THE OHIO STATE UNIVERSITY

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Optimizing Data Freshness, Throughput, and Delay in Multi-Server Information-Update Systems

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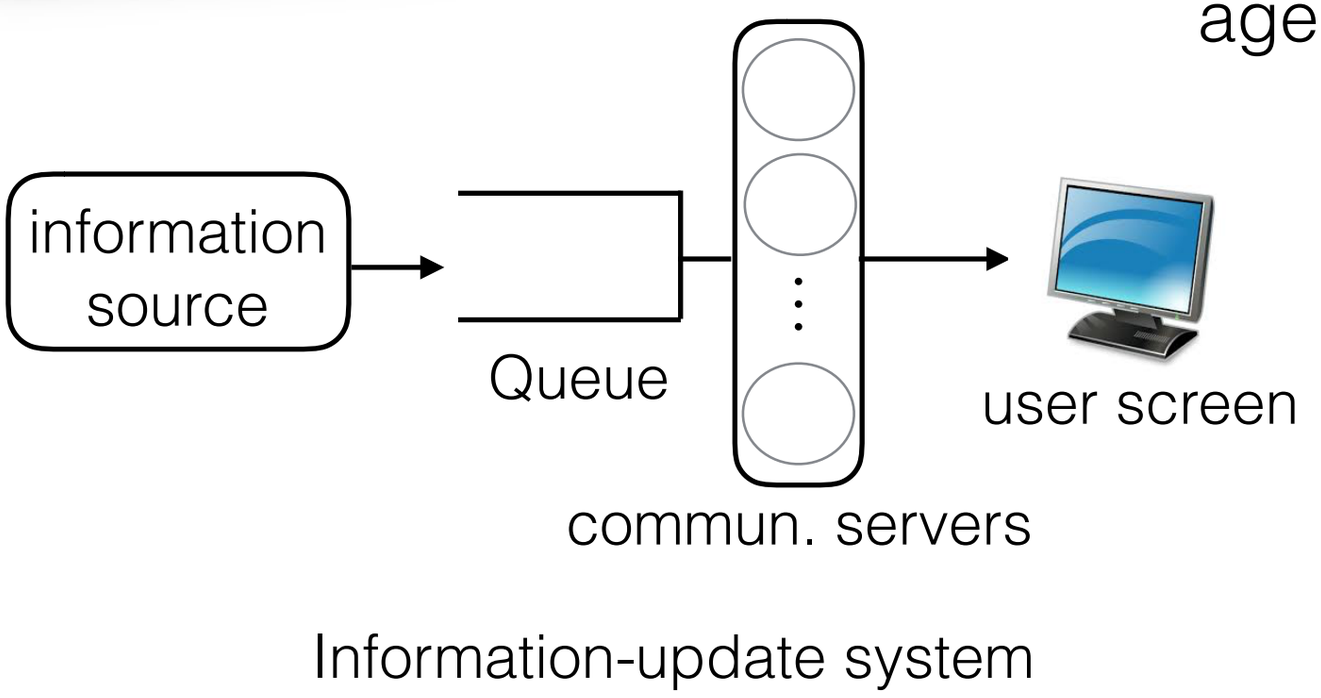
Joint work with Yin Sun, Ness B. Shroff

Departments of ECE,
The Ohio State University

Nov. 7th 2016

What is the Age of Information?



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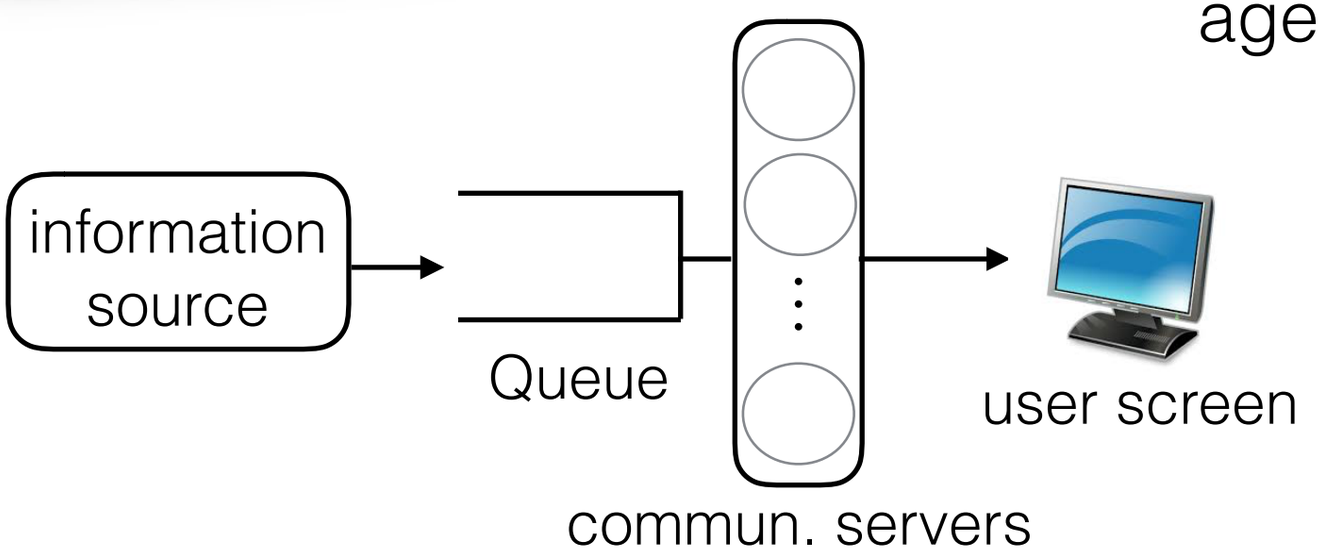

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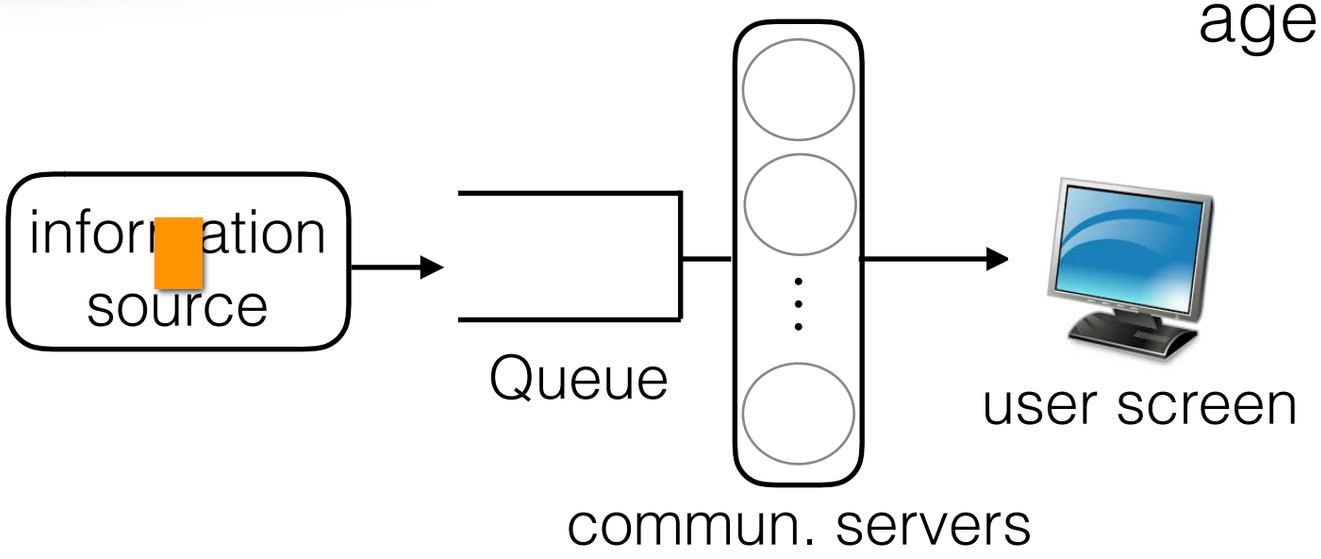

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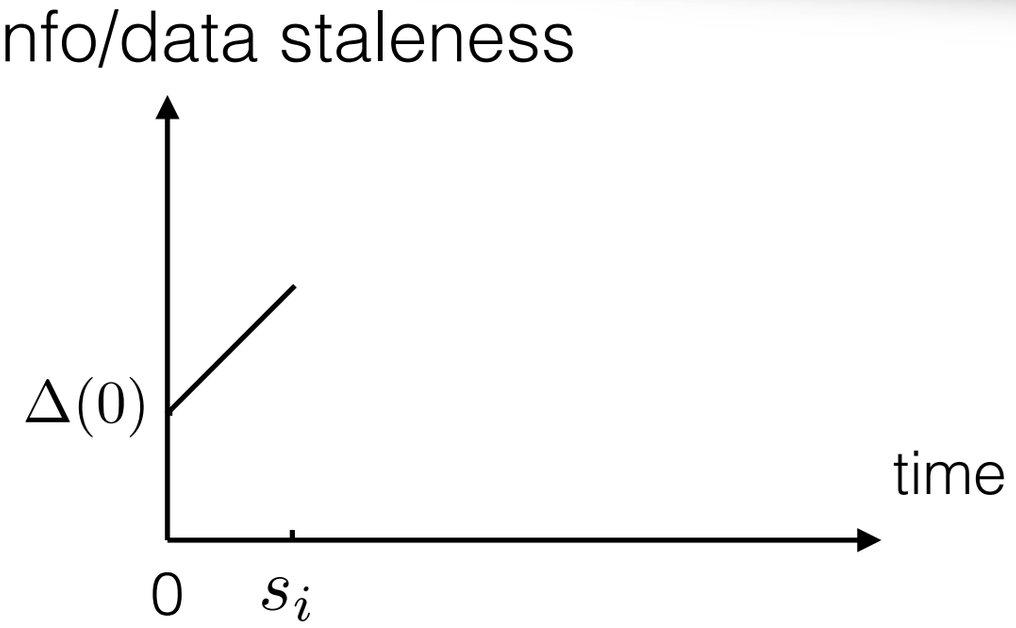

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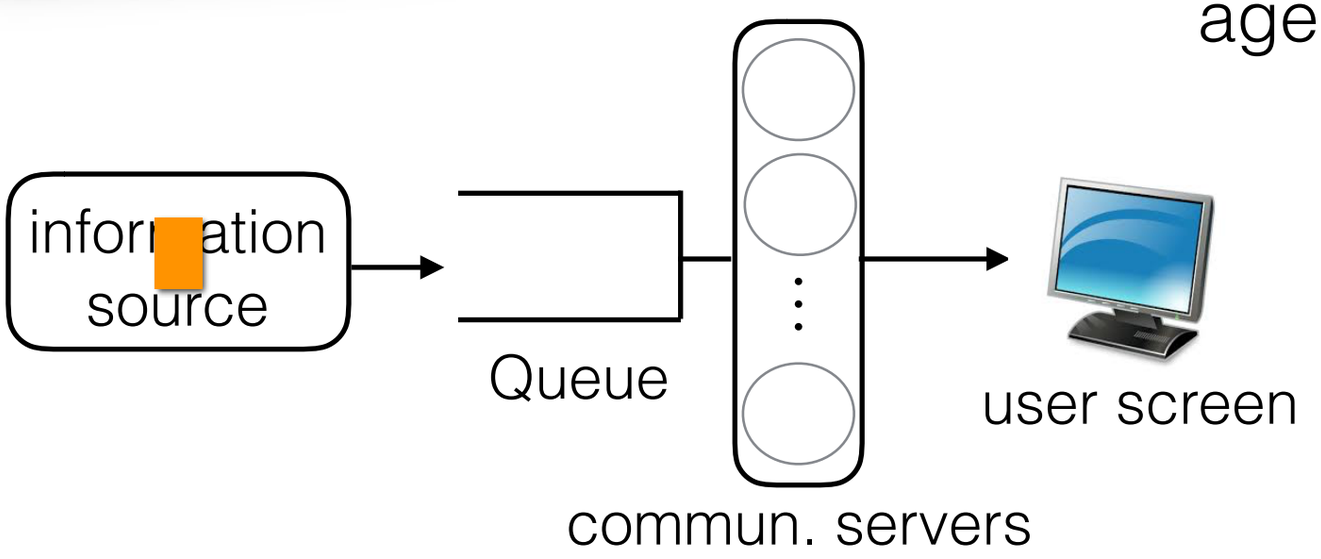

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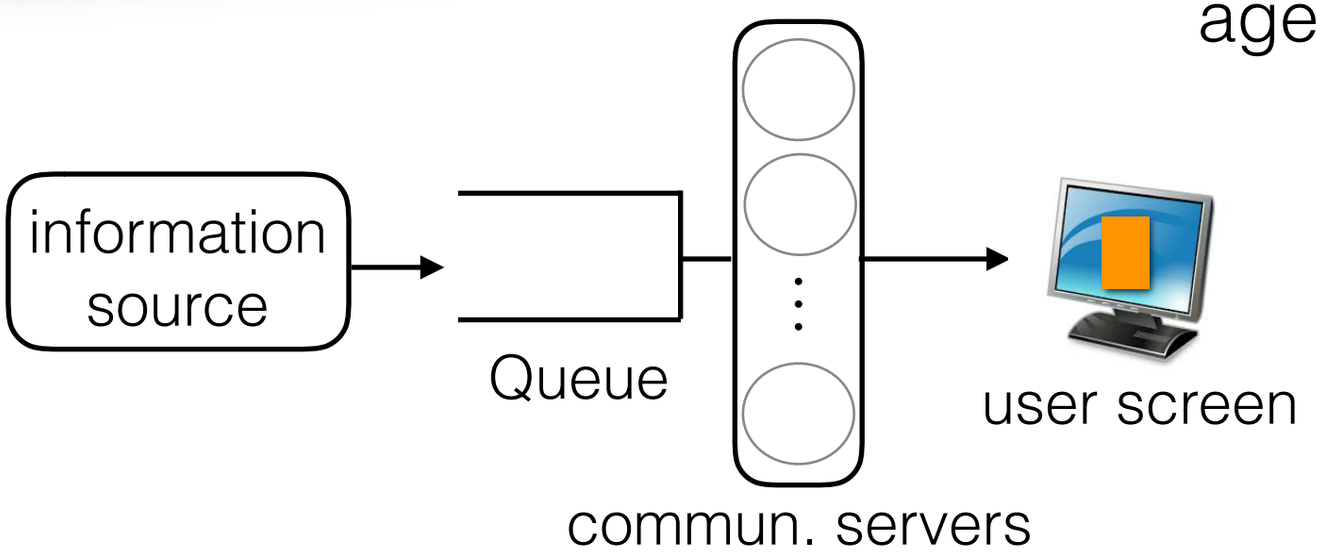

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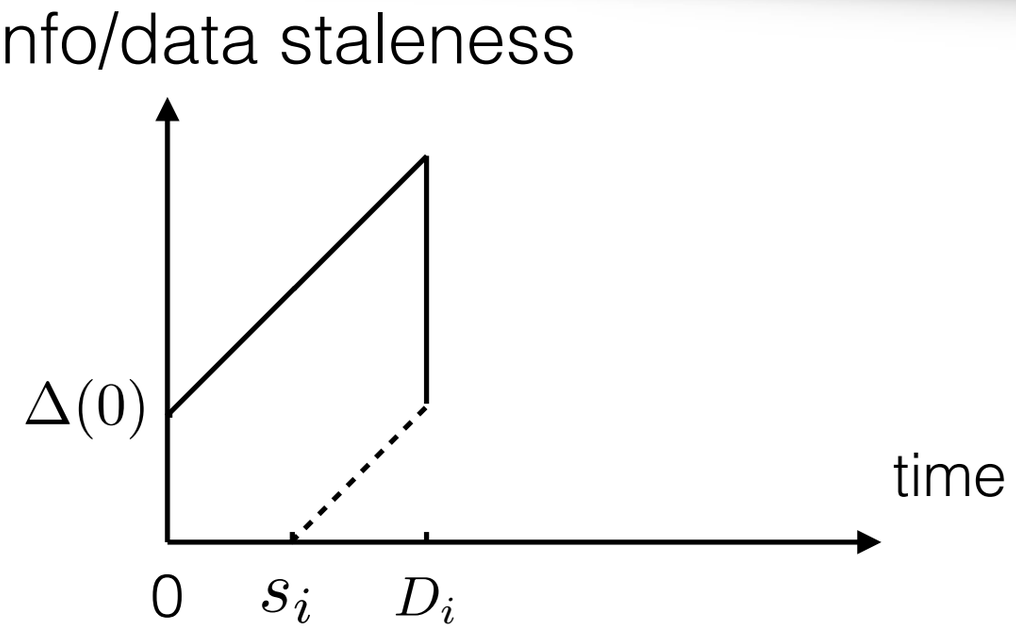

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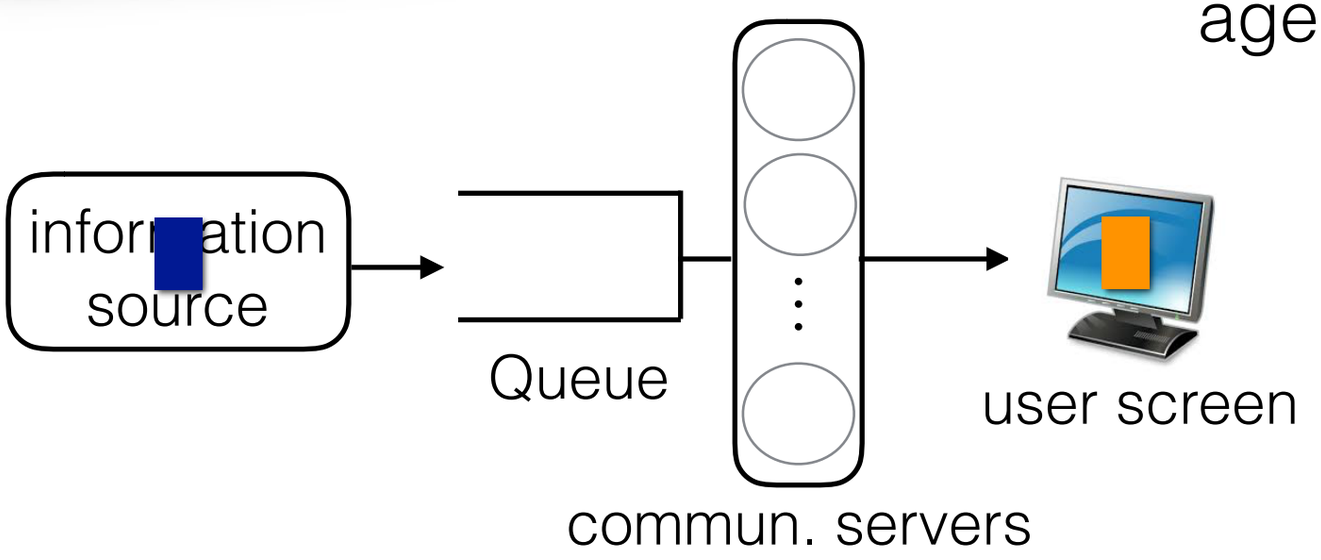

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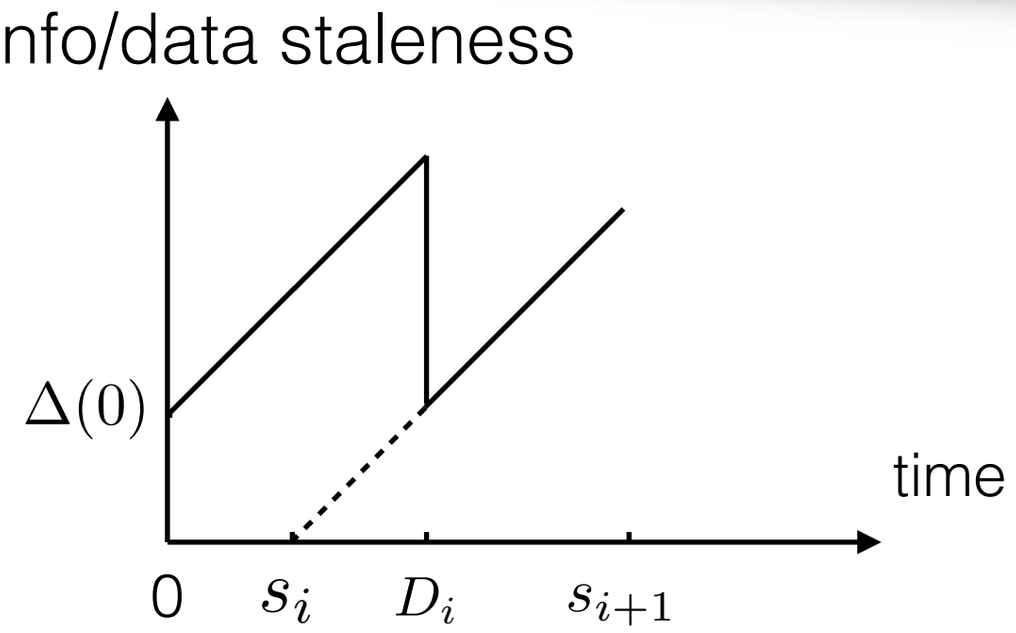

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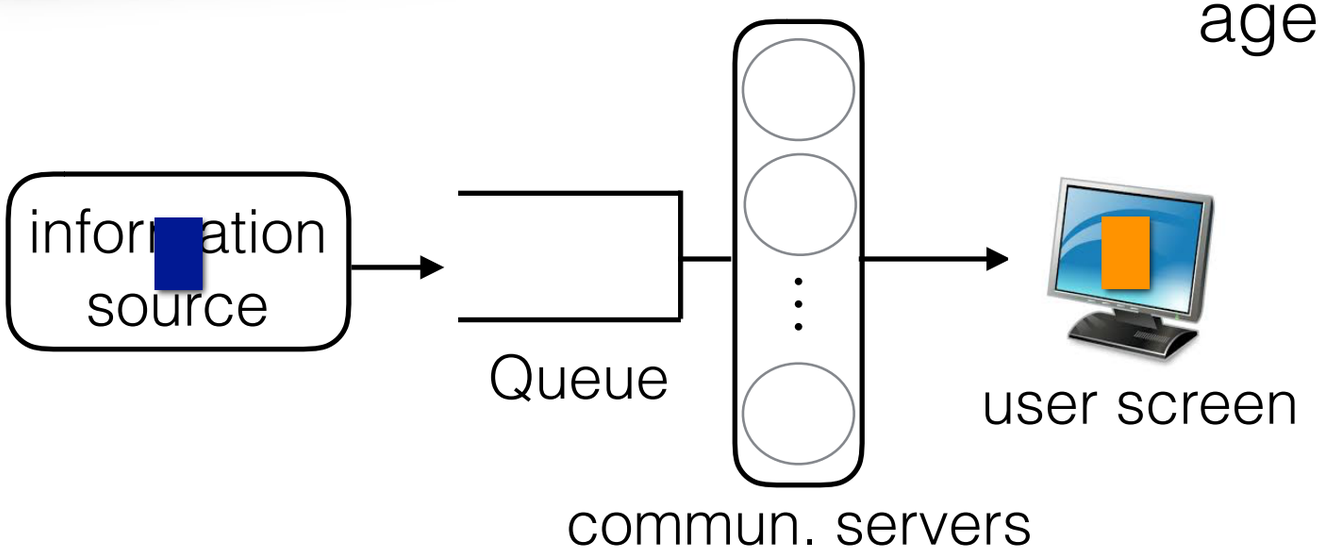

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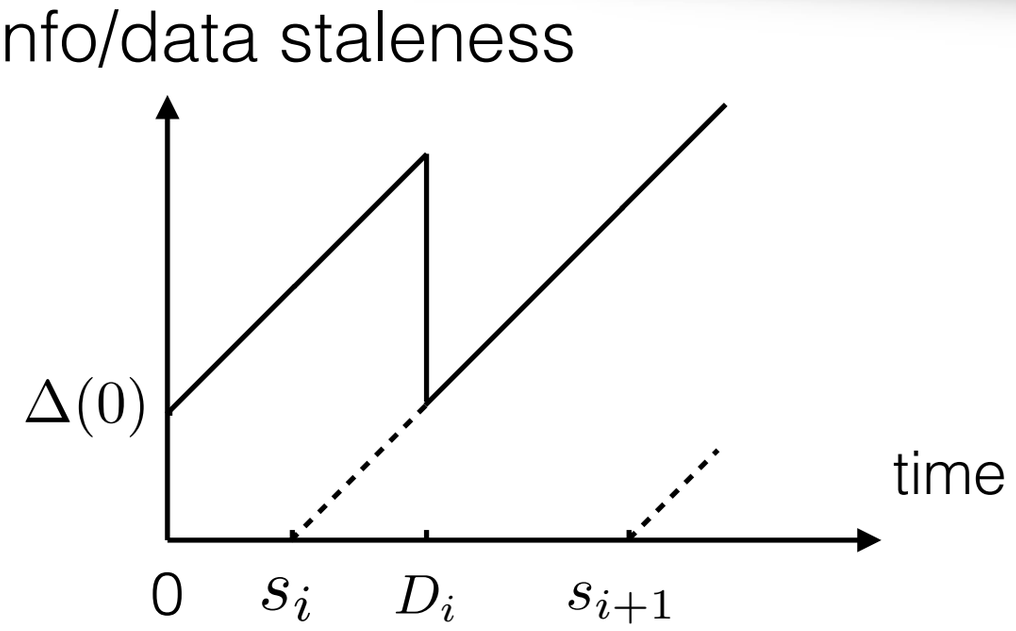

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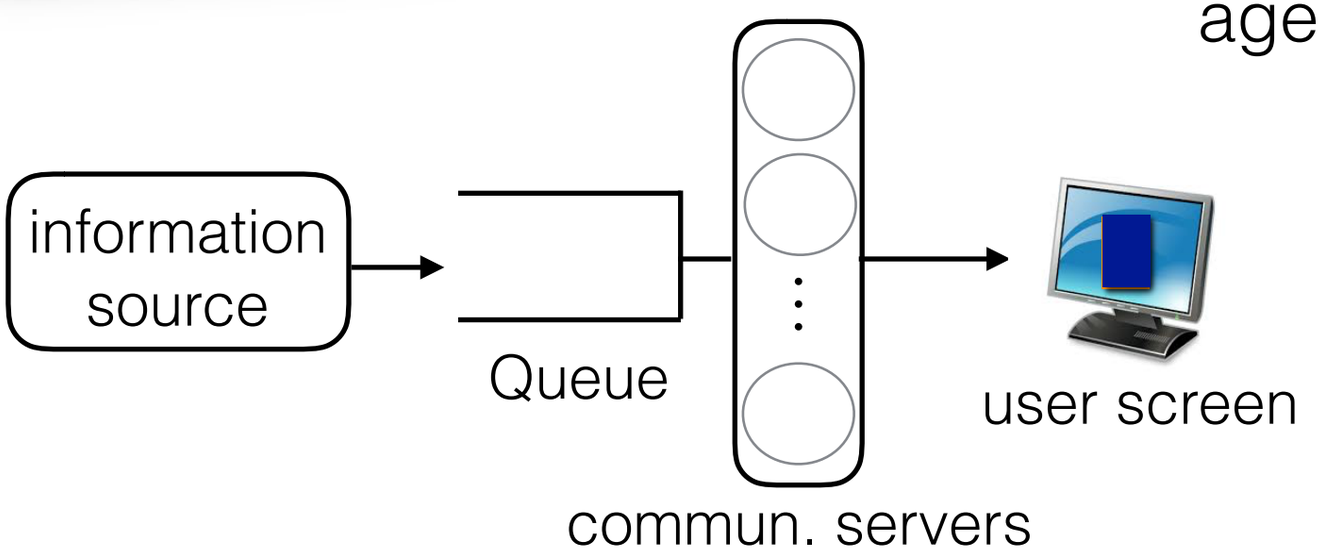

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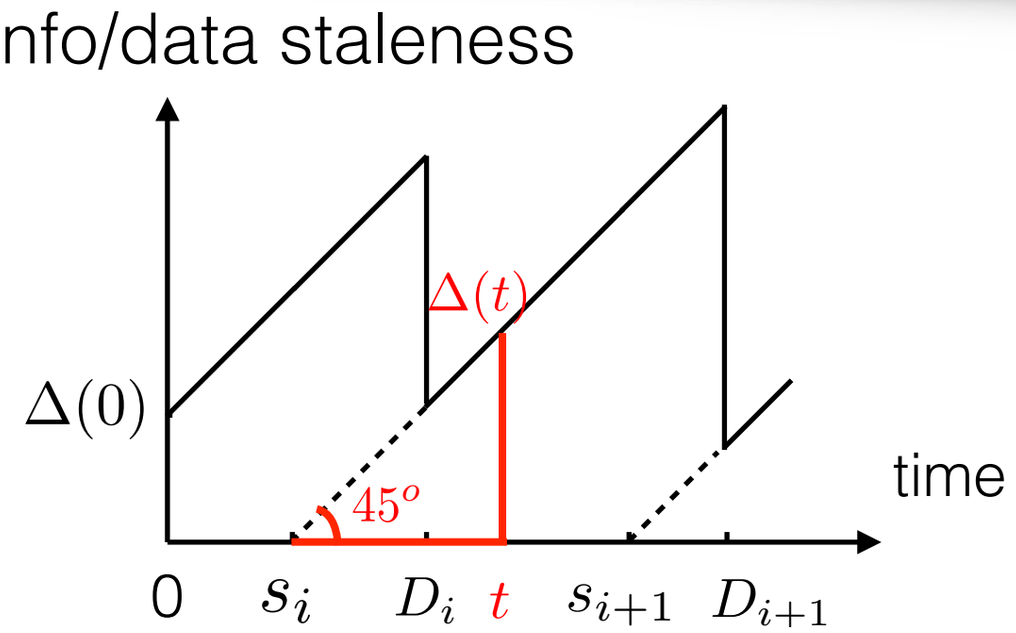

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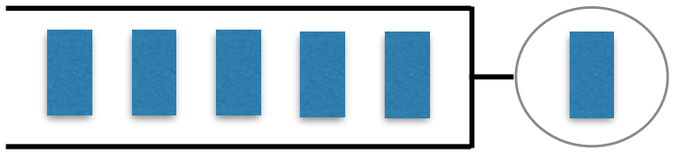
Difference between Delay & Age



High arrival rate

Low arrival rate

Age is different from delay. [Kaul, Yates, Gruteser, Infocom 2012]

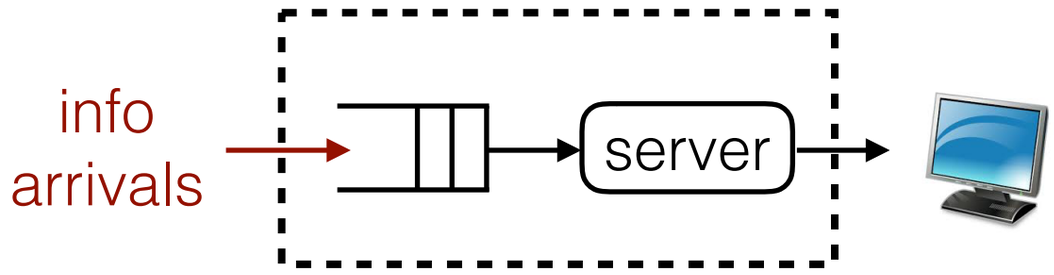
- High arrival rate: long waiting time, packets becomes stale
- Low arrival rate: short queue, but infrequent updates

} High age

Age is not monotonic wrt queue length

- For delay, Little's Law says: $L = \lambda W$
- Delay grows linearly wrt queue length

Open Questions



Enqueue-and-forward model

- **Age Characterization and Age Reduction:**
  - M/M/1, M/D/1, D/M/1 [Kaul, Yates, Gruteser 2012]
  - M/M/2 [Kam, Kompella, Ephremides 2014]
  - Multi-sources [Yates, Kaul ISIT 2012] [Huang, Modiano 2015]
  - Packet management, LCFS [Kam, Kompella, Ephremides 2013, 2014]
  - Channel state info [Costa, Valentin, Ephremides, 2015]
  - LCFS (single server) with & without preemption [Kaul, Yates, Gruteser 2012]

- **Question 1:** Which policy is **age-optimal**?

Open Questions (cont.)

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- **Information Updates**
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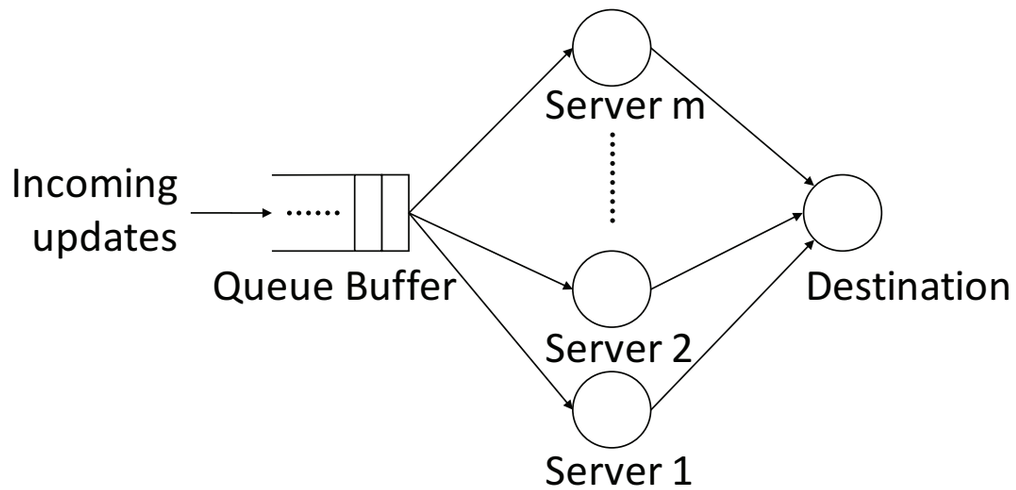
- **Question 2**: Is there a policy which simultaneously optimizes **age**, **throughput**, and **delay**?

  - This presentation will answer these two questions

System Model



- An information-update system with
  - **m** *i.i.d.* servers
  - Queue with buffer size B
- Packet i is generated at time s_i,and arrives at time a_i ($s_1 \leq s_2 \leq \ldots$)
  - **Arbitrary** arrival process (including **non-stationary**)
  - Update packets can arrive **out of order** (e.g., $a_i > a_{i+1}$ but $s_i < s_{i+1}$)
- The set of all causal policies is denoted by Π

Definitions

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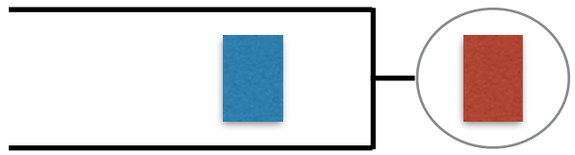
- **Definition. Service Preemption**: At any time



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- **Definition. Stochastic Ordering**: Let X and Y be two random variables. Then, X is said to be **stochastically smaller** than Y (denoted as $X \leq_{\text{st}} Y$), if

$$\mathbb{P}\{X > x\} \leq \mathbb{P}\{Y > x\}, \quad \forall x \in \mathbb{R}.$$

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- **In other words**: $X \leq_{\text{st}} Y$ **iff** there exist two random variable $\hat{X}$ and $\hat{Y}$, defined on the same probability space, such that

$$\hat{Y} =_{\text{st}} Y \quad \& \quad \hat{X} =_{\text{st}} X$$

and

$$\mathbb{P}\{\hat{X} \leq \hat{Y}\} = 1$$

(here $=_{\text{st}}$ denotes equality in law)

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- **Definition. Stochastic Ordering of Stochastic Processes:** Two random processes $\{X(t), t \in [0,\infty)\}$ and $\{Y(t), t \in [0,\infty)\}$ satisfies $\{X(t), t \in [0,\infty)\} \leq_{\text{st}} \{Y(t), t \in [0,\infty)\}$ **iff** there exist two random processes $\{\hat{X}(t), t \in [0,\infty)\}$ and $\{\hat{Y}(t), t \in [0,\infty)\}$, defined on the same probability space, such that

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- **Definition. Age Optimality:** A policy $\gamma \in \Pi$ is said to be age-optimal, if for all $\pi \in \Pi$

$$\{\Delta_\gamma(t), t \in [0, \infty)\} \leq_{\text{st}} \{\Delta_\pi(t), t \in [0, \infty)\}.$$

Policy P Algorithm

- Policy P: **Preemptive Last Generated First Served (LGFS)**

- Arrival

- Departure

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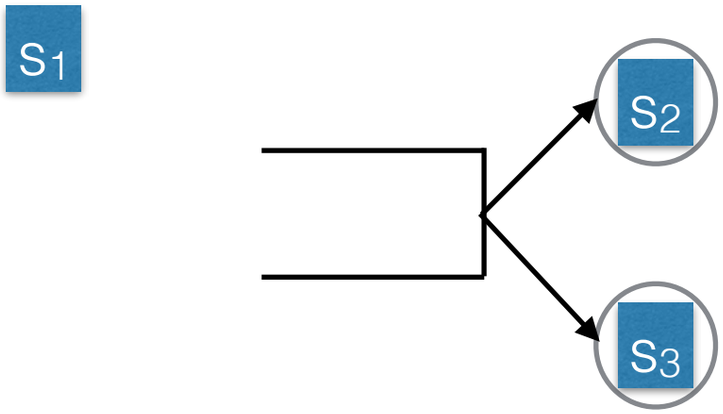

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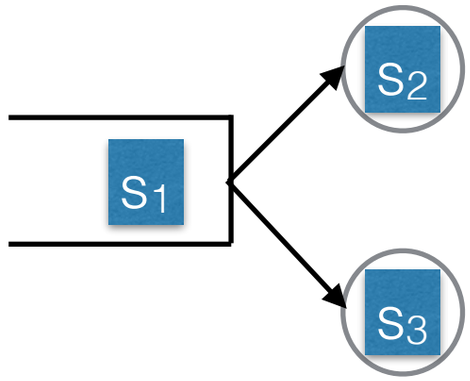

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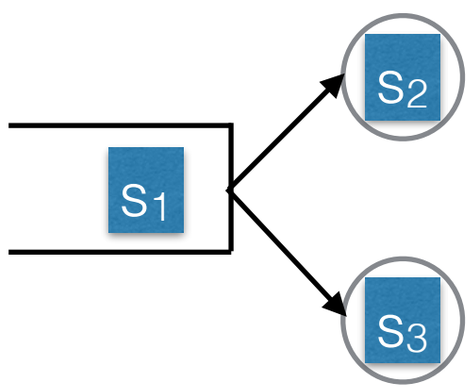

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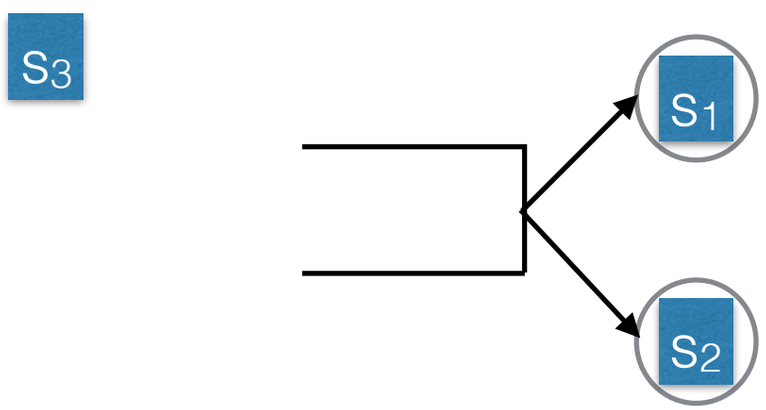

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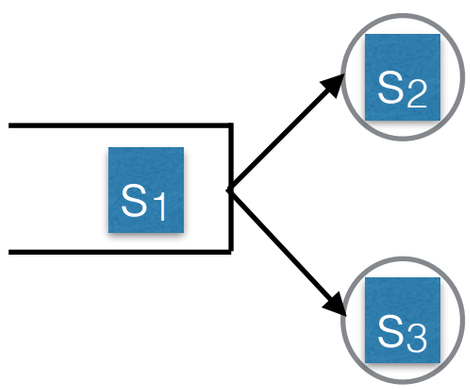

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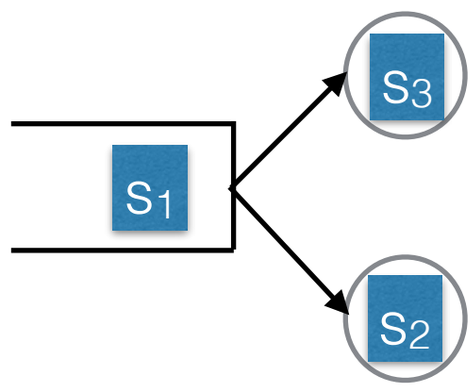

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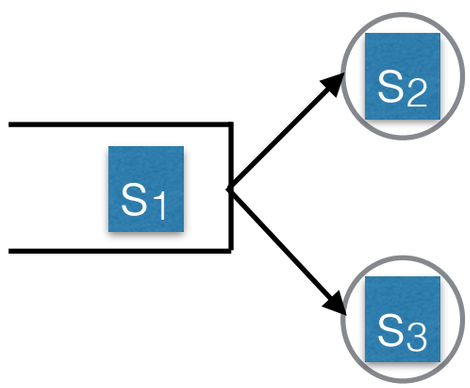

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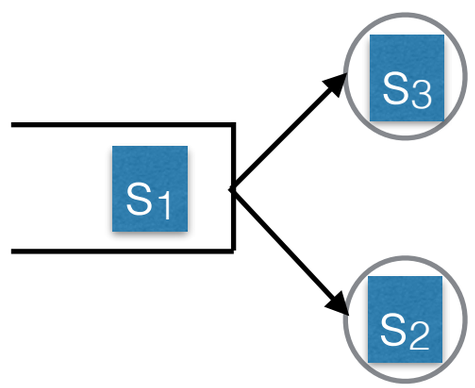

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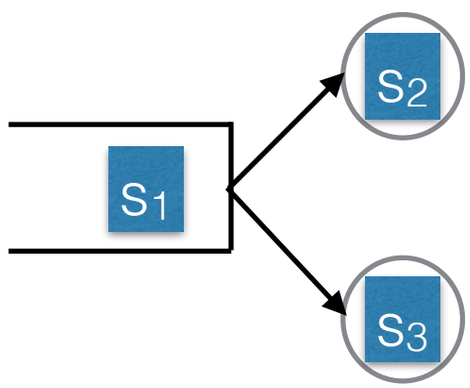

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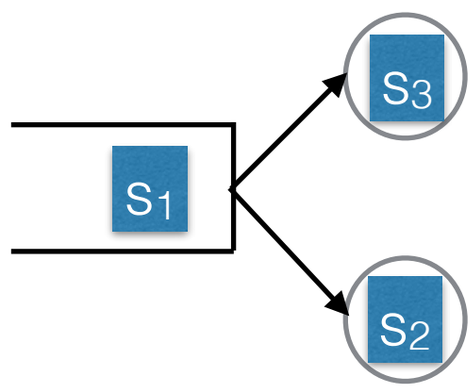

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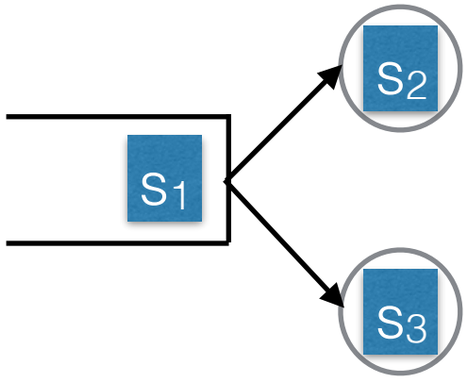

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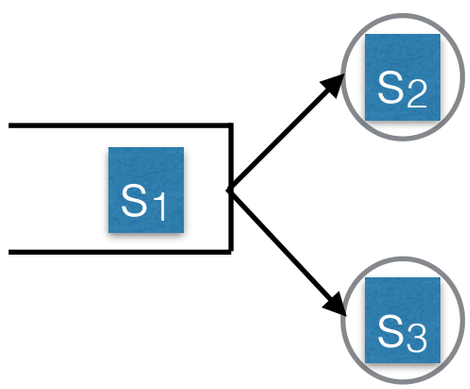

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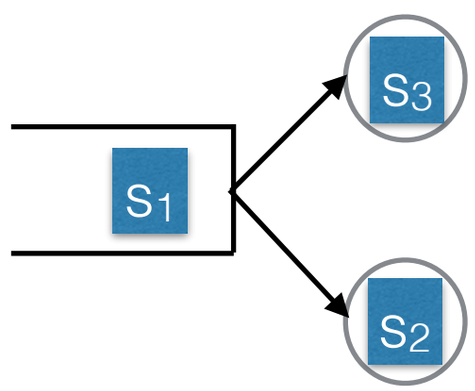

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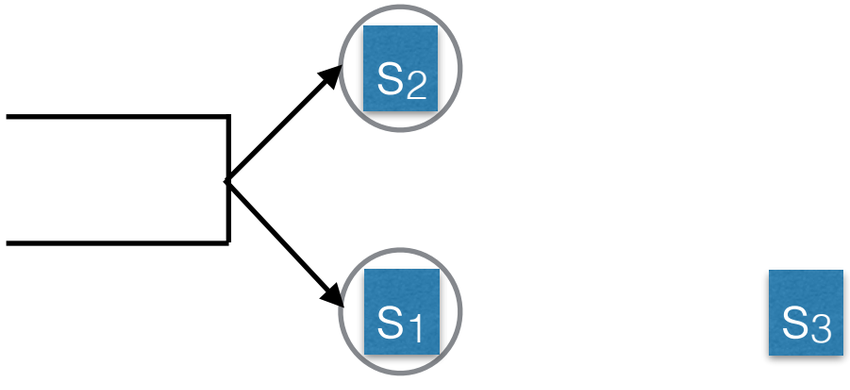

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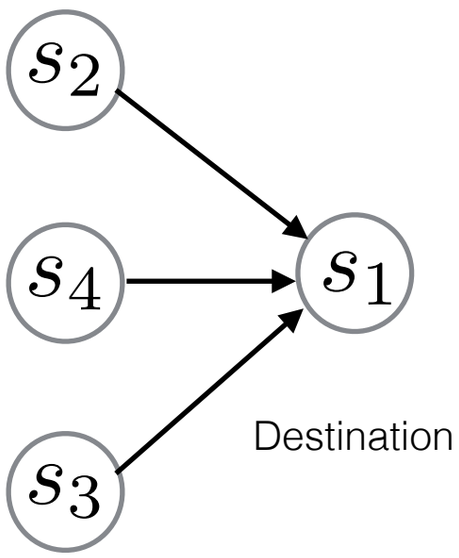

3-Servers system

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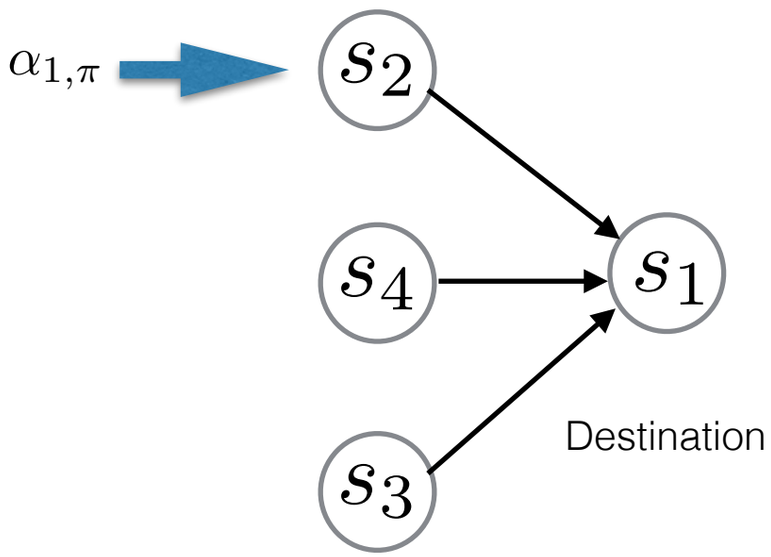

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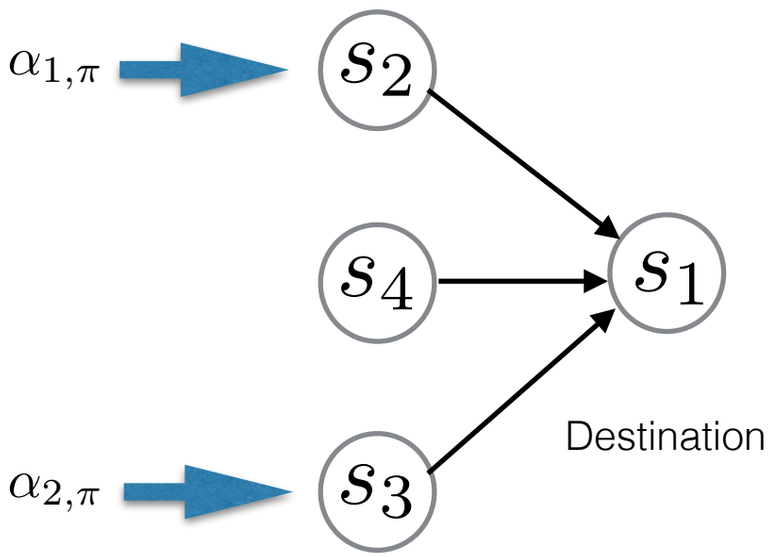

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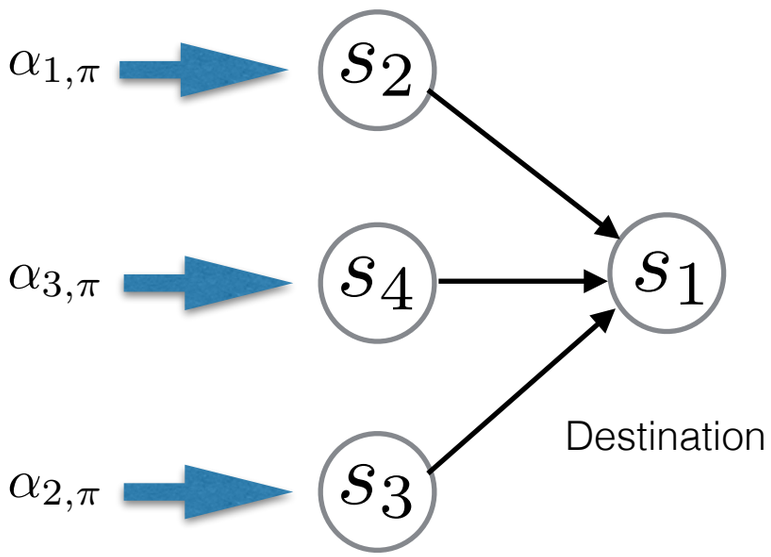

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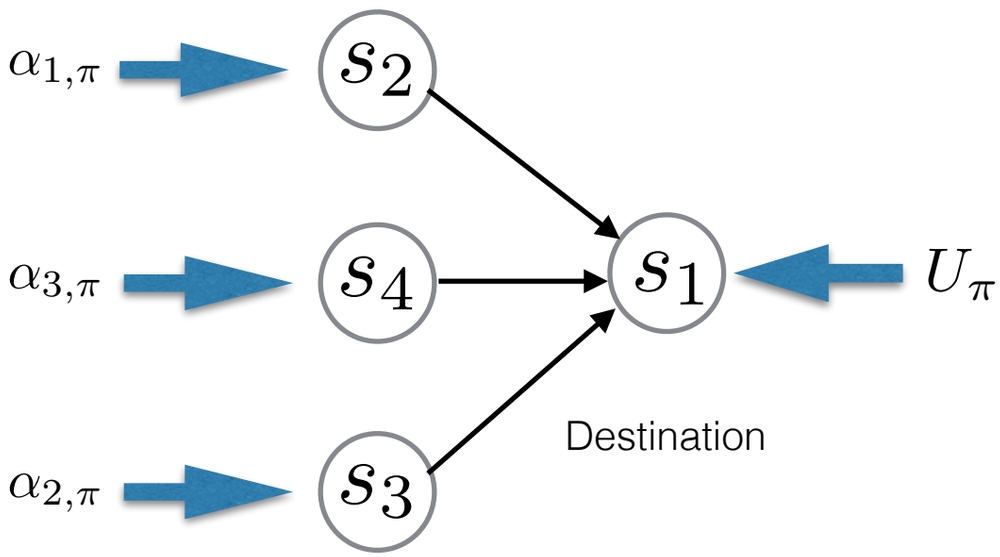

3-Servers system

Proof sketch

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$$\{\Delta_P(t), t \in [0,\infty)\} \leq_{\text{st}} \{\Delta_\pi(t), t \in [0,\infty)\}, \ \forall \pi \in \Pi$$

Corollaries

- **Corollary**: The age performance of the **preemptive LGFS** policy remains the same for any queue size $B \geq 0$
    - Stale packets are stored back in the queue.

- **Corollary**: For the same system setting as Theorem 1, the preemptive LGFS policy **minimizes**:
    1. The time-average age
    2. Average peak age
    3. Time-average age penalty

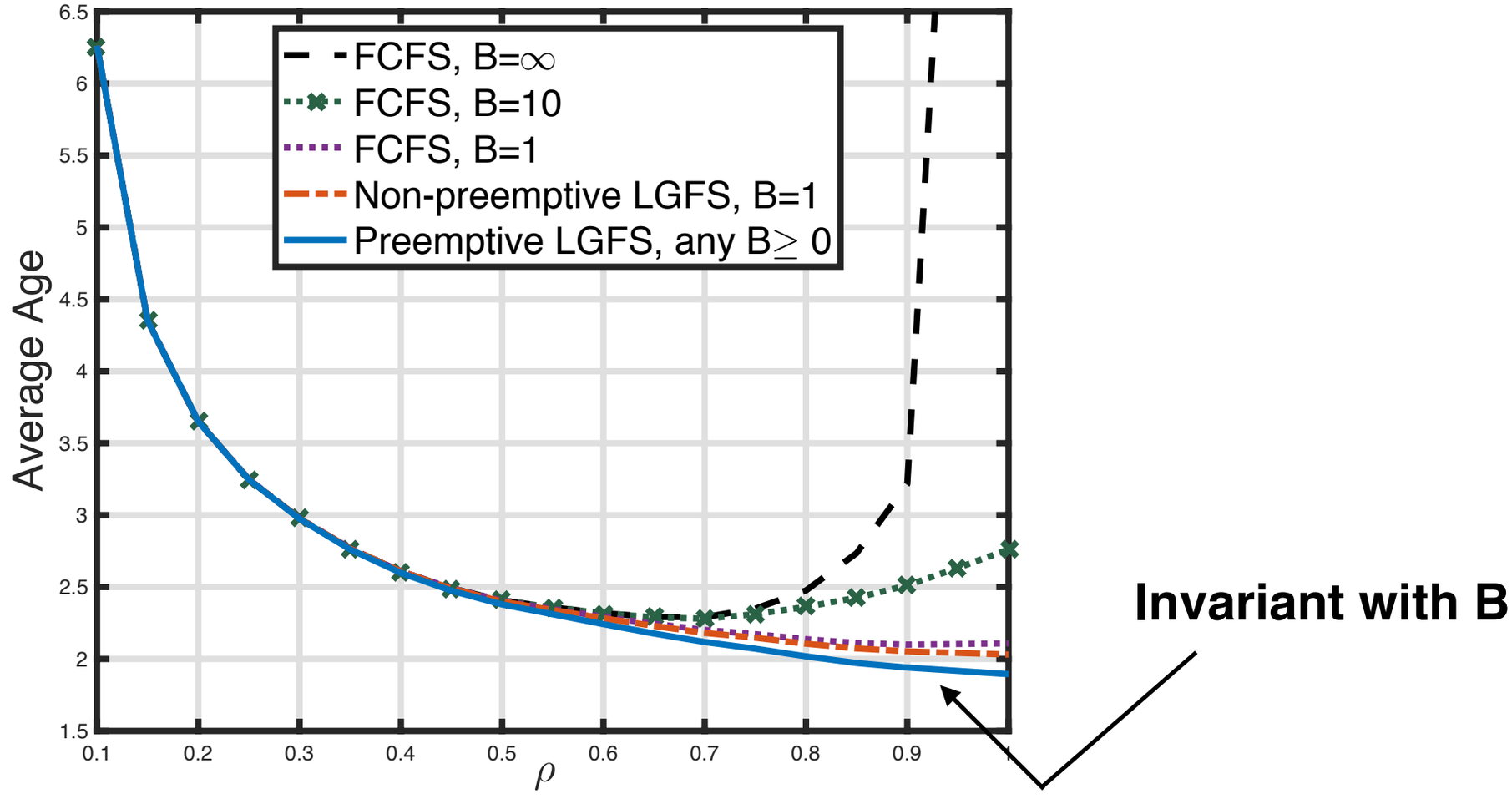
- These age metrics are increasing function of the age process.

Theorem

Theorem 2: For
1. i.i.d. exponential service time distribution
2. *A*ny packet generation and arrival times
3. Infinite buffer size $B = \infty$

The **preemptive LGFS** policy is **throughput-optimal** and **mean-delay-optimal** among all policies in Π .

Simulation Result



- $m = 5$ servers, buffer size B
- inter-generation times: *i.i.d.* **Erlang-2 distribution**
- $(a_i - s_i)$ is modeled to be either **1** or **100** with **equal probability**

Observations:
1. Preemptive LGFS **outperforms** all other policies.
2. The age performance of the preemptive LGFS is **invariant** for any B
3. **FCFS**: The age gets worse as B increases.

Summary & Future Work

Exponential service time.

- The **preemptive LGFS** optimizes **age**, **throughput** and **delay** among all causally feasible policies for
  - **Arbitrary** packet generation times $s_1, s_2, \ldots$
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Service time dist.	Exponential	NWU Hyperexponential distribution	NBU Erlang distribution
Preemptive LGFS	Age-optimal (any arrival orders)	Not age-optimal in the same policy space	
Non-preemptive LGFS	Near age-optimal		Near age-optimal


Thank
You!