Computer Engineering Department
Faculty of Engineering
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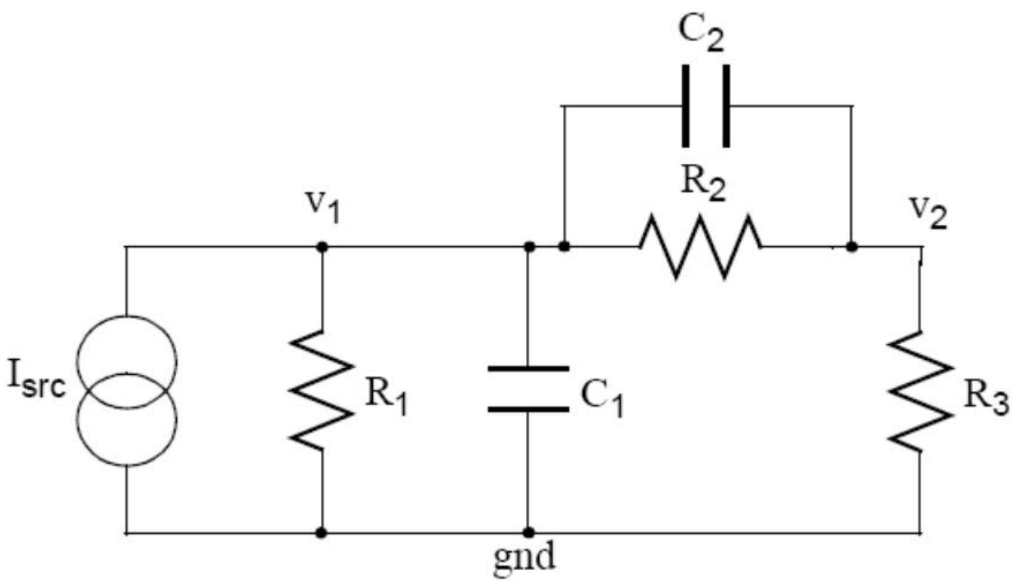
Modeling and Simulation Lab# 3: SPICE Simulation

Agenda

- SPICE Simulator
- Modified Nodal analysis

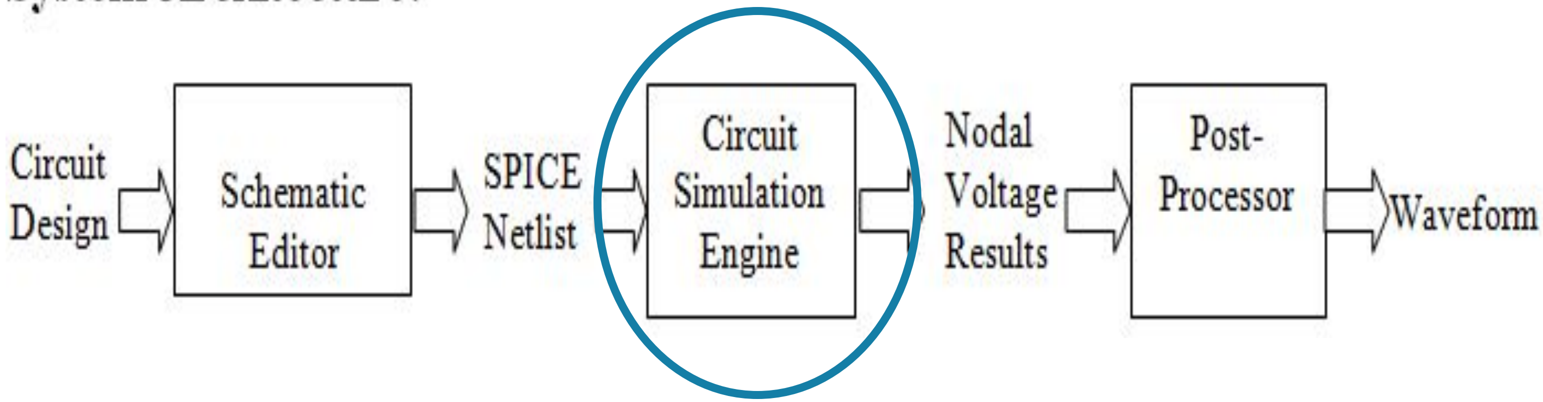
SPICE Simulator

It takes an analog circuit schematic, simulates, calculates and plots the voltage at each node, and the current produced by each voltage source.



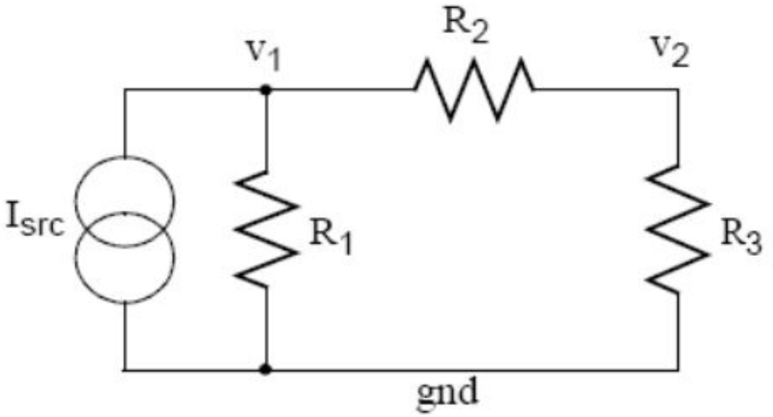
SPICE Simulator

System Architecture:



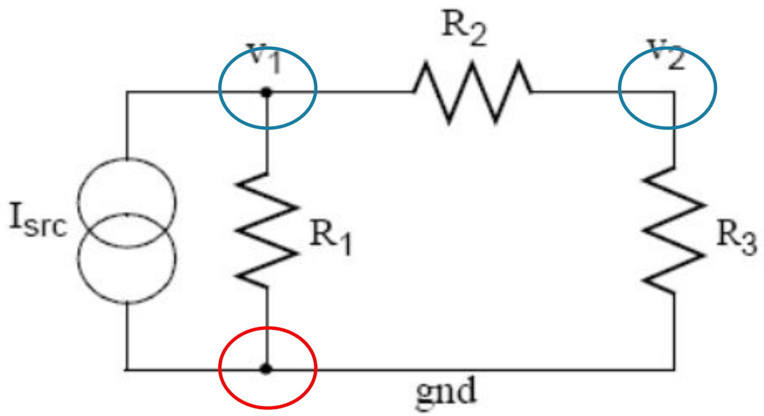
Circuit Simulation Engine

Requirement: Find V1,V2



Simulation solution steps:

1. Find Nodes in the circuit
   - 2 Nodes
   - 1 ground node



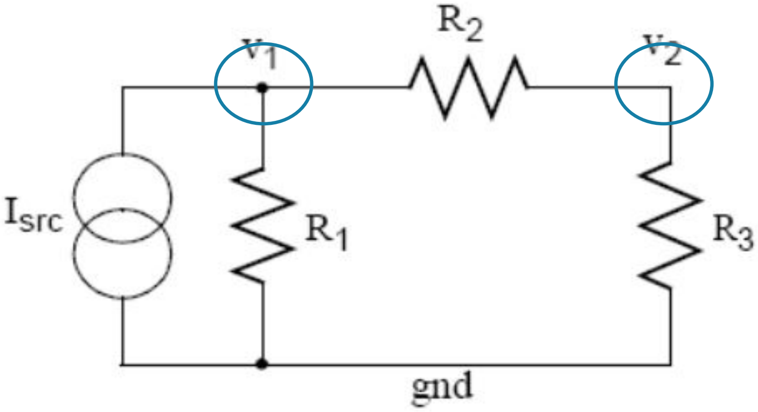
Circuit Simulation Engine

Simulation solution steps[Cont’]:

2. Make KCL at each node.

$$\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} = i_{src}$$

$$\frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$



-> Replace Resistance with conductance G=1/R

$$G_1 v_1 + G_2(v_1 - v_2) = i_{src}$$
$$G_2(v_1 - v_2) = G_3 v_2$$

$$\Rightarrow$$

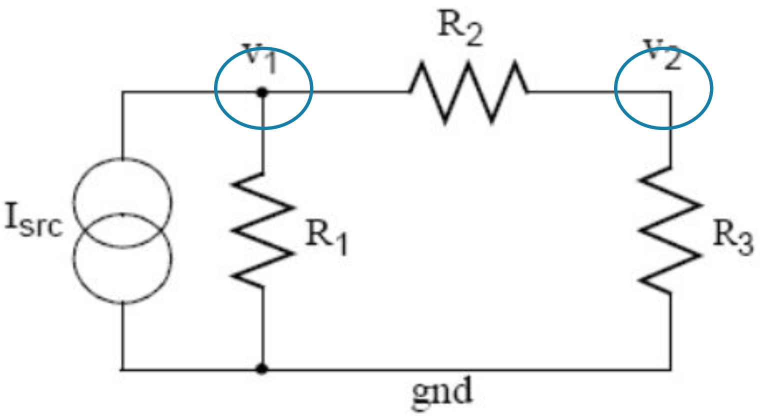
$$(G_1 + G_2)v_1 - G_2 v_2 = i_{src}$$
$$G_2 v_1 - (G_2 + G_3)v_2 = 0$$

Circuit Simulation Engine

Simulation solution steps[Cont']:

3. Create conductance matrix.

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ G_2 & -(G_2 + G_3) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_{src} \\ 0 \end{bmatrix}$$



$$\boxed{Gv=I}$$

4. Solve using by a simple matrix manipulation.

$$\boxed{v = G^{-1}I}$$

Modified Nodal analysis (MNA)

Straightforward analog circuit solver technique based on KCL and KVL.

MNA applied to a circuit with only passive elements (resistors) and independent current and voltage sources results in a matrix equation of the form:

$$\boxed{Ax=z}$$

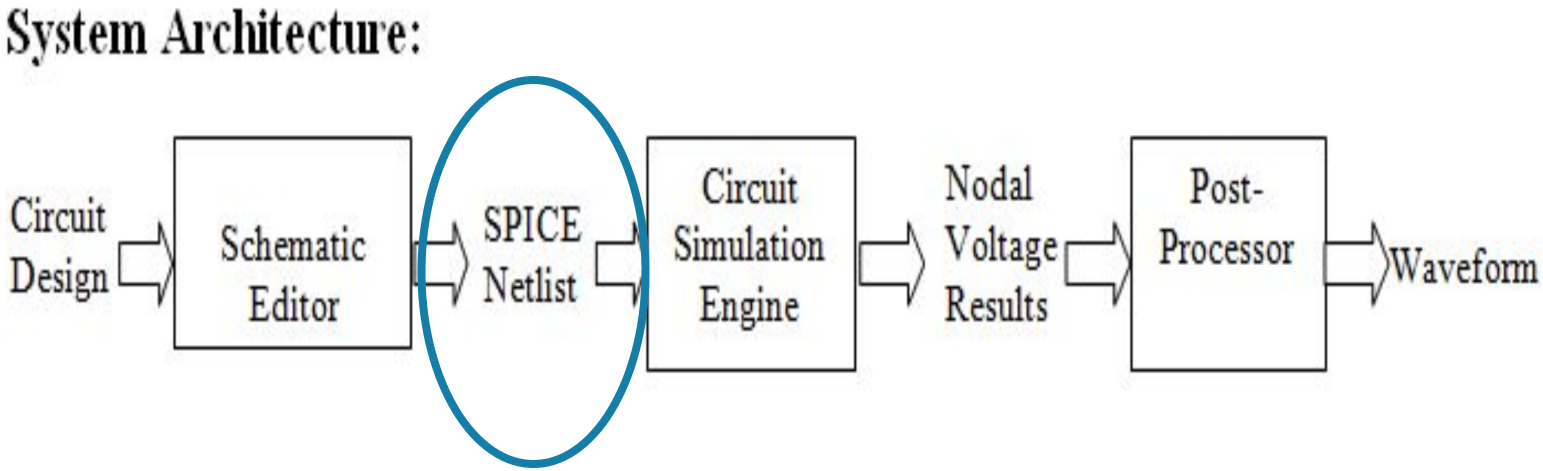
- A is a matrix consists only of known quantities.
- Z is a vector holds only known quantities
- X is a vector holds the unknown quantities (nodal voltages and source currents)

The circuit is solved by a simple matrix manipulation:

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{z}$$

SPICE Netlist

1. <u>Find Nodes in the circuit</u>



Netlist Format

Component_Type | Node1 | Node2 | Value | Initial_Value

- Component_Type stands for the type of the component. It can be one of the following:

Voltage Source "Vsrc".

Current Source "Isrc".

Resistance "R".

Inductor "I".

Capacitor "C".

Netlist Format[Cont']

Node1 and Node2 stands for the nodes' numbers to which the component is connected to. (for uni-polar components as Vsrc, Node1 is the +ve port, Node2 is the –ve port)

Value is the physical value of the component.

Initial_Value is the initial current or voltage that is observed on the component at time=0.

Circuit Simulation Engine

Solve Ax=z

The Matrix A

The A matrix will be developed as the combination of 4 smaller matrices, G, B, C, and D.

$$\mathbf{A} = \begin{bmatrix} \mathbf{G} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$$

The **A** matrix is $(m+n)$x$(m+n)$ **(*n* is the number of nodes, and *m* is the number of independent voltage sources).**

The G matrix

$$A = \begin{bmatrix} G & B \\ C & D \end{bmatrix}$$

The **G** matrix is *nxn* and is determined by the interconnections between the passive circuit elements (resistors)

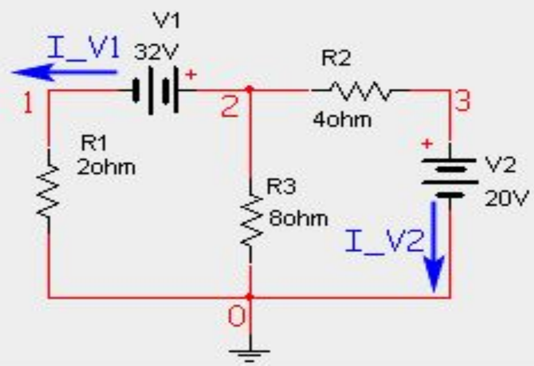
<u>The G matrix is an *nxn* matrix formed in two steps:</u>

1-Each element in the diagonal matrix is equal to the sum of the conductance of each element connected to the corresponding node.

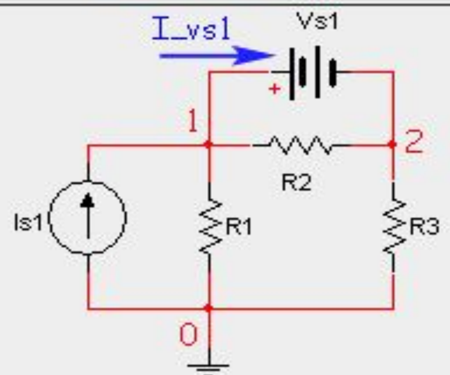
2-The off diagonal elements are the negative conductance of the element connected to the pair of corresponding node.

Example:

Case 1

Circuit Diagram	MNA Equations
	$G = \begin{bmatrix} \frac{1}{R_1} & 0 & 0 \\ 0 & \frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2} \end{bmatrix}$

Case 2

Circuit Diagram	MNA Equations
	$G = \begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3} \end{bmatrix}$

The B matrix

$$A = \begin{bmatrix} G & B \\ C & D \end{bmatrix}$$

The **B** matrix is *nxm* and is determined by the connection of the voltage sources.

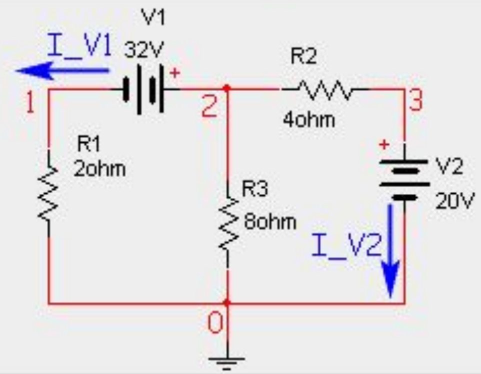
<u>The B matrix is an *nxm* matrix with only 0, 1 and -1 elements</u>

Each location in the matrix corresponds to a particular voltage source and a node.

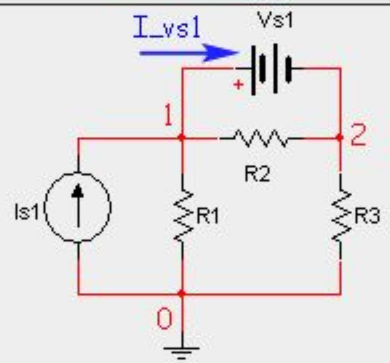
- If the positive terminal of the *i*th voltage source is connected to node *k*, then the element (*k,i*) in the **B** matrix is a 1.
- If the negative terminal of the *i*th voltage source is connected to node *k*, then the element (*k,i*) in the **B** matrix is a -1.
- Otherwise, elements of the **B** matrix are zero.

Example

Case 1

Circuit Diagram	MNA Equations
	$\mathbf{B}=\begin{bmatrix}-1 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}$

Case 2

Circuit Diagram	MNA Equations
	$\mathbf{B}=\begin{bmatrix}1\\ -1\end{bmatrix}$

The C & D matrices

$$\mathbf{A} = \begin{bmatrix} \mathbf{G} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$$

- the **C** matrix is *mxn* and is determined by the connection of the voltage sources.**It is the transpose of the B matrix**.
- The **D** matrix is *mxm* and is zero if only independent sources are considered.
- These are not the case when dependent sources are present.

$$\text{Case 1:}\ \mathbf{C} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Case 2:}\ \mathbf{C} = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$\text{Case 1:}\ \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{Case 2:}\ \mathbf{D} = \begin{bmatrix} 0 \end{bmatrix}$$

Circuit Simulation Engine

Solve Ax=z

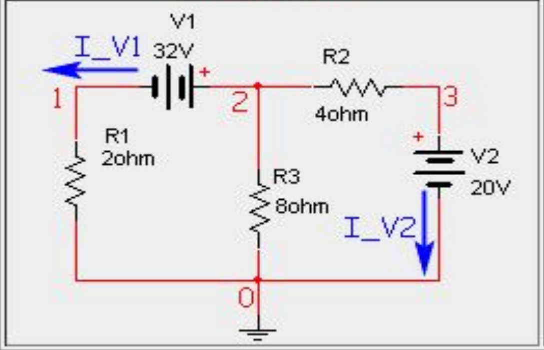
The X matrix:

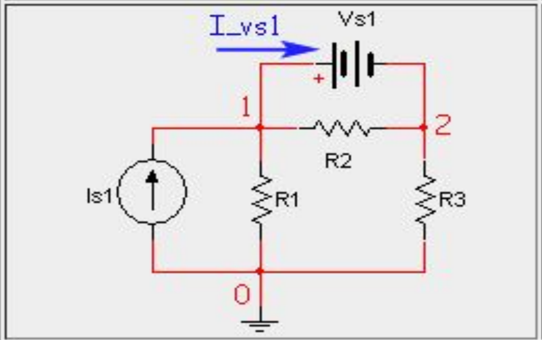
The **x** matrix holds our unknown quantities and will be developed as the combination of 2 smaller matrices **v** and **j**.

$$\mathbf{x} = \begin{bmatrix} \mathbf{v} \\ \mathbf{j} \end{bmatrix}$$

- The **v** matrix is *n*x1 and hold the unknown voltages. Each element in **v** corresponds to the voltage at the equivalent node in the circuit
- The **j** matrix is *m*x1 and holds the unknown currents through the voltage sources.

Example

Case 1	
Circuit Diagram	MNA Equations
	$\mathbf{v}=\begin{bmatrix} v\_1 \\ v\_2 \\ v\_3 \end{bmatrix}, \mathbf{j}=\begin{bmatrix} i\_V1 \\ i\_V2 \end{bmatrix}, \quad \mathbf{x}=\begin{bmatrix} \mathbf{v} \\ \mathbf{j} \end{bmatrix}=\begin{bmatrix} v\_1 \\ v\_2 \\ v\_3 \\ i\_V1 \\ i\_V2 \end{bmatrix}$

Case 2	
Circuit Diagram	MNA Equations
	$\mathbf{v}=\begin{bmatrix} v\_1 \\ v\_2 \end{bmatrix}, \mathbf{j}=\begin{bmatrix} i\_Vs1 \end{bmatrix}, \quad \mathbf{x}=\begin{bmatrix} \mathbf{v} \\ \mathbf{j} \end{bmatrix}=\begin{bmatrix} v\_1 \\ v\_2 \\ i\_Vs1 \end{bmatrix}$

Circuit Simulation Engine

Solve Ax=z

The Z matrix:

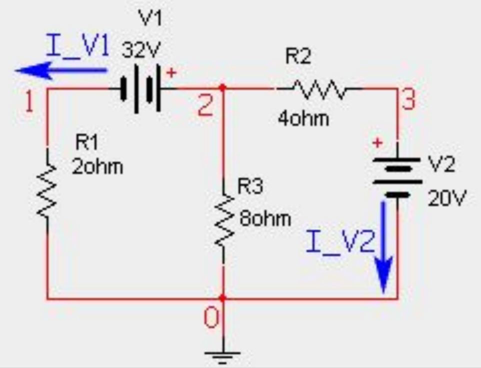
The z matrix holds our independent voltage and current sources and will be developed as the combination of 2 smaller matrices i and e.

$$\mathbf{z} = \begin{bmatrix} \mathbf{i} \\ \mathbf{e} \end{bmatrix}$$

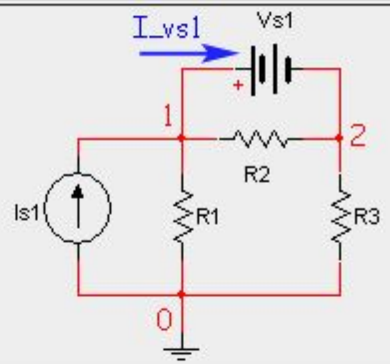
- The **i** matrix is *n*x1 and contains the sum of the currents through the passive elements into the corresponding node (either zero, or the sum of independent current sources).
- The **e** matrix is *m*x1 and holds the values of the independent voltage sources.

Example

Case 1

Circuit Diagram	MNA Equations
	$\mathbf{i}=\begin{bmatrix}0\\0\\0\end{bmatrix}, \mathbf{e}=\begin{bmatrix}V1\\V2\end{bmatrix}, \quad \mathbf{z}=\begin{bmatrix}0\\0\\0\\V1\\V2\end{bmatrix}$

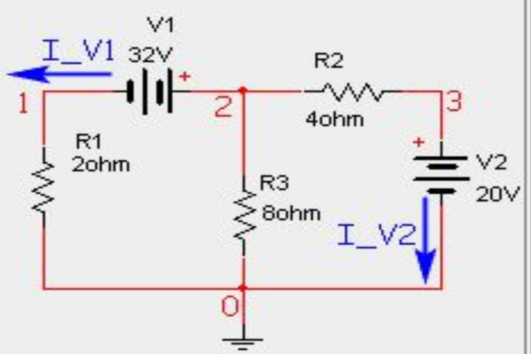
Case 2

Circuit Diagram	MNA Equations
	$\mathbf{i}=\begin{bmatrix}Is1\\0\end{bmatrix}, \mathbf{e}=\begin{bmatrix}Vs1\end{bmatrix}, \quad \mathbf{z}=\begin{bmatrix}Is1\\0\\Vs1\end{bmatrix}$

Putting all Together

Case 1

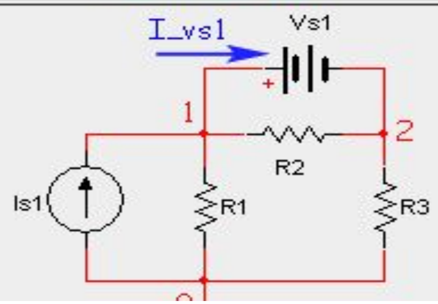
Circuit Diagram	MNA Equations



$$\begin{bmatrix} \frac{1}{R_1} & 0 & 0 & -1 & 0 \\ 0 & \frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & 1 & 0 \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2} & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v\_1 \\ v\_2 \\ v\_3 \\ i\_V1 \\ i\_V2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V1 \\ V2 \end{bmatrix}$$

Case 2

Circuit Diagram	MNA Equations



$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} & 1 \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3} & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} v\_1 \\ v\_2 \\ i\_Vs1 \end{bmatrix} = \begin{bmatrix} Is1 \\ 0 \\ Vs1 \end{bmatrix}$$

MNA stamp

Resistor MNA Stamp:

$$\begin{array}{cccc} & V+ & V- & RHS \\ V+ & G & -G & 0 \\ V- & -G & G & 0 \end{array}$$

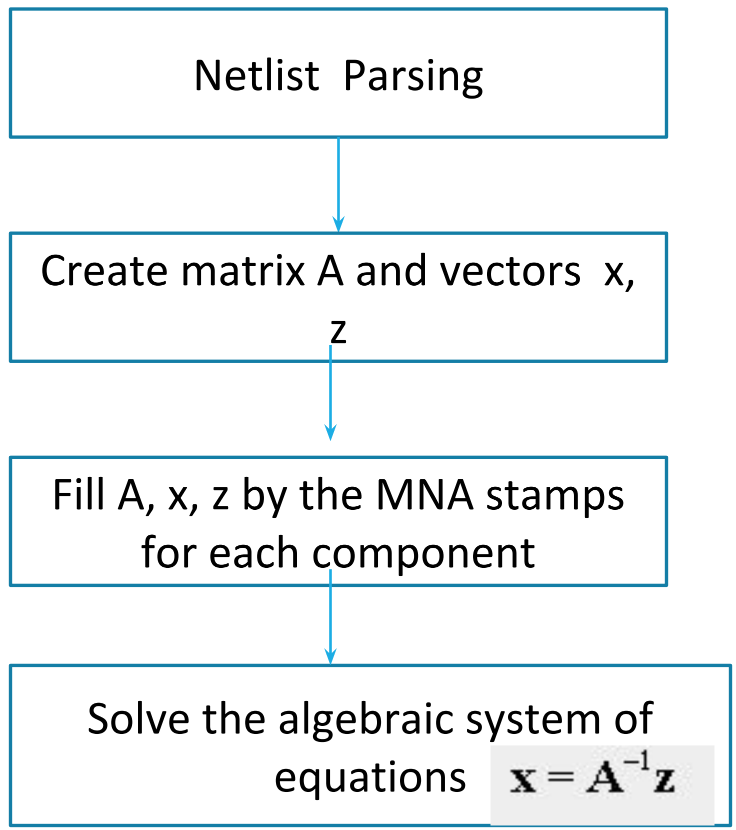
Current Source MNA Stamp:

$$\begin{array}{cccc} & V+ & V- & RHS \\ V+ & o & o & I_r \\ V- & o & o & -I_r \end{array}$$

Voltage Source MNA Stamp:

$$\begin{array}{ccccc} & V+ & V- & Ir & RHS \\ V+ & 0 & 0 & 1 & 0 \\ V- & 0 & 0 & -1 & 0 \\ r & 1 & -1 & 0 & V_r \end{array}$$

Recap-System Flow chart




Any
Questions