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"ASSIGNMENT: 2"

Question: 1

part. (a)

$$\lim_{(x,y)\to(2,1)} \frac{x^2-2xy}{x^2-4y^2}$$

$$= \frac{(2)^2-2(2)(1)}{(2)^2-4(1)^2} = \frac{4-4}{4-4}$$

$$= \frac{0}{0}$$

let $x^2-4y^2=0$

$x=4y$

$y=x/4$

$$\lim_{x\to 0} \frac{x^2-2(x)(x)/16}{x^2-4(x/4)^2}$$

$$\lim_{x\to 0} \frac{x^2-\frac{x^2}{8}}{x^2-4(x^2/16)}$$

$$= \lim_{x\to 0} \frac{\frac{8x^2-x^2}{8}}{\frac{x^2-4x^2}{4}}$$

$$= \lim_{x\to 0} \frac{8x^2-x^2}{2(x^2-4x^2)} = \lim_{x\to 0} \frac{7x^2}{-6x^2}$$

$$= 7/6$$

part (b)

$$\lim_{x,y\to(0,0)} \frac{x-4y}{6y+7x}$$

$$6y = 7x.$$

$$y = \frac{7}{6}x$$

$$\lim_{x\to 0} \frac{x-4\left(\frac{7x}{6}\right)}{7+7(x)}$$

$$\lim_{x\to 0} \frac{x-\frac{28x}{6}}{7x+7} = \lim \frac{\frac{6x-28x}{6}}{14x}$$

$$= \lim_{x\to 0} \frac{-22x}{6(14x)} = \lim_{x\to 0} \frac{-11x}{42x}$$

$$= \frac{-11}{42}$$

part (c).

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^6}{xy^3}$$

Let $y = mx.$

$$\frac{x^2-m^6x^6}{x(m^3x^3)}$$

$$\frac{x^2(1-m^6x^3)}{x\ m^3}$$

$$\frac{1-m^6x^3}{x^2 m^3}$$

part (d)

$$\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3-ze^2y}{6x+2y-3z}$$

$$= \frac{(-1)^3-(4)e^2(0)}{6(-1)+0-3(4)}$$

$$= \frac{-1-0}{-6-12} = \frac{-1}{-18}$$

$$= \frac{1}{18}$$

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Question No : 2

Determine $D_{\vec{u}} f$ for the given function in the indicated direction:-

a) $f(x,y) = \cos\left(\frac{x}{y}\right)$ in the direction of $\vec{v} = (3, -4)$

Sol

$$\vec{v} = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$$

$$\vec{v} = i\frac{\partial\left(\cos\left(\frac{x}{y}\right)\right)}{\partial x} + j\,\partial\left(\cos\left(\frac{x}{y}\right)\right)$$

$$\bar{v} = i \cdot \frac{\left(-\sin\frac{x}{y}\right)}{y} + \hat{j}\left[\left(-\sin\left(\frac{x}{y}\right)x \cdot -y^{-2}\right)\right]$$

$$\bar{v} = \left[\frac{-\sin\left(\frac{x}{y}\right)}{y}\right]\hat{i} + \hat{j}\left[\frac{x\sin\left(\frac{x}{y}\right)}{y^2}\right]$$

$$\vec{v} = \left[\frac{-\sin\left(\frac{3}{-4}\right)}{-4}\right]\hat{i} + \hat{j}\left[\frac{3\sin\left(\frac{3}{-4}\right)}{(-4)^2}\right]$$

$$= \frac{1}{4}\left[-0.0131)\hat{i} + \left(\frac{3}{16}(-0.0131)\right)\hat{j}\right.$$

$$\vec{v} = \frac{-0.0131\,\hat{i}}{4} - \frac{0.0393}{16}\hat{j}$$

part b:-

$f(x,y,z) = x^2y^3 - 4xz \qquad \vec{v} = (-1,2,0)$

$$\vec{v} = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$\vec{v} = \hat{i}\frac{\partial}{\partial x}(x^2y^3 - 4xz) + \hat{j}\frac{\partial}{\partial y}(x^2y^3 - 4xz) + \hat{k}\frac{\partial}{\partial z}(x^2y^3 - 4xz)$$

$$= \hat{i}(2xy^3 - 4z) + \hat{j}(3x^2y^2) + \hat{k}(-4x)$$

$$\vec{v} = \hat{i}\left[2(-1)(2)^3 - 4(0)\right] + \hat{j}\left[3(-1)^2(2)^2\right] + \hat{k}\left[-4(-1)\right]$$

$$\vec{v} = \hat{i}(-2 \times 8) + (+3 \times 4)\hat{j} + 4\hat{k}$$

$$\vec{v} = -16\hat{i} + 12\hat{j} + 4\hat{k}$$

Question 03

:- Determine directional derivative of $f(x,y,z) = 4xy^2e^{3xz})$ at $(3,-1,0)$ in the direction of $\vec{v}(-1,4,2)$

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$f(x,y,z) = 4xy^2 e^{3xz}$

$\bar{v} = -i + 4\hat{j} + 2\hat{k}$

$$\vec{v} = \frac{-i + 4\hat{j} + 2\hat{k}}{\sqrt{(-1)^2 + (4)^2 + (2)^2}} = \frac{-\hat{i} + 4\hat{j} + 2\hat{k}}{\sqrt{1+16+4}}$$

$$= \frac{-\hat{i} + 4\hat{j} + 2\hat{k}}{\sqrt{21}}$$

grad f at $(3, -1, 0)$

$$\text{grad } f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$= \hat{i}\frac{\partial}{\partial x} 4xy^2 e^{3xz} + \hat{j}\frac{\partial}{\partial x}(4xy^2 e^{3xz}) + \hat{k}\left(\frac{\partial}{\partial z}(4xy^2 e^{3xz})\right)$$

$$= \hat{i}(4y^2 \cdot e^{3xz} + 4xy^2 e^{3xz} \cdot 3z) + \hat{j}(4xe^{3xz} \cdot 2y) + \hat{k}(4xy^2 e^{3xz} \cdot 3x)$$

$$= \hat{i}(4y^2 e^{3xz} + 12xy^2 z e^{3xz}) + \hat{j}(8xy e^{3xz}) + \hat{k}(12x^2y^2 e^{3xz})$$

$$= e^{3xz}\left[\hat{i}(4y^2 + 12xy^2 z) + \hat{j}(8xy) + \hat{k}(12x^2y^2)\right]$$

$$= e^{3(3)(0)}\left[\hat{i}\left[4(-1)^2 + 12(3)(-1)^2(0)\right] + \hat{j}\left[8(3)(-1)\right] + \hat{k}\left[12(3)^2(-1)^2\right]\right]$$

$$= e^0\left[4\hat{i} - 24\hat{j} + 108\hat{k}\right]$$

$$= 4\hat{i} - 24\hat{j} + 108\hat{k}$$

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direction derivate = $\hat{a} \cdot \text{grad } f$

$$= \frac{1}{\sqrt{21}} \left\{ (-\hat{i} + 4\hat{j} + 2\hat{k}) \cdot (4\hat{i} - 24\hat{j} + 108\hat{k}) \right]$$

$$= \frac{1}{\sqrt{21}} \left[-4 - 96 + 216 \right)$$

$$= \frac{1}{\sqrt{21}} (116)$$

$$= \frac{116}{\sqrt{21}}$$

Question No: 04

Find maximum rate of change of the function at the indicated point and direction in which this rate of change occurs.

(a) $f(x,y) = \sqrt{x^2+y^2}$ at $(-2,3)$

$\nabla f(x,y) = |\text{grad } f|$

$$\text{grad } f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y}$$

$$= \hat{i}\frac{\partial}{\partial x}(x^2+y^2)^{1/2} + \hat{j}\frac{\partial (x^2+y^2)^{1/2}}{\partial y}$$

$$= \hat{i}\left[\frac{1}{\cancel{2}\sqrt{x^2+y^2}} \cdot \cancel{2}x\right] + \hat{j}\left[\frac{1}{\cancel{2}\sqrt{x^2+y^2}} \cdot \cancel{2}y\right]$$

$$= \hat{i}\left[\frac{x}{\sqrt{x^2+y^2}}\right] + \hat{j}\left[\frac{y}{\sqrt{x^2+y^2}}\right]$$

$$= \frac{-2\hat{i}}{\sqrt{4+9}} + \hat{j}\frac{3}{\sqrt{4+9}}$$

$$= \frac{-2\hat{i}}{\sqrt{13}} + \hat{j}\frac{3}{\sqrt{13}}$$

$$|\text{grad} f| = \sqrt{\left(\frac{-2}{\sqrt{13}}\right)^2 + \left(\frac{3}{\sqrt{13}}\right)^2}$$

$$= \sqrt{\frac{4}{13} + \frac{9}{13}} = \sqrt{\frac{\cancel{13}}{\cancel{13}}}$$

$$= \sqrt{1} = 1$$

direction at which the rate of change occur: $\frac{\nabla \text{grad} f}{|\nabla \text{grad} f|} = \frac{\frac{-2\hat{i}}{\sqrt{13}} + \frac{3\hat{j}}{\sqrt{13}}}{1}$

$$= \frac{-2\hat{i}}{\sqrt{13}} + \frac{3\hat{j}}{\sqrt{13}}$$

part b:

$f(x,y,z) = e^x \cos(y-2z)$ at $(4,-2,$

$\nabla f(x,y,z) = |\text{grad} f|$

$$\text{grad} f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$= \hat{i}\frac{\partial}{\partial x}[e^x\cos(y-2z)] + \hat{j}\frac{\partial}{\partial y}[e^x\cos(y$$

$$+ \hat{k}\,\partial\,[e^x\cos(4-2z)]]$$

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$$= \hat{i}\left[e^{x}\cos(y-2z)\right] + \hat{j}\left[e^{x}\cdot -\sin(y-2z)\right) + \hat{k}\left[e^{x}\cdot -\sin(y-2z)\cdot -2\right]$$

$$= \hat{i}\left[e^{x}\cos(y-2z)\right] + \hat{j}\left[-e^{x}\sin(y-2z)\right) + \hat{k}\left[e^{x}\,2\sin(y-2z)\right]$$

$$e^{x}\left[\hat{i}\left(\cos(y-2z)\right) + \hat{j}\left[-\sin(y-2z)\right] + \hat{k}\left(2\sin(y-2z)\right)\right]$$

$$= e^{4}\left[\hat{i}\left[\cos(-2-2(0)\right] + \hat{j}\left(\sin(-2-2(0)\right] + \hat{k}\left(2\sin(-2-2(0)\right]\right.$$

$$= 5.4.6\left[\hat{i}\left(\cos(-2)\right) + \hat{j}\left(\sin(-2)\right) + \hat{k}\left(2\sin(-2\right.\right.$$

$$= 5.46\left[(1\hat{i}) + \hat{j}(-0.035) + \hat{k}(-0.07)\right)$$

$$= 54.6\hat{i} - 1.911\hat{j} - 3.8\hat{k}$$

$$|\nabla \text{grad} f| = \sqrt{(54.6)^2 + (-1.911)^2 + (-3.8)^2}$$

$$=$$

$$\boxed{\nabla \text{grad} f = 54.8}$$

The direction at which this rate of change occur $= \dfrac{\nabla \text{grad} f}{|\nabla \text{grad} f|}$

$$= \frac{54.6\hat{i} - 1.911\hat{j} - 3.8\hat{k}}{54.8}$$

$$= \frac{54.6}{54.8}\hat{i} - \frac{1.911}{54.8}\hat{j} - \frac{3.8}{54.8}\hat{k}$$

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Question No: 07

a) $z = \frac{x^2 - w}{y^4}$, $x = t^3 + 7$

$y = \cos(2t)$. $w = 4t$

$\frac{dz}{dt} = ?$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{y} \cdot \frac{dy}{dt} + \frac{\partial z}{\partial w} \cdot \frac{dw}{dt}$$

$\frac{dz}{dt} = \frac{\partial x}{y^4}$, $\frac{\partial z}{\partial y} = (x^2 - w) y^{-4}$

$\frac{\partial z}{\partial y} = (x^2 - w) - 4y^{-5}$

$\frac{dx}{dt} - 3t^2$, $\frac{\partial z}{\partial y} = \frac{-4(x^2 - w)}{y^5}$

$$\frac{dy}{dt} - \frac{d}{dt} \cos 2t = -\sin 2t \cdot 2$$
$$= -2 \sin 2t$$

$\frac{\partial z}{\partial w} - \frac{-1}{y^4}$ $\frac{dw}{dt} = 4$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial z}{\partial w} \cdot \frac{dw}{dt}$$
$$= \frac{2x^2}{y^4} \cdot 3t^2 + \left(\frac{-4(x^2 - w)}{y^5} \cdot -2 \sin 2t \right)$$
$$+ \cdot \left(\frac{-1}{y^4} \right) \cdot 4$$

$$= \frac{6x^2t^2}{y^4} + \frac{8(x^2 - w) \sin 2t}{y^5} - \frac{4w}{y^4}$$

b) $z = x^2y^4 - 2y$, $y = \sin(x^2)$

$\frac{dz}{dx} = ?$

$\frac{dz}{dx} = \frac{\partial z}{\partial y} \cdot \frac{dy}{dx}$

$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(x^2y^4 - 2y)$

$= (4y^3x^2 - 2)$

$= (4x^2y^3 - 2)$

$\frac{dy}{dx} = \frac{d}{dx}\sin(x^2) = \cos x^2 \cdot 2x$

$= 2x\cos x^2$

$\frac{dz}{dx} = \frac{\partial z}{\partial y} \cdot \frac{dy}{dx}$

$= (4x^2y^3 - 2)(2x\cos x^2$

$= 8x^3y^3\cos x^2 - 4x\cos x^2$

part (c)

Compute $\frac{dy}{dx}$ for the following equation

$x^2y^4 - 3 = \sin(xy)$

differentiate w.r.t with x

$\frac{d}{dx}(x^2y^4 - 3) = \frac{d}{dx}\sin(x \cdot y)$

$2xy^4 + x^2 4y^3\frac{dy}{dx} = y\cos(xy)$

$2xy^4 + 4x^2y^3\frac{dy}{dx} = 4\cos(xy)$

$4x^2y^3\frac{dy}{dx} = 4\cos(xy) - 2xy^4$

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$$\frac{dy}{dx} = \frac{\cancel{y}(\cos(xy) - 2xy^3)}{4x^2y^{\cancel{x}}}$$

$$\frac{dy}{dx} = \frac{\{\cos(xy) - 2xy^3\}}{4x^2y^2}$$

Question No: 5

Complute div $\vec{F}$ and cur $\vec{F}$

a) $\vec{F} = x^2y\,\hat{i} - (z^3 - 3x)\hat{j} + 4y^2\hat{k}$

$\vec{F} = (x^2y, -(z^3-3x), 4y^2)$

$$\nabla \cdot \vec{F} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$$

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x} x^2y + \frac{\partial}{\partial y}(z^3 - 3x) + \frac{\partial}{\partial z} 4y^2$$

$$\nabla \cdot F = 2xy\,\hat{i} - 0\,\hat{j} + 0$$

$$\text{Div} f = \nabla f \cdot f$$
$$= (2xy\,\hat{i}) \cdot [x^2y\,\hat{i} - (z^3-3x)\hat{j} + 4y^2\hat{k}]$$
$$= 2x^3y^2$$

$$\text{Curv} = \nabla f \times f$$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & -(z^3 - 3x) & 4y^2 \end{vmatrix}$$

$= -\hat{j}\,(\,0\cdot y$

$$= \hat{i}\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -(z^3-3x) & 4y^2 \end{vmatrix} - \hat{j}\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2y & 4y^2 \end{vmatrix} + \hat{k}\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & -(z^3-3x) \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(4y^2) + \frac{\partial}{\partial z}(z^3-3x)\right] - \hat{j}\left[\frac{\partial}{\partial x}4y^2 - \frac{\partial}{\partial z}x^2y\right] + \hat{k}\left[\frac{\partial}{\partial x}-(z^3-3x) - \frac{\partial}{\partial y}(x^2y)\right]$$

$$= \hat{i}(8y + 3z^2) - \hat{j}(0-0) + \hat{k}(-(-3) - x^2)$$

$$= (8y + 3z^2)\hat{i} + (3 - x^2)\hat{k}$$

Part b:-

$$\vec{F} = (8x + 2z^2)\hat{i} + \frac{x^3y^2}{2}\hat{j} - (z - 7x)\hat{k}$$

$\text{div} = \nabla f \cdot f$

$$\nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$= \hat{i}\frac{\partial}{\partial x}(8x + 2z^2) + \hat{j}\frac{\partial}{\partial y}\left(\frac{x^3y^2}{2}\right) + \hat{k}\frac{\partial}{\partial z}-(z-7x$$

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$$= 8\hat{i} + \frac{x^3 \cdot 2y}{z}\hat{j} + -1\hat{k}$$

$$= 8\hat{i} + \frac{2x^3y}{z}\hat{j} - \hat{k}$$

$$= 8\hat{i} + \frac{2x^3y}{z}\hat{j} - \hat{k} \cdot (8x+2z)\hat{i} + \left(\frac{x^3y^2}{z}\right)\hat{j} - (z-7x)\hat{k}$$

$$= 16x + 16z + 2x^6y^4/z + z - 7x$$

$$= 9x + 17z + \frac{2x^6y^4}{z}$$

curve $\nabla f \times f$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x+2z^2 & \frac{x^3y^2}{z} & -(z-7x) \end{vmatrix}$$

$$\hat{i}\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ \frac{x^3y^2}{z} & -(z-7x) \end{vmatrix} - \hat{j}\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ 3x+2z^2 & -(z-7x) \end{vmatrix} + \hat{k}\begin{vmatrix} \partial/\partial x & \partial/ \\ 3x+2z^2 & x \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y} -(z-7x) - \frac{\partial}{\partial z}\left(\frac{x^3y^2}{z}\right)\right] - \hat{j}\left[\frac{\partial}{\partial x} -(+z-7x) - \frac{\partial}{\partial z}\right.$$

$$+\hat{k}\left[\frac{\partial}{\partial x}\frac{x^3y^2}{z} - \frac{\partial}{\partial y}\, 3x+2z^2\right]$$

$$= \hat{i}\left[0 - x^3y^2 \cdot -1z^{-2}\right] - \hat{j}\left[+7 - 4z\right) + \hat{k}\left[\; - 0\right]$$

$$= \hat{i}\,\frac{x^3y^2}{z^2} - \hat{j}\,(7-4z) + \left(\frac{3x^2y^2}{z}\right)\hat{k}$$

Question No: 06

Determine if the vector field is

a) conservative.

a $\vec{F} = x^2 y \hat{i}$ -

$\vec{F} = \left(4x^2 + \frac{3x^2 y}{z^2}\right)\hat{i} + \left(8xy + \frac{x^3}{z^2}\right)\hat{j} + (11 - 2$

The vector field is conservative if a only if

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, $\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$; $\frac{\partial M}{\partial z} = \frac{\partial}{\partial}$

$$\vec{F} = \underbrace{\left(4y^2 + \frac{3x^2 y}{z^2}\right)}_{M}\hat{i} + \underbrace{\left(8xy + \frac{x^3}{z^2}\right)}_{N}\hat{j} + \left(11 - \frac{2x^3 y}{z^3}\right)\hat{k} \Rightarrow P$$

$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z}$, $\frac{\partial N}{\partial x} = 8y - \frac{3x^2}{z^2}$

$\frac{\partial N}{\partial z} = x^3 z^{-2} = x^3(-2)z^{-3} = \frac{-2x^3}{z^3}$

$\frac{\partial P}{\partial y} = \frac{-2x^3}{z^3}$

$\frac{\partial M}{\partial z} = 4x^2 + \frac{3x^2 y}{z^2} = 3x^2 y(-2)z^{-3}$

$= \frac{-6x^2 y}{z^3}$

$\frac{\partial P}{\partial x} = \frac{\partial\left(11 - \frac{2x^3 y}{z^3}\right)}{\partial x} = \frac{-6x^2 y}{z^3}$

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Hence:-

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y} = \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

Since the vector field is conservative.

(b) $\vec{F} = 6x\hat{i} + (2x - y^2)\hat{j} + (6z - x^3)\hat{k}$

The vector field is conservative if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$\vec{F} = \underbrace{6x}_{M}\hat{i} + (\underbrace{2x - y^2}_{N})\hat{j} + (\underbrace{6z - x^3}_{P})\hat{k}$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(6x) = 0, \quad \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2x - y^2)$$
$$= 2$$

$$\frac{\partial N}{\partial z} = \frac{\partial}{\partial z}(2x - y^2) = 0, \quad \frac{\partial P}{\partial y} = \frac{\partial}{\partial y}(6z - x^3)$$

$$\frac{\partial M}{\partial z} = \frac{\partial}{\partial z}6x = 0, \quad \frac{\partial P}{\partial x} = \frac{\partial}{\partial x}(6z - x^3)$$
$$= -3x^2$$

So,

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} \neq \frac{\partial P}{\partial x}$$

Since, the vector field is not conserv