Propositional Logic

Commutative laws	$P \vee Q \equiv Q \vee P$ $P \wedge Q \equiv Q \wedge P$
Associative laws	$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$ $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$
Distributive laws:	$(P \vee Q) \wedge (P \vee R) \equiv P \vee (Q \wedge R)$ $(P \wedge Q) \vee (P \wedge R) \equiv P \wedge (Q \vee R)$
Identity	$P \vee F \equiv P,\ P \wedge T \equiv P$
Negation	$P \vee \sim P \equiv T$ (excluded middle) $P \wedge \sim P \equiv F$ (contradiction)
Double negation	$\sim(\sim P) \equiv P$
Idempotent laws	$P \vee P \equiv P$ $P \wedge P \equiv P$
De Morgan's Laws	$\sim(P \vee Q) \equiv \sim P \wedge \sim Q$ $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$
Universal bound laws (Domination)	$P \vee T \equiv T$ $P \wedge F \equiv F$
Absorption Laws	$P \vee (P \wedge Q) \equiv P$ $P \wedge (P \vee Q) \equiv P$
Negation of **T** and **F**	$\sim T \equiv F, \sim F \equiv T$

Example

- Consider the problem of representing the following information:
  - Every person is mortal.
  - Confucius is a person.
  - Confucius is mortal.
- How can these sentences be represented so that we can infer the third sentence from the first two?

Solution

P = "person"; Q = "mortal"; R = "Confucius"

$P \to Q$; $R \to P$; $R \to Q$

- To represent other individuals we must introduce separate symbols for each one, with some way to represent the fact that all individuals who are "people" are also "mortal"

Quiz

- Construct a truth table for the following compound proposition:

$$(p \vee q) \to (p \wedge q).$$

- Let p stand for the proposition "I bought a lottery ticket" and q for "I won the jackpot". Express the following as natural English sentences:
  - $\neg p$
  - $p \vee Q$
  - $p \wedge q$
  - $p \to Q$
  - $\neg p \to \neg q$
  - $\neg p \vee (p \wedge q)$

First-Order Logic

- is also known as First-order Predicate Calculus, the Lower Predicate Calculus, Quantification Theory, and Predicate Logic.
- First-order logic is distinguished from propositional logic by its use of quantified variables as :
  - Quanitfier → ∃ | ∀ where ∀ x asserts that a sentence is true for all values of variable x for example "all man loves soccer" could be represented as ∀x man(x)→love (x, soccer).
  - While ∃x asserts that a sentence is true for at least one value of a variable x for example "Haneen is not the youngest girl" could be represented as ∃x younger (x, Haneen).

First-Order Logic

- Function symbols, which map individuals to individuals
  - father-of(Mary) = John
  - color-of(Sky) = Blue
- Predicate symbols, which map individuals to truth values
  - greater(5,3)
  - green(Grass)
  - color(Grass, Green)

- Example
  - (∀x) student(x) → smart(x) means "All students are smart"
  - (∀x)student(x)∧smart(x) means "Everyone in the world is a student and is smart"
  - (∃x) student(x) → smart(x) means " there is a student who is smart"

Examples

Every gardener likes the sun.

$\forall x$ gardener(x) $\rightarrow$ likes(x,Sun)

You can fool some of the people all of the time.

$\exists x\ \forall t$ person(x) $\wedge$time(t) $\rightarrow$ can-fool(x,t)

You can fool all of the people some of the time.

$\forall x\ \exists t$ (person(x) $\rightarrow$ time(t) $\wedge$can-fool(x,t))

$\forall x$ (person(x) $\rightarrow \exists t$ (time(t) $\wedge$can-fool(x,t)))

All purple mushrooms are poisonous.

$\forall x$ (mushroom(x) $\wedge$ purple(x)) $\rightarrow$ poisonous(x)

No purple mushroom is poisonous.

$\neg\exists x$ purple(x) $\wedge$ mushroom(x) $\wedge$ poisonous(x)

$\forall x$ (mushroom(x) $\wedge$ purple(x)) $\rightarrow \neg$poisonous(x)

There are exactly two purple mushrooms.

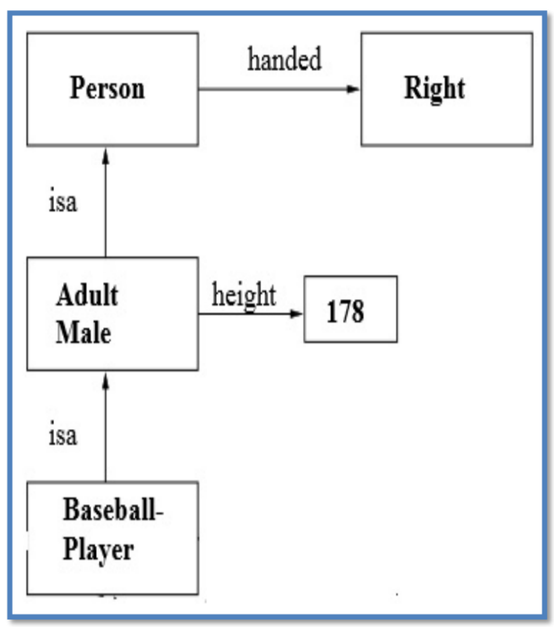
$\exists x\ \exists y$ mushroom(x) $\wedge$ purple(x) $\wedge$ mushroom(y) $\wedge$ purple(y) ^ $\neg$(x=y)

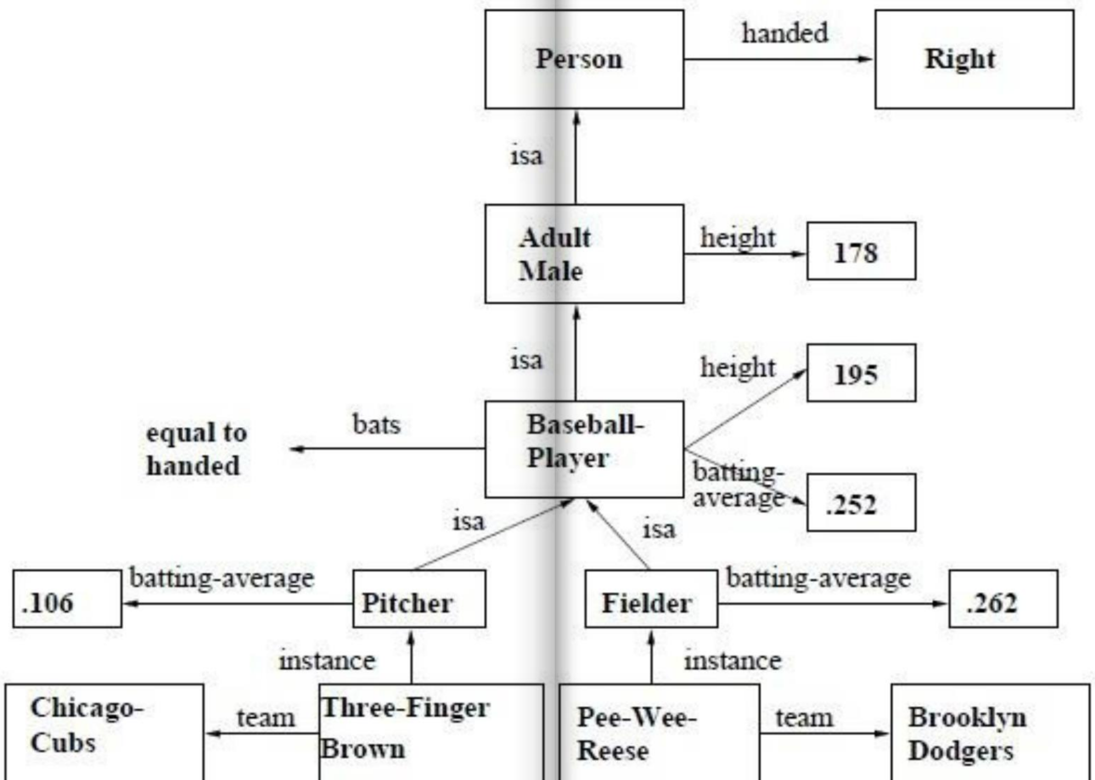
Clinton is not tall.

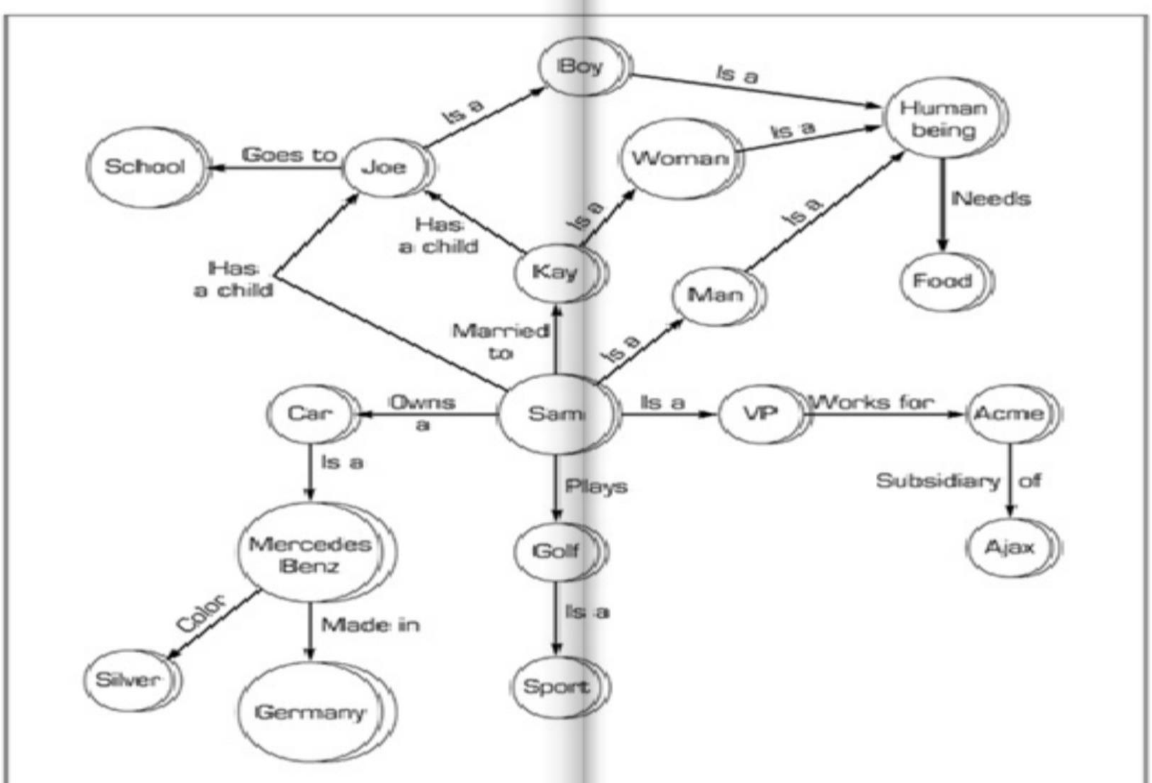
$\neg$tall(Clinton)

Semantic Networks

- humans tend to store and manipulate knowledge in Graphs.
- Graphs are very easy to store inside programs.
- **Semantic Network** is a graph structure for representing knowledge in patterns of interconnected nodes and arcs.




Person
handed
Right
isa
Adult Male
height
178
isa
height
195
equal to handed
bats
Baseball-Player
batting-average
.252
isa
isa
.106
batting-average
Pitcher
Fielder
batting-average
.262
instance
instance
Chicago-Cubs
team
Three-Finger Brown
Pee-Wee-Reese
team
Brooklyn Dodgers


Boy
Is a
Human being
Is a
School
Goes to
Joe
Woman
Is a
Needs
Has a child
Is a
Is a
Has a child
Kay
Man
Food
Married to
Is a
Car
Owns a
Sam
Is a
VP
Works for
Acme
Is a
Plays
Subsidiary of
Mercedes Benz
Golf
Ajax
Color
Made in
Is a
Silver
Germany
Sport

Propositional Logic

- Propositional logic is declarative
- Propositional logic allows partial / disjunctive / negated information unlike most data structures and databases
- Propositional logic has very limited expressive power
- The symbols in this logic are:

$\wedge$ ...and	[conjunction]
$\vee$ ...or	[disjunction]
$\Rightarrow$...implies	[implication / conditional]
$\Leftrightarrow$..is equivalent	[biconditional]
$\neg$ ...not	[negation]

Propositional Logic

- Logical constants: true, false
- Propositional symbols: P, Q, S, ... (atomic sentences)
- Wrapping parentheses: (…)
- We can summarize the meanings of these connectives using

p	Q	¬q	p^q	p∨q	p→q	p⇔q
T	T	F	T	T	T	T
T	F	T	F	T	F	F
F	T	F	F	T	T	F
F	F	T	F	F	T	T

Propositional Logic

- If " it is hot" and " the sun is shining" then "kids should wear hats"
  - P means "It is hot"
  - Q means "the sun is shining"
  - R means "kids should wear hats"

P ^ Q → R

Propositional Logic

- A sentence is a Well Formed Formula when:
  - A symbol is a sentence
  - If S is a sentence, then $\neg S$ is a sentence
  - If S is a sentence, then (S) is a sentence
  - If S and T are sentences, then $(S \vee T)$, $(S \wedge T)$, $(S \rightarrow T)$, and $(S \Leftrightarrow T)$ are sentences
  - A sentence results from a finite number of applications of the above rules