Comparison of Equation-Based and Non-Equation-Based Approaches for Transient Modeling of a Vapor Compression Cycle

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ABSTRACT

Transient modeling methods can also be classified into two categories based on equation specifications. The first category of methods are non-equation based methods, in which one has to describe the underlying physics with fundamental equations, formulate residual equations and provide an algorithm to solve the underlying nonlinear equations. The second category of methods are equation based methods, wherein the engineer only has to specify the underlying physics, that holds for each time step, in the form of equations and use an established modeling platform or a solver which will handle the formulation and solution of underlying nonlinear differential algebraic equations. There are two modeling platforms that are widely emerging as an industry standard for transient modeling and control algorithm development. First set of platforms are based on the Modelica® language and the second set are based on MATLAB®. Three commercially available modeling environments based on these platforms are chosen for comparison in this study. In this paper, a transient system model for a simple vapor compression cycle is developed in the three environments (Modelica®, Simulink® and SimScape™). The heat exchanger models are based on the finite volume method. The compressor model is an efficiency-based model. The expansion device is modeled using an empirical correlation. Three tools demonstrate quite different modeling concepts. Comparison of simulation results obtained by three tools shows good agreement, but the computation time varies significantly due to underlying model reduction and solution scheme used.

1. INTRODUCTION

In modern day refrigeration and air-conditioning industries, numerical simulation has been widely used for the design and optimization of advanced products in response to the increasing pressure of cost reduction and high energy efficiency standards. With the aid of simulation tools, design engineers can evaluate a not-yet-existing product on computers instead of building real systems and running expensive tests in the lab.

Vapor compression system (VCC) simulation can fall into one of two categories: steady-state and transient. Transient simulations of vapor compression systems inherently differ from the steady state simulations. Steady-state oriented models put a large amount of effort into getting heat transfer and pressure drop equations correct, while transient simulations are mainly used for control design purposes. Therefore, it is the system dynamic trends during startup, shutdown and the switching between capacity control modes that matter most in transient simulations.

Traditionally, to develop a transient simulation model for VCCs, it is crucial for engineers to devise a smart system solution scheme to combine all the component models together according to the relationship among component parameters to compute system transients given certain constraints and under certain operating condition. This process involves the integration of an ordinary differential equation (ODE) solver into the modeling code that describes the physics. Under this circumstance, the engineers are expected to be familiar with the working principles of ODE solvers and to be capable of handling a great many numerical issues encountered during simulation. Inevitably, this process results in additional burden on the engineers because they not only need to focus on the physical modeling, but also need to devote much effort on how to solve the mathematical equations generated by the models.

Considering the inconvenience of this traditional modeling procedure, more and more modeling efforts are now resorting to commercial modeling tools with robust ODE solvers embeded in. The advantage of using commercial modeling tools is that modeling engineers can focus more on the underlying physics. In general, there are three commerncial modeling platforms that are widely emerging as an industry standard for transient modeling and control algorithm development, i.e. Simulink®, Modelica® and Simscape™.

Simulink is a block-based environment for multidomain simulation and model-based design for dynamic systems. Some transient modeling tools have been developed based on Matlab®/Simulink® (Thermosys, 2007; Thermolib, 2009). Thermosys, originally developed by University of Illinois and now jointly with Texas A&M University, is a transient modeling tool for air conditioning and refrigeration systems; whereas Thermolib is a third-party commercial toolbox of Matlab/Simulink for modeling and simulation of thermodynamic systems, and it has been used in a variety of applications including HVAC&R, energy generation, automobile and aerospace. In the recent years, quite a few studies based on Thermosys have been carried out (Rasmussen, 2002; Rasmussen and Alleyne, 2004; Rasmussen, 2005; Rasmussen *et al*., 2005; McKinley and Alleyne, 2008; Li and Alleyne, 2010). These studies were focused on control purposes, the moving boundary method was chosen for heat exchanger modeling owing to the fact that it is a good compromise between the computational cost and the accuracy of providing physical insights into system transients.

Modelica is a publicly available, object-oriented, equation based language for modeling large, complex physical systems. Models in Modelica are described by differential, algebraic and discrete equations (Modelica Association, 1996). Several Modelica-based simulation packages have been developed, e.g. the Thermo-Fluid Library (Eborn, 2001; Tummescheit, 2002), the AirConditioning Library (Modelon, 2007), the Modelica_Fluid Library (Casella *et al*., 2006), the HITLib (Videla and Lie, 2006), the TIL (Richter, 2008), the FluidFlow (Ljubijankic *et al*., 2009). Meanwhile, a series of papers have been published to address the transient modeling of thermo-fluid systems using Modelica (Jesen and Tummescheit, 2002; Casella and Schiavo, 2003; Elmqvist *et al*., 2003; Pfafferott and Schmitz, 2004; Tummescheit *et al*., 2005; Li *et al*., 2010).

Simscape (MathWorks, 2009) is a relatively new platform for equation-based modeling. It is an extension of Simulink for modeling complex interactions in multi-domain physical systems. There appears to be no evidence yet in open literature of using Simscape in the transient modeling of vapor compression systems (Trčka and Hensen, 2010).

The present study individually establishes transient simulation for a simple vapor compression system in these commercially available modeling environments. The three tools are based on quite different modeling concepts. Comparison of simulation results obtained by three tools shows good agreement, but the computation time varies significantly due to underlying model reduction and solution scheme used.

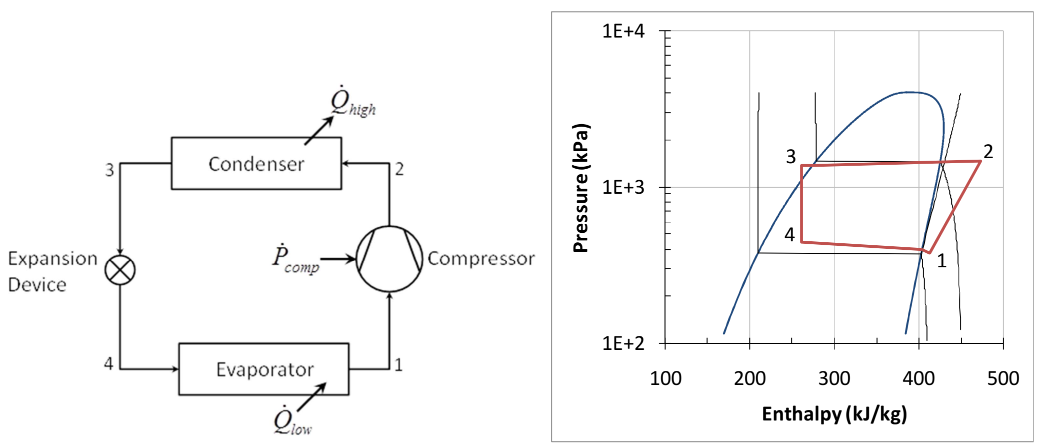

Figure 1: Schematic of a vapor compression system (Winkler, 2009)

2. SYSTEM DESCRIPTION

The studied system and its P-h diagram are shown in Figure 1. The hot vapor discharged by the compressor flows through the condenser coil where cooler air removes the heat from the vapor until it condenses into a subcooled liquid when exiting the condenser. Then the liquid refrigerant goes through the expansion valve where the pressure and temperature decrease abruptly. This process results in a vapor-liquid mixture flowing through the evaporator coil where the liquid evaporates. The resulting refrigerant vapor returns to the compressor inlet, and is then compressed and discharged at a higher temperature, thus completing a thermodynamic cycle.

3. MATHEMATICAL MODELS

In vapor compression systems, compressor, condenser, evaporator and expansion device are four essential components. The models for these four components are described hereinafter.

3.1 Compressor

In the transient simulation, the compressor is generally treated as a quasi-steady state component because the timescales associated with the variation of the compressor mass flow rate are very small compared to timescales associated with heat exchanger and charge distribution. Therefore, a full-scaled physics-based compressor model that captures the detailed transients during the compression process is not necessary considering the computation cost (Winkler, 2009). As a result, an efficiency-based compressor model is used in the study.

The mass flow rate is given by

$$\dot{m} = V_d N/60 \eta_V \rho_s \tag{1}$$

The discharge enthalpy is a function of the isentropic efficiency

$$h_d = \frac{h_{is,d} - h_s}{\eta_{is}} \tag{2}$$

If an adiabatic compression process is assumed, the compression power is estimated by

$$W = \dot{m}(h_d - h_s) \tag{3}$$

3.2 Orifice

Same as the compressor model, the dynamics of the valve itself are neglected. The mass flow rate through the valve is determined by the flow coefficient, the flow area, the inlet density and the pressure drop across the valve

$$\dot{m} = C_v A \sqrt{\rho_{in} \Delta p} \tag{4}$$

The throttling process is an isenthalpic one, hence the mass and energy conservation equations are

$$\dot{m}_o = \dot{m}_i \tag{5}$$
$$h_o = h_i \tag{6}$$

3.3 Heat Exchangers

Transient heat exchanger models generally fall into one of two categories; namely phase-independent finite volume models and phase-dependent moving boundary models (Bendapudi *et al.*, 2008). In finite volume models, the heat exchanger is subdivided into a fixed number of constant volumes and the manner in which the heat exchanger is subdivided is independent of the refrigerant phase. In moving boundary models, the heat exchanger is subdivided based on the location of the phase transition points. The length of each region changes with time as the transition points move throughout the heat exchanger. Compared with finite volume models, moving boundary models are more efficient in terms of computation because they utilize fewer control volumes. However, when it comes to simulation of fast transient behavior, such as startup and shut-down operations, moving boundary models tend to have difficulty in making a continuous, smooth and stable transition in the solving process when phase change occurs.

In general, the heat exchanger is analyzed by dividing it into three control volumes, i.e. refrigerant flow, finned wall and the air stream. The conservation laws are applied to each of these three control volumes. The models are mainly based on the following assumptions:

1) The number of tubes for each circuit is the same. Each circuit is treated as a long single-pass tube.
2) The air side is modeled as an incompressible fluid and thus the dynamics are negligible.
3) The refrigerant properties are considered uniform on the transverse section of the heat exchangers.
4) The axial heat conduction in the refrigerant flow direction is ignored.
5) In the two phase region, the liquid and the vapor are in thermodynamic equilibrium.
6) The potential energy and kinetic energy of the refrigerant are not taken into account.
7) The tubes and fins are modeled as a lumped section and thus the metal temperature is uniform.
8) The air passing over the evaporator is assumed to be in a dry condition, and the effects of dehumidification are not taken into account.

Based on the above assumptions, the governing equations for each control volume can be established.

<u>Refrigerant side</u>

$$V\left[\left(\frac{\partial\rho_r}{\partial p_r}\right)_h \frac{dp_r}{dt} + \left(\frac{\partial\rho_r}{\partial h_r}\right)_p \frac{dh_r}{dt}\right] = \dot{m}_{r,i} - \dot{m}_{r,o} \tag{7}$$

$$V\left[h_r\left(\frac{\partial\rho_r}{\partial p_r}\right)_h - 1\right]\frac{dp_r}{dt} + V\left[h_r\left(\frac{\partial\rho_r}{\partial h_r}\right)_p + \rho_r\right]\frac{dh_r}{dt} = \dot{m}_{r,i}h_i - \dot{m}_{r,o}h_o - Q_{rw} \tag{8}$$

$$p_{r,i} - p_{r,o} = f(\dot{m}) \tag{9}$$

<u>Finned wall</u>

$$\left(MC_p\right)\frac{dT_w}{dt} = Q_{rw} - Q_{wa} \tag{10}$$

<u>Air side</u>

$$T_{a,o} = T_w - \left(T_w - T_{a,i}\right)exp\left(-\frac{\eta_f\alpha_o A_T}{\dot{m}_a c_{p,a}}\right) \tag{11}$$

A staggered grid scheme is used to discretize the equations on the refrigerant side because it can give better convergence properties in cases of dealing with pressure gradient. Explicitly, mass and energy balance are calculated in control volumes, whereas the momentum balance is calculated between control volumes (Tummescheit, 2002).

4. MODEL IMPLEMENTATION

Figure 2 shows the Simulink model of a basic vapor compression system with four components. It can be noted that the boundary conditions for each component model are the inlet refrigerant mass flow rate, the inlet refrigerant enthalpy and the outlet refrigerant pressure. The outputs of each component model are the outlet refrigerant mass flow rate and the outlet refrigerant enthalpy, which are parts of the boundary conditions of the next downstream component model. As a result, the components can be seamlessly connected together in an arbitrary manner regardless of whether a component is flow equipment like compressors and valves or heat transfer equipment like heat exchangers.

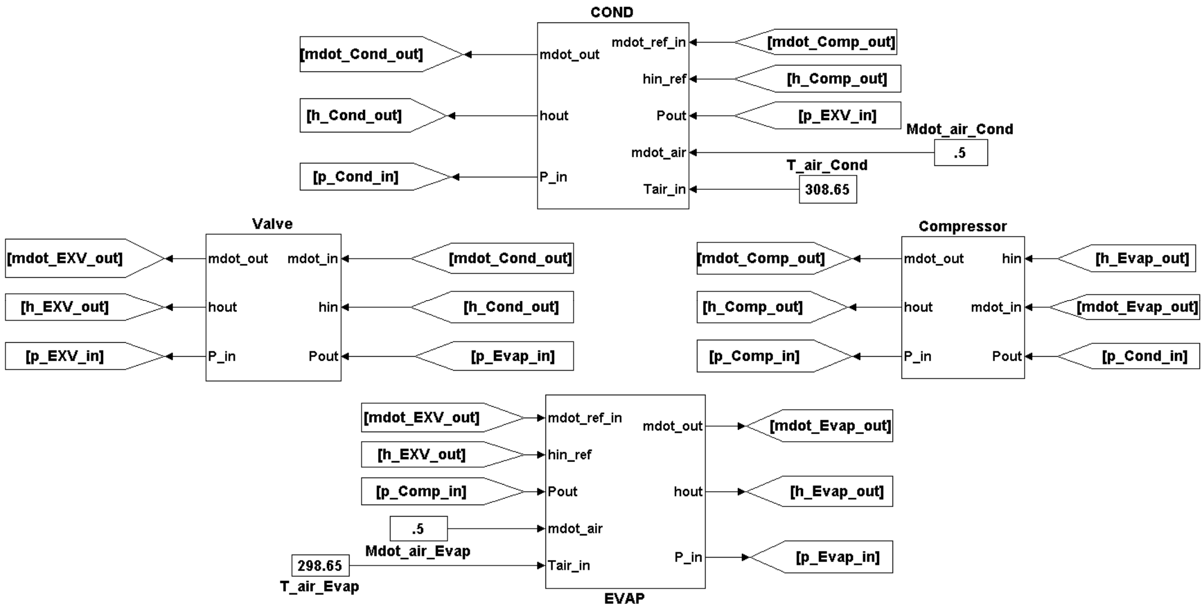

Figure 2: Simulink model of a simple vapor compression cycle

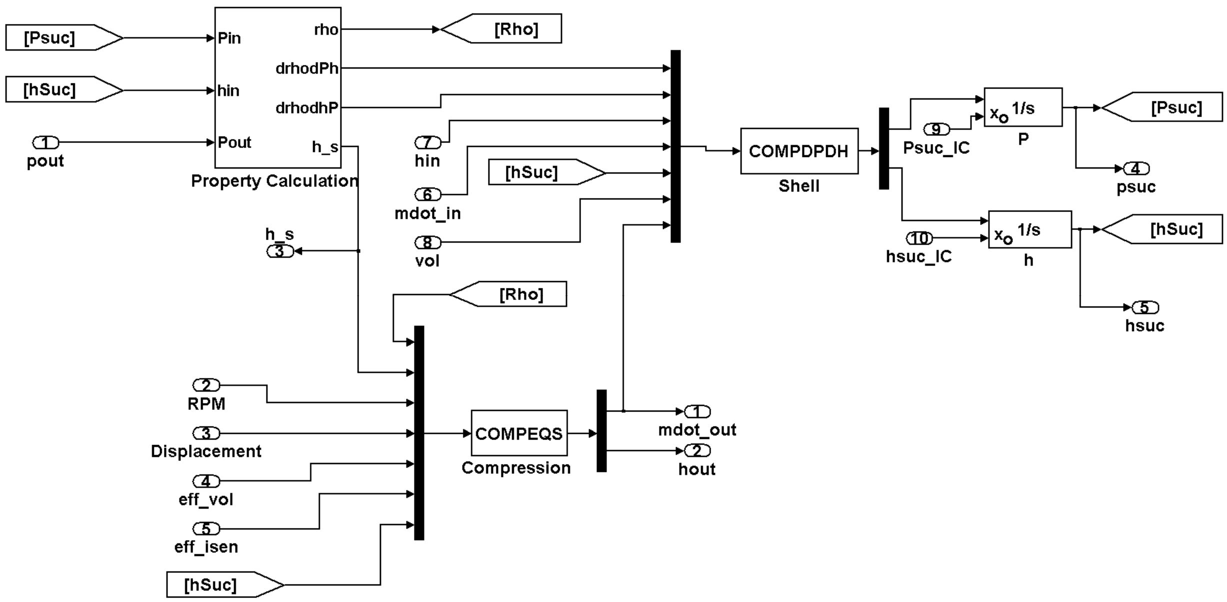

Figure 3: Simulink compressor model

Figure 3 takes the compressor model as an example to illustrate how a component model is constructed in Simulink. The heat exchanger model is not shown in this paper due to its complexity and space constraint. It can be seen that the compressor model consists of three parts. The first part is the free volume inside the compressor shell. This part is treated as a lumped section on which mass and energy conservations are posed and the resulting equations are described in the Shell module. The second part is the compression process. As a matter of fact, this part is used to calculate the mass flow rate through the compressor and other relevant physical quantities. This part is realized by the Compression module. The last part is the associated refrigerant property calculations which are incorporated in the Property Calculation module. It must be pointed out that there is no easy way to directly call refrigerant property routines in the Simulink code. Therefore, S-functions have been used in the Simulink engine to be able to interact with the C++ refrigerant property routines.

Figure 4 depicts how a vapor compression system looks like in Simscape. One of the most prominent differences between the Simulink model and the Simscape model is that a strict input-output relationship is defined in the Simulink model. Inputs are determined outside the model whereas the outputs are determined by factors inside the model. That is how the causal modeling approach normally works. The causal modeling method works pretty well for control systems, however, it brings great difficulties when it comes to the modeling of physical systems because it is necessary to deduce the data/signal flow in advance in order to get the connection relationship of all the elements correct. This makes model reuse very difficult and the system topology is not easy to be retained when changes in the physical structure are made.

However, a different modeling approach is used in the Simscape model, i.e. acausal modeling. In the model, no signal flow, i.e. cause-and-effect flow, is specified (Fritzson, 2003). The connection relationship between components only represents the physical topology and does not provide any other special meaning. Kirchhoff's first law is applied at the nodes where components are connected, i.e. all through variables (mass flow rate and enthalpy flow rate) need to sum to zero and all the across variables (pressure and enthalpy) should be equal.

As a comparison, Figure 5 shows how the compressor model is established in Simscape. The model is also comprised of three parts like the Simulink compressor model. It must be pointed out that again there is also no easy way to carry out the refrigerant property calculation and therefore Simulink S-functions are used. That is the reason why the casual modeling method is still used when passing around the refrigerant properties among different modules.

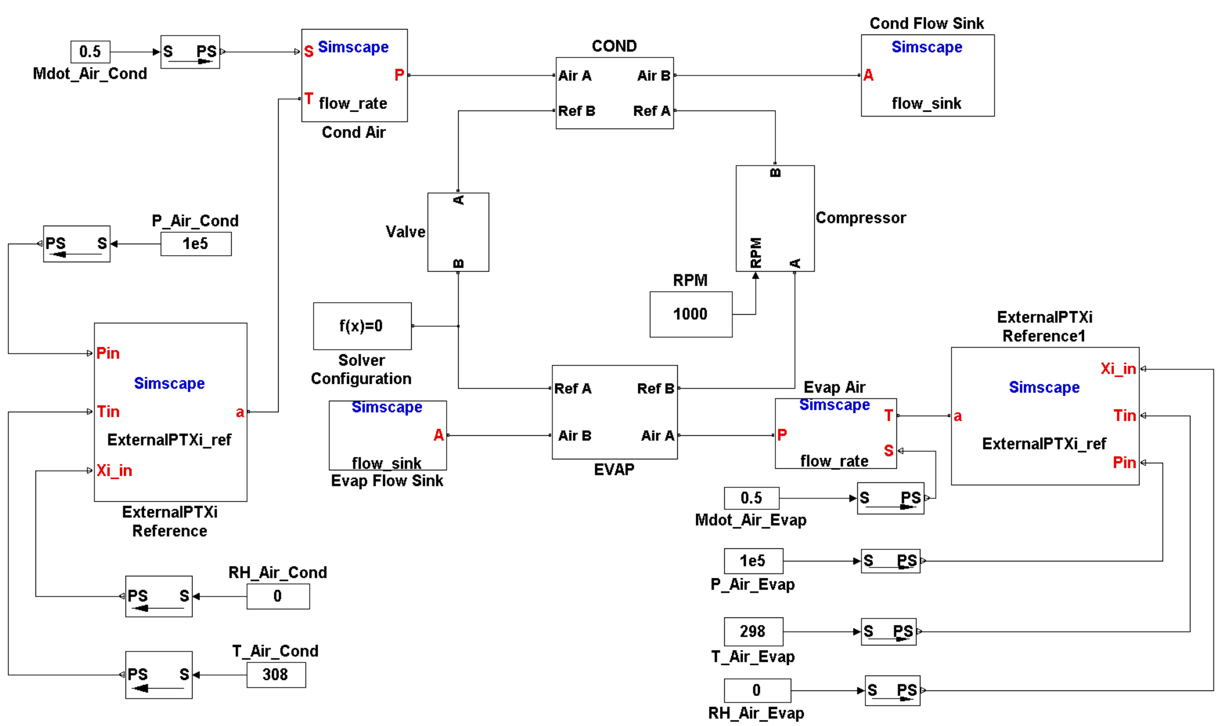

Figure 4: Simscape model of a simple vapor compression cycle

As indicated previously, the heat exchanger is discretized using the staggered grid scheme on the refrigerant side. The mass and energy balance are calculated in control volume models, whereas the momentum balance is calculated in flow models. Each component model consists of a series of control volume models and flow models which are connected in an alternating sequence. The basic ideas here are very similar to what is described in (Tummescheit, 2002). More details can be found in this reference. Figure 6 shows the interface for a control volume model which accepts the heat transfer area, heat transfer coefficients and the initial state as inputs and calculates the derivatives of the pressure and the mean enthalpy of the control volume. The interface of a flow model is shown in Figure 7. Detailed explanation of the pressure drop calculation will be presented in the next section.

For the sake of brevity, the paper will not show the schematic of Modelica model because Modelica has been widely used nowadays and it is pretty easy for readers to find a Modelica model (Pfafferott and Schmitz, 2004; Richter, 2008; Li *et al*., 2010) for a vapor compression system and its components.

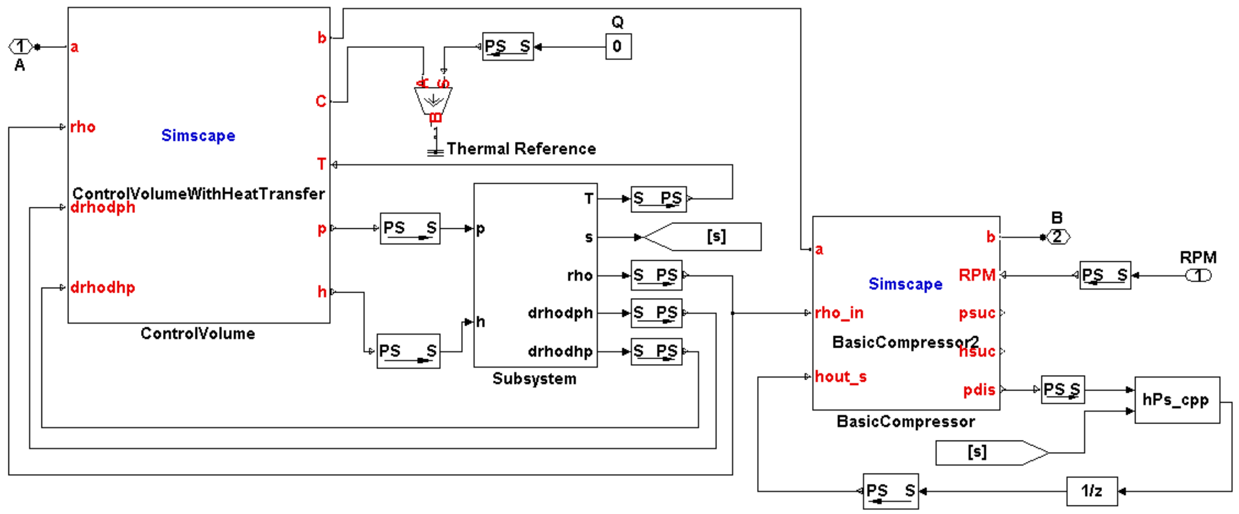

Figure 5: Simscape compressor model

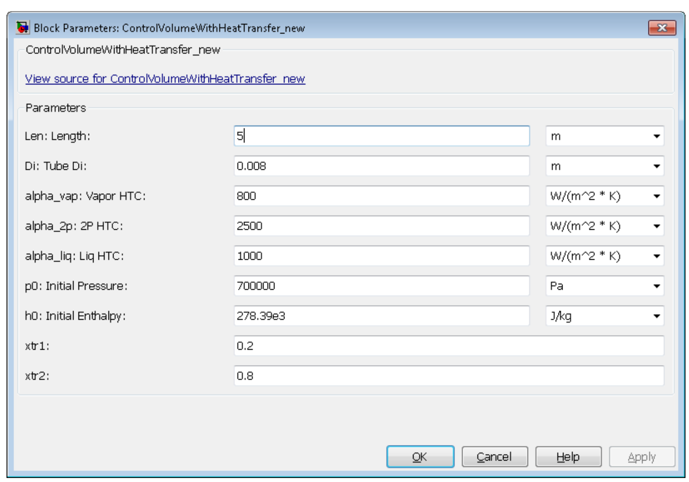

Figure 6: Interface of the control volume model in Simscape

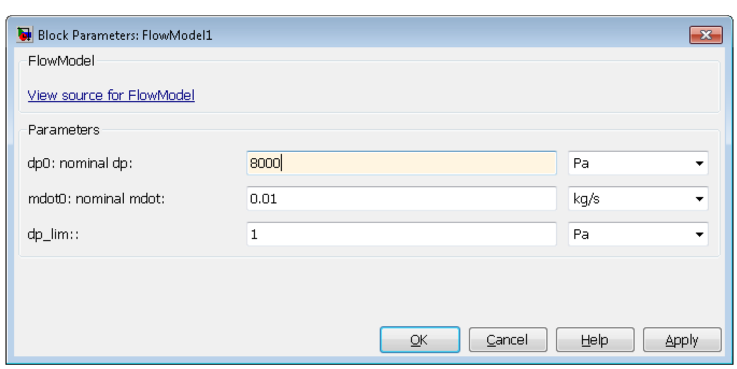

Figure 7: Interface of the flow model in Simscape

5. CASE STUDY AND DISCUSSIONS

The present study tries to simulate the startup transients of a simple vapor compression system in these three modeling platforms, respectively. Since the study is rather about the evaluation and comparison of different modeling approaches than presenting the mathematic modeling strategy, the component models are greatly simplified.

The heat transfer coefficients area not calculated based on empirical correlations. They are assumed to be constants for each regime on the refrigerant side and the air side. A first order smooth function is applied at the phase transitions ($0 < x < 0.2$ and $0.8 < x < 1$). The function ensured a smooth transition in heat transfer to avoid model failure or a slow computation speed.

The frictional pressure drop is a simple function of mass flow rate and is given by

$$\Delta p = k\dot{m}|\dot{m}| \tag{12}$$

It is assumed that the flow is only turbulent. The constant k is defined by providing Δp and *m_dot* for nominal flow conditions (Elmqvist *et al*., 2003).

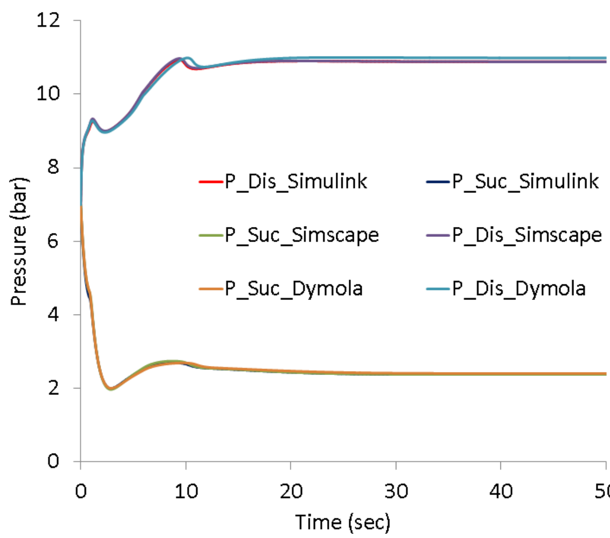

Figure 8: Pressure change

Figure 8 shows the change in the suction and discharge pressure during startup. It can be seen that the pressure transients are exactly the same except the Dymola result is slightly different from that of Simulink and Simscape. This mainly comes from two aspects. Firstly, these three modeling environments use different integration solvers. ODE45 was used in the Simulink and Simscape models whereas DASSL was used in the Dymola model.

Secondly, refrigerant property routines cause slight differences. In the Simulink and Simscape models, constant specific heat for the air was used to avoid calling the external air property routines. While in the Dymola model, the air side property package was based on the MoistAir package from the AirConditioning Library (Modelon, 2007). Meanwhile, the refrigerant property calculation in the Simulink and Simscape models were based on curve-fitted polynomials, whereas the refrigerant property package from the AirConditioning Library (Modelon, 2007) was used in the Dymola model.

In terms of the computation time, ODE45 and ODE15s were compared in the Simulink and the Simscape models. For the Simulink model, these two solvers resulted in very different computation efficiency. ODE45 seemed much more efficient for the Simulink model; the computation time took 297 seconds. However, it took 825 seconds for

ODE15s to solve the case. For the Simscape model, these two solvers showed minute difference. ODE15s spent 125 seconds whereas ODE45 spent 124 seconds. For the Dymola model, the overall computation time was 32 seconds.

Since the paper is not meant to focus on the mathematical modeling of vapor compression systems and the discussion of whether the transients make sense or not, other system transient behavior is not shown here.

6. CONCLUDING REMARKS

The present study individually established transient simulation for a simple vapor compression system in three commercially available modeling environments, i.e. Simulink, Simscape and Modelica. These three platforms demonstrate quite different modeling concepts. Simulink is based on the causal modeling approach wherein a strict input-output relationship must be defined in the model; whereas the Simscape model and the Modelica model are based on the acausal modeling approach wherein no signal flow is needed to be specified. Comparison of simulation results obtained by three tools shows good agreement. The discrepancy is mainly due to different property routines and how they are handled. Meanwhile, the computation time varies significantly due to underlying model reduction and solution scheme used.

NOMENCLATURE

A	area	η	efficiency
C_p	specific heat	α	heat transfer coefficient
h	enthalpy	Δ	difference
k	constant		
M	mass	Subscripts	
p	pressure	a	air
Q	heat transfer rate	d	discharge
T	temperature	i	inlet
t	time	o	outlet
V	volume	r	refrigerant
W	power	s	suction
ρ	density	w	wall
$\dot{m}$	mass flow rate		

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