HW2 Stat-comp (due Sunday Oct. 26th 8pm in D2L)

1) Predicting confidence bands for logistic regression using Bootsrap

Include here your code and results from INCLASS-10

2) Maximum likelihood estimation and inference with the exponential distribution

The density function of an exponential random variable is

$f(x_i|\lambda) = \lambda e^{-\lambda x_i}$

where $x_i \geq 0$ is the random variable, and $\lambda > 0$ is a rate parameter.

The expected value and variance of the random variables are $E[X] = \frac{1}{\lambda}$ and $Var[X] = \frac{1}{\lambda^2}$.

The following code simulates 50 IID draws from an exponential distribution

```r
set.seed(195021)
x=rexp(n=50,rate=2)
```

The maximum likelihood estimate of λ has a closed form, indeed

$L(\lambda|x) = \lambda^n e^{-\lambda n\bar{x}}$

Thus, $l(\lambda|x) = nlog(\lambda) - \lambda n\bar{x}$, therefore

$\frac{dl}{d\lambda} = \frac{n}{\lambda} - \lambda n\bar{x}$. Setting this derivative equal to zero, and solving for $\hat{\lambda}$ gives $\hat{\lambda} = \frac{1}{\bar{x}}$

Using numerical optimization to estimate λ:

Since $\lambda > 0$, we need to be careful using `optim()` because this function may report an estimate smaller than zero. Furthermore, for models involving a single parameter, `optimize()` is preferred relative to `optim()`; `optimize()` allows you to provide an interval for the optimization.

2.1) Use `optimize()` to estimate λ compare your estimate with $\frac{1}{\bar{x}}$.

2.2 Use numerical methods to proivde an approximate 95% CI for your estimate.

Hint: `optimize()` does not provide a Hessian. However, you can use the `hessian()` function of the `numDeriv` R-package to obtain a numerical approximation to the second order derivative of the logLikelihood at the ML estiamte. To install this package you can use

```r
install.packages(pkg='numDeriv',repos='https://cran.r-project.org/')
```

3) Bootstrap

3.1) Use 10,000 bootstrap samples to estimate the SE. Compare your results with those reported in the previous question.

3.2) Report 95% CI assuming normality using the SE from question 2 and the SE from 3.1, and compare these CIs with those obtained with the percentile method (i.e., applying `quantile(x=bootstrap_estimates,prob=c(.025,.975))` to the bootstrap samples).

3.3) Compare the estimate obtained with the sample, with the average Bootstrap estimate. Do we have any evidence that the estimator may be biased?