

- ASSIGNMENT No # 01 -

MULTIVARIABLE CALCULUS

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Submitted to :

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Question # 01 :-

The points A, B, C have vectors

$$4\hat{i} + 4\hat{j} + \hat{k}$$

$$-4\hat{i} + 3\hat{j} - 4\hat{k}$$

$$4\hat{i} - \hat{j} - 2\hat{k} \quad \text{respectively,}$$

relate to the origin O.

- (a) Find the equation of the plane ABC, giving yr. answers in the form:

$$ax + by + cz = 0.$$

SOLUTION:-

We have A (4, -4, 1)

B (-4, 3, -4)

C (4, -1, -2)

$$\begin{aligned} \overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA} \\ &= \begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix} \end{aligned}$$

$$= -8\hat{i} + 7\hat{j} - 5\hat{k}$$

$$\begin{aligned}\vec{AC} &= \vec{OC} - \vec{OA} \\ &= \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ -3 \end{bmatrix} \\ &= 3\hat{j} - 3\hat{k}\end{aligned}$$

$$\begin{aligned}n = \vec{AB} \times \vec{AC} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{vmatrix} \\ &= \hat{i}(-4+15) - (24-0)\hat{j} + (-34)\hat{k} \\ &\Rightarrow \hat{i} + 4\hat{j} + 4\hat{k}\end{aligned}$$

$$\begin{aligned}d = \vec{OA} \cdot n &= (4\hat{i} - 4\hat{j} + \hat{k}) \cdot (\hat{i} + 4\hat{j} + 4\hat{k}) \\ &= 4 - 16 + 4 = -8\end{aligned}$$

Eqn of plane.

$$r \cdot n = d.$$

$$r(\hat{i} + 4\hat{j} + 4\hat{k}) = -8.$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 4\hat{j} + 4\hat{k}) = -8$$

$$x + 4y + 4z = -8$$

$$x + 4y + 4z + 8 = 0.$$

- (b) Find p the perpendicular distance from O to the plane ABC

$$\begin{aligned}\text{perp distance} &= \frac{d}{|n|} = \frac{-8}{\sqrt{1^2 + 4^2 + 4^2}} \\ &= \frac{8}{3.3} = 1.39\end{aligned}$$

- (c) The point D has position vector $2\hat{i} + 3\hat{j} - 3\hat{k}$. Find the coordinates of the point of intersection of the line OD with the plane ABC.

$$\text{Line OD: } r = a + \lambda b$$

$$\Rightarrow r = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}$$

as same value of λ .

$$\begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = \varepsilon$$

$$\Rightarrow 2\lambda = 1 \quad 3\lambda = 4 \quad -3\lambda = 4$$

$$\Rightarrow 2\lambda = -4$$

$$\lambda = -2$$

$$r = \begin{pmatrix} 2(-2) \\ 0(-2) \\ -3(-2) \end{pmatrix} = (-4, 0, 6)$$

Question # 02 :-

The position vector of the point.

$$A = [7\hat{i} + 4\hat{j} - \hat{k}] \quad B = [11\hat{i} + 3\hat{j}]$$

$$C = [2\hat{i} + 6\hat{j} + 13\hat{k}] \quad D = [2\hat{i} + 7\hat{j} + \lambda\hat{k}]$$

- a) Given that the shortest distance between the line AB and the line CD is 3. Show that $\lambda^2 - 5\lambda + 4 = 0$.

$$\vec{r}_1 = \vec{OA} - \vec{OB}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= \begin{bmatrix} 11 \\ 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$\vec{r}_2 = \vec{OC} - \vec{OD}$$

$$\vec{CD} = \vec{OD} - \vec{OC}$$

$$= \begin{bmatrix} 2 \\ 7 \\ \lambda \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ 13 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \lambda - 13 \end{bmatrix}$$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 1 \\ 0 & 1 & \lambda - 13 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} -1 & 1 \\ 1 & \lambda-3 \end{vmatrix} - \hat{j} \begin{vmatrix} 4 & 1 \\ 0 & \lambda-3 \end{vmatrix} + \hat{k} \begin{vmatrix} 4 & -1 \\ 0 & 1 \end{vmatrix}$$

$$= \hat{i}(-(\lambda-3)) - \hat{j}(4(\lambda-3)) + \hat{k}(4)$$

$$= (2-\lambda)\hat{i} - \hat{j}(4\lambda-12) + 4\hat{k}$$

$$a_2 - a_1 = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}$$

$$|b_1 \times b_2| = \sqrt{(2-\lambda)^2 + (4\lambda-12)^2 + (4)^2}$$

$$= \sqrt{4 + \lambda^2 - 4\lambda + 16\lambda^2 + 144 - 9\lambda + 16}$$

$$= \sqrt{17\lambda^2 - 100\lambda + 164}$$

$$(b_1 \times b_2) \cdot (a_1 - a_2) = \left[(2-\lambda)\hat{i} - \hat{j}(4\lambda-12) + 4\hat{k} \right] \cdot \begin{bmatrix} -5 \\ 12 \\ 4 \end{bmatrix}$$

$$= -5(2-\lambda) - 2(4\lambda-12) + 16$$

$$= -10 + 5\lambda - 8\lambda + 24 + 16$$

$$= 30 - 3\lambda$$

$$d = \frac{(b_1 \times b_2) \cdot (a_1 - a_2)}{|b_1 \times b_2|}$$

$$\Rightarrow \frac{30 - 3\lambda}{\sqrt{17\lambda^2 - 100\lambda + 164}}$$

Taking square on b/s.

$$9 = \frac{(30-3\lambda)^2}{17\lambda^2 - 100\lambda + 164} = \frac{900 + 9\lambda^2 - 180\lambda}{17\lambda^2 - 100\lambda + 164}$$

$$9(17\lambda^2 - 100\lambda + 164) = 900 + 9\lambda^2 - 180\lambda$$

$$153\lambda^2 - 900\lambda + 1476 = 900 + 9\lambda^2 - 180\lambda = 0$$

$$144\lambda^2 - 720\lambda + 576 = 0$$

$$4(36\lambda^2 - 180\lambda + 144) = 0$$

$$36\lambda^2 - 180\lambda + 144 = 0$$

$$4(9\lambda^2 - 45\lambda + 36) = 0.$$

$$9\lambda^2 - 45\lambda + 36 = 0.$$

$$\lambda^2 - 5\lambda + 4 = 0.$$

$$(\lambda - 4)(\lambda - 1) = 0.$$

$$\boxed{\lambda = 4} \quad \boxed{\lambda = 1}$$

put $\lambda = 4$ in eqn (1).

$$14^2 - 5(4) + 4 = 0.$$

$$16 - 20 + 4 = 0$$

$$0 = 0.$$

put $\lambda = 1$ in eqn (1).

$$11^2 - 5(1) + 4 = 0.$$

$$1 - 5 + 4 = 0.$$

$$0 = 0.$$

- (PART : 2) -

Let π_1 be the plane ABD when $\lambda = 1$

π_2 be the plane ABD when $\lambda = 4$

(ii)

Write down an eqn of π_1 giving your answer in the form.

$$r = a + sb + tc.$$

$$\vec{r}_1 = \vec{r}_1 = \vec{a} + s\vec{b} + t\vec{c}$$

where $\lambda = 1$

$$\vec{AD} = \vec{OD} - \vec{OA}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 0 \end{bmatrix}$$

$$\pi_1 = 7(14\hat{j} - \hat{k}) + 5(14\hat{i} - \hat{j} + \hat{k}) + 1(-5\hat{i} + 3\hat{j} + 12\hat{k})$$

(ii)

Find an eqn of π_2 giving your answer in the form:

$$ax + by + cz = d$$

$$\lambda_2 = r_2 = \overrightarrow{OA} + \lambda \overrightarrow{AB} + \mu \overrightarrow{AD}$$

$$\text{when } \lambda = 4$$

$$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$$

$$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$$

$$= \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}$$

$$\lambda_2 = r_2 = \overrightarrow{OA} + \lambda \overrightarrow{AB} + \mu \overrightarrow{AD}$$

$$= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} +$$

$$\mu \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}$$

$$x = 7 + 4\lambda - 5\mu \rightarrow \textcircled{i}$$

$$y = 7 - \lambda + 3\mu \rightarrow \textcircled{ii}$$

$$z = -1 + \lambda + 5\mu \rightarrow \textcircled{iii}$$

$$x = 7 + 4\lambda - 5u \quad \text{--- (i)}$$

$$y = 7 - \lambda + 3u \quad \text{--- (ii)}$$

$$z = -1 + \lambda + 5u \quad \text{--- (iii)}$$

$$\text{eq (ii)} + \text{eq (iii)}$$

$$y = 4 - \lambda + 3u$$

$$z = -1 + \lambda + 5u$$

$$y + z = 3 + 8u \quad \text{--- (iv)}$$

Multiply eq (ii) by 2

$$y = 4 - \lambda + 3u$$

$$14$$

$$4y = 16 - 4\lambda + 12u \quad \text{--- (v)}$$

$$\text{eq (i)} + \text{eq (v)}$$

$$x = 7 + 4\lambda - 5u$$

$$4y = 16 - 4\lambda + 12u$$

$$x + 4y = 23 + 7u$$

$$x + 4y - 23 = u$$

put the values in eq (iv)

$$y + z = 3 + 8(x + 4y - 23)$$

$$y + z = 3 + 8x + 32y - 184$$

Multiply both side by 7

$$7y + 7z = 21 + 8x + 32y - 184$$

$$7y + 7z - 21 - 8x - 32y + 184 = 0$$

$$-8x - 25y + 7z + 163 = 0$$

$$-(8x + 25y - 7z - 163) = 0$$

$$8x + 25y - 7z = 163$$

$$\vec{r}_1 = \vec{OA} + \lambda \vec{AB} + \mu \vec{AD}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} -5 \\ 3 \\ 2 \end{bmatrix}$$

$$x = 7 + 4\lambda - 5\mu \quad \text{--- (i)}$$

$$y = 4 - \lambda + 3\mu \quad \text{--- (ii)}$$

$$z = -1 + \lambda + 2\mu \quad \text{--- (iii)}$$

$$\text{eq (ii) + eq (iii)}$$

$$y = 4 - \lambda + 3\mu$$

$$z = -1 + \lambda + 2\mu$$

$$y + z = 3 + 5\mu \quad \text{--- (iv)}$$

Multiply eq (ii) by 4

$$4y = 16 - 4\lambda + 12\mu \quad \text{--- (v)}$$

Add eq (i) and eq (v)

$$x = 7 + 4\lambda - 5\mu$$

$$4y = 16 - 4\lambda + 12\mu$$

$$x + 4y = 23 + 7\mu$$

$$x + 4y - 23 = \mu$$

$$\cdot \quad 7$$

$$y + z = 3 + 5 \left(\frac{x + 4y - 23}{7} \right)$$

$$y + z = 3 + \frac{5x + 20y - 115}{7}$$

Multiply both side by 7

$$7y + 7z = 21 + 5x + 20y - 115$$

$$7y + 7z - 21 = 5x - 20y + 115 - c$$

$$-5x - 13z + 7z + 94 = 0$$

$$-(5x + 13y - 7z - 94) = 0$$

$$5x + 13y - 7z = 94$$

$$\theta = \cos^{-1} \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|}$$

$$\vec{n}_1 \cdot \vec{n}_2 = \begin{bmatrix} 5 & 13 & -7 \end{bmatrix} \cdot \begin{bmatrix} 8 & 25 & -7 \end{bmatrix}$$

$$= 40 + 325 - 49 = 414$$

$$|\vec{n}_1| = \sqrt{5^2 + 13^2 + (-7)^2} = 9\sqrt{3}$$

$$|\vec{n}_2| = \sqrt{8^2 + 25^2 + (-7)^2} = 3\sqrt{82}$$

$$\theta = \cos^{-1} \left| \frac{414}{9\sqrt{3} \times 3\sqrt{82}} \right|$$

$$\theta = \cos^{-1} \left(\frac{414}{27\sqrt{246}} \right)$$

$$\theta = 12.15^\circ$$

Question No: 05

part (a)

If $P = (-2, -1)$ and $Q = (-6, -3)$ are the end point of diameter of a circle, find the equation of the circle.

mid point

$$\left(\frac{-2-6}{2} \right) \left(\frac{-1-3}{2} \right)$$

$$\frac{-8}{2}, \frac{-4}{2}$$

$$(-4, -2)$$

equation of circle $= (x-h)^2 + (y-k)^2 = r^2$ — (1)

$$(x+4)^2 + (y+2)^2 = r^2$$

$$\text{Let } (x, y) = (-2, -1)$$

$$(-2+4)^2 + (-1+2)^2 = r^2$$

$$(2)^2 + (1)^2 = r^2$$

$$4 + 1 = r^2$$

$$5 = r^2$$

$$r^2 = 5$$

put the values.

$$(x+4)^2 + (y+2)^2 = 5$$

$$\therefore \text{Ans}$$

part (b)

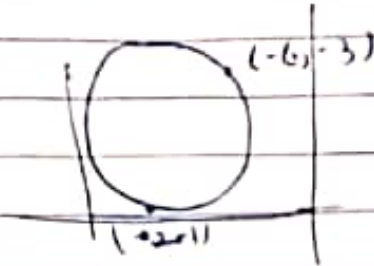
If the circle pass through $(4, 0)$ and $(0, 2)$ and center at y -axis then find the radius of the circle.

so/

equation of circle

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\text{let } x = a, y = b$$



at point $(4, 0)$

$$(4)^2 + (-b)^2 = r^2$$

$$16 + b^2 = r^2 \quad \text{--- (I)}$$

at point $(0, 2)$

$$(0)^2 + (2-b)^2 = r^2$$

$$(2-b)^2 = r^2 \quad \text{--- (II)}$$

compare eq (I) and eq (II)

$$16 + b^2 = (2-b)^2$$

$$16 + b^2 = 4 - 4b + b^2$$

$$16 + \cancel{b^2} - \cancel{b^2} - 4 + 4b = 0$$

$$12 + 4b = 0$$

$$4b = -12$$

$$\boxed{b = -3}$$

Put the values in eq (I)

$$\text{So, } r^2 = (4)^2 + (-3)^2$$

$$r^2 = 16 + 9$$

$$r = \pm 5$$

~~radii~~ neglect -ve

$$\boxed{r = 5}$$

part C :-

Find the equation of directrix of parabola $y^2 = 100x$

$$y^2 = 100x$$

compare with

$$y^2 = 4ax$$

$$4a = 100$$

$$\boxed{a = 25}$$

equation of directrix = $x = -a$

$$x = -25$$

Question # 03.

Let t be a +ve constant.
The line l passes through the

(a) Find the value of t .

$$L_1 = t\hat{i} + \hat{j} \quad -2\hat{i} - \hat{j}$$

$$L_2 = \hat{j} + t\hat{k} \quad -2\hat{j} + \hat{k}$$

The shortest distance between l_1 and l_2 is $\sqrt{21}$.

$$r_1 = a_1 + \lambda b_1$$

$$r_2 = a_2 + \mu b_2$$

$$r_1 = t\hat{i} + \hat{j} + \lambda(-2\hat{i} - \hat{j})$$

$$L_1 = \vec{r}_1 = \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

$$L_2 = \vec{r}_2 = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + \mu \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$r_1 = t\hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j})$$

$$L_1 = \vec{r}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -10 \\ 0 \end{bmatrix}$$

$$L_2 = \vec{r}_2 = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + \mu \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$0 = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$\therefore a_2 - a_1 = \text{norm vector}$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} -2 & 0 \\ 0 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} -2 & -1 \\ 0 & -2 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(-2) + \hat{k}(4)$$

$$= -\hat{i} + 2\hat{j} + 4\hat{k}$$

$$|b_1 \times b_2| = \sqrt{(-1)^2 + (2)^2 + (4)^2}$$

$$= \sqrt{1 + 4 + 16}$$

$$= \sqrt{21}$$

$$(a_2 - a_1) = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} - \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix}$$

$$= -t\hat{i} + t\hat{k}$$

$$D = (-\hat{i} + 2\hat{j} + 4\hat{k}) \cdot (-t\hat{i} + t\hat{k})$$

$$\sqrt{21} = \frac{t + 4t}{\sqrt{21}}$$

$$21 = t + 4t$$

$$21 = 5t$$

$$\boxed{t = \frac{21}{5}}$$

Part b

$$\vec{r}_1 = \frac{21}{5}\hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j})$$

$$\vec{r}_2 = \hat{j} - \frac{21}{5}\hat{k} + \mu(-2\hat{j} + \hat{k})$$

$$\vec{r} = \vec{OA} + \lambda\vec{AB} + \mu\vec{AC}$$

$$\vec{r} = -\frac{21}{5}\hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j}) +$$

part c:-

$$\lambda_2 = 5x - 6y + 5z = 0$$

$$\lambda_2 = \frac{x-0}{0}, \lambda_2 = \frac{y-1}{-2}$$

$$\lambda_2 = \frac{2-4 \cdot 2}{1}$$

From L_2 direction vector is

$$= \{0, -2, 1\}$$

From π_2 normal vector is

$$= (5, -6, 7)$$

$$\cos \theta = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{|a| |b|}$$

$$(a \cdot b) = \begin{bmatrix} 0 \\ 1 \\ 21 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$$= -6 + 29.4 = 23.4$$

$$|a| = \sqrt{(1)^2 + (21)^2} = \sqrt{1 + 441}$$

$$|a| = 4.3$$

$$|b| = \sqrt{(5)^2 + (6)^2 + (7)^2}$$

$$|b| = 10.49$$

$$\theta = \cos^{-1} \left(\frac{23.4}{4.3 \times 10.49} \right)$$

$$\theta = \cos^{-1} \frac{23.4}{45.11}$$

$$\theta = 59.34^\circ$$

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part d

$$\vec{A}_1 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix}, \vec{A}_2 = \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$$\vec{A}_1 \cdot \vec{A}_2 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$$= -21 - 6 = -27$$

$$|\vec{A}_1| = \sqrt{\left(\frac{-21}{5}\right)^2 + (1)^2} = \sqrt{17.64 + 1} = 4.3$$

$$|\vec{A}_2| = \sqrt{5^2 + (-6)^2 + 7^2} = 10.49$$

$$\theta = \cos^{-1} \frac{-27}{4.3 \times 10.49}, \therefore \theta = 126.78$$

acute angle: $\phi = 180 - 126.78$

$$\boxed{\phi = 53.23^\circ}$$

Question No: 05

part (a)

If $P = (-2, -1)$ and $Q = (-6, -3)$ are the end point of diameter of a circle, find the equation of the circle.

Mid point

$$\left(\frac{-2-6}{2} \right) \left(\frac{-1-3}{2} \right)$$

$$\frac{-8}{2}, \frac{-4}{2}$$

$$(-4, -2)$$

equation of circle $= (x-h)^2 + (y-k)^2 = r^2$ — (1)

$$(x+4)^2 + (y+2)^2 = r^2$$

$$\text{Let } (x, y) = (-2, -1)$$

$$(-2+4)^2 + (-1+2)^2 = r^2$$

$$(2)^2 + (1)^2 = r^2$$

$$4 + 1 = r^2$$

$$5 = r^2$$

$$r^2 = 5$$

put the values.

$$(x+4)^2 + (y+2)^2 = 5$$

∴

part (b)

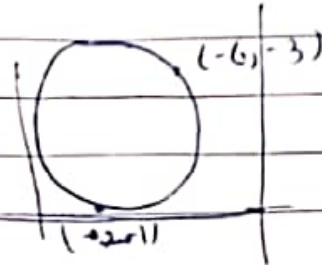
If the circle pass through $(4, 0)$ and $(0, 2)$ and center at y -axis then find the radius of the circle.

sol/

equation of circle

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\text{let } x = a, y = b$$



at point $(4, 0)$

$$(4)^2 + (-b)^2 = r^2$$

$$16 + b^2 = r^2 \quad \text{--- (I)}$$

at point $(0, 2)$

$$(0)^2 + (2-b)^2 = r^2$$

$$(2-b)^2 = r^2 \quad \text{--- (II)}$$

compare eq (I) and eq (II)

$$16 + b^2 = (2-b)^2$$

$$16 + b^2 = 4 - 4b + b^2$$

$$16 + b^2 - b^2 - 4 + 4b = 0$$

$$12 + 4b = 0$$

$$4b = -12$$

$$b = -3$$

Put the values in eq (I)

$$\text{So, } r^2 = (4)^2 + (-3)^2$$

$$r^2 = 16 + 9$$

$$r = \pm 5$$

~~radius~~ neglect -ve

$$r = 5$$

part C :-

Find the equation of directrix of parabola $y^2 = 100x$

$$y^2 = 100x$$

compare with

$$y^2 = 4ax$$

$$4a = 100$$

$$a = 25$$

equation of directrix = $x = -a$

$$x = -25$$

Part (d)

Find the equation of the axis of the parabola $x^2 = 24y$

$$x^2 = 24y$$

compare with

$$x^2 = 4ay$$

$$4a = 24$$

$$a = 8$$

So, focus is $F(a, 0) = F(8, 0)$

and equation of directrix is

$$x = -a$$

$$\boxed{x = -8}$$

part e:-

What is the major axis length for

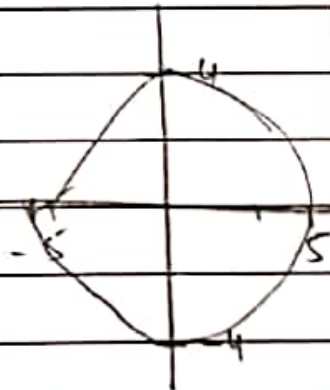
$$\text{ellipse } \left(\frac{x}{25}\right)^2 + \left(\frac{y}{16}\right)^2 = 1$$

compare with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2}{5^2} + \frac{y^2}{4^2}$$

$$a = 5$$

$$b = 4$$



$$c = \sqrt{a^2 - b^2} = \sqrt{25 - 16} = \pm 3$$

$$F_1 = (3, 0)$$

$$F_2 = (-3, 0)$$

$$\text{length of major axis} = 2a$$

$$= 2(5) = 10$$

Part (f)

If : length of major axis is 10 and minor is 8 and major axis is along x-axis then find the equation of ellipse.

$$\text{major axis} = 10$$

$$\text{minor axis} = 8$$

$$2a = 10$$

$$a = 5$$

$$2b = 8$$

$$b = 4$$

Equation of ellipse along x-axis.

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$