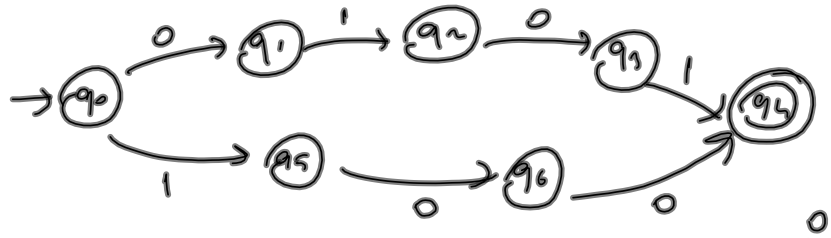
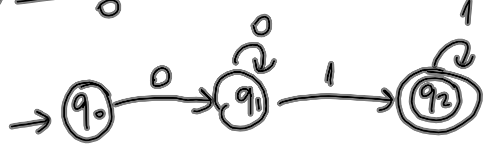
## Düzenli Kümeler, Düzenli Deyimler

Her sonlu otomatanın tanıdığı bir stringler kümesi vardır.
Sonlu otomatalar tarafından tanınan kümelere "Düzenli Kümeler" denir.



$P_1 = \{0101, 100\}$

$P_2 = \{0^n 1^m \mid n \geq 1, m \geq 1\}$



$P_2 = \{01, 001, 0111, 0011, \ldots\}$

$P_3 = \{$ab alt stringini içeren stringler$\}$

$P_4 = \{$Eşit sayıda 0 ve 1 içeren stringler$\}$

$P_4 = \{01, 10, 0101, 00011011, \ldots\}$

* $P_1$, $P_2$ ve $P_3$ düzenli kümelerdir. $P_4$ ise düzenli küme değildir.

**Düzenli Deyim:** Düzenli kümeleri biçimsel olarak tanımlamak için kullanılan bir dildir. $\{a, b, c\}$ alfabesindeki düzenli deyimler şöyle tanımlanabilir:

1. Alfabedeki her simge düzenli bir deyimdir.

   $a = \{a\}$ $b = \{b\}$

2. $\lambda$ ve $\emptyset$ düzenli bir deyimdir.

   $\lambda = \{\lambda\}$ $\emptyset = \{\}$

3. Eğer $P$ ve $Q$ düzenli birer deyim ise $P+Q$, $P.Q$ ve $P^*$ da düzenli bir deyimdir.

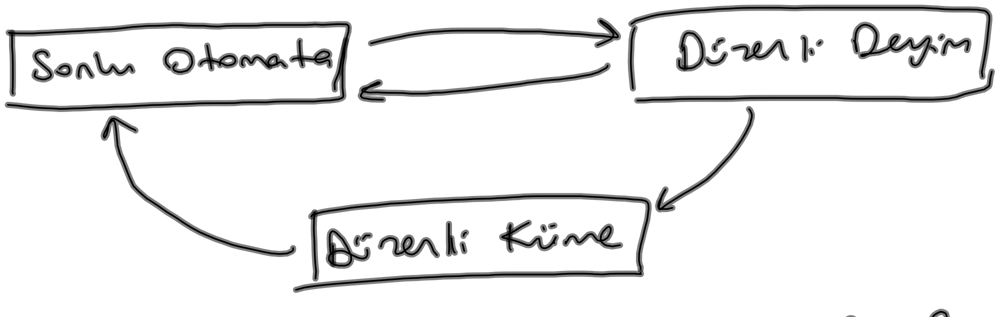
   $+ \rightarrow \cup$, birleşim
   $\cdot \rightarrow$ sonuna ekleme
   $* \rightarrow$ kapanış (closure)

$P^* = \{\lambda, P, PP, PPP, PPPP, \ldots\} = \{\lambda, P, P^2, P^3, \ldots\}$

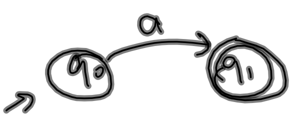
$P^+ = \{P, PP, PPP, \ldots\} = PP^*$

| Düzenli Deyim | Düzenli Küme |
|---|---|
| $00+11$ | $\{00, 11\}$ |
| $a(b+c)$ | $\{ab, ac\}$ |
| $a^*$ | $\{\lambda, a, aa, \ldots\}$ |
| $(0+1)^*$ | $\{\lambda, 0, 1, 001, 101, 100010, \ldots\}$ |
| $a(bb+cc)d^*$ | $\{abb, abbd, abbdd, acc, accd, \ldots$ |
| $a^*b^*$ | $\{\lambda, a, b, ab, aaab, abb, \ldots\}$ |
| $(ab)^*$ | $\{\lambda, ab, abab, ababab, \ldots\}$ |
| $\emptyset$ | $\{\ \}$ |

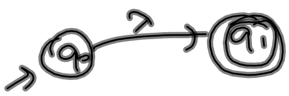
## Düzenli Deyim, Düzenli Küme ve Sonlu Otomata İlişkisi




1) $P = a$ ise




2) $P = \lambda$ ise



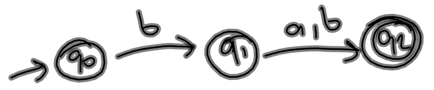

3) $P = a+b$ ise $\{a, b\}$


4) $P = ab$ ise

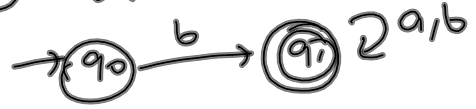

5) $P = a^*$

ÖRN: Düzenli deyimlerin sonlu otomata karşılığını bulunuz.

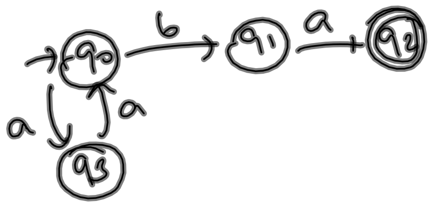
① $b(a+b) = \{ba, bb\}$



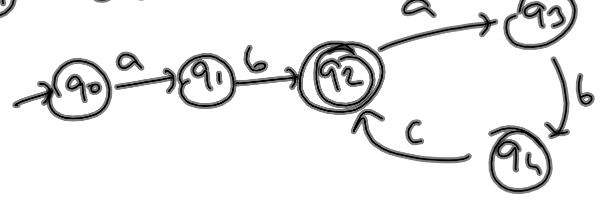
② $b(a+b)^*$



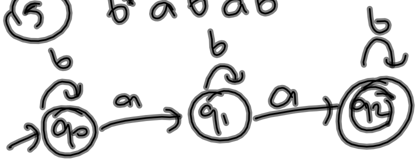
③ $(aa)^* ba$



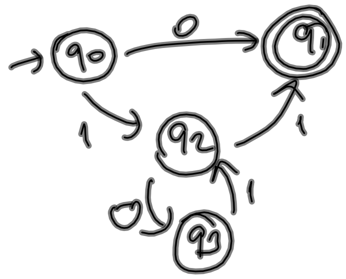
④ $ab(abc)^*$



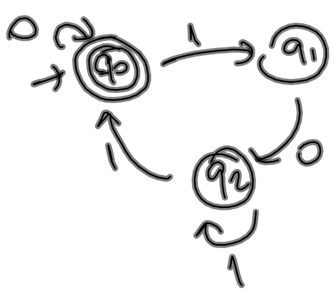
⑤ $b^* a b^* a b^*$



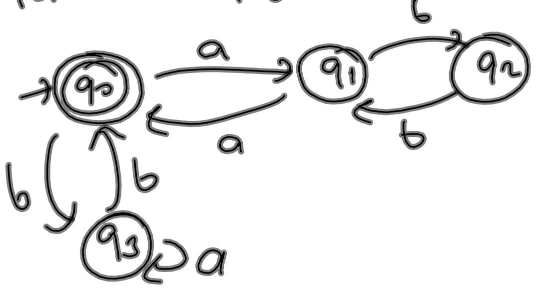
⑥ $0 + 1(01)^* 1$



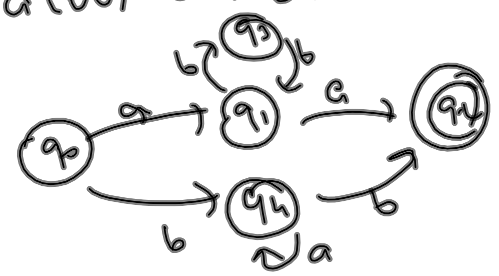
⑦ $(0+101^*1)^*$



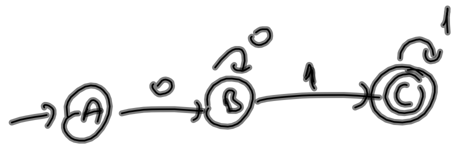
⑧ $(a(bb)^*a + ba^*b)^*$



⑨ $a(bb)^*a + ba^*b$



## Sonlu Otomatadan Düzenli Deyim Bulma



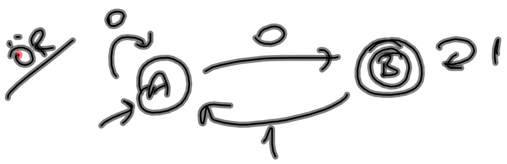
$A = \lambda$

$B = A0 + B0 = A00^*$

$C = B1 + C1 = B11^*$

**Önerme:** P, Q ve R düzenli birer deyim olsun.
Eğer $R = Q + RP$ ise $R = QP^*$ olarak yazılabilir.

Ör/



FA ⇒ RE ?

$A = \lambda + A0 + B1$

$\underbrace{B}_{R} = \underbrace{A0}_{Q} + \underbrace{B}_{R}\underbrace{1}_{P}$

$B = A01^*$

$B = A01^* = (0+01^*1)^*01^*$

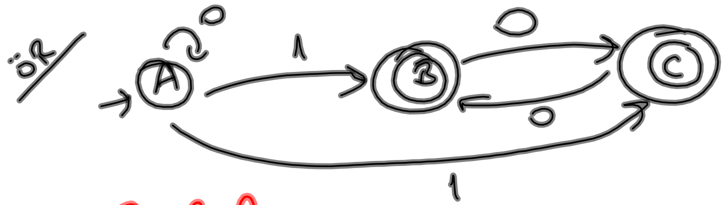
$A = \lambda + A0 + B1$

$A = \lambda + A0 + A01^*1$

$\underbrace{A}_{R} = \underbrace{\lambda}_{Q} + \underbrace{A}_{R}\underbrace{(0+01^*1)}_{P}$

$A = \lambda \cdot (0+01^*1)^*$

Ör/



FA ⇒ RE ?

RE(B) + RE(C)

$\underbrace{A}_{R} = \underbrace{\lambda}_{Q} + \underbrace{A}_{R}\underbrace{0}_{P} = \lambda . 0^* = 0^*$

$B = A1 + C0 = 0^*1 + C0$

$C = A1 + B0 = 0^*1 + B0$

$C = 0^*1 + (0^*1 + C0)0$

$C = 0^*1 + 0^*10 + C00$

$C = \underbrace{(0^*1 + 0^*10)(00)^*}_{X}$

$B = 0^*1 + C0$

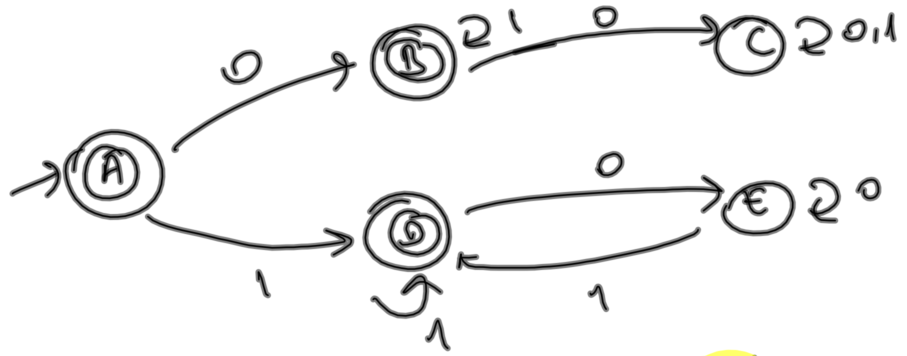
$B = 0^*1 + (0^*1 + B0)0$

$\underbrace{B}_{R} = \underbrace{0^*1 + 0^*10}_{Q} + \underbrace{B}_{R}\underbrace{00}_{P}$

$B = \underbrace{(0^*1 + 0^*10)(00)^*}_{Y}$

$\boxed{X+Y}$

ör/



FA ⇒ RE

$A = \lambda$

$B = A0 + B1$

$C = B0 + C0 + C1$

$D = A1 + D1 + E1$

$E = D0 + E0$

$E = D00^*$

$B = 0 + B1 = 01^*$

$D = A1 + D1 + E1$

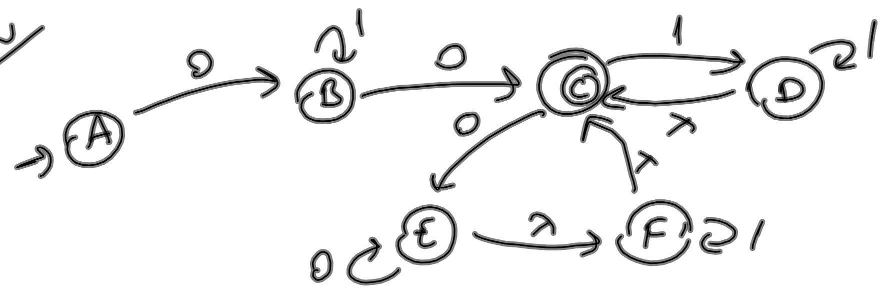
$D = 1 + D1 + D00^*1$

$\overset{R}{D} = \overset{Q}{1} + \overset{R}{D}\,\overset{P}{(1+00^*1)}$

$D = 1(1+00^*1)^*$

$$\lambda + 01^* + 1(1+00^*1)^*$$

ödev



1) λ-geçişlerini yok ediniz.
2) NFA'ya ait RE'yi bulunuz.
3) NFA'yı DFA'ya çeviriniz.