Introduction to Digital Filters

Classification of Digital Filters:

I) Regarding duration of impulse response:

(IIR)

Infinite Impulse Response

$h(n) \equiv$ Infinite number of samples

ex: $h(n) = 10 \left(\frac{1}{2}\right)^n u(n)$

$\updownarrow$

$$H(z) = 10 \frac{z}{z - \frac{1}{2}}$$

(Zeros & poles)

(FIR)

Finite Impulse Response

$h(n) \equiv$ Finite number of samples

ex: $h(n) = \{\underset{\uparrow}{1}, 2, 3\}$

$h(n) = \delta(n) + 2\delta(n-1) + 3\delta(n-2)$

$\updownarrow$

$$H(z) = 1 + 2z^{-1} + 3z^{-2}$$

No poles (Always stable)

II) Regarding the type of realization

Non-Recursive Realization

$\Downarrow$

$y(n)$ depends on i/p only $x(n)$.

$\Downarrow$

FIR $\boxed{D.E}$. $y(n) = a_0 x(n) + a_1 x(n-1) + \dots$

$$H(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} \dots$$

$a_i = \checkmark$, $b_i = 0$

ex:

$$y(n) = 2x(n) + 3x(n-1) + 2x(n-2)$$

$$H(z) = 2 + 3z^{-1} + 2z^{-2}$$

$\therefore h(n) = \{2, 3, 2\}$ FIR

Recursive Realization

$\Downarrow$

$y(n)$ depends on its previous values $[y(n-1), y(n-2) \dots]$

$\Downarrow$

IIR $y(n) + b_1 y(n-1) \dots = a_0 x(n) \dots$

$$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2} \dots}$$

$a_i = \checkmark$

$b_i = \checkmark$

ex:

$$y(n) + 2y(n-1) + \tfrac{1}{2} y(n-2) = 3x(n) + 2x(n-1)$$

$$H(z) = \frac{3 + 2z^{-1}}{1 + 2z^{-1} + \frac{1}{2} z^{-2}}$$

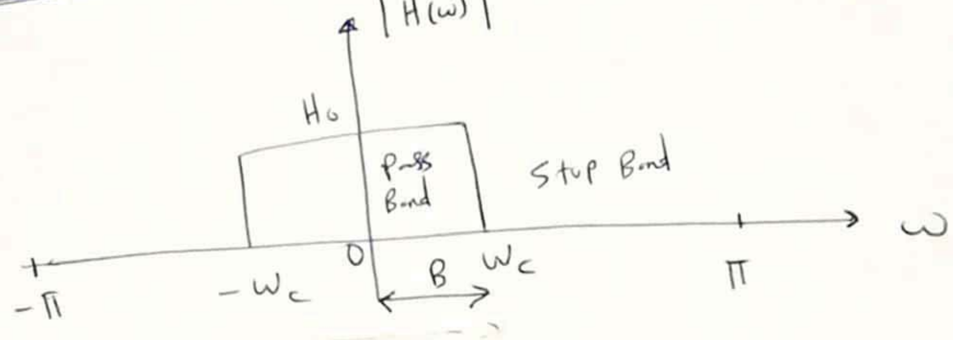
Recursive Realization (IIR)

III Regarding the magnitude of frequency response

Note: $H(\omega)$ = frequency response, $H(Z)$ = transfer function

$$H(\omega) = H(Z)\Big|_{Z=e^{j\omega}}$$

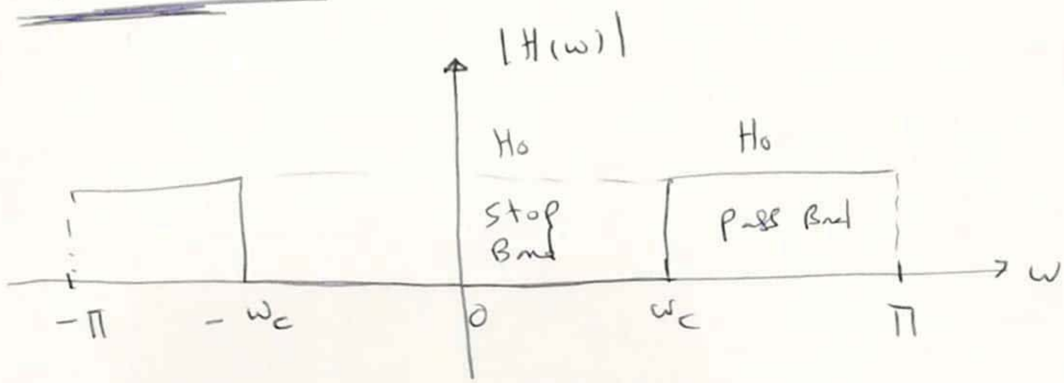
① Low-pass Filter (LPF):



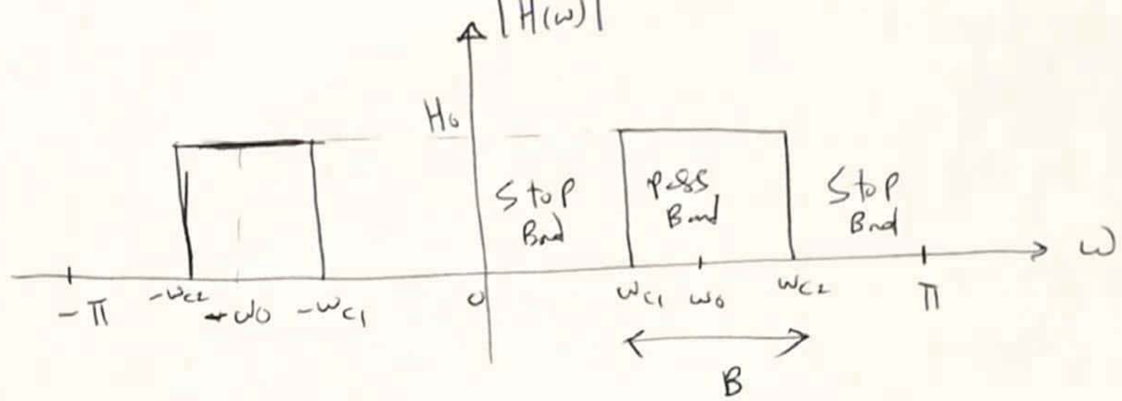
B: Bandwidth ω_c: cut-off frequency

H_0: gain (Constant) or unity gain

② High-pass Filter (HPF):



③ Band-pass Filter (BPF):

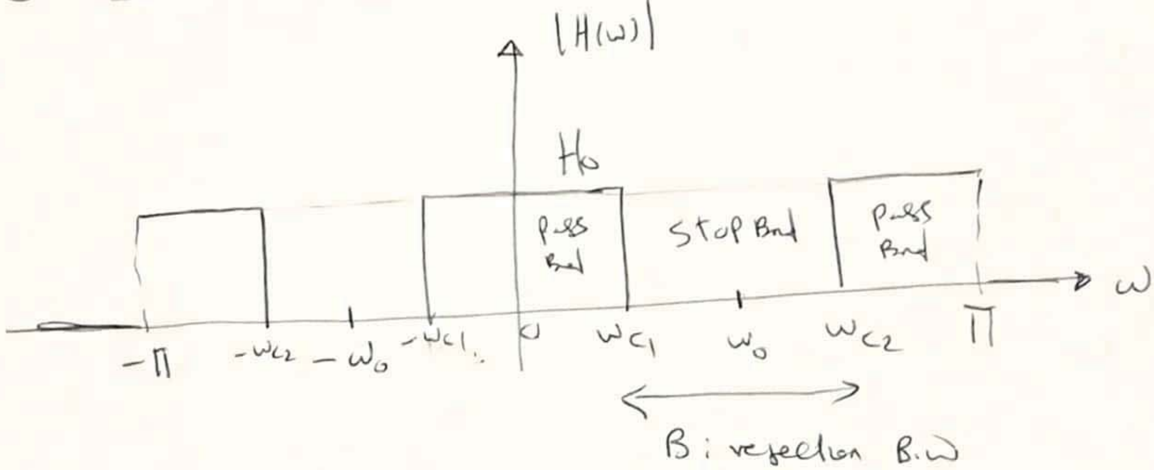


B: Filter Bandwidth

ω_{c1}, ω_{c2}: Cutoff frequencies

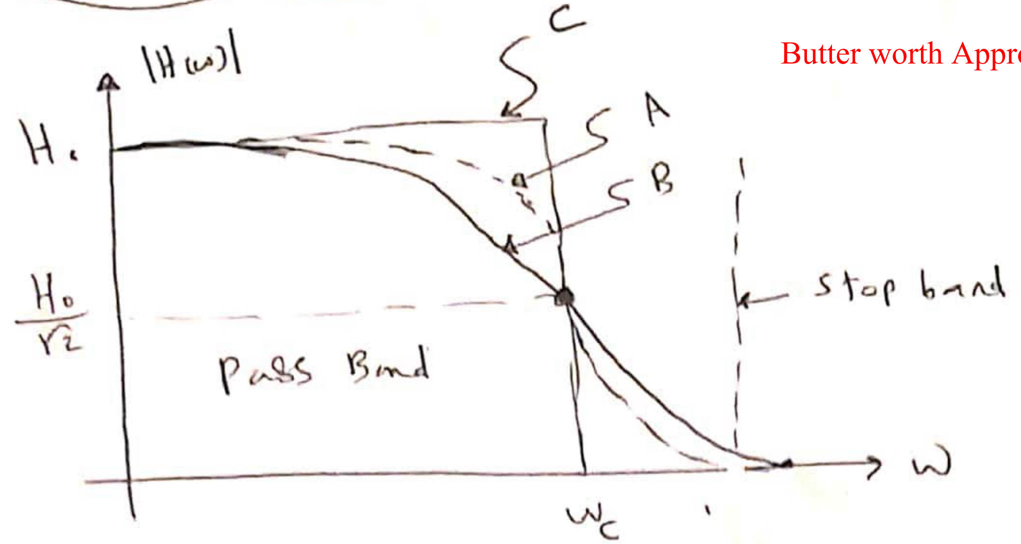
H_0: gain (constant) or unity.

④ Band-Stop Filter (BSF):



LPF Approximations

Butter worth Approximation



---- order (A)

order (B)

order (C)

order A > order B

order (C) = ∞

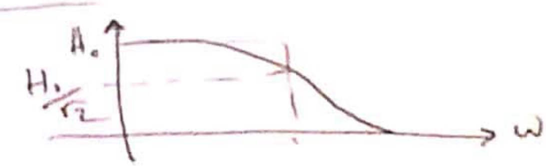
[Ideal LPF]

Butter worth Approximation

$$|H(\omega)|^2 = \frac{H_0^2}{1 + \left(\frac{\omega}{\omega_c}\right)^{2N}}$$

There are different approximations:

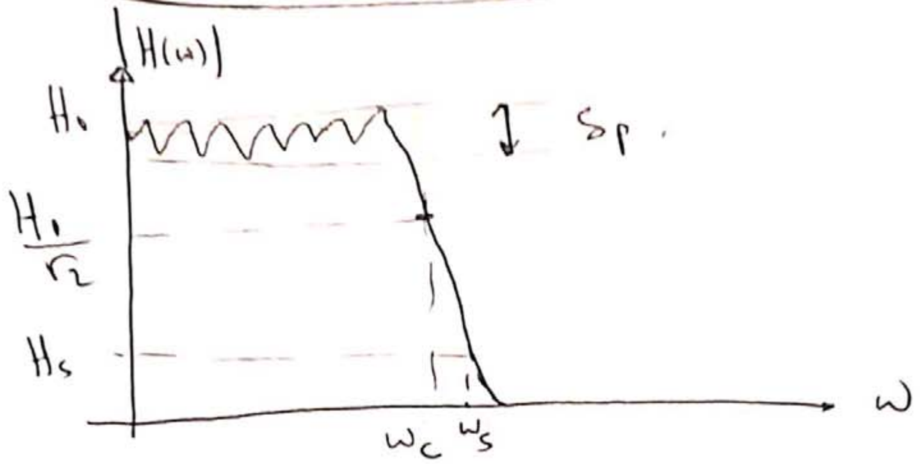
[1] Butter worth:



- Maximal Flat Response in pass Band [advantage]
- high Transition Bandwidth [disadvantage]

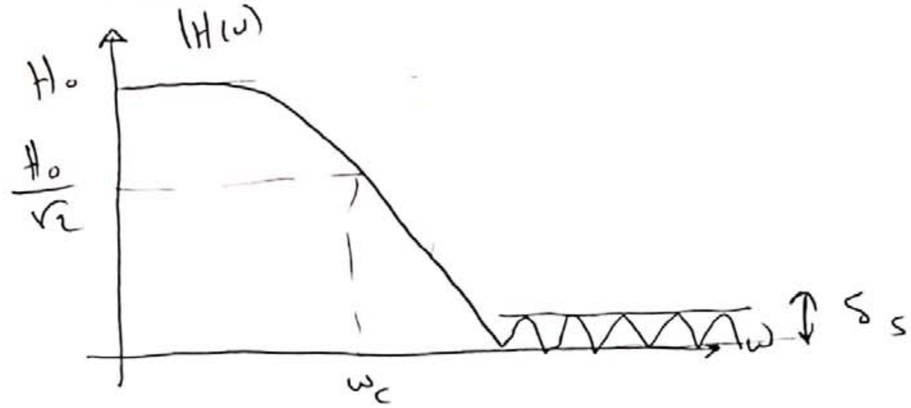
Scanned by CamScanner

2 Chebychev I : Type I



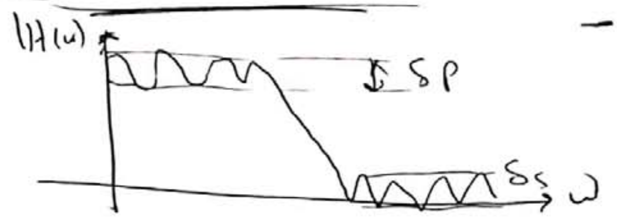
- pass Band equiripples [ripples in pass Band] disadvant
- Transition Bw is medium [Δw is medium]

3 Chebychev Type II :



- Stop Band equiripples [ripples in Stop Band] disadvantage
- Δw medium

4 Elliptic :



- Equiripples in both pass & stop bands [disadvantage]
- Δw is the smallest [advantage]

Structure for Realization of Filters

Realization ≡ Flow graph

- FIR digital Filters
  - Direct Form I
  - Simplified Form
- IIR digital Filters
  - Parallel Form
  - Cascaded Form
  - Direct Form II (Canonic Form)
  - Direct Form I

(I) IIR digital Filters

D.E : General Form of IIR Digital Filters

$$y(n) + b_1\, y(n-1) + b_2\, y(n-2) \cdots = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2) \cdots$$

$$H(z) = \frac{x \text{ Coefficients}}{y \text{ Coefficients}} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} \cdots}{1 + b_1 z^{-1} + b_2 z^{-2} \cdots}$$

① Direct Form I

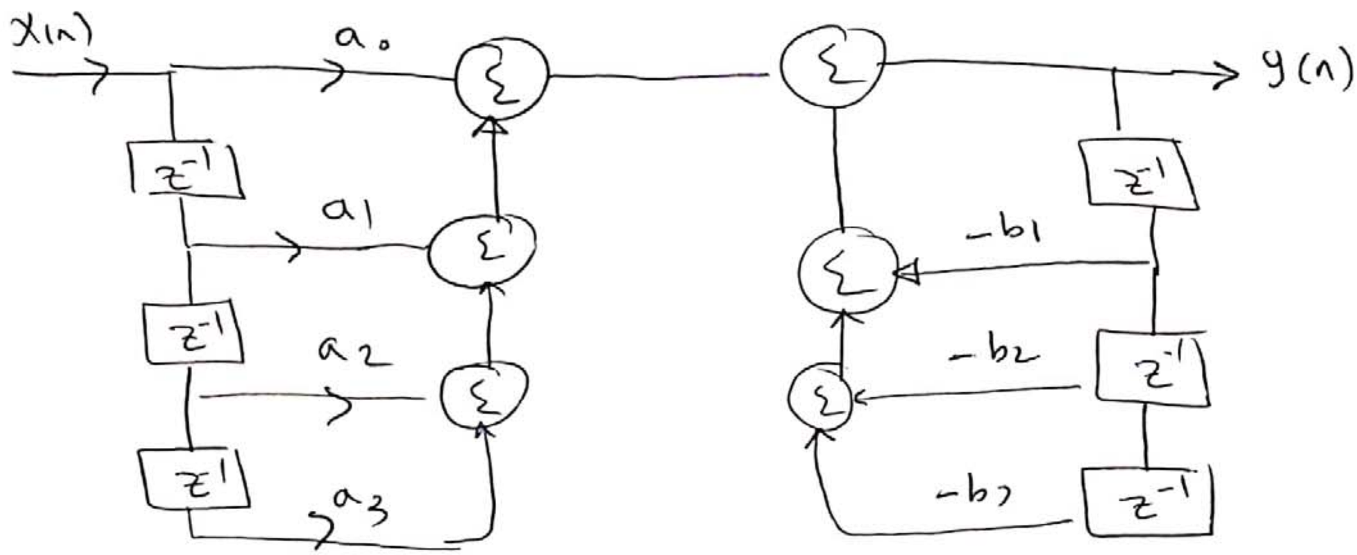
Recall: $H(z) = \dfrac{a_0 + a_1 z^{-1} + a_2 z^{-2} \cdots}{b_0 + b_1 z^{-1} + b_2 z^{-3} \cdots}$

or

D.E: $y(n) + b\,y(n-1) + b_2\,y(n-2) \cdots = a_0\,x(n) + a_1\,x(n-1) \cdots$

This can be written as:

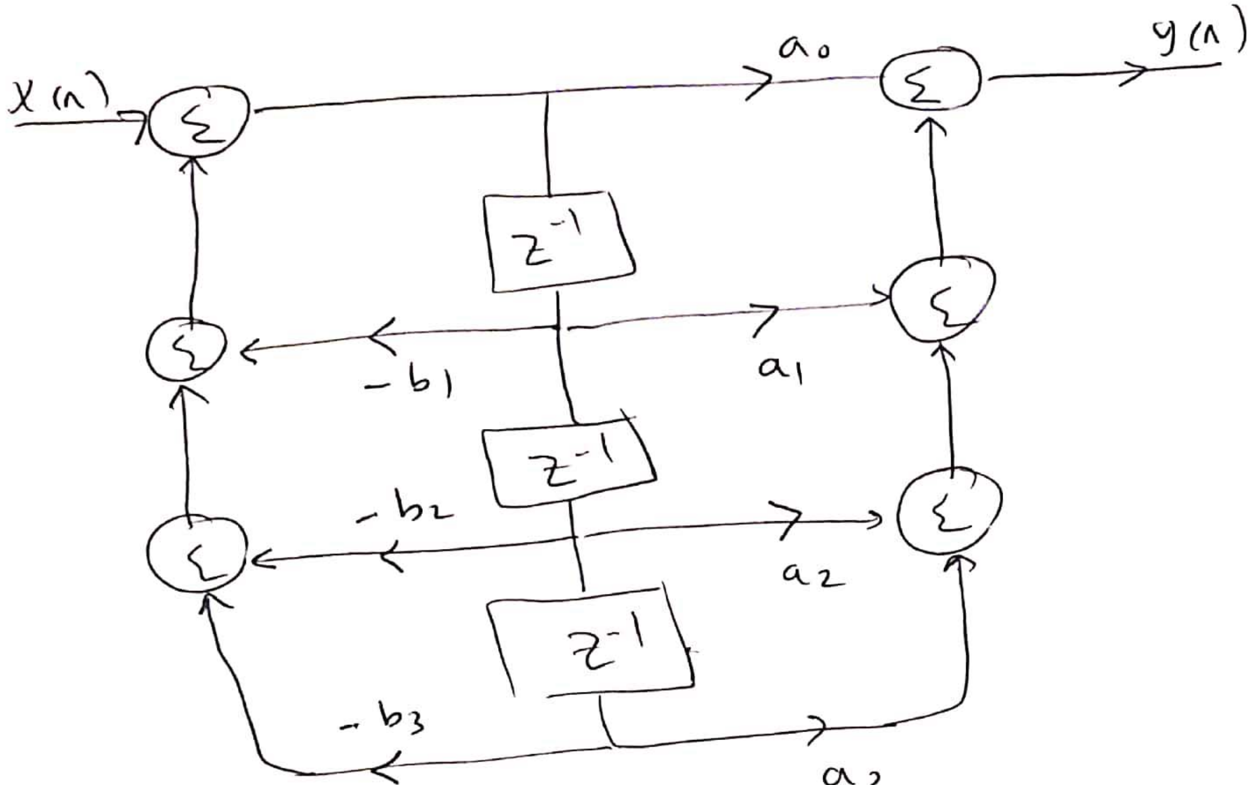
$$y(n) = -b_1\,y(n-1) - b_2\,y(n-2) \cdots + a_0\,x(n) + a_1\,x(n-1) + a_2\,x(n-2) \cdots$$



(2) Direct Form (II) [Canonic Form] Flow graph

→ minimize the memory by using minium number of delay elements

D.E: $y(n) + b_1 y(n-1) + b_2 y(n-2) \cdots = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2) \cdots$



→ It can be noted that we have used $\frac{1}{2}$ number of delay units used in Direct form I.

3 Cascaded Form

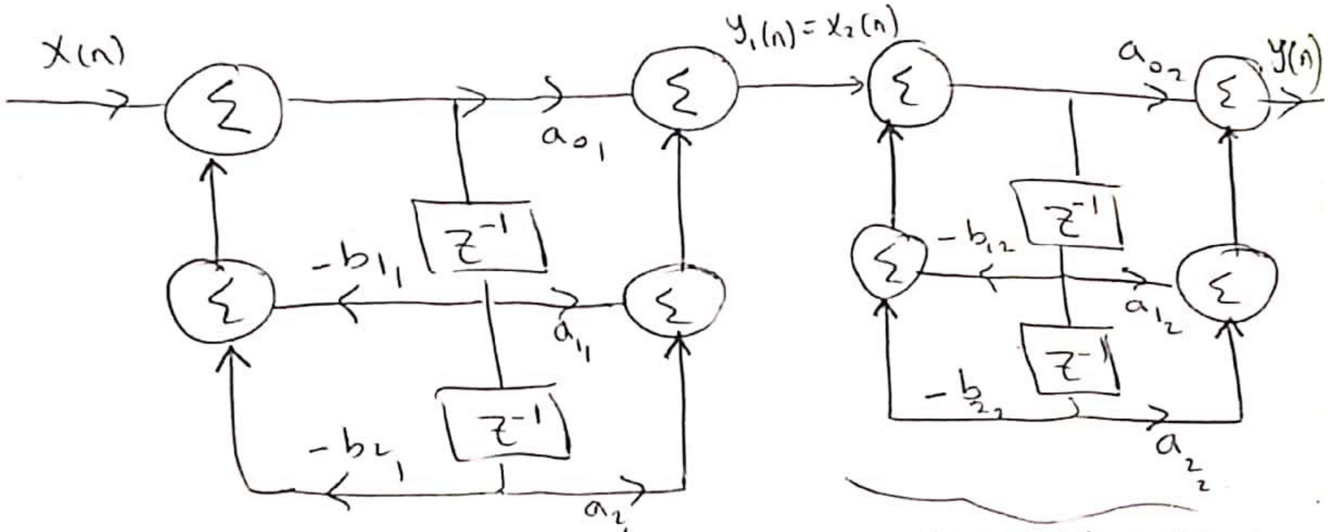
For high order Filters, we can write

$$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z) \text{ --- --- }$$

For ex order of Filter $n = 4$, using 2 sections Cascaded

$$H(z) = \boxed{\frac{a_{0_1} + a_{1_1} z^{-1} + a_{2_1} z^{-2}}{1 + b_{1_1} z^{-1} + b_{2_1} z^{-2}}} \cdot \boxed{\frac{a_{0_2} + a_{1_2} z^{-1} + a_{2_2} z^{-2}}{1 + b_{1_2} z^{-1} + b_{2_2} z^{-2}}}$$

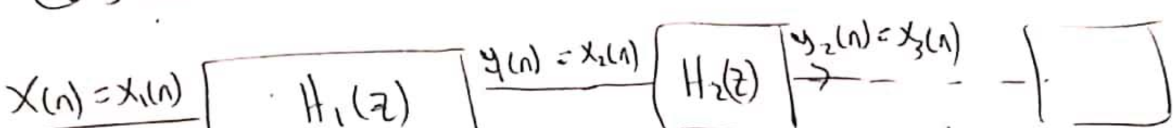
Section (1) Section (2)



section (1) in Direct form II

section (2) in Direct form (II)

→ Cascaded Form:



and SoN

Steps for Cascade Realization (flow graph)

1) factorize both numerator and denominator of $H(z)$

2) Divide $H(z)$ into sections such that

$$H(z) = H_1(z) \cdot H_2(z) \cdots$$

3) draw each one in cascade [each section may be drawn using Direct Form I or II]

→ Cascade Realization is relatively slow

$\boxed{4}$ **Parallel Form** : ⇒ Faster than Cascade Realization

In this case the transfer function $H(z)$ can be written as:

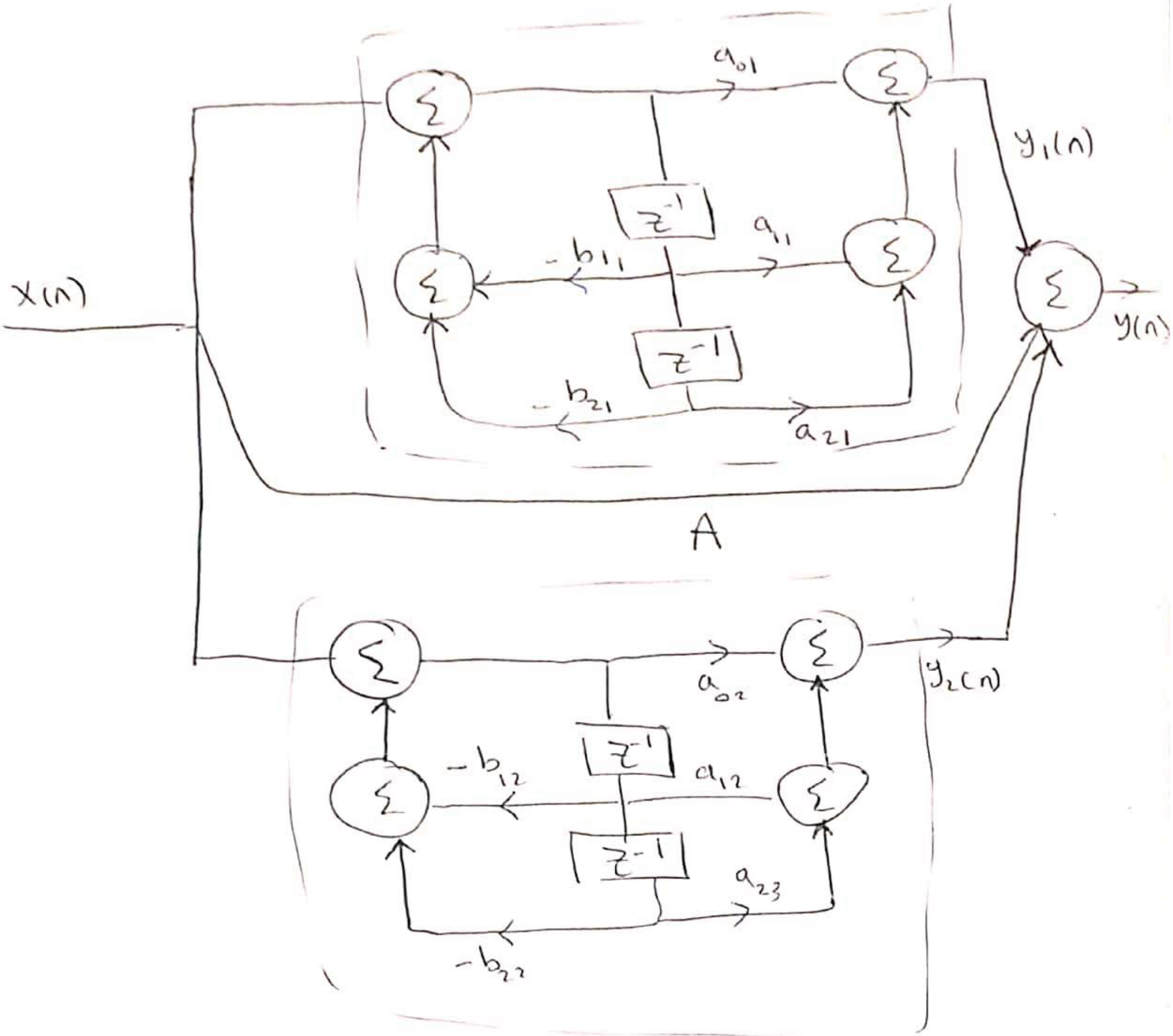
$$H(z) = H_1(z) + H_2(z) + \cdots$$

or

$$H(z) = \underset{\substack{\uparrow \\ \text{Constant.}}}{A} + H_1(z) + H_2(z) \cdots$$

ex: $H(z) = H_1(z) + H_2(z) + A$

where $H_1(z) = \dfrac{a_{01} + a_{11} z^{-1} + a_{21} z^{-2}}{1 + b_{11} z^{-1} + b_{21} z^{-2}}$, $H_2(z) = \dfrac{a_{02} + a_{12} z^{-1} + a_{22} z^{-2}}{1 + b_{12} z^{-1} + b_{22} z^{-2}}$



Steps for parallel Realization (Flow graph)

1) If Degree of numerator $\geq$ degree of denominator of $H(z)$ $\Rightarrow$ make Long Division

2) If Degree of numerator $<$ degree of denominator of $H(z)$

$\Downarrow$

factorize the denominator provided that no complex roots

3) After denominator factorization, Assume Constants for partial fractions

4) Find Constants $<$ Covering / Substitution

5) Now, we have

$$H(z) = H_1(z) + H_2(z) - \;—\; —$$

6) draw it

ex: For the Following system:

$$y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7\, x(n) - 0.252\, x(n-2)$$

1) Specify if this Filter IIR or FIR.

2) Implement this system using Direct Form (I), Direct Form II, Cascaded, Parallel.

Sol

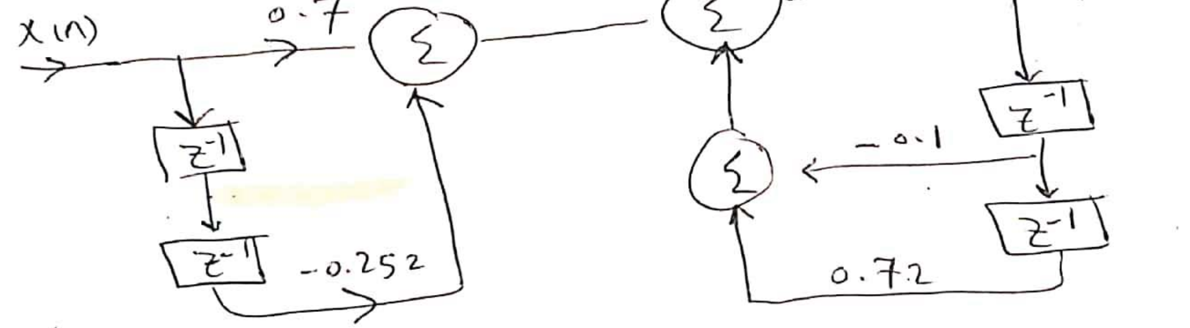
Put it in General Form:

$$y(n) + 0.1\, y(n-1) - 0.72\, y(n-2) = 0.7\, x(n) - 0.252\, x(n-2)$$

as $y(n)$ Depends on $y(n-1)$, $y(n-2)$. $\Rightarrow \boxed{IIR}$

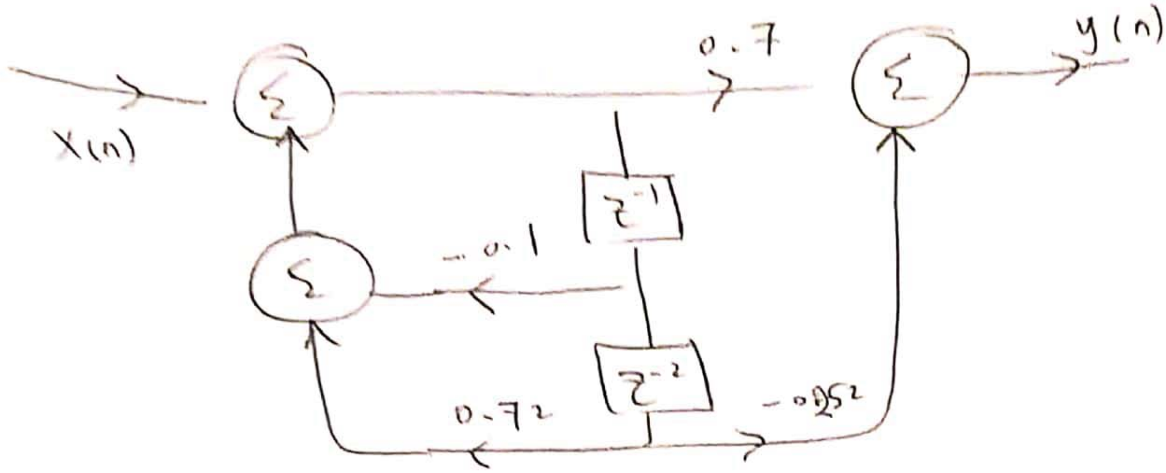
$$H(z) = \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

1) Direct Form I



Scanned by CamScanner

2) Direct Form II :



3) $\boxed{\text{CasCaded Form}}$

Steps :

1 - factorize numerator & denominator → multiply by $\frac{z^2}{z^2}$

$$H(z) = \frac{0.7 z^2 - 0.252}{z^2 + 0.1 z - 0.72} = \frac{0.7\left[z^2 - \frac{0.252}{0.7}\right]}{z^2 + 0.1 z - 0.72}$$

$$z^2 - \frac{0.252}{0.7} = (z - 0.6)(z + 0.6)$$ Calculator factorize using

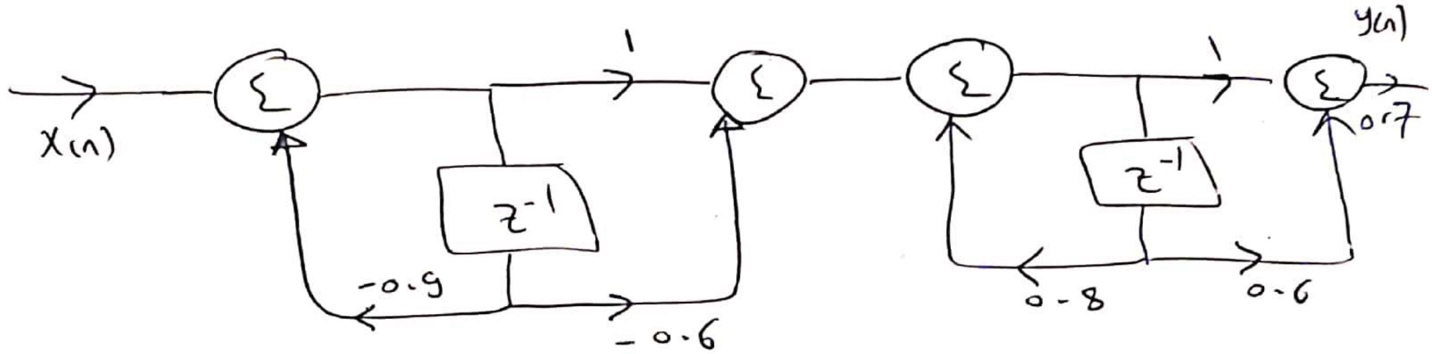
$$z^2 + 0.1 z - 0.72 = (z - 0.8)(z + 0.9)$$

$$H(z) = 0.7 \frac{(z - 0.6)(z + 0.6)}{(z - 0.8)(z + 0.9)} * \frac{z^{-2}}{z^{-2}}$$

$$H(z) = 0.7 \frac{(1 - 0.6 z^{-1})(1 + 0.6 z^{-1})}{(1 - 0.8 z^{-1})(1 + 0.9 z^{-1})}$$

Take any bracket from numerator & denominator to form $H_1(z)$ & the same for $H_2(z)$

$$H(z) = 0.7 \cdot \underbrace{\frac{(1-0.6z^{-1})}{(1+0.9z^{-1})}}_{H_1(z)} \cdot \underbrace{\frac{(1+0.6z^{-1})}{(1-0.8z^{-1})}}_{H_2(z)}$$



4 parallel:

In parallel, you should work with negative degrees to make the realization.

Steps:

$$H(z) = \frac{0.7(z^2 - 0.36)}{z^2 + 0.1z - 0.72}$$

multiply by $\frac{z^{-2}}{z^{-2}}$

$$H(z) = 0.7 \frac{(1 - 0.36z^{-2})}{(1 + 0.1z^{-1} - 0.72z^{-2})}$$

Same Degree for numerator & denominator

⇓

Long Division

$$\begin{array}{r|l} & 0.5 \text{ result} \\ \hline -0.72z^{-2} + 0.1z^{-1} + 1 & -0.36z^{-2} + 1 \\ & \underline{\mp 0.36z^{-2} \mp 0.05z^{-1} \mp 0.5} \\ & -0.05z^{-1} + 0.5 \end{array}$$

Should be in order

remainder

Scanned by CamScanner

$$H(z) = 0.7\left[0.5 + \frac{0.5 - 0.05z^{-1}}{1 + 0.1z^{-1} - 0.72z^{-2}}\right]$$

Factorize denominator:

$$1 + 0.1z^{-1} - 0.72z^{-2} = 0 \longrightarrow z^2 + 0.1z - 0.72 = 0$$

$$(z + 0.9)(z - 0.8) = 0$$

$$(1 + 0.9z^{-1})(1 - 0.8z^{-1}) = 0$$

Partial Fractions

$$\Longrightarrow \frac{0.5 - 0.05z^{-1}}{(1 + 0.9z^{-1})(1 - 0.8z^{-1})}$$

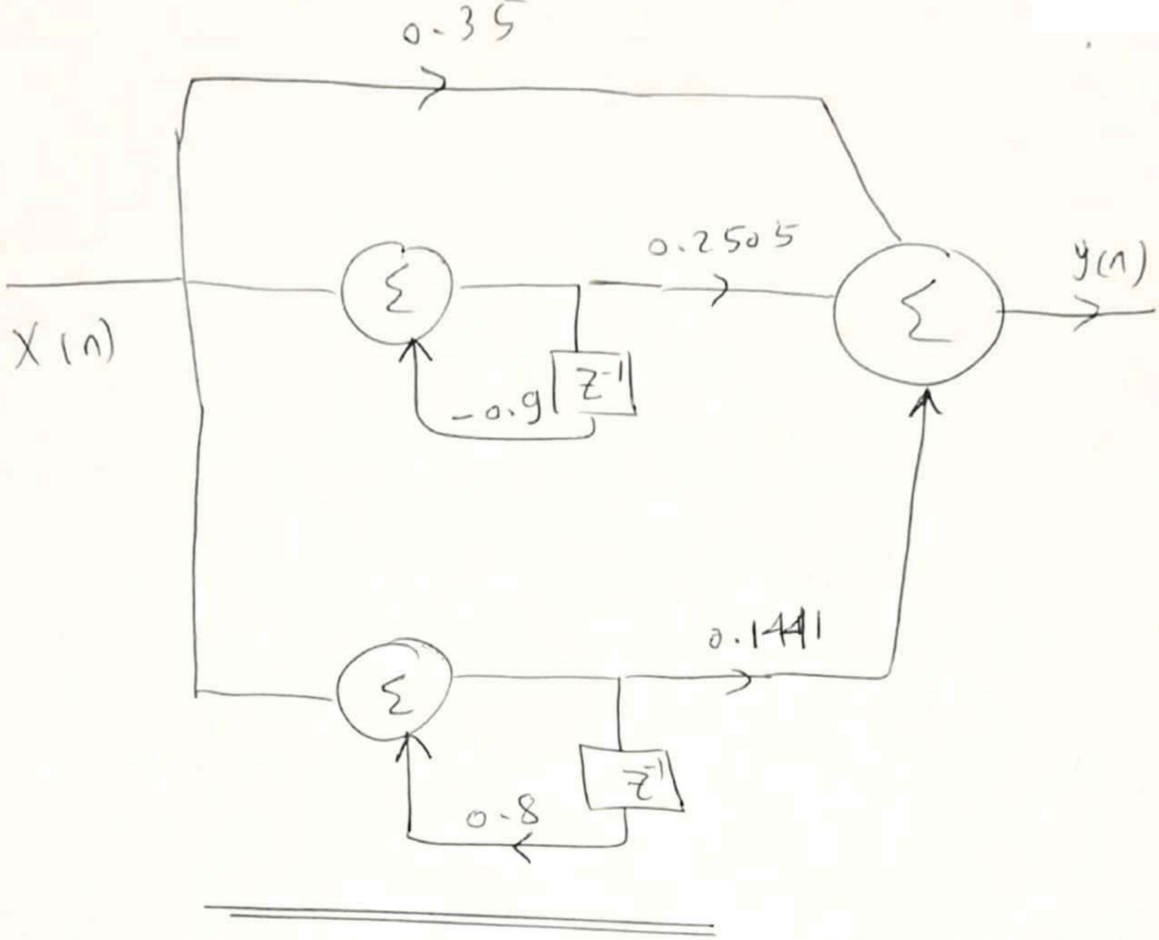
$$= \frac{A}{(1 + 0.9z^{-1})} + \frac{B}{(1 - 0.8z^{-1})}, \text{ let } z^{-1} = x$$

$$A = \left.\frac{0.5 - 0.05x}{1 - 0.8x}\right|_{x = \frac{-1}{0.9}} = \boxed{\frac{5}{17}}$$

$$B = \left.\frac{0.5 - 0.05x}{(1 + 0.9x)}\right|_{x = \frac{1}{0.9}} = \boxed{\frac{7}{34}}$$

$$H(z) = 0.7\left[0.5 + \frac{5/17}{1 + 0.9z^{-1}} + \frac{7/34}{1 - 0.8z^{-1}}\right]$$

$$H(z) = 0.35 + \frac{0.2059}{1 + 0.9z^{-1}} + \frac{0.1441}{1 - 0.8z^{-1}}$$



0.35
0.2505
y(n)
X(n)
Σ
Σ
-0.9
z^{-1}
0.1441
Σ
z^{-1}
0.8

Note

IF you have zeros or poles "imaginary" $\Rightarrow$ You should have the complex conjugate & you should use 2nd order sections in the cascade realization to avoid any imaginary values

Example: Implement using cascade realization

$$H(z) = \frac{(z-2)(z+3)\,[z+0.5-j0.5]\,[z+0.5+j0.5]}{(z-0.2)(z+0.7)\,[z-(-0.5-j0.5)]\,[z-(0.5+j0.5)]}$$

Solution

We will use 2nd order cascaded sections due to imaginary poles & zeros.

let $H_1(z) = \dfrac{(z-2)(z+3)}{(z-0.2)(z+0.7)} = \dfrac{z^2+z-6}{z^2+0.5z-0.14}$

$$\boxed{H_1(z) = \frac{1+z^{-1}-6z^{-2}}{1+0.5z^{-1}-0.14z^{-2}}}$$

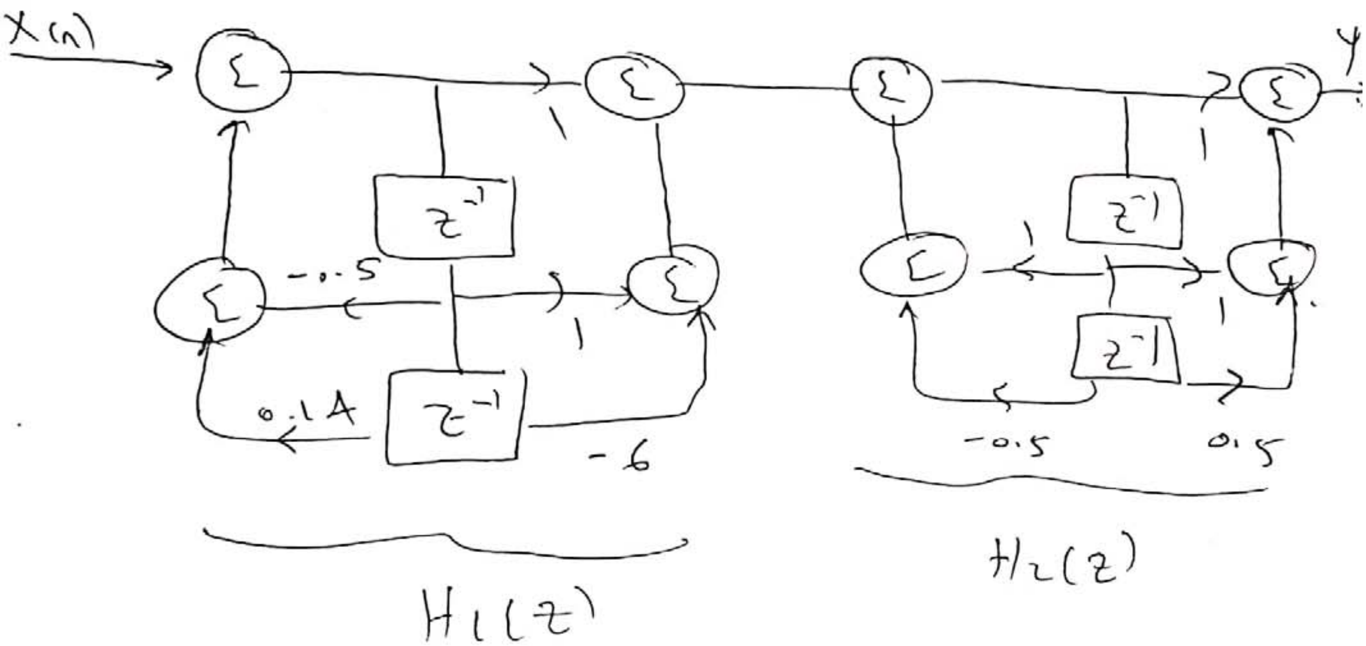
$$H_2(z) = \frac{[z-(-0.5+j0.5)]\,[z-(-0.5-j0.5)]}{[z-(-0.5-j0.5)]\,[z-(0.5+j0.5)]}$$

take imaginary together

$$H_2(z) = \frac{z^2+z+0.5}{z^2-z+0.5} = \frac{1+z^{-1}+0.5z^{-2}}{1-z^{-1}+0.5z^{-2}}$$

$$H(z) = H_1(z)\cdot H_2(z)$$

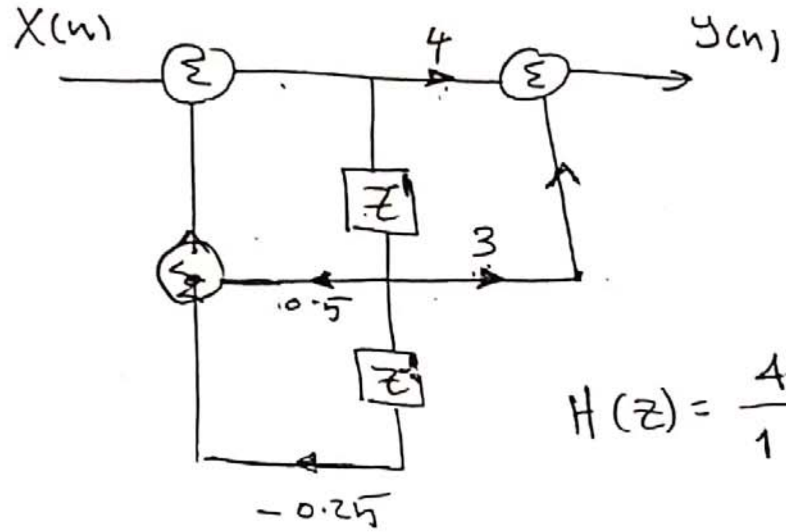
$$= \left[\frac{1+z^{-1}-6z^{-2}}{1+0.5z^{-1}-0.14z^{-2}}\right]\cdot\left[\frac{1+z^{-1}+0.5z^{-2}}{1-z^{-1}+0.5z^{-2}}\right]$$



Scanned by CamScanner

Examples

Find D.E. of the system having :- the following Block diagram & Transfer Function



$$H(z) = \frac{4 + 3z^{-1}}{1 - 0.5z^{-1} + 0.25z^{-2}}$$

Solution from Canonic form (Direct form II)

X-Coff

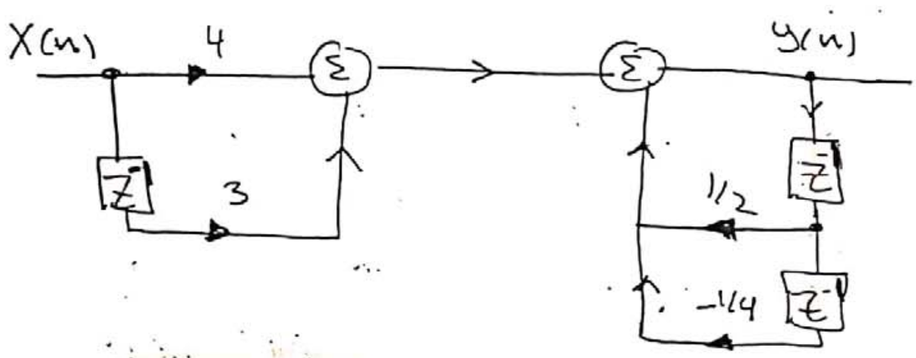
$$H(z) = \frac{4 + 3z^{-1}}{1 - 0.5z^{-1} + 0.25z^{-2}}$$

Y-Coff

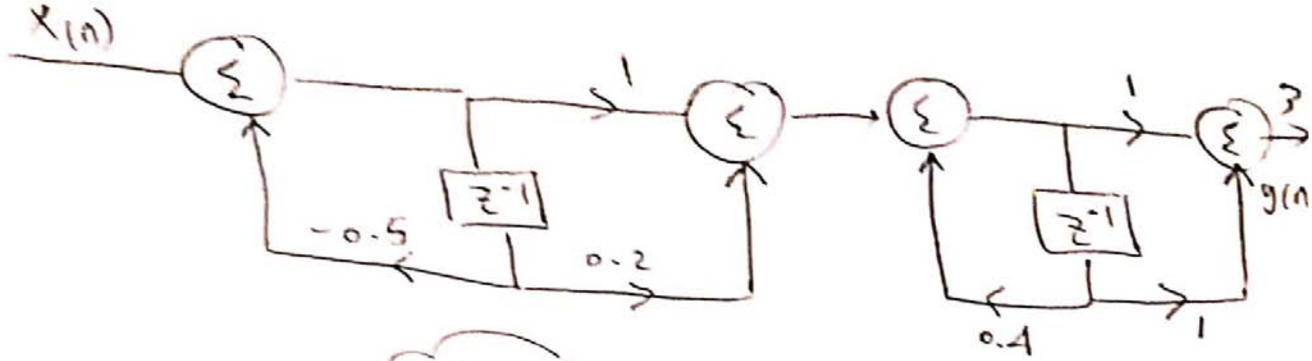
$$\Downarrow$$

$$\therefore\ y(n) - \frac{1}{2}y(n-1) + \frac{1}{4}y(n-2) = 4X(n) + 3X(n-1)$$

ex : describe the above System using direct form(I)



ex: For the Following system, Find Transfer Function & D.E



Sol

it is Cascaded Realization (2 sections 1st order)

$$H(z) = 3 H_1(z) H_2(z)$$

$$= 3 \left[\frac{1 + 0.2 z^{-1}}{1 + 0.5 z^{-1}} \right] \left[\frac{1 + z^{-1}}{1 - 0.4 z^{-1}} \right]$$

$$H(z) = \frac{3 (1 + 0.2 z^{-1})(1 + z^{-1})}{(1 + 0.5 z^{-1})(1 - 0.4 z^{-1})}$$

multiply brackets to get D.E

$$H(z) = \frac{3 + 3.6 z^{-1} + 0.6 z^{-2}}{1 + 0.1 z^{-1} - 0.2 z^{-2}}$$

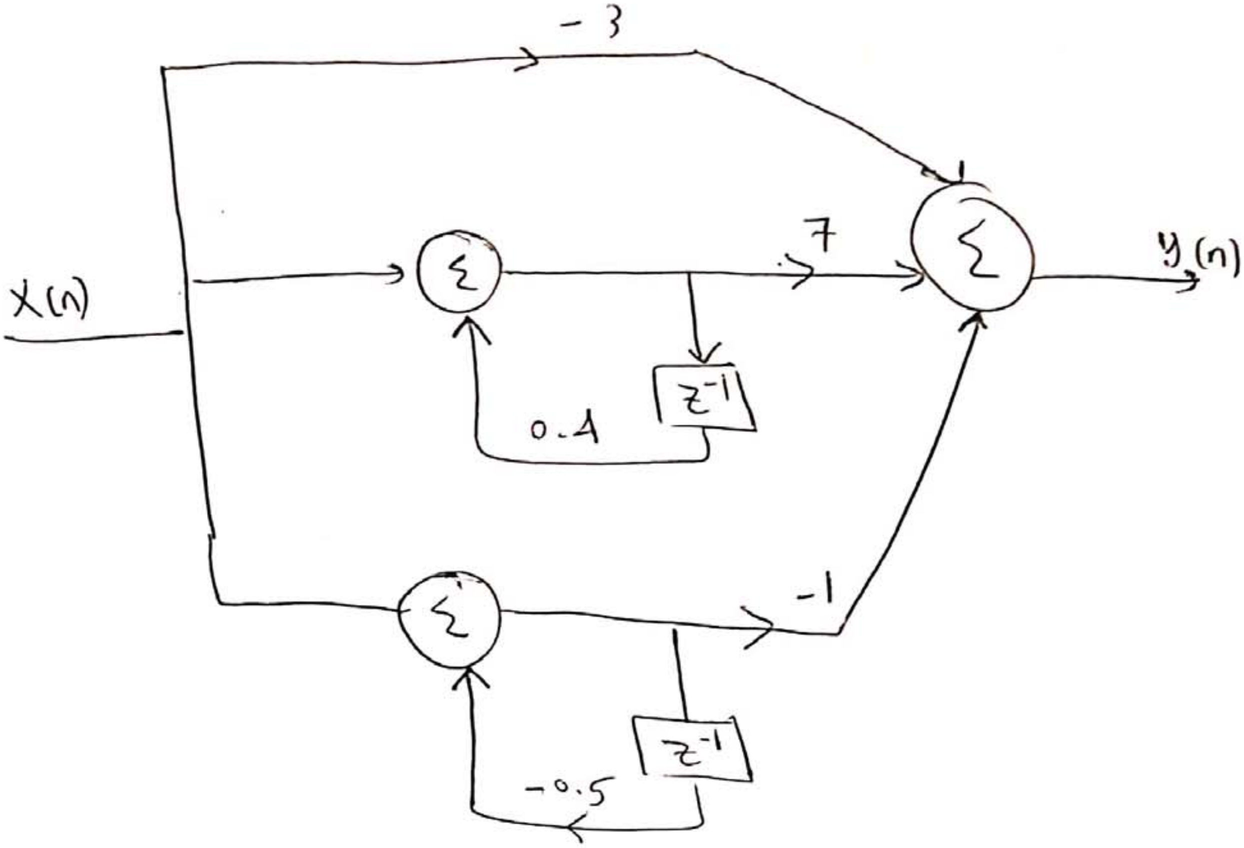
$a_0 = 3$, $a_1 = 3.6$, $a_2 = 0.6$

$b_1 = 0.1$, $b_2 = -0.2$

D.E : $y(n) + 0.1\, y(n-1) - 0.2\, y(n-2)$

$= 3\, x(n) + 3.6\, x(n-1) + 0.6\, x(n-2)$

For the Following system, Find Transfer Function & D.E & Type of Filter.



Sol

it is parallel Realization

$$H(z) = -3 + \frac{7}{1 - 0.4 z^{-1}} - \frac{1}{1 + 0.5 z^{-1}}$$

$$\Downarrow$$

$$H(z)=\frac{-3(1-0.4z^{-1})(1+0.5z^{-1})+7(1+0.5z^{-1})-(1-0.4z^{-1})}{(1-0.4z^{-1})(1+0.5z^{-1})}$$

$$H(z)=\frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

$$\Downarrow$$

D.E: $y(n)+0.1\,y(n-1)-0.2\,y(n-2)=3\,x(n)+3.6\,x(n-1)+0.6\,x(n-2)$

$$\Downarrow$$

IIR Filter.

Recall

(I) How to get $H(Z)$ · Transfer Function of system)

① given $h(n) \longrightarrow H(z) = Z\{h(n)\}$

② given: D.E in General Form

$$y(n) + a_1 y(n-1) + a_2 y(n-2) \; --- = b_0 x(n) + b_1 x(n-1) --$$

$$\Downarrow$$

$$H(z) = \frac{x \text{ Coeff}}{y \text{ Coeff}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} --}{1 + a_1 z^{-1} + a_2 z^{-2} --}$$

③ given: Block diagram (Flow graph) $\longrightarrow$ Direct Form I, II, Parallel, CasCade

$$\Downarrow$$

Know $H(z)$ & D.E

④ given: zeros $z_1, z_2 --$, Poles $P_1, P_2 --$, gain $= K$

$$H(z) = K \frac{(z-z_1)(z-z_2) ---}{(z-P_1)(z-P_2) ---}$$

(II) How we can use $H(z)$?

Usage of $H(z)$

$x(n) = \ldots$, Req: $y(n)$

expression

1- $Y(z) = X(z)\,H(z)$

2- $y(n) = Z^{-1}\{Y(z)\}$.

For any input to the filter we can get the output.

Know Filter Stability

1- IF all poles of $H(z) < 1$

$|P| < 1$

$\Downarrow$

Stable

2- IF any poles $|P| > 1$

or

more than pole equal $= 1$

UnStable

3- only one pole $= 1$ & others < 1

marginally Stable

$\Downarrow$

Know filter stability

Consider the discrete - time - system described by D.E.

$$y(n) = 4\,x(n) - \frac{1}{2}x(n-1) - \frac{1}{2}x(n-2) + \frac{1}{4}y(n-1) + \frac{1}{4}y(n-2) - \frac{1}{16}y(n-3).$$

a) Find Transfer Function of system.

b) Discuss stability.

c) Depict the parallel form representation for the Transfer Function using First-order sections implemented as a Canonic Form

Sol

(a) write D.E in General Form:

$$y(n) - \frac{1}{4}y(n-1) - \frac{1}{4}y(n-2) + \frac{1}{16}y(n-3) = 4\,x(n) - \frac{1}{2}x(n-1) - \frac{1}{2}x(n-2)$$

$$H(z) = \frac{X \div Y \text{ both}}{Y \div Y \text{ both}} = \frac{1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{1}{4}z^{-2} + \frac{1}{16}z^{-3}}$$

(b) To discuss stability

we should get poles

$$\rightarrow 1 - \frac{1}{4}z^{-1} - \frac{1}{4}z^{-2} + \frac{1}{16}z^{-3} = 0 \qquad * z^3$$

$$z^3 - \frac{1}{4}z^2 - \frac{1}{4}z + \frac{1}{16} = 0$$

Solve by Calculator [Mode → Eq → 3rd Degree]

$$z_1 = -0.5 \; , \; z_2 = 0.5 \; , \; z_3 = 0.25$$

$$|P_1| = 0.5 < 1 \; , \; |P_2| = 0.5 < 1 \; , \; |P_3| = 0.25 < 1$$

all |poles| < 1 → $\boxed{\text{Stable system}}$

(c) Req: parallel realization

(Canonic Form ⇓ Partial Fractions , each section 1st order) poles: 0.5, -0.5, 0.25

$$(z-0.5)(z+0.5)(z-0.25)=0 \quad * z^{-3}$$

$$\left(1-\tfrac{1}{2}z^{-1}\right)\left(1+\tfrac{1}{2}z^{-1}\right)\left(1-\tfrac{1}{4}z^{-1}\right)=0$$

$$H(z)=\frac{4-\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$\Downarrow$ partial Fractions

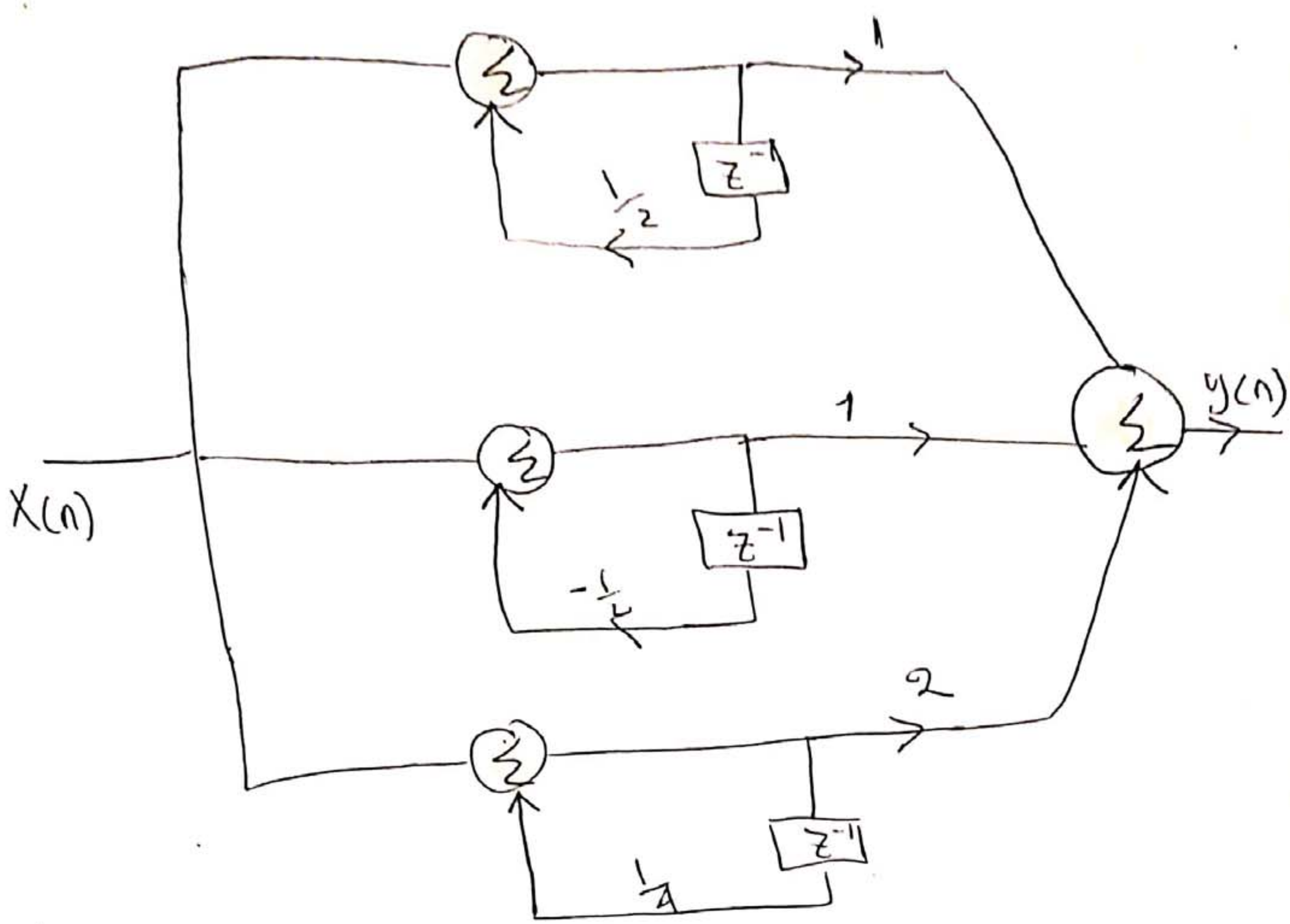
$$H(z)=\frac{A}{1-\frac{1}{2}z^{-1}}+\frac{B}{1+\frac{1}{2}z^{-1}}+\frac{C}{1-\frac{1}{4}z^{-1}}$$

$$A=\left.\frac{4-\frac{1}{2}x-\frac{1}{2}x^2}{\left(1+\frac{1}{2}x\right)\left(1-\frac{1}{4}x\right)}\right|_{x=2}\quad \frac{4-1-2}{2*\frac{1}{2}}=1$$

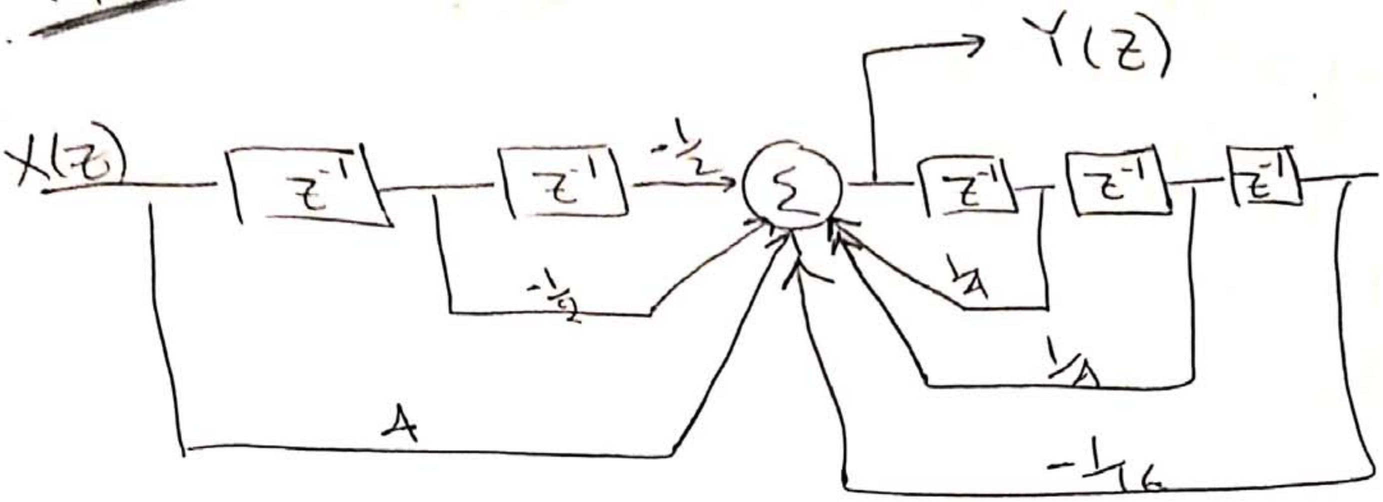
$$B=\left.\frac{4-\frac{1}{2}x-\frac{1}{2}x^2}{\left(1-\frac{1}{2}x\right)\left(1-\frac{1}{4}x\right)}\right|_{x=-2}=\frac{4+1-2}{2*\frac{3}{2}}=1$$

$$C=\left.\frac{4-\frac{1}{2}x-\frac{1}{2}x^2}{\left(1-\frac{1}{2}x\right)\left(1+\frac{1}{2}x\right)}\right|_{x=4}=\frac{4-2-8}{(-1)(3)}=\frac{-6}{-3}=2$$

$$\therefore H(z)=\frac{1}{1-\frac{1}{2}z^{-1}}+\frac{1}{1+\frac{1}{2}z^{-1}}+\frac{2}{1-\frac{1}{4}z^{-1}}$$


1
1/2
X(n)
1
y(n)
-1/2
2
1/4
z^{-1}
z^{-1}
z^{-1}

Exam



a) Find Transfer Function of system.

b) Discuss stability.

c) Depict the parallel Form representation For the transfer Function using First order sections implemented as a Canonic Form

Sol

$$Y(n) = 4\,x(n) - \frac{1}{2}x(n-1) - \frac{1}{2}x(n-2) + \frac{1}{4}y(n-1) + \frac{1}{4}y(n-2) - \frac{1}{16}y(n-3) \quad (IIR)$$

put in Form: $$y(n) - \frac{1}{4}y(n-1) - \frac{1}{4}y(n-2) + \frac{1}{16}y(n-3) = 4\,x(n) - \frac{1}{2}x(n-1) - \frac{1}{2}x(n-2)$$

" Same previous problem "

ex :

Consider the discrete - time system described by D.E :-

$$y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n) + \frac{1}{3} x(n-1)$$

1. Find Transfer Function of system
2. Implement this system using Direct Form I, Direct Form II.
3. Implement the system in cascade realization
4. Implement the system in parallel realization

Sol

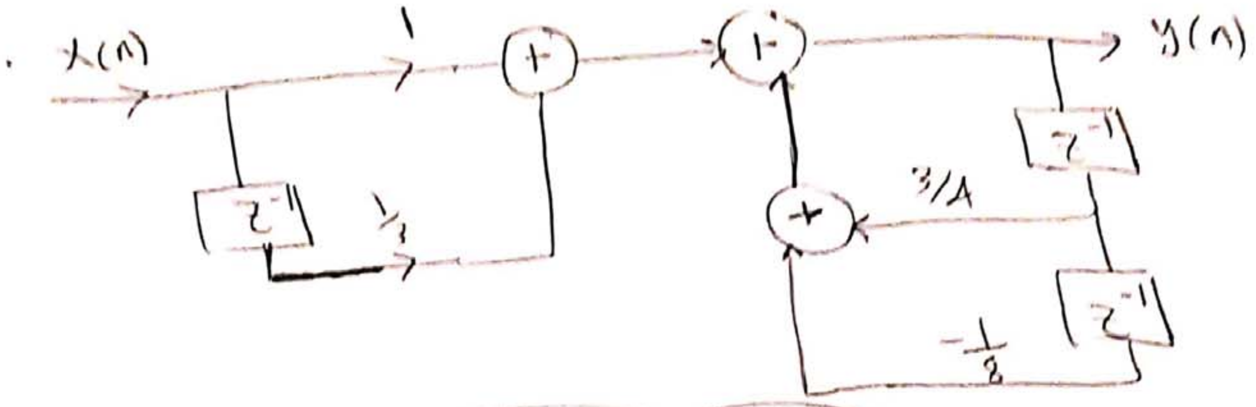
Write D.E in General Form :-

$$y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n) + \frac{1}{3} x(n-1)$$

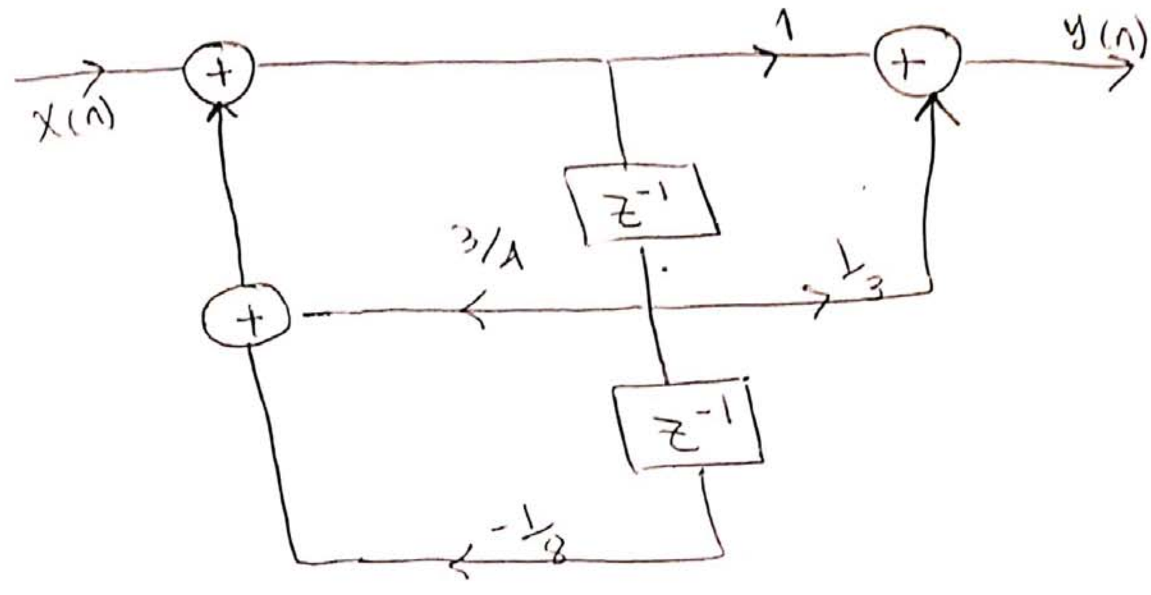
$$H(z) = \frac{x \text{ Coeff}}{y \text{ Coeff}} = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

(9)

Direct Form I



Direct Form II : Canonic Form



Cascade Form

$$H(z) = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

Factorize Denominator

$1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2} = 0 \xrightarrow{* z^2} z^2 - \frac{3}{4} z + \frac{1}{8} = 0$ — Factorize denominator

$(z - \frac{1}{2})(z - \frac{1}{4}) = 0$ by Calculator

$* z^{-2}$

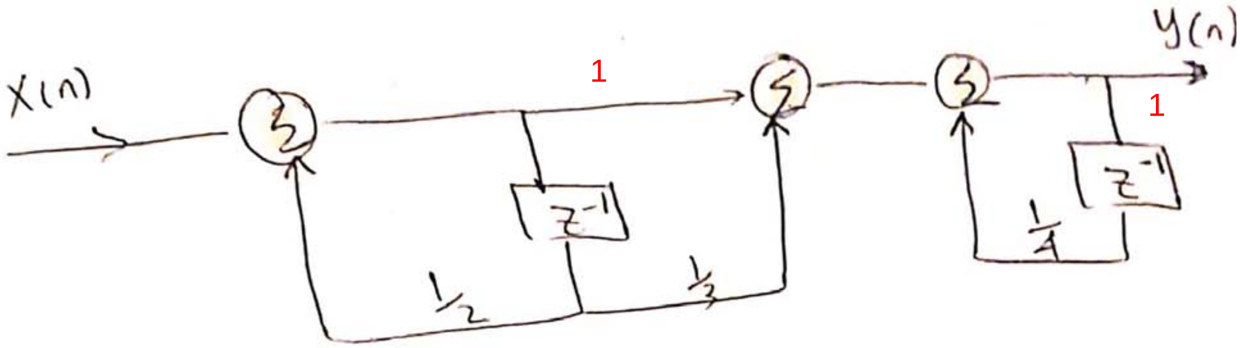
$$(1 - \frac{1}{2} z^{-1})(1 - \frac{1}{4} z^{-1}) = 0$$

$$H(z) = \frac{(1 + \frac{1}{3} z^{-1})}{(1 - \frac{1}{2} z^{-1})(1 - \frac{1}{4} z^{-1})}$$

$$\therefore H(z) = \underbrace{\frac{(1 + \frac{1}{3} z^{-1})}{(1 - \frac{1}{2} z^{-1})}}_{H_1(z)} \cdot \underbrace{\frac{1}{(1 - \frac{1}{4} z^{-1})}}_{H_2(z)}$$

Scanned by CamScanner

$\therefore\ H(z) = H_1(z) \cdot H_2(z)$ "Cascade Realization"



④ Parallel Form

$$H(z) = \frac{1 + \frac{1}{3}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

Degree of numerator < Degree of denomat

Partial Fractions

Partial Fractions

$$H(z) = \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 - \frac{1}{4}z^{-1}}$$

$$A = \left.\frac{1 + \frac{1}{3}X}{1 - \frac{1}{4}X}\right|_{X=2} = \frac{1 + \frac{2}{3}}{1 - \frac{1}{2}} = \frac{10}{3}$$

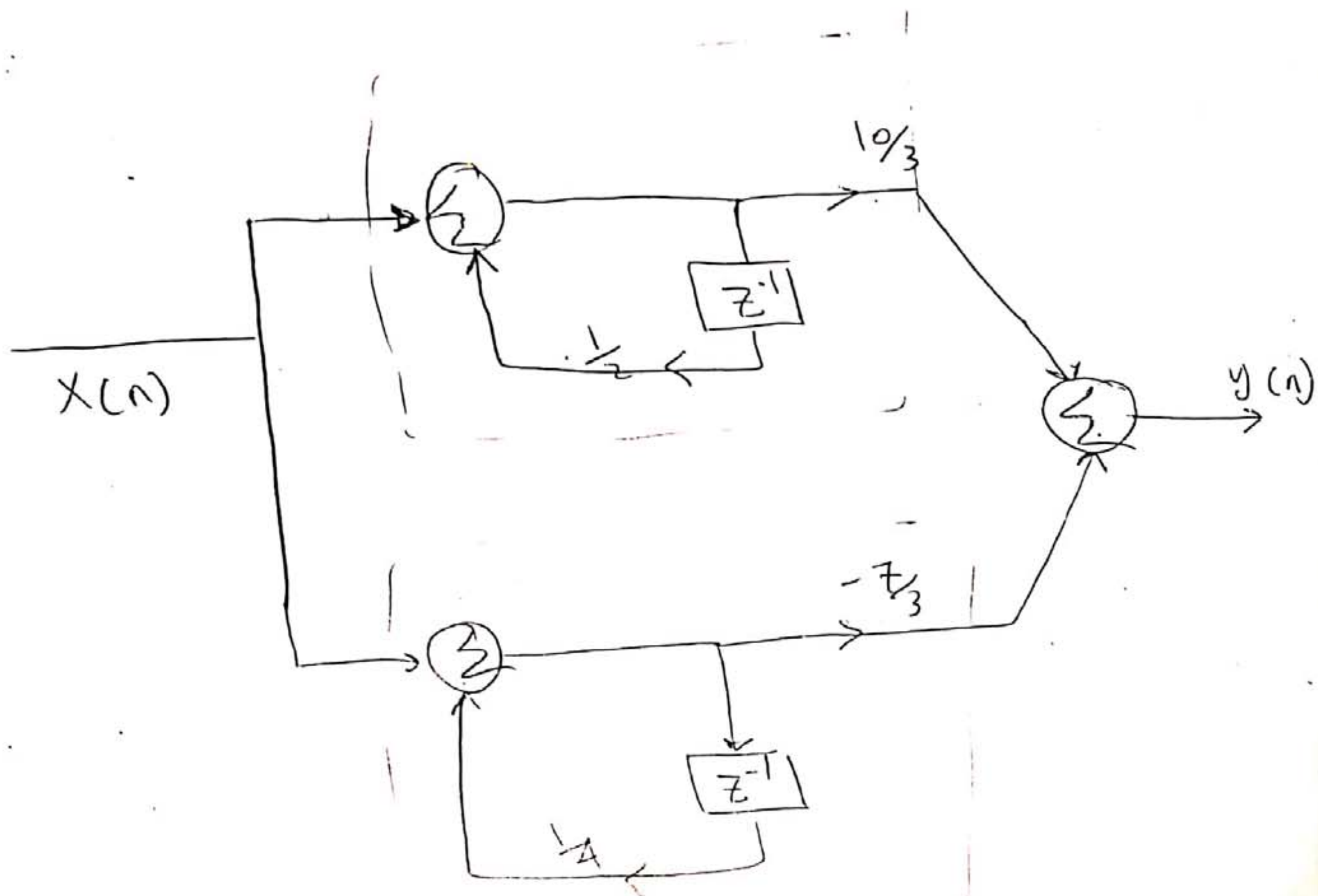
where $X = z^{-1}$

$$B = \left. \frac{1 + \frac{1}{3}x}{1 - \frac{1}{2}x} \right|_{x=4} = -\frac{7}{3}$$

$$H(z) = \frac{10/3}{\left(1 - \frac{1}{2}z^{-1}\right)} - \frac{7/3}{\left(1 - \frac{1}{4}z^{-1}\right)}$$

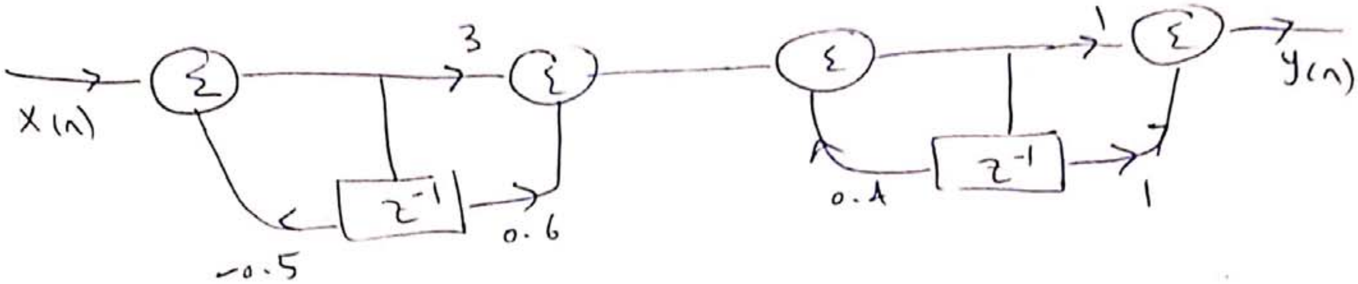
→ Now, we will Implement parallel realization

(& each section Direct Form II)



Example:

LTI system with the below realization



(i) what is the type of realization?

(ii) is it IIR or FIR system? Find D.E?

(iii) Implement using parallel

Solution

(i) This realization is Cascade Realization

$$H(z) = \frac{(3 + 0.6 z^{-1})}{(1 + 0.5 z^{-1})} \cdot \frac{(1 + z^{-1})}{(1 - 0.4 z^{-1})}$$

$$\downarrow$$

$$H(z) = \frac{3 + 3.6 z^{-1} + 0.6 z^{-2}}{1 + 0.1 z^{-1} - 0.2 z^{-2}}$$

(ii) IIR System

D.E: $y(n) + 0.1 y(n-1) - 0.2 y(n-2) = 3 x(n) + 3.6 x(n-1) + 0.6 x(n-2)$

(iii) In order to implement parallel realization
$\Downarrow$ [use <u>negative powers</u> in all steps]

$$H(z) = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

Degree of num = Degree of Den.

Long Division

$$\begin{array}{r|l} & -3 \\ \hline -0.2z^{-2} + 0.1z^{-1} + 1 & 0.6z^{-2} + 3.6z^{-1} + 3 \\ & \mp 0.6z^{-2} \pm 0.3z^{-1} \pm 3 \\ \hline & 3.9z^{-1} + 6 \end{array}$$

$$\therefore H(z) = -3 + \frac{3.9z^{-1} + 6}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

paratial Fractions

$$\frac{3.9z^{-1} + 6}{1 + 0.1z^{-1} - 0.2z^{-2}} = \frac{6 + 3.9z^{-1}}{(1 - 0.4z^{-1})(1 + 0.5z^{-1})}$$

$$1 + 0.1z^{-1} - 0.2z^{-2} = 0 \longrightarrow z^2 + 0.1z - 0.2 = 0$$

$$(z - 0.4)(z + 0.5) = 0$$

$$(1 - 0.4z^{-1})(1 + 0.5z^{-1})$$

$$\therefore \frac{6+3.9z^{-1}}{(1-0.4z^{-1})(1+0.5z^{-1})} = \frac{A}{1-0.4z^{-1}} + \frac{B}{1+0.5z^{-1}}$$

$$A = \left.\frac{6+3.9z^{-1}}{1+0.5z^{-1}}\right|_{z^{-1}=\frac{1}{0.4}} = 7$$

$$B = \left.\frac{6+3.9z^{-1}}{1-0.4z^{-1}}\right|_{z^{-1}=-2} = -1$$

$$\therefore H(z) = -3 + \frac{7}{1-0.4z^{-1}} - \frac{1}{1+0.5z^{-1}}$$

Parallel Realization