# Final Lab Exam - Comp 304 - Spring 24-25

## Task 1: Elimination of $\varepsilon$-Productions

**Question:** Given a context-free grammar $G$, eliminate all $\varepsilon$-productions (productions of the form $A \to \varepsilon$) from $G$ to obtain an equivalent $\varepsilon$-free grammar $G'$ that generates the same language except possibly for the empty string if $S \to \varepsilon$ was originally present.

**Algorithm:**

1. Copy all non $\varepsilon$-productions from $G$ to the new grammar.

2. Find all nullable symbols

   (a) Initialize a nullable set $N$ with all non-terminals $A$ where $A \to \varepsilon$.

   (b) For all productions $A \to X_1 X_2 \ldots X_k$. If all $X_i \in N$, add $A$ to $N$.

   (c) Repeat the above step until $N$ does not change.

3. For each productions $A \to \alpha$ (where $\alpha$ is a sequence of terminals and non-terminals), generate new productions:

   (a) For each subset of nullable symbols in $\alpha$, create a new production by removing those nullable symbols.

   (b) Exclude the case where all symbols are removed (empty RHS), unless $A$ is the start symbol and $\varepsilon$ is in the language.

4. Remove all original $\varepsilon$-productions (except possibly $S \to \varepsilon$ if language contains $\varepsilon$).

**Example:**

| Input | Step 1 | Step 2 | Step 3 | Output |
|---|---|---|---|---|
| $S \to A\,B\,C \mid D$ <br> $A \to a\,A \mid \varepsilon$ <br> $B \to b\,B \mid \varepsilon$ <br> $C \to c$ <br> $D \to \varepsilon$ | $S \to A\,B\,C \mid D$ <br> $A \to a\,A$ <br> $B \to b\,B$ <br> $C \to c$ | $N = \{A, B, D, S\}$ | $S \to ABC$ <br> $\mid BC \mid AC \mid C$ <br> $\mid \varepsilon$ <br> $A \to aA \mid a$ <br> $B \to bB \mid b$ <br> $C \to c$ | $S \to ABC \mid BC \mid AC \mid C \mid \varepsilon$ <br> $A \to aA \mid a$ <br> $B \to bB \mid b$ <br> $C \to c$ |
| $S \to a\,S\,b\,S$ <br> $\mid b\,S\,a\,S$ <br> $\mid \varepsilon$ | | | | $S \to a\,S\,b\,S$ <br> $\mid a\,b\,S$ <br> $\mid a\,S\,b$ <br> $\mid a\,b$ <br> $\mid b\,S\,a\,S$ <br> $\mid b\,a\,S$ <br> $\mid b\,S\,a$ <br> $\mid b\,a$ <br> $\mid \varepsilon$ |