I from choice theory as in the preceding section, we can work with a second indicator of θ, i.e., a second measurement M_2. Suppose that either by limit operations ($P(X, Z) \to 0$ or $P(X, Z) \to 1$ along certain sequences in its support) or some randomization we observe triplets (Y_0, M_1, M_2), (Y_1, M_1, M_2) but not Y_0 and Y_1 together. We can still identify the joint distribution of (Y_0, Y_1).

Example 1 applies to this case with only trivial modifications. We can identify all of the variances and covariances of the factor model as well as the factor loadings up to one normalization. Thus we can identify the joint distribution of (Y_0, Y_1). Since the (M_1, M_2) are assumed to be observed and their scale is known, we can identify the variances of M_1 and M_2 directly. In this example, we do not need to use any of the apparatus of discrete-choice theory except to govern the limit operations that control for selection.

There are other ways to construct the joint distributions that do not require a proxy M that may be extended to the model. Access to panel data on earnings affords identification. One way, that motivates our analysis of *ex ante* vs. *ex post* returns developed later, is given next.

EXAMPLE 3 *(Two (or more) periods of panel data on outcomes).* Suppose that for each person we have two periods of outcome data in one counterfactual state or the other. Thus we observe $(Y_{0,1}, Y_{0,2})$ or $(Y_{1,1}, Y_{1,2})$ but never both pairs of vectors together for the same person. We also observe choices. We assume that $Y_{j,t} = \mu_{j,t}(X) + U_{j,t}$, $j = 0, 1$, $t = 1, 2$, and write

$$U_{1,t} = \alpha_{1,t}\theta + \varepsilon_{1,t} \quad \text{and} \quad U_{0,t} = \alpha_{0,t}\theta + \varepsilon_{0,t}$$

to obtain

$$Y_{1,t} = \mu_{1,t}(X) + \alpha_{1,t}\theta + \varepsilon_{1,t}, \quad t = 1, 2,$$

$$Y_{0,t} = \mu_{0,t}(X) + \alpha_{0,t}\theta + \varepsilon_{0,t}, \quad t = 1, 2.$$

In the context of a schooling choice model as analyzed by Carneiro, Hansen and Heckman (2001, 2003) and Cunha, Heckman and Navarro (2005, 2006), if we assume that the interest rate is zero and that agents maximize the present value of their income, the index generating choices is

$$I = (Y_{1,2} + Y_{1,1}) - (Y_{0,2} + Y_{0,1}) - C,$$
$$D = \mathbf{1}[I \geqslant 0],$$

where C was defined previously, and

$$I = \mu_{1,1}(X) + \mu_{1,2}(X) - \mu_{0,1}(X) - \mu_{0,2}(X) - \mu_C(Z) + U_{1,1} + U_{1,2} - U_{0,1} - U_{0,2} - U_C.$$

We assume no proxy—just two periods of panel data. The multiple periods of earnings serve as the proxy.

Under normality, application of the standard normal selection model allows us to identify $\mu_{1,t}(X)$ for $t = 1, 2$; $\mu_{0,t}(X)$ for $t = 1, 2$ and $\mu_{1,1}(X) + \mu_{1,2}(X) - \mu_{0,1}(X) - \mu_{0,2}(X) - \mu_C(Z)$, the latter up to a scalar σ_{U_I} where

$$U_I = U_{1,1} + U_{1,2} - U_{0,1} - U_{0,2} - U_C.$$

Following our discussion of Example 1, we can recover the scale σ_{U_I} if there are variables in X that are not in Z such that $(\mu_{1,1}(X) + \mu_{1,2}(X) - (\mu_{0,1}(X) + \mu_{0,2}(X)))$ can be varied independently from $\mu_C(Z)$. To simplify the analysis, we assume that this condition holds.[24]

From normality, we can recover the joint distributions of $(I, Y_{1,1}, Y_{1,2})$ and $(I, Y_{0,1}, Y_{0,2})$ but not directly the joint distribution of $(I, Y_{1,1}, Y_{1,2}, Y_{0,1}, Y_{0,2})$. Thus, conditioning on X and Z, we can recover the joint distribution of $(U_I, U_{0,1}, U_{0,2})$ and $(U_I, U_{1,1}, U_{1,2})$ but apparently not that of $(U_I, U_{0,1}, U_{0,2}, U_{1,1}, U_{1,2})$. However, under our factor structure assumptions, this joint distribution can be recovered as we next show.

From the available data, we can identify the following covariances:

$$\begin{aligned}
\operatorname{Cov}(U_I, U_{1,2}) &= (\alpha_{1,2} + \alpha_{1,1} - \alpha_{0,2} - \alpha_{0,1} - \alpha_C)\alpha_{1,2}\sigma_\theta^2, \\
\operatorname{Cov}(U_I, U_{1,1}) &= (\alpha_{1,2} + \alpha_{1,1} - \alpha_{0,2} - \alpha_{0,1} - \alpha_C)\alpha_{1,1}\sigma_\theta^2, \\
\operatorname{Cov}(U_I, U_{0,1}) &= (\alpha_{1,2} + \alpha_{1,1} - \alpha_{0,2} - \alpha_{0,1} - \alpha_C)\alpha_{0,1}\sigma_\theta^2, \\
\operatorname{Cov}(U_I, U_{0,2}) &= (\alpha_{1,2} + \alpha_{1,1} - \alpha_{0,2} - \alpha_{0,1} - \alpha_C)\alpha_{0,2}\sigma_\theta^2, \\
\operatorname{Cov}(U_{1,1}, U_{1,2}) &= \alpha_{1,1}\alpha_{1,2}\sigma_\theta^2, \\
\operatorname{Cov}(U_{0,1}, U_{0,2}) &= \alpha_{0,1}\alpha_{0,2}\sigma_\theta^2.
\end{aligned}$$

[24] If not, then $\mu_C(Z)$, $\sigma^2_{\varepsilon_C}$ and α_C are only identified up to normalizations.

If we normalize $\alpha_{0,1} = 1$ (recall that one normalization is needed to set the scale of θ), we can form the ratios

$$\frac{\operatorname{Cov}(U_I, U_{1,2})}{\operatorname{Cov}(U_I, U_{0,1})} = \alpha_{1,2}, \qquad \frac{\operatorname{Cov}(U_I, U_{1,1})}{\operatorname{Cov}(U_I, U_{0,1})} = \alpha_{1,1},$$
$$\frac{\operatorname{Cov}(U_I, U_{0,2})}{\operatorname{Cov}(U_I, U_{0,1})} = \alpha_{0,2}.$$

From these coefficients and the remaining covariances, using $\operatorname{Cov}(U_{1,1}, U_{1,2})$ and/or $\operatorname{Cov}(U_{0,1}, U_{0,2})$, we identify σ_θ^2. Thus if the factor loadings are nonzero, we can identify σ_θ^2 from two relationships, both of which are identified:

$$\frac{\operatorname{Cov}(U_{1,1}, U_{1,2})}{\alpha_{1,1}\alpha_{1,2}} = \sigma_\theta^2$$

and

$$\frac{\operatorname{Cov}(U_{0,1}, U_{0,2})}{\alpha_{0,1}\alpha_{0,2}} = \sigma_\theta^2.$$

Since we know $\alpha_{1,1}\alpha_{2,2}$ and $\alpha_{0,1}\alpha_{0,2}$, we can recover σ_θ^2 from $\operatorname{Cov}(U_{1,1}, U_{1,2})$ and $\operatorname{Cov}(U_{0,1}, U_{0,2})$. We can also recover α_C since we know σ_θ^2, $\alpha_{1,2} + \alpha_{1,1} - \alpha_{0,2} - \alpha_{0,1} - \alpha_C$, and $\alpha_{1,1}$, $\alpha_{1,2}$, $\alpha_{0,1}$, $\alpha_{0,2}$. We can form (conditional on X)

$$\operatorname{Cov}(Y_{1,1}, Y_{0,1}) = \alpha_{1,1}\alpha_{0,1}\sigma_\theta^2; \quad \operatorname{Cov}(Y_{1,2}, Y_{0,1}) = \alpha_{1,2}\alpha_{0,1}\sigma_\theta^2;$$
$$\operatorname{Cov}(Y_{1,1}, Y_{0,2}) = \alpha_{1,1}\alpha_{0,2}\sigma_\theta^2 \quad \text{and} \quad \operatorname{Cov}(Y_{1,2}, Y_{0,2}) = \alpha_{1,2}\alpha_{0,2}\sigma_\theta^2.$$

We can identify $\mu_C(Z)$ from the schooling choice equation since we know $\mu_{0,1}(X)$, $\mu_{0,2}(X)$, $\mu_{1,1}(X)$, $\mu_{1,2}(X)$ and we have assumed that there are some Z not in X so that σ_{U_I} is identified. Thus we can identify the joint distribution of $(Y_{0,1}, Y_{0,2}, Y_{1,1}, Y_{1,2}, C)$.

These examples extend to nonnormal and nonparametric models. The key idea to constructing joint distributions of counterfactuals using the analysis of Cunha and Heckman (2008) and Cunha, Heckman and Navarro (2005, 2006) is *not* the factor structure for unobservables although it is convenient. The crucial idea is the assumption that a low dimensional set of random variables generates the dependence across outcomes. Other low dimensional representations such as the ARMA model or the dynamic factor structure model [see Sargent and Sims (1977)] can also be used. Cunha and Heckman (2007a) and Cunha, Heckman and Schennach (2007) extend factor models to more general frameworks where the θ evolve over time as in state space models. The factor structure model presented in this section is easy to exposit and has been used to estimate joint distributions of counterfactuals. We present some examples in a later subsection. That subsection reviews recent work that generalizes the analysis of this section to derive *ex ante* and *ex post* outcome distributions, and measure the fundamental uncertainty facing agents in the labor market. With these methods it is possible to compute the distributions of both *ex ante* and *ex post* returns to treatments. Before presenting a more general analysis, we relate factor models to matching models.

2.7.2. Relationship to matching

If the analyst knew θ and could condition on it, the analyst would obtain the conditional-independence assumption of matching, (M-1), in Chapter 71:

(U-1) $(Y_0, Y_1) \perp\!\!\!\perp D \mid X, Z, \theta$.

This is also the general control function assumption (U-1) in Chapter 71.

The approach developed by Aakvik, Heckman and Vytlacil (2005), Carneiro, Hansen and Heckman (2001, 2003), Cunha, Heckman and Navarro (2005, 2006), and Cunha and Heckman (2007b, 2007c, 2008) extends matching and treats θ as an unobservable. It uses proxies for θ and identifies the distribution of θ under the following assumption:

(U-2) $\theta \perp\!\!\!\perp X, Z$.

Thus the factor approach is a version of matching on unobservables, where the unobserved match variables are integrated out.

2.7.3. Nonparametric extensions

The analysis of the generalized Roy model developed in Appendix B of Chapter 70 establishes conditions under which it is possible to nonparametrically identify the joint distribution of (Y_0, I, M) given X, Z and the joint distribution of (Y_1, I, M) given X, Z, where we also allow the functions determining M to be nonparametrically determined.[25] These conditions can be extended to provide identification of the distributions of (Y_0, I, M) and (Y_1, I, M) where M is observed for all persons treated or not whereas Y_0 and Y_1 are observed only if $D = 0$ or $D = 1$, respectively. The identification conditions are also easily extended to account for vector Y_0 and Y_1 (e.g., $Y_0 = (Y_{0,1}, Y_{0,2})$ and $Y_1 = (Y_{1,1}, Y_{1,2})$) as our third example in Section 2.7.1 reveals. We present a general theorem for the identification of state-contingent outcomes free of selection bias in the next section and in Appendix B of this chapter. With the state-contingent distributions nonparametrically identified, we can apply factor analysis to identify the factor loadings because we identify the required covariances as a by-product of our nonparametric analysis.

With the α_j (or $\alpha_{i,j}$) in hand, we can nonparametrically identify the distribution of θ and the ε_j (or $\varepsilon_{i,j}$) for the different models assuming mutual independence between θ and all of the components of ε_j (or $\varepsilon_{i,j}$) using Kotlarski's Theorem [Kotlarski (1967), Prakasa-Rao (1992)]. That theorem states that, for any pair of random variables T_1, T_2 generated by a common random variable θ, we can nonparametrically identify the distribution of θ and the associated components of errors: ε_1 and ε_2. Stated precisely:

[25] Recall that, depending on the assumptions discussed in Section 2.7.1, the scale of I may, or may not, be identified.

THEOREM 1. *If*

$$T_1 = \theta + \varepsilon_1$$

and

$$T_2 = \theta + \varepsilon_2$$

and $(\theta, \varepsilon_1, \varepsilon_2)$ *are mutually independent, the means of all three generating random variables are finite and are normalized to* $E(\varepsilon_1) = E(\varepsilon_2) = 0$*, and the random variables possess nonvanishing* (a.e.) *characteristic functions, then the densities of* $(\theta, \varepsilon_1, \varepsilon_2)$, $g_\theta(\theta)$, $g_1(\varepsilon_1)$, $g_2(\varepsilon_2)$*, respectively, are identified.*

PROOF. See Kotlarski (1967). See also Prakasa-Rao (1992). □

Applied to our context, consider the first two equations of a vector of indicators M which are stochastically dependent only through θ. We write

$$M_1 = \lambda_1 \theta + \varepsilon_1, \quad \text{where } \lambda_1 = 1,$$
$$M_2 = \lambda_2 \theta + \varepsilon_2, \quad \text{where } \lambda_2 \neq 0.$$

By the preceding analysis, we can identify λ_2 (subject to a normalization $\lambda_1 = 1$) from factor models. Thus we can rewrite these equations as

$$M_1 = \theta + \varepsilon_1,$$
$$\frac{M_2}{\lambda_2} = \theta + \varepsilon_2^*,$$

where $\varepsilon_2^* = \varepsilon_2/\lambda_2$. Applying Kotlarski's Theorem, we can nonparametrically identify the densities $g_\theta(\theta)$, $g_1(\varepsilon_1)$ and $g_2(\varepsilon_2^*)$. Since we know λ_2, we can nonparametrically identify $g_2(\varepsilon_2)$. Schennach (2004), Hu and Schennach (2006), and Cunha, Heckman and Schennach (2007) weaken many of the strong independence conditions to mean independence assumptions. Carneiro, Hansen and Heckman (2003) extend the analysis of this section to the case of vector θ.

2.8. General models

The analysis of Carneiro, Hansen and Heckman (2003), Cunha, Heckman and Navarro (2005, 2006) and Cunha and Heckman (2007c, 2008) generalizes the analysis of the preceding sections to consider vectors of outcomes (Y_0 and Y_1), vectors of measurements (M) and more general choice equations. We summarize that work here. This analysis feeds directly into our analysis of dynamic treatment effects and dynamic discrete choice presented in Section 3.

Our analysis has three components: (1) Identifying the choice of treatment equation and hence evaluation of treatments as perceived by agents; (2) Identifying the joint distributions of outcomes and measurements in each treatment state s, $s = 1, \ldots, \bar{S}$, where $\bar{S}$ is the number of treatment states; and (3) Identifying the joint distribution of outcomes across treatment states. Only the third step requires a factor structure. Step 1 is conventional nonparametric discrete-choice analysis. Step 2 solves the selection problem using nonparametric methods. Step 3 solves the evaluation problem using factor models.

Conditions for nonparametric identification of discrete-choice models are presented in Matzkin (1992, 1993, 1994) and in her contribution to this Handbook (Chapter 73). Appendix B of Chapter 70 presents a nonparametric proof of identification of choice equations as part of a nonparametric analysis of choice and outcome equations for a general static discrete-choice model. Carneiro, Hansen and Heckman (2003) present a parallel analysis for an ordered choice model.[26] Heckman and Navarro (2007) present an identification analysis that is used in this section and in Section 3. We now establish an extension of the theorem proved in Appendix B of Chapter 70 to account for vectors of outcomes and for associated vectors of measurements. This provides a solution to the selection problem.

2.8.1. Steps 1 and 2: Solving the selection problem within each treatment state

Associated with each treatment s, $s = 1, \ldots, \bar{S}$, is a vector of outcomes of length $\bar{A}$,

$$Y\bigl(s, X, U(s)\bigr) = \bigl(Y\bigl(1, s, X, U(1, s)\bigr), \ldots, Y\bigl(a, s, X, U(a, s)\bigr), \ldots,\\ Y\bigl(\bar{A}, s, X, U(\bar{A}, s)\bigr)\bigr).$$

They depend on observables X and unobservables $U(s) = (U(1, s), \ldots, U(a, s), \ldots, U(\bar{A}, s))$, where the observability distinction is made from the point of view of the econometrician. The X may also have a- and s-specific subvectors, but for the sake of notational simplicity we do not make this explicit. We can make the list of outcomes s-dependent, but only at the cost of notational complexity. Elements of $Y(s, X, U(s))$ are outcomes associated with receiving treatment s. They are factual outcomes if treatment s is actually selected, which we denote by $D(s) = 1$. Outcomes corresponding to treatments s' that are not selected—we denote this by $D(s') = 0$—are counterfactuals. The outcome variables are not necessarily what the agent thinks will happen when he or she chooses treatment s, but rather what actually happens. The treatments s may be associated with stages that are not necessarily identical with real time events, although this framework can be used in our analysis of dynamic choices evolving in real time that is presented in Section 3.

Henceforth, whenever we have random variables with multiple arguments $R_0(s, Q_0, \ldots)$ or $R_1(a, s, Q_0, \ldots)$ where the argument list begins with treatment state s or both age a and state s (perhaps followed by other arguments $Q_0, \ldots$), we will make use of several condensed notations: (a) dropping the first argument as we collect the

[26] Cunha, Heckman and Navarro (2007) present a nonparametric identification analysis of the ordered choice model. They also establish that it imposes the absence of option values.

components into vectors $R_0(Q_0, \ldots)$ or $R_1(s, Q_0, \ldots)$ of length $\bar{S}$ or $\bar{A}$, respectively, and (b) going further in the case of R_1, dropping the s argument as we collect the vectors $R_1(s, Q_0, \ldots)$ into a single $\bar{S} \times \bar{A}$ array $R_1(Q_0, \ldots)$, but also (c) suppressing one or more of the other arguments and writing $R_1(a, s)$ or $R_1(a, s, Q_0)$ instead of $R_1(a, s, Q_0, Q_1, \ldots)$, etc. This notation is sufficiently rich to represent the life cycle of outcomes for persons who receive treatment s. We use this notation in the remainder of this section and in Section 3.

Following Carneiro, Hansen and Heckman (2003), the variables in $Y(a, s, X, U(a, s))$ may include discrete, continuous or mixed discrete-continuous components. For the discrete or mixed discrete-continuous cases, we assume that latent continuous variables cross thresholds to generate the discrete components. Durations can be generated by latent index models associated with each outcome crossing thresholds analogous to the model we develop in Section 3 below, in the discussion surrounding Equation (3.11). In this framework, we can model the effect of attaining s years of schooling on durations of unemployment or durations of employment.

We decompose $Y(a, s)$ into continuous and discrete components:

$$Y(a, s) = \begin{bmatrix} Y_c(a, s) \\ Y_d(a, s) \end{bmatrix}.$$

Associated with the jth component of $Y_d(a, s)$, $Y_{d,j}(a, s)$ is a latent variable $Y^*_{d,j}(a, s)$. We define

$$Y_{d,j}(a, s) = \mathbf{1}\big(Y^*_{d,j}(a, s) \geqslant 0\big).$$[27]

From standard results in the discrete-choice literature, without additional information, we can only know $Y^*_{d,j}(a, s)$ up to scale.

We assume an additively separable model for the continuous variables and latent continuous indices. Making the X explicit, we write

$$\begin{aligned} &Y_c(a, s, X) = \mu_c(a, s, X) + U_c(a, s), \\ &Y^*_d(a, s, X) = \mu_d(a, s, X) - U_d(a, s), \\ &\quad 1 \leqslant s \leqslant \bar{S},\ 1 \leqslant a \leqslant \bar{A}. \end{aligned}$$

We array the $Y_c(a, s, X)$ into a matrix $Y_c(s, X)$ and the $Y^*_d(a, s, X)$ into a matrix $Y^*_d(s, X)$. We decompose these vectors into components corresponding to the means $\mu_c(s, X)$, $\mu_d(s, X)$ and the unobservables $U_c(s)$, $U_d(s)$. Thus

$$\begin{aligned} &Y_c(s, X) = \mu_c(s, X) + U_c(s), \\ &Y^*_d(s, X) = \mu_d(s, X) - U_d(s). \end{aligned}$$

[27] Extensions to nonbinary discrete outcomes are straightforward. Thus we could entertain, at greater notational cost, a multinomial outcome model at each age a for each counterfactual state.

$Y_d^*(s, X)$ generates $Y_d(s, X)$. Using our condensed notation, we write

$$Y_c(X) = \mu_c(X) + U_c,$$
$$Y_d^*(X) = \mu_d(X) - U_d.$$

Following Carneiro, Hansen and Heckman (2003), Cunha, Heckman and Navarro (2005, 2006) and Cunha and Heckman (2007b, 2007c, 2008), we may also have a system of measurements with both discrete and continuous components. The measurements are not s-indexed. They are the same for each treatment state.[28] We write the equations for the measurements in an additively separable form, in a fashion comparable to those of the outcomes. The equations for the continuous measurements and latent indices producing discrete measurements are

$$M_c(a, X) = \mu_{c,M}(a, X) + U_{c,M}(a),$$
$$M_d^*(a, X) = \mu_{d,M}(a, X) - U_{d,M}(a),$$

where the discrete variable corresponding to the jth index in $M_d^*(a, X)$ is

$$M_{d,j}(a, X) = \mathbf{1}\big(M_{d,j}^*(a, X) \geqslant 0\big).$$

The measurements play the role of indicators unaffected by the process being studied. We array $M_c(a, X)$ and $M_d^*(a, X)$ into matrices $M_c(X)$ and $M_d^*(X)$. We array $\mu_{c,M}(a, X)$, $\mu_{d,M}(a, X)$ into matrices $\mu_{c,M}(X)$ and $\mu_{d,M}(X)$. We array the corresponding unobservables into $U_{c,M}$ and $U_{d,M}$. In this notation,

$$M_c(X) = \mu_{c,M}(X) + U_{c,M},$$
$$M_d^*(X) = \mu_{d,M}(X) - U_{d,M}.$$

In the notation of Appendix B of Chapter 70, write the utility valuation of treatment state s as

$$R(s, Z) = \mu_R(s, Z) - V(s), \quad s = 1, \ldots, \bar{S}.$$

Collect $R(s, Z)$, $s = 1, \ldots, \bar{S}$, into a vector

$$R(Z) = \big(R(1, Z), \ldots, R(\bar{S}, Z)\big).$$

Collect $\mu_R(s, Z)$, $s = 1, \ldots, \bar{S}$, into a vector

$$\mu_R(Z) = \big(\mu_R(1, Z), \ldots, \mu_R(\bar{S}, Z)\big).$$

Collect $V(s)$, $s = 1, \ldots, \bar{S}$, into a vector

$$V = \big(V(1), \ldots, V(\bar{S})\big).$$

[28] Thus measurements are not causally affected by treatment. Measurements that are causally affected by treatment can be included in the model as outcomes using the analysis of Hansen, Heckman and Mullen (2004).

$D(s) = 1$ (state s is selected) if

$$s = \underset{j=1,\dots,\bar{S}}{\operatorname{argmax}} \{R(j, Z)\}.$$

Otherwise $D(s) = 0$.

$$\sum_{j=1}^{\bar{S}} D(j) = 1.$$

Define

$$V^s = \big(V(s) - V(1), \dots, V(s) - V(\bar{S})\big),$$
$$\mu_R^s(Z) = \big(\mu_R(s, Z) - \mu_R(1, Z), \dots, \mu_R(s, Z) - \mu_R(\bar{S}, Z)\big), \quad s = 1, \dots, \bar{S}.$$

These contrast vectors are standard in discrete-choice theory, where utilities in treatment state s are compared with utilities in other treatment states. We assume that we have access to a large i.i.d. sample from the distribution of $(Y_c, Y_d, M_c, M_d, \{D(s)\}_{s=1}^{\bar{S}})$.[29]

We now state a basic theorem that solves the selection problem (Step 2) for the general model of this section. We draw on the work of Matzkin (1992, 1993, 1994) and Chapter 73 of this Handbook to provide a general characterization of nonparametric functions and their identifiability. We define the Matzkin class of functions in Appendix B and use it in the next proof. They include all of the familiar linear-in-parameters functional forms for discrete choice as well as a variety of other classes of functions that can be identified under conditions specified in her papers.

THEOREM 2. *The joint distribution of $(U_c(s), U_d(s), U_{c,M}, U_{d,M}, V^s)$ is identified along with the functions $(\mu_c(s, X), \mu_d(s, X), \mu_{c,M}(X), \mu_{d,M}(X), \mu_R^s(Z))$ (the components of $\mu_d(s, X)$ and $\mu_{d,M}(X)$ over the supports admitted by the supports of the errors) if, for $s = 1, \dots, \bar{S}$,*

(i) $E[U_c(s)] = E[U_{c,M}] = 0$. *$(U_c(s), U_d(s), U_{c,M}, U_{d,M}, V^s)$ are continuous random variables with support $(\underline{U}_c(s), \overline{U}_c(s)) \times (\underline{U}_d(s), \overline{U}_d(s)) \times (\underline{U}_{c,M}, \overline{U}_{c,M}) \times (\underline{U}_{d,M}, \overline{U}_{d,M}) \times \mathbb{R}^{s-1}$. These conditions are assumed to apply within each component of each subvector. The joint system is thus variation free for each component with respect to every other component.*

(ii) $(U_c(s), U_d(s), U_{c,M}, U_{d,M}, V^s) \perp\!\!\!\perp (X, Z)$.

(iii) $\operatorname{Supp}(\mu_R^s(Z), X) = \operatorname{Supp}(\mu_R^s(Z)) \times \operatorname{Supp}(X)$.

(iv) $\operatorname{Supp}(\mu_d(s, X), \mu_{d,M}(X)) \supseteq \operatorname{Supp}(U_d(s), U_{d,M})$.

(v) *$\mu_c(s, X)$, $\mu_{c,M}(X)$ and $\mu_R(Z)$ are continuous functions. The components of the $\mu_d(s, X)$ and $\mu_{d,M}(X)$ belong to the Matzkin class of functions given in Appendix B. $\mu_R^1(z)$ is known for $z \in \widetilde{\mathcal{Z}}$ with $\widetilde{\mathcal{Z}} \subseteq \operatorname{Supp}(Z)$ such that $\{\mu_R^1(z); z \in \widetilde{\mathcal{Z}}\} = \mathbb{R}^{\bar{S}-1}$. $\mu_R(1, Z)$ is known.*

[29] We can allow for dependence across individuals by invoking appropriate limit laws for dependent random variables.

PROOF. See Appendix B.[30] $\square$

This proof presents conditions for producing a selection-bias free joint distribution of $(Y_c(s, X),\ Y_d(s, X),\ M_c(X),\ M_d(X),\ V^s)$, $s = 1, \dots, \bar{S}$ conditionally on X which are the inputs for our factor analysis to which we now turn.

2.8.2. *Step 3: Constructing counterfactual distributions using factor models*

The analysis of the preceding section presented conditions under which subjective relative evaluations of treatment outcomes from choice functions and objective outcome distributions in state s, $s = 1, \dots, \bar{S}$, can be identified. Missing is an analysis of identification of joint outcome distributions. In this subsection, we generalize the analysis of Section 2.7 to present conditions under which joint distributions can be identified in a multifactor setting.

Theorem 2 gives conditions under which the distributions of $(U_c(s), U_d(s), U_{c,M}, U_{d,M}, V^s)$, $s = 1, \dots, \bar{S}$, are identified. If we factor analyze these errors, we can identify the joint distributions of these vectors across treatment states. We write in the case of vector θ,

$$\begin{aligned}
U_c(s) &= \alpha'_{c,s}\theta + \varepsilon_c(s),\\
U_d(s) &= \alpha'_{d,s}\theta + \varepsilon_d(s),\\
U_{c,M} &= \alpha'_{c,M}\theta + \varepsilon_{c,M},\\
U_{d,M} &= \alpha'_{d,M}\theta + \varepsilon_{d,M},\\
V^s &= \alpha'_{V^s}\theta + \varepsilon_V(s),
\end{aligned} \tag{2.7}$$

or more compactly, using the notation

$$\begin{aligned}
U(s) &= \bigl(U_c(s), U_d(s), U_{c,M}, U_{d,M}, V^s\bigr),\\
\varepsilon(s) &= \bigl(\varepsilon_c(s), \varepsilon_d(s), \varepsilon_{c,M}, \varepsilon_{d,M}, \varepsilon_V(s)\bigr),
\end{aligned}$$

we may write the preceding system as a system of equations:

$$U(s) = \Lambda(s)\theta + \varepsilon(s), \quad s = 1, \dots, \bar{S}, \tag{2.8}$$

where the components of $\varepsilon = (\varepsilon(1), \dots, \varepsilon(\bar{S}))$ are mutually independent and $\varepsilon \perp\!\!\!\perp \theta$. The factor loadings may differ across treatment states. All of the dependence among outcomes and measurements and the choice indicators $\{D(s)\}_{s=1}^{\bar{S}}$ is generated by dependence on common factors θ. The outcome, choice, and measurement equations all contribute to the $U(s)$ and are a source of information on the distribution of θ.

The same principles guide identifiability in this system of equations as in the one-factor models analyzed in Section 2.7. With enough measurements, outcomes and

[30] Matzkin (1993) presents alternative sets of conditions for identifiability of the choice model.

choices relative to the dimensionality of θ, it is possible to identify the joint distribution of outcomes across counterfactual states.

Identification problems in factor analysis were first clearly stated by Anderson and Rubin (1956). If, for example, there are $L(s)$ components of $U(s)$ and θ is $K \times 1$, $\varepsilon(s)$ is $L(s) \times 1$ and $\Lambda(s)$ is $L(s) \times K$. Even if the θ_i, $i = 1, \ldots, K$, are mutually independent, the model of Equation (2.8) is underidentified. To see this, note that $\operatorname{Cov}(U(s)) = \Lambda(s)\Sigma_\theta \Lambda'(s) + D_{\varepsilon(s)}$, where Σ_θ is a matrix of the variances of the factors, assumed to be diagonal in this example, $D_{\varepsilon(s)}$ is a diagonal matrix of the variances of the uniquenesses.[31] We have identified $\operatorname{Cov}(U(s))$, the discrete components up to scale, but we do not directly observe θ or $\varepsilon(s)$. Any orthogonal transformation applied to $\Lambda(s)$ is consistent with the same $\operatorname{Cov}(U(s))$.

Without restrictions on $\Lambda(s)$, and on the dependence structure among the components of θ, identification of the model is not possible. Conventional factor-analytic models make assumptions to identify parameters. The diagonals of $\operatorname{Cov}(U(s))$ combine elements of $D_{\varepsilon(s)}$ with parameters from the rest of the model. Once those other parameters are determined, the diagonals identify $D_{\varepsilon(s)}$. Accordingly, one can only rely on the $L(s)(L(s)-1)/2$ non-diagonal elements to identify the K variances (assuming $\theta_i \perp\!\!\!\perp \theta_j$, $\forall i \neq j$), and the $L(s) \times K$ factor loadings. Since the scale of each θ_i is arbitrary, one factor loading devoted to each factor must be normalized to set the scale. Typically the normalization is unity. Accordingly, we require as a necessary condition for identification of the variances and parameters of (2.8) for a given s

$$\underbrace{\frac{L(s)(L(s)-1)}{2}}_{\substack{\text{Number of off-diagonal}\\ \text{covariance elements}}} \geqslant \underbrace{\big(\big(L(s) \times K\big) - K\big)}_{\text{Number of unrestricted } \Lambda} + \underbrace{K.}_{\text{Variances of } \theta}$$

$$\iff \quad L(s) \geqslant 2K + 1. \tag{2.9}$$

Anderson and Rubin (1956), Chamberlain (1975), Carneiro, Hansen and Heckman (2003), Hansen, Heckman and Mullen (2004), Cunha, Heckman and Navarro (2005) and Cunha, Heckman and Schennach (2007) present alternative normalizations and identification assumptions for models with multiple factors. Carneiro, Hansen and Heckman (2003) and Cunha, Heckman and Schennach (2006, 2007) use information from higher moments to identify the model.[32] Many of the identifying assumptions in various empirical literatures such as the literature on earnings dynamics are motivated by appeals to empirical conventions and to economic theory [see, e.g., Cunha and Heckman (2007b, 2007c, 2008)]. Case-specific analyses are necessary to provide economically interpretable identifying assumptions. Access to measurements facilitates this task.

[31] The uniquenesses are the $\varepsilon(s)$ in Equation (2.8).

[32] See also Bonhomme and Robin (2004). Note that restrictions across the s-systems facilitate identification.

2.9. Distinguishing ex ante from ex post returns

The analysis of the preceding sections presents tools for estimating joint distributions of outcomes and subjective valuations of outcomes across counterfactual states. It is silent about the information that agents possess about expected returns at the time they make their program participation decisions. Uncertainty and the dynamics of information revelation are not systematically incorporated in the current literature on treatment effects. As noted in Chapter 70 of this Handbook, anticipated (*ex ante*) returns may differ from realized (*ex post*) returns and understanding these differences is important for computing the welfare gains to program participation, the regret that agents may experience about participating or not participating in a program, and the option value of social programs. In addition, subjective evaluations are not a part of the literature on statistical treatment effects.

In a medical trial [see, e.g., Chan and Hamilton (2006)], the patient will not only value the medical treatment but he/she will also consider the medical benefits or costs (pain and suffering) connected with the treatment. Agents may be pleasantly or unpleasantly surprised by arrival of information during a course of therapy, and this information revision will affect choices of future treatment. Knowing agent preferences and perceptions is helpful in determining compliance and patient welfare. In an analysis of job training programs, agents may be disappointed, *ex post* about the treatment they have received [Heckman and Smith (1998)].

Empirical analyses of the "returns to education" that have extensively used *IV* methods focus exclusively on the *ex post* returns to education rather than the *ex ante* returns that motivate agent schooling decisions. As Hicks (1946, p. 179) puts it,

> "Ex post *calculations of capital accumulation have their place in economic and statistical* history*; they are a useful measuring-rod for economic progress; but they are of no use to theoretical economists who are trying to find out how the system works, because they have no significance for conduct*."

This section presents some recent results on the identification of agent information sets and *ex ante* and *ex post* distributions of outcomes. It builds on and synthesizes work by Carneiro, Hansen and Heckman (2001, 2003), Cunha, Heckman and Navarro (2005, 2006) and Cunha and Heckman (2007b, 2007c, 2008).

To motivate the main ideas underlying this approach, consider the problem of estimating the return to an activity. It could be schooling or the installation of a new technology. The problem can be cast as a prototypical generalized Roy model with two sectors and solutions to it apply to many related problems. Let D denote different choices. $D = 0$ denotes choice of sector 0 and $D = 1$ denotes choice of sector 1. In a schooling example this could represent high school ($D = 0$) and college ($D = 1$). Each person chooses to be in one or the other sector but cannot be in both. Let the two potential outcomes be represented by the pair (Y_0, Y_1), only one of which is observed by the analyst for any agent. Denote by C the direct cost of choosing sector 1. In a schooling example these would include tuition and nonpecuniary costs of attending college expressed in monetary values.

Y_1 is the *ex post* present value of making the choice 1, discounted over horizon $\bar{T}$ for a person choosing at a fixed age, assumed for convenience to be one,

$$Y_1 = \sum_{t=1}^{\bar{T}} \frac{Y_{1,t}}{(1+r)^{t-1}},$$

and Y_0 is the *ex post* present value of making the choice 0 at age one,

$$Y_0 = \sum_{t=1}^{\bar{T}} \frac{Y_{0,t}}{(1+r)^{t-1}},$$

where r is the one-period risk-free interest rate. Y_1 and Y_0 can be constructed from time series of *ex post* potential outcome streams in the two states: $(Y_{0,1}, \ldots, Y_{0,\bar{T}})$ and $(Y_{1,1}, \ldots, Y_{1,\bar{T}})$. A practical problem is that we only observe one or the other of these streams for any person. This is the fundamental program evaluation problem. In addition, we observe these streams selectively, i.e., for those who chose $D = 0$ or $D = 1$, respectively.

The variables Y_1, Y_0, and C are *ex post* realizations of returns and costs, respectively. At the time agents make their choices, these random variables may only be partially known to the agent. Using the information set notation introduced in Section 2.6 of Chapter 70, let $\mathcal{I}_A$ denote the information set of an agent at the time the choice is made, which is time period $t = 1$ in our notation. Under a complete markets assumption with all risks diversifiable (so that there is risk-neutral pricing) or under a perfect foresight model with unrestricted borrowing or lending but full repayment, the decision rule governing sectoral choices at decision time 1 is

$$D = \begin{cases} 1, & \text{if } E(Y_1 - Y_0 - C \mid \mathcal{I}_A) \geqslant 0, \\ 0, & \text{otherwise.}^{33} \end{cases} \tag{2.10}$$

Under perfect foresight, the postulated information set would include Y_1, Y_0, and C. Under either model of information, the decision rule is simple: one chooses sector 1 if the expected gains from doing so are greater than or equal to the expected costs. Thus under either set of assumptions, a separation theorem governs choices. Agents maximize expected wealth independently of their consumption decisions over time.[34]

[33] If there are aggregate sources of risk, full insurance would require a linear utility function.

[34] The decision rule is more complicated in the absence of full risk diversifiability and depends on the curvature of utility functions, the availability of markets to spread risk, and possibilities for storage. [See Heckman, Lochner and Todd (2006) for a more extensive discussion.] In these more realistic economic settings, the components of earnings and costs required to forecast the gain to the choice depend on higher moments than the mean. In this section, we use a model with a simple market setting to motivate the identification analysis of a more general environment analyzed elsewhere [Carneiro, Hansen and Heckman (2003)].

Suppose that we seek to determine $\mathcal{I}_A$. This is a difficult task. Typically we can only partially identify $\mathcal{I}_A$ and generate a list of candidate variables that belong to the information set. We can usually only estimate the distributions of the unobservables in $\mathcal{I}_A$ (from the standpoint of the econometrician) and not individual realizations of the unobservables.

Cunha, Heckman and Navarro (2005, 2006) and Cunha and Heckman (2007b, 2007c) exploit covariances between choices and realized outcomes that arise under different information structures to test which information structure characterizes the data, building on the analysis of Carneiro, Hansen and Heckman (2003). To see how the method works, we simplify the exposition to a two-choice framework. In Section 3 of this contribution, we extend this analysis to multiple choices in a dynamic setting.

Suppose, contrary to what is possible, that the analyst observes Y_0, Y_1, and C for each person. Such information would come from an ideal data set in which the evaluation problem is solved and we could observe two different lifetime outcome streams for the same person as well as the costs they pay for choosing sector 1. From such information, we could construct $Y_1 - Y_0 - C$. If we knew the information set $\mathcal{I}_A$ of the agent that governs choices, we could also construct $E(Y_1 - Y_0 - C \mid \mathcal{I}_A)$. Under the correct model of expectations, we could form the residual

$$\zeta_{\mathcal{I}_A} = (Y_1 - Y_0 - C) - E(Y_1 - Y_0 - C \mid \mathcal{I}_A),$$

and from the *ex ante* choice decision, we could determine whether D depends on $\zeta_{\mathcal{I}_A}$. It should not if we have specified $\mathcal{I}_A$ correctly.

A test for correct specification of candidate information set $\widetilde{\mathcal{I}}_A$ for an agent is a test of whether D depends on $\zeta_{\widetilde{\mathcal{I}}_A}$, where

$$\zeta_{\widetilde{\mathcal{I}}_A} = (Y_1 - Y_0 - C) - E(Y_1 - Y_0 - C \mid \widetilde{\mathcal{I}}_A).$$

More precisely, the information set is valid if $D \perp\!\!\!\perp \zeta_{\widetilde{\mathcal{I}}_A} \mid \widetilde{\mathcal{I}}_A$. A test of misspecification of $\widetilde{\mathcal{I}}_A$ is a test of whether the coefficient of $\zeta_{\widetilde{\mathcal{I}}_A}$ in the choice equation is statistically significantly different from zero.

More generally, $\widetilde{\mathcal{I}}_A$ is the correct information set if $\zeta_{\widetilde{\mathcal{I}}_A}$ does not help to predict schooling. One can search among candidate information sets $\widetilde{\mathcal{I}}_A$ to determine which ones satisfy the requirement that the generated $\zeta_{\widetilde{\mathcal{I}}_A}$ does not predict D and what components of $Y_1 - Y_0 - C$ (and $Y_1 - Y_0$) are predictable at the age schooling decisions are made for the specified information set. This procedure is motivated by a Sims (1972) version of a Wiener–Granger causality test. There may be several information sets that satisfy this property.[35] For a properly specified $\widetilde{\mathcal{I}}_A$, $\zeta_{\widetilde{\mathcal{I}}_A}$ should not cause (predict) schooling choices. The components of $\zeta_{\widetilde{\mathcal{I}}_A}$ that are unpredictable are intrinsic components of uncertainty at the date the choice represented by D is made.

[35] Thus different combinations of variables may contain the same information. The issue of the existence of a smallest information set is a technical one concerning a minimum σ-algebra that satisfies the conditions used to define $\mathcal{I}_A$.

It is difficult to determine the exact content of $\mathcal{I}_A$ known to each agent. If we could, we would perfectly predict D given our decision rule. More realistically, we might find variables that proxy $\mathcal{I}_A$ or their distribution. This strategy is pursued in Cunha, Heckman and Navarro (2005, 2006) for a two-choice model, and is generalized by Cunha and Heckman (2007b) and Heckman and Navarro (2007). We now present an example of this approach. We consider identification of information sets as well as identification of the psychic costs of treatment.

2.9.1. An approach based on factor structures

Consider the following model for $\bar{T}$ periods. Write outcomes in each counterfactual state as

$$Y_{0,t} = \mu_{0,t}(X_t) + U_{0,t},$$
$$Y_{1,t} = \mu_{1,t}(X_t) + U_{1,t}, \quad t = 1, \dots, \bar{T}.$$

We let costs of picking sector 1 be defined as

$$C = \mu_C(Z) + U_C.$$

Assume that the horizon of the agent ends at period $\bar{T}$.

Suppose that there exists a vector of mutually independent factors

$$\theta = (\theta_1, \theta_2, \dots, \theta_K).$$

Under the factor assumption, the error term in outcomes in period t for an agent can be represented in the following manner:

$$U_{0,t} = \alpha_{0,t}\theta + \varepsilon_{0,t},$$
$$U_{1,t} = \alpha_{1,t}\theta + \varepsilon_{1,t},$$

where $\alpha_{0,t}$ and $\alpha_{1,t}$ are now $1 \times K$ vectors and θ is a $K \times 1$ vector. The $\varepsilon_{0,t}$, $\varepsilon_{1,t}$, and θ are mutually independent. We can also decompose the cost function C in a similar fashion:

$$C = \mu_C(Z) + \alpha_C\theta + \varepsilon_C.$$

All of the statistical dependence across potential outcomes and costs is generated by θ, X, and Z. Thus, if we could match on θ (as well as X and Z), we could use matching to infer the distribution of counterfactuals and capture all of the dependence across the counterfactual states through θ. Carneiro, Hansen and Heckman (2001, 2003), Cunha, Heckman and Navarro (2005, 2006) and Cunha and Heckman (2007b, 2007c, 2008) allow for the possibility that not all of the required elements of θ are observed.

The parameters α_C and $\alpha_{s,t}$, for $s = 0, 1$, and $t = 1, \dots, \bar{T}$ are the factor loadings. ε_C is independent of the θ and the other ε components. In this notation, the choice equation can be written as:

$$D^* = E\left(\sum_{t=1}^{\bar{T}} \frac{(\mu_{1,t}(X_t) + \alpha_{1,t}\theta + \varepsilon_{1,t}) - (\mu_{0,t}(X_t) + \alpha_{0,t}\theta + \varepsilon_{0,t})}{(1+r)^{t-1}} - \big(\mu_C(Z) + \alpha_C\theta + \varepsilon_C\big) \,\Big|\, \mathcal{I}_A \right),$$

$$D = 1 \quad \text{if } D^* \geqslant 0; \qquad D = 0 \quad \text{otherwise.} \tag{2.11}$$

The first term in the summation inside the parentheses is discounted outcomes in state 1 minus discounted outcomes in state 0. The second term in the expression is the cost.

Equation (2.11) entails counterfactual comparisons. Even if the outcomes associated with one choice are observed over the horizon using panel data, the outcomes in the counterfactual state are not. After the choice is made, some components of the X_t, the θ, and the ε_t may be revealed (e.g., unemployment rates, macroshocks) to both the observing economist and the agent, although different components may be revealed to each and at different times.

Examining alternative information sets, one can determine which ones produce models for outcomes that fit the data best in terms of producing a model that predicts date $t = 1$ choices and at the same time passes the test for misspecification of predicted earnings and costs described in the previous subsection. Some components of the error terms of the outcome equations may be known or not known at the date schooling choices are made. The unforecastable components are intrinsic uncertainty. The forecastable information is called heterogeneity.[36]

To formally characterize an empirical procedure to test for and measure the importance of uncertainty, it is useful to introduce some additional notation. Let $\odot$ denote the Hadamard product, $a \odot b = (a_1 b_1, \ldots, a_L b_L)$, for vectors a and b of length L. This is a componentwise multiplication of vectors to produce a vector. Let κ_{X_t}, $t = 1, \ldots, \bar{T}$, κ_Z, κ_θ, κ_{ε_t}, κ_{ε_C}, denote coefficient vectors associated with the X_t, $t = 1, \ldots, \bar{T}$, the Z, the θ, the $\varepsilon_{1,t} - \varepsilon_{0,t}$, and the ε_C, respectively. For a proposed information set $\widetilde{\mathcal{I}}_A$ which may or may not be the true information set on which agents act, define the proposed choice index $\tilde{D}^*$ in the following way. For simplicity write $\mu_{1,t}(X_t) = X_t\beta_{1,t}$, $\mu_{0,t}(X_t) = X_t\beta_{0,t}$, and $\mu_C(Z) = Z\gamma$. Then

$$\tilde{D}^* = \sum_{t=1}^{\bar{T}} \frac{E(X_t \mid \widetilde{\mathcal{I}}_A)}{(1+r)^{t-1}}(\beta_{1,t} - \beta_{0,t}) + \sum_{t=1}^{\bar{T}} \frac{[X_t - E(X_t \mid \widetilde{\mathcal{I}}_A)]}{(1+r)^{t-1}}(\beta_{1,t} - \beta_{0,t}) \odot \kappa_{X_t}$$
$$+ \left[\sum_{t=1}^{\bar{T}} \frac{(\alpha_{1,t} - \alpha_{0,t})}{(1+r)^{t-1}} - \alpha_C \right] E(\theta \mid \widetilde{\mathcal{I}}_A)$$

[36] The term ‘heterogeneity’ is somewhat unfortunate. This term includes trends common across all people (e.g., macrotrends). The real distinction they are making is between components of realized outcomes forecastable by agents at the time they make their choices vs. components that are not forecastable.

$$
\begin{aligned}
&+\left\{\left[\sum_{t=1}^{\bar{T}}\frac{(\alpha_{1,t}-\alpha_{0,t})}{(1+r)^{t-1}}-\alpha_C\right]\odot\kappa_\theta\right\}\big[\theta-E(\theta\mid\widetilde{\mathcal{I}}_A)\big]\\
&+\sum_{t=1}^{\bar{T}}\frac{E(\varepsilon_{1,t}-\varepsilon_{0,t}\mid\widetilde{\mathcal{I}}_A)}{(1+r)^{t-1}}+\sum_{t=1}^{\bar{T}}\frac{[(\varepsilon_{1,t}-\varepsilon_{0,t})-E(\varepsilon_{1,t}-\varepsilon_{0,t}\mid\widetilde{\mathcal{I}}_A)]}{(1+r)^{t-1}}\kappa_{\varepsilon_t}\\
&-E(Z\mid\widetilde{\mathcal{I}}_A)\gamma-\big[Z-E(Z\mid\widetilde{\mathcal{I}}_A)\big]\gamma\odot\kappa_Z-E(\varepsilon_C\mid\widetilde{\mathcal{I}}_A)\\
&-\big[\varepsilon_C-E(\varepsilon_C\mid\widetilde{\mathcal{I}}_A)\big]\kappa_{\varepsilon_C}.
\end{aligned}
\tag{2.12}
$$

Fit a choice model based on the proposed information set. Estimate the parameters of the model including the κ parameters. The κ parameters will be estimated to be nonzero in a choice equation if a proposed information set is not the actual information set used by agents. This particular decomposition for $\tilde{D}^*$ assumes that agents know the β, the γ, and the α.[37] If this assumption is not correct, the presence of additional unforecastable components due to unknown coefficients affects the interpretation of the estimates. A test of no misspecification of information set $\widetilde{\mathcal{I}}_A$ is a joint test of the hypothesis that the κ are all zero. That is, when $\widetilde{\mathcal{I}}_A = \mathcal{I}_A$ then the proposed choice index $\tilde{D}^* = D^*$. In a model with a correctly specified information set, the components associated with zero κ_j are the unforecastable elements or the elements which, even if known to the agent, are not acted on in making schooling choices.

To illustrate the method of Cunha, Heckman and Navarro (2005), assume that the X_t, the Z, the ε_C, the $\beta_{1,t}$, $\beta_{0,t}$, the $\alpha_{1,t}$, $\alpha_{0,t}$, and α_C are known to the agent at the time decisions about D are being made, and that the $\varepsilon_{j,t}$ are unknown, and that the agents set them at their mean values of zero. We can infer which components of the θ are known and acted on in making decisions if we postulate that some components of θ are known perfectly at date $t = 1$ while others are not known at all, and their forecast values have mean zero given $\mathcal{I}_A$.

If there is an element of the vector θ, say θ_2 (factor 2), that has nonzero loadings (coefficients) in the choice equation and a nonzero loading on one or more potential future outcomes, then one can say that at the time the choice is made, the agent knows the unobservable captured by factor 2 that affects future outcomes. If θ_2 does not enter the choice equation but explains future outcomes, then θ_2 is unknown (not predictable by the agent) at the age decisions are made. An alternative interpretation is that the second component of

$$
\left[\sum_{t=1}^{\bar{T}}\frac{(\alpha_{1,t}-\alpha_{0,t})}{(1+r)^{t-1}}-\alpha_C\right]
$$

is zero, i.e., that even if the component is known, it is not acted on. Analysts can only test for what the agent knows and acts on.

[37] Cunha, Heckman and Navarro (2005) and Cunha and Heckman (2007b) relax this assumption.

One plausible scenario is that ε_C is known to the agent, since costs are assumed to be incurred up front, but that the future $\varepsilon_{1,t}$ and $\varepsilon_{0,t}$ are not and have mean zero. If there are components of the $\varepsilon_{j,t}$ that are predictable at age $t = 1$, they will induce additional dependence between D and future outcomes that will pick up additional factors beyond those initially specified. The procedure can be generalized to consider all components of the outcome equations. Using this procedure, the analyst can test the predictive power of each subset of the possible information set at the date the decision is being made. The approach allows the analyst to determine which components of θ and $\{\varepsilon_{0,t}, \varepsilon_{1,t}\}_{t=1}^{\bar{T}}$ are known and acted on at the time decisions are made.

Statistical decompositions do not tell us which components of error variance are known at the time agents make their decisions. A model of expectations and choices is needed. If some of the components of $\{\varepsilon_{0,t}, \varepsilon_{1,t}\}_{t=1}^{\bar{T}}$ are known to the agent at the date decisions are made and enter decision equation (2.11), then additional dependence between D and future $Y_1 - Y_0$ due to the $\{\varepsilon_{0,t}, \varepsilon_{1,t}\}_{t=1}^{\bar{T}}$, beyond that due to θ, would be estimated. Our version of the Sims test can in principle detect these components.

It is helpful to contrast the dependence between D and future $Y_{0,t}$, $Y_{1,t}$ arising from θ and the dependence between D and the $\{\varepsilon_{0,t}, \varepsilon_{1,t}\}_{t=1}^{\bar{T}}$. Some of the θ in the *ex post* outcomes equation may not appear in the choice equation. Under other information sets, some additional dependence between D and $\{\varepsilon_{0,t}, \varepsilon_{1,t}\}_{t=1}^{\bar{T}}$ may arise. The contrast between the sources generating realized outcomes and the sources generating dependence between D and realized outcomes is the essential idea in inferring the information in the agent's information set when decisions are being made. The method can be generalized to deal with nonlinear preferences and imperfect market environments.[38] We next show how to operationalize this method and identify psychic costs and agent information sets. This econometric analysis is followed by some empirical applications of this methodology.

2.9.2. *Operationalizing the method*

In order to see how to operationalize the method, we draw on the work of Cunha and Heckman (2007b). Assume normality to simplify the analysis. The normality assumption plays no essential role in the analysis and is relaxed below. Our empirical examples in fact show the estimated models to be highly nonnormal.

The key idea underlying this approach is to have more measurement, outcome and choice equations than components in θ. These are the necessary conditions for identification encapsulated in inequality (2.9). Here we assume that we have multiple periods of data on outcomes associated with each treatment state s, $s = 1, \ldots, \bar{S}$, as well as measurement equations. We assume a two-factor example and show how to test

[38] See Carneiro, Hansen and Heckman (2003), Cunha and Heckman (2007c) and the survey in Heckman, Lochner and Todd (2006).

whether factors that predict post-treatment earnings appear in the choice equation. For specificity, one can think of the choice as schooling (high school vs. college), and the outcomes as earnings.

2.9.3. The estimation of the components in the information set

We show how we can determine the unobservable components of the information set $\mathcal{I}_A$ of the agent at the time of the choice by exploring the convenient structure provided by the factor models. Assume that X, Z, ε_C, and the factor loadings and parameters of cost equations and outcome equations are in the information set $\mathcal{I}_A$. We can test for what is in agent's decision sets using the Sims test described in Section 2.9.1. To conserve on notation, we define factor loadings on each factor in (2.12) using the condensed expression

$$\alpha_{k,D} = \sum_{t=1}^{\bar{T}} \left(\frac{1}{1+r}\right)^{t-1} (\alpha_{k,1,t} - \alpha_{k,0,t}) - \alpha_{k,C} \quad \text{for } k = 1, \dots, K. \tag{2.13}$$

Suppose that for a two-factor ($K = 2$) model, θ_1 and θ_2 are in the agent's information set $\mathcal{I}_A$ but $\varepsilon_{s,t}$ is not. If the null hypothesis that θ_1 and θ_2 are in $\mathcal{I}_A$ is true, we may write the choice index D^* as:

$$D^* = \mu_D(X, Z) + \alpha_{1,D}\theta_1 + \alpha_{2,D}\theta_2 + \varepsilon_C. \tag{2.14}$$

The choice index is written in terms of structural parameters using (2.10). From our analysis of Step 2, we can identify $\mu_D(X, Z)$ and $\beta_{s,t}$ for all s and t. Given observations on X and Z, we can obtain from data on outcomes, (Y, X, D, Z), the covariance between the terms $D^* - \mu_D(X, Z)$ and $Y_{1,1} - X\beta_{1,1}$. Under the null hypothesis that θ_1 and θ_2 are both in the agents' information sets, this covariance is equal to

$$\mathrm{Cov}\big(D^* - \mu_D(X, Z), Y_{1,1} - \mu_{1,1}(X)\big) = \alpha_{1,D}\alpha_{1,1,1}\sigma^2_{\theta_1} + \alpha_{2,D}\alpha_{2,1,1}\sigma^2_{\theta_2}. \tag{2.15}$$

We seek to test the null that θ_1 and θ_2 are in $\mathcal{I}_A$ against alternative hypotheses. To fix ideas, consider the alternative assumption that θ_1 is in $\mathcal{I}_A$ but θ_2 is not, and maintain that $E[\theta_2 \mid \mathcal{I}_A] = 0$. If the alternative is valid, the choice index (2.14) may be written as

$$D^* = \mu_D(X, Z) + \alpha_{1,D}\theta_1 + \varepsilon_C. \tag{2.16}$$

In this case, the covariance between the terms $D^* - \mu_D(X, Z)$ and $Y_{1,1} - \mu_{1,1}(X)$ satisfies

$$\mathrm{Cov}\big(D^* - \mu_D(X, Z), Y_{1,1} - \mu_{1,1}(X)\big) = \alpha_{1,D}\alpha_{1,1,1}\sigma^2_{\theta_1}, \tag{2.17}$$

and the difference between the choice generated by the null and the alternative hypotheses is the term $\alpha_{2,D}\alpha_{2,1,1}\sigma^2_{\theta_2}$ that appears in (2.15) but not in (2.17). This insight allows us to redefine the Sims test by generating parameters κ_{θ_1} and κ_{θ_2} to satisfy:

$$\mathrm{Cov}\big(D^* - \mu_D(X,Z), Y_{1,1} - \mu_{1,1}(X)\big) - \kappa_{\theta_1}\alpha_{1,D}\alpha_{1,1,1}\sigma^2_{\theta_1} - \kappa_{\theta_2}\alpha_{2,D}\alpha_{2,1,1}\sigma^2_{\theta_2} = 0.$$

It is easy to see how we can rewrite the test in terms of κ_{θ_1} and κ_{θ_2}. We conclude that agents know and act on the information contained in factors 1 and 2, so that θ_1 and θ_2 are in $\mathcal{I}_A$, if we reject both $\kappa_{\theta_1} = 0$ and $\kappa_{\theta_2} = 0$. Parallel tests can be conducted for other components of realized earnings.

It remains to be shown that we can actually identify all of the parameters of the model, in particular, the function $\mu_D(X,Z)$, the parameters β and α in the test and earnings equations, the distribution of the factors, F_θ, as well as the distribution of idiosyncratic components F_ε in the measurement, outcomes and cost equations.

We start by analyzing the measurement equations which in the context of a schooling choice problem could be test score equations. We assume that the measurement equations only depend on θ_1 and not the other factors. In an analysis of college choices, test scores are typically available for all agents before their decisions are made, and they proxy ability. By assumption, there is no selection bias in observations on the measurement equations. We can identify the mean outcome equations $\mu_{M,n}(X)$, $n = 1, \ldots, N$, where N is the number of measurements.

Given knowledge of these parameters, we can construct differences $M_n - \mu_{M,n}(X)$ and compute the covariances, as in the case of three measurements:

$$\mathrm{Cov}\big(M_1 - \mu_{M,1}(X), M_2 - \mu_{M,2}(X)\big) = \alpha_1^M\alpha_2^M\sigma^2_{\theta_1}, \tag{2.18}$$

$$\mathrm{Cov}\big(M_1 - \mu_{M,1}(X), M_3 - \mu_{M,3}(X)\big) = \alpha_1^M\alpha_3^M\sigma^2_{\theta_1}, \tag{2.19}$$

$$\mathrm{Cov}\big(M_2 - \mu_{M,2}(X), M_3 - \mu_{M,3}(X)\big) = \alpha_2^M\alpha_3^M\sigma^2_{\theta_1}. \tag{2.20}$$

The left-hand sides of (2.18), (2.19), and (2.20) can be computed from sample moments. The right-hand sides of (2.18), (2.19), and (2.20) are implications of the factor model, assuming measurements are dependent only through θ_1. We need to normalize one of the factor loadings. Let $\alpha_1^M = 1$. If we take the ratio of (2.20) to (2.18), we identify α_3^M. Analogously, the ratio of (2.20) to (2.19) allows us to recover α_2^M. Given the normalization of $\alpha_1^M = 1$ and identification of α_2^M, we recover $\sigma^2_{\theta_1}$ from (2.18). Finally, we can identify the variance of ε_k^M from the variance of $M_k - \mu_{M,k}$. Because the factor θ_1 and uniquenesses ε_k are independently normally distributed random variables, we have identified their distribution. Normality plays no crucial role here. Our analysis in Section 2.7.3 shows how this analysis can be made fully nonparametric under the conditions of Theorem 1.

2.9.4. Outcome and choice equations

Establishing the identification of the joint distribution of outcomes requires more work because of the evaluation problem. We only observe one stream of outcomes for each agent, corresponding to outcomes associated with treatment D. It is at this stage of the analysis that focusing the discussion on normally distributed factors and uniquenesses becomes helpful for understanding how identification can be secured. We can use the closed-form solutions developed in the traditional econometric literature to reduce the identification problem to the identification of a few parameters. However, the analysis does not require normality.

All of the dependence among $U_{0,t}$, $U_{1,t}$, and U_C is captured through the factors θ_1 and θ_2. To establish identification most transparently, assume that they are normally distributed with the following mean and covariance matrix:

$$\begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix} \sim N\left(\begin{pmatrix}0\\ 0\end{pmatrix}, \begin{bmatrix}\sigma^2_{\theta_1} & 0\\ 0 & \sigma^2_{\theta_2}\end{bmatrix}\right).$$

Because of the loadings $\alpha_{1,s,t}$, $\alpha_{2,s,t}$, $\alpha_{1,C}$, and $\alpha_{2,C}$ the factors θ can affect $U_{0,t}$, $U_{1,t}$, and U_C differently. By adopting the factor structure representation, we are not imposing, for example, perfect ranking in the sense that the best in the distribution of earnings in sector s at period t is the best (or the worst) in the distribution of earnings in sector s' at period t' as in the models of rank invariance surveyed in Section 2.5. The joint distribution of the earnings $Y_{0,t}$, $Y_{1,t}$ conditional on X is:

$$\begin{aligned}&\begin{bmatrix}Y_{0,t}\\ Y_{1,t}\end{bmatrix} \Bigm| X\\ &\sim N\left(\begin{bmatrix}\mu_{0,t}(X)\\ \mu_{1,t}(X)\end{bmatrix}, \begin{bmatrix}\alpha^2_{1,0,t}\sigma^2_{\theta_1}+\alpha^2_{2,0,t}\sigma^2_{\theta_2}+\sigma^2_{\varepsilon_{0,t}} & \alpha_{1,0,t}\alpha_{1,1,t}\sigma^2_{\theta_1}+\alpha_{2,0,t}\alpha_{2,1,t}\sigma^2_{\theta_2}\\ \alpha_{1,0,t}\alpha_{1,1,t}\sigma^2_{\theta_1}+\alpha_{2,0,t}\alpha_{2,1,t}\sigma^2_{\theta_2} & \alpha^2_{1,1,t}\sigma^2_{\theta_1}+\alpha^2_{2,1,t}\sigma^2_{\theta_2}+\sigma^2_{\varepsilon_{1,t}}\end{bmatrix}\right).\end{aligned} \tag{2.21}$$

The joint distribution of $\left[\begin{smallmatrix}Y_{0,t}\\ Y_{1,t}\end{smallmatrix}\right]$ and $\left[\begin{smallmatrix}Y_{0,t'}\\ Y_{1,t'}\end{smallmatrix}\right]$ is fully determined by the means of each vector, the variance matrix of each vector, and the covariance matrix

$$\begin{aligned}&\operatorname{Cov}\left(\begin{bmatrix}Y_{0,t}\\ Y_{1,t}\end{bmatrix}, \begin{bmatrix}Y_{0,t'}\\ Y_{1,t'}\end{bmatrix} \Bigm| X\right)\\ &\quad= \begin{bmatrix}\alpha_{1,0,t}\alpha_{1,0,t'}\sigma^2_{\theta_1}+\alpha_{2,0,t}\alpha_{2,0,t'}\sigma^2_{\theta_2} & \alpha_{1,0,t}\alpha_{1,1,t'}\sigma^2_{\theta_1}+\alpha_{2,0,t}\alpha_{2,1,t'}\sigma^2_{\theta_2}\\ \alpha_{1,0,t}\alpha_{1,1,t'}\sigma^2_{\theta_1}+\alpha_{2,0,t}\alpha_{2,1,t'}\sigma^2_{\theta_2} & \alpha_{1,1,t}\alpha_{1,1,t'}\sigma^2_{\theta_1}+\alpha_{2,1,t}\alpha_{2,1,t'}\sigma^2_{\theta_2}\end{bmatrix},\end{aligned} \tag{2.22}$$

for all $t \neq t'$. If we determine the means and the covariances across all of the t, t' under a normality assumption, we fully specify the joint distributions of $(Y_{0,1}, \ldots, Y_{0,\bar{T}}, Y_{1,1}, \ldots, Y_{1,\bar{T}})$ and $(Y_{0,1}, \ldots, Y_{0,\bar{T}}, Y_{1,1}, \ldots, Y_{1,\bar{T}}, D^*)$. As a result, identification of the joint distributions reduces to the identification of the functions $\mu_{0,t}(X)$, $\mu_{1,t}(X)$, $\alpha_{k,s,t}$, $\alpha_{k,D}$, $\sigma_{\varepsilon_{s,t}}$, σ_{ε_C} (possibly up to scale) and $\sigma^2_{\theta_j}$ for $s = 0, 1$; $t = 1, \ldots, \bar{T}$ and $j = 1, 2$, and $k = 1, 2$. This also entails identification of the distributions of θ_1 and θ_2 as well as the parameters associated with the choice equation. Using the methods discussed in Sections 2.7 and 2.8, we can relax the normality assumption. The factor structure is essential to model the dependence across observations. The factors can be nonnormal. We now show an example of how to secure identification.

From the observed data and the factor structure it follows that

$$\begin{aligned} E(Y_{1,t} \mid X, Z, D = 1) = \mu_{1,t}(X) + \alpha_{1,1,t} E[\theta_1 \mid X, Z, D = 1] \\ + \alpha_{2,1,t} E[\theta_2 \mid X, Z, D = 1] \\ + E[\varepsilon_{1,t} \mid X, Z, D = 1]. \end{aligned} \tag{2.23}$$

The event $D = 1$ is the event $D^* = E(\sum_{t=1}^{\bar{T}}(\frac{1}{1+r})^{t-1}(Y_{1,t} - Y_{0,t}) - C \mid \mathcal{I}_A) \geqslant 0$. For simplicity, assume that r is known by the analyst. It can be identified along with the other parameters.[39]

It is important to distinguish the role played by the factors θ from the role played by the uniquenesses $\varepsilon_{s,t}$. We assume in this example that the $\varepsilon_{s,t}$ are unknown to the agent at the time choices are made. If not, those components would fail the Sims test for their exclusion from the choice equation, and would be in agent information sets. By definition, the terms that affect the covariance between future outcomes and choices are what is in the information set at the time choices are made. Under our assumptions and initial specification of the information set,

$$\begin{aligned} & E\left(\sum_{t=1}^{\bar{T}} \left(\frac{1}{1+r} \right)^{t-1} (Y_{1,t} - Y_{0,t}) - C \,\middle|\, \mathcal{I}_A \right) \\ & \quad = \mu_D(X, Z) + \alpha_{1,D}\theta_1 + \alpha_{2,D}\theta_2 - \varepsilon_C. \end{aligned}$$

Let V_D be the linear combination of the three independent normal random variables in the decision rule:

$$V_D = \alpha_{1,D}\theta_1 + \alpha_{2,D}\theta_2 - \varepsilon_C.$$

Then, $V_D \sim N(0, \sigma_{V_D}^2)$, with $\sigma_{V_D}^2 = \alpha_{1,D}^2\sigma_{\theta_1}^2 + \alpha_{2,D}^2\sigma_{\theta_2}^2 + \sigma_{\varepsilon_c}^2$ and

$$D = 1 \quad \Leftrightarrow \quad V_D \geqslant -\mu_D(X, Z). \tag{2.24}$$

We now use standard normal sample selection arguments to establish identifiability. If we use representation (2.24) in place of $D = 1$ in Equation (2.23) and use the fact that $\varepsilon_{s,t}$ is independent of X, Z, and V_D, it follows that

$$\begin{aligned} E(Y_{1,t} \mid X, Z, D = 1) = \mu_{1,t}(X) + \alpha_{1,1,t} E\big[\theta_1 \mid X, Z, V_D \geqslant -\mu_D(X, Z)\big] \\ + \alpha_{2,1,t} E\big[\theta_2 \mid X, Z, V_D \geqslant -\mu_D(X, Z)\big]. \end{aligned} \tag{2.25}$$

Second, because θ_1, θ_2 and V_D are normal random variables, we can use the projection property for normal random variables to break θ_j into statistically independent components predictable by V_D and components that are not predictable:

$$\theta_j = \frac{\operatorname{Cov}(\theta_j, V_D)}{\operatorname{Var}(V_D)} V_D + \nu_j \quad \text{for } j = 1, 2, \tag{2.26}$$

[39] See the discussion and references in Section 3.

where ν_j is a mean zero, normal random variable independent of V_D. Because $\operatorname{Cov}(\theta_1, V_D) = \sigma^2_{\theta_1}\alpha_{1,D}$ and $\operatorname{Cov}(\theta_2, V_D) = \sigma^2_{\theta_2}\alpha_{2,D}$, it follows that:

$$E\big[\theta_1 \mid X, Z, V_D \geqslant -\mu_D(X,Z)\big] = \frac{\sigma^2_{\theta_1}\alpha_{1,D}}{\sigma^2_{V_D}} E\big[V_D \mid X, Z, V_D \geqslant -\mu_D(X,Z)\big],$$

$$E\big[\theta_2 \mid X, Z, V_D \geqslant -\mu_D(X,Z)\big] = \frac{\sigma^2_{\theta_2}\alpha_{2,D}}{\sigma^2_{V_D}} E\big[V_D \mid X, Z, V_D \geqslant -\mu_D(X,Z)\big].$$

From the standard normal selection formulae presented in Appendix C of Chapter 70,

$$E\big(Y_{1,t} \mid X, Z, V_D \geqslant -\mu_D(X,Z)\big) = \mu_{1,t}(X) + \pi_{1,t}\frac{\phi\big(\frac{\mu_D(X,Z)}{\sigma_{V_D}}\big)}{\Phi\big(\frac{\mu_D(X,Z)}{\sigma_{V_D}}\big)}, \tag{2.27}$$

where ϕ is the density, Φ is the cdf of the unit normal, and

$$\pi_{1,t} = \frac{\operatorname{Cov}(U_{1,t}, V_D)}{(\operatorname{Var}(V_D))^{\frac{1}{2}}} = \frac{\alpha_{1,D}\,\alpha_{1,1,t}\,\sigma^2_{\theta_1} + \alpha_{2,D}\,\alpha_{2,1,t}\,\sigma^2_{\theta_2}}{\sigma_{V_D}}.$$

Following the same steps, we can derive a similar expression for mean observed earnings in sector "0":

$$E\big(Y_{0,t} \mid X, Z, V_D < -\mu_D(X,Z)\big) = \mu_{0,t}(X) - \pi_{0,t}\frac{\phi\big(\frac{\mu_D(X,Z)}{\sigma_{V_D}}\big)}{\Phi\big(-\frac{\mu_D(X,Z)}{\sigma_{V_D}}\big)}.\text{[40]} \tag{2.28}$$

Standard arguments show that we can identify $\mu_{0,t}(X)$, $\mu_{1,t}(X)$, $\pi_{0,t}$, and $\pi_{1,t}$. Given identification of $\beta_{s,t}$ for all s and t, we can construct the differences $Y_{s,t} - \mu_{s,t}(X)$ and compute the covariances:

$$\operatorname{Cov}\big(M_1 - \mu_{M,1}(X), Y_{0,t} - \mu_{0,t}(X)\big) = \alpha_{1,0,t}\sigma^2_{\theta_1}, \tag{2.29}$$

$$\operatorname{Cov}\big(M_1 - \mu_{M,1}(X), Y_{1,t} - \mu_{1,t}(X)\big) = \alpha_{1,1,t}\sigma^2_{\theta_1}. \tag{2.30}$$

The left-hand sides of (2.29) and (2.30) are identified from sample moments. The right-hand sides are implied by the factor model and the assumption that the measurements depend only on factor 1. We determined $\sigma^2_{\theta_1}$ from the analysis of the test scores. From Equations (2.29) and (2.30) we can recover $\alpha_{1,0,t}$ and $\alpha_{1,1,t}$ for all t. Note that we can also identify the $\frac{\alpha_{1,C}}{\sigma_{V_D}}$ by computing the covariance:

$$\operatorname{Cov}\bigg(M_1 - \mu_{M,1}(X), \frac{D^* - \mu_D(X,Z)}{\sigma_{V_D}}\bigg) = \frac{\sum_{t=1}^{\bar{T}}(\frac{1}{1+r})^{t-1}(\alpha_{1,1,t} - \alpha_{1,0,t}) - \alpha_{1,C}}{\sigma_{V_D}}\sigma^2_{\theta_1}. \tag{2.31}$$

[40] $\pi_{0,t} = (\alpha_{1,D}\alpha_{1,0,t}\sigma^2_{\theta_1} + \alpha_{2,D}\alpha_{2,0,t}\sigma^2_{\theta_2})/\sigma_{V_D}$.

Using (2.29) and (2.30), we can identify $\alpha_{1,1,t}$ and $\alpha_{1,0,t}$ for all t. The only remaining term to be identified is the ratio $\frac{\alpha_{1,C}}{\sigma_{V_D}}$, which we can obtain from the covariance equation (2.31).

With enough panel data on outcomes, we can also identify the parameters related to factor θ_2, such as $\alpha_{2,s,t}$ and $\sigma^2_{\theta_2}$. To see this, first normalize $\alpha_{2,0,1} = 1$ and compute the covariances:

$$\operatorname{Cov}\bigl(Y_{0,1} - \mu_{0,1}(X), Y_{0,2} - \mu_{0,2}(X)\bigr) - \alpha_{1,0,1}\alpha_{1,0,2}\sigma^2_{\theta_1} = \alpha_{2,0,2}\sigma^2_{\theta_2}, \tag{2.32}$$

$$\begin{aligned}
&\operatorname{Cov}\left(Y_{0,1} - \mu_{0,1}(X), \frac{D^* - \mu_D(X,Z)}{\sigma_{V_D}}\right) \\
&\quad - \frac{\alpha_{1,0,1}\sigma^2_{\theta_1} \sum_{t=1}^{\bar{T}}((\frac{1}{1+r})^{t-1}(\alpha_{1,1,t} - \alpha_{1,0,t}) - \alpha_{1,C})}{\sigma_{V_D}} \\
&= \frac{\sigma^2_{\theta_2} \sum_{t=1}^{\bar{T}}((\frac{1}{1+r})^{t-1}(\alpha_{2,1,t} - \alpha_{2,0,t}) - \alpha_{2,C})}{\sigma_{V_D}},
\end{aligned} \tag{2.33}$$

$$\begin{aligned}
&\operatorname{Cov}\left(Y_{0,2} - \mu_{0,2}(X), \frac{D^* - \mu_D(X,Z)}{\sigma_{V_D}}\right) \\
&\quad - \frac{\alpha_{1,0,2}\sigma^2_{\theta_1} \sum_{t=1}^{\bar{T}}((\frac{1}{1+r})^{t-1}(\alpha_{1,1,t} - \alpha_{1,0,t}) - \alpha_{1,C})}{\sigma_{V_D}} \\
&= \frac{\alpha_{2,0,2}\sigma^2_{\theta_2} \sum_{t=1}^{\bar{T}}((\frac{1}{1+r})^{t-1}(\alpha_{2,1,t} - \alpha_{2,0,t}) - \alpha_{2,C})}{\sigma_{V_D}}.
\end{aligned} \tag{2.34}$$

The left-hand sides of (2.32), (2.33), and (2.34) are identified from sample moments. If we compute the ratio of (2.34) to (2.33) we can recover $\alpha_{2,0,2}$. From (2.32), we can recover $\sigma^2_{\theta_2}$. From the covariances from the earnings associated with $s = 1$,

$$\operatorname{Cov}\bigl(Y_{1,1} - \mu_{1,1}(X), Y_{1,2} - \mu_{1,2}(X)\bigr) - \alpha_{1,1,1}\alpha_{1,1,2}\sigma^2_{\theta_1} = \alpha_{2,1,1}\alpha_{2,1,2}\sigma^2_{\theta_2}, \tag{2.35}$$

$$\begin{aligned}
&\operatorname{Cov}\left(Y_{1,1} - \mu_{1,1}(X), \frac{D^* - \mu_D(X,Z)}{\sigma_{V_D}}\right) \\
&\quad - \frac{\alpha_{1,1,1}\sigma^2_{\theta_1} \sum_{t=1}^{\bar{T}}((\frac{1}{1+r})^{t-1}(\alpha_{1,1,t} - \alpha_{1,0,t}) - \alpha_{1,C})}{\sigma_{V_D}} \\
&= \frac{\alpha_{2,1,1}\sigma^2_{\theta_2} \sum_{t=1}^{\bar{T}}((\frac{1}{1+r})^{t-1}(\alpha_{2,1,t} - \alpha_{2,0,t}) - \alpha_{2,C})}{\sigma_{V_D}},
\end{aligned} \tag{2.36}$$

$$
\begin{aligned}
\operatorname{Cov}&\left(Y_{1,2}-\mu_{1,2}(X), \frac{D^*-\mu_D(X,Z)}{\sigma_{V_D}}\right)\\
&\quad -\frac{\alpha_{1,1,2}\sigma_{\theta_1}^2\sum_{t=1}^{\bar{T}}((\frac{1}{1+r})^{t-1}(\alpha_{1,1,t}-\alpha_{1,0,t})-\alpha_{1,C})}{\sigma_{V_D}}\\
&=\frac{\alpha_{2,1,2}\sigma_{\theta_2}^2\sum_{t=1}^{\bar{T}}((\frac{1}{1+r})^{t-1}(\alpha_{2,1,t}-\alpha_{2,0,t})-\alpha_{2,C})}{\sigma_{V_D}}. \qquad (2.37)
\end{aligned}
$$

Taking the ratios of (2.37) to (2.35) and (2.36) to (2.35) and assuming nonzero denominators, we obtain $\alpha_{2,1,2}$ and $\alpha_{2,1,1}$ respectively. Finally, we use the information in $\operatorname{Var}(Y_{0,t} \mid X, Z, D = 0)$ and $\operatorname{Var}(Y_{1,t} \mid X, Z, D = 1)$ to compute $\sigma^2_{\varepsilon_{0,t}}$ and $\sigma^2_{\varepsilon_{1,t}}$, respectively. Thus we can identify all of the elements that characterize the joint distribution as specified in (2.21) and can construct the counterfactual joint distributions. Using the factor loadings identified within each treatment group, we can form the covariance (2.22) and identify the joint distribution of $(Y_{0,1}, \ldots, Y_{0,\bar{T}}, Y_{1,1}, \ldots, Y_{1,\bar{T}})$, and, in a similar fashion, the joint distribution of $(Y_{0,1}, \ldots, Y_{0,\bar{T}}, Y_{1,1}, \ldots, Y_{1,\bar{T}}, D^*)$.

Our use of normality in this example is merely for expositional convenience. As established in Section 2.7.3 and in Section 2.8, all we require is the factor structure assumption (2.6). We can nonparametrically identify all means and distributions of unobservables as a consequence of Theorem 2. The covariances are a by-product of a general nonparametric identification analysis. We next consider two applications of the method. In the context of an analysis of college choice and earnings, they show examples of how to use panel data to identify agent information sets, regret, intrinsic uncertainty and *ex ante* and *ex post* distributions, and the psychic costs facing agents at the time they make their schooling decisions.

2.10. Two empirical studies

This subsection presents two applications of the factor methodology exposited in this section. We draw on work by Cunha and Heckman (2007b, 2008). The computational algorithms used to compute the estimates are described in Carneiro, Hansen and Heckman (2003), Cunha, Heckman and Navarro (2005, 2006) and Cunha and Heckman (2007b, 2008). Geweke and Keane (2001) present relevant background on the Bayesian computational methods used to produce the estimates reported here.

Using data from the National Longitudinal Sample of Youth (NLSY79) on lifetime earnings, ability and college choices for white males, Cunha and Heckman (2007b) estimate a six-factor model ($K = 6$). The θ are assumed to be mutually independent. Agents are assumed to know ε_C, the coefficients of the factors and the regression coefficients, but not the ε's in the earnings equation. They can update their expectation of θ after choices are made, as in the normal model presented in the preceding section. The θ are estimated as mixtures of normals and there is strong evidence that most of the components are nonnormal. Using the Sims testing procedure described in Section 2.9, Cunha and Heckman conclude that three factors $(\theta_1, \theta_2, \theta_3)$ are in agents' information sets at the age college going decisions are made.

Table 4
Ex ante conditional distributions for the NLSY79 (college earnings Y_1 conditional on high school earnings Y_0)

High school	College									
	1	2	3	4	5	6	7	8	9	10
1	0.2995	0.1685	0.1114	0.0789	0.0570	0.0413	0.0393	0.0431	0.0471	0.1137
2	0.2273	0.2119	0.1597	0.1271	0.0907	0.0678	0.0450	0.0288	0.0180	0.0236
3	0.1532	0.1840	0.1656	0.1472	0.1146	0.0914	0.0642	0.0434	0.0230	0.0132
4	0.1110	0.1368	0.1492	0.1474	0.1418	0.1184	0.0882	0.0588	0.0334	0.0148
5	0.0748	0.1100	0.1244	0.1413	0.1459	0.1403	0.1172	0.0836	0.0462	0.0162
6	0.0494	0.0866	0.1146	0.1204	0.1371	0.1399	0.1283	0.1242	0.0736	0.0258
7	0.0306	0.0582	0.0904	0.1094	0.1264	0.1436	0.1506	0.1430	0.1064	0.0414
8	0.0236	0.0348	0.0531	0.0769	0.0989	0.1252	0.1638	0.1799	0.1676	0.0761
9	0.0264	0.0262	0.0316	0.0459	0.0651	0.0929	0.1308	0.1784	0.2431	0.1594
10	0.0457	0.0182	0.0214	0.0216	0.0321	0.0446	0.0772	0.1176	0.2291	0.3925

Notes: $\Pr(d_i < Y_1 < d_{i+1} \mid d_j < Y_0 < d_{j+1}, \mathcal{I})$ where d_i is the ith decile of the college lifetime *ex ante* earnings distribution and d_j is the jth decile of the high school *ex ante* lifetime earnings distribution. The agent fixes unknown θ at their means. The information set includes $\{\theta_1, \theta_2, \theta_3\}$. Correlation $(Y_1, Y_0) = 0.1666$.
Source: Cunha and Heckman (2007b).

Table 4 presents the estimated *ex ante* conditional distributions of the college earnings conditional on high school earnings in the overall population. They show a mild positive correlation that is far from the perfect dependence across potential outcomes assumed by the rank invariance approaches discussed in Section 2.5. Table 5 shows the *ex post* joint distribution of college earnings after all components of θ and the ε are realized.[41] The dependence across potential outcomes in the *ex post* distribution is stronger than that in the *ex ante* distribution.

Table 6 documents that there are substantial unpredictable components in college Y_1, high school Y_0 and the returns $(Y_1 - Y_0)$ distributions after conditioning on X, Z at the time agents make their schooling decisions. Figures 1–3 plot the distributions of total residual and unforecastable components of $Y_1 - Y_0$, Y_1 and Y_0, respectively, where forecasts are measured from the date college decisions are made (age 17). There are substantial components of uncertainty that are distinct from variability observed in the data. *Ex post*, many agents regret their choices (see Table 7). Only 3.1% of those who attend college regret that decision, while 7.5% of those who do not proceed beyond high school regret not attending college.[42]

[41] It assumes that, *ex post*, agents perfectly observe all potential outcome streams. More realistically, agents would only know one stream or the other.
[42] This calculation is for a stationary environment and ignores the secular growth in the mean earning gap between college and high school graduates that is documented by Katz and Autor (1999). Accounting for the growth in this gap substantially reduces the regret of those going to college and raises the regret of those who stopped at high school.

Table 5

Ex post conditional distributions for the NLSY79 (college earnings Y_1 conditional on high school earnings Y_0)

High school	College									
	1	2	3	4	5	6	7	8	9	10
1	0.2118	0.1614	0.1188	0.0932	0.0782	0.0654	0.0532	0.0554	0.0651	0.0974
2	0.1684	0.1777	0.1557	0.1213	0.1038	0.0862	0.0640	0.0516	0.0417	0.0296
3	0.1374	0.1676	0.1464	0.1390	0.1244	0.0954	0.0754	0.0577	0.0333	0.0234
4	0.1080	0.1336	0.1433	0.1378	0.1213	0.1115	0.0980	0.0746	0.0475	0.0243
5	0.0787	0.1105	0.1232	0.1335	0.1345	0.1291	0.1144	0.0862	0.0614	0.0286
6	0.0656	0.1028	0.1149	0.1201	0.1276	0.1330	0.1250	0.0998	0.0823	0.0288
7	0.0548	0.0779	0.0842	0.1097	0.1196	0.1224	0.1410	0.1331	0.1132	0.0441
8	0.0428	0.0507	0.0741	0.0880	0.0994	0.1224	0.1410	0.1585	0.1539	0.0693
9	0.0416	0.0436	0.0474	0.0577	0.0803	0.1001	0.1277	0.1728	0.1939	0.1348
10	0.0386	0.0204	0.0269	0.0292	0.0339	0.0520	0.0704	0.1155	0.1945	0.4186

Notes: $\Pr(d_i < Y_1 < d_{i+1} \mid d_j < Y_0 < d_{j+1}, \mathcal{I})$ where d_i is the ith decile of the college lifetime *ex post* earnings distribution and d_j is the jth decile of the high school *ex post* lifetime earnings distribution. Individual fixes unknown θ at their means. The information set includes $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6\}$. Correlation $(Y_1, Y_0) = 0.2842$.
Source: Cunha and Heckman (2007b).

Table 6

Uncertainty at age 17 about future returns

	College	High school	Returns
Total residual variance*	709.7487	507.2910	906.0066
Variance of unforecastable components*	372.3509	272.3596	432.8733

Source: Cunha and Heckman (2007b).
*After conditioning on X, Z.

Table 7

Percentage that regret schooling choices

Percentage of high school graduates who regret not graduating from college	0.0749
Percentage of college graduates who regret graduating from college	0.0311

Notes: *Ex post* people know their "luck" components (i.e., the uncertain $\varepsilon_{s,t}$ for each schooling group s for all ages t on their earnings equations) when making their schooling decisions. These calculations are for a stationary environment and ignore the growth in the mean of college distribution experienced in recent decades.
Source: Cunha and Heckman (2007b).

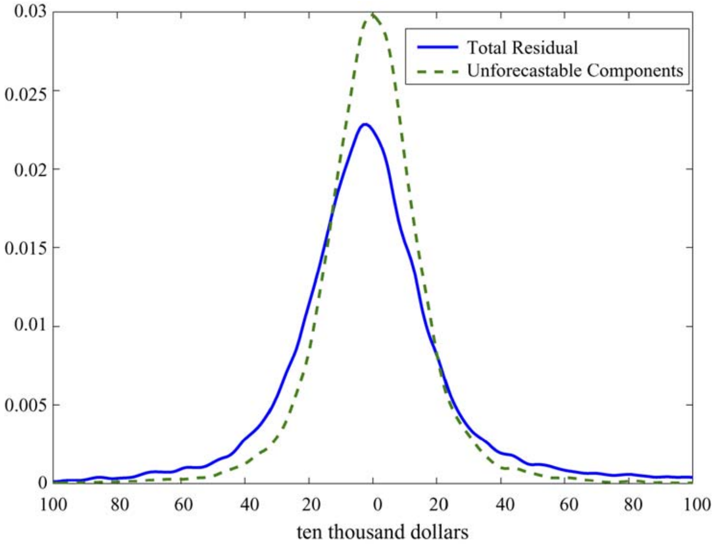

Figure 1. The densities of total residual vs. unforecastable components. Returns to college vs. high school (NLSY79). In this figure, we plot the density of the total residual (the solid curve) against the density of unforecastable components (the dashed curve) for the present value of returns to college from ages 22 to 41. The present value of returns to college is calculated using a 5% interest rate.
Source: Cunha and Heckman (2007b).

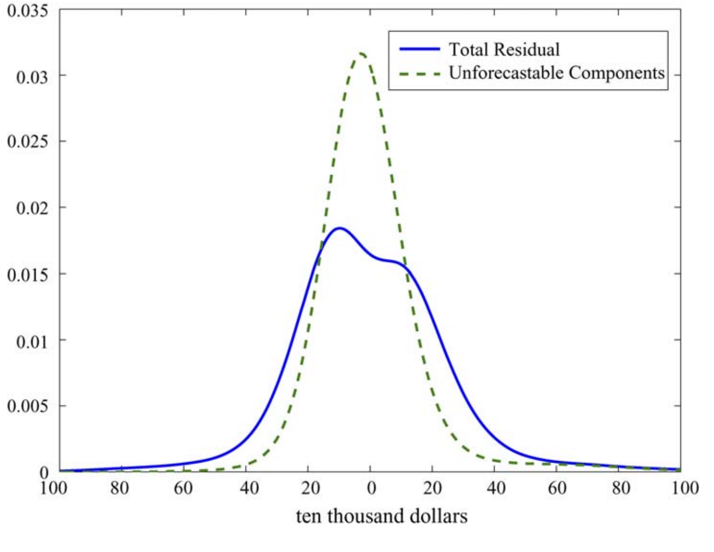

Figure 2. The densities of total residual vs. unforecastable components in present value of high school earnings. In this figure, we plot the density of the total residual (the solid curve) against the density of unforecastable components (the dashed curve) for the present value of high school earnings from ages 22 to 41. The present value of earnings is calculated using a 5% interest rate. *Source*: Cunha and Heckman (2007b).

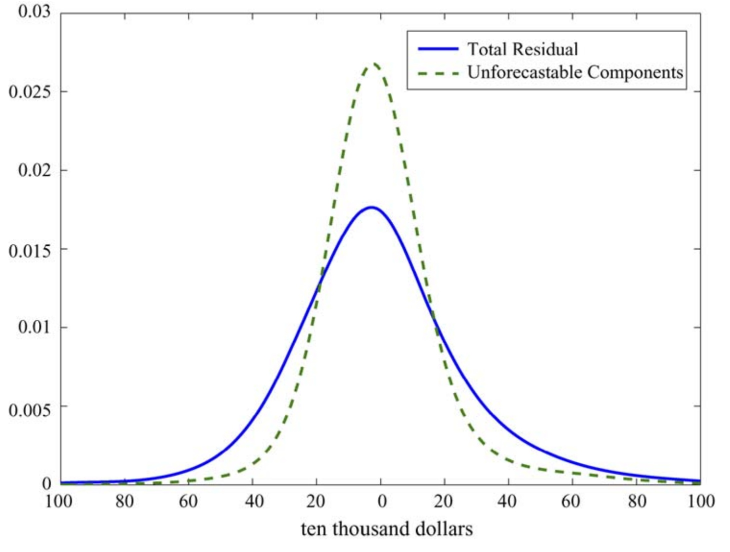

Figure 3. The densities of total residual vs. unforecastable components in present value of college earnings. In this figure, we plot the density of the total residual (the solid curve) against the density of unforecastable components (the dashed curve) for the present value of college earnings from ages 22 to 41. The present value of earnings is calculated using a 5% interest rate. *Source*: Cunha and Heckman (2007b).

Selection on the first three factors is illustrated in Figures 4–6. Factor one is associated with ability as measured by a test score (θ_1 in the examples of the previous sections). The factors sort on the basis of schooling choices. Cunha and Heckman (2007b) show that accounting for nonnormality of the factors is empirically important.

Table 8 presents the selection-corrected mean rates of return to 4 years of college. It is close to 10% for college goers, 8.25% for those who choose to stop their education at high school and 8.75% for those who are at the margin of indifference between attending high school and going to college. Matching assumption (M-1), which requires that average returns equal marginal returns, is not supported by these estimates. For further details on these estimates, see Cunha and Heckman (2007b).

To show the possibilities for a more nuanced approach to policy evaluation that is possible, we draw on a second, earlier, paper by Cunha and Heckman (2008). While this research is superceded by the richer empirical analysis in Cunha and Heckman (2007b), it illustrates the potential of the method exposited in this chapter.[43] They estimate a

[43] Cunha and Heckman (2007b) use many more periods of panel data, have many more measurements and estimate a six-factor model. Cunha and Heckman (2008) use many fewer periods, have a lower dimension $L(s)$ in the notation of condition (2.9) and determine that $K = 2$ fits the data they analyze.

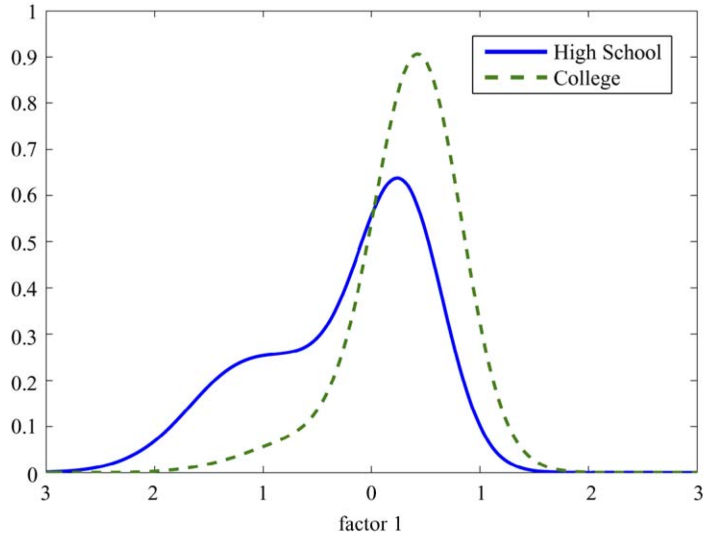

Figure 4. Densities of factor 1 by schooling level (NLSY79). The solid line plots the density of the factor for high school graduates. The dashed line plots the density of the factor for college graduates. *Source*: Cunha and Heckman (2007b).

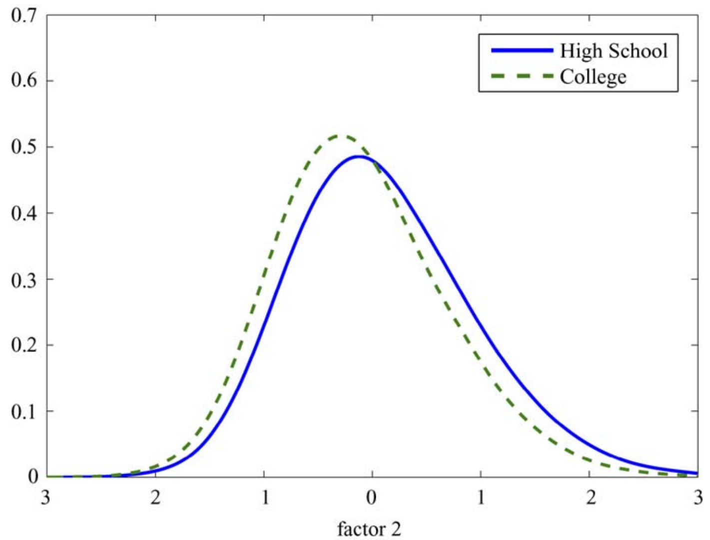

Figure 5. Densities of factor 2 by schooling level (NLSY79). The solid line plots the density of the factor for high school graduates. The dashed line plots the density of the factor for college graduates. *Source*: Cunha and Heckman (2007b).

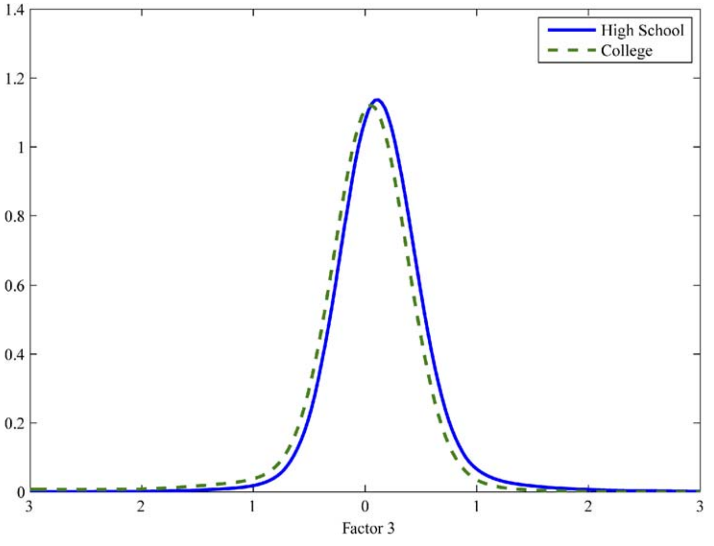

Figure 6. Densities of factor 3 by schooling level (NLSY79). The solid line plots the density of the factor for high school graduates. The dashed line plots the density of the factor for college graduates. *Source*: Cunha and Heckman (2007b).

Table 8
Mean rates of return to college by schooling group (NLSY79)

Schooling group	Mean returns	Standard error
High school graduates	0.3095	0.0113
College graduates	0.3994	0.0129
Individuals at the margin	0.3511	0.0535

Source: Cunha and Heckman (2007b).

two-factor model. Figures 7 and 8 plot the densities of the present value of earnings and the associated counterfactual distribution for college graduates ($D = 1$, Figure 7) and high school graduates ($D = 0$, Figure 8). Gross rates of return ($\frac{Y_1 - Y_0}{Y_0}$) are plotted in Figure 9 for both high school and college graduates.

The overlap in the factual and counterfactual distributions for each schooling level is substantial. The returns to college for high school graduates are substantial. One reason why such large monetary returns to college are not realized is shown in Figure 10 which plots the psychic costs (C) of attending college. Confirming earlier findings by Carneiro, Hansen and Heckman (2003) and Cunha, Heckman and Navarro (2005), there are substantial psychic costs of attending school for the high school graduates (see Figure 10).

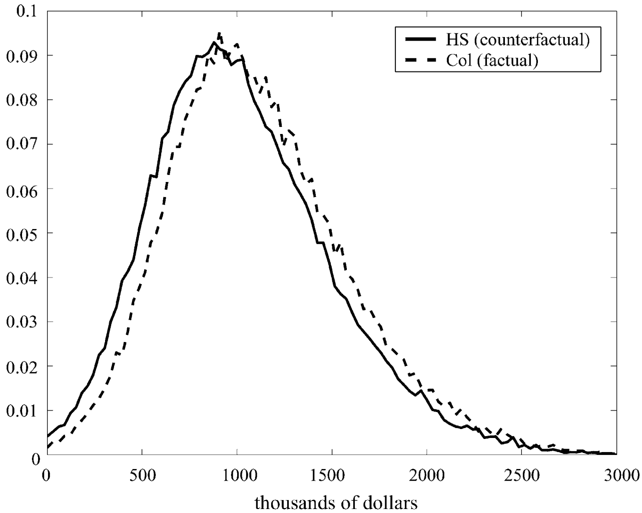

Figure 7. Density of present value of earnings in the college sector. Let Y_1 denote present value of earnings (discounted at a 3% interest rate) in the college sector. Let $f_1(y_1)$ denote its density function. The dashed line plots the predicted Y_1 density conditioned on choosing college, that is, $f_1(y_1 \mid D = 1)$, while the solid line shows the counterfactual density function of Y_0 for those agents that are actually college graduates, that is, $f_0(y_0 \mid D = 1)$. *Source*: Cunha and Heckman (2008).

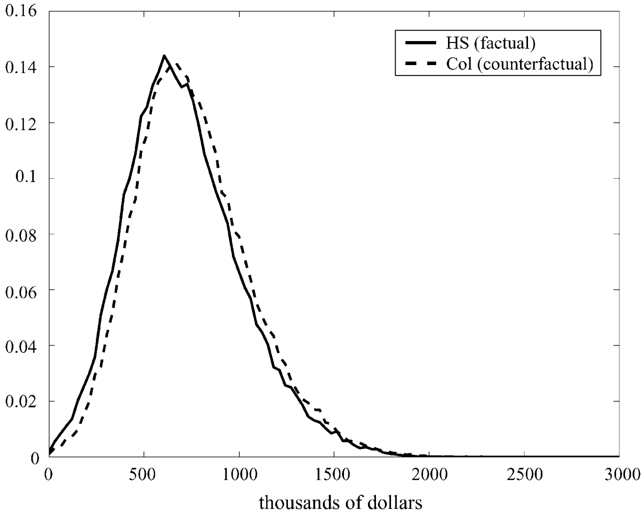

Figure 8. Density of present value of earnings in the high school sector. Let Y_0 denote present value of earnings (discounted at a 3% interest rate) in the high school sector. Let $f_0(y_0)$ denote its density function. The solid curve plots the predicted Y_0 density conditioned on choosing high school, that is, $f_0(y_0 \mid D = 0)$, while the dashed line shows the counterfactual density function of Y_1 for those agents that are high-school graduates, that is, $f_1(y_1 \mid D = 0)$. *Source*: Cunha and Heckman (2008).

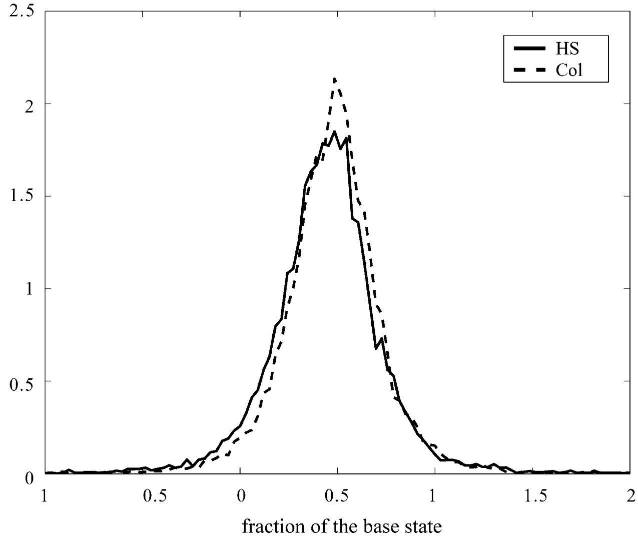

Figure 9. Density of *ex post* returns to college by schooling level chosen. Let Y_0, Y_1 denote the present value of earnings in high school and college sectors, respectively. Define *ex post* returns to college as the ratio $R = (Y_1 - Y_0)/Y_0$. Let $f(r)$ denote the density function of random variable R. The solid line is the density of *ex post* returns to college for high school graduates, that is, $f(r \mid D = 0)$. The dashed line is the density of *ex post* returns to college for college graduates, that is, $f(r \mid D = 1)$. *Source*: Cunha and Heckman (2008).

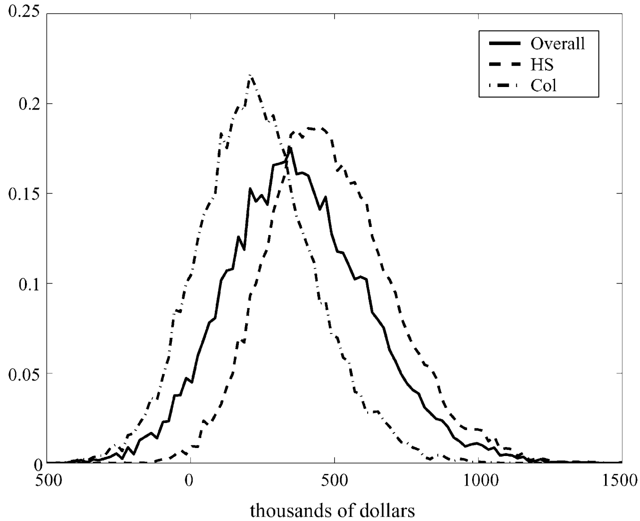

Figure 10. Density of monetary value of psychic cost both overall and by schooling level. In this figure we plot the monetary value of psychic costs, which we denote by C. It is defined as: $C = Z\gamma + \theta_1\alpha_{1,C} + \theta_2\alpha_{2,C} + \varepsilon_C$. The contribution of ability to the costs of attending college, in monetary value, is $\theta_1\alpha_{1,C}$. *Source*: Cunha and Heckman (2008).

A comparison of Figures 9 and 10 is revealing. There are small differences in objective returns to college between those who go to college and those who do not. However, the subjective returns (inclusive of psychic costs) are substantially different. This evidence of large subjective costs highlights the value of the econometric approach to the evaluation of social programs, and the importance of the distinction between objective and subjective outcomes in interpreting choices and outcomes.

As an example of the power of these methods to evaluate the consequences of policy on income inequality, Cunha and Heckman (2008) analyze a cross-subsidized tuition policy indexed by family income level. The traditional approach to policy evaluation compares overall income distributions before and after a policy change is implemented. Although this approach can be justified by certain axiomatic approaches [see, e.g., Foster and Sen (1997), and Cowell (2000)], it does not present a very accurate summary of the true distributional consequences of policies.

Cunha and Heckman (2008) construct joint distributions of outcomes within policy regimes (treatment and no treatment or schooling and no schooling) and joint distributions of outcomes ($Y = DY_1 + (1 - D)Y_0$) across policy regimes. The policy they analyze is as follows. A prospective student whose family income at age 17 is below the mean is allowed to attend college free of charge. The policy is self financing within each schooling cohort. To pay for this policy, persons attending college with family income above the mean pay a tuition charge equal to the amount required to cover the costs of the students from lower income families as well as their own.

Total tuition raised covers the cost Q of educating each student. Thus if there are N_P poor students and N_R rich students, total costs are $(N_P + N_R)Q$. For the proposed policy, the poor pay nothing. So each rich person is charged a tuition

$$T = Q\left(1 + \frac{N_P}{N_R}\right).$$

To determine T, notice that there is a unique tuition level T such that

$$T = Q\left(1 + \frac{N_P(T)}{N_R(T)}\right),$$

with $N_P(T)$ and $N_R(T)$ the numbers of poor and rich people attending college if the rich pay a fee T. They iterate to find the unique self-financing T. Notice that $N_P(T)$, the number of poor people who attend college when tuition is zero, is the same for all values of T ($N_P(T) = N_P(0)$ for all T). N_R is sensitive to the tuition level charged.

Figure 11 shows that the marginal distributions of overall income in both the pre-policy state and the post-policy state are essentially identical. Under the standard anonymity postulate used to evaluate income distributions [see Foster and Sen (1997), and Cowell (2000)], we would judge these two situations as equally good using Lorenz measures or second order stochastic dominance. Cunha and Heckman (2008) move beyond anonymity and analyze the effect that the policy has on what Fields (2003) calls "positional" mobility.

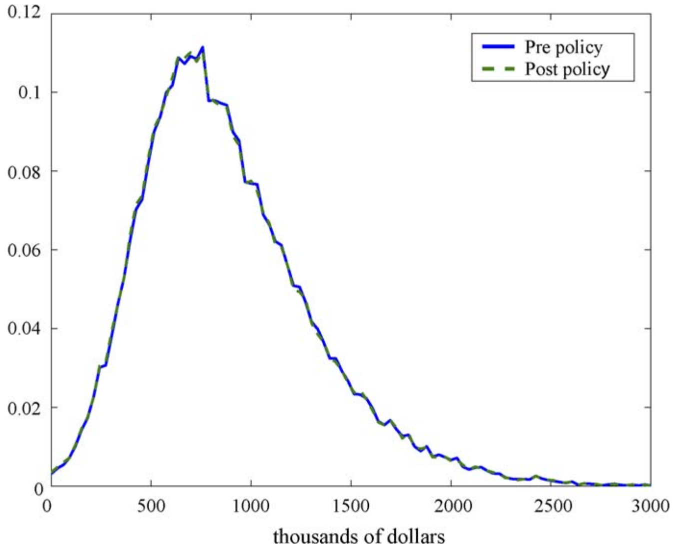

Figure 11. Density of present value of lifetime earnings before and after implementing cross-subsidy policy. Let Y^A, Y^B denote the observed present value of earnings pre and post policy, respectively. Define $f(y^A)$, $g(y^B)$ as the marginal densities of present value of earnings pre and post policy. In this figure, we plot $f(y^A)$, $g(y^B)$. *Source*: Cunha and Heckman (2007a).

Panel 1 of Table 9 presents this analysis by describing how the 9.2% of the people who are affected by the policy move between deciles of the distribution of income. These statistics describe movements from one income distribution in the initial regime to another income distribution associated with the new regime. The policy affects more people at the top deciles than at the lower deciles. Around half of the people affected who start at the first decile remain at the first decile. People in the middle deciles are spread both up and down and a large proportion of people in the upper deciles is moved into a lower position (only sixteen percent of those starting on the top decile (the first) remain there after the policy is implemented). Moving beyond the anonymity postulate (which instructs us to examine only marginal distributions), we learn much more about the effects of the policy on different groups by looking at joint distributions.

Thus far, we have focused on constructing and interpreting the joint distribution of outcomes across the two policy regimes. If outcomes under both regimes are observed, these comparisons could be made using panel data. No use of econometric analysis would be necessary. However, the methods discussed by Cunha and Heckman (2007b) will apply if either or both policy regimes are unobserved but are proposed. Taking advantage of the fact that we can identify not only joint distributions of earnings over policy regimes but also over counterfactual states within regimes we can learn a great

Table 9
Mobility of people affected by cross-subsidizing tuition

Fraction by decile of origin	Deciles of origin	Probability of moving to a different decile of the lifetime earnings distribution									
		1	2	3	4	5	6	7	8	9	10
Panel 1											
Overall. Fraction of total population who switch schooling levels: 0.0923											
0.0730	1	0.5680	0.2052	0.1245	0.0647	0.0288	0.0076	0.0012	0.0000	0.0000	0.0000
0.0869	2	0.2079	0.1712	0.1715	0.1690	0.1585	0.0870	0.0322	0.0025	0.0002	0.0000
0.0957	3	0.1148	0.1489	0.0935	0.1137	0.1573	0.1888	0.1387	0.0409	0.0034	0.0000
0.1001	4	0.0619	0.1557	0.0910	0.0534	0.0764	0.1615	0.2084	0.1557	0.0360	0.0000
0.1035	5	0.0296	0.1495	0.1387	0.0630	0.0304	0.0571	0.1411	0.2456	0.1396	0.0055
0.1053	6	0.0066	0.0959	0.1726	0.1471	0.0520	0.0142	0.0415	0.1671	0.2605	0.0425
0.1087	7	0.0006	0.0336	0.1411	0.1956	0.1269	0.0420	0.0082	0.0348	0.2346	0.1827
0.1092	8	0.0000	0.0046	0.0519	0.1765	0.2211	0.1495	0.0388	0.0034	0.0513	0.3029
0.1104	9	0.0000	0.0000	0.0055	0.0421	0.1570	0.2733	0.2302	0.0447	0.0014	0.2459
0.1071	10	0.0000	0.0000	0.0000	0.0002	0.0041	0.0517	0.2082	0.3242	0.2490	0.1626
Panel 2											
High school. Fraction of total population who switch from high school to college due to the policy: 0.0450											
0.1014	1	0.4049	0.2618	0.1817	0.0958	0.0427	0.0112	0.0018	0.0000	0.0000	0.0000
0.1282	2	0.0382	0.1220	0.2176	0.2325	0.2200	0.1210	0.0448	0.0035	0.0003	0.0000
0.1372	3	0.0023	0.0188	0.0692	0.1536	0.2244	0.2701	0.1984	0.0584	0.0049	0.0000
0.1370	4	0.0000	0.0016	0.0088	0.0368	0.1116	0.2417	0.3123	0.2332	0.0540	0.0000
0.1288	5	0.0000	0.0000	0.0007	0.0052	0.0277	0.0903	0.2324	0.4047	0.2300	0.0090
0.1125	6	0.0000	0.0000	0.0000	0.0004	0.0024	0.0151	0.0792	0.3209	0.5004	0.0816
0.1019	7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0009	0.0101	0.0761	0.5133	0.3997
0.0798	8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0067	0.1440	0.8493
0.0559	9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0032	0.9968
0.0173	10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
Panel 3											
College. Fraction of total population who switch from college to high school due to the policy: 0.0473											
0.0460	1	0.9066	0.0878	0.0056	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0477	2	0.6423	0.2972	0.0534	0.0062	0.0009	0.0000	0.0000	0.0000	0.0000	0.0000
0.0562	3	0.3763	0.4510	0.1501	0.0211	0.0015	0.0000	0.0000	0.0000	0.0000	0.0000
0.0649	4	0.1860	0.4648	0.2559	0.0868	0.0059	0.0007	0.0000	0.0000	0.0000	0.0000
0.0794	5	0.0753	0.3801	0.3518	0.1522	0.0347	0.0059	0.0000	0.0000	0.0000	0.0000
0.0985	6	0.0138	0.2001	0.3602	0.3064	0.1059	0.0133	0.0004	0.0000	0.0000	0.0000
0.1152	7	0.0011	0.0618	0.2598	0.3603	0.2337	0.0766	0.0066	0.0000	0.0000	0.0000
0.1371	8	0.0000	0.0071	0.0807	0.2744	0.3436	0.2323	0.0603	0.0015	0.0000	0.0000
0.1623	9	0.0000	0.0000	0.0073	0.0559	0.2084	0.3628	0.3056	0.0593	0.0008	0.0000
0.1926	10	0.0000	0.0000	0.0000	0.0002	0.0044	0.0561	0.2260	0.3519	0.2702	0.0911

Notes: Cross subsidy consists in making tuition zero for people with family income below average and making the budget balance by raising tuition for college students with family income above the average. For example, we read from the first panel row 1, column 1 that 7.3% of the people who switch schooling levels come from the lowest decile. Out of those, 56.8% are still in the first decile after the policy while 2.88% jump to the fifth decile. Panel 2 has the same interpretation but it only looks at people who switch from high school to college while panel 3 looks at individuals who switch from college to high school.
Source: Cunha and Heckman (2008).

Table 10
Mobility of people affected by cross-subsidizing tuition (extracted from Table 9)

Fraction of the total population who switch schooling levels: 0.0923
Pre-policy choice

	Fraction of high school graduates	
	Do not switch	Become college graduates
High school	0.9197	0.0803
	Fraction of college graduates	
	Do not switch	Become high school graduates
College	0.8923	0.1077

Note: Cross subsidy consists of making tuition zero for people with family income below average and making the budget balance by raising tuition for college students with family income above the average.
Source: Cunha and Heckman (2008).

deal more about the effects of this policy, whether or not policy regimes are observed.[44] In this way, one solves problems P-1, P-2, and P-3 stated in Chapter 70.

Panels 2 and 3 of Table 9 reveal that not only 9.2% of the population is affected by the policy, but that actually about half of them moved from high school into college (4.5% of the population) and half moved from college into high school (4.7% percent of the population). This translates into saying that, of those affected by the policy, 92% of the high school graduates stay in high school in the post-policy regime while only 89% of college graduates stay put. (See Table 10.) Thus the policy is slightly biased against college attendance. We can form the joint distributions of lifetime earnings by initial schooling level. Figure 12 breaks out some of the evidence implicit in Table 9. Panels 2 and 3 of Table 9 show that the policy affects very few high school graduates at the top end of the income distribution (only 1.7% of those affected come from the 10th percentile) and a lot of college graduates in the same situation (19% of college graduates affected come from the top decile). We can also see that the policy tends to move high school graduates up in the income distribution and moves college graduates down.

As another example of the generality of our method and the new insight into income mobility induced by policy that it provides, we can determine where people come from and where they end up at in the counterfactual distributions of earnings. Table 11 shows where in the pre-policy distribution of high school earnings persons induced to go to college come from and where in the post-policy distribution of college earnings they go to. Most people stay in their decile or move to closely adjacent ones. Given that some people benefit from the policy while others lose, it is not clear whether society as a

[44] It is implausible that analysts would have panel data on policy regimes where under one regime a person goes to school and under another he does not.

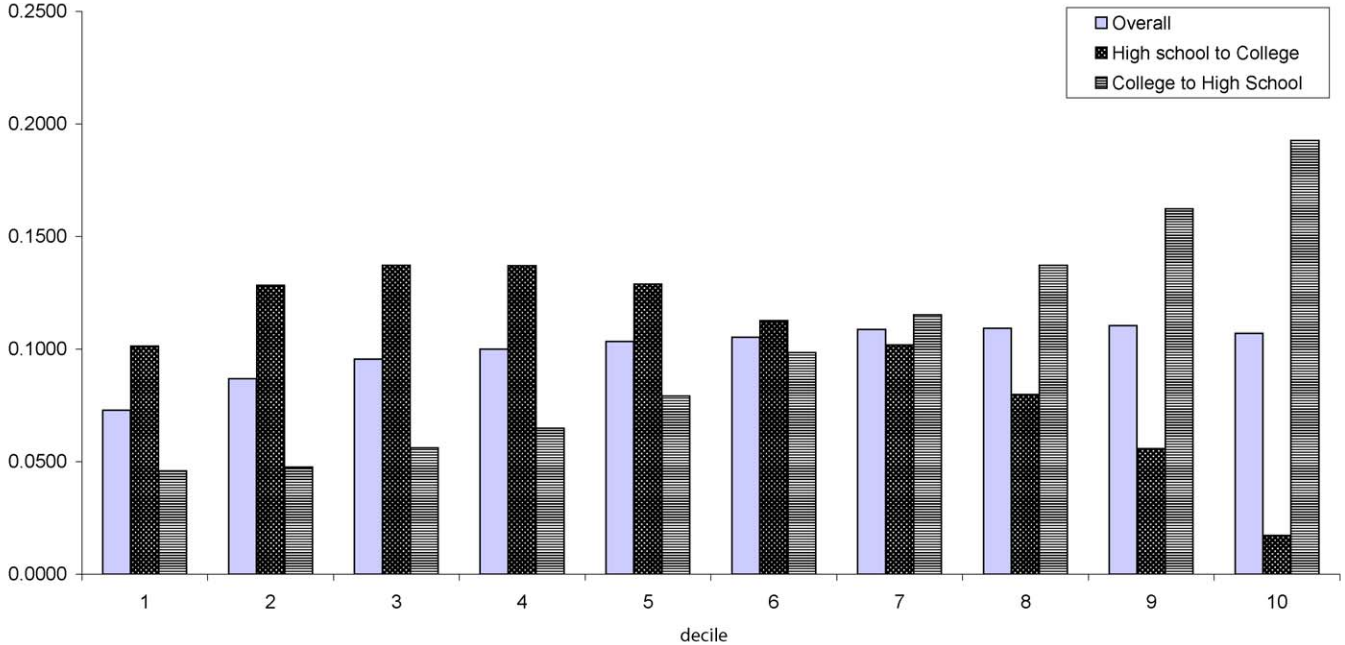

Figure 12. Fraction of people who switch schooling levels when tuition is cross subsidized by decile of origin from the lifetime earnings distribution. Cross subsidy consists in making tuition zero for people with family income below average and making the budget balance by raising tuition for college students with family income above the average. *Source*: Cunha and Heckman (2008).

Table 11
Mobility of people affected by cross-subsidizing tuition across counterfactual distributions

Panel 1
High school. Fraction of total population who switch from high school to college due to the policy: 0.0450

Fraction by decile of origin in the pre-policy high school distribution	Deciles of origin	Probability of moving to a different decile of the post-policy college lifetime earnings distribution									
		1	2	3	4	5	6	7	8	9	10
0.0668	1	0.8563	0.1272	0.0145	0.0021	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0813	2	0.4046	0.4112	0.1491	0.0296	0.0055	0.0000	0.0000	0.0000	0.0000	0.0000
0.0910	3	0.1488	0.3544	0.3059	0.1419	0.0445	0.0039	0.0005	0.0000	0.0000	0.0000
0.1000	4	0.0401	0.2343	0.3096	0.2490	0.1234	0.0379	0.0053	0.0004	0.0000	0.0000
0.1049	5	0.0089	0.0713	0.2081	0.3053	0.2348	0.1282	0.0365	0.0068	0.0000	0.0000
0.1060	6	0.0004	0.0202	0.0950	0.2155	0.2761	0.2416	0.1273	0.0239	0.0000	0.0000
0.1064	7	0.0000	0.0033	0.0243	0.0896	0.1888	0.3026	0.2662	0.1155	0.0096	0.0000
0.1118	8	0.0000	0.0004	0.0016	0.0159	0.0630	0.1690	0.3220	0.3228	0.1024	0.0028
0.1140	9	0.0000	0.0000	0.0000	0.0016	0.0043	0.0293	0.1227	0.3271	0.4568	0.0582
0.1176	10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0027	0.0333	0.2626	0.7014

Panel 2
College. Fraction of total population who switch from college to high school due to the policy: 0.0473

Fraction by decile of origin in the pre-policy college distribution	Deciles of origin	Probability of moving to a different decile of the post-policy high school lifetime earnings distribution									
		1	2	3	4	5	6	7	8	9	10
0.1098	1	0.5505	0.2962	0.1141	0.0318	0.0062	0.0012	0.0000	0.0000	0.0000	0.0000
0.1059	2	0.1076	0.3257	0.2937	0.1789	0.0716	0.0204	0.0016	0.0004	0.0000	0.0000
0.1039	3	0.0180	0.1473	0.2776	0.2657	0.1833	0.0857	0.0200	0.0024	0.0000	0.0000
0.1016	4	0.0004	0.0355	0.1535	0.2349	0.2866	0.1890	0.0847	0.0150	0.0004	0.0000
0.1016	5	0.0000	0.0050	0.0467	0.1503	0.2654	0.2705	0.1903	0.0668	0.0050	0.0000
0.0983	6	0.0000	0.0000	0.0091	0.0513	0.1678	0.2683	0.2972	0.1786	0.0276	0.0000
0.0980	7	0.0000	0.0000	0.0000	0.0087	0.0463	0.1609	0.3071	0.3387	0.1362	0.0022
0.0956	8	0.0000	0.0000	0.0000	0.0004	0.0044	0.0430	0.1560	0.4020	0.3617	0.0324
0.0967	9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0009	0.0127	0.1337	0.5355	0.3173
0.0885	10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0034	0.0915	0.9051

Notes: Cross subsidy consists in making tuition zero for people with family income below average and making the budget balance by raising tuition for college students with family income above the average. For example, we read from the first panel row 1, column 1 that 6.68% of the people who switch from high school to college come from the lowest decile of the pre-policy high school distribution. Out of those, 85.63% are still in the first decile of the post-policy college earnings distribution after the policy is implemented while 1.45% "jump" to the third decile. Panel 2 has the same interpretation but it only looks at people who switch from college to high school.
Source: Cunha and Heckman (2008).

Table 12
Voting outcome of proposing cross-subsidizing tuition

Fraction of the total population who switch schooling levels: 0.0923	
Average pre-policy lifetime earnings*	920.55
Average post-policy lifetime earnings*	905.96
Fraction of the population who vote	
Yes	0.0716
No	0.6152
Indifferent	0.3132

Note: Cross subsidy consists of making tuition zero for people with family income below average and making the budget balance by raising tuition for college students with family income above the average.
Source: Cunha and Heckman (2008).
*In thousands of dollars.

whole values this policy positively or not. An advantage of examining the joint distribution of outcomes is that it allows us to calculate the effect that the policy has on welfare. An individual's relative utility is not only given by earnings but also by the monetary value of psychic costs. We can predict how people would vote if the policy analyzed in this section were proposed. Table 12 shows the result of such an exercise. The policy lowers the mean earnings for people affected by it. Most people not indifferent to the policy would vote against it.

We next turn to the development of models for the timing of treatment choice. The models that distinguish *ex ante* from *ex post* outcomes discussed in this section of the chapter have an implicit dynamics. Agents make decisions under one information set. That information set is revised in light of subsequent flows of information. The outcomes realized after the choice is made will differ in general from the outcomes that are anticipated. However, in this section, choices are one shot. While this framework advances models that ignore uncertainty, it does not capture the rich dynamics that comes from updating information in real time. We next consider models that analyze the choice of the timing of treatment and the consequences of the choices. Analyses of decisions about the timing of dropping out of school, the timing of initiating or terminating a medical treatment, when to end a period of unemployment, and the consequences of such decisions raise new issues to which we now turn.

3. Dynamic models[45]

We now develop econometric and statistical models for the choice of timing of treatment and the consequences of alternative treatment times on subjective and objective

45 This section draws in part on Abbring and Heckman (2008) and the papers they cite.

outcomes. The analysis presented in this section extends the analysis of multiple treatments and treatment choices presented in Chapter 71 by explicitly considering dynamics and information updating. We first develop some main ideas in a framework with general dynamic treatments. We subsequently focus on the choice of the timing of a single treatment which may have very different consequences when implemented in different periods. The same treatments administered at different times can be thought of as different treatments. Thus, dropping out of school at grade 11 may have different consequences than dropping out at grade 10. Starting chemotherapy eight months after diagnosis of the onset of cancer may have different consequences than chemotherapy starting after one month. There is a close affinity between econometric models for discrete choice and models for the analysis of the choice of treatment times which is developed in this section.

The plan of this section is as follows. Section 3.1 briefly reviews the policy evaluation problem extensively discussed by Heckman and Vytlacil in Chapter 70 and discusses the treatment-effects approach to policy evaluation. It establishes the notation used in the rest of this section. Section 3.2 reviews an approach to the analysis of dynamic treatment effects developed in statistics based on a sequential randomization assumption that is popular in biostatistics [Robins (1997), Gill and Robins (2001), Lok (2007)] and has been applied in economics [see Fitzenberger, Osikominu and Völter (2006) and Lechner and Miquel (2002)]. This is a dynamic version of matching. We relate the assumptions justifying this approach to the assumptions underlying the econometric dynamic discrete-choice literature based on Rust's (1987) conditional-independence condition which, as discussed in Section 3.4.5 below, is frequently invoked in the structural econometrics literature. We note the limitations of the dynamic matching treatment-effects approach in accounting for dynamic information accumulation. In Sections 3.3 and 3.4, we discuss two econometric approaches for the analysis of treatment times that allow for nontrivial dynamic selection on unobservables. Section 3.3 discusses the continuous-time event-history approach to policy evaluation developed by Abbring and Van den Berg (2003b, 2005) and Abbring (2008). Section 3.4 introduces an approach that builds on and extends the discrete-time dynamic discrete-choice literature. Like the analysis of Abbring and Van den Berg, it does not rely on the conditional-independence assumptions used in dynamic matching. This part of our survey is based on the work of Heckman and Navarro (2007). The approach exposited in this section generalizes the factor model approach exposited in Section 2 to a dynamic setting. The two complementary approaches surveyed in this section span the existing econometric literature on dynamic treatment effects.

3.1. Policy evaluation and treatment effects

3.1.1. The evaluation problem

We review the evaluation problem discussed in Chapter 70 using a succinct notation employed in the analysis of this section. Let Ω be the set of agent types. It is the sample space of a probability space $(\Omega, \mathcal{I}, \mathbb{P})$, and all choices and outcomes are random variables defined on this probability space. Each agent type $\omega \in \Omega$ represents a single agent in a particular state of nature. We could distinguish variation between agents from within-agent randomness by taking $\Omega = J \times \tilde{\Omega}$, with J the set of agents and $\tilde{\Omega}$ the set of possible states of nature. However, we do not make this distinction explicit in this section, and often simply refer to agents instead of agent types.[46]

Consider a policy that targets the allocation of each agent in Ω to a single treatment from a set $\mathcal{S}$. In the most basic binary version, $\mathcal{S} = \{0, 1\}$, where "1" represents "treatment", such as a training program, and "0" some baseline, "control" program. Alternatively, $\mathcal{S}$ could take a continuum of values, e.g., $\mathbb{R}_+ = [0, \infty)$, representing, e.g., unemployment benefit levels, or duration of time in a program.

A policy $p = (a, \tau) \in \mathcal{A} \times \mathcal{T} = \mathcal{P}$ consists of a planner's rule $a : \Omega \to \mathcal{B}$ for allocating constraints and incentives to agents, and a rule $\tau : \Omega \times \mathcal{A} \to \mathcal{S}$ that generates agent treatment choices for a given constraint allocation a. This framework allows agent ω's treatment choice to depend both on the constraint assignment mechanism a—in particular, the distribution of the constraints in the population—and on the constraints $a(\omega) \in \mathcal{B}$ assigned to agent ω.[47]

The randomness in the planner's constraint assignment a may reflect heterogeneity of agents as observed by the planner, but it may also be due to explicit randomization. For example, consider profiling on background characteristics of potential participants in the assignment a to treatment eligibility. If the planner observes some background characteristics on individuals in the population of interest, she could choose eligibility status to be a deterministic function of those characteristics and, possibly, some other random variable under her control by randomization. This includes the special case in which the planner randomizes persons into eligibility. We denote the information set generated by the variables observed by the planner when she assigns constraints, including those generated through deliberate randomization, by $\mathcal{I}_P$.[48] The planner's information set $\mathcal{I}_P$ determines how precisely she can target agents ω when assigning constraints. The variables in the information set fully determine the constraints assignment a.

Subsequent to the planner's constraints assignment a, each agent ω chooses treatment $\tau(\omega, a)$. We assume that agents know the constraint assignment mechanism a in place. However, agents do not directly observe their types ω, but only observe realizations $I_A(\omega)$ of some random variables I_A. For given $a \in \mathcal{A}$, agent ω's treatment

[46] For example, we could have $\Omega = [0, 1]$ indexing the population of agents, with $\mathbb{P}$ being Lebesgue measure on $[0, 1]$. Alternatively, we could take $\Omega = [0, 1] \times \tilde{\Omega}$ and have $[0, 1]$ represent the population of agents and $\tilde{\Omega}$ states of nature.

[47] In Chapter 70, the dependence of agent ω's treatment choice τ on the constraints $a(\omega)$ was made explicit by defining τ on $\Omega \times \mathcal{A} \times \mathcal{B}$, and subsequently restricting τ to $\{(\omega, a, b) \in \Omega \times \mathcal{A} \times \mathcal{B}\colon a(\omega) = b\}$. Because the constraints $b = a(\omega)$ assigned are already encoded in a and ω, we can drop the constraints b from τ assigned without loss of generality. In the dynamic context of this chapter, this convention simplifies the discussion of dynamic information accumulation.

[48] Formally, $\mathcal{I}_P$ is a sub-σ-algebra of $\mathcal{I}$ and a is assumed to be $\mathcal{I}_P$-measurable.

choice $\tau(\omega, a)$ can only depend on ω through his observations $I_A(\omega)$. Typically, $I_A(\omega)$ includes the variables used by the planner in determining $a(\omega)$, so that agents know the constraints that they are facing. Other components of $I_A(\omega)$ may be determinants of preferences and outcomes. Variation in $I_A(\omega)$ across ω may thus reflect preference heterogeneity, heterogeneity in the assigned constraints, and heterogeneity in outcome predictors. We use $\mathcal{I}_A$ to denote the information set generated by I_A.[49] An agent's information set $\mathcal{I}_A$ determines how precisely the agent can tailor his treatment choice to his type ω. For expositional convenience, we assume that agents know more when choosing treatment than what the planner knows when assigning constraints, so that $\mathcal{I}_A \supseteq \mathcal{I}_P$. One consequence is that agents observe the constraints $a(\omega)$ assigned to them, as previously discussed. In turn, the econometrician may not have access to all of the information that is used by the agents when they choose treatment.[50] In this case, $\mathcal{I}_A \nsubseteq \mathcal{I}_E$, where $\mathcal{I}_E$ denotes the econometrician's information set.

We define $s_p(\omega)$ as the treatment selected by agent ω under policy p. With $p = (a, \tau)$, we have that $s_p(\omega) = \tau(\omega, a)$. The random variable $s_p : \Omega \to \mathcal{S}$ represents the allocation of agents to treatments implied by policy p.[51] Randomness in this allocation reflects both heterogeneity in the planner's assignment of constraints and the agent's heterogeneous responses to this assignment. One extreme case arises if the planner assigns agents to treatment groups and agents perfectly comply, so that $\mathcal{B} = \mathcal{S}$ and $s_p(\omega) = \tau(\omega, a) = a(\omega)$ for all $\omega \in \Omega$. In this case, all variation of s_p is due to heterogeneity in the constraints $a(\omega)$ across agents ω. At the other extreme, agents do not respond at all to the incentives assigned by mechanisms in $\mathcal{A}$, and $\tau(a, \omega) = \tau(a', \omega)$ for all $a, a' \in \mathcal{A}$ and $\omega \in \Omega$. In general, there are policies that have a nontrivial (that is, nondegenerate) constraint assignment a, where at least some agents respond to the assigned constraints a in their treatment choice, $\tau(a, \omega) \neq \tau(a', \omega)$ for some $a, a' \in \mathcal{A}$ and $\omega \in \Omega$.

We seek to evaluate a policy p in terms of some outcome Y_p, for example, earnings. For each $p \in \mathcal{P}$, Y_p is a random variable defined on the population Ω. We index outcomes by a policy subscript in order to simplify the notation. To avoid notational confusion, we will not use treatment subscripts in this section. The evaluation can focus

[49] Formally, $\mathcal{I}_A$ is a sub-σ-algebra of $\mathcal{I}$ – the σ-algebra generated by I_A – and $\omega \in \Omega \mapsto \tau(\omega, a) \in \mathcal{S}$ should be $\mathcal{I}_A$-measurable for all $a \in \mathcal{A}$. The possibility that different agents have different information sets is allowed for because a distinction between agents and states of nature is implicit. As suggested in the introduction to this section, we can make it explicit by distinguishing a set J of agents and a set $\tilde{\Omega}$ of states of nature and writing $\Omega = J \times \tilde{\Omega}$. For expositional convenience, let J be finite. We can model that agents observe their identity j by assuming that the random variable J_A on Ω that reveals their identity, that is $J_A(j, \tilde{\omega}) = j$, is in their information set $\mathcal{I}_A$. If agents, in addition, observe some other random variable V on Ω, then the information set $\mathcal{I}_A$ generated by (J_A, V) can be interpreted as providing each agent $j \in J$ with perfect information about his identity j and with the agent-j-specific information about the state of nature $\tilde{\omega}$ encoded in the random variable $\tilde{\omega} \mapsto V(j, \tilde{\omega})$ on $\tilde{\Omega}$.

[50] See the discussion by Heckman and Vytlacil in Chapter 71, Sections 2 and 9, of their contribution to this Handbook.

[51] Formally, $\{s_p\}_{p \in \mathcal{A} \times \mathcal{T}}$ is a stochastic process indexed by p.

on objective outcomes Y_p, on the subjective valuation $R(Y_p)$ of Y_p by the planner or the agents, or on both types of outcomes. The evaluation can be performed relative to a variety of information sets reflecting different actors (the agent, the planner and the econometrician) and the arrival of information in different time periods. Thus, the randomness of Y_p may represent both (*ex ante*) heterogeneity among agents known to the planner when constraints are assigned (that is, variables in $\mathcal{I}_P$) and/or heterogeneity known to the agents when they choose treatment (that is, information in $\mathcal{I}_A$), as well as (*ex post*) shocks that are not foreseen by the policy maker or by the agents. An information-feasible (*ex ante*) policy evaluation by the planner would be based on some criterion using the distribution of Y_p conditional on $\mathcal{I}_P$. The econometrician can assist the planner in computing this evaluation if the planner shares her *ex ante* information and $\mathcal{I}_P \subseteq \mathcal{I}_E$. We discussed *ex ante* and *ex post* evaluations in Section 2 in the context of a one shot model. It is also discussed in Chapter 70 of this Handbook. In this section, we discuss information revelation and *ex ante* and *ex post* evaluations in a dynamic setting.

Suppose that we have data on outcomes Y_{p_0} under policy p_0 with corresponding treatment assignment s_{p_0}. Consider an intervention that changes the policy from the actual p_0 to some counterfactual p' with associated treatments $s_{p'}$ and outcomes $Y_{p'}$. This could involve a change in the planner's constraint assignment from a_0 to a' for given $\tau_0 = \tau'$, a change in the agent choice rule from τ_0 to τ' for given $a_0 = a'$, or both.

The policy evaluation problem involves contrasting $Y_{p'}$ and Y_{p_0} or functions of these outcomes. For example, if the outcome of interest is mean earnings, we might be interested in some weighted average of $E[Y_{p'} - Y_{p_0} \mid \mathcal{I}_P]$, such as $E[Y_{p'} - Y_{p_0}]$. The special case where $\mathcal{S} = \{0, 1\}$ and $s_{p'} = a' = 0$ generates the effect of abolishing the program.[52] Implementing such a policy requires that the planner be able to induce all agents into the control group by assigning constraints $a' = 0$. In particular, as discussed in Chapter 71, Section 10, this assumes that there are no substitute programs available to agents that are outside the planner's control.

For notational convenience, write $S = s_{p_0}$ for treatment assignment under the actual policy p_0 in place. Cross-sectional microdata typically provide a random sample from the joint distribution of (Y_{p_0}, S).[53] Clearly, without further assumptions, such data do not identify the effects of the policy shift from p_0 to p'. This identification problem becomes even more difficult if we do not seek to compare the counterfactual policy p' with the actual policy p_0, but rather with another counterfactual policy p'' that also has never been observed. A leading example is the binary case in which $0 < \Pr(S = 1) < 1$, but we seek to know the effects of $s_{p'} = 0$ (universal nonparticipation) and $s_{p''} = 1$ (universal treatment), where neither policy has ever been observed in place. As we have

[52] Such a widespread policy would likely have general equilibrium effects. In this section, we will abstract from these by invoking invariance assumptions (PI-1)–(PI-4) discussed in Chapter 70. Section 4 discusses general equilibrium effects.

[53] Notice that a random sample of outcomes under a policy may entail nonrandom selection of treatments as individual agents select individual treatments given τ and the constraints they face assigned by a.

stressed repeatedly in Chapters 70 and 71 of this Handbook, determining the average treatment effect (ATE) is often a difficult task.

The standard microeconometric approach to the policy evaluation problem assumes that the (subjective and objective) outcomes for any individual agent are the same across all policy regimes for any particular treatment assigned to the individual [see, e.g., Heckman, LaLonde and Smith (1999)]. The invariance assumptions (PI-1)–(PI-4) that justify this practice are presented in Chapter 70. They simplify the task of evaluating policy p to determining (i) the assignment s_p of treatments under policy p and (ii) treatment effects for individual outcomes. Even within this simplified framework, there are still two difficult, and distinct, problems in identifying treatment effects on individual outcomes:

(A) *The Evaluation Problem*: *that we observe an agent in one treatment state and seek to determine that agent's outcomes in another state*; *and*

(B) *The Selection Problem: that the distributions of outcomes for the agents we observe in a given treatment state are not the marginal population distributions that would be observed if agents were randomly assigned to the state.*

The assignment mechanism s_p of treatments under counterfactual policies p is straightforward in the case where the planner assigns agents to treatment groups and agents fully comply, so that $s_p = a$. More generally, an explicit model of agent treatment choices is needed to derive s_p for counterfactual policies p. An explicit model of agent treatment choices can also be helpful in addressing the selection problem, and in identifying agent subjective valuations of outcomes. We now formalize the notation for the treatment-effect approach that we will use in this section.

3.1.2. *The treatment-effect approach*

For each agent $\omega \in \Omega$, let $y(s, X(\omega), U(\omega))$ be the potential outcome when the agent is assigned to treatment $s \in \mathcal{S}$. Here, X and U are covariates that are not causally affected by the treatment or the outcomes.[54,55] In the language of Kalbfleisch and Prentice (1980) and Leamer (1985), we say that such covariates are "external" to the causal model. X is observed by the econometrician (that is, in $\mathcal{I}_E$) and U is not.

Recall that s_p is the assignment of agents to treatments under policy p. For all policies p that we consider, the outcome Y_p is linked to the potential outcomes by the

[54] This is the "no feedback" condition (A-6) presented in Chapter 71. The condition requires that X and U are the same fixing $S = s$ for all s. See Haavelmo (1943), Pearl (2000), or the discussion in Chapter 70.

[55] Note that this framework is rich enough to capture the case in which potential outcomes depend on treatment-specific unobservables as in Sections 2 and 3.4, because these can be simply stacked in U and subsequently selected by y. For example, in the case where $\mathcal{S} = \{0, 1\}$ we can write $y(s, X, (U_0, U_1)) = sy_1(X, U_1) + (1 - s)y_0(X, U_0)$ for some y_0 and y_1. A specification without treatment-dependent unobservables is more tractable in the case of continuous treatments in Section 3.2 and, in particular, continuous treatment times in Section 3.3.

consistency condition $Y_p = y(s_p, X, U)$. This condition follows from the invariance assumptions presented in Chapter 70. It embodies the assumption that an agent's outcome only depends on the treatment assigned to the agent and not separately on the mechanism used to assign treatments. This excludes (strategic) interactions between agents and equilibrium effects of the policy.[56] It ensures that we can specify individual outcomes y from participating in programs in $\mathcal{S}$ independently of the policy p and treatment assignment s_p. Economists say that y is autonomous, or structurally invariant with respect to the policy environment [see Frisch (1938), Hurwicz (1962), and our discussion of structure and invariance in Chapter 70]. [57] With this notation in hand, we now turn to the dynamic policy evaluation problem.

3.1.3. Dynamic policy evaluation

Interventions often have consequences that span over many periods. Policy interventions at different points in time can be expected to affect not only current outcomes, but also outcomes at other points in time. The same policy implemented at different time periods may have different consequences. Moreover, policy assignment rules often have nontrivial dynamics. The assignment of programs at any point in time can be contingent on the available data on past program participation, intermediate outcomes and covariates.

The dynamic policy evaluation problem can be formalized in a fashion similar to the way we formalized the static problem in Chapter 70 and in Section 3.1.1. In this subsection, we analyze a discrete-time finite-horizon model. We consider continuous-time models in Section 3.3. The possible treatment assignment times are $1, \dots, \bar{T}$. We do not restrict the set $\mathcal{S}$ of treatments. We allow the same treatment to be assigned on multiple occasions. In general, the set of available treatments at each time t may depend on time t and on the history of treatments, outcomes, and covariates. For expositional convenience, we will only make this explicit in Sections 3.3 and 3.4, where we focus on the timing of a single treatment.

We define a dynamic policy $p = (a, \tau) \in \mathcal{A} \times \mathcal{T} = \mathcal{P}$ as a dynamic constraint assignment rule $a = \{a_t\}_{t=1}^{\bar{T}}$ with a dynamic treatment choice rule $\tau = \{\tau_t\}_{t=1}^{\bar{T}}$. At each time t, the planner assigns constraints $a_t(\omega)$ to each agent $\omega \in \Omega$, using information in the time-t policy-p information set $\mathcal{I}_P(t, p) \subseteq \mathcal{I}$. The planner's information set $\mathcal{I}_P(t, p)$ could be based on covariates and random variables under the planner's control, as well as past choices and realized outcomes. We denote the sequence of planner's information sets by $\mathcal{I}_P(p) = \{\mathcal{I}_P(t, p)\}_{t=1}^{\bar{T}}$. We assume that the planner does not

[56] See Pearl (2000), Heckman (2005), or the discussion in Chapter 70.

[57] See also Aldrich (1989) and Hendry and Morgan (1995). Rubin's (1986) stable-unit-treatment-value assumption is a version of the classical invariance assumptions of econometrics [see Abbring (2003), for discussion of this point, and the discussion in Chapter 70].

forget any information she once had, so that her information improves over time and $\mathcal{I}_P(t, p) \subseteq \mathcal{I}_P(t+1, p)$ for all t.[58]

Each agent ω chooses treatment $\tau_t(\omega, a)$ given their information about ω at time t under policy p and given the constraint assignment mechanism $a \in \mathcal{A}$ in place. We assume that agents know the constraint assignment mechanism a in place. At time t, under policy p, agents infer their information about their type ω from random variables $I_A(t, p)$ that may include preference components and determinants of constraints and future outcomes. $\mathcal{I}_A(t, p)$ denotes the time-t policy-p information set generated by $I_A(t, p)$ and $\mathcal{I}_A(p) = \{\mathcal{I}_A(t, p)\}_{t=1}^{\bar{T}}$. We assume that agents are increasingly informed as time goes by, so that $\mathcal{I}_A(t, p) \subseteq \mathcal{I}_A(t+1, p)$.[59] For expositional convenience, we also assume that agents know more than the planner at each time t, so that $\mathcal{I}_P(t, p) \subseteq \mathcal{I}_A(t, p)$.[60] Because all determinants of past and current constraints are in the planner's information set $\mathcal{I}_P(t, p)$, this implies that agents observe $(a_1(\omega), \ldots, a_t(\omega))$ at time t. Usually, they do not observe all determinants of their future constraints $(a_{t+1}(\omega), \ldots, a_{\bar{T}}(\omega))$.[61] Thus, the treatment choices of the agents may be contingent on past and current constraints, their preferences, and on their predictions of future outcomes and constraints given their information $\mathcal{I}_A(t, p)$ and given the constraint assignment mechanism a in place.

Extending the notation for the static case, we denote the assignment of agents to treatment τ_t at time t implied by a policy p by the random variable $s_p(t)$ defined so that $s_p(\omega, t) = \tau_t(\omega, a)$. We use the shorthand s_p^t for the vector $(s_p(1), \ldots, s_p(t))$ of treatments assigned up to and including time t under policy p, and write $s_p = s_p^{\bar{T}}$. The assumptions made so far about the arrival of information imply that treatment assignment $s_p(t)$ can only depend on the information $\mathcal{I}_A(t, p)$ available to agents at time t.[62]

Because past outcomes typically depend on the policy p, the planner's information $\mathcal{I}_P(p)$ and the agents' information $\mathcal{I}_A(p)$ will generally depend on p as well. In the treatment-effect framework that we develop in the next section, at each time t different policies may have selected different elements in the set of potential outcomes in the past. The different elements reveal different aspects of the unobservables underlying past and future outcomes. We will make assumptions that limit the dependence of information sets on policies in the context of the treatment-effects approach developed in the next section.

Objective outcomes associated with policies p are expressed as a vector of time-specific outcomes $Y_p = (Y_p(1), \ldots, Y_p(\bar{T}))$. The components of this vector may

[58] Formally, the information $\mathcal{I}_P(p)$ that accumulates for the planner under policy p is a filtration in $\mathcal{I}$, and a is a stochastic process that is adapted to $\mathcal{I}_P(p)$.

[59] Formally, the information $\mathcal{I}_A(p)$ that accumulates for the agents is a filtration in $\mathcal{I}$.

[60] If agents are strictly better informed, and $\mathcal{I}_P(t, p) \subset \mathcal{I}_A(t, p)$, it is unlikely that the planner catches up and learns the agent's information with a delay (e.g., $\mathcal{I}_A(t, p) \subseteq \mathcal{I}_P(t+1, p)$) unless agent's choices and outcomes reveal all their private information.

[61] Formally, $a_1, \ldots, a_t$ are $\mathcal{I}_A(t, p)$-measurable, but $a_{t+1}, \ldots, a_{\bar{T}}$ are not.

[62] Formally, $\{s_p(t)\}_{t=1}^{\bar{T}}$ is a stochastic process that is adapted to $\mathcal{I}_A(p)$.

also be vectors. We denote the outcomes from time 1 to time t under policy p by $Y_p^t = (Y_p(1), \ldots, Y_p(t))$. We analyze both subjective and objective evaluations of policies in Section 3.4, where we consider more explicit economic models. Analogous to our analysis of the static case, we cannot learn about the outcomes $Y_{p'}$ that would arise under a counterfactual policy p' from data on outcomes Y_{p_0} and treatments $s_{p_0} = S$ under a policy $p_0 \neq p'$ without imposing further structure on the problem.[63] We follow the approach exposited for the static case and assume policy invariance of individual outcomes under a given treatment. These are the invariance assumptions (PI-1)–(PI-4) presented in Chapter 70. They reduce the evaluation of a dynamic policy p to identifying (i) the dynamic assignment s_p of treatments under policy p and (ii) the dynamic treatment effects on individual outcomes. We focus our discussion on the fundamental evaluation problem and the selection problem that haunt inference about treatment effects. In the remainder of the section, we review alternative approaches to identifying dynamic treatment effects, and some approaches to modeling dynamic treatment choice. We first analyze methods recently developed in statistics.

3.2. Dynamic treatment effects and sequential randomization

In a series of papers, Robins extends the static Neyman–Rubin model based on selection on observables discussed in Chapter 71 to a dynamic setting [see, e.g., Robins (1997), and the references therein]. He does not consider agent choice or subjective evaluations. Here, we review his extension, discuss its relationship to dynamic choice models in econometrics, and assess its merits as a framework for economic policy analysis. We follow the exposition of Gill and Robins (2001), but add some additional structure to their basic framework to exposit the connection of their approach to the dynamic approach pursued in econometrics.

3.2.1. Dynamic treatment effects

3.2.1.1. Dynamic treatment and dynamic outcomes To simplify the exposition, suppose that $\mathcal{S}$ is a finite discrete set.[64] Recall that, at each time t and for given p, treatment assignment $s_p(t)$ is a random variable that only depends on the agent's information $\mathcal{I}_A(t, p)$, which includes personal knowledge of preferences and determinants of constraints and outcomes. To make this dependence explicit, suppose that external covariates Z, observed by the econometrician (that is, variables in $\mathcal{I}_E$), and unobserved external covariates V_1 that affect treatment assignment are revealed to the agents at time 1. Then, at the start of each period $t \geqslant 2$, past outcomes $Y_p(t-1)$ corresponding

[63] If outcomes under different policy regimes are informative about the same technology and preferences, for example, then the analyst and the agent could learn about the ingredients that produce counterfactual outcomes in all outcome states.

[64] All of the results presented in this subsection extend to the case of continuous treatments. We will give references to the appropriate literature in subsequent footnotes.

to the outcomes realized under treatment assignment s_p and external unobserved covariates V_t enter the agent's information set.[65] In this notation, $\mathcal{I}_A(1, p)$ is the information $\sigma(Z, V_1)$ conveyed to the agent by (Z, V_1) and, for $t \geqslant 2$, $\mathcal{I}_A(t, p) = \sigma(Y_p^{t-1}, Z, V^t)$, with $V^t = (V_1, \dots, V_t)$. In the notation of the previous subsection, $I_A(1, p) = (Z, V_1)$ and, for $t \geqslant 2$, $I_A(t, p) = (Y_p^{t-1}, Z, V^t)$. Among the elements of $I_A(t, p)$ are the determinants of the constraints faced by the agent up to t, which may or may not be observed by the econometrician.

We attach *ex post* potential outcomes $Y(t, s) = y_t(s, X, U_t)$, $t = 1, \dots, \bar{T}$, to each treatment sequence $s = (s(1), \dots, s(\bar{T}))$. Here, X is a vector of observed (by the econometrician) external covariates and U_t, $t = 1, \dots, \bar{T}$, are vectors of unobserved external covariates. Some components of X and U_t may be in agent information sets. We denote $Y^t(s) = (Y(1, s), \dots, Y(t, s))$, $Y(s) = Y^{\bar{T}}(s)$, and $U = (U_1, \dots, U_{\bar{T}})$. As in the static case, potential outcomes y are assumed to be invariant across policies p, which ensures that $Y_p(t) = y_t(s_p, X, U_t)$. In the remainder of this section, we keep the dependence of outcomes on observed covariates X implicit and suppress all conditioning on X.

We assume no causal dependence of outcomes on future treatment:[66]

(NA) *For all* $t \geqslant 1$, $Y(t, s) = Y(t, s')$ *for all* s, s' *such that* $s^t = (s')^t$,

where $s^t = (s(1), \dots, s(t))$ and $(s')^t = (s'(1), \dots, s'(t))$. Abbring and Van den Berg (2003b) and Abbring (2003) define this as a "no-anticipation" condition. It requires that outcomes at time t (and before) be the same across policies that allocate the same treatment up to and including t, even if they allocate different treatments after t. In the structural econometric models discussed in Sections 3.2.2 and 3.4 below, this condition is trivially satisfied if all state variables relevant to outcomes at time t are included as inputs in the outcome equations $Y(t, s) = y_t(s, U_t)$, $t = 1, \dots, \bar{T}$.

Because Z and V_1 are assumed to be externally determined, and therefore not affected by the policy p, the initial agent information set $\mathcal{I}_A(1, p) = \sigma(Z, V_1)$ does not depend on p. Agent ω has the same initial data $(Z(\omega), V_1(\omega))$ about his type ω under all policies p. Thus, $\mathcal{I}_A(1, p) = \mathcal{I}_A(1, p')$ is a natural benchmark information set for an *ex ante* comparison of outcomes at time 1 among different policies. For $t \geqslant 2$, (NA) implies that actual outcomes up to time $t - 1$ are equal between policies p and p', $Y_p^{t-1} = Y_{p'}^{t-1}$, if the treatment histories coincide up to time $t - 1$ so that $s_p^{t-1} = s_{p'}^{t-1}$. Together with the assumption that Z and V^t are externally determined, it follows that agents have the same time-t information set structure about ω under policies p and p',

65 Note that any observed covariates that are dynamically revealed to the agents can be subsumed in the outcomes.

66 For statistical inference from data on the distribution of (Y_{p_0}, S, Z), these equalities only need to hold on events $\{\omega \in \Omega\colon\ S^t(\omega) = s^t\}$, $t \geqslant 1$, respectively.

$\mathcal{I}_A(t, p) = \sigma(Y_p^{t-1}, Z, V^t) = \sigma(Y_{p'}^{t-1}, Z, V^t) = \mathcal{I}_A(t, p')$, if $s_p^{t-1} = s_{p'}^{t-1}$.[67,68] In this context, $\mathcal{I}_A(t, p) = \mathcal{I}_A(t, p')$ is a natural information set for an *ex ante* comparison of outcomes from time t onwards between any two policies p and p' such that $s_p^{t-1} = s_{p'}^{t-1}$.

With this structure on the agent information sets in hand, it is instructive to review the separate roles in determining treatment choice of information about ω and knowledge about the constraint assignment rule a. First, agent ω's time-t treatment choice $s_p(\omega, t) = \tau_t(\omega, a)$ may depend on distributional properties of a, for example the share of agents assigned to particular treatment sequences, and on the past and current constraints $(a_1(\omega), \ldots, a_t(\omega))$ that were actually assigned to them. We have assumed both to be known to the agent. Both may differ between policies, even if the agent information about ω is fixed across the policies. Second, agent ω's time-t treatment choice may depend on agent ω's predictions of future constraints and outcomes. A forward-looking agent ω will use observations of his covariates $Z(\omega)$ and $V^t(\omega)$ and past outcomes $Y_p^{t-1}(\omega)$ to infer his type ω and subsequently predict future external determinants $(U_t(\omega), \ldots, U_{\bar{T}}(\omega))$ of his outcomes and $(V_{t+1}(\omega), \ldots, V_{\bar{T}}(\omega))$ of his constraints and treatments. In turn, this information updating allows agent ω to predict his future potential outcomes $(Y(t, s, \omega), \ldots, Y(\bar{T}, s, \omega))$ and, for a given policy regime p, his future constraints $(a_{t+1}(\omega), \ldots, a_{\bar{T}}(\omega))$, treatments $(s_p(t+1, \omega), \ldots, s_p(\bar{T}, \omega))$, and realized outcomes $(Y_p(t, \omega), \ldots, Y_p(\bar{T}, \omega))$. Under different policies, the agent may gather different information on his type ω and therefore come up with different predictions of the external determinants of his future potential outcomes and constraints. In addition, even if the agent has the same time-t predictions of the external determinants of future constraints and potential outcomes, he may translate these into different predictions of future constraints and outcomes under different policies.

Assumption (NA) requires that current potential outcomes are not affected by future treatment. Justifying this assumption requires specification of agent information about future treatment and agent behavior in response to that information. Such an interpretation requires that we formalize how information accumulates for agents across treatment sequences s and s' such that $s^t = (s')^t$ and $(s_{t+1}, \ldots, s_{\bar{T}}) \neq (s'_{t+1}, \ldots, s'_{\bar{T}})$. To this end, consider policies p and p' such that $s_p = s$ and $s_{p'} = s'$. These policies produce the same treatment assignment up to time t, but are different in the future. We have previously shown that, even though the time-t agent information about ω is the

67 If $s_p^{t-1}(\omega) = s_{p'}^{t-1}(\omega)$ only holds for ω in some subset $\Omega_{t-1} \subset \Omega$ of agents, then $Y_p^{t-1}(\omega) = Y_{p'}^{t-1}(\omega)$ only for $\omega \in \Omega_{t-1}$, and information coincides between p and p' only for agents in Ω_{t-1}. Formally, let Ω_{t-1} be the set $\{\omega \in \Omega\colon s_p^{t-1}(\omega) = s_{p'}^{t-1}(\omega)\}$ of agents that share the same treatment up to and including time $t-1$. Then, Ω_{t-1} is in the agent's information set under both policies, $\Omega_{t-1} \in \mathcal{I}_A(t, p) \cap \mathcal{I}_A(t, p')$. Moreover, the partitioning of Ω_{t-1} implied by $\mathcal{I}_A(t, p)$ and $\mathcal{I}_A(t, p')$ is the same. To see this, note that the collections of all sets in, respectively, $\mathcal{I}_A(t, p)$ and $\mathcal{I}_A(t, p')$ that are weakly included in Ω_{t-1} are identical σ-algebras on Ω_{t-1}.

68 Notice that the realizations of the random variables $Y_{p'}^{t-1}, Z, V^t$ may differ among agents.

same under both policies, $\mathcal{I}_A(t, p) = \mathcal{I}_A(t, p')$, agents may have different predictions of future constraints, treatments and outcomes because the policies may differ in the future and agents know this. The policy-invariance conditions (PI-1)–(PI-4) of Chapter 70 ensure that time-t potential outcomes are nevertheless the same under each policy. This requires that potential outcomes be determined externally, and are not affected by agent actions in response to different predictions of future constraints, treatments and outcomes.

In general, different policies in $\mathcal{P}$ will produce different predictions of future constraints, treatment and outcomes. In the dynamic treatment-effects framework, this may affect outcomes indirectly through agent treatment choices. If potential outcomes are directly affected by agent's forward-looking decisions, then the invariance conditions (PI-1)–(PI-4) of Chapter 70 underlying the treatment-effects framework will be violated. Section 3.2.3 illustrates this issue, and the no-anticipation condition, with some examples.

3.2.1.2. Identification of treatment effects Suppose that the econometrician has data that allows her to estimate the joint distribution of (Y_{p_0}, S, Z) of outcomes, treatments and covariates under some policy p_0, where again $S = s_{p_0}$. These data are not enough to identify dynamic treatment effects.

To secure identification, Gill and Robins (2001) invoke a dynamic version of the matching assumption (conditional independence) which relies on sequential randomization:[69]

(M-2) *For all treatment sequences s and all t,*

$$S(t) \perp\!\!\!\perp \big(Y(t, s), \ldots, Y(\bar{T}, s)\big) \mid \big(Y_{p_0}^{t-1}, S^{t-1} = s^{t-1}, Z\big),$$

where the conditioning set $(Y_{p_0}^0, S^0 = s^0, Z)$ for $t = 1$ should be simply stated as Z.

Equivalently,

$$S(t) \perp\!\!\!\perp (U_t, \ldots, U_{\bar{T}}) \mid \big(Y_{p_0}^{t-1}, S^{t-1}, Z\big)$$

for all t without further restricting the data. Sequential randomization allows the $Y_{p_0}(t)$ to be "dynamic confounders"—variables that are affected by past treatment and that affect future treatment assignment.

The sequence of conditioning information sets appearing in the sequential randomization assumption, $\mathcal{I}_E(1) = \sigma(Z)$ and, for $t \geqslant 2$, $\mathcal{I}_E(t) = \sigma(Y_{p_0}^{t-1}, S^{t-1}, Z)$, is a filtration $\mathcal{I}_E$ of the econometrician's information set $\sigma(Y_{p_0}, S, Z)$. Note that $\mathcal{I}_E(t) \subseteq \mathcal{I}_A(t, p_0)$ for each t. If treatment assignment is based on strictly more information than $\mathcal{I}_E$, so

[69] Formally, we need to restrict attention to sequences s in the support of S. Throughout this section, we will assume this and related support conditions hold.

that agents know strictly more than the econometrician and act on their superior information, (M-2) is likely to fail if that extra information also affects outcomes. This point is made in a static setting in Chapter 71.

Together with the no-anticipation condition (NA), which is a condition on outcomes and distinct from (M-2), the dynamic potential-outcome model set up so far is a natural dynamic extension of the Neyman–Rubin model for a static (stratified) randomized experiment.

Under assumption (M-2) that the actual treatment assignment S is sequentially randomized, we can sequentially identify the causal effects of treatment from the distribution of the data (Y_{p_0}, S, Z) and construct the distribution of the potential outcomes $Y(s)$ for any treatment sequence s in the support of S.

Consider the case in which all variables are discrete. No-anticipation condition (NA) ensures that potential outcomes for a treatment sequence s equal actual (under policy p_0) outcomes up to time $t-1$ for agents with treatment history s^{t-1} up to time $t-1$. Formally, $Y^{t-1}(s) = Y_{p_0}^{t-1}$ on the set $\{S^{t-1} = s^{t-1}\}$. Using this, sequential randomization assumption (M-2) can be rephrased in terms of potential outcomes: for all s and t,

$$S(t) \perp\!\!\!\perp \bigl(Y(t,s), \dots, Y(\bar{T}, s)\bigr) \mid \bigl(Y^{t-1}(s), S^{t-1} = s^{t-1}, Z\bigr).$$

In turn, this implies that, for all s and t,

$$\begin{aligned}&\Pr\bigl(Y(t,s) = y(t) \mid Y^{t-1}(s) = y^{t-1}, S^t = s^t, Z\bigr)\\ &\quad = \Pr\bigl(Y(t,s) = y(t) \mid Y^{t-1}(s) = y^{t-1}, Z\bigr),\end{aligned} \tag{3.1}$$

where $y^{t-1} = (y(1), \dots, y(t-1))$ and $y = y^{\bar{T}}$. From Bayes' rule and (3.1), it follows that

$$\begin{aligned}&\Pr\bigl(Y(s) = y \mid Z\bigr)\\ &\quad = \Pr\bigl(Y(1,s) = y(1) \mid Z\bigr) \prod_{t=2}^{\bar{T}} \Pr\bigl(Y(t,s) = y(t) \mid Y^{t-1}(s) = y^{t-1}, Z\bigr)\\ &\quad = \Pr\bigl(Y(1,s) = y(1) \mid S(1) = s(1), Z\bigr)\\ &\qquad \times \prod_{t=2}^{\bar{T}} \Pr\bigl(Y(t,s) = y(t) \mid Y^{t-1}(s) = y^{t-1}, S^t = s^t, Z\bigr).\end{aligned}$$

Invoking (NA), in particular $Y(t,s) = Y_{p_0}(t)$ and $Y^{t-1}(s) = Y_{p_0}^{t-1}$ on $\{S^t = s^t\}$, produces

$$\begin{aligned}&\Pr\bigl(Y(s) = y \mid Z\bigr)\\ &\quad = \Pr\bigl(Y_{p_0}(1) = y(1) \mid S(1) = s(1), Z\bigr)\\ &\qquad \times \prod_{t=2}^{\bar{T}} \Pr\bigl(Y_{p_0}(t) = y(t) \mid Y_{p_0}^{t-1} = y^{t-1}, S^t = s^t, Z\bigr).\end{aligned} \tag{3.2}$$

This is a version of Robins' (1997) "g-computation formula".[70,71] We can sequentially identify each component on the left-hand side of the first expression, and hence identify the counterfactual distributions. This establishes identification of the distribution of $Y(s)$ by expressing it in terms of objects that can be identified from data. Identification is exact (or "tight") in the sense that the identifying assumptions, no anticipation and sequential randomization, do not restrict the factual data and are therefore not testable [Gill and Robins (2001, Section 6)].[72]

EXAMPLE 4. Consider a two-period ($\bar{T} = 2$) version of the model in which agents take either "treatment" (1) or "control" (0) in each period. Then, $S(1)$ and $S(2)$ have values in $\mathcal{S} = \{0, 1\}$. The potential outcomes in period t are $Y(t, (0, 0))$, $Y(t, (0, 1))$, $Y(t, (1, 0))$ and $Y(t, (1, 1))$. For example, $Y(2, (0, 0))$ is the outcome in period 2 in the case that the agent is assigned to the control group in each of the two periods. Using Bayes' rule, it follows that

$$\begin{aligned}
&\Pr\bigl(Y(s) = y \mid Z\bigr) \\
&\quad = \Pr\bigl(Y(1, s) = y(1) \mid Z\bigr) \Pr\bigl(Y(2, s) = y(2) \mid Y(1, s) = y(1), Z\bigr).
\end{aligned} \tag{3.3}$$

The g-computation approach to constructing $\Pr(Y(s) = y \mid Z)$ from data replaces the two probabilities in the right-hand side with probabilities of the observed (by the econometrician) variables (Y_{p_0}, S, Z). First, note that $\Pr(Y(1, s) = y(1) \mid Z) = \Pr(Y(1, s) = y(1) \mid S(1) = s(1), Z)$ by (M-2). Moreover, (NA) ensures that potential outcomes in period 1 do not depend on the treatment status in period 2, so that

$$\Pr\bigl(Y(1, s) = y(1) \mid Z\bigr) = \Pr\bigl(Y_{p_0} = y(1) \mid S(1) = s(1), Z\bigr).$$

70 Gill and Robins (2001) present versions of (NA) and (M-2) for the case with more general distributions of treatments, and prove a version of the g-computation formula for the general case. For a random vector X and a function f that is integrable with respect to the distribution of X, let $\int_{x \in A} f(x) \Pr(X \in dx) = E[f(X)\mathbf{1}(X \in A)]$. Then,

$$\begin{aligned}
\Pr\bigl(Y(s) \in A \mid Z\bigr) = \int_{y \in A} &\Pr\bigl(Y_{p_0}(\bar{T}) \in dy(\bar{T}) \mid Y_{p_0}^{\bar{T}-1} = y^{\bar{T}-1}, S^{\bar{T}} = s^{\bar{T}}, Z\bigr) \\
&\vdots \\
&\times \Pr\bigl(Y_{p_0}(2) \in dy(2) \mid Y_{p_0}(1) = y(1), S^2 = s^2, Z\bigr) \\
&\times \Pr\bigl(Y_{p_0}(1) \in dy(1) \mid S(1) = s(1), Z\bigr),
\end{aligned}$$

where A is a set of $Y(s)$. The right-hand side of this expression is almost surely unique under regularity conditions presented by Gill and Robins (2001).

71 An interesting special case arises if the outcomes are survival indicators, that is if $Y_{p_0}(t) = 1$ if the agent survives up to and including time t and $Y_{p_0}(t) = 0$ otherwise, $t \geqslant 1$. Then, no anticipation (NA) requires that treatment after death does not affect survival, and the g-computation formula simplifies considerably [Abbring (2003)].

72 Gill and Robins' (2001) analysis only involves causal inference on a final outcome (i.e., our $Y(s, \bar{T})$) and does not invoke the no-anticipation condition. However, their proof directly applies to the case studied in this chapter.

Similarly, subsequently invoking (NA) and (M-2), then (M-2), and then (NA), gives

$$\begin{aligned}
&\Pr\bigl(Y(2,s) = y(2) \mid Y(1,s) = y(1), Z\bigr) \\
&\quad = \Pr\bigl(Y(2,s) = y(2) \mid Y_{p_0}(1), S(1) = s(1), Z\bigr) \quad \bigl(\text{by (NA) and (M-2)}\bigr) \\
&\quad = \Pr\bigl(Y(2,s) = y(2) \mid Y_{p_0}(1), S = s, Z\bigr) \quad \bigl(\text{by (M-2)}\bigr) \\
&\quad = \Pr\bigl(Y_{p_0}(2) = y(2) \mid Y_{p_0}(1), S = s, Z\bigr). \quad \bigl(\text{by (NA)}\bigr)
\end{aligned}$$

Substituting these equations into the right-hand side of (3.3) gives the g-computation formula,

$$\begin{aligned}
\Pr\bigl(Y(s) = y \mid Z\bigr) = {} & \Pr\bigl(Y_{p_0}(1) = y(1) \mid S(1) = s(1), Z\bigr) \\
& \times \Pr\bigl(Y_{p_0}(2) = y(2) \mid Y_{p_0}(1) = y(1), S = s, Z\bigr).
\end{aligned}$$

Note that the right-hand side does not generally reduce to $\Pr(Y_{p_0} = y \mid S = s, Z)$. This would require the stronger, static matching condition $S \perp\!\!\!\perp Y(s) \mid Z$, which we have not assumed here.

Matching on pre-treatment covariates is a special case of the g-computation approach. Suppose that the entire treatment path is assigned independently of potential outcomes given pre-treatment covariates Z or, more precisely, $S \perp\!\!\!\perp Y(s) \mid Z$ for all s. This implies sequential randomization (M-2), and directly gives identification of the distributions of $Y(s) \mid Z$ and $Y(s)$. The matching assumption imposes no restriction on the data since $Y(s)$ is only observed if $S = s$. The no-anticipation condition (NA) is not required for identification in this special case because no conditioning on S^t is required. Matching on pre-treatment covariates is equivalent to matching in a static model. The distribution of $Y(s) \mid Z$ is identified without (NA), and assuming it to be true would impose testable restrictions on the data. In particular, it would imply that treatment assignment cannot be dependent on past outcomes given Z. The static matching assumption is not likely to hold in applications where treatment is dynamically assigned based on information on intermediate outcomes. This motivates an analysis based on the more subtle sequential randomization assumption. An alternative approach, developed in Section 3.4, is to explicitly model and identify the evolution of the unobservables.

Gill and Robins claim that their sequential randomization and no-anticipation assumptions are "neutral", "for free", or "harmless". As we will argue later, from an economic perspective, some of the model assumptions, notably the no-anticipation assumption, can be interpreted as substantial behavioral/informational assumptions. For example, Heckman and Vytlacil (2005, and Chapter 70 of this Handbook) and Heckman and Navarro (2004) show how matching imposes the condition that marginal and average returns are equal. Because of these strong assumptions, econometricians sometimes phrase their "neutrality" result more negatively as a nonidentification result [Abbring and Van den Berg (2003b)], since it is possible that (M-2) and/or (NA) may not hold.

3.2.2. Policy evaluation and dynamic discrete-choice analysis

3.2.2.1. The effects of policies Consider a counterfactual policy p' such that the corresponding allocation of treatments $s_{p'}$ satisfies sequential randomization, as in (M-2):

(M-3) *For all treatment sequences s and all t,*

$$s_{p'}(t) \perp\!\!\!\perp \big(Y(t,s),\dots,Y(\bar{T},s)\big) \mid \big(Y_{p'}^{t-1}, s_{p'}^{t-1} = s^{t-1}, Z\big).$$

The treatment assignment rule $s_{p'}$ is equivalent to what Gill and Robins (2001) call a "randomized plan". The outcome distribution under such a rule cannot be constructed by integrating the distributions of $\{Y(s)\}$ with respect to the distribution of $s_{p'}$, because there may be feedback from intermediate outcomes into treatment assignment. Instead, under the assumptions of the previous subsection and a support condition, we can use a version of the g-computation formula for randomized plans given by Gill and Robins to compute the distribution of outcomes under the policy p':[73]

$$\begin{aligned}
\Pr(Y_{p'} = y \mid Z) = \sum_{s\in\mathcal{S}} & \Pr\big(Y_{p_0}(1) = y(1) \mid S(1) = s(1), Z\big) \\
& \times \Pr\big(s_{p'}(1) = s(1) \mid Z\big) \\
& \times \prod_{t=2}^{\bar{T}} \Big[\Pr\big(Y_{p_0}(t) = y(t) \mid Y_{p_0}^{t-1} = y^{t-1}, S^t = s^t, Z\big) \\
& \times \Pr\big(s_{p'}(t) = s(t) \mid Y_{p'}^{t-1} = y^{t-1}, s_{p'}^{t-1}(1) = s^{t-1}, Z\big)\Big].
\end{aligned} \tag{3.4}$$

[73] The corresponding formula for the case with general treatment distributions is

$$\begin{aligned}
\Pr(Y_{p'} \in A \mid Z) = \int_{y\in A}\int_{s\in\mathcal{S}} & \Pr\big(Y_{p_0}(\bar{T}) \in dy(\bar{T}) \mid Y_{p_0}^{\bar{T}-1} = y^{\bar{T}-1}, S^{\bar{T}} = s^{\bar{T}}, Z\big) \\
& \times \Pr\big(s_{p'}(\bar{T}) \in ds(\bar{T}) \mid Y_{p'}^{\bar{T}-1} = y^{\bar{T}-1}, s_{p'}^{\bar{T}-1} = s^{\bar{T}-1}, Z\big) \\
\vdots \qquad & \\
& \times \Pr\big(Y_{p_0}(2) \in dy(2) \mid Y_{p_0}(1) = y(1), S(1) = s(1), Z\big) \\
& \times \Pr\big(s_{p'}(2) \in ds(2) \mid Y_{p'}(1) = y(1), s_{p'}(1) = s(1), Z\big) \\
& \times \Pr\big(Y_{p_0}(1) \in dy(1) \mid S(1) = s(1), Z\big) \Pr\big(s_{p'}(1) \in ds(1) \mid Z\big).
\end{aligned}$$

The support condition on $s_{p'}$ requires that, for each t, the distribution of $s_{p'}(t) \mid (Y_{p'}^{t-1} = y^{t-1}, s_{p'}^{t-1} = s^{t-1}, Z = z)$ is absolutely continuous with respect to the distribution of $S(t) \mid (Y_{p_0}^{t-1} = y^{t-1}, S^{t-1} = s^{t-1}, Z = z)$ for almost all (y^{t-1}, s^{t-1}, z) from the distribution of $(Y_{p_0}^{t-1}, S^{t-1}, Z)$.

In the special case of static matching on Z, so that $s_{p'} \perp\!\!\!\perp U \mid Z$, this simplifies to integrating the distribution of $Y_{p_0} \mid (S = s, Z)$ over the distribution of $s_{p'} \mid Z$:[74]

$$\Pr(Y_{p'} = y \mid Z) = \sum_{s \in \mathcal{S}} \Pr(Y_{p_0} = y \mid S = s, Z) \Pr(s_{p'} = s \mid Z).$$

3.2.2.2. Policy choice and optimal policies We now consider the problem of choosing a policy p that is optimal according to some criterion. This problem is both of normative interest and of descriptive interest if actual policies are chosen to be optimal. We could, for example, study the optimal assignment a' of constraints and incentives to agents. Alternatively, we could assume that agents pick τ to maximize their utilities, and use the methods discussed in this section to model τ.

Under the policy invariance assumptions that underlie the treatment-effects approach, p only affects outcomes through its implied treatment allocation s_p. Thus, the problem of choosing an optimal policy boils down to choosing an optimal treatment allocation s_p under informational and other constraints specific to the problem at hand. For example, suppose that the planner and the agents have the same information, $\mathcal{I}_P(p) = \mathcal{I}_A(p)$, the planner assigns eligibility to a program by a, and agents fully comply, so that $\mathcal{B} = \mathcal{S}$ and $s_p = a$. Then, s_p can be any rule from $\mathcal{A}$ and is adapted to $\mathcal{I}_P(p) = \mathcal{I}_A(p)$.

For expositional convenience, we consider the optimal choice of a treatment assignment s_p adapted to the agent's information $\mathcal{I}_A(p)$ constructed earlier. We will use the word "agents" to refer to the decision maker in this problem, even though it can also apply to the planner's decision problem. An econometric approach to this problem is to estimate explicit dynamic choice models with explicit choice-outcome relationships. One emphasis in the literature is on Markovian discrete-choice models that satisfy Rust's (1987) conditional-independence assumption [see Rust (1994)]. Other assumptions are made in the literature and we exposit them in Section 3.4.

Here, we explore the use of Rust's (1987) model as a model of treatment choice in a dynamic treatment-effects setting. In particular, we make explicit the additional structure that Rust's model, and in particular his conditional-independence assumption, imposes on Robins' dynamic potential-outcomes model. We follow Rust (1987) and focus on a finite treatment (control) space $\mathcal{S}$. In the notation of our model, payoffs are determined by the outcomes Y_p, treatment choices s_p, the "cost shocks" V, and the covariates Z. Rust (1987) assumes that $\{Y_p(t-1), V_t, Z\}$ is a controlled first-order Markov process, with initial condition $Y_p(0) \equiv 0$ and control s_p.[75] As before, V_t and Z

[74] In the general case this condition becomes

$$\Pr(Y_{p'} \in A \mid Z) = \int_{s \in \mathcal{S}} \Pr(Y_{p_0} \in A \mid S = s, Z) \Pr(s_{p'} \in ds \mid Z).$$

[75] Rust (1987) assumes an infinite-horizon, stationary environment. Here, we present a finite-horizon version to facilitate a comparison with the dynamic potential-outcomes model and to link up with the analysis in Section 3.4.

are not causally affected by choices, but $Y_p(t)$ may causally depend on current and past choices. The agents choose a treatment assignment rule s_p that maximizes

$$E\left[\sum_{t=1}^{\bar{T}} \Upsilon_t\{Y_p(t-1), V_t, s_p(t), Z\} + \Upsilon_{\bar{T}+1}\{Y_p(\bar{T}), Z\} \;\middle|\; \mathcal{I}_A(1)\right], \tag{3.5}$$

for some (net and discounted) utility functions Υ_t and $\mathcal{I}_A(1) = \mathcal{I}_A(1, p)$, which is independent of p. $\Upsilon_{\bar{T}+1}\{Y_p(\bar{T}), Z\}$ is the terminal value. Under standard regularity conditions on the utility functions, we can solve backward for the optimal policy s_p. Because of Rust's Markov assumption, s_p has a Markovian structure,

$$s_p(t) \perp\!\!\!\perp (Y_p^{t-2}, V^{t-1}) \mid [Y_p(t-1), V_t, Z],$$

for $t = 2, \ldots, \bar{T}$, and $\{Y_p(t-1), V_t, Z\}$ is a first-order Markov process. Note that Z enters the model as an observed (by the econometrician) factor that shifts net utility. A key assumption embodied in the specification of (3.5) is time-separability of utility. Rust (1987), in addition, imposes separability between observed and unobserved state variables. This assumption plays no essential role in expositing the core ideas in Rust, and we will not make it here.

Rust's (1987) conditional-independence assumption imposes two key restrictions on the decision problem. It is instructive to consider these restrictions in isolation from Rust's Markov restriction. We make the model's causal structure explicit using the potential-outcomes notation. Note that the model has a recursive causal structure—the payoff-relevant state is controlled by current and past choices only—and satisfies no-anticipation condition (NA). Setting $Y(0, s) \equiv 0$ for specificity, and ignoring the Markov restriction, Rust's conditional-independence assumption requires, in addition to the assumption that there are no direct causal effects of choices on V, that

$$Y(s, t) \perp\!\!\!\perp V^t \mid [Y^{t-1}(s), Z], \tag{3.6}$$

$$V_{t+1} \perp\!\!\!\perp V^t \mid [Y^t(s), Z] \tag{3.7}$$

for all s and t. As noted by Rust (1987, p. 1011) condition (3.6) ensures that the observed (by the econometrician) controlled state evolves independently of the unobserved payoff-relevant variables. It is equivalent to [Florens and Mouchart (1982)][76]

(M-4) $[Y(s, t), \ldots, Y(s, \bar{T})] \perp\!\!\!\perp V^t \mid [Y^{t-1}(s), Z]$ *for all t and s.*

In turn, (M-4) implies (M-2) and is equivalent to the assumption that (M-3) holds for all $s_{p'}$.[77]

[76] Note that (3.6) is a Granger (1969) noncausality condition stating that, for all s and conditional on Z, V does not cause $Y(s)$.

[77] If V has redundant components, that is components that do not nontrivially enter any assignment rule s_p, (M-4) imposes more structure, but structure that is irrelevant to the decision problem and its empirical analysis.

Condition (3.7) excludes serial dependence of the unobserved payoff-relevant variables conditional on past outcomes. In contrast, Robins' g-computation framework allows for such serial dependence, provided that sequential randomization holds if serial dependence is present. For example, if $V \perp\!\!\!\perp U \mid Z$, then (M-2) and its variants hold without further assumptions on the time series structure of V_t.

The first-order Markov assumption imposes additional restrictions on potential outcomes. These restrictions are twofold. First, potential outcomes follow a first-order Markov process. Second, $s(t)$ only directly affects the Markov transition from $Y(t, s)$ to $Y(t+1, s)$. This strengthens the no-anticipation assumption presented in Section 3.2.1.1. The Markov assumption also requires that V_{t+1} only depends on $Y(s, t)$, and not on $Y^{t-1}(s)$, given $Y(s, t)$.

In applications, we may assume that actual treatment assignment S solves the Markovian decision problem. Together with specifications of Υ_t, this further restricts the dynamic choice-outcome model. Alternatively, one could make other assumptions on S and use (3.5) to define and find an optimal, and typically counterfactual, assignment rule $s_{p'}$.

Our analysis shows that the substantial econometric literature on the structural empirical analysis of Markovian decision problems under conditional independence can be applied to policy evaluation under sequential randomization. Conversely, methods developed for potential-outcomes models with sequential randomization can be applied to learn about aspects of dynamic discrete-choice models. Murphy (2003) develops methods to estimate an optimal treatment assignment rule using Robins' dynamic potential-outcomes model with sequential randomization (M-3).

3.2.3. The information structure of policies

One concern about methods for policy evaluation based on the potential-outcomes model is that potential outcomes are sometimes reduced form representations of dynamic models of agent's choices. A policy maker choosing optimal policies typically faces a population of agents who act on the available information, and their actions in turn affect potential outcomes. For example, in terms of the model of Section 3.2.2, a policy may change financial incentives—the $b \in \mathcal{B}$ assigned through a could enter the net utilities Υ_t—and leave it to the agents to control outcomes by choosing treatment. In econometric policy evaluation, it is therefore important to carefully model the information $\mathcal{I}_A$ that accumulates to the agents in different program states and under different policies, separately from the policy maker's information $\mathcal{I}_P$.

This can be contrasted with common practice in biostatistics. Statistical analyses of the effects of drugs on health are usually concerned with the physician's (planner's) information and decision problem. Gill and Robins' (2001) sequential randomization assumption, for example, is often justified by the assumption that physicians base their treatment decisions on observable (by the analyst) information only. This literature, however, often ignores the possibility that many variables known to the physician may not be known to the observing statistician and that the agents being given drugs alter the protocols.

Potential outcomes will often depend on the agent's information. Failure to correctly model the information will often lead to violation of (NA) and failure of invariance. Potential outcomes may therefore not be valid inputs in a policy evaluation study. A naive specification of potential outcomes would only index treatments by actual participation in, e.g., job search assistance or training programs. Such a naive specification is incomplete in the context of economies inhabited by forward-looking agents who make choices that affect outcomes. In specifying potential outcomes, we should not only consider the effects of actual program participation, but also the effects of the information available to agents about the program and policy. We now illustrate this point.

EXAMPLE 5. Black et al. (2003) analyze the effect of compulsory training and employment services provided to unemployment insurance (UI) claimants in Kentucky on the exit rate from UI and earnings. In the program they study, letters are sent out to notify agents some time ahead whether they are selected to participate in the program. This information is recorded in a database and available to them. They can analyze the letter as part of a program that consists of information provision and subsequent participation in training. The main empirical finding of their paper is that the threat of future mandatory training conveyed by the letters is more effective in increasing the UI exit rate than training itself.

The data used by Black et al. (2003) are atypical of many economic data sets, because the data collectors carefully record the information provided to agents. This allows Black et al. to analyze the effects of the provision of information along with the effects of actual program participation. In many econometric applications, the information on the program under study is less rich. Data sets may provide information on actual participation in training programs and some background information on how the program is administered. Typically, however, the data do not record all of the letters sent to agents and do not record every phone conversation between administrators and agents. Then, the econometrician needs to make assumptions on how this information accumulates for agents. In many applications, knowledge of specific institutional mechanisms of assignment can be used to justify specific informational assumptions.

EXAMPLE 6. Abbring, Van den Berg and Van Ours (2005) analyze the effect of punitive benefits reductions, or sanctions, on Dutch UI on re-employment rates. In the Netherlands, UI claimants have to comply with certain rules concerning search behavior and registration. If a claimant violates these rules, a sanction may be applied. A sanction is a punitive reduction in benefits for some period of time and may be accompanied by increased levels of monitoring by the UI agency.[78] Abbring, Van den Berg and Van

[78] See Grubb (2000) for a review of sanction systems in the OECD.

Ours (2005) use administrative data and know the re-employment duration, the duration at which a sanction is imposed if a sanction is imposed, and some background characteristics for each UI case.

Without prior knowledge of the Dutch UI system, an analyst might make a variety of informational assumptions. One extreme is that UI claimants know at the start of their UI spells that their benefits will be reduced at some specific duration if they are still claiming UI at that duration. This results in a UI system with entitlement periods that are tailored to individual claimants and that are set and revealed at the start of the UI spells. In this case, claimants will change their labor-market behavior from the start of their UI spell in response to the future benefits reduction [e.g., Mortensen (1977)]. At another extreme, claimants receive no prior signals of impending sanctions and there are no anticipatory effects of actual benefits reductions. However, agents may still be aware of the properties of the sanctions process and to some extent this will affect their behavior. Abbring, Van den Berg and Van Ours (2005) analyze a search model with these features. Abbring and Van den Berg (2003b) provide a structural example where the data cannot distinguish between these two informational assumptions. We discuss this example further in Section 3.3.1. Abbring, Van den Berg and Van Ours (2005) use institutional background information to argue in favor of the second informational assumption as the one that characterizes their data.

If data on information provision are not available and simplifying assumptions on the program's information structure cannot be justified, the analyst needs to model the information that accumulates to agents as an unobserved determinant of outcomes. This is the approach followed, and further discussed, in Section 3.4.

The information determining outcomes typically includes aspects of the policy. In Example 5, the letter announcing future training will be interpreted differently in different policy environments. If agents are forward looking, the letter will be more informative under a policy that specifies a strong relation between the letter and mandatory training in the population than under a policy that allocates letters and training independently. In Example 6, the policy is a monitoring regime. Potential outcomes are UI durations under different sanction times. A change in monitoring policy changes the value of unemployment. In a job-search model with forward looking agents, agents will respond by changing their search effort and reservation wage, and UI duration outcomes will change. In either example, potential outcomes are not invariant to variation in the policy. In the terminology of Hurwicz (1962), the policy is not "structural" with regard to potential outcomes and violates invariance assumptions (PI-1)–(PI-4) presented in Chapter 70. One must control for the effects of agents' information.

3.2.4. *Selection on unobservables*

In econometric program evaluations, (sequentially) randomized assignment is unlikely to hold. We illustrate this in the models developed in Section 3.4. Observational data are characterized by a lot of heterogeneity among agents, as documented by the empirical examples in Section 2 and in Heckman, LaLonde and Smith (1999). This heterogeneity is unlikely to be fully captured by the observed variables in most data sets. In a dynamic context, such unmeasured heterogeneity leads to violations of the assumptions of Gill and Robins (2001) and Rust (1987) that choices represent a sequential randomization. This is true even if the unmeasured variables only affect the availability of slots in programs but not outcomes directly. If agents are rational, forward-looking and observe at least some of the unmeasured variables that the econometrician does not, they will typically respond to these variables through their choice of treatment and through their investment behavior. In this case, the sequential randomization condition fails.

For the same reason, identification based on instrumental variables is relatively hard to justify in dynamic models [Hansen and Sargent (1980), Rosenzweig and Wolpin (2000), Abbring and Van den Berg (2005)]. If the candidate instruments only vary across persons but not over time for the same person, then they are not likely to be valid instruments because they affect expectations and future choices and may affect current potential outcomes. Instead of using instrumental variables that vary only across persons, we require instruments based on unanticipated person-specific shocks that affect treatment choices but not outcomes at each point in time. In the context of continuously assigned treatments, the implied data requirements seem onerous. To achieve identification, Abbring and Van den Berg (2003b) focus on regressor variation rather than exclusion restrictions in a sufficiently smooth model of continuous-time treatment effects. We discuss their analysis in Section 3.3. Heckman and Navarro (2007) show that curvature conditions, not exclusion restrictions, that result in the same variables having different effects on choices and outcomes in different periods, are motivated by economic theory and can be exploited to identify dynamic treatment effects in discrete time without literally excluding any variables. We discuss their analysis in Section 3.4. We now consider a formulation of the analysis in continuous time.

3.3. The event-history approach to policy analysis

The discrete-time models just discussed in Section 3.2 have an obvious limitation. Time is continuous and many events are best described by a continuous-time model. There is a rich field of continuous-time event-history analysis that has been adapted to conduct policy evaluation analysis.[79] For example, the effects of training and counseling on unemployment durations and job stability have been analyzed by applying event-history methods to data on individual labor-market and training histories [Ridder (1986), Card and Sullivan (1988), Gritz (1993), Ham and LaLonde (1996), Eberwein, Ham and LaLonde (1997), Bonnal, Fougère and Sérandon (1997)]. Similarly, the moral hazard effects of unemployment insurance have been studied by analyzing the effects of time-varying benefits on labor-market transitions [e.g., Meyer (1990), Abbring, Van den Berg

[79] Abbring and Van den Berg (2004) discuss the relation between the event-history approach to program evaluation and more standard latent-variable and panel-data methods, with a focus on identification issues.

and Van Ours (2005), Van den Berg, Van der Klaauw and Van Ours (2004)]. In fields like epidemiology, the use of event-history models to analyze treatment effects is widespread [see, e.g., Andersen et al. (1993), Keiding (1999)].

The event-history approach to program evaluation is firmly rooted in the econometric literature on state dependence (lagged dependent variables) and heterogeneity [Heckman and Borjas (1980), and Heckman (1981a)]. Event-history models along the lines of Heckman and Singer (1984, 1986) are used to jointly model transitions into programs and transitions into outcome states. Causal effects of programs are modelled as the dependence of individual transition rates on the individual history of program participation. Dynamic selection effects are modelled by allowing for dependent unobserved heterogeneity in both the program and outcome transition rates.

Without restrictions on the class of models considered, true state dependence and dynamic selection effects cannot be distinguished.[80] Any history dependence of current transition rates can be explained both as true state dependence and as the result of unobserved heterogeneity that simultaneously affects the history and current transitions. This is a dynamic manifestation of the problem of drawing causal inference from observational data. In applied work, researchers avoid this problem by imposing additional structure. A typical, simple, example is a mixed semi-Markov model in which the causal effects are restricted to program participation in the previous spell [e.g., Bonnal, Fougère and Sérandon (1997), see Section 3.3.2]. There is a substantial literature on the identifiability of state-dependence effects and heterogeneity in duration and event-history models that exploit such additional structure [see Heckman and Taber (1994), and Van den Berg (2001), for reviews]. Here, we provide discussion of some canonical cases.

3.3.1. Treatment effects in duration models

3.3.1.1. Dynamically assigned binary treatments and duration outcomes We first consider the simplest case of mutual dependence of events in continuous time, involving only two binary events. This case is sufficiently rich to capture the effect of a dynamically assigned binary treatment on a duration outcome. Binary events in continuous time can be fully characterized by the time at which they occur and a structural model for their joint determination is a simultaneous-equations model for durations. We develop such a model along the lines of Abbring and Van den Berg (2003b). This model is an extension, with general marginal distributions and general causal and spurious dependence of the durations, of Freund's (1961) bivariate exponential model.

Consider two continuously-distributed random durations Y and S. We refer to one of the durations, S, as the time to treatment and to the other duration, Y, as the outcome duration. Such an asymmetry arises naturally in many applications. For example, in Abbring, Van den Berg and Van Ours's (2005) study of unemployment insurance, the

[80] See Heckman and Singer (1986).

treatment is a punitive benefits reduction (sanction) and the outcome re-employment. The re-employment process continues after imposition of a sanction, but the sanctions process is terminated by re-employment. The current exposition, however, is symmetric and unifies both cases. It applies to both the asymmetric setup of the sanctions example and to applications in which both events may causally affect the other event.

Let $Y(s)$ be the potential outcome duration that would prevail if the treatment time is externally set to s. Similarly, let $S(y)$ be the potential treatment time resulting from setting the outcome duration to y. We assume that *ex ante* heterogeneity across agents is fully captured by observed covariates X and unobserved covariates V, assumed to be external and temporally invariant. Treatment causally affects the outcome duration through its hazard rate. We denote the hazard rate of $Y(s)$ at time t for an agent with characteristics (X, V) by $\theta_Y(t \mid s, X, V)$. Similarly, outcomes affect the treatment times through its hazard $\theta_S(t \mid y, X, V)$. Causal effects on hazard rates are produced by recursive economic models driven by point processes, such as search models. We provide an example below, and further discussion in Section 3.3.3.

Without loss of generality, we partition V into (V_S, V_Y) and assume that $\theta_Y(t \mid s, X, V) = \theta_Y(t \mid s, X, V_Y)$ and $\theta_S(t \mid y, X, V) = \theta_S(t \mid y, X, V_S)$. Intuitively, V_S and V_Y are the unobservables affecting, respectively, treatment and outcome, and the joint distribution of (V_S, V_Y) is unrestricted. In particular, V_S and V_Y may have elements in common.

The corresponding integrated hazard rates are defined by $\Theta_Y(t \mid s, X, V_Y) = \int_0^t \theta_Y(u \mid s, X, V_Y)\,du$ and $\Theta_S(t \mid y, X, V_S) = \int_0^t \theta_S(u \mid y, X, V_S)\,du$. For expositional convenience, we assume that these integrated hazards are strictly increasing in t. We also assume that they diverge to ∞ as $t \to \infty$, so that the duration distributions are non-defective.[81] Then, $\Theta_Y(Y(s) \mid s, X, V_Y)$ and $\Theta_S(S(y) \mid y, X, V_S)$ are unit exponential for all $y, s \in \mathbb{R}_+$.[82] This implies the following model of potential outcomes and treatments,[83]

$$Y(s) = y(s, X, V_Y, \varepsilon_Y) \quad \text{and} \quad S(y) = s(y, X, V_S, \varepsilon_S),$$

for some unit exponential random variables ε_Y and ε_S that are independent of (X, V), $y = \Theta_Y^{-1}$, and $s = \Theta_S^{-1}$.

[81] Abbring and Van den Berg (2003b) allow for defective distributions, which often have structural interpretations. For example, some women never have children and some workers will never leave a job. See Abbring (2002) for discussion.

[82] Let $T \mid X$ be distributed with density $f(t \mid X)$, non-defective cumulative distribution function $F(t \mid X)$, and hazard rate $\theta(t \mid X) = f(t \mid X)/[1 - F(t \mid X)]$. Then, $\int_0^T \theta(t \mid X)\,dt = -\ln[1 - F(T \mid X)]$ is a unit exponential random variable that is independent of X.

[83] The causal hazard model only implies that the distributions of ε_Y and ε_S are invariant across assigned treatments and outcomes, respectively; their realizations may not be. This is sufficient for the variation of $y(s, X, V_Y, \varepsilon_Y)$ with s and of $s(y, X, V_S, \varepsilon_S)$ with y to have a causal interpretation. The further restriction that the random variables ε_Y and ε_S are invariant is made for simplicity, and is empirically innocuous. See Abbring and Van den Berg (2003b) for details and Freedman (2004) for discussion.

The exponential errors ε_Y and ε_S embody the *ex post* shocks that are inherent to the individual hazard processes, that is the randomness in the transition process after conditioning on covariates X and V and survival. We assume that $\varepsilon_Y \perp\!\!\!\perp \varepsilon_S$, so that $\{Y(s)\}$ and $\{S(y)\}$ are only dependent through the observed and unobserved covariates (X, V). This conditional-independence assumption is weaker than the conditional-independence assumption underlying the analysis of Section 3.2 and used in matching, because it allows for conditioning on the invariant unobservables V. It shares this feature with the discrete-time models developed in Section 3.4 and is a version of matching on unobserved variables discussed in Section 2.

We assume a version of the no-anticipation condition of Section 3.2.1: for all $t \in \mathbb{R}_+$,

$$\theta_Y(t \mid s, X, V_Y) = \theta_Y(t \mid s', X, V_Y) \quad \text{and} \quad \theta_S(t \mid y, X, V_S) = \theta_S(t \mid y', X, V_S)$$

for all $s, s', y, y' \in [t, \infty)$. This excludes effects of anticipation of the treatment on the outcome. Similarly, there can be no anticipation effects of future outcomes on the treatment hazard.

EXAMPLE 7. Consider a standard search model describing the job search behavior of an unemployed individual [e.g., Mortensen (1986)] with characteristics (X, V). Job offers arrive at a rate $\lambda > 0$ and are random draws from a given distribution F. Both λ and F may depend on (X, V), but for notational simplicity we suppress all explicit representations of conditioning on (X, V) throughout this example. An offer is either accepted or rejected. A rejected offer cannot be recalled at a later time. The individual initially receives a constant flow of unemployment-insurance benefits. However, the individual faces the risk of a sanction—a permanent reduction of his benefits to some lower, constant level—at some point during his unemployment spell. During the unemployment spell, sanctions arrive independently of the job-offer process at a constant rate $\mu > 0$. The individual cannot foresee the exact time a sanction is imposed, but he knows the distribution of these times.[84] The individual chooses a job-acceptance rule as to maximize his expected discounted lifetime income. Under standard conditions, this is a reservation-wage rule: at time t, the individual accepts each wage of $\underline{w}(t)$ or higher. The corresponding re-employment hazard rate is $\lambda(1 - F(\underline{w}(t)))$. Apart from the sanction, which is not foreseen and arrives at a constant rate during the unemployment spell, the model is stationary. This implies that the reservation wage is constant, say equal to $\underline{w}_0$, up to and including time s, jumps to some lower level $\underline{w}_1 < \underline{w}_0$ at time s and stays constant at $\underline{w}_1$ for the remainder of the unemployment spell if benefits would be reduced at time s.

The model is a version of the simultaneous-equations model for durations. To see this, let Y be the re-employment duration and S the sanction time. The potential-outcome

[84] This is a rudimentary version of the search model with punitive benefits reductions, or sanctions, of Abbring, Van den Berg and Van Ours (2005). The main difference is that in the present version of the model the sanctions process cannot be controlled by the agent.

hazards are

$$\theta_Y(t \mid s) = \begin{cases} \lambda_0 & \text{if } 0 \leqslant t \leqslant s, \\ \lambda_1 & \text{if } t > s, \end{cases}$$

where $\lambda_0 = \lambda[1 - F(\underline{w}_0)]$ and $\lambda_1 = \lambda[1 - F(\underline{w}_1)]$, and clearly $\lambda_1 \geqslant \lambda_0$. Similarly, the potential-treatment time hazards are $\theta_S(t \mid y) = \mu$ if $0 \leqslant t \leqslant y$, and 0 otherwise. Note that the no-anticipation condition follows naturally from the recursive structure of the economic decision problem in this case in which we have properly accounted for all relevant components of agent information sets. Furthermore, the assumed independence of the job offer and sanction processes at the individual level for given (X, V) implies that $\varepsilon_Y \perp\!\!\!\perp \varepsilon_S$.

The actual outcome and treatment are related to the potential outcomes and treatments by $S = S(Y)$ and $Y = Y(S)$. The no-anticipation assumption ensures that this system has a unique solution (Y, S) by imposing a recursive structure on the underlying transition processes. Without anticipation effects, current treatment and outcome hazards only depend on past outcome and treatment events, and the transition processes evolve recursively [Abbring and Van den Berg (2003b)]. Together with a distribution $G(\cdot \mid X)$ of $V \mid X$, this gives a nonparametric structural model of the distribution of $(Y, S) \mid X$ that embodies general simultaneous causal dependence of Y and S, dependence of (Y, X) on observed covariates X, and general dependence of the unobserved errors V_Y and V_S.

There are two reasons for imposing further restrictions on this model. First, it is not identified from data on (Y, S, X). Take a version of the model with selection on unobservables ($V_Y \not\perp\!\!\!\perp V_S \mid X$) and consider the distribution of $(Y, S) \mid X$ generated by this version of the model. Then, there exists an alternative version of the model that satisfies both no-anticipation and $V_Y \perp\!\!\!\perp V_S \mid X$, and that generates the same distribution of $(Y, S) \mid X$ [Abbring and Van den Berg (2003b, Proposition 1)]. In other words, for each version of the model with selection on unobservables and anticipation effects, there is an observationally-equivalent model version that satisfies no-anticipation and conditional randomization. This is a version of the nonidentification result discussed in Section 3.2.1.

Second, even if we ensure nonparametric identification by assuming no-anticipation and conditional randomization, we cannot learn about the agent-level causal effects embodied in y and s without imposing even further restrictions. At best, under regularity conditions we can identify $\theta_Y(t \mid s, X) = E[\theta_Y(t \mid s, X, V_Y) \mid X, Y(s) \geqslant t]$ and $\theta_S(t \mid y, X) = E[\theta_S(t \mid y, X, V_S) \mid X, S(y) \geqslant t]$ from standard hazard regressions [e.g., Andersen et al. (1993), Fleming and Harrington (1991)]. Thus we can identify the distributions of $Y(s) \mid X$ and $S(y) \mid X$, and therefore solve the selection problem if we are only interested in these distributions. However, if we are also interested in the causal effects on the corresponding hazard rates for given X, V, we face an additional dynamic selection problem. The hazards of the identified distributions of $Y(s) \mid X$ and $S(y) \mid X$ only condition on observed covariates X, and not on unobserved covariates V, and are confounded with dynamic selection effects [Heckman and Borjas (1980), Heckman and Singer (1986), Meyer (1996), Abbring and Van den Berg (2005)]. For example, the difference between $\theta_Y(t \mid s, X)$ and $\theta_Y(t \mid s', X)$ does not only reflect agent-level differences between $\theta_Y(t \mid s, X, V_Y)$ and $\theta_Y(t \mid s', X, V_Y)$, but also differences in the subpopulations of survivors $\{X, Y(s) \geqslant t\}$ and $\{X, Y(s') \geqslant t\}$ on which the hazards are computed.

In the next two subsections, we discuss what can be learned about treatment effects in duration models under additional model restrictions. We take the no-anticipation assumption as fundamental. As explained in Section 3.2, this requires that we measure and include in our model all relevant information needed to define potential outcomes. However, we relax the randomization assumption. We first consider Abbring and Van den Berg's (2003b) analysis of identification without exclusion restrictions. They argue that these results are useful, because exclusion restrictions are hard to justify in an inherently dynamic setting with forward-looking agents. Abbring and Van den Berg (2005) further clarify this issue by studying inference for treatment effects in duration models using a social experiment. We discuss what can be learned from such experiments at the end of this section.

3.3.1.2. Identifiability without exclusion restrictions Abbring and Van den Berg consider an extension of the multivariate Mixed Proportional Hazard (MPH) model [Lancaster (1979)] in which the hazard rates of $Y(s) \mid (X, V)$ and $S(y) \mid (X, V)$ are given by

$$\theta_Y(t \mid s, X, V) = \begin{cases} \lambda_Y(t)\phi_Y(X)V_Y & \text{if } t \leqslant s, \\ \lambda_Y(t)\phi_Y(X)\delta_Y(t, s, X)V_Y & \text{if } t > s \end{cases} \quad \text{and} \tag{3.8}$$

$$\theta_S(t \mid y, X, V) = \begin{cases} \lambda_S(t)\phi_S(X)V_S & \text{if } t \leqslant y, \\ \lambda_S(t)\phi_S(X)\delta_S(t, y, X)V_S & \text{if } t > y, \end{cases} \tag{3.9}$$

respectively, and $V = (V_Y, V_S)$ is distributed independently of X. The baseline hazards $\lambda_Y : \mathbb{R}_+ \to (0, \infty)$ and $\lambda_S : \mathbb{R}_+ \to (0, \infty)$ capture duration dependence of the individual transition rates. The integrated hazards are $\Lambda_Y(t) = \int_0^t \lambda_Y(\tau)\, d\tau < \infty$ and $\Lambda_S(t) = \int_0^t \lambda_S(\tau)\, d\tau < \infty$ for all $t \in \mathbb{R}_+$. The regressor functions $\phi_Y : \mathcal{X} \to (0, \infty)$ and $\phi_S : \mathcal{X} \to (0, \infty)$ are assumed to be continuous, with $\mathcal{X} \subset \mathbb{R}^q$ the support of X. In empirical work, these functions are frequently specified as $\phi_Y(x) = \exp(x'\beta_Y)$ and $\phi_S(x) = \exp(x'\beta_S)$ for some parameter vectors β_Y and β_S. We will not make such parametric assumptions. Note that the fact that both regressor functions are defined on the same domain $\mathcal{X}$ is not restrictive, because each function ϕ_Y and ϕ_S can "select" certain elements of X by being trivial functions of the other elements. In the parametric example, the vector β_Y would only have nonzero elements for those regressors that matter to the outcome hazard. The functions δ_Y and δ_S capture the causal effects. Note that $\delta_Y(t, s, X)$ only enters $\theta_Y(t \mid s, X, V)$ at durations $t > s$, so that the model satisfies no anticipation of treatment. Similarly, it satisfies no anticipation of outcomes and has a recursive causal structure as required by the no-anticipation assumption. If $\delta_Y = 1$, treatment is ineffective; if δ_Y is larger than 1, it stochastically reduces the remaining outcome duration.

Note that this model allows δ_Y and δ_S to depend on elapsed duration t, past endogenous events, and the observed covariates X, but not on V. Abbring and Van den Berg also consider an alternative model that allows δ_Y and δ_S to depend on unobservables in a general way, but not on past endogenous events.

Abbring and Van den Berg show that these models are nonparametrically identified from single-spell data under the conditions for the identification of competing risks models based on the multivariate MPH model given by Abbring and Van den Berg (2003a). Among other conditions are the requirements that there is some independent local variation of the regressor effects in both hazard rates and a finite-mean restriction on V, and are standard in the analysis of multivariate MPH models. With multiple-spell data, most of these assumptions, and the MPH structure, can be relaxed [Abbring and Van den Berg (2003b)].

The models can be parameterized in a flexible way and estimated by maximum likelihood. Typical parameterizations involve linear-index structures for the regressor and causal effects, a discrete distribution G, and piecewise-constant baseline hazards λ_S and λ_Y. Abbring and Van den Berg (2003c) develop a simple graphical method for inference on the sign of $\ln(\delta_Y)$ in the absence of regressors. Abbring, Van den Berg and Van Ours (2005) present an empirical application.

3.3.1.3. Inference based on instrumental variables The concerns expressed in Section 3.2.4 about the validity of exclusion restrictions in dynamic settings carry over to event-history models.

EXAMPLE 8. A good illustration of this point is offered by the analysis of Eberwein, Ham and LaLonde (1997), who study the effects of a training program on labor-market transitions. Their data are particularly nice, as potential participants are randomized into treatment and control groups at some baseline point in time. This allows them to estimate the effect of intention to treat (with training) on subsequent labor-market transitions. This is directly relevant to policy evaluation in the case that the policy involves changing training enrollment through offers of treatment which may or may not be accepted by agents.

However, Eberwein et al. are also interested in the effect of actual participation in the training program on post-program labor-market transitions. This is a distinct problem, because compliance with the intention-to-treat protocol is imperfect. Some agents in the control group are able to enroll in substitute programs, and some agents in the treatment group choose never to enroll in a program at all. Moreover, actual enrollment does not take place at the baseline time, but is dispersed over time. Those in the treatment group are more likely to enroll earlier. This fact, coupled with the initial randomization, suggests that the intention-to-treat indicator might be used as an instrument for identifying the effect of program participation on employment and unemployment spells.

The dynamic nature of enrollment into the training program, and the event-history focus of the analysis complicate matters considerably. Standard instrumental-variables methods cannot be directly applied. Instead, Eberwein et al. use a parametric duration model for pre- and post-program outcomes that excludes the intention-to-treat indicator from directly determining outcomes. They specify a duration model for training enrollment that includes an intention-to-treat indicator as an explanatory variable, and specify a model for labor-market transitions that excludes the intention-to-treat indicator and imposes a no-anticipation condition on the effect of actual training participation on labor-market transitions. Such a model is consistent with an environment in which agents cannot perfectly foresee the actual training time they will be assigned and in which they do not respond to information about this time revealed by their assignment to an intention-to-treat group. This is a strong assumption. In a search model with forward-looking agents, for example, such information would typically affect the *ex ante* values of unemployment and employment. Then, it would affect the labor-market transitions before actual training enrollment through changes in search efforts and reservation wages, unless these are both assumed to be exogenous. An assumption of perfect foresight on the part of the agents being studied only complicates matters further.

Abbring and Van den Berg (2005) study what can be learned about dynamically assigned programs from social experiments if the intention-to-treat instrument cannot be excluded from the outcome equation. They develop bounds, tests for unobserved heterogeneity, and point-identification results that extend those discussed in this section.[85]

3.3.2. *Treatment effects in more general event-history models*

It is instructive to place the causal duration models developed in Section 3.3.1 in the more general setting of event-history models with state dependence and heterogeneity. We do this following Abbring's (2008) analysis of the mixed semi-Markov model.

3.3.2.1. The mixed semi-Markov event-history model The model is formulated in a fashion analogous to the frameworks of Heckman and Singer (1986). The point of departure is a continuous-time stochastic process assuming values in a finite set $\mathcal{S}$ at each point in time. We will interpret realizations of this process as agents' event histories of transitions between states in the state space $\mathcal{S}$.

Suppose that event histories start at real-valued random times T_0 in an $\mathcal{S}$-valued random state S_0, and that subsequent transitions occur at random times $T_1, T_2, \ldots$ such that $T_0 < T_1 < T_2 < \cdots$. Let S_l be the random destination state of the transition at T_l. Taking the sample paths of the event-history process to be right-continuous, we have that S_l is the state occupied in the interval $[T_l, T_{l+1})$.

[85] In the special case that a static treatment, or treatment plan, is assigned at the start of the spell, standard instrumental-variables methods may be applied. See Abbring and Van den Berg (2005).

Suppose that heterogeneity among agents is captured by vectors of time-constant observed covariates X and unobserved covariates V.[86] In this case, state dependence in the event-history process for given individual characteristics X, V has a causal interpretation.[87] We structure such state dependence by assuming that the event-history process conditional on X, V is a time-homogeneous semi-Markov process. Conditional on X, V the length of a spell in a state and the destination state of the transition ending that spell depend only on the past through the current state. In our notation, $(\Delta T_l, S_l) \perp\!\!\!\perp \{(T_i, S_i),\ i = 0, \ldots, l-1\} \mid S_{l-1}, X, V$, where $\Delta T_l = T_l - T_{l-1}$ is the length of spell l. Also, the distribution of $(\Delta T_l, S_l) \mid S_{l-1}, X, V$ does not depend on l. Note that, conditional on X, V, $\{S_l,\ l \geqslant 0\}$ is a time-homogeneous Markov chain under these assumptions.

Nontrivial dynamic selection effects arise because V is not observed. The event-history process conditional on observed covariates X only is a mixed semi-Markov process. If V affects the initial state S_0, or transitions from there, subpopulations of agents in different states at some time t typically have different distributions of the unobserved characteristics V. Therefore, a comparison of the subsequent transitions in two such subpopulations does not only reflect state dependence, but also sorting of agents with different unobserved characteristics into the different states they occupy at time t.

We model $\{(\Delta T_l, S_l),\ l \geqslant 1\} \mid T_0, S_0, X, V$ as a repeated competing risks model. Due to the mixed semi-Markov assumption, the latent durations corresponding to transitions into the possible destination states in the lth spell only depend on the past through the current state S_{l-1}, conditional on X, V. This implies that we can fully specify the repeated competing risks model by specifying a set of origin-destination-specific latent durations, with corresponding transition rates. Let T^l_{jk} denote the latent duration corresponding to the transition from state j to state k in spell l. We explicitly allow for the possibility that transitions between certain (ordered) pairs of states may be impossible. To this end, define the correspondence $\mathcal{Q} : \mathcal{S} \to \sigma(\mathcal{S})$ assigning to each $s \in \mathcal{S}$ the set of all destination states to which transitions are made from s with positive probability.[88] Here, $\sigma(\mathcal{S})$ is the set of all subsets of $\mathcal{S}$ (the "power set" of $\mathcal{S}$). Then, the length of spell l is given by $\Delta T_l = \min_{s \in \mathcal{Q}(S_{l-1})} T^l_{S_{l-1}s}$, and the destination state by $S_l = \arg\min_{s \in \mathcal{Q}(S_{l-1})} T^l_{S_{l-1}s}$.

We take the latent durations to be mutually independent, jointly independent from T_0, S_0, and identically distributed across spells l, all conditional on X, V. This reflects

86 We restrict attention to time-invariant observed covariates for expositional convenience. The analysis can easily be adapted to more general time-varying external covariates. Restricting attention to time-constant regressors is a worst-case scenario for identification. External time variation in observed covariates aids identification [Heckman and Taber (1994)].

87 We could make this explicit by extending the potential-outcomes model of Section 3.3.1 to the general event-history setup. However, this would add a lot of complexity, but little extra insight.

88 Throughout this section, we assume that $\mathcal{Q}$ is known. It is important to note, however, that $\mathcal{Q}$ can actually be identified trivially in all cases considered.

both the mixed semi-Markov assumption and the additional assumption that all dependence between the latent durations corresponding to the competing risks in a given spell l is captured by the observed regressors X and the unobservables V. This is a standard assumption in econometric duration analysis, which, with the semi-Markov assumption, allows us to characterize the distribution of $\{(\Delta T_l, S_l),\ l \geqslant 1\} \mid T_0, S_0, X, V$ by specifying origin-destination-specific hazards $\theta_{jk}(t \mid X, V)$ for the marginal distributions of $T_{jk}^l \mid X, V$.

We assume that the hazards $\theta_{jk}(t \mid X, V)$ are of the mixed proportional hazard (MPH) type:[89]

$$\theta_{jk}(t \mid X, V) = \begin{cases} \lambda_{jk}(t)\phi_{jk}(X)V_{jk} & \text{if } k \in \mathcal{Q}(j), \\ 0 & \text{otherwise.} \end{cases} \tag{3.10}$$

The baseline hazards $\lambda_{jk} : \mathbb{R}_+ \to (0, \infty)$ have integrated hazards $\Lambda_{jk}(t) = \int_0^t \lambda_{jk}(\tau)\, d\tau < \infty$ for all $t \in \mathbb{R}_+$. The regressor functions $\phi_{jk} : \mathcal{X} \to (0, \infty)$ are assumed to be continuous. Finally, the $(0, \infty)$-valued random variable V_{jk} is the scalar component of V that affects the transition from state j to state k. Note that we allow for general dependence between the components of V. This way, we can capture, for example, that agents with lower re-employment rates have higher training enrollment rates.

This model fully characterizes the distribution of the transitions $\{(\Delta T_l, S_l),\ l \geqslant 1\}$ conditional on the initial conditions T_0, S_0 and the agents' characteristics X, V. A complete model of the event histories $\{(T_l, S_l),\ l \geqslant 0\}$ conditional on X, V would in addition require a specification of the initial conditions T_0, S_0 for given X, V. It is important to stress here that T_0, S_0 are the initial conditions of the event-history process itself, and should not be confused with the initial conditions in a particular sample (which we will discuss below). In empirical work, interest in the dependence between start times T_0 and characteristics X, V is often limited to the observation that the distribution of agent's characteristics may vary over cohorts indexed by T_0. The choice of initial state S_0 may in general be of some interest, but is often trivial. For example, we could model labor-market histories from the calendar time T_0 at which agents turn 15 onwards. In an economy with perfect compliance to a mandatory schooling up to age 15, the initial state S_0 would be "(mandatory) schooling" for all. Therefore, we will not consider a model of the event history's initial conditions, but instead focus on the conditional model of subsequent transition histories.

Because of the semi-Markov assumption, the distribution of $\{(\Delta T_l, S_l),\ l \geqslant 1\} \mid T_0, S_0, X, V$ only depends on S_0, and not T_0. Thus, T_0 only affects observed event histories through cohort effects on the distribution of unobserved characteristics V. The initial state S_0, on the other hand, may both have causal effects on subsequent transitions and be informative on the distribution of V. For expositional clarity, we assume

[89] Proportionality can be relaxed if we have data on sufficiently long event-histories. See Honoré (1993) and Abbring and Van den Berg (2003a, 2003b) for related arguments for various multi-spell duration models.

that $V \perp\!\!\!\perp (T_0, S_0, X)$. This is true, for example, if all agents start in the same state, so that S_0 is degenerate, and V is independent of the start date T_0 and the observed covariates X.

An econometric model for transition histories conditional on the observed covariates X can be derived from the model of $\{(\Delta T_l, S_l),\ l \geqslant 1\} \mid S_0, X, V$ by integrating out V. The exact way this should be done depends on the sampling scheme used. Here, we focus on sampling from the population of event-histories. We assume that we observe the covariates X, the initial state S_0, and the first L transitions from there. Then, we can model these transitions for given S_0, X by integrating the conditional model over the distribution of V.

Abbring (2008) discusses more complex, and arguably more realistic, sampling schemes. For example, when studying labor-market histories we may randomly sample from the stock of the unemployed at a particular point in time. Because the unobserved component V affects the probability of being unemployed at the sampling date, the distribution of $V \mid X$ in the stock sample does not equal its population distribution. This is again a dynamic version of the selection problem. Moreover, in this case we typically do not observe an agent's entire labor-market history from T_0 onwards. Instead, we may have data on the time spent in unemployment at the sampling date and on labor-market transitions for some period after the sampling date. This "initial conditions problem" complicates matters further [Heckman (1981b)].

In the next two subsections, we first discuss some examples of applications of the model and then review a basic identification result for the simple sampling scheme above.

3.3.2.2. Applications to program evaluation Several empirical papers study the effect of a single treatment on some outcome duration or set of transitions. Two approaches can be distinguished. In the first approach, the outcome and treatment processes are explicitly and separately specified. The second approach distinguishes treatment as one state within a single event-history model with state dependence.

The first approach is used in a variety of papers in labor economics. Eberwein, Ham and LaLonde (1997) specify a model for labor-market transitions in which the transition intensities between various labor-market states (not including treatment) depend on whether someone has been assigned to a training program in the past or not. Abbring, Van den Berg and Van Ours (2005) and Van den Berg, Van der Klaauw and Van Ours (2004) specify a model for re-employment durations in which the re-employment hazard depends on whether a punitive benefits reduction has been imposed in the past. Similarly, Van den Berg, Holm and Van Ours (2002) analyze the duration up to transition into medical trainee positions and the effect of an intermediate transition into a medical assistant position (a "stepping-stone job") on this duration. In all of these papers, the outcome model is complemented with a hazard model for treatment choice.

These models fit into the framework of Section 3.3.1 or a multi-state extension thereof. We can rephrase the class of models discussed in Section 3.3.1 in terms of a simple event-history model with state dependence as follows. Distinguish three states, untreated (O), treated (P) and the exit state of interest (E), so that $\mathcal{S} = \{O, P, E\}$. All subjects start in O, so that $S_0 = O$. Obviously, we do not want to allow for all possible transitions between these three states. Instead, we restrict the correspondence $\mathcal{Q}$ representing the possible transitions as follows:

$$\mathcal{Q}(s) = \begin{cases} \{P, E\} & s = O, \\ \{E\} \quad \text{if} & s = P, \\ \emptyset & s = E. \end{cases}$$

State dependence of the transition rates into E captures treatment effects in the sense of Section 3.3.1. Not all models in Abbring and Van den Berg (2003b) are included in the semi-Markov setup discussed here. In particular, in this paper we do not allow the transition rate from P to E to depend on the duration spent in O. This extension with "lagged duration dependence" [Heckman and Borjas (1980)] would be required to capture one variant of their model.

The model for transitions from "untreated" (O) is a competing risks model, with program enrollment (transition to P) and employment (E) competing to end the untreated spell. If the unobservable factor V_{OE} that determines transitions to employment and the unobservable factor V_{OP} affecting program enrollment are dependent, then program enrollment is selective in the sense that the initial distribution of V_{OE}—and also typically that of V_{PE}—among those who enroll at a given point in time does not equal its distribution among survivors in O up to that time.[90]

The second approach is used by Gritz (1993) and Bonnal, Fougère and Sérandon (1997), among others. Consider the following simplified setup. Suppose workers are either employed (E), unemployed (O), or engaged in a training program (P). We can now specify a transition process among these three labor-market states in which a causal effect of training on unemployment and employment durations is modeled as dependence of the various transition rates on the past occurrence of a training program in the labor-market history. Bonnal, Fougère and Sérandon (1997) only have limited information on agents' labor-market histories before the sample period. Partly to avoid difficult initial conditions problems, they restrict attention to "first order lagged occurrence dependence" [Heckman and Borjas (1980)] by assuming that transition rates only depend on the current and previous state occupied. Such a model is not directly covered by the semi-Markov model, but with a simple augmentation of the state space it can be covered. In particular, we have to include lagged states in the state space on which the transition process is defined. Because there is no lagged state in the event-history's first spell, initial states should be defined separately. So, instead of just distinguishing states in $\mathcal{S}^* = \{E, O, P\}$, we distinguish augmented states in $\mathcal{S} = \{(s, s') \in (\mathcal{S}^* \cup \{I\}) \times \mathcal{S}^*\colon\ s \neq s'\}$. Then, (I, s), $s \in \mathcal{S}^*$, denote the initial states, and $(s, s') \in \mathcal{S}$ the augmented state of an agent who is currently in s' and came from $s \neq s'$. In order to preserve the interpretation of the model as a model of lagged

[90] Note that, in addition, the survivors in O themselves are a selected subpopulation. Because V affects survival in O, the distribution of V among survivors in O is not equal to its population distribution.

occurrence dependence, we have to exclude certain transitions by specifying

$$\mathcal{Q}(s, s') = \{(s', s''),\ s'' \in \mathcal{S}^* \setminus \{s'\}\}.$$

This excludes transitions to augmented states that are labeled with a lagged state different from the origin state. Also, it ensures that agents never return to an initial state. For example, from the augmented state (O, P)—previously unemployed and currently enrolled in a program—only transitions to augmented states (P, s'')—previously enrolled in a program and currently in s''—are possible. Moreover, it is not possible to be currently employed and transiting to initially unemployed, (I, O). Rather, an employed person who loses her job would transit to (E, O)—currently unemployed and previously employed.

The effects of, for example, training are now modeled as simple state-dependence effects. For example, the effect of training on the transition rate from unemployment to employment is simply the contrast between the individual transition rate from (E, O) to (O, E) and the transition rate from (P, O) to (O, E). Dynamic selection into the augmented states (E, O) and (P, O), as specified by the transition model, confounds the empirical analysis of these training effects. Note that due to the fact that we have restricted attention to first-order lagged occurrence dependence, there are no longer-run effects of training on transition rates from unemployment to employment.

3.3.2.3. Identification without exclusion restrictions In this section, we sketch a basic identification result for the following sampling scheme. Suppose that the economist randomly samples from the population of event-histories, and that we observe the first $\bar{L}$ transitions (including destinations) for each sampled event-history, with the possibility that $\bar{L} = \infty$.[91] Thus, we observe a random sample of $\{(T_l, S_l),\ l \in \{0, 1, \dots, \bar{L}\}\}$, and X.

First note that we can only identify the determinants of θ_{jk} for transitions (j, k) that occur with positive probability among the first $\bar{L}$ transitions. Moreover, without further restrictions, we can only identify the joint distribution of a vector of unobservables corresponding to (part of) a sequence of transitions that can be observed among the first $\bar{L}$ transitions.

With this qualification, identification can be proved by extending Abbring and Van den Berg's (2003a) analysis of the MPH competing risks model to the present setting. This analysis assumes that transition rates have an MPH functional form. Identification again requires specific moments of V to be finite, and independent local variation in the regressor effects.

3.3.3. A structural perspective

Without further restrictions, the causal duration model of Section 3.3.1 is versatile. It can be generated as the reduced form of a wide variety of continuous-time economic

[91] Note that this assumes away econometric initial conditions problems of the type previously discussed.

models driven by point processes. Leading examples are sequential job-search models in which job-offer arrival rates, and other model parameters, depend on agent characteristics (X, V) and policy interventions [see, e.g., Mortensen (1986), and Example 7].

The MPH restriction on this model, however, is hard to justify from economic theory. In particular, nonstationary job-search models often imply interactions between duration and covariate effects; the MPH model only results under strong assumptions [Heckman and Singer (1986), Van den Berg (2001)]. Similarly, an MPH structure is hard to generate from models in which agents learn about their individual value of the model's structural parameters, that is about (X, V), through Bayesian updating.

An alternative class of continuous-time models, not discussed in this chapter, specifies durations as the first time some Gaussian or more general process crosses a threshold. Such models are closely related to a variety of dynamic economic models. They have attracted recent attention in statistics [see, e.g., Aalen and Gjessing (2004)]. Abbring (2007) analyzes identifiability of "mixed hitting-time models", continuous-time threshold-crossing models in which the parameters depend on observed and unobserved covariates, and discusses their link with optimizing models in economics. This is a relatively new area of research, and a full development is beyond the scope of this paper. It extends to a continuous-time framework the dynamic threshold-crossing model developed in Heckman (1981a, 1981b) that is used in the next subsection of this chapter.

We now discuss a complementary discrete-time approach where it is possible to make many important economic distinctions that are difficult to make in the setting of continuous-time models and to avoid some difficult measure-theoretic problems. In the structural version, it is possible to specify precisely agent information sets in a fashion that is not possible in conventional duration models.

3.4. Dynamic discrete choice and dynamic treatment effects

Heckman and Navarro (2007) and Cunha, Heckman and Navarro (2007) present econometric models for analyzing time to treatment and the consequences of the choice of a particular treatment time. Treatment may be a medical intervention, stopping schooling, opening a store, conducting an advertising campaign at a given date or renewing a patent. Associated with each treatment time, there can be multiple outcomes. They can include a vector of health status indicators and biomarkers; lifetime employment and earnings consequences of stopping at a particular grade of schooling; the sales revenue and profit generated from opening a store at a certain time; the revenues generated and market penetration gained from an advertising campaign; or the value of exercising an option at a given time. Heckman and Navarro (2007) unite and contribute to the literatures on dynamic discrete choice and dynamic treatment effects. For both classes of models, they present semiparametric identification analyses. We summarize their work in this section. It is a natural extension of the framework for counterfactual analysis of multiple treatments developed in Section 2 to a dynamic setting. It is formulated in discrete time, which facilitates the specification of richer unobserved and observed covariate processes than those entertained in the continuous-time framework of Abbring and Van den Berg (2003b).

Heckman and Navarro extend the literature on treatment effects to model choices of treatment times and the consequences of choice and link the literature on treatment effects to the literature on precisely formulated structural dynamic discrete-choice models generated from index models crossing thresholds. They show the value of precisely formulated economic models in extracting the information sets of agents, in providing model identification, in generating the standard treatment effects and in enforcing the nonanticipating behavior condition (NA) discussed in Section 3.2.1.[92]

They establish the semiparametric identifiability of a class of dynamic discrete-choice models for stopping times and associated outcomes in which agents sequentially update the information on which they act. They also establish identifiability of a new class of reduced form duration models that generalize conventional discrete-time duration models to produce frameworks with much richer time series properties for unobservables and general time-varying observables and patterns of duration dependence than conventional duration models. Their analysis of identification of these generalized models requires richer variation driven by observables than is needed in the analysis of the more restrictive conventional models. However, it does not require conventional period-by-period exclusion restrictions, which are often difficult to justify. Instead, they rely on curvature restrictions across the index functions generating the durations that can be motivated by dynamic economic theory.[93] Their methods can be applied to a variety of outcome measures including durations.

The key to their ability to identify structural models is that they supplement information on stopping times or time to treatment with additional information on measured consequences of choices of time to treatment as well as measurements. The dynamic discrete-choice literature surveyed in Rust (1994) and Magnac and Thesmar (2002) focuses on discrete-choice processes with general preferences and state vector evolution equations, typically Markovian in nature. Rust's (1994) paper contains negative results on nonparametric identification of discrete-choice processes. Magnac and Thesmar (2002) present some positive results on nonparametric identification if certain parameters or distributions of unobservables are assumed to be known. Heckman and Navarro (2007) produce positive results on nonparametric identification of a class of dynamic discrete-choice models based on expected income maximization developed in labor economics by Flinn and Heckman (1982), Keane and Wolpin (1997) and Eckstein and Wolpin (1999). These frameworks are dynamic versions of the Roy model. Heckman and Navarro (2007) show how use of cross-equation restrictions joined with data on supplementary measurement systems can undo Rust's nonidentification result. We exposit

[92] Aakvik, Heckman and Vytlacil (2005), Heckman, Tobias and Vytlacil (2001, 2003), Carneiro, Hansen and Heckman (2001, 2003) and Heckman and Vytlacil (2005) show how standard treatment effects can be generated from structural models.

[93] See Heckman and Honoré (1989) for examples of such an identification strategy in duration models. See also Cameron and Heckman (1998).

their work and the related literature in this section. With their structural framework, they can distinguish objective outcomes from subjective outcomes (valuations by the decision maker) in a dynamic setting. Applying their analysis to health economics, they can identify the causal effects on health of a medical treatment as well as the associated subjective pain and suffering of a treatment regime for the patient.[94] Attrition decisions also convey information about agent preferences about treatment.[95]

They do not rely on the assumption of conditional independence of unobservables with outcomes, given observables, that is used throughout much of the dynamic discrete-choice literature and the dynamic treatment literature surveyed in Section 3.2.[96] As noted in Section 3.1, sequential conditional-independence assumptions underlie recent work on reduced form dynamic treatment effects.[97] The semiparametric analysis of Heckman and Navarro (2007) based on factors generalizes matching to a dynamic setting. In their paper, some of the variables that would produce conditional independence and would justify matching if they were observed, are treated as unobserved match variables. They are integrated out and their distributions are identified.[98] They consider two classes of models. We review both.

3.4.1. Semiparametric duration models and counterfactuals

Heckman and Navarro (2007), henceforth HN, develop a semiparametric index model for dynamic discrete choices that extends conventional discrete time duration analysis. They separate out duration dependence from heterogeneity in a semiparametric framework more general than conventional discrete-time duration models. They produce a new class of reduced form models for dynamic treatment effects by adjoining time-to-treatment outcomes to the duration model. This analysis builds on Heckman (1981a, 1981b, 1981c).

Their models are based on a latent variable for choice at time s,

$$I(s) = \Psi\bigl(s, Z(s)\bigr) - \eta(s),$$

where the $Z(s)$ are observables and $\eta(s)$ are unobservables from the point of view of the econometrician. Treatments at different times may have different outcome consequences which they model after analyzing the time to treatment equation. Define $D(s)$ as an indicator of receipt of treatment at date s. Treatment is taken the first time $I(s)$

[94] See Chan and Hamilton (2006) for a structural dynamic empirical analysis of this problem.

[95] See Heckman and Smith (1998). Use of participation data to infer preferences about outcomes is developed in Heckman (1974b).

[96] See, e.g., Rust (1987), Manski (1993), Hotz and Miller (1993) and the papers cited in Rust (1994).

[97] See, e.g., Gill and Robins (2001) and Lechner and Miquel (2002).

[98] For estimates based on this idea see Carneiro, Hansen and Heckman (2003), Aakvik, Heckman and Vytlacil (2005), Cunha and Heckman (2007b, 2008), Cunha, Heckman and Navarro (2005, 2006), and Heckman and Navarro (2005).

becomes positive. Thus,

$$D(s) = \mathbf{1}\big[I(s) \geqslant 0,\ I(s-1) < 0,\ \ldots,\ I(1) < 0\big],$$

where the indicator function $\mathbf{1}[\cdot]$ takes the value of 1 if the term inside the braces is true.[99] They derive conditions for identifying a model with general forms of duration dependence in the time to treatment equation using a large sample from the distribution of (D, Z).

3.4.1.1. Single spell duration model Individuals are assumed to start spells in a given (exogenously determined) state and to exit the state at the beginning of time period S.[100] S is thus a random variable representing total completed spell length. Let $D(s) = 1$ if the individual exits at time s, $S = s$, and $D(s) = 0$ otherwise. In an analysis of drug treatments, S is the discrete-time period in the course of an illness at the beginning of which the drug is administered. Let $\bar{S}$ $(< \infty)$ be the upper limit on the time the agent being studied can be at risk for a treatment. It is possible in this example that $D(1) = 0, \ldots, D(\bar{S}) = 0$, so that a patient never receives treatment. In a schooling example, "treatment" is not schooling, but rather dropping out of schooling.[101] In this case, $\bar{S}$ is an upper limit to the number of years of schooling, and $D(\bar{S}) = 1$ if $D(1) = 0, \ldots, D(\bar{S} - 1) = 0$.

The duration model can be specified recursively in terms of the threshold-crossing behavior of the sequence of underlying latent indices $I(s)$. Recall that $I(s) = \Psi(s, Z(s)) - \eta(s)$, with $Z(s)$ being the regressors that are observed by the analyst. The $Z(s)$ can include expectations of future outcomes given current information in the case of models with forward-looking behavior. For a given stopping time s, let $D^s = (D(1), \ldots, D(s))$ and designate by $d(s)$ and d^s values that $D(s)$ and D^s assume. Thus, $d(s)$ can be zero or one and d^s is a sequence of s zeros or a sequence containing $s - 1$ zeros and a single one. Denote a sequence of all zeros by (0), regardless of its length. Then,

$$\begin{aligned} &D(1) = \mathbf{1}\big[I(1) \geqslant 0\big] \quad \text{and} \\ &D(s) = \begin{cases} \mathbf{1}[I(s) \geqslant 0] & \text{if } D^{s-1} = (0), \\ 0 & \text{otherwise,} \end{cases} \qquad s = 2, \ldots, \bar{S}. \end{aligned} \tag{3.11}$$

For $s = 2, \ldots, \bar{S}$, the indicator $\mathbf{1}[I(s) \geqslant 0]$ is observed if and only if the agent is still at risk of treatment, $D^{s-1} = (0)$. To identify period s parameters from period s outcomes, one must condition on all past outcomes and control for any selection effects.

[99] This framework captures the essential feature of any stopping time model. For example, in a search model with one wage offer per period, $I(s)$ is the gap between market wages and reservation wages at time s. See, e.g., Flinn and Heckman (1982). This framework can also approximate the explicit dynamic discrete-choice model analyzed in Section 3.4.2.

[100] Thus we abstract from the initial conditions problem discussed in Heckman (1981b).

[101] In the drug treatment example, S may designate the time a treatment regime is completed.

Let $Z = (Z(1), \ldots, Z(\bar{S}))$, and let $\eta = (\eta(1), \ldots, \eta(\bar{S}))$.[102] Assume that Z is statistically independent of η. Heckman and Navarro (2007) assume that $\Psi(s, Z(s)) = Z(s)\gamma_s$. We deal with a more general case. $\Psi(Z) = (\Psi(1, Z(1)), \ldots, \Psi(\bar{S}, Z(\bar{S})))$. We let Ψ denote the abstract parameter. Depending on the values assumed by $\Psi(s, Z(s))$, one can generate very general forms of duration dependence that depend on the values assumed by the $Z(s)$. HN allow for period-specific effects of regressors on the latent indices generating choices.

This model is the reduced form of a general dynamic discrete-choice model. Like many reduced form models, the link to choice theory is not clearly specified. It is not a conventional multinomial choice model in a static (perfect certainty) setting with associated outcomes.

3.4.1.2. Identification of duration models with general error structures and duration dependence Heckman and Navarro (2007) establish semiparametric identification of the model of Equation (3.11) assuming access to a large sample of i.i.d. (D, Z) observations. Let $Z^s = (Z(1), \ldots, Z(s))$. Data on (D, Z) directly identify the conditional probability $\Pr(D(s) = d(s) \mid Z^s, D^{s-1} = (0))$ a.e. $F_{Z^s|D^{s-1}=(0)}$ where $F_{Z^s|D^{s-1}=(0)}$ is the distribution of Z^s conditional on previous choices $D^{s-1} = (0)$. Assume that $(\Psi, F_\eta) \in \Phi \times \mathcal{H}$, where F_η is the distribution of η and $\Phi \times \mathcal{H}$ is the parameter space. The goal is to establish conditions under which knowledge of $\Pr(D(s) = d(s) \mid Z, D^{s-1} = (0))$ a.e. $F_{Z|D^{s-1}=(0)}$ allows the analyst to identify a unique element of $\Phi \times \mathcal{H}$. They use a limit strategy that allows them to recover the parameters by conditioning on large values of the indices of the preceding choices. This identification strategy is widely used in the analysis of discrete choice.[103]

They establish sufficient conditions for the identification of model (3.11). We prove the following more general result:

THEOREM 3. *For the model defined by Equation* (3.11)*, assume the following conditions*:

(i) $\eta \perp\!\!\!\perp Z$.
(ii) η *is an absolutely continuous random variable on* $\mathbb{R}^{\bar{S}}$ *with support* $\prod_{s=1}^{\bar{S}}(\underline{\eta}(s), \bar{\eta}(s))$, *where* $-\infty \leqslant \underline{\eta}(s) < \bar{\eta}(s) \leqslant +\infty$ *for all* $s = 1, \ldots, \bar{S}$.
(iii) *The* $\Psi(s, Z(s))$ *are members of the Matzkin class of functions defined in Appendix* B.1, $s = 1, \ldots, \bar{S}$.

[102] A special case of the general model arises when $\eta(s)$ has a factor model representation as analyzed in Section 2. We will use such a representation when we adjoin outcomes to treatment times later in this section.

[103] See, e.g., Manski (1988), Heckman (1990), Heckman and Honoré (1989, 1990), Matzkin (1992, 1993), Taber (2000), and Carneiro, Hansen and Heckman (2003). A version of the strategy of this proof was first used in psychology where agent choice sets are eliminated by experimenter manipulation. The limit set argument effectively uses regressors to reduce the choice set confronting agents. See Falmagne (1985) for a discussion of models of choice in psychology.

(iv) $\mathrm{Supp}(\Psi^{s-1}(Z), Z(s)) = \mathrm{Supp}(\Psi^{s-1}(Z)) \times \mathrm{Supp}(Z(s))$, $s = 2, \ldots, \bar{S}$.
(v) $\mathrm{Supp}(\Psi(Z)) \supseteq \mathrm{Supp}(\eta)$.

Then F_η and $\Psi(Z)$ are identified, where the $\Psi(s, Z(s))$, $s = 1, \ldots, \bar{S}$, are identified over the relevant support admitted by (ii).

PROOF. We sketch the proof for $\bar{S} = 2$. The result for general $\bar{S}$ follows by a recursive application of this argument. Consider the following three probabilities.

(a) $\Pr\bigl(D(1) = 1 \mid Z = z\bigr) = \displaystyle\int_{\underline{\eta}(1)}^{\Psi(1,z(1))} f_{\eta(1)}(u)\,du.$

(b) $\Pr\bigl(D(2) = 1, D(1) = 0 \mid Z = z\bigr)$
$$= \int_{\underline{\eta}(2)}^{\Psi(2,z(2))} \int_{\Psi(1,z(1))}^{\bar{\eta}(1)} f_{\eta(1),\eta(2)}(u_1, u_2)\,du_1\,du_2.$$

(c) $\Pr\bigl(D(2) = 0, D(1) = 0 \mid Z = z\bigr)$
$$= \int_{\Psi(2,z(2))}^{\bar{\eta}(2)} \int_{\Psi(1,z(1))}^{\bar{\eta}(1)} f_{\eta(1),\eta(2)}(u_1, u_2)\,du_1\,du_2.$$

The left-hand sides are observed from data on those who stop in period 1 (a); those who stop in period 2 (b); and those who terminate in the "0" state in period 2 (c). From Matzkin (1992), under our conditions on the class of functions Φ, which are stronger than hers, we can identify $\Psi(1, z(1))$ and $F_{\eta(1)}$ from (a). Using (b), we can fix $z(2)$ and vary $\Psi(1, z(1))$. From (iv) and (v) there exists a limit set $\tilde{\mathcal{Z}}_1$, possibly dependent on $z(2)$, such that $\lim_{z(1)\to\tilde{\mathcal{Z}}_1} \Psi(1, z(1)) = \underline{\eta}(1)$. Thus we can construct

$$\Pr\bigl(D(2) = 0 \mid Z = z\bigr) = \int_{\Psi(2,z(2))}^{\bar{\eta}(2)} f_{\eta(2)}(u_2)\,du_2$$

and identify $\Psi(2, z(2))$ and $F_{\eta(2)}(\eta(2))$. Using the $\Psi(1, z(1))$, $\Psi(2, z(2))$, one can trace out the joint distribution $F_{\eta(1),\eta(2)}$ over its support. Under the Matzkin conditions, identification is achieved on a nonnegligible set. The proof generalizes in a straightforward way to general $\bar{S}$. □

Observe that if the $\eta(s)$ are bounded by finite upper and lower limits, we can only determine the $\Psi(s, Z(s))$ over the limits so defined. Consider the first step of the proof. Under the Matzkin conditions, $F_{\eta(1)}$ is known. From assumption (ii) we can determine

$$\Psi\bigl(1, z(1)\bigr) = F_{\eta(1)}^{-1}\bigl(\Pr\bigl(D(1) = 1 \mid Z = z\bigr)\bigr),$$

but only over the support $(\underline{\eta}(1), \bar{\eta}(1))$. If the support of $\eta(1)$ is $\mathbb{R}$, we determine $\Psi(1, z(1))$ for all $z(1)$. Heckman and Navarro (2007) analyze the special case $\Psi(s, Z(s)) = Z(s)\gamma_s$ and invoke sequential rank conditions to identify γ_s, even over limited supports. They also establish that the limit sets are nonnegligible in this case so that standard definitions of identifiability [see, e.g., Matzkin (1992)] will be satisfied.[104] Construction of the limit set $\widetilde{Z}_s$, $s = 1, \ldots, \bar{S}$, depends on the functional form specified for the $\Psi(s, z(s))$. For the linear-in-parameters case $\Psi(s, z(s)) = Z(s)\gamma_s$, they are obtained by letting arguments get big or small. Matzkin (1992) shows how to establish the limit sets for functions in her family of functions.

A version of Theorem 3 with $\Psi(s, Z(s)) = Z_s\gamma_s$ that allows dependence between Z and η^s except for one component can be proved using the analysis of Lewbel (2000) and Honoré and Lewbel (2002).[105]

The assumptions of Theorem 3 will be satisfied if there are transition-specific exclusion restrictions for Z with the required properties. As noted in Section 3.3, in models with many periods, this may be a demanding requirement. Very often, the Z variables are time invariant and so cannot be used as exclusion restrictions. Corollary 1 in HN, for the special case $\Psi(s, Z(s)) = Z(s)\gamma_s$, tells us that the HN version of the model can be identified even if there are no conventional exclusion restrictions and the $Z(s)$ are the *same* across all time periods if sufficient structure is placed on how the γ_s vary with s. Variations in the values of γ_s across time periods arise naturally in finite-horizon dynamic discrete-choice models where a shrinking horizon produces different effects of the same variable in different periods. For example, in Wolpin's (1987) analysis of a search model, the value function depends on time and the derived decision rules weight the same invariant characteristics differently in different periods. In a schooling model, parental background and resources may affect education continuation decisions differently at different stages of the schooling decision. The model generating equation (3.11) can be semiparametrically identified without transition-specific exclusions if the duration dependence is sufficiently general. For a proof, see Corollary 1 in Heckman and Navarro (2007).

The conditions of Theorem 3 are somewhat similar to the conditions on the regressor effects needed for identification of the continuous-time event-history models in Section 3.3. One difference is that the present analysis requires independent variation of the regressor effects over the support of the distribution of the unobservables generating outcomes. The continuous-time analysis based on the functional form of the mixed proportional hazard model (MPH) as analyzed by Abbring and Van den Berg (2003a) only requires local independent variation.

Theorem 3 and Corollary 1 in HN have important consequences. The $\Psi(s, Z(s))$, $s = 1, \ldots, \bar{S}$, can be interpreted as duration dependence parameters that are modified by the $Z(s)$ and that vary across the spell in a more general way than is permitted in

[104] Heckman and Navarro (2007) prove their theorem for a model where $D(s) = \mathbf{1}[I(s) \leqslant 0]$ if $D^{s-1} = (0)$, $s = 2, \ldots, \bar{S}$. Our formulation of their result is consistent with the notation in this chapter.

[105] HN discuss a version of such an extension at their website. Lewbel's conditions are very strong. To account for general forms of dependence between Z and η^s requires modeling the exact form of the dependence. Nonparametric solutions to this problem remain an open question in the literature on dynamic discrete choice. One solution is to assume functional forms for the error terms, but in general, this is not enough to identify the model without further restrictions imposed. See Heckman and Honoré (1990).

mixed proportional hazards (MPH), generalized accelerated failure time (GAFT) models or standard discrete-time hazard models.[106] Duration dependence in conventional specifications of duration models is usually generated by variation in model intercepts. The regressors are allowed to interact with the duration dependence parameters. In the specifications justified by Theorem 3, the "heterogeneity" distribution F_η is identified for a general model. No special "permanent-transitory" structure is required for the unobservables although that specification is traditional in duration analysis. Their explicit treatment of the stochastic structure of the duration model is what allows HN to link in a general way the unobservables generating the duration model to the unobservables generating the outcome equations that are introduced in the next section. Such an explicit link is not currently available in the literature on continuous-time duration models for treatment effects surveyed in Section 3.3, and is useful for modelling selection effects in outcomes across different treatment times. Their outcomes can be both discrete and continuous and are not restricted to be durations.

Under conditions given in Corollary 1 of HN, no period-specific exclusion conditions are required on the Z. Hansen and Sargent (1980) and Abbring and Van den Berg (2003b) note that period-specific exclusions are not natural in reduced form duration models designed to approximate forward-looking life cycle models. Agents make current decisions in light of their forecasts of future constraints and opportunities, and if they forecast some components well, and they affect current decisions, then they are in $Z(s)$ in period s. Corollary 1 in HN establishes identification without such exclusions. HN adjoin a system of counterfactual outcomes to their model of time to treatment to produce a model for dynamic counterfactuals. We summarize that work next.

3.4.1.3. Reduced form dynamic treatment effects This section reviews a reduced form approach to generating dynamic counterfactuals developed by HN. They apply and extend the analysis of Carneiro, Hansen and Heckman (2003) and Cunha, Heckman and Navarro (2005, 2006) to generate *ex post* potential outcomes and their relationship with the time to treatment indices $I(s)$ analyzed in the preceding subsection. With reduced form models, it is difficult to impose restrictions from economic theory or to make distinctions between *ex ante* and *ex post* outcomes. In the structural model developed below, these and other distinctions can be made easily.

The reduced form model's specification closely follows the exposition of Section 2.8.1. Associated with each treatment s, $s = 1, \dots, \bar{S}$, is a vector of $\bar{T}$ outcomes,

$$\begin{aligned} &Y\big(s, X, U(s)\big) \\ &\quad = \Big(Y\big(1, s, X, U(1, s)\big), \dots, Y\big(t, s, X, U(t, s)\big), \dots, Y\big(\bar{T}, s, X, U(\bar{T}, s)\big)\Big). \end{aligned}$$

In this section, treatment time s is synonymous with treatment state s in Section 2. Outcomes depend on covariates X and $U(s) = (U(1, s), \dots, U(t, s), \dots, U(\bar{T}, s))$

[106] See Ridder (1990) for a discussion of these models.

that are, respectively, observable and unobservable by the econometrician. Elements of $Y(s, X, U(s))$ are outcomes associated with stopping or receiving treatment at the beginning of period s. They are factual outcomes if treatment s is actually selected ($S = s$ and $D(s) = 1$). Outcomes corresponding to treatments s' that are not selected ($D(s') = 0$) are counterfactuals. The outcomes associated with each treatment may be different, and indeed the treatments administered at different times may be different.

The components $Y(t, s, X, U(t, s))$ of the vector $Y(s, X, U(s))$ can be interpreted as the outcomes revealed at age t, $t = 1, \dots, \bar{T}$, and may themselves be vectors. The reduced form approach presented in this section is not sufficiently rich to capture the notion that agents revise their anticipations of components of $Y(s, X, U(s))$, $s = 1, \dots, \bar{S}$, as they acquire information over time. This notion is systematically developed using the structural model discussed below in Section 3.4.2.

The treatment "times" may be stages that are not necessarily connected with real times. Thus s may be a schooling level. The correspondence between stages and times is exact if each stage takes one period to complete. Our notation is more flexible, and time and periods can be defined more generally. Our notation in this section accommodates both cases.

In this section of the chapter, we use the condensed notation introduced in Section 2.8.1. This notation is sufficiently rich to represent the life cycle of outcomes for persons who receive treatment at s. Thus, in a schooling example, the components of this vector may include life cycle earnings, employment, and the like associated with a person with characteristics $X, U(s)$, $s = 1, \dots, \bar{S}$, who completes s years of schooling and then forever ceases schooling. It could include earnings while in school at some level for persons who will eventually attain further schooling as well as post-school earnings.

We measure age and treatment time on the same time scale, with origin 1, and let $\bar{T} \geqslant \bar{S}$. Then, the $Y(t, s, X, U(t, s))$ for $t < s$ are outcomes realized while the person is in school at age t (s is the time the person will leave school; t is the current age) and before "treatment" (stopping schooling) has occurred. When $t \geqslant s$, these are post-school outcomes for treatment with s years of schooling. In this case, $t - s$ is years of post-school experience. In the case of a drug trial, the $Y(t, s, X, U(t, s))$ for $t < s$ are measurements observed before the drug is taken at s and if $t \geqslant s$, they are the post-treatment measurements.

Following Carneiro, Hansen and Heckman (2003) and our analysis in Section 2, the variables in $Y(t, s, X, U(t, s))$ may include discrete, continuous or mixed discrete-continuous components. For the discrete or mixed discrete-continuous cases, HN assume that latent continuous variables cross thresholds to generate the discrete components. Durations can be generated by latent index models associated with each outcome crossing thresholds analogous to the model presented in Equation (3.11). In this framework, for example, we can model the effect of attaining s years of schooling on durations of unemployment or durations of employment.

The reduced form analysis in this section does not impose restrictions on the temporal (age) structure of outcomes across treatment times in constructing outcomes and specifying identifying assumptions. Each treatment time can have its own age path of outcomes pre and post treatment. Outcomes prior to treatment and outcomes after treatment are treated symmetrically and both may be different for different treatment times. In particular, HN can allow earnings at age t for people who receive treatment at some future time s' to differ from earnings at age t for people who receive treatment at some future time s'', $\min(s', s'') > t$ even after controlling for U and X.

This generality is in contrast with the analyses of Robins (1997) and Gill and Robins (2001) discussed in Section 3.2 and the analysis of Abbring and Van den Berg (2003b) discussed in Section 3.3. These analyses require exclusion of such anticipation effects to secure identification, because their models attribute dependence of treatment on past outcomes to selection effects. The sequential randomization assumption (M-2) underlying the work of Gill and Robins allows treatment decisions $S(t)$ at time t to depend on past outcomes $Y_{p_0}^{t-1}$ in a general way. Therefore, without additional restrictions, it is not possible to also identify *causal* (anticipatory) effects of treatment $S(t)$ on $Y_{p_0}^{t-1}$. The no-anticipation condition (NA) excludes such effects and secures identification in their framework.[107] It is essential for applying the conditional-independence assumptions in deriving the g-computation formula.

HN's very different approach to identification allows them to incorporate anticipation effects. As in their analysis of the duration model, they assume that there is an exogenous source of independent variation of treatment decisions, independent of past outcomes. Any variation in current outcomes with variation in future treatment decisions induced by this exogenous source cannot be due to selection effects (since they explicitly control for the unobservables) and is interpreted as anticipatory effects of treatment in their framework. However, their structural analysis naturally excludes such effects (see Section 3.4.2 below). Therefore, a natural interpretation of the ability of HN to identify anticipatory effects is that they have overidentifying restrictions that allow them to test their model and, if necessary, relax their assumptions.

In a model with uncertainty, agents act on and value *ex ante* outcomes. The model developed below in Section 3.4.2 distinguishes *ex ante* from *ex post* outcomes. The

[107] The role of the no-anticipation assumption in Abbring and Van den Berg (2003b) is similar. However, their main analysis assumes an asymmetric treatment-outcome setup in which treatment is not observed if it takes place after the outcome transition. In that case, the treatment time is censored at the outcome time. In this asymmetric setup, anticipatory effects of treatment on outcomes cannot be identified because the econometrician cannot observe variation of outcome transitions with future treatment times. This point may appear to be unrelated to the present discussion, but it is not. As was pointed out by Abbring and Van den Berg (2003b), and in Section 3.3, the asymmetric Abbring and Van den Berg (2003b) model can be extended to a fully symmetric bivariate duration model in which treatment hazards may be causally affected by the past occurrence of an outcome event just like outcomes may be affected by past treatment events. This model could be used to analyze data in which both treatment and outcome times are fully observed. In this symmetric setup, any dependence in the data of the time-to-treatment hazard on past outcome events is interpreted as an effect of outcomes on future treatment decisions, and not an anticipatory effect of treatment on past outcomes. If one does not restrict the effects of outcomes on future treatment, without further restrictions, the data on treatments occurring after the outcome event carry no information on anticipatory effects of treatment on outcomes and they face an identification problem similar to that in the asymmetric case.

model developed in this section cannot because, within it, it is difficult to specify the information sets on which agents act or the mechanism by which agents forecast and act on $Y(s, X, U(s))$ when they are making choices.

One justification for not making an *ex ante–ex post* distinction is that the agents being modeled operate under perfect foresight even though econometricians do not observe all of the information available to the agents. In this framework, the $U(s)$, $s = 1, \ldots, \bar{S}$, are an ingredient of the econometric model that accounts for the asymmetry of information between the agent and the econometrician studying the agent.

Without imposing assumptions about the functional structure of the outcome equations, it is not possible to nonparametrically identify counterfactual outcome states $Y(s, X, U(s))$ that have never been observed. Thus, in a schooling example, HN assume that analysts observe life cycle outcomes for some persons for each stopping time (level of final grade completion) and our notation reflects this.[108] However, analysts do not observe $Y(s, X, U(s))$ for all s for anyone. A person can have only one stopping time (one completed schooling level). This observational limitation creates the "fundamental problem of causal inference".[109]

In addition to this problem, there is the standard selection problem that the $Y(s, X, U(s))$ are only observed for persons who stop at s and not for a random sample of the population. The selected distribution may not accurately characterize the population distribution of $Y(s, X, U(s))$ for persons selected at random. Note also that without further structure, we can only identify treatment responses within a given policy environment. In another policy environment, where the rules governing selection into treatment and/or the outcomes from treatment may be different, the same time to treatment may be associated with entirely different responses.[110] We now turn to the HN analysis of identification of outcome and treatment time distributions.

3.4.1.4. Identification of outcome and treatment time distributions We assume access to a large i.i.d. sample from the distribution of $(S, Y(S, X, U(S)), X, Z)$, where S is the stopping time, X are the variables determining outcomes and Z are the variables determining choices. We also know $\Pr(S = s \mid Z = z)$ for $s = 1, \ldots, \bar{S}$, from the data. For expositional convenience, we first consider the case of scalar outcomes $Y(S, X, U(S))$. An analysis for vector $Y(S, X, U(S))$ is presented in HN and is discussed below.

Consider the analysis of continuous outcomes. HN analyze more general cases. Their results extend the analyses of Heckman and Honoré (1990), Heckman (1990) and Carneiro, Hansen and Heckman (2003) by considering choices generated by a stopping

[108] In practice, analysts can only observe a portion of the life cycle after treatment. See the discussion on pooling data across samples in Cunha, Heckman and Navarro (2005) to replace missing life cycle data.

[109] See Holland (1986) or Gill and Robins (2001).

[110] This is the problem of general equilibrium effects, and leads to violation of the policy invariance conditions. See Heckman, Lochner and Taber (1998a), Heckman, LaLonde and Smith (1999) or Abbring and Van den Berg (2003b) for discussion of this problem.

time model. To simplify the notation in this section, assume that the scalar outcome associated with stopping at time s can be written as $Y(s) = \mu(s, X) + U(s)$, where $Y(s)$ is shorthand for $Y(s, X, U(s))$. $Y(s)$ is observed only if $D(s) = 1$ where the $D(s)$ are generated by the model analyzed in Theorem 3. Write $I(s) = \Psi(s, Z(s)) - \eta(s)$. Assume that the $\Psi(s, Z(s))$ belong to the Matzkin class of functions described in Appendix B. We use the condensed representations I, $\Psi(Z)$, η, Y, $\mu(X)$ and U as described in Section 2.8.1, and in the previous subsection.

Heckman and Navarro permit general stochastic dependence within the components of U, within the components of η and across the two vectors. They assume that (X, Z) are independent of (U, η). Each component of (U, η) has a zero mean. The joint distribution of (U, η) is assumed to be absolutely continuous.

With "sufficient variation" in the components of $\Psi(Z)$, one can identify $\mu(s, X)$, $[\Psi(1, Z(1)), \dots, \Psi(s, Z(s))]$ and the joint distribution of $U(s)$ and η^s. This enables the analyst to identify average treatment effects across all stopping times, since one can extract $E(Y(s) - Y(s') \mid X = x)$ from the marginal distributions of $Y(s)$, $s = 1, \dots, \bar{S}$.

THEOREM 4. *Write* $\Psi^s(Z) = (\Psi(1, Z(1), \dots, \Psi(s, Z(s)))$. *Assume in addition to the conditions in Theorem 3 that*

(i) $E[U(s)] = 0$. $(U(s), \eta^s)$ *are continuous random variables with support* $\mathrm{Supp}(U(s)) \times \mathrm{Supp}(\eta^s)$ *with upper and lower limits* $(\overline{U}(s), \bar{\eta}^s)$ *and* $(\underline{U}(s), \underline{\eta}^s)$, *respectively,* $s = 1, \dots, \bar{S}$. *These conditions hold for each component of each subvector. The joint system is thus variation free for each component with respect to every other component.*

(ii) $(U(s), \eta^s) \perp\!\!\!\perp (X, Z)$, $s = 1, \dots, \bar{S}$ *(independence).*

(iii) $\mu(s, X)$ *is a continuous function,* $s = 1, \dots, \bar{S}$.

(iv) $\mathrm{Supp}(\Psi(Z), X) = \mathrm{Supp}(\Psi(Z)) \times \mathrm{Supp}(X)$.

Then one can identify $\mu(s, X)$, $\Psi^s(Z)$ $F_{\eta^s, U(s)}$, $s = 1, \dots, \bar{S}$, *where* $\Psi(Z)$ *is identified over the support admitted by condition* (ii) *of Theorem* 3.

PROOF. See Appendix C. □

Appendix D, which extends Heckman and Navarro (2007), states and proves the more general Theorem D.1 for vector outcomes and both discrete and continuous variables that is parallel to the proof of Theorem 2 for the static model.

Theorem 4 does not identify the joint distribution of $Y(1), \dots, Y(\bar{S})$ because analysts observe only one of these outcomes for any person. Observe that exclusion restrictions in the arguments of the choice of treatment equation are not required to identify the counterfactuals. What is required is independent variation of arguments which might be achieved by exclusion conditions but can be obtained by other functional restrictions (see HN, Corollary 1, for example). One can identify the $\mu(s, X)$ (up to constants) without the limit set argument. Thus one can identify certain features of the model without using the limit set argument. See HN.

The proof of Theorem 4 in Appendix C covers the case of vector $Y(s, X, U(s))$ where each component is a continuous random variable. The analysis in Appendix D allows for age-specific outcomes $Y(t, s, X, U(t, s))$, $t = 1, \dots, \bar{T}$, where Y can be a vector of outcomes. In particular, HN can identify age-specific earnings flows associated with multiple sources of income.

As a by-product of Theorem 4, one can construct various counterfactual distributions of $Y(s)$ for agents with index crossing histories such that $D(s) = 0$ (that is, for whom $Y(s)$ is not observed). Define $B(s) = \mathbf{1}[I(s) \geqslant 0]$, $B^s = (B(1), \dots, B(s))$, and let b^s denote a vector of possible values of B^s. $D(s)$ was defined as $B(s)$ if $B^{s-1} = (0)$ and 0 otherwise. Theorem 4 gives conditions under which the counterfactual distribution of $Y(s)$ for those with $D(s') = 1$, $s' \neq s$, can be constructed. More generally, it can be used to construct

$$\Pr\bigl(Y(s) \leqslant y(s) \mid B^{s'} = b^{s'}, X = x, Z = z\bigr)$$

for all of the $2^{s'}$ possible sequences $b^{s'}$ of $B^{s'}$ outcomes up to $s' \leqslant s$. If $b^{s'}$ equals a sequence of $s' - 1$ zeros followed by a one, then $B^{s'} = b^{s'}$ corresponds to $D(s') = 1$. The event $B^{s'} = (0)$ corresponds to $D^{s'} = (0)$, i.e., $S > s'$. For all other sequences $b^{s'}$, $B^{s'} = b^{s'}$ defines a subpopulation of the agents with $D(s'') = 1$ for some $s'' < s'$ and multiple index crossings. For example, $B^{s'} = (0, 1, 0)$ corresponds to $D(2) = 1$ and $I(3) < 0$. This defines a subpopulation that takes treatment at time 2, but that would not take treatment at time 3 if it would not have taken treatment at time 2.[111] It is tempting to interpret such sequences with multiple crossings as corresponding to multiple entry into and exit from treatment. However, this is inconsistent with the stopping time model (3.11), and would require extension of the model to deal with recurrent treatment. Whether a threshold-crossing model corresponds to a structural model of treatment choice is yet another issue, which is taken up in the next section and is also addressed in Cunha, Heckman and Navarro (2007).

The counterfactuals that are identified by fixing different $D(s') = 1$ for different treatment times s' in the general model of HN have an asymmetric aspect. HN can generate $Y(s)$ distributions for persons who are treated at s or before. Without further structure, they cannot generate the distributions of these random variables for people who receive treatment at times after s.

The source of this asymmetry is the generality of duration model (3.11). At each stopping time s, HN acquire a new random variable $\eta(s)$ which can have arbitrary dependence with $Y(s)$ and $Y(s')$ for all s and s'. From Theorem 4, HN can identify the dependence between $\eta(s')$ and $Y(s)$ if $s' \leqslant s$. They cannot identify the dependence between $\eta(s')$ and $Y(s)$ for $s' > s$ without imposing further structure on the unobservables.[112] Thus one can identify the distribution of college outcomes for high school graduates who do not go on to college and can compare these to outcomes for high

[111] Cunha, Heckman and Navarro (2007) develop an ordered choice model with stochastic thresholds.

[112] One possible structure is a factor model which is applied to this problem in the next section.

school graduates, so they can identify the parameter "treatment on the untreated." However, one cannot identify the distribution of high school outcomes for college graduates (and hence treatment on the treated parameters) without imposing further structure.[113] Since one can identify the marginal distributions under the conditions of Theorem 4, one can identify pairwise average treatment effects for all s, s'.

It is interesting to contrast the model identified by Theorem 4 with a conventional static multinomial discrete-choice model with an associated system of counterfactuals, as presented in Appendix B of Chapter 70 and analyzed in Section 2 of this chapter. Using standard tools, it is possible to establish semiparametric identification of the conventional static model of discrete choice joined with counterfactuals and to identify all of the standard mean counterfactuals. For that model there is a fixed set of unobservables governing all choices of states. Thus the analyst does not acquire new unobservables associated with each stopping time as occurs in a dynamic model. Selection effects for $Y(s)$ depend on the unobservables up to s but not later innovations. Selection effects in a static discrete-choice model depend on a fixed set of unobservables for all outcomes. With suitable normalizations, HN identify the joint distributions of choices and associated outcomes without the difficulties, just noted, that appear in the reduced form dynamic model. HN develop models for discrete outcomes including duration models.

3.4.1.5. Using factor models to identify joint distributions of counterfactuals From Theorem 4 and its generalizations reported in HN, one can identify joint distributions of outcomes for each treatment time s and the index generating treatment times. One cannot identify the joint distributions of outcomes across treatment times. Moreover, as just discussed, one cannot, in general, identify treatment on the treated parameters.

As reviewed in Section 2, Aakvik, Heckman and Vytlacil (2005) and Carneiro, Hansen and Heckman (2003) show how to use factor models to identify the joint distributions across treatment times and recover the standard treatment parameters. HN use their approach to identify the joint distribution of $Y = (Y(1), \dots, Y(\bar{S}))$.

The basic idea underlying this approach is to use joint distributions for outcomes measured at each treatment time s along with the choice index to construct the joint distribution of outcomes across treatment choices. To illustrate how to implement this intuition, suppose that we augment Theorem 4 by appealing to Theorem 2 in Carneiro, Hansen and Heckman (2003) or the extension of Theorem 4 proved in Appendix D to identify the joint distribution of the vector of outcomes at each stopping time along with $I^s = (I(1), \dots, I(s))$ for each s. For each s, we may write

$$Y\big(t, s, X, U(t, s)\big) = \mu(t, s, X) + U(t, s), \quad t = 1, \dots, \bar{T},$$
$$I(s) = \Psi\big(s, Z(s)\big) - \eta(s).$$

[113] In the schooling example, one can identify treatment on the treated for the final category $\bar{S}$ since $D^{\bar{S}-1} = (0)$ implies $D(\bar{S}) = 1$. Thus at stage $\bar{S} - 1$, one can identify the distribution of $Y(\bar{S} - 1)$ for persons for whom $D(0) = 0, \dots, D(\bar{S} - 1) = 0$, $D(\bar{S}) = 1$. Hence, if college is the terminal state, and high school the state preceding college, one can identify the distribution of high school outcomes for college graduates.

The scale of $\Psi(s, Z(s))$ is determined from the Matzkin (1994) conditions presented in Appendix B. If we specify the Matzkin functions only up to scale, we determine the functions up to scale and make a normalization. From Theorem 4, we can identify the joint distribution of $(\eta(1), \ldots, \eta(s), U(1, s), \ldots, U(\bar{T}, s))$.

To review these concepts and their application to the model discussed in this section, suppose that we adopt a one-factor model where θ is the factor. It has mean zero. The errors can be represented by

$$\begin{aligned} \eta(s) &= \varphi_s \theta + \varepsilon_{\eta(s)}, \\ U(t, s) &= \alpha_{t,s} \theta + \varepsilon_{t,s}, \quad t = 1, \ldots, \bar{T},\ s = 1, \ldots, \bar{S}. \end{aligned}$$

The θ are independent of all of the $\varepsilon_{\eta(s)}$, $\varepsilon_{t,s}$ and the ε's are mutually independent mean zero disturbances. The φ_s and $\alpha_{t,s}$ are factor loadings. Since θ is an unobservable, its scale is unknown. One can set the scale of θ by normalizing one-factor loading, say $\alpha_{\bar{T},\bar{S}} = 1$. From the joint distribution of $(\eta^s, U(s))$, one can identify σ_θ^2, $\alpha_{t,s}$, φ_s, $t = 1, \ldots, \bar{T}$, for $s = 1, \ldots, \bar{S}$, using the same argument as presented in Section 2.8. A sufficient condition is $\bar{T} \geqslant 3$, but this ignores possible additional information from cross-system restrictions. From this information, one can form for $t \neq t'$ or $s \neq s''$ or both,

$$\operatorname{Cov}\bigl(U(t, s), U(t', s'')\bigr) = \alpha_{t,s} \alpha_{t',s''} \sigma_\theta^2,$$

even though the analyst does not observe outcomes for the same person at two different stopping times. In fact, one can construct the joint distribution of $(U, \eta) = (U(1), \ldots, U(\bar{S}), \eta)$. From this joint distribution, one can recover the standard mean treatment effects as well as the joint distributions of the potential outcomes. One can determine the percentage of participants at treatment time s who benefit from participation compared to what their outcomes would be at other treatment times. One can perform a parallel analysis for models for discrete outcomes and durations. The analysis can be generalized to multiple factors in precisely the same way as described in Section 2.8. Conventional factor analysis assumes that the unobservables are normally distributed. Carneiro, Hansen and Heckman (2003) establish nonparametric identifiability of the θ's and the ε's and their analysis of nonparametric identifiability applies here.

Theorem 4, strictly applied, actually produces only one scalar outcome along with one or more choices for each stopping time, although the proof of the extended Theorem 4 in Appendix D is for a vector-outcome model with both discrete and continuous outcomes.[114] If vector outcomes are not available, access to a measurement system M that assumes the same values for each stopping time can substitute for the need for vector outcomes for Y. Let M_j be the jth component of this measurement system. Write

$$M_j = \mu_{j,M}(X) + U_{j,M}, \quad j = 1, \ldots, J,$$

where $U_{j,M}$ are mean zero and independent of X.

114 HN analyze the vector-outcome case.

Suppose that the $U_{j,M}$ have a one-factor structure so $U_{j,M} = \alpha_{j,M}\theta + \varepsilon_{j,M}$, $j = 1, \ldots, J$, where the $\varepsilon_{j,M}$ are mean zero, mutually independent random variables, independent of the θ. Adjoining these measurements to the one outcome measure $Y(s)$ can substitute for the measurements of $Y(t, s)$ used in the previous example. In an analysis of schooling, the M_j can be test scores that depend on ability θ. Ability is assumed to affect outcomes $Y(s)$ and the choice of treatment times indices.

To extend a point made in Section 2 to the framework for dynamic treatment effects, the factor models implement a matching on unobservables assumption, $\{Y(s)\}_{s=1}^{\bar{S}} \perp\!\!\!\perp S \mid X, Z, \theta$. HN allow for the θ to be unobserved variables and present conditions under which their distributions can be identified.

3.4.1.6. Summary of the reduced form model A limitation of the reduced form approach pursued in this section is that, because the underlying model of choice is not clearly specified, it is not possible without further structure to form, or even define, the marginal treatment effect analyzed in Heckman and Vytlacil (1999, 2001, 2005, and Chapters 70 and 71 in this Handbook) or Heckman, Urzua and Vytlacil (2006). The absence of well defined choice equations is problematic for the models analyzed thus far in this section of our chapter, although it is typical of many statistical treatment effect analyses.[115] In this framework, it is not possible to distinguish objective outcomes from subjective evaluations of outcomes, and to distinguish *ex ante* from *ex post* outcomes. Another limitation of this analysis is its strong reliance on large support conditions on the regressors coupled with independence assumptions. Independence can be relaxed following Lewbel (2000) and Honoré and Lewbel (2002). The large support assumption plays a fundamental role here and throughout the entire evaluation literature.

HN develop an explicit economic model for dynamic treatment effects that allows analysts to make these and other distinctions. They extend the analysis presented in this subsection to a more precisely formulated economic model. They explicitly allow for agent updating of information sets. A well posed economic model enables economists to evaluate policies in one environment and accurately project them to new environments as well as to accurately forecast new policies never previously experienced. We now turn to an analysis of a more fully articulated structural econometric model.

3.4.2. A sequential structural model with option values

This section analyzes the identifiability of a structural sequential optimal stopping time model. HN use ingredients assembled in the previous sections to build an economically interpretable framework for analyzing dynamic treatment effects. For specificity, Heckman and Navarro focus on a schooling model with associated earnings outcomes

[115] Heckman (2005) and the analysis of Chapters 70 and 71 point out that one distinctive feature of the economic approach to program evaluation is the use of choice theory to define parameters and evaluate alternative estimators.

that is motivated by the research of Keane and Wolpin (1997) and Eckstein and Wolpin (1999). They explicitly model costs and build a dynamic version of a Roy model. We briefly survey the literature on dynamic discrete choice in Section 3.4.5 below.

In the model of this section, it is possible to interpret the literature on dynamic treatment effects within the context of an economic model; to allow for earnings while in school as well as grade-specific tuition costs; to separately identify returns and costs; to distinguish private evaluations from "objective" *ex ante* and *ex post* outcomes and to identify persons at various margins of choice. In the context of medical economics, HN consider how to identify the pain and suffering associated with a treatment as well as the distribution of benefits from the intervention. They also model how anticipations about potential future outcomes associated with various choices evolve over the life cycle as sequential treatment choices are made.

In contrast to the analysis of Section 3.4.1, the identification proof for their dynamic choice model works in reverse starting from the last period and sequentially proceeding backward. This approach is required by the forward-looking nature of dynamic choice analysis and makes an interesting contrast with the analysis of identification for the reduced form models which proceeds forward from initial period values.

HN use limit set arguments to identify the parameters of outcome and measurement systems for each stopping time $s = 1, \ldots, \bar{S}$, including means and joint distributions of unobservables. These systems are identified without invoking any special assumptions about the structure of model unobservables. When they invoke factor structure assumptions for the unobservables, they identify the factor loadings associated with the measurements (as defined in Section 3.4.1) and outcomes. They also nonparametrically identify the distributions of the factors and the distributions of the innovations to the factors. With the joint distributions of outcomes and measurements in hand for each treatment time, HN can identify cost (and preference) information from choice equations that depend on outcomes and costs (preferences). HN can also identify joint distributions of outcomes across stopping times. Thus they can identify the proportion of people who benefit from treatment. Their analysis generalizes the one shot decision models of Cunha and Heckman (2007b, 2008), Cunha, Heckman and Navarro (2005, 2006) to a sequential setting.

All agents start with one year of schooling at age 1 and then sequentially choose, at each subsequent age, whether to continue for another year in school. New information arrives at each age. One of the benefits of staying in school is the arrival of new information about returns. Each year of schooling takes one year of age to complete. There is no grade repetition. Once persons leave school, they never return.[116] As a consequence, an agent's schooling level equals her age up to the time $S \leqslant \bar{S}$ she leaves school. After that, ageing continues up to age $\bar{T} \geqslant \bar{S}$, but schooling does not. We again denote

[116] It would be better to derive such stopping behavior as a feature of a more general model with possible recurrence of states. Cunha, Heckman and Navarro (2007) develop general conditions under which it is optimal to stop and never return.

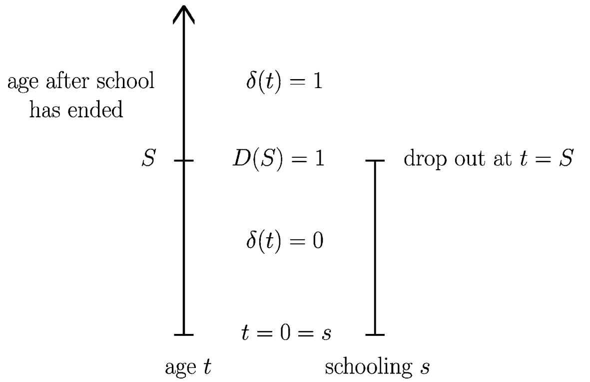

Figure 13. Evolution of grades and age.

$D(s) = \mathbf{1}(S = s)$ for all $s \in \{1, \ldots, \bar{S}\}$. Let $\delta(t) = 1$ if a person has left school at or before age t; $\delta(t) = 0$ if a person is still in school. Figure 13 shows the evolution of age and grades, and clarifies the notation used in this section.

A person's earnings at age t depend on her *current* schooling level s and whether she has left school on or before age t ($\delta(t) = 1$) or not ($\delta(t) = 0$). Thus,

$$Y\bigl(t, s, \delta(t), X\bigr) = \mu\bigl(t, s, \delta(t), X\bigr) + U\bigl(t, s, \delta(t)\bigr). \tag{3.12}$$

Note that $Y(t, s, 0, X)$ is only meaningfully defined if $s = t$, in which case it denotes the earnings of a person as a student at age and schooling level s. More precisely, $Y(s, s, 0, X)$ denotes the earnings of an individual with characteristics X who is still enrolled in school at age and schooling level s and goes on to complete at least $s + 1$ years of schooling. The fact that earnings in school depend only on the current schooling level, and not on the final schooling level obtained, reflects the no-anticipation condition (NA). $U(t, s, \delta(t))$ is a mean zero shock that is unobserved by the econometrician but may, or may not, be observed by the agent. $Y(t, s, 1, X)$ is meaningfully defined only if $s \leqslant t$, in which case it denotes the earnings at age t of an agent who has decided to stop schooling at s.

The direct cost of remaining enrolled in school at age and schooling level s is

$$C\bigl(s, X, Z(s)\bigr) = \Phi\bigl(s, X, Z(s)\bigr) + W(s)$$

where X and $Z(s)$ are vectors of observed characteristics (from the point of view of the econometrician) that affect costs at schooling level s, and $W(s)$ are mean zero shocks that are unobserved by the econometrician that may or may not be observed by the agent. Costs are paid in the period before schooling is undertaken. The agent is assumed to know the costs of making schooling decisions at each transition. The agent is also assumed to know the X and $Z = (Z(1), \ldots, Z(\bar{S} - 1))$ from age 1.[117]

The optimal schooling decision involves comparisons of the value of continuing in school for another year and the value of leaving school forever at each age and schooling level $s \in \{1, \ldots, \bar{S} - 1\}$. We can solve for these values, and the optimal schooling decision, by backward recursion.

The agent's expected reward of stopping schooling forever at level and age s (i.e., receiving treatment s) is given by the expected present value of her remaining lifetime earnings:

$$R(s, I_s) = E\left(\sum_{j=0}^{\bar{T}-s} \left(\frac{1}{1+r}\right)^j Y(s + j, s, 1, X) \;\middle|\; I_s \right), \tag{3.13}$$

where I_s are the state variables generating the age-s-specific information set $\mathcal{I}_s$.[118] They include the schooling level attained at age s, the covariates X and Z, as well as all other variables known to the agent and used in forecasting future variables. Assume a fixed, nonstochastic, interest rate r.[119] The continuation value at age and schooling level s given information I_s is denoted by $K(s, I_s)$.

At $\bar{S}-1$, when an individual decides whether to stop or continue on to $\bar{S}$, the expected reward from remaining enrolled and continuing to $\bar{S}$ (i.e., the continuation value) is the earnings while in school less costs plus the expected discounted future return that arises from completing $\bar{S}$ years of schooling:

$$\begin{aligned} K(\bar{S} - 1, I_{\bar{S}-1}) = {} & Y(\bar{S} - 1, \bar{S} - 1, 0, X) - C\big(\bar{S} - 1, X, Z(\bar{S} - 1)\big) \\ & + \frac{1}{1+r} E\big(R(\bar{S}, I_{\bar{S}}) \mid I_{\bar{S}-1}\big) \end{aligned}$$

where $C(\bar{S} - 1, X, Z(\bar{S} - 1))$ is the direct cost of schooling for the transition to $\bar{S}$. This expression embodies the assumption that each year of school takes one year of age. $I_{\bar{S}-1}$ incorporates all of the information known to the agent.

The value of being in school just before deciding on continuation at age and schooling level $\bar{S} - 1$ is the larger of the two expected rewards that arise from stopping at $\bar{S} - 1$ or continuing one more period to $\bar{S}$:

$$V(\bar{S} - 1, I_{\bar{S}-1}) = \max\big\{R(\bar{S} - 1, I_{\bar{S}-1}), K(\bar{S} - 1, I_{\bar{S}-1})\big\}.$$

More generally, at age and schooling level s this value is

[117] These assumptions can be relaxed and are made for convenience. See Carneiro, Hansen and Heckman (2003), Cunha, Heckman and Navarro (2005) and Cunha and Heckman (2007b) for a discussion of selecting variables in the agent's information set.

[118] We only consider the agent's information set here, and drop the subscript A for notational convenience.

[119] This assumption is relaxed in HN who present conditions under which r can be identified.

$$
\begin{aligned}
V(s, I_s) &= \max\left\{R(s, I_s), K(s, I_s)\right\} \\
&= \max\left\{R(s, I_s), \begin{pmatrix} Y(s, s, 0, X) - C(s, X, Z(s)) \\ + \frac{1}{1+r} E(V(s+1, I_{s+1}) \mid I_s) \end{pmatrix}\right\}.^{120}
\end{aligned}
$$

Following the exposition of the reduced form decision rule in Section 3.4.1, define the decision rule in terms of a first passage of the "index" $R(s, I_s) - K(s, I_s)$,

$$
\begin{aligned}
D(s) = \mathbf{1}\big[&R(s, I_s) - K(s, I_s) \geqslant 0,\ R(s-1, I_{s-1}) - K(s-1, I_{s-1}) < 0, \ldots, \\
&R(1, I_1) - K(1, I_1) < 0\big].
\end{aligned}
$$

An individual stops at the schooling level at the first age where this index becomes positive. From data on stopping times, one can nonparametrically identify the conditional probability of stopping at s,

$$
\Pr(S = s \mid X, Z) = \Pr\left(\begin{matrix} R(s, I_s) - K(s, I_s) \geqslant 0, \\ R(s-1, I_{s-1}) - K(s-1, I_{s-1}) < 0, \ldots, \\ R(1, I_1) - K(1, I_1) < 0 \end{matrix} \;\middle|\; X, Z\right).
$$

HN use factor structure models based on the θ introduced in Section 3.4.1 to define the information updating structure. Agents learn about different components of θ as they evolve through life. The HN assumptions allow for the possibility that agents may know some or all the elements of θ at a given age t regardless of whether or not they determine earnings at or before age t. Once known, they are not forgotten. As agents accumulate information, they revise their forecasts of their future earnings prospects at subsequent stages of the decision process. This affects their decision rules and subsequent choices. Thus HN allow for learning which can affect both pre-treatment outcomes and post-treatment outcomes.[121,122] All dynamic discrete choice models make some assumptions

[120] This model allows no recall and is clearly a simplification of a more general model of schooling with option values. Instead of imposing the requirement that once a student drops out the student never returns, it would be useful to derive this property as a feature of the economic environment and the characteristics of individuals. Cunha, Heckman and Navarro (2007) develop such conditions. In a more general model, different persons could drop out and return to school at different times as information sets are revised. This would create further option value beyond the option value developed in the text that arises from the possibility that persons who attain a given schooling level can attend the next schooling level in any future period. Implicit in this analysis of option values is the additional assumption that persons must work at the highest level of education for which they are trained. An alternative model allows individuals to work each period at the highest wage across all levels of schooling that they have attained. Such a model may be too extreme because it ignores the costs of switching jobs, especially at the higher educational levels where there may be a lot of job-specific human capital for each schooling level. A model with these additional features is presented in Heckman, Urzua and Yates (2007).

[121] This type of learning about unobservables can be captured by HN's reduced form model, but not by Abbring and Van den Berg's (2003b) single-spell mixed proportional hazards model. Their model does not allow for time-varying unobservables. Abbring and Van den Berg develop a multiple-spell model that allows for time-varying unobservables. Moreover, their nonparametric discussion of (NA) and randomization does not exclude the sequential revelation to the agent of a general finite number of unobserved factors although they do not systematically develop such a model.

[122] It is fruitful to distinguish models with exogenous arrival of information (so that information arrives at each age t independent of any actions taken by the agent) from information that arrives as a result of choices

about the updating of information and any rigorous identification analysis of this class of models must test among competing specifications of information updating.

Variables unknown to the agent are integrated out by the agent in forming expectations over future outcomes. Variables known to the agent are treated as constants by the agents. They are integrated out by the econometrician to control for heterogeneity. These are separate operations except for special cases. In general, the econometrician knows less than what the agent knows. The econometrician seeks to identify the distributions of the variables in the agent information sets that are used by the agents to form their expectations as well as the distributions of variables known to the agent and treated as certain quantities by the agent but not known by the econometrician. Determining which elements belong in the agent's information set can be done using the methods exposited in Cunha, Heckman and Navarro (2005) and Cunha and Heckman (2007b) who consider testing what components of X, Z, ε as well as θ are in the agent's information set (see Section 2). We briefly discuss this issue at the end of the next section.[123] HN establish semiparametric identification of the model assuming a given information structure. Determining the appropriate information structure facing the agent and its evolution is an essential aspect of identifying any dynamic discrete-choice model.

Observe that agents with the same information variables I_t at age t have the same expectations of future returns, and the same continuation and stopping values. They make the same investment choices. Persons with the same *ex ante* reward, state and preference variables have the same *ex ante* distributions of stopping times. *Ex post*, stopping times may differ among agents with identical *ex ante* information. Controlling for I_t, future realizations of stopping times do not affect past rewards. This rules out the problem that the future can cause the past, which may happen in HN's reduced form model. It enforces the (NA) condition of Abbring and Van den Berg. Failure to accurately model I_t produces failure of (NA).

HN establish semiparametric identification of their model without period-by-period exclusion restrictions. Their analysis extends Theorems 3 and 4 to an explicit choice-theoretic setting. They use limit set arguments to identify the joint distributions of earnings (for each treatment time s across t) and any associated measurements that do not depend on the stopping time chosen. For each stopping time, they construct the means of earnings outcomes at each age and of the measurements and the joint distributions of the unobservables for earnings and measurements. Factor analyzing the joint distributions of the unobservables, under conditions specified in Carneiro, Hansen and Heckman (2003), they identify the factor loadings, and nonparametrically identify the distributions of the factors and the independent components of the error terms in the earnings and measurement equations. Armed with this knowledge, they use choice data

by the agent. The HN model is in the first class. The models of Miller (1984) or Pakes (1986) are in the second class. See our discussion in Section 3.4.5.

[123] The HN model of learning is clearly very barebones. Information arrives exogenously across ages. In the factor model, all agents who advance to a stage get information about additional factors at that stage of their life cycles but the realizations of the factors may differ across persons.

to identify the distribution of the components of the cost functions that are not directly observed. They construct the joint distributions of outcomes across stopping times. They also present conditions under which the interest rate r is identified.

In their model, analysts can distinguish period by period *ex ante* expected returns from *ex post* realizations by applying the analysis of Cunha, Heckman and Navarro (2005). See the survey in Heckman, Lochner and Todd (2006) for a discussion of this approach or recall our analysis in Section 2. Because they link choices to outcomes through the factor structure assumption, they can also distinguish *ex ante* preference or cost parameters from their *ex post* realizations. *Ex ante*, agents may not know some components of θ. *Ex post*, they do. All of the information about future rewards and returns is embodied in the information set $\mathcal{I}_t$. Unless the time of treatment is known with perfect certainty, it cannot cause outcomes prior to its realization.

The analysis of HN is predicated on specification of the agent's information sets. This information set should be carefully distinguished from that of the econometrician. Cunha, Heckman and Navarro (2005) present methods for determining which components of future outcomes are in the information sets of agents at each age, $\mathcal{I}_t$. If there are components unknown to the agent at age t, under rational expectations, agents form their value functions used to make schooling choices by integrating out the unknown components using the distributions of the variables in their information sets. Components that are known to the agent are treated as constants by the individual in forming the value function but as unknown variables by the econometrician and their distribution is estimated. The true information set of the agent is determined from the set of possible specifications of the information sets of agents by picking the specification that best fits the data on choices and outcomes penalizing for parameter estimation. If neither the agent nor the econometrician knows a variable, the econometrician identifies the determinants of the distribution of the unknown variables that is used by the agent to form expectations. If the agent knows some variables, but the econometrician does not, the econometrician seeks to identify the distribution of the variables, but the agent treats the variables as known constants.

HN can identify all of the treatment parameters including pairwise ATE, the marginal treatment effect (MTE) for each transition (obtained by finding mean outcomes for individuals indifferent between transitions), all of the treatment on the treated and treatment on the untreated parameters and the population distribution of treatment effects by applying the analysis of Carneiro, Hansen and Heckman (2003) and Cunha, Heckman and Navarro (2005) to this model. Their analysis can be generalized to cover the case where there are vectors of contemporaneous outcome measures for different stopping times. See HN for proofs and details.[124]

[124] The same limitations regarding independence assumptions between the regressors and errors discussed in the analysis of reduced forms apply to the structural model.

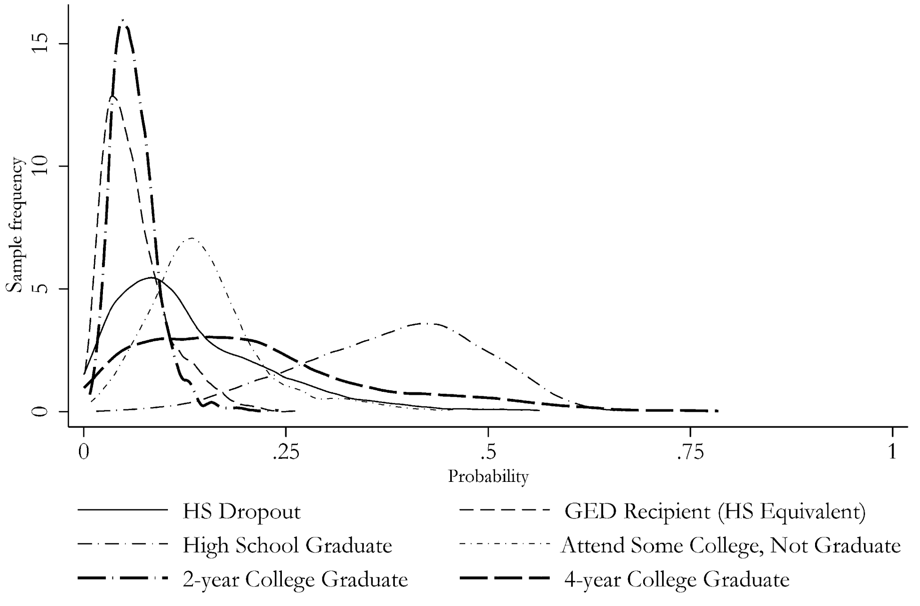

Figure 14. Sample distribution of schooling attainment probabilities for males from the National Longitudinal Survey of Youth. *Source*: Heckman, Stixrud and Urzua (2006).

3.4.3. Identification at infinity

Heckman and Navarro (2007), and many other researchers, rely on identification at infinity to obtain their main identification results. As noted in Chapter 71, identification at infinity is required to identify the average treatment effect (ATE) using IV and control function methods and in the reduced form discrete-time models developed in the previous subsections. While this approach is controversial, it is also testable. In any sample, one can plot the distributions of the probability of each state (exit time) to determine if the identification conditions are satisfied in any sample. Figure 14, taken from Heckman, Stixrud and Urzua (2006), shows such plots for a six-state static schooling model that they estimate. To identify the marginal outcome distributions for each state, the support of the state probabilities should be the full unit interval. The identification at infinity condition is clearly not satisfied in their data.[125] Only the empirical distribution of the state probability of graduating from a four year college comes even close to covering the full unit interval. Thus, their empirical results rely on parametric assumptions, and ATE and the marginal distributions of outcomes are nonparametrically nonidentified in their data without invoking additional structure.

[125] One can always argue that they are satisfied in an infinite sample that has not yet been realized. That statement has no empirical content.

3.4.4. Comparing reduced form and structural models

The reduced form model analyzed in Section 3.4.1 is typical of many reduced form statistical approaches within which it is difficult to make important conceptual distinctions. Because agent choice equations are not modeled explicitly, it is hard to use such frameworks to formally analyze the decision makers' expectations, costs of treatment, the arrival of information, the content of agent information sets and the consequences of the arrival of information for decisions regarding time to treatment as well as outcomes. Key behavioral assumptions are buried in statistical assumptions. It is difficult to distinguish *ex post* from *ex ante* valuations of outcomes in the reduced form models. Cunha, Heckman and Navarro (2005), Carneiro, Hansen and Heckman (2003) and Cunha and Heckman (2007b, 2008) present analyses that distinguish *ex ante* anticipations from *ex post* realizations.[126] In reduced form models, it is difficult to make the distinction between private evaluations and preferences (e.g., "costs" as defined in this section) from objective outcomes (the Y variables).

Statistical and reduced form econometric approaches to analyzing dynamic counterfactuals appeal to uncertainty to motivate the stochastic structure of models. They do not explicitly characterize how agents respond to uncertainty or make treatment choices based on the arrival of new information [see Robins (1989, 1997), Lok (2007), Gill and Robins (2001), Abbring and Van den Berg (2003b), and Van der Laan and Robins (2003)]. The structural approach surveyed in Section 3.4.2 and developed by HN allows for a clear treatment of the arrival of information, agent expectations, and the effects of new information on choice and its consequences. In an environment of imperfect certainty about the future, it rules out the possibility of the future causing the past once the effects of agent information are controlled for.

The structural model developed by HN allows agents to learn about new factors (components of θ) as they proceed sequentially through their life cycles. It also allows agents to learn about other components of the model [see Cunha, Heckman and Navarro (2005)]. Agent anticipations of when they will stop and the consequences of alternative stopping times can be sequentially revised. Agent anticipated payoffs and stopping times are sequentially revised as new information becomes available. The mechanism by which agents revise their anticipations is modeled and identified. See Cunha, Heckman and Navarro (2005, 2006), Cunha and Heckman (2007b, 2008) and the discussion in Section 2 for further discussion of these issues and Heckman, Lochner and Todd (2006) for a partial survey of recent developments in the literature.

The clearest interpretation of the models in the statistical literature on dynamic treatment effects is as *ex post* selection-corrected analyses of distributions of events that have occurred. In a model of perfect certainty, where *ex post* and *ex ante* choices and outcomes are identical, the reduced form approach can be interpreted as approximating clearly specified choice models. In a more general analysis with information arrival and

[126] See the summary of this literature in Heckman, Lochner and Todd (2006).

agent updating of information sets, the nature of the approximation is less clear cut. Thus the current reduced form literature is unclear as to which agent decision-making processes and information arrival assumptions justify the conditional sequential randomization assumptions widely used in the dynamic treatment effect literature [see, e.g., Gill and Robins (2001), Lechner and Miquel (2002), Lok (2007), Robins (1989, 1997), Van der Laan and Robins (2003)]. Section 3.2.2 provides some insight by highlighting the connection to the conditional-independence assumption often employed in the structural dynamic discrete-choice literature [see Rust (1987), and the survey in Rust (1994)]. Reduced form approaches are not clear about the source of the unobservables and their relationship with conditioning variables. It would be a valuable exercise to exhibit which structural models are approximated by various reduced form models. In the structural analysis, this specification emerges as part of the analysis, as our discussion of the stochastic properties of the unobservables presented in the preceding section makes clear.

The HN analysis of both structural and reduced form models relies heavily on limit set arguments. They solve the selection problem in limit sets. The dynamic matching models of Gill and Robins (2001) and Lok (2007) solve the selection problem by invoking recursive conditional-independence assumptions. In the context of the models of HN, they assume that the econometrician knows the θ or can eliminate the effect of θ on estimates of the model by conditioning on a suitable set of variables. The HN analysis entertains the possibility that analysts know substantially less than the agents they study. It allows for some of the variables that would make matching valid to be unobservable. As we have noted in early subsections, versions of recursive conditional-independence assumptions are also used in the dynamic discrete-choice literature [see the survey in Rust (1994)]. The HN factor models allow analysts to construct the joint distribution of outcomes across stopping times. This feature is missing from the statistical treatment effect literature.

Both HN's structural and reduced form models of treatment choice are stopping time models. Neither model allows for multiple entry into and exit from treatment, even though agents in these models would like to reverse their treatment decisions for some realizations of their index if this was not too costly (or, in the case of the reduced form model, if the index thresholds for returning would not be too low).[127] Cunha, Heckman and Navarro (2007) derive conditions on structural stopping models from a more basic model that entertains the possibility of return from dropout states but which nonetheless exhibits the stopping time property. The HN identification strategy relies on the nonrecurrent nature of treatment. Their identification strategy of using limit sets can be applied to a recurrent model provided that analysts confine attention to subsets of (X, Z) such that in those subsets the probability of recurrence is zero.

[127] Recall that treatment occurs if the index turns positive. If there are costs to reversing this decision, agents would only reverse their decision if the index falls below some negative threshold. The stopping time assumption is equivalent to the assumption that the costs of reversal are prohibitively large, or that the corresponding threshold is at the lower end of the support of the index.

3.4.5. A short survey of dynamic discrete-choice models

Table 13 presents a brief summary of the models used to analyze dynamic discrete choices. Rust (1994) presents a widely cited nonparametric nonidentification theorem for dynamic discrete-choice models. It is important to note the restrictive nature of his negative results. He analyzes a recurrent-state infinite-horizon model in a stationary environment. He does not use any exclusion restrictions or cross outcome-choice restrictions. He uses a general utility function. He places no restrictions on period-specific utility functions such as concavity or linearity nor does he specify restrictions connecting preferences and outcomes. One can break Rust's nonidentification result with additional information.

Magnac and Thesmar (2002) present an extended comment on Rust's analysis including positive results for identification when the econometrician knows the distributions of unobservables, assumes that unobservables enter period-specific utility functions in an additively separable way and is willing to specify functional forms of utility functions or other ingredients of the model, as do Pakes (1986), Keane and Wolpin (1997), Eckstein and Wolpin (1999), and Hotz and Miller (1988, 1993). Magnac and Thesmar (2002) also consider the case where one state (choice) is absorbing [as do Hotz and Miller (1993)] and where the value functions are known at the terminal age ($\bar{T}$) [as do Keane and Wolpin (1997) and Belzil and Hansen (2002)]. In HN, each treatment time is an absorbing state. In a separate analysis, Magnac and Thesmar consider the case where unobservables from the point of view of the econometrician are correlated over time (or age t) and choices (s) under the assumption that the distribution of the unobservables is known. They also consider the case where exclusion restrictions are available. Throughout their analysis, they maintain that the distribution of the unobservables is known both by the agent and the econometrician.

HN provide a semiparametric identification of a finite-horizon finite-state model with an absorbing state with semiparametric specifications of reward and cost functions.[128] Given that rewards are in value units, the scale of their utility function is fixed. Choices are not invariant to arbitrary affine transformations so that one source of nonidentifiability in Rust's analysis is eliminated. They can identify the error distributions nonparametrically given their factor structure. They do not have to assume either the functional form of the unobservables or knowledge of the entire distribution of unobservables.

HN present a fully specified structural model of choices and outcomes motivated by, but not identical to, the analyses of Keane and Wolpin (1994, 1997) and Eckstein and Wolpin (1999). In their setups, outcome and cost functions are parametrically specified. Their states are recurrent while those of HN are absorbing. In their model, once an agent drops out of school, the agent does not return. In the Keane–Wolpin model, an

[128] Although their main theorems are for additively separable reward and cost functions, it appears that additive separability can be relaxed using the analysis of Matzkin (2003).

Table 13
Comparisons among papers in the literature on dynamic discrete-choice models

	Use outcomes along with discrete choices?	Finite or infinite horizon	Recurrent states	Stationary environment	Temporal correlation of unobserved shocks	Information updating	Nonparametric or parametric identification	Terminal value assumed to be known	Cross-equation restrictions?[1]
Flinn and Heckman (1982)	Yes (wages)	Infinite	Yes	Yes	Temporal independence given heterogeneity	Arrival of independent shocks	Nonparametric	No	Yes
Miller (1984)	Yes (wages)	Infinite	Yes	Yes	Bayesian normal learning induces dependence	Bayesian learning, arrival of independent shocks	Parametric	No	Yes
Pakes (1986)	No (use cost data to identify discrete choice)	Finite	No	No	AR-1 dependence on unobservables	Arrival of independent shocks	Parametric[2]	Yes	No
Wolpin (1984)	No	Finite	Yes	No	Temporal independence	Temporal independence	Parametric	Yes	No
Wolpin (1987)	Yes	Finite	No	No	Independent shocks	Arrival of independent shocks	Parametric	No	Yes
Wolpin (1992)	Yes (wages)	Finite	Yes	No	Renewal process for shocks; job-specific shocks independent across jobs	Arrival of independent shocks (from new jobs)	Parametric	Yes	Yes
Rust (1987)	Yes[3]	Infinite	Yes	Yes	Shocks conditionally independent given state variables	Arrival of independent shocks	Parametric	No	No
Hotz and Miller (1993)	No	Infinite	Yes	Yes	Shocks conditionally independent given state variables	Synthetic cohort assumption	Parametric	Yes	No

(*continued on next page*)

Table 13
(*continued*)

	Use outcomes along with discrete choices?	Finite or infinite horizon	Recurrent states	Stationary environment	Temporal correlation of unobserved shocks	Information updating	Nonparametric or parametric identification	Terminal value assumed to be known	Cross-equation restrictions?[1]
Manski (1993)	No	Infinite	Yes	Yes	Shocks conditionally independent given state variables	Synthetic cohort assumption	Nonparametric	No	No
Keane and Wolpin (1997)	Yes	Finite	Yes	No	Shocks temporally independent given initial condition	Shocks temporally independent	Parametric	Yes	Yes
Taber (2000)	No	Finite (2 periods)	No	No	General dependence	General dependence	Nonparametric	No	No
Magnac and Thesmar (2002)	Yes[3]	Both finite and infinite	Yes	Yes	Conditional independence given state variables in main case	Conditional dependence	Conditional nonparametric	No	No
Heckman and Navarro (2007)	Yes	Finite	No	No	General dependence (updating)	Serially correlated updating of states	Nonparametric	No	Yes

[1]Cross equation means restrictions used between outcome and choice equations.

[2]Pakes and Simpson (1989) sketch a nonparametric proof of this model.

[3]There is an associated state vector equation which can be interpreted as an outcome equation.

agent who drops out can return. Keane and Wolpin do not establish identification of their model whereas HN establish semiparametric identification of their model. They analyze models with more general times series processes for unobservables. In both the HN and Keane–Wolpin frameworks, agents learn about unobservables. In the Keane–Wolpin framework, such learning is about temporally independent shocks that do not affect agent expectations about returns relevant to possible future choices. The information just affects the opportunity costs of current choices. In the HN framework, learning affects agent expectations about future returns as well as opportunity costs.

The HN model extends previous work by Carneiro, Hansen and Heckman (2003) and Cunha and Heckman (2007b, 2008), Cunha, Heckman and Navarro (2006, 2005) by considering explicit multiperiod dynamic models with information updating. They consider one-shot decision models with information updating and associated outcomes.

Their analysis is related to that of Taber (2000). Like Cameron and Heckman (1998), both HN and Taber use identification-in-the-limit arguments.[129] Taber considers identification of a two period model with a general utility function whereas in Section 3.4.2, we discuss how HN consider identification of a specific form of the utility function (an earnings function) for a multiperiod maximization problem. As in HN, Taber allows for the sequential arrival of information. His analysis is based on conventional exclusion restrictions, but the analysis of HN is not. They use outcome data in conjunction with the discrete dynamic choice data to exploit cross-equation restrictions, whereas Taber does not.

The HN treatment of serially correlated unobservables is more general than any discussion that appears in the current dynamic discrete choice and dynamic treatment effect literature. They do not invoke the strong sequential conditional-independence assumptions used in the dynamic treatment effect literature in statistics [Gill and Robins (2001), Lechner and Miquel (2002), Lok (2007), Robins (1989, 1997)], nor the closely related conditional temporal independence of unobserved state variables given observed state variables invoked by Rust (1987), Hotz and Miller (1988, 1993), Manski (1993) and Magnac and Thesmar (2002) (in the first part of their paper) or the independence assumptions invoked by Wolpin (1984).[130] HN allow for more general time series dependence in the unobservables than is entertained by Pakes (1986), Keane and Wolpin (1997) or Eckstein and Wolpin (1999).[131]

[129] Pakes and Simpson (1989) sketch a proof of identification of a model of the option values of patents that is based on limit sets for an option model.

[130] Manski (1993) and Hotz and Miller (1993) use a synthetic cohort effect approach that assumes that young agents will follow the transitions of contemporaneous older agents in making their life cycle decisions. The synthetic cohort approach has been widely used in labor economics at least since Mincer (1974). Manski and Hotz and Miller exclude any temporally dependent unobservables from their models. See Ghez and Becker (1975), MaCurdy (1981) and Mincer (1974) for applications of the synthetic cohort approach. For empirical evidence against the assumption that the earnings of older workers are a reliable guide to the earnings of younger workers in models of earnings and schooling choices for recent cohorts of workers, see Heckman, Lochner and Todd (2006).

[131] Rust (1994) provides a clear statement of the stochastic assumptions underlying the dynamic discrete-choice literature up to the date of his survey.

Like Miller (1984) and Pakes (1986), HN explicitly model, identify and estimate agent learning that affects expected future returns.[132] Pakes and Miller assume functional forms for the distributions of the error process and for the serial correlation pattern about information updating and time series dependence. The HN analysis of the unobservables is nonparametric and they estimate, rather than impose, the stochastic structure of the information updating process.

Virtually all papers in the literature, including the HN analysis, invoke rational expectations. An exception is the analysis of Manski (1993) who replaces rational expectations with a synthetic cohort assumption that choices and outcomes of one group can be observed (and acted on) by a younger group. This assumption is more plausible in stationary environments and excludes any temporal dependence in unobservables. In recent work, Manski (2004) advocates use of elicited expectations as an alternative to the synthetic cohort approach.

While HN use rational expectations, they estimate, rather than impose the structure of agent information sets. Miller (1984), Pakes (1986), Keane and Wolpin (1997), and Eckstein and Wolpin (1999) assume that they know the law governing the evolution of agent information up to unknown parameters.[133] Following the procedure presented in Cunha and Heckman (2007b, 2008), Cunha, Heckman and Navarro (2005, 2006) and Navarro (2005), HN can test for which factors (θ) appear in agent information sets at different stages of the life cycle and they identify the distributions of the unobservables nonparametrically.

The HN analysis of dynamic treatment effects is comparable, in some aspects, to the recent continuous-time event-history approach of Abbring and Van den Berg (2003b) previously analyzed. Those authors build a continuous time model of counterfactuals for outcomes that are durations. They model treatment assignment times using a continuous-time duration model.

The HN analysis is in discrete time and builds on previous work by Heckman (1981a, 1981c) on heterogeneity and state dependence that identifies the causal effect of employment (or unemployment) on future employment (or unemployment).[134] They model time to treatment and associated vectors of outcome equations that may be discrete, continuous or mixed discrete-continuous. In a discrete-time setting, they are able to generate a variety of distributions of counterfactuals and economically motivated parameters. They allow for heterogeneity in responses to treatment that has a general time series structure.

As noted in Section 3.4.4, Abbring and Van den Berg (2003b) do not identify explicit agent information sets as HN do in their paper and as is done in Cunha, Heckman

[132] As previously noted, the previous literature assumes learning only about current costs.

[133] They specify *a priori* particular processes of information arrival as well as which components of the unobservables agents know and act on, and which components they do not.

[134] Heckman and Borjas (1980) investigate these issues in a continuous-time duration model. See also Heckman and MaCurdy (1980).

and Navarro (2005), and they do not model learning about future rewards. Their outcomes are restricted to be continuous-time durations. The HN framework is formulated in discrete time, which facilitates the specification of richer unobserved and observed covariate processes than those entertained in the continuous-time framework of Abbring and Van den Berg (2003b). It is straightforward to attach a vector of treatment outcomes in the HN model that includes continuous outcomes, discrete outcomes and durations expressed as binary strings.[135] At a practical level, the approach often can produce very fine-grained descriptions of continuous-time phenomena by using models with many finite periods. Clearly a synthesis of the event-history approach with the HN approach would be highly desirable. That would entail taking continuous-time limits of the discrete-time models. It is a task that awaits completion.

Flinn and Heckman (1982) utilize information on stopping times and associated wages to derive cross-equation restrictions to partially identify an equilibrium job search model for a stationary economic environment where agents have an infinite horizon. They establish that the model is nonparametrically nonidentified. Their analysis shows that use of outcome data in conjunction with data on stopping times is not sufficient to secure nonparametric identification of a dynamic discrete-choice model, even when the reward function is linear in outcomes unlike the reward functions in Rust (1987) and Magnac and Thesmar (2002). Parametric restrictions can break their nonidentification result. Abbring and Campbell (2005) exploit such restrictions, together with cross-equation restrictions on stopping times and noisy outcome measures, to prove identification of an infinite-horizon model of firm survival and growth with entrepreneurial learning. Alternatively, nonstationarity arising from finite horizons can break their nonidentification result [see Wolpin (1987)]. The HN analysis exploits the finite-horizon backward-induction structure of our model in conjunction with outcome data to secure identification and does not rely on arbitrary period by period exclusion restrictions. They substantially depart from the assumptions maintained in Rust's nonidentification theorem (1994). They achieve identification by using cross-equation restrictions, linearity of preferences and additional measurements, and exploiting the structure of their finite-horizon nonrecurrent model. Nonstationarity of regressors greatly facilitates identification by producing both exclusion and curvature restrictions which can substitute for standard exclusion restrictions.

3.5. Summary of the state of the art in analyzing dynamic treatment effects

This section has surveyed new methods for analyzing the dynamic effects of treatment. We have compared and contrasted the statistical dynamic treatment approach based on sequential conditional-independence assumptions that generalize matching to a dynamic panel setting to approaches developed in econometrics. We compared and

[135] Abbring (2008) considers nonparametric identification of mixed semi-Markov event-history models that extends his work with Van den Berg. See Section 3.3.

contrasted a continuous-time event-history approach developed by Abbring and Van den Berg (2003b) to discrete time reduced form and structural models developed by Heckman and Navarro (2007), and Cunha, Heckman and Navarro (2005).

4. Accounting for general equilibrium, social interactions, and spillover effects

The treatment-control paradigm motivates the modern treatment effect literature. Outcomes of persons who are "treated" are compared to outcomes of those who are not. The "untreated" are assumed to be completely unaffected by who else gets treatment. This assumption is embodied in invariance assumptions (PI-2) and (PI-4) in Chapter 70. In the "Rubin" model, (PI-2) is one component of his "SUTVA" assumption.[136]

In any social setting, this assumption is very strong, and many economists have built models to account for various versions of social interactions and their consequences for policy evaluation. The literature on general equilibrium policy analysis is vast and the details of particular approaches are difficult to synthesize in a concise way. In this section, we make a few general points and offer some examples where accounting for general equilibrium effects has substantial consequences for the evaluation of public policy. Note that there are also cases where accounting for general equilibrium has little effect on policy evaluations. One cannot say that a full-fledged empirical general equilibrium analysis is an essential component of every evaluation. However, ignoring general equilibrium and social interactions can be perilous.

It is fruitful to distinguish interactions of agents through market mechanisms, captured by the literature on general equilibrium analysis, from social interactions. Social interactions are a type of direct externality in which the actions of one agent directly affect the actions (preferences, constraints, technology) of other agents.[137] The former type of interaction is captured by general equilibrium models. The second type of interaction is captured in the recent social interactions literature. Within the class of equilibrium models where agents interact through markets, there is a full spectrum of possible interactions from partial equilibrium models where agent interactions in some markets are modeled, to full fledged general equilibrium models where all interactions are modeled.

The social interactions literature is explicitly microeconomic in character, since it focuses on the effects of individuals (or groups) on other individuals. The traditional general equilibrium literature is macroeconomic in its focus and deals with aggregates. A more recent version moves beyond the representative consumer paradigm and considers heterogeneity and the impact of policy on individuals. We first turn to versions of the empirical general equilibrium literature.

[136] Recall the discussion in Chapter 70, Section 4.4.

[137] This distinction is captured in neoclassical general equilibrium models by the contrast between pecuniary and nonpecuniary externalities.

4.1. General equilibrium policy evaluation

There is a large literature on empirical general equilibrium models applied to trade, public finance, finance, macroeconomics, energy policy, industrial organization, and labor economics. The essays in Kehoe, Srinivasan and Whalley (2005) present a rich collection of empirical general equilibrium models and references to a large body of related work. Much of the traditional general equilibrium analysis analyzes representative models using aggregate data.

Lewis (1963) is an early study of the partial equilibrium spillover effects of unionism on the wages of nonunion workers.[138] Leading examples of empirical general equilibrium studies based on the representative consumer paradigm are Auerbach and Kotlikoff (1987), Hansen and Sargent (1980), Huggett (1993), Jorgenson and Slesnick (1997), Jorgenson and Yun (1990), Kehoe, Srinivasan and Whalley (2005), Krusell and Smith (1998), Kydland and Prescott (1982), Shoven and Whalley (1977). There are many other important studies and this list is intended to be illustrative, and not exhaustive. Jorgenson and Slesnick (1997) give precise conditions for aggregation of microdata into macro aggregates that can be used to identify clearly defined economic parameters and policy criteria.

These models provide specific frameworks for analyzing policy interventions. Their specificity is a source of controversy because so many components of the social system need to be accounted for, and so often there is little professional consensus on these components and their empirical importance. Being explicit has its virtues and stimulates research promoting improved understanding of mechanisms and parameters. However, rhetorically, this clarity can be counterproductive. By sweeping implicit assumptions under the rug, the treatment effect literature appears to some to offer a universality and generality that is absent from the general equilibrium approach, in which mechanisms of causation and agent interaction are more clearly specified.

There is a large and often controversial literature about the sources of parameter estimates for the representative agent models. The "calibration vs. estimation debate" concerns the best way to secure parameters for these models [see Kydland and Prescott (1996), Hansen and Heckman (1996), and Sims (1996)]. Dawkins, Srinivasan and Whalley (2001) present a useful guide to this literature. Browning, Hansen and Heckman (1999) discuss the sources of the estimates for a variety of prototypical general equilibrium frameworks. In this section, we discuss the smaller body of literature that links general equilibrium models to microdata to evaluate public policy.

4.2. General equilibrium approaches based on microdata

A recent example of general equilibrium analysis applied to policy problems is the study of Heckman, Lochner and Taber (1998a, 1998b, 1998c), who consider the evaluation of

[138] He does not consider the effect of unionism on product prices or other factor markets besides the labor market.

tuition subsidy programs in a general equilibrium model of human capital accumulation with both schooling and on the job training, and with heterogeneous skills in which prices are flexible. We first discuss their model and then turn to other frameworks. Their model is an overlapping generations empirical general equilibrium model with heterogeneous agents across and within generations which generalizes the analysis of Auerbach and Kotlikoff (1987) by introducing human capital and by synthesizing micro- and macrodata analysis.

The standard microeconomic evaluation of tuition policy on schooling choices estimates the response of college enrollment to tuition variation using geographically dispersed cross-sections of individuals facing different tuition rates. These estimates are then used to determine how subsidies to tuition will raise college enrollment. The impact of tuition policies on earnings are evaluated using a schooling–earnings relationship fit on pre-intervention data and do not account for the enrollment effects of the taxes raised to finance the tuition subsidy. Kane (1994), Dynarski (2000), and Cameron and Heckman (1998, 2001) exemplify this approach. This approach is neither partial equilibrium or general equilibrium in character. It entirely ignores market interactions.

The danger in this widely used practice is that what is true for policies affecting a small number of individuals, as studied by social experiments or as studied in the microeconomic "treatment effect" literature, may not be true for policies that affect the economy at large. A national tuition-reduction policy may stimulate substantial college enrollment and will also likely reduce skill prices. However, agents who account for these changes will not enroll in school at the levels calculated from conventional procedures which ignore the impact of the induced enrollment on skill prices. As a result, standard policy evaluation practices are likely to be misleading about the effects of tuition policy on schooling attainment and wage inequality. The empirical question is to determine the extent to which this is true. Heckman, Lochner and Taber (1998a, 1998b, 1998c) show that conventional practices in the educational evaluation literature lead to estimates of enrollment responses that are ten times larger than the long-run general equilibrium effects, which account for the effect of policy on all factor markets. They improve on current practice in the "treatment effects" literature by considering both the gross benefits of the program and the tax costs of financing the policy as borne by different groups.

Evaluating the general equilibrium effects of a national tuition policy requires more information than the tuition-enrollment parameter that is the centerpiece of the micro policy analyses, which ignore any equilibrium effects. Policy proposals of all sorts typically extrapolate well outside the range of known experience and ignore the effects of induced changes in skill quantities on skill prices. To improve on current practice, Heckman, Lochner and Taber (1998a) use microdata to identify an empirically estimated rational expectations, perfect foresight overlapping-generations general equilibrium framework for the pricing of heterogeneous skills and the adjustment of capital. It is based on an empirically grounded theory of the supply of schooling and post-school human capital, where different schooling levels represent different skills. Individuals differ in their learning ability and in initial endowments of human capital.

Household saving behavior generates the aggregate capital stock, and output is produced by combining the stocks of different types of human capital with physical capital. Factor markets are competitive, and it is assumed that wages are set in flexible, competitive markets. Their model explains the pattern of rising wage inequality experienced in the United States in the past 30 years. They apply their framework to evaluate tuition policies that attempt to increase college enrollment.

They present two reasons why the "treatment effect" framework that ignores the general equilibrium effects of tuition policy is inadequate. First, the conventional treatment parameters depend on who in the economy is "treated" and who is not. Second, these parameters do not measure the full impact of the program. For example, increasing tuition subsidies may increase the earnings of uneducated individuals who do not take advantage of the subsidy. They become more scarce after the policy is implemented. The highly educated are taxed to pay for the subsidy, and depending on how taxes are collected this may affect their investment behavior. In addition, more competitors for educated workers enter the market as a result of the policy, and their earnings are depressed. Conventional methods ignore the effect of the policy on nonparticipants operating through changes in equilibrium skill prices and on taxes. In order to account for these effects, it is necessary to conduct a general equilibrium analysis.

The analysis of Heckman, Lochner and Taber (1998a, 1998b, 1998c) has important implications for the widely-used difference-in-differences estimator. If the tuition subsidy changes the aggregate skill prices, the decisions of nonparticipants will be affected. The "no treatment" benchmark group is affected by the policy and the difference-in-differences estimator does not identify the effect of the policy for anyone compared to a no treatment state.

Using their estimated model, Heckman, Lochner and Taber (1998c) simulate the effects of a revenue-neutral \$500 increase in college tuition subsidy on top of existing programs that is financed by a proportional tax, on enrollment in college and wage inequality. They start from a baseline economy that describes the US in the mid-1980s and that produces wage growth profiles and schooling enrollment and capital stock data that match micro- and macroevidence. The microeconomic treatment effect literature predicts an increase in college attendance of 5.3 percent. This analysis holds college and high school wage rates fixed. This is the standard approach in the microeconomic "treatment effect" literature.

When the policy is evaluated in a general equilibrium setting, the estimated effect falls to 0.46 percent. Because the college–high school wage ratio falls as more individuals attend college, the returns to college are less than when the wage ratio is held fixed. Rational agents understand this effect of the tuition policy on skill prices and adjust their college-going behavior accordingly. Policy analysis of the type offered in the "treatment effect" literature ignores equilibrium price adjustment and the responses of rational agents to the policies being evaluated. Their analysis shows substantial attenuation of the effects of tuition policy on capital and on the stocks of the different skills in their model compared to a treatment effect model. They show that their results are robust to a variety of specifications of the economic model.

Table 14
Simulated effects of $5000 tuition subsidy on different groups. Steady state changes in present value of lifetime wealth (in thousands of US dollars)

Group (proportion)[1]	After-tax earnings using base tax (1)	After-tax earnings (2)	After-tax earnings net of tuition (3)	Utility[2] (4)
High School–High School (0.528)	9.512	−0.024	−0.024	−0.024
High School–College (0.025)	−4.231	−13.446	1.529	1.411
College–High School (0.003)	−46.711	57.139	−53.019	−0.879
College–College (0.444)	−7.654	−18.204	0.420	0.420

[1]The groups correspond to each possible counterfactual. For example, the "High School–High School" group consists of individuals who would not attend college in either steady state, and the "High School–College" group would not attend college in the first steady state, but would in the second, etc.
[2]Column (1) reports the after-tax present value of earnings in thousands of 1995 US dollars discounted using the after-tax interest rate where the tax rate used for the second steady state is the base tax rate. Column (2) adds the effect of taxes, column (3) adds the effect of tuition subsidies and column (4) includes the nonpecuniary costs of college in dollar terms.
Source: Heckman, Lochner and Taber (1998b).

They also analyze short run effects. When they simulate the model with rational expectations, the short-run college enrollment effects in response to the tuition policy are also very small, as agents anticipate the effects of the policy on skill prices and calculate that there is little gain from attending college at higher rates. Under myopic expectations, the short-run enrollment effects are much closer to the estimated treatment effects. With learning on the part of agents, but not perfect foresight, there is still a substantial gap between treatment and general equilibrium estimates. The sensitivity of policy estimates to model specification is a source of concern and a stimulus to research. The treatment effect literature ignores these issues.

Heckman, Lochner and Taber (1998a, 1998b, 1998c) also consider the impact of a policy change on discounted earnings and utility and decompose the total effects into benefits and costs, including tax costs for each group. Table 14 compares outcomes in two steady states: (a) the benchmark steady state and (b) the steady state associated with the new tuition policy.[139] The row "High School–High School" reports the change in a variety of outcome measures for those persons who would be in high school under either the benchmark or new policy regime; the "High School–College" row reports the change in the same measures for high school students in the benchmark state who are induced to attend college by the new policy; the "College–High School" outcomes refer

[139] Given that the estimated schooling response to a $500 subsidy is small, Heckman, Lochner and Taber instead use a $5000 subsidy for the purpose of exploring general equilibrium effects on earnings. Current college tuition subsidy levels are this high or higher at many colleges in the US.

to those persons in college in the benchmark economy who only attend high school after the policy; and so forth. Because agents choose sectors, there is spillover from one sector to another.

By the measure of the present value of earnings, some of those induced to change are worse off. Contrary to the monotonicity assumption built into the LATE parameter discussed in Chapters 70 and 71, and defined in this context as the effect of the tuition subsidy on the earnings of those induced by it to go to college, Heckman, Lochner and Taber find that the tuition policy produces a two-way flow. Some people who would have attended college in the benchmark regime no longer do so. The rest of society is also affected by the policy—again, contrary to the implicit assumption built into LATE that only those who change status are affected by the policy. People who would have gone to college without the policy and continue to do so after the policy are financially worse off for two reasons: (a) the price of their skill is depressed and (b) they must pay higher taxes to finance the policy. However, they now receive a tuition subsidy and for this reason, on net, they are better off both financially and in terms of utility. Those who abstain from attending college in both steady states are worse off. They pay higher taxes, and do not get the benefits of a college education. Those induced to attend college by the policy are better off in terms of utility. Note that neither category of non-changers is a natural benchmark for a difference-in-differences estimator. The movement in their wages before and after the policy is due to the policy and cannot be attributed to a benchmark "trend" that is independent of the policy.

Table 15 presents the impact of a \$5000 tuition policy on the log earnings of individuals with ten years of work experience for different definitions of treatment effects. The treatment effect version given in the first column holds skill prices constant at initial steady state values. The general equilibrium version given in the second column allows prices to adjust when college enrollment varies. Consider four parameters initially defined in a partial equilibrium context. The *average treatment effect* is defined for a randomly selected person in the population in the benchmark economy and asks how that person would gain in wages by moving from high school to college. The parameter *treatment on the treated* is defined as the average gain over their non-college alternative of those who attend college in the benchmark state. The parameter *treatment on the untreated* is defined as the average gain over their college wage received by individuals who did not attend college in the benchmark state. The *marginal treatment effect* is defined for individuals who are indifferent between going to college or not. This parameter is a limit version of the LATE parameter under the assumptions presented in Chapter 71. Column (2) presents the general equilibrium version of *treatment on the treated*. It compares the earnings of college graduates in the benchmark economy with what they would earn if no one went to college.[140] The treatment on the

[140] In the empirical general equilibrium model of Heckman, Lochner and Taber (1998a, 1998b, 1998c), the Inada conditions for college and high school are not satisfied for the aggregate production function and the marginal product of each skill group when none of it is utilized is a bounded number. If the Inada conditions were satisfied, this counterfactual and the counterfactual treatment on the untreated would not be defined.

Table 15
Treatment effect parameters: treatment effect and general equilibrium difference in log earnings, college graduates vs. high school graduates at 10 years of work experience

Parameter	Prices fixed[1] (1)	Prices vary[2] (2)	Fraction of sample[3] (3)
Average treatment effect (ATE)	0.281	1.801	100%
Treatment on treated (TT)	0.294	3.364	44.7%
Treatment on untreated (TUT)	0.270	−1.225	55.3%
Marginal treatment effect (MTE)	0.259	0.259	–
LATE[4] \$5000 subsidy:			
Partial equilibrium	0.255	–	23.6%
GE (H.S. to College) (LATE)	0.253	0.227	2.48%
GE (College to H.S.) (LATER)	0.393	0.365	0.34%
GE net (TLATE)	–	0.244	2.82%
LATE[4] \$500 subsidy:			
Partial equilibrium	0.254	–	2.37%
GE (H.S. to College) (LATE)	0.250	0.247	0.24%
GE (College to H.S.) (LATER)	0.393	0.390	0.03%
GE net (TLATE)	–	0.264	0.27%

[1]In column (1), prices are held constant at their initial steady state levels when wage differences are calculated.
[2]In column (2), we allow prices to adjust in response to the change in schooling proportions when calculating wage differences.
[3]For each row, column (3) presents the fraction of the sample over which the parameter is defined.
[4]The LATE group gives the effect on earnings for persons who would be induced to attend college by a tuition change. In the case of GE, LATE measures the effect on individuals induced to attend college when skill prices adjust in response to quantity movements among skill groups. The treatment effect LATE measures the effect of the policy on those induced to attend college when skill prices are held at the benchmark level.
Source: Heckman, Lochner and Taber (1998b).

untreated parameter is defined analogously by comparing what high school graduates in the benchmark economy would earn if everyone in the population were forced to go to college. The *average treatment effect* compares the average earnings in a world in which everyone attends college versus the earnings in a world in which nobody attends college. Such dramatic policy shifts produce large estimated effects. In contrast, the general equilibrium marginal treatment effect parameter considers the gain to attending college for people on the margin of indifference between attending college and only attending high school. In this case, as long as the mass of people in the indifference set is negligible, the standard treatment effect and general equilibrium parameters are the same.

The final set of parameters considered by Heckman, Lochner and Taber (1998b) are versions of the LATE parameter. This parameter depends on the particular intervention being studied and its magnitude. The standard version of LATE is defined on the outcomes of individuals induced to attend college, assuming that skill prices do not change.

The general equilibrium version is defined for the individuals induced to attend college when prices adjust in response to the policy. In this general equilibrium model, the two LATE parameters are quite close to each other and are also close to the marginal treatment effect.[141] General equilibrium effects change the group over which the parameter is defined compared to the standard treatment effect case. For a $5000 subsidy, there are substantial price effects and the standard treatment effect parameter differs substantially from the general equilibrium version.

Heckman, Lochner and Taber (1998a, 1998b, 1998c) also present standard treatment effect and general equilibrium estimates for two extensions of the LATE concept: LATER (the effect of the policy on those induced to attend only high school rather than go to college)—Reverse LATE—and TLATE (the effect of the policy on all of those induced to change whichever direction they flow). LATER is larger than LATE, indicating that those induced to drop out of college have larger gains from dropping out than those induced to enter college have from entering. TLATE is a weighted average of LATE and LATER with weights given by the relative proportion of people who switch in each direction.

The general equilibrium impacts of tuition on college enrollment are an order of magnitude smaller than those reported in the literature on microeconometric treatment effects. The assumptions used to justify the LATE parameter in a microeconomic setting do not carry over to a general equilibrium framework. Policy changes, in general, induce two-way flows and violate the monotonicity—or one-way flow—assumption of LATE. Heckman, Lochner and Taber (1998b) extend the LATE concept to allow for the two-way flows induced by the policies. They present a more comprehensive approach to program evaluation by considering both the tax and benefit consequences of the program being evaluated and placing the analysis in a market setting. Their analysis demonstrates the possibilities of the general equilibrium approach and the limitations of the microeconomic "treatment effect" approach to policy evaluation.

4.2.1. Subsequent research

Subsequent research by Blundell et al. (2004), Duflo (2004), Lee (2005), and Lee and Wolpin (2006) estimate—or estimate and calibrate—general equilibrium models for the effects of policies on labor markets. Lee (2005) assumes that occupational groups are perfect substitutes and that people can costlessly switch between skill categories. These assumptions neutralize any general equilibrium effects. They are relaxed and shown to be inconsistent with data from US labor markets in Lee and Wolpin (2006).

Lee and Wolpin (2006) assume adaptive expectations rather than rational expectations. Heckman, Lochner and Taber (1998a) establish the sensitivity of the policy evaluations to specifications of expectations. Duflo (2004) demonstrates the importance

[141] The latter is a consequence of the discrete-choice framework for schooling choices analyzed in the Heckman, Lochner and Taber (1998b) model. See Chapter 71.

of general equilibrium effects on wages for the evaluation of a large scale schooling program in Indonesia. However, accounting for general equilibrium does not affect her estimates of the rate of return of schooling.

4.2.2. Equilibrium search approaches

Equilibrium search models are another framework for studying market level interactions among agents. Search theory as developed by Mortensen and Pissarides (1994) and Pissarides (2000) has begun to be tested on microdata [see Van den Berg (1999)]. It accounts for direct and indirect effects without imposing full equilibrium price adjustment. Some versions of search theory allow for wage flexibility through a bargaining mechanism while other approaches assume rigid wages. Search theory produces an explicit theory of unemployment. Davidson and Woodbury (1993) consider direct and indirect effects of a bonus scheme to encourage unemployed workers to find jobs more quickly using a Mortensen–Pissarides (1994) search model in which prices are fixed. Their model is one of displacement with fixed prices.

More recent studies of equilibrium search models in which wages are set through bargaining that have been used for policy analysis include papers by Lise, Seitz and Smith (2005a, 2005b) and Albrecht, Van den Berg and Vroman (2005). Lise, Seitz and Smith (2005a) present a careful synthesis of experimental and nonexperimental data combining estimation and calibration. They provide evidence on labor-market feedback effects associated with job subsidy schemes. In their analysis, accounting for general equilibrium feedback reverses the cost–benefit evaluations of a job subsidy program in Canada. Albrecht, Van den Berg and Vroman (2005) demonstrate important equilibrium effects of an adult education program on employment and job vacancies, showing a skill bias of the programs.

4.3. Analyses of displacement

Newly trained workers from a job training program may displace previously trained workers if wages are inflexible, as they are in many European countries. For some training programs in Europe, substantial displacement effects have been estimated [Organization for Economic Cooperation and Development (1993), Calmfors (1994)]. If wages are flexible, the arrival of new trained workers to the market tends to lower the wages of previously trained workers but does not displace any worker.

Even if the effect of treatment on the treated is positive, nonparticipants may be worse off as a result of the program compared to what they would have experienced in a no program state. Nonparticipants who are good substitutes for the new trainees are especially adversely affected. Complementary factors benefit. These spillover effects can have important consequences for the interpretation of traditional evaluation parameters. The benchmark “no treatment” state is affected by the program and invariance assumption (PI-2) presented in Chapter 70 is violated.

To demonstrate these possibilities in a dramatic way, consider the effect of a wage subsidy for employment in a labor market for low-skill workers. Assume that firms act to minimize their costs of employment. Wage subsidies operate by taking nonemployed persons and subsidizing their employment at firms. Firms who employ the workers receive the wage subsidy.

Many active labor-market policies have a substantial wage-subsidy component. Suppose that the reason for nonemployment of low-skill workers is that minimum wages are set too high. This is the traditional justification for wage subsidies.[142] If the number of subsidized workers is less than the number of workers employed at the minimum wage, a wage subsidy financed from lump sum taxes has no effect on total employment in the low wage sector because the price of labor for the marginal worker hired by firms is the minimum wage. It is the same before and after the subsidy program is put in place. Thus the marginal worker is unsubsidized both before and after the subsidy program is put in place.

The effects of the program are dramatic on the individuals who participate in it. Persons previously nonemployed become employed as firms seek workers who carry a wage subsidy. Many previously-employed workers become nonemployed as their employment is not subsidized. There are no effects of the wage subsidy program on GDP unless the taxes raised to finance the program have real effects on output. Yet there is substantial redistribution of employment. Focusing solely on the effects of the program on subsidized workers greatly overstates its beneficial impact on the economy at large.

In order to estimate the impact of the program on the overall economy, it is necessary to look at outcomes for both participants and nonparticipants. Only if the benefits accruing to previously-nonemployed participants are adopted as the appropriate evaluation criterion would the effect of treatment on the treated be a parameter of interest. Information on both participants and nonparticipants affected by the program is required to estimate the net gain in earnings and employment resulting from the program.

In the case of a wage subsidy, comparing the earnings and employment of subsidized participants during their subsidized period to their earnings and employment in the presubsidized period can be a very misleading estimator of the total impact of the program. So is a cross-section comparison of participants and nonparticipants. In the example of a subsidy in the presence of a minimum wage, the before–after estimate of the gain exceeds the cross-section estimate unless the subsidy is extended to a group of nonemployed workers as large as the number employed at the minimum wage. For subsidy coverage levels below this amount, some proportion of the unsubsidized employment is paid the minimum wage. Under these circumstances, commonly-used evaluation estimators produce seriously misleading estimates of program impacts.

The following example clarifies and extends these points to examine the effect of displacement on conventional estimators. Let N be the number of participants in the low-wage labor market. Let N_E be the number of persons employed at the minimum

[142] See, e.g., Johnson (1979), or Johnson and Layard (1986).

wage M and let N_S be the number of persons subsidized. Subsidization operates solely on persons who would otherwise have been nonemployed and had no earnings. Assume $N_E > N_S$. Therefore, the subsidy has no effect on total employment in the market, because the marginal cost of labor to a firm is still the minimum wage. Workers with the subsidy are worth more to the firm by the amount of the subsidy S. Firms would be willing to pay up to $S + M$ per subsidized worker to attract them.

The estimated wage gain using a before–after comparison for subsidized participants is:

$$\textit{Before–After}: \quad \underbrace{(S + M)}_{\text{after}} - \underbrace{(0)}_{\text{before}} = S + M,$$

because all subsidized persons earn a zero wage prior to the subsidy. For them, the program is an unmixed blessing. The estimated wage gain using cross-section comparisons of participants and nonparticipants is:

$$\begin{aligned} \textit{Cross-Section}: \quad & \underbrace{S + M}_{\text{participant's wage}} - \underbrace{M}_{\substack{\text{nonparticipant's}\\ \text{wage}}} \times \left(\frac{N_E - N_S}{N - N_S}\right) \\ &= S + M \underbrace{\left(\frac{N - N_E}{N - N_S}\right)}_{(<1)} < S + M. \end{aligned}$$

Since $N_E > N_S$, the before–after estimator is larger than the cross-section estimator. The widely used difference-in-differences estimator compares the before–after outcome measure for participants to the before–after outcome measure for nonparticipants:

$$\begin{aligned} \textit{Difference-in-Differences}: \quad & (S + M - 0) - M\left(\frac{N_E - N_S}{N - N_S} - \frac{N_E}{N - N_S}\right) \\ &= S + M\left(\frac{N}{N - N_S}\right) > S + M. \end{aligned}$$

The gain estimated from the difference-in-differences estimator exceeds the gain estimated from the before–after estimator which in turn exceeds the gain estimated from the cross-section estimator. The "no treatment" benchmark in the difference-in-differences model is contaminated by treatment. The estimate of employment creation obtained from the three estimators is obtained by setting $M = 1$ and $S = 0$ in the previous expressions. This converts those expressions into estimates of employment gains for the different groups used in their definition.

None of these estimators produces a correct assessment of wage or employment gain for the economy at large. Focusing only on direct participants causes analysts to lose sight of overall program impacts. Only an aggregate analysis of the economy as a whole, or random samples of the entire economy, would produce the correct assessment that no wage increase or job creation is produced by the program. The problem of indirect effects poses a major challenge to conventional micro methods used in evaluation research that focus on direct impacts instead of total impacts, and demonstrates the need for program evaluations to utilize market-wide data and general equilibrium methods.

Calmfors (1994) presents a comprehensive review of the issues that arise in evaluating active labor-market programs and an exhaustive list of references on theoretical and empirical work on this topic. He distinguishes a number of indirect effects including *displacement effects* (jobs created by one program at the expense of other jobs), *deadweight effects* (subsidizing hiring that would have occurred in the absence of the program), *substitution effects* (jobs created for a certain category of workers replace jobs for other categories because relative wage costs have changed) and *tax effects* (the effects of taxation required to finance the programs on the behavior of everyone in society). A central conclusion of this literature is that the estimates of program impact from the microeconomic treatment effect literature provide incomplete information about the full impacts of active labor-market programs. The effect of a program on participants may be a poor approximation to the total effect of the program, as our simple example has shown. Blundell et al. (2004) present evidence on substitution and displacement for an English active labor-market program.

4.4. Social interactions

There is a growing empirical literature on social interactions. Brock and Durlauf (2001) and Durlauf and Fafchamps (2005) present comprehensive surveys of the methods and evidence from this emerging field. Instead of being market mediated, as in search and general equilibrium models, the social interactions considered in this literature are at the individual or group level which can include family interactions through transfers. Linkages through family and other social interactions undermine the sharp treatment-control separation assumed in the microeconomic treatment effect literature.

A recent paper by Angelucci and De Giorgi (2006) illustrates this possibility. They analyze the effect of the Progressa program in Mexico on both treated and untreated families. Progressa paid families to send their children to school. They present evidence that noneligible families received transfers from the eligible families and altered their saving and consumption behavior. Thus, through the transfer mechanism, the "untreated" receive treatment. However, they show no general equilibrium effects of the program on the product and labor markets that they study.

4.5. Summary of general equilibrium approaches

Many policies affect both "treatment" groups and indirectly affect "control" groups through market and social interactions. Reliance on microeconomic treatment effect approaches to evaluate such policies can produce potentially misleading estimates. The analysis of Heckman, Lochner and Taber (1998a) and the later work by Albrecht, Van den Berg and Vroman (2005), Blundell et al. (2004), Duflo (2004), Angelucci and De

Giorgi (2006), Lee (2005), Lise, Seitz and Smith (2005a, 2005b), and Lee and Wolpin (2006) indicate that ignoring indirect effects can produce misleading policy evaluations.

The cost of this enhanced knowledge is the difficulty in assembling all of the behavioral parameters required to conduct a general equilibrium evaluation. From a long run standpoint, these costs are worth incurring. Once a solid knowledge base is put in place, a more trustworthy framework for policy evaluation will be available, one that will offer an economically-justified framework for accumulating evidence across studies and will motivate empirical research by microeconomists to provide better empirical foundations for general equilibrium policy analyses.

5. Summary

This chapter extends the traditional static *ex post* literature on mean treatment effects to consider the identification of distributions of treatment effects, the identification of *ex ante* and *ex post* distributions of treatment effects, the measurement of uncertainty facing agents and the analysis of subjective valuations of programs. We also survey methods for identifying dynamic treatment effects with information updating by agents, using both explicitly formulated economic models and less explicit approaches. We discuss general equilibrium policy evaluation and evaluation of models with social interactions.

Appendix A: Deconvolution

To see how to use (CON-1) and (M-1) to identify $F(y_0, y_1 \mid X)$, note that

$$Y = Y_0 + D\Delta.$$

From $F_Y(y \mid X, D = 0)$, we identify $F_0(y_0 \mid X)$ as a consequence of matching assumption (M-1). From $F_Y(y \mid X, D = 1)$ we identify $F_1(y_1 \mid X) = F_{Y_0+\Delta}(y_0 + \Delta \mid X)$. If Y_0 and Y_1 have densities, then, as a consequence of (CON-1) and (M-1), the densities satisfy

$$f_1(y_1 \mid X) = f_\Delta(\Delta \mid X) * f_0(y_0 \mid X)$$

where "$*$" denotes convolution. The characteristic functions of Y_0, Y_1 and Δ are related in the following way:

$$E\left(e^{i\ell Y_1} \mid X\right) = E\left(e^{i\ell\Delta} \mid X\right)E\left(e^{i\ell Y_0} \mid X\right).$$

Since we can identify $F_1(y_1 \mid X)$, we know its characteristic function. By a similar argument, we can recover $E(e^{i\ell Y_0} \mid X)$. Thus, from

$$E\left(e^{i\ell\Delta} \mid X\right) = \frac{E(e^{i\ell Y_1} \mid X)}{E(e^{i\ell Y_0} \mid X)},$$

and by the inversion theorem,[143] we can recover the density $f_\Delta(\Delta \mid X)$. We know the joint density

$$f_{\Delta,Y_0}(\Delta, y_0 \mid X) = f_\Delta(\Delta \mid X) f_0(y_0 \mid X).$$

From the definition of Δ, we obtain

$$f_\Delta(y_1 - y_0 \mid X) f_0(y_0 \mid X) = f(y_1, y_0 \mid X).$$

Thus we can recover the full joint distribution of outcomes and the distribution of gains.

Under assumption (M-1), assumption (CON-1) is testable. The ratio of two characteristic functions is not necessarily a characteristic function. If it is not, the estimated density f_Δ recovered from the ratio of the characteristic functions need not be positive and the estimated variance of Δ can be negative.[144]

Appendix B: Matzkin conditions and proof of Theorem 2

We prove Theorem 2. We first present a review of the conditions Matzkin (1992) imposes for identification of nonparametric discrete choice models which are used in this proof.

B.1. The Matzkin conditions

Consider a binary choice model, $D = \mathbf{1}[\varphi(Z) > V]$, where Z is observed and V is unobserved. Let φ^* denote the true φ and let F_V^* denote the true cdf of V. Let $\mathcal{Z} \subseteq \mathbb{R}^K$ denote the support of Z. Let $\mathcal{H}$ denote the set of monotone increasing functions from $\mathbb{R}$ into $[0, 1]$. Assume:

(i) $\varphi \in \Phi$, where Φ is a set of real valued, continuous functions defined over $\mathcal{Z}$, which is also assumed to be the domain of definition of φ, and the true function is $\varphi^* \in \Phi$. There exists a subset $\widetilde{\mathcal{Z}} \subseteq \mathcal{Z}$ such that (a) for all $\varphi, \varphi' \in \Phi$, and all $z \in \widetilde{\mathcal{Z}}$, $\varphi(z) = \varphi'(z)$, and (b) for all $\varphi \in \Phi$ and all t in the range space of $\varphi^*(z)$ for $z \in \mathcal{Z}$, there exists a $\tilde{z} \in \widetilde{\mathcal{Z}}$ such that $\varphi(\tilde{z}) = t$. In addition, φ^* is strictly increasing in the Kth coordinate of Z.

(ii) $Z \perp\!\!\!\perp V$.

(iii) The Kth component of Z possesses a Lebesgue density conditional on the other components of Z.

[143] See, e.g., Kendall and Stuart (1977).

[144] For the ratio of characteristic functions, $r(\ell)$, to be a characteristic function, it must satisfy the requirement that $r(0) = 1$, that $r(\ell)$ is continuous in ℓ and $r(\ell)$ is nonnegative definite. This identifying assumption can be tested using the procedures developed in Heckman, Robb and Walker (1990).

(iv) F_V^* is strictly increasing on the support of $\varphi^*(Z)$. Matzkin (1992) notes that if one assumes that V is absolutely continuous, and the other conditions hold, one can relax the condition that φ^* is strictly increasing in one coordinate (listed in (i)) and the requirement in (iii).

Then (φ^*, F_V^*) is identified within $\Phi \times \mathcal{H}$, where F_V^* is identified on the support of $\varphi^*(Z)$.

Matzkin establishes identifiability for the following alternative representations of functional forms that satisfy condition (i) for exact identification for $\varphi(Z)$.

1. $\varphi(Z) = Z\gamma$, $\|\gamma\| = 1$ or $\gamma_1 = 1$.
2. $\varphi(z)$ is homogeneous of degree one and attains a given value α at $z = z^*$ (e.g., cost functions).
3. The $\varphi(Z)$ are least concave functions that attain common values at two points in their domain.
4. The $\varphi(Z)$ are additively separable functions:
   (a) Functions additively separable into a continuous monotone increasing function and a continuous monotone increasing function which is concave and homogeneous of degree one;
   (b) Functions additively separable into the value of one variable and a continuous, monotone increasing function of the remaining variables;
   (c) A set of functions additively separable in each argument [see Matzkin (1992, Example 5, p. 255)].

We now prove Theorem 2.

B.2. *Proof of Theorem 2*

PROOF. Proof of the identifiability of the joint distribution of V^s and $\mu_R^s(Z)$ follows from Matzkin (1993), Theorem 2. See also the proof presented in Chapter 70 (Appendix B) of this Handbook. We condition on the event $D(s) = 1$. From the data on $Y_c(s, X)$, $Y_d(s, X)$, $M_c(X)$, $M_d(X)$ for $D(s) = 1$, and the treatment selection probabilities, we can construct the left-hand side of the following equation:

$$\Pr\left(\begin{array}{c} Y_c(s,X) \leqslant y_c, \mu_d(s,X) \leqslant U_d(s), \\ M_c(X) \leqslant m_c, \mu_{d,M}(X) \leqslant U_{d,M} \end{array} \;\middle|\; D(s) = 1, X = x, Z = z \right)$$
$$\times \Pr\big(D(s) = 1 \mid X = x, Z = z\big)$$
$$= \int_{\underline{U}_c(s)}^{y_c - \mu_c(s,x)} \int_{\mu_d(s,x)}^{\overline{U}_d(s)} \int_{\underline{U}_{c,M}}^{m_c - \mu_{c,M}(x)} \int_{\mu_{d,M}(x)}^{\overline{U}_{d,M}} \int_{\underline{V}^s(1)}^{\mu_R(s,z) - \mu_R(1,z)} \cdots$$
$$\int_{\underline{V}^s(\bar{S})}^{\mu_R(s,z) - \mu_R(\bar{S},z)} f_{U_c(s), U_d(s), U_{c,M}, U_{d,M}, V^s}\big(u_c(s), u_d(s), u_{c,M}, u_{d,M},$$
$$v(s) - v(1), v(s) - v(\bar{S})\big)$$

$$\cdot\, d\big(v(s) - v(\bar{S})\big) \cdots d\big(v(s) - v(1)\big)\, du_{d,M}\, du_{c,M}\, du_d(s)\, du_c(s). \tag{B.1}$$

Parallel expressions can be derived for the other possible values of $M_d(X)$ and $Y_d(s, X)$. We obtain the selection-bias free distribution of $Y_c(s, X), Y_d(s, X), M_c(X), M_d(X)$ given X, $\Pr(Y_c(s, X) \leqslant y_c, Y_d(s, X) = y_d, M_c(X) \leqslant m_c, M_d(X) = m_d \mid X)$, from $\Pr(Y_c(s, X) \leqslant y_c, Y_d(s, X) = y_d, M_c(X) \leqslant m_c, D(s) = 1 \mid X, Z = z)$ for $z \to \mathcal{Z}_s$, a limit set, possibly dependent on X, such that $\lim_{z \to \mathcal{Z}_s} \Pr(D(s) = 1 \mid X, Z = z) = 1$. This produces the $\mu_c(s, X)$, $\mu_{c,M}(X)$ directly and the $\mu_d(s, X)$, $\mu_{d,M}(X)$ using the analysis of Matzkin (1992, 1993, 1994) for the class of Matzkin functions defined in Appendix B.1. Varying the $y_c - \mu_c(s, X)$, $\mu_d(s, X)$, $m_c - \mu_{c,M}(X)$, $\mu_{d,M}(X)$, $\mu_R^s(Z)$, under the conditions of the theorem we can trace out the joint distribution of $(U_c(s), U_d(s), U_{c,M}, U_{d,M}, V^s)$ for each $s = 1, \ldots, \bar{S}$. $\square$

As a consequence of (ii), we can identify $\mu_c(s, X)$, $\mu_{c,M}(X)$ directly from the means of the limit outcome distributions. We can thus identify all pairwise average treatment effects

$$E\big(Y_c(s, X) \mid X = x\big) - E\big(Y_c(s', X) \mid X = x\big)$$

for all s, s' and any other linear functionals derived from the distributions of the continuous variables defined at s and s'. Identification of the means and distributions of the latent variables giving rise to the discrete outcomes is more subtle, but standard [see Carneiro, Hansen and Heckman (2003)]. With one continuous regressor among the X, one can identify the marginal distributions of the $U_d(s)$ and the $U_{d,M}$. To identify the joint distributions of $U_d(s)$ and $U_{d,M}$ one must use condition (iv) component by component.

Thus for system s, suppose that there are $N_{d,s}$ discrete outcome components with associated means $\mu_{d,j}(s, X)$ and error terms $U_{d,j}(s)$, $j = 1, \ldots, N_{d,s}$. As a consequence of condition (iv) of this theorem, $\mathrm{Supp}(\mu_d(s, X)) \supseteq \mathrm{Supp}(U_d(s))$. We thus can trace out the joint distribution of $U_d(s)$ and identify it (up to scale if we specify the Matzkin class only up to scale). By a parallel argument for the measurements, we can identify the joint distribution of $U_{d,M}$. Let $N_{d,M}$ be the number of discrete measurements. From condition (iv), we obtain $\mathrm{Supp}(\mu_{d,M}(X)) \supseteq \mathrm{Supp}(U_{d,M})$. Under these conditions, we can trace out the joint distribution of $U_{d,M}$ and identify it (up to scale for Matzkin class of functions specified up to scale) within the limit sets. In the general case, we can vary each limit of the integral in (B.1) and similar integrals for the other possible values of the discrete measurements and outcomes independently and trace out the full joint distribution of $(U_c(s), U_d(s), U_{c,M}, U_{d,M}, V^s)$. For further discussion, see the analysis in Carneiro, Hansen and Heckman (2003, Theorem 3).

Appendix C: Proof of Theorem 4

PROOF. From Theorem 3, we obtain identifiability of $\Psi^s(Z)$ and the joint distribution of η^s. From the data on $Y(s, X)$, for $D(s) = 1$, and from the time to treatment probabilities, we can construct the left-hand side of the following equation:

$$\begin{aligned}
&\Pr\bigl(Y(s, X) \leqslant y \bigm| D(s) = 1, X = x, Z = z\bigr)\\
&\quad \times \Pr\bigl(D(s) = 1 \mid X = x, Z = z\bigr)\\
&= \int_{\underline{U}(s)}^{y-\mu(s,x)} \int_{\underline{\eta}(s)}^{\Psi(s,z(s))} \int_{\Psi(s-1,z(s-1))}^{\bar{\eta}(s-1)} \cdots\\
&\quad \int_{\Psi(1,z(1))}^{\bar{\eta}(1)} f_{U(s),\eta^s}\bigl(u(s), \eta(1), \dots, \eta(s)\bigr)\, d\eta(1) \cdots d\eta(s)\, du(s).
\end{aligned} \tag{C.1}$$

Under assumption (iv), for all $x \in \operatorname{Supp}(X)$, we can vary the $\Psi(j, Z(j))$, $j = 1, \dots, s$, and obtain a limit set $\mathcal{Z}_s$, possibly dependent on X, such that $\lim_{z \to \mathcal{Z}_s} \Pr(D(s) = 1 \mid X = x, Z = z) = 1$. We can identify the joint distribution of $Y(s, X)$, free of selection bias in this limit set for all $s = 1, \dots, \bar{S}$. We know the limit sets given the functional forms in Matzkin (1992, 1993, 1994) with the leading case being $\Psi(s, Z(s)) = Z(s)\gamma_s$. From the analysis of Theorem 3, we achieve identifiability on nonnegligible sets.

As a consequence of (ii), we can identify $\mu(s, X)$ directly from the means of the limit outcome distributions. We can thus identify all pairwise average treatment effects $E(Y(s, X) \mid X = x) - E(Y(s', X) \mid X = x)$ for all s, s' and any other linear functionals derived from the distributions of the continuous variables defined at s and s'.

In the general case, we can vary each limit of the integral in (C.1) independently and trace out the full joint distribution of $(U(s), \eta(1), \dots, \eta(s))$. For further discussion, see the analysis in Carneiro, Hansen and Heckman (2003, Theorem 3). Note the close parallel to the proof of Theorem 2. The key difference between the two proofs is the choice equation. In Theorem 2, the choice of treatment equation is a conventional multivariate discrete-choice model. In Theorem 3, it is the reduced form dynamic model extensively analyzed in Heckman and Navarro (2007). □

Appendix D: Proof of a more general version of Theorem 4

This appendix states and proves a more general version of Theorem 4. Use $Y(t, s)$ as shorthand for $Y(t, s, X, U(t, s))$. Ignore (for notational simplicity) the mixed discrete-continuous outcome case. One can build that case from the continuous and discrete cases and for the sake of brevity we do not analyze it here. We also do not analyze duration outcomes although it is straightforward to do so. Decompose $Y(t, s)$ into discrete and continuous components:

$$Y(t, s) = \begin{bmatrix} Y_c(t, s) \\ Y_d(t, s) \end{bmatrix}.$$

Associated with the jth component of $Y_d(t,s)$, $Y_{d,j}(t,s)$, is a latent variable $Y^*_{d,j}(t,s)$. Define, as in Theorem 2,

$$Y_{d,j}(t,s) = \mathbf{1}\bigl(Y^*_{d,j}(t,s) \geqslant 0\bigr).\text{[145]}$$

From standard results in the discrete-choice literature, without additional information, one can only identify $Y^*_{d,j}(t,s)$ up to scale.

Assume an additively separable model for the continuous variables and latent continuous indices. Making the X explicit, we obtain

$$\begin{aligned}
&Y_c(t,s,X) = \mu_c(t,s,X) + U_c(t,s),\\
&Y^*_d(t,s,X) = \mu_d(t,s,X) - U_d(t,s),\\
&\quad 1 \leqslant s \leqslant \bar{S},\ 1 \leqslant t \leqslant \bar{T}.
\end{aligned}$$

Array the $Y_c(t,s,X)$ into a matrix $Y_c(s,X)$ and the $Y^*_d(t,s,X)$ into a matrix $Y^*_d(s,X)$. Decompose these vectors into components corresponding to the means $\mu_c(s,X)$, $\mu_d(s,X)$ and the unobservables $U_c(s)$, $U_d(s)$. Thus

$$\begin{aligned}
&Y_c(s,X) = \mu_c(s,X) + U_c(s),\\
&Y^*_d(s,X) = \mu_d(s,X) - U_d(s).
\end{aligned}$$

$Y^*_d(s,X)$ generates $Y_d(s,X)$. To simplify the notation, make use of the condensed forms $Y_c(X)$, $Y^*_d(X)$, $\mu_c(X)$, $\mu_d(X)$, U_c and U_d as described in the text. In this notation,

$$\begin{aligned}
&Y_c(X) = \mu_c(X) + U_c,\\
&Y^*_d(X) = \mu_d(X) - U_d.
\end{aligned}$$

Following Carneiro, Hansen and Heckman (2003) and Cunha and Heckman (2007b, 2008), Cunha, Heckman and Navarro (2005, 2006), one may also have a system of measurements with both discrete and continuous components. The measurements are not s-indexed. They are the same for each stopping time. Write the equations for the measurements in an additively separable form, in a fashion comparable to those of the outcomes. The equations for the continuous measurements and latent indices producing discrete measurements are

$$\begin{aligned}
&M_c(t,X) = \mu_{c,M}(t,X) + U_{c,M}(t),\\
&M^*_d(t,X) = \mu_{d,M}(t,X) - U_{d,M}(t),
\end{aligned}$$

where the discrete variable corresponding to the jth index in $M^*_d(t,X)$ is

$$M_{d,j}(t,X) = \mathbf{1}\bigl(M^*_{d,j}(t,X) \geqslant 0\bigr).$$

[145] Extensions to nonbinary discrete outcomes are straightforward. Thus one could entertain, at greater notational cost, a multinomial outcome model at each age t for each counterfactual state, building on the analysis of Appendix B in Chapter 70.

The measurements play the role of indicators unaffected by the process being studied. We array $M_c(t, X)$ and $M_d^*(t, X)$ into matrices $M_c(X)$ and $M_d^*(X)$. We array $\mu_{c,M}(t, X)$, $\mu_{d,M}(t, X)$ into matrices $\mu_{c,M}(X)$ and $\mu_{d,M}(X)$. We array the corresponding unobservables into $U_{c,M}$ and $U_{d,M}$. Thus we write

$$M_c(X) = \mu_{c,M}(X) + U_{c,M},$$
$$M_d^*(X) = \mu_{d,M}(X) - U_{d,M}.$$

We use the notation of Section 3.4.1 to write $I(s) = \Psi(s, Z(s)) - \eta(s)$ and collect $I(s)$, $\Psi(s, Z(s))$ and $\eta(s)$ into vectors I, $\Psi(Z)$, η. We define $\eta^s = (\eta(1), \ldots, \eta(s))$ and $\Psi^s(Z) = (\Psi(1, Z(1)), \ldots, \Psi(s, Z(s)))$. Using this notation, we extend the analysis of Carneiro, Hansen and Heckman (2003) to identify our model assuming that we have a large i.i.d. sample from the distribution of (Y_c, Y_d, M_c, M_d, I).

THEOREM D.1. *Assuming the conditions of Theorem 3 hold, for $s = 1, \ldots, \bar{S}$, the joint distribution of $(U_c(s), U_d(s), U_{c,M}, U_{d,M}, \eta^s)$ is identified along with the mean functions $(\mu_c(s, X), \mu_d(s, X), \mu_{c,M}(X), \mu_{d,M}(X), \Psi^s(Z))$ (the components of $\mu_d(s, X)$ and $\mu_{d,M}(X)$ over the supports admitted by the supports of the errors) if*

(i) $E[U_c(s)] = E[U_{c,M}] = 0$. *$(U_c(s), U_d(s), U_{c,M}, U_{d,M}, \eta^s)$ are continuous random variables with support*: $\operatorname{Supp}(U_c(s)) \times \operatorname{Supp}(U_d(s)) \times \operatorname{Supp}(U_{c,M}) \times \operatorname{Supp}(U_{d,M}) \times \operatorname{Supp}(\eta^s)$ *with upper and lower limits $(\overline{U}_c(s), \overline{U}_d(s), \overline{U}_{c,M}, \overline{U}_{d,M}, \bar{\eta}^s)$ and $(\underline{U}_c(s), \underline{U}_d(s), \underline{U}_{c,M}, \underline{U}_{d,M}, \underline{\eta}^s)$ respectively. These conditions are assumed to apply within each component of each subvector. The joint system is thus variation free for each component with respect to every other component.*

(ii) $(U_c(s), U_d(s), U_{c,M}, U_{d,M}, \eta^s) \perp\!\!\!\perp (X, Z)$.

(iii) $\operatorname{Supp}(\Psi(Z), X) = \operatorname{Supp}(\Psi(Z)) \times \operatorname{Supp}(X)$.

(iv) $\operatorname{Supp}(\mu_d(s, X), \mu_{d,M}(X)) \supseteq \operatorname{Supp}(U_d(s), U_{d,M})$.

(v) *$\mu_c(s, X)$ and $\mu_{c,M}(X)$ are continuous functions. The components of the $\mu_d(s, X)$ and $\mu_{d,M}(X)$ satisfy the Matzkin conditions developed in Appendix* B.1.

PROOF. We use the proof of Theorem 3 to identify $\Psi^s(Z)$ and the distributions of η^s, $s = 1, \ldots, \bar{S}$. From the data on $Y_c(s, X)$, $Y_d(s, X)$, $M_c(X)$, $M_d(X)$ for $D(s) = 1$, and from the time to treatment probabilities, we can construct the left-hand side of the following equation:

$$\begin{aligned}
&\Pr\left(\begin{matrix} Y_c(s, X) \leqslant y_c, \mu_d(s, X) \leqslant U_d(s), \\ M_c(X) \leqslant m_c, \mu_{d,M}(X) \leqslant U_{d,M} \end{matrix} \;\middle|\; D(s) = 1, X = x, Z = z \right) \\
&\quad \times \Pr\big(D(s) = 1 \mid X = x, Z = z\big)
\end{aligned}$$

$$= \int_{\underline{U}_c(s)}^{y_c-\mu_c(s,x)} \int_{\mu_d(s,x)}^{\overline{U}_d(s)} \int_{\underline{U}_{c,M}}^{m_c-\mu_{c,M}(x)} \int_{\mu_{d,M}(x)}^{\overline{U}_{d,M}} \int_{\underline{\eta}(s)}^{\Psi(s,z(s))} \int_{\Psi(s-1,z(s-1))}^{\bar{\eta}(s-1)} \cdots$$

$$\int_{\Psi(1,z(1))}^{\bar{\eta}(1)} f_{U_c(s),U_d(s),U_{c,M},U_{d,M},\eta^s}\big(u_c(s), u_d(s), u_{c,M}, u_{d,M}, \eta(1), \ldots, \eta(s)\big)$$

$$\cdot\, d\eta(1) \cdots d\eta(s)\, du_{d,M}\, du_{c,M}\, du_d(s)\, du_c(s). \tag{D.1}$$

We can construct distributions for the other configurations of conditioning events defining the discrete dependent variables (i.e., $\mu_d(s,X) > U_d(s)$, $\mu_{d,M}(X) > U_{d,M}$; $\mu_d(s,X) > U_d(s)$, $\mu_{d,M}(X) < U_{d,M}$; $\mu_d(s,X) \leqslant U_d(s)$, $\mu_{d,M}(X) > U_{d,M}$).

Under assumption (iii), for all $x \in \operatorname{Supp}(X)$, we can vary the $\Psi(j, z(j))$, $j = 1, \ldots, s$, and obtain a limit set $\mathcal{Z}_s$, possibly dependent on X, such that $\lim_{z\to\mathcal{Z}_s} \Pr(D(s) = 1 \mid X = x, Z = z) = 1$. We can use (D.1) and parallel distributions for the other configurations for the discrete dependent variables to identify the joint distribution of $Y_c(s,X)$, $Y_d(s,X)$, $M_c(X)$, $M_d(X)$ free of selection bias for all $s = 1, \ldots, \bar{S}$ in these limit sets. We identify the parameters of $Y_d(s,X)$, $s = 1, \ldots, \bar{S}$, and $M_d(X)$. We know the limit sets given the functional forms for the $\Psi(s, Z(s))$, $s = 1, \ldots, \bar{S}$, presented in B.1 or in Matzkin (1992, 1993, 1994).

As a consequence of (ii), we can identify $\mu_c(s,X)$, $\mu_{c,M}(X)$ directly from the means of the limit outcome distributions. We can thus identify all pairwise average treatment effects

$$E\big(Y_c(s,X) \mid X = x\big) - E\big(Y_c(s',X) \mid X = x\big)$$

for all s, s' and any other linear functionals derived from the distributions of the continuous variables defined at s and s'. Identification of the means and distributions of the latent variables giving rise to the discrete outcomes is more subtle. The required argument is standard. With one continuous regressor among the X, one can identify the marginal distributions of the $U_d(s)$ and the $U_{d,M}$ (up to scale if the Matzkin functions are only specified up to scale). To identify the joint distributions of $U_d(s)$ and $U_{d,M}$, one can invoke (iv).

Thus for system s, suppose that there are $N_{d,s}$ discrete outcome components with associated means $\mu_{d,j}(s,X)$ and error terms $U_{d,j}(s)$, $j = 1, \ldots, N_{d,s}$. As a consequence of condition (iv) of this theorem, $\operatorname{Supp}(\mu_d(s,X)) \supseteq \operatorname{Supp}(U_d(s))$. We thus can trace out the joint distribution of $U_d(s)$ and identify it (up to scale if we specify the Matzkin class only up to scale). By a parallel argument for the measurements, we can identify the joint distribution of $U_{d,M}$. Let $N_{d,M}$ be the number of discrete measurements. From condition (iv), we obtain $\operatorname{Supp}(\mu_{d,M}(X)) \supseteq \operatorname{Supp}(U_{d,M})$. Under these conditions, we can trace out the joint distribution of $U_{d,M}$ and identify it (up to scale for the Matzkin class of functions specified up to scale) within the limit sets. From assumption (v), we obtain identification on nonnegligible sets.

We can vary each limit of the integral in (D.1) independently and trace out the full joint distribution of $(U_c(s), U_d(s), U_{c,M}, U_{d,M}, \eta(1), \ldots, \eta(s))$ using the parameters determined from the marginals. For further discussion, see the analysis in Carneiro, Hansen and Heckman (2003, Theorem 3). We obtain identifiability on nonnegligible sets by combining the conditions in Theorem 3 with those in condition (v).

□

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