Basic operations on signals

Operations on time index (n) "independent variable"	operations on X(n) "Dependent variable"

I) operations on X(n) :-

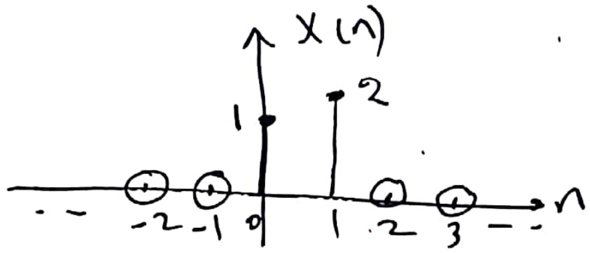
1) Amplitude Scaling:

$$y(n) = c\,x(n)$$

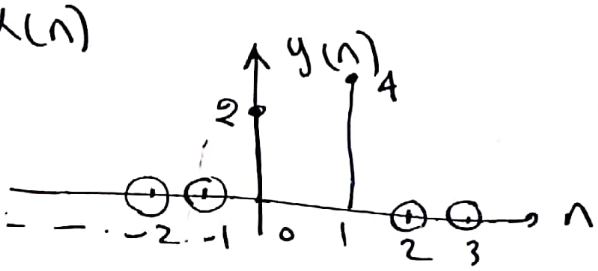
$c > 1$: Amplification

$0 < c < 1$ Attenuation

ex:



$$y(n) = 2\,x(n)$$



2) Addition:

$$y(n) = x_1(n) \pm x_2(n)$$

Add the corresponding samples [Both $x_1(n)$ & $x_2(n)$ should have same sampling time]

③ Multiplication:

$$y(n) = x_1(n) \cdot x_2(n)$$

Multiply the corresponding samples [Both $x_1(n)$ & $x_2(n)$ should have the same sampling time]

II- Operations on "n"

1 Time Scaling:

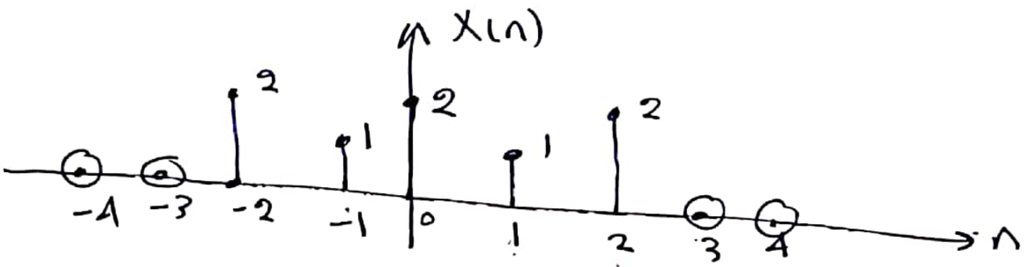
$$y(n) = x(kn)$$

← نقسم محور n على K , نشيل اي time index كسور .

IF $K > 1$: down sampling "compression"

IF $K < 1$: UP Sampling

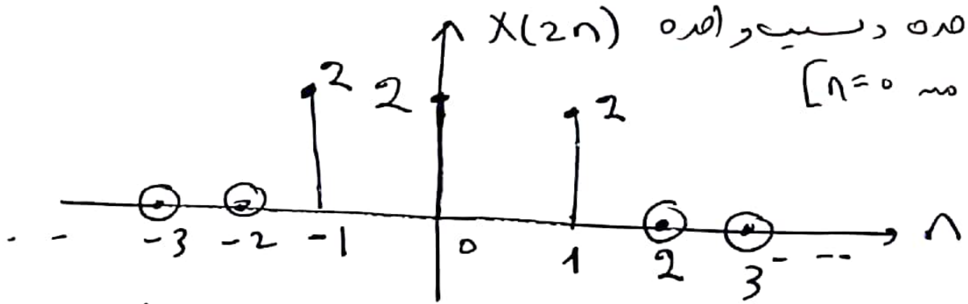
ex:



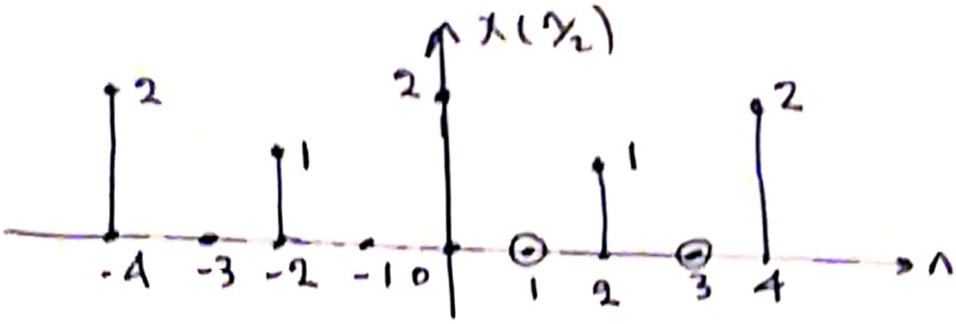
Sketch $x(2n)$, $x(n/2)$

Sol

ناخد واحده و سيب واحده
[بداية من n=0]

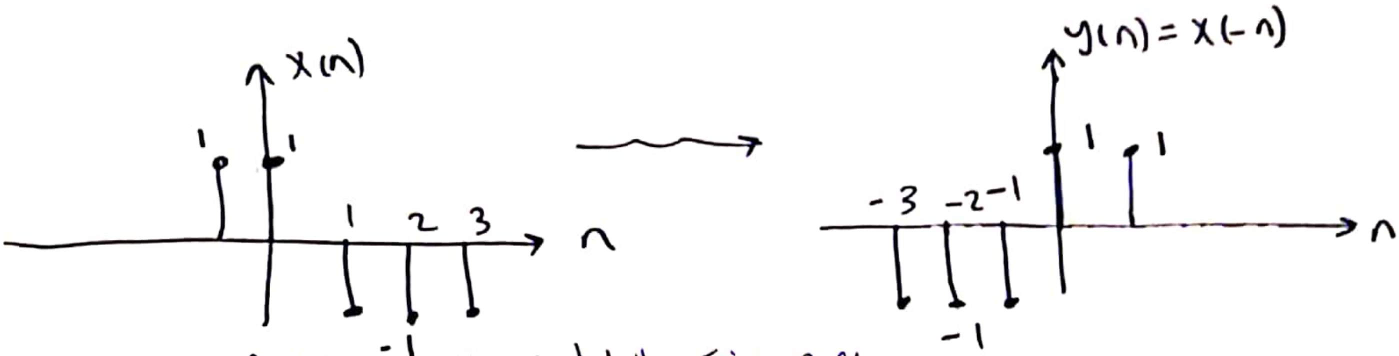


note that there are samples lost in time scaling.



② Reflection (time - reversal)

$$y(n) = x(-n)$$



Even signal is not affected by Time Reflection.

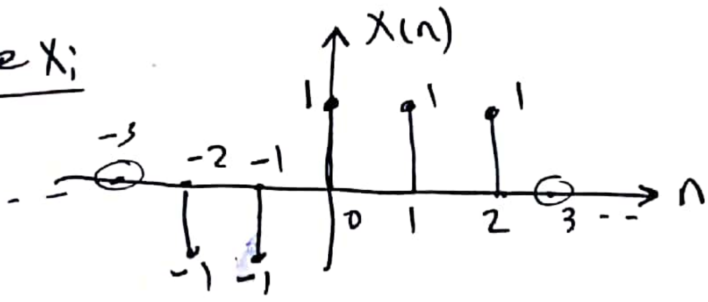
③ Time shift :

$$y(n) = x(n \pm m)$$, m: must be <u>integer</u>

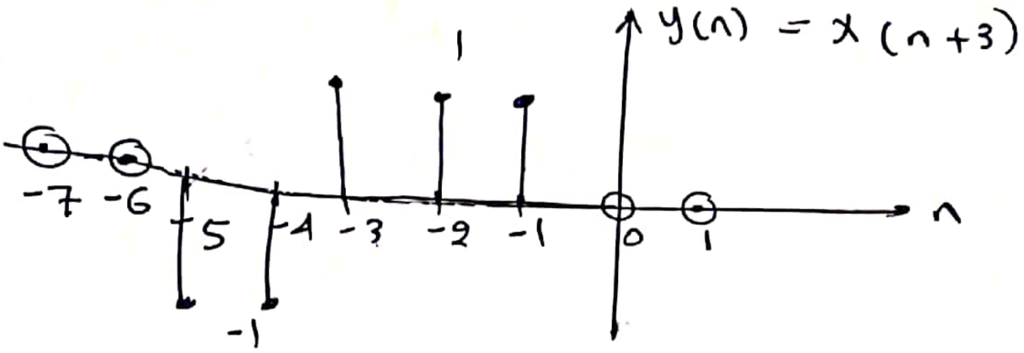
$x(n+m)$ — shift To left ←

$x(n-m)$ — shift To right →

<u>ex:</u>



Find $y(n) = x(n+3)$



4 Time Scaling & Time shift

$$y(n) = x(an - b)$$

methode I

$$y(n) = x(an-b)$$

From the right

1) shift by "b"

2) Scaling "a"

methode II

$$y(n) = x\left[a\left(n - \frac{b}{a}\right)\right]$$

From the left

1- Scaling "a"

2 - Time shift "$\frac{b}{a}$"

Condition: b/a should be integer

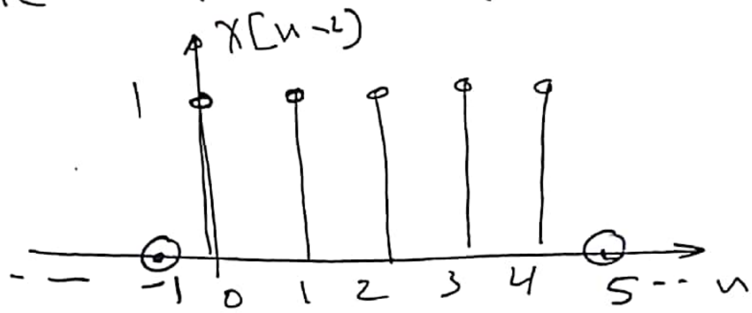
Sketch $y[n] = x[3n-2]$, where

$$x[n] = \begin{cases} 1 & -2 \le n < 3 \\ 0 & \text{o.w} \end{cases}$$

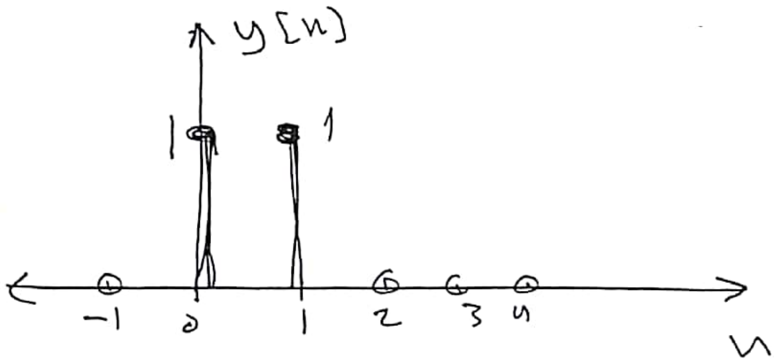
Sol

For $y[n] = x[3n-2]$

[1] time shift to right by 2



[2] Time Scaling by 3 [Start with n=0: take one and remove the next 2 and so on

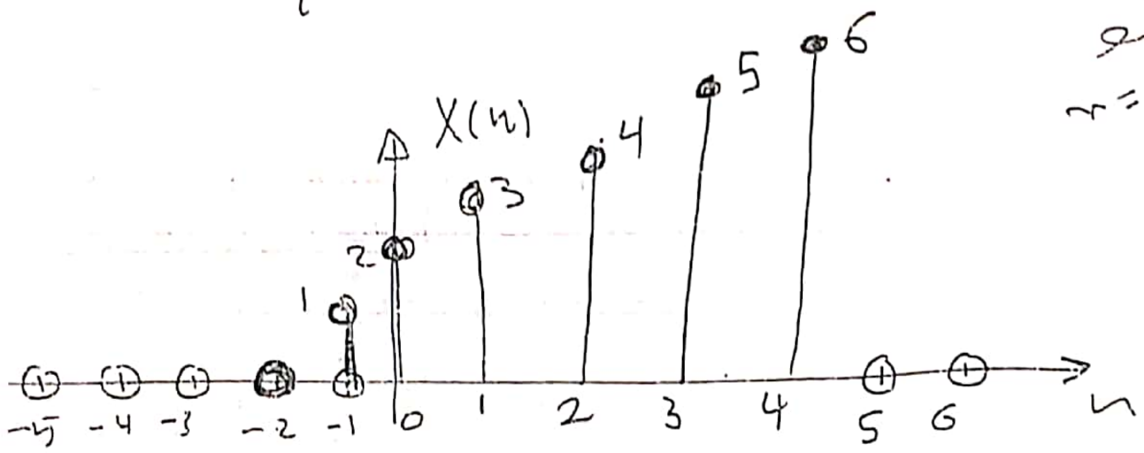


ex(4) Sketch

$$x(n) = \begin{cases} n+2 & -2 \le n < 5 \\ 0 & \text{o.w} \end{cases}$$

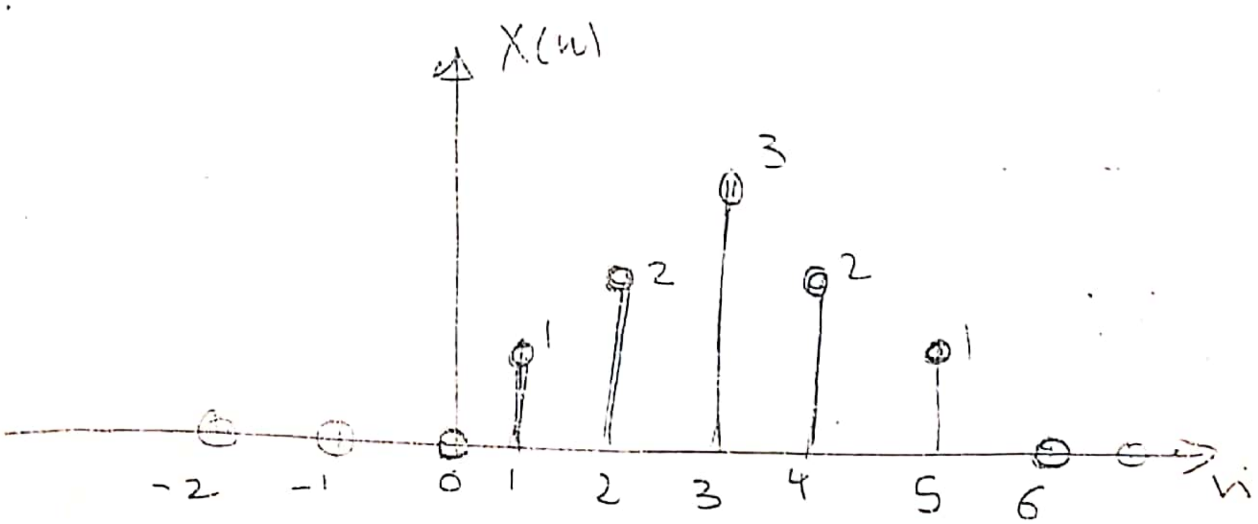
$<$

$\therefore$ نقف عند $n=4$



ex(5) Sketch

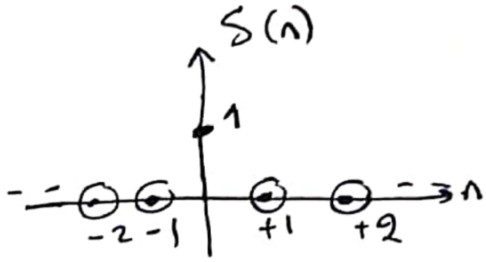
$$X(n) = \begin{cases} n & 0 \le n < 3 \rightarrow \text{هقف عند } n=2 \\ 6-n & 3 \le n < 7 \rightarrow \text{هقف عند } n=6 \\ 0 & \text{o.w} \end{cases}$$



Some Famous discrete signals

[I] Delta signal: $\delta(n)$ "Impulse signal"

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0. \end{cases}$$



properties:

i- $\delta(n)\, x(n) = x(0)\, \delta(n)$

ex: $\longrightarrow \delta(n-n_0)\, x(n) = x(n_0)\, \delta(n-n_0)$

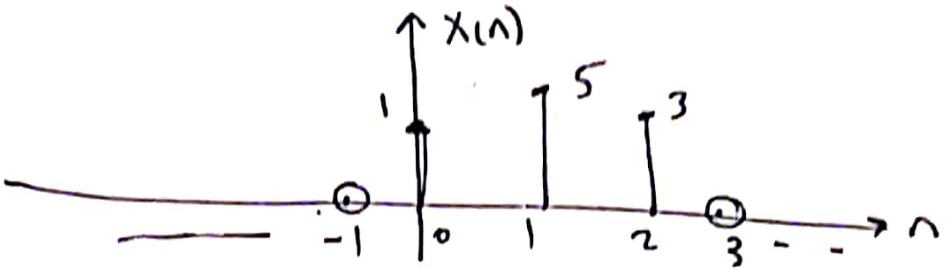
$\longrightarrow \delta(n-1)\, x(n) = x(1)\, \delta(n-1)$

iii- $\displaystyle\sum_{n=-\infty}^{\infty} \delta(n-n_0) = 1$

iv- any discrete signal can be written using Delta function

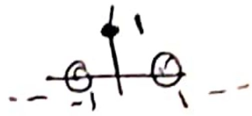
ex:

$$x(n) = \begin{cases} 1, & n = 0 \\ 5, & n = 1 \\ 3, & n = 2 \\ 0, & o.w \end{cases} \Longrightarrow x(n) = \delta(n) + 5\,\delta(n-1) + 3\,\delta(n-2)$$



ex:

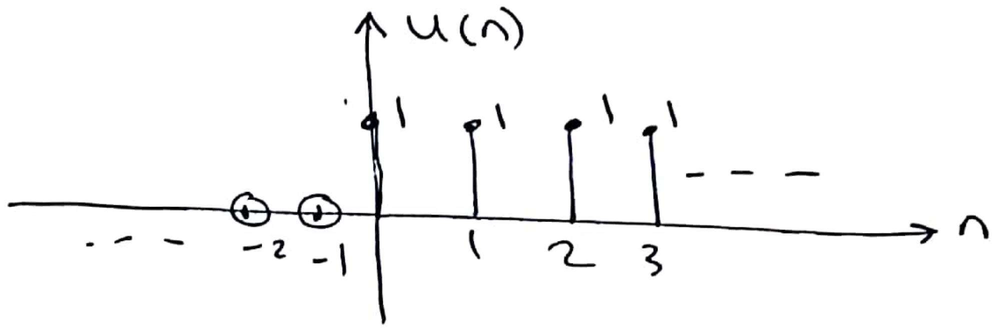
$x(n)\,\delta(n-1) = x(1)\,\delta(n-1) = 5\,\delta(n-1)$ 

$\rightarrow$ $x(n)\cdot\delta(n) = x(0)\,\delta(n) = \delta(n)$ 

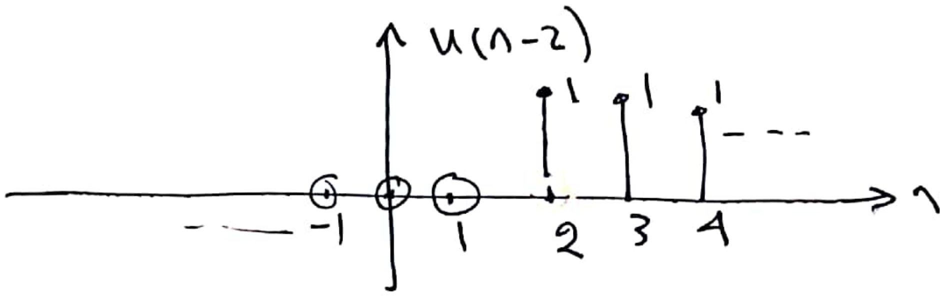
$x(n)\cdot\delta(n+3) = x(-3)\,\delta(n+3) = \text{zero}$

$\boxed{2}$ Unit Step: u(n)

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

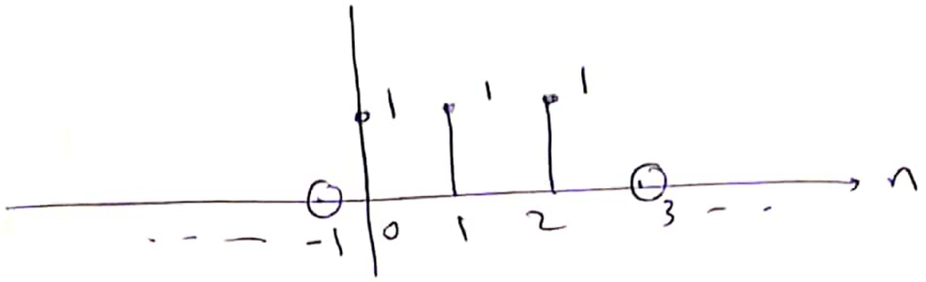


ex: sketch $u(n-2)$

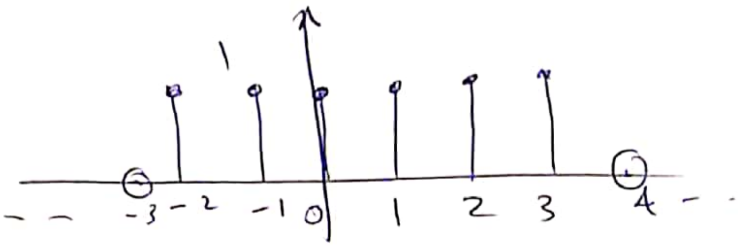


Notes

1) sketch $u(n) - u(n-3)$



2) sketch $u(n+2) - u(n-4)$



note $u(n) - u(n-3) = 1, \quad n = 0, 1, 2$ ← ٣ لاحظ

$u(n) - u(n-2) = 1, \quad n = 0, 1$ ← ٢

<u>Generally</u> :- $u(n-a) - u(n-b) = 1, \quad a \leq n < b$

<u>ex</u>: $u(n+2) - u(n-4) = 1, \quad -2 \leq n \leq 3$

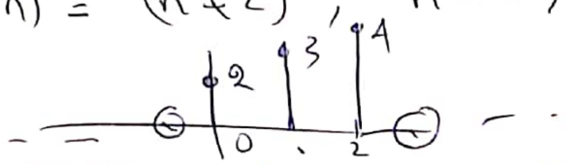
or

$-2 \leq n < 4$

→ note $x(n) = (n+2)\left[u(n) - u(n-3)\right]$

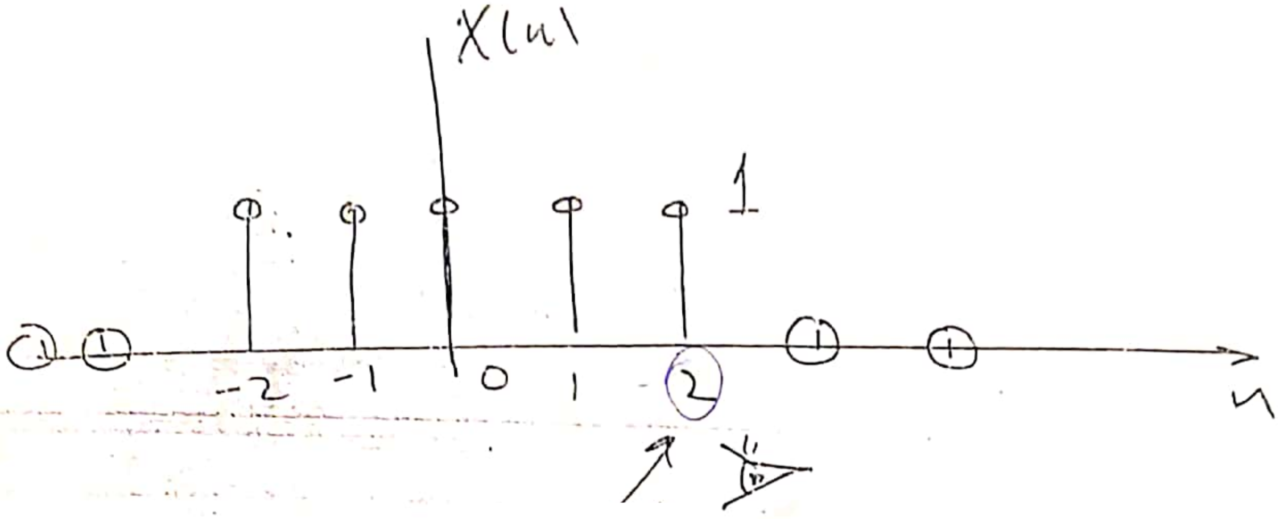
$\Downarrow$

$x(n) = (n+2), \quad n = 0, 1, 2.$



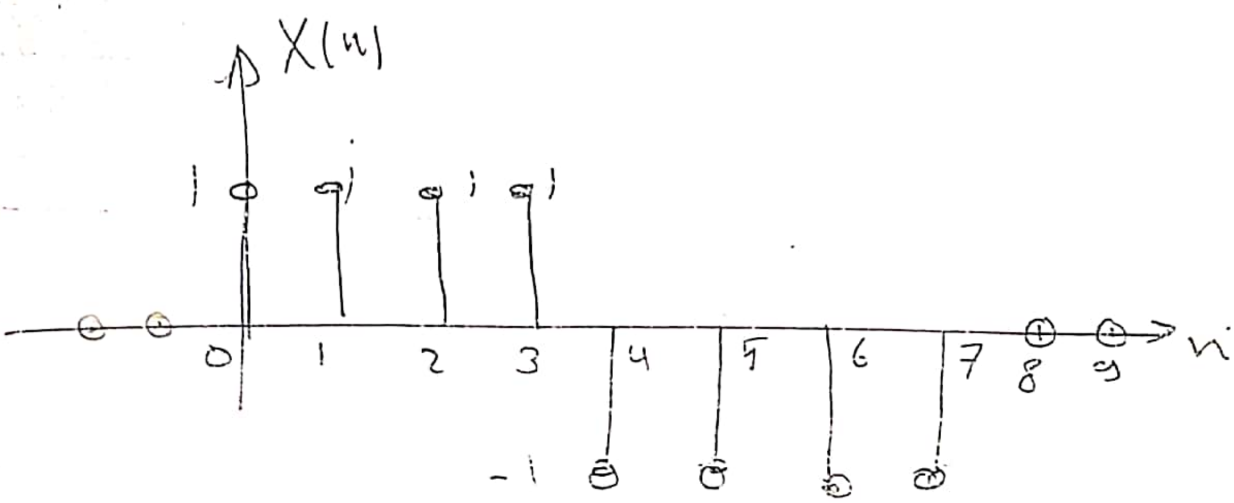
ex (2) Sketch

$$x(n) = u(n+2) - u(n-3)$$



ex (3) Sketch

$$X(n) = u(n) - 2u(n-4) + u(n-8)$$



Ex(6) Sketch

a) $x(n) = 2[u(n+2) - u(n-12)]$

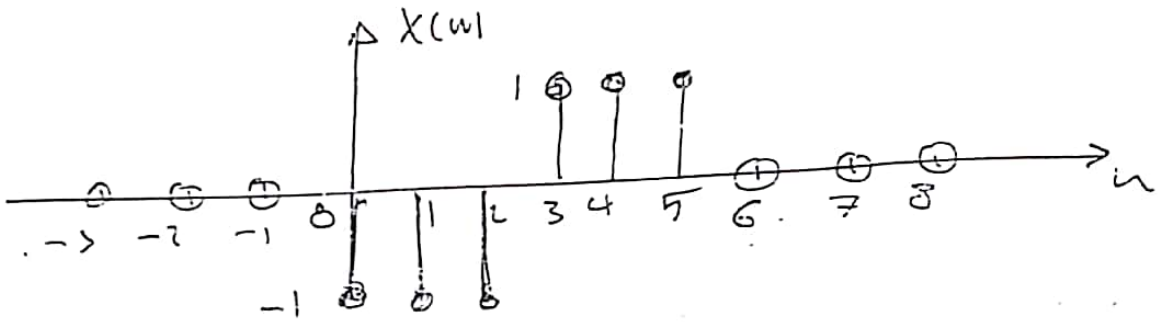
b) $x(n) = -u(n) + 2u(n-3) - u(n-6)$

c) $x(n) = 2u(n) - u(n+3) - u(n-4)$

Soln

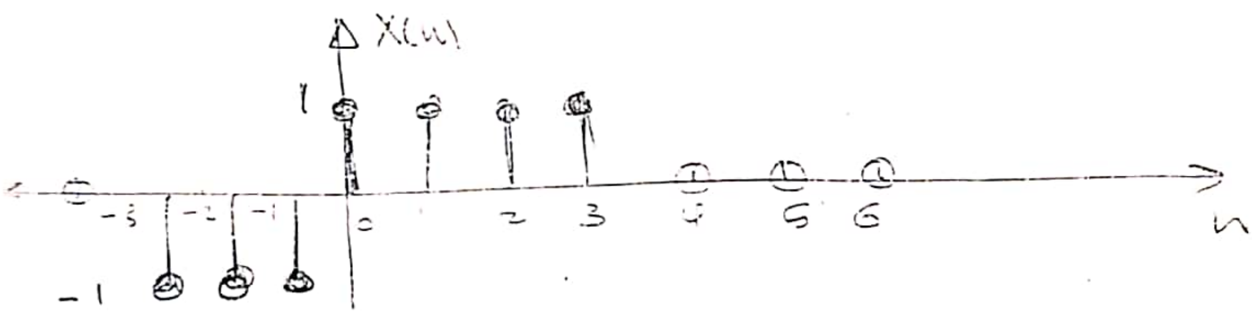
a)

b)



c) $x(n) = -u(n+3) + 2u(n) - u(n-4)$

Sort them first

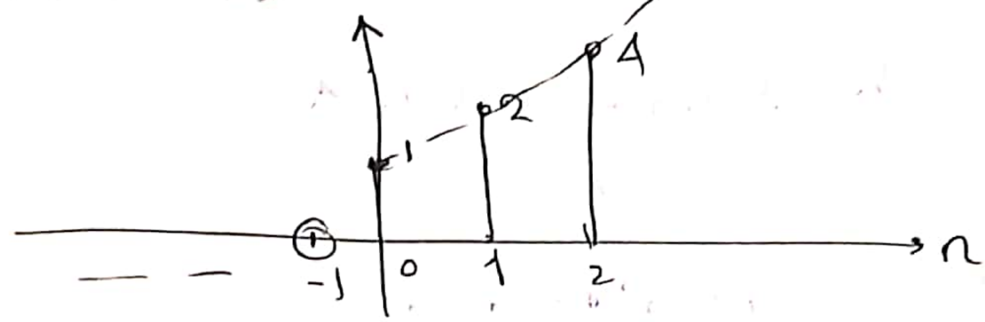


3 Exponential sequence

$$x(n) = (a)^n$$

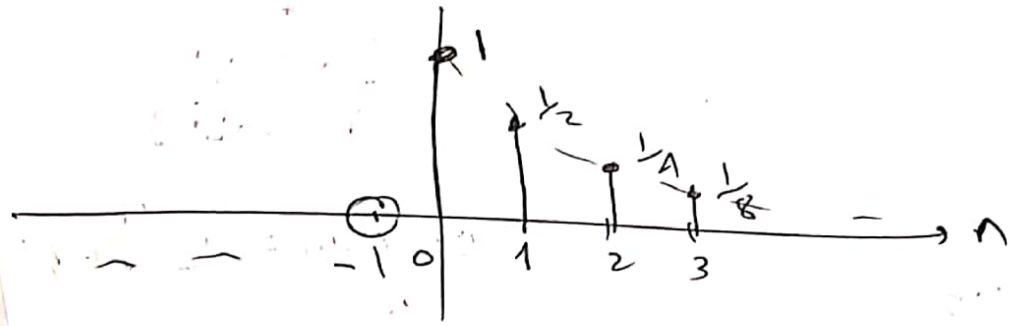
(i) $\boxed{a > 1}$ Growing "Increasing"

<u>ex:</u> $x(n) = (2)^n u(n) = (2)^n , \; n \geq 0$



(ii) $\boxed{0 < a < 1}$ Decaying "decreasing"

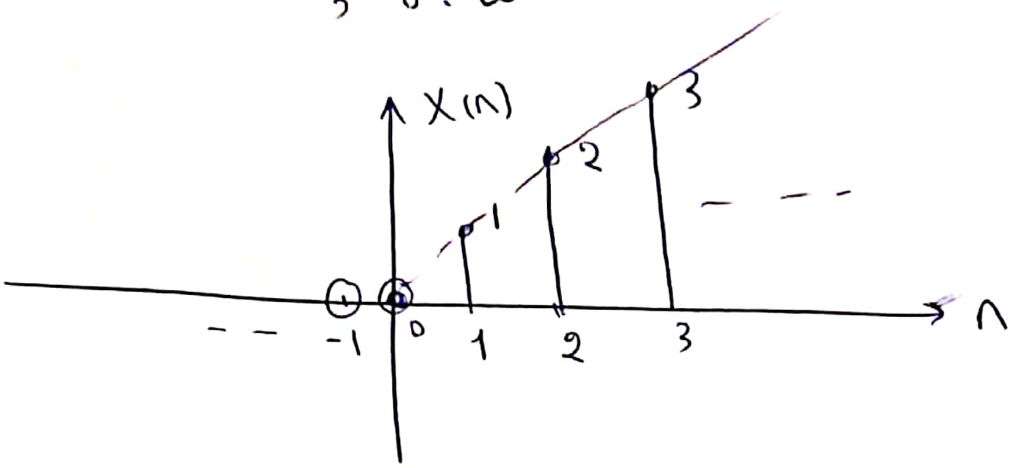
<u>ex:</u> $x(n) = \left(\frac{1}{2}\right)^n u(n) = \left(\frac{1}{2}\right)^n , \; n \geq 0$



4) Ramp signals

$$x(n) = \begin{cases} n, & n \geq 0 \\ 0, & o.w \end{cases}$$

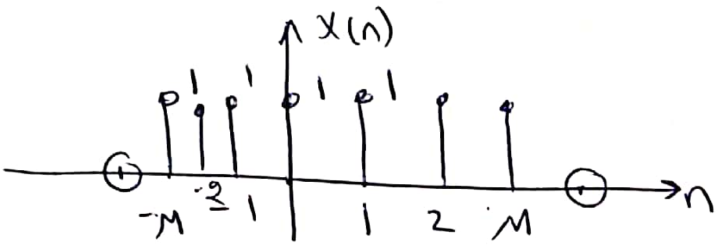
or Can be written : $n\,u(n)$



5) Rectangular signal:

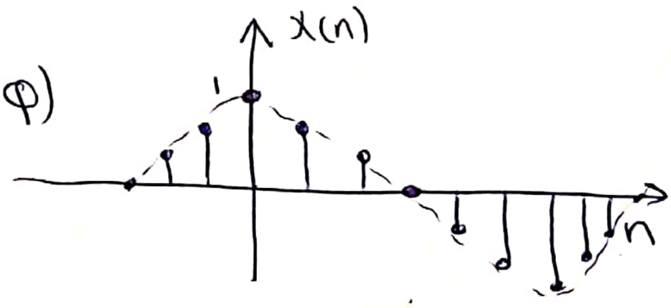
$$x(n) = \begin{cases} 1, & -M \leq n \leq M \\ 0, & o.w \end{cases}$$

Pulse width $= M - (-M) + 1$
$= 2M + 1$



6) Sinsoidal signals:-

$$x(n) = \cos(\Omega n + \varphi)$$



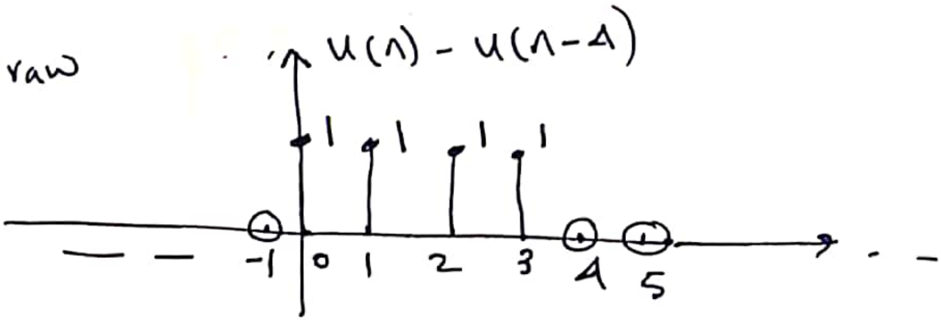
If $x(n) = (4-n)\left[u(n) - u(n-4)\right]$

1) Find Energy of $x(n)$

2) Find and sketch $y(n) = x(3-2n)$

SoL

$\boxed{1}$ 1st draw



$x(n) = (4-n)$, $n = 0, 1, 2, 3.$

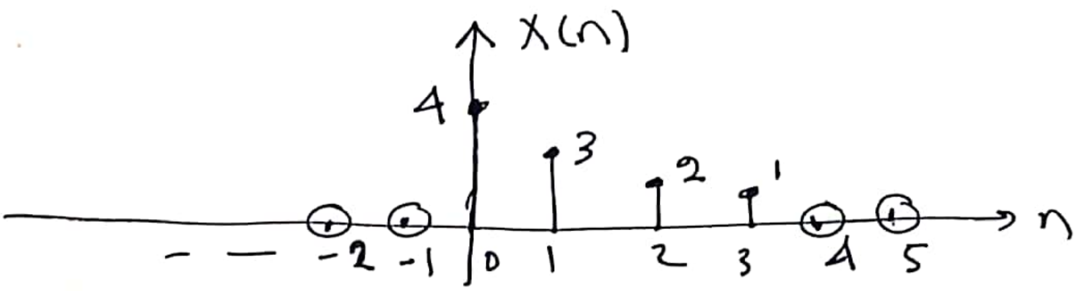
$\boxed{n=0} \longrightarrow x(0) = 4 * 1 = 4$

$\boxed{n=1} \longrightarrow x(1) = 3 * 1 = 3$

$\boxed{n=2} \longrightarrow x(2) = 2 * 1 = 2$

$\boxed{n=1} \longrightarrow x(3) = 1 * 1 = 1$

& rest samples = zero



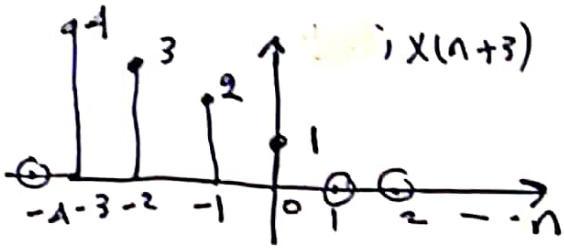
$$\text{Energy} = E_t = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= (4)^2 + (3)^2 + (2)^2 + (1)^2 = 30$$

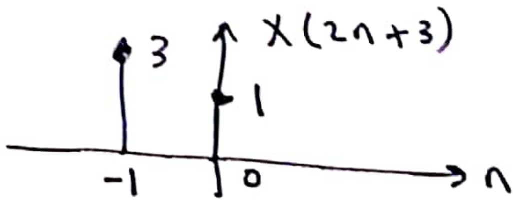
if Req: $P_{av} = 0$ as Energy signal "time limited"

$\boxed{2}$ $y(n) = x(3-2n) = x(-2n+3)$

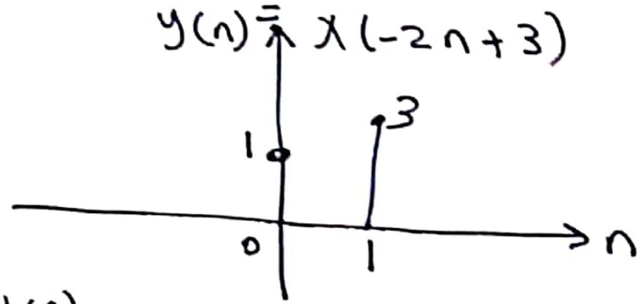
1) shift by (3) To left



2) scaling by (2)



3) reflection



Req: expersion For $y(n)$.

$$\boxed{y(n) = \delta(n) + 3\delta(n-1)}$$

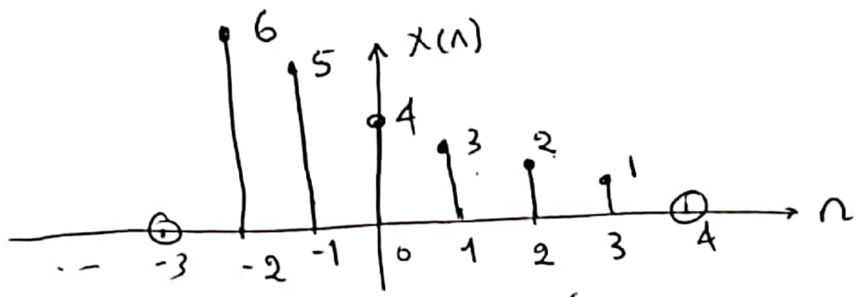
ex

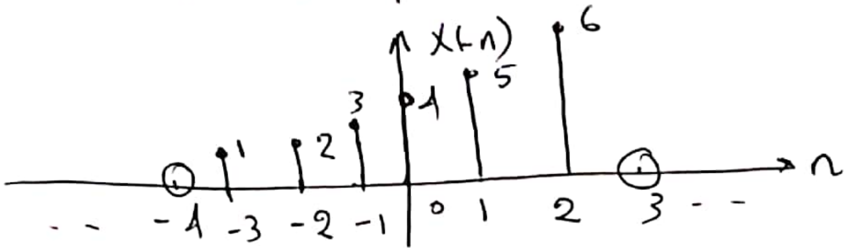
$$x(n) = (4-n), \quad -2 \le n < 4$$

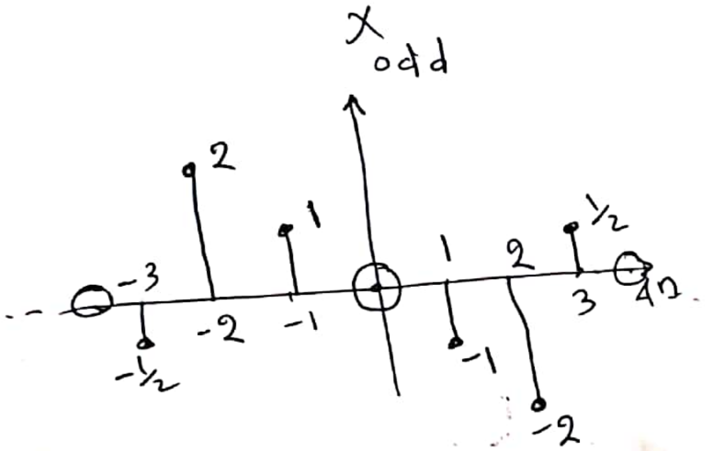
Write $x(n)$ as $x(n) = x_{odd}(n) + x_{even}(n)$

Solution

$$x_{odd}(n) = \frac{1}{2}[x(n) - x(-n)], \quad x_{even}(n) = \frac{1}{2}[x(n) + x(-n)]$$







[odd]

[even]

Notes

1- $|n| < a \longrightarrow -a < n < a$

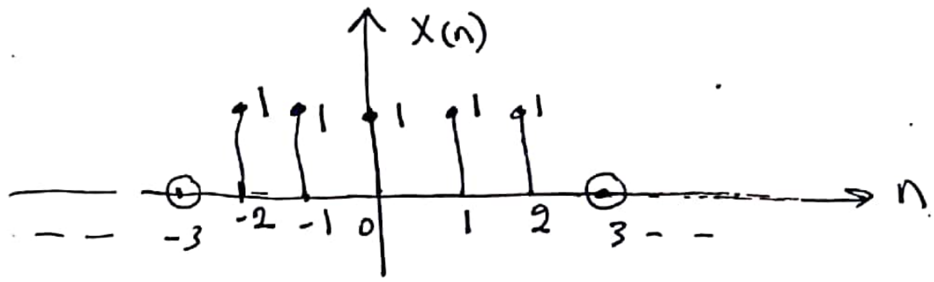
2- $|n| > a \longrightarrow \begin{cases} n > a \\ \& \\ n < -a \end{cases}$

$\boxed{\text{ex}}$ $x(n) = \begin{cases} 1, & |n| \leq 2 \\ 0, & |n| > 2 \end{cases}$

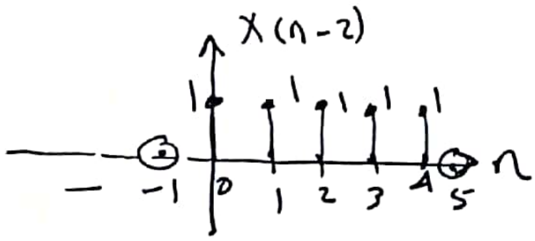
Find $y(n) = 3\,x(n-2)$

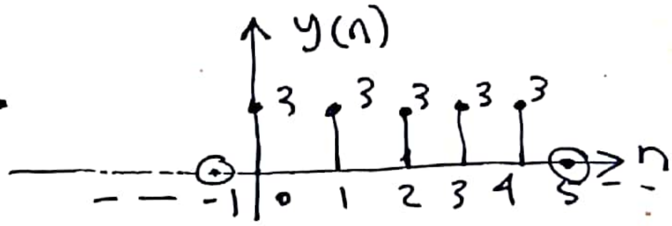
SoL

$x(n) = \begin{cases} 1, & -2 \leq n \leq 2 \\ 0, & n > 2 \ \& \ n < -2 \end{cases}$

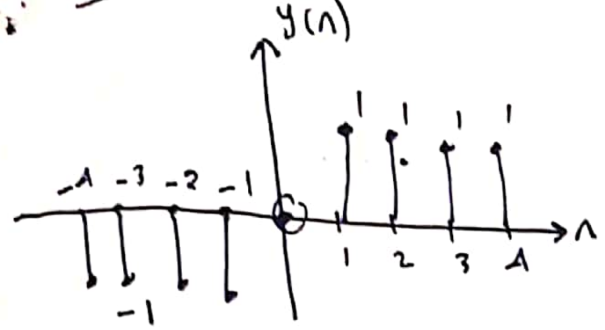


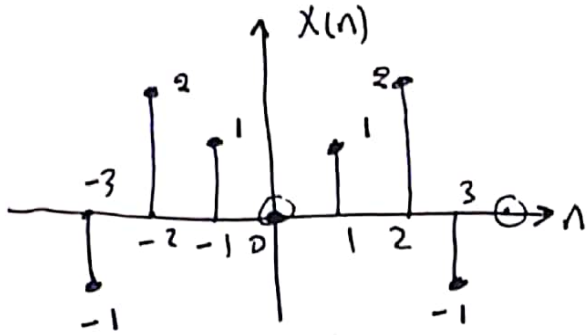
Req: $y(n) = 3\,x(n-2) \longrightarrow$ shift(2) ∴ then ×3





ex:



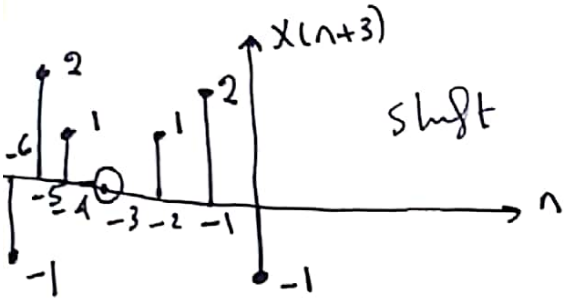


Find 1- $x(3-n) \cdot y(n)$

2- $x(n-2) \cdot y(6-n)$

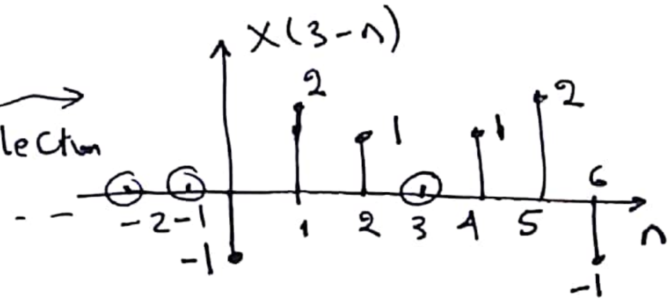
Sol

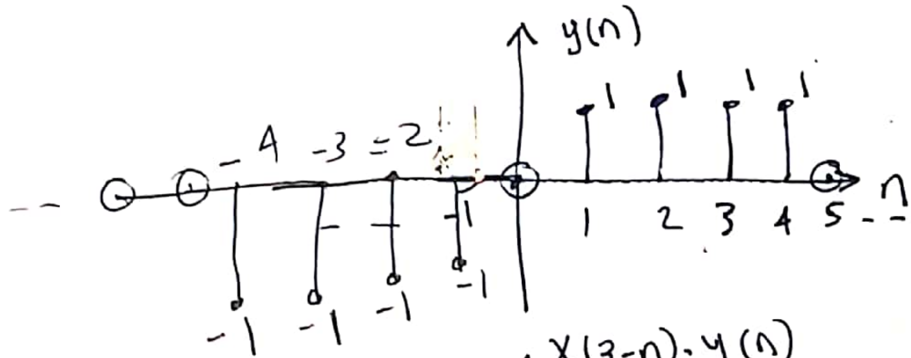
$\boxed{1}$ $x(3-n) = x(-n+3)$ shift 3 left
reflection

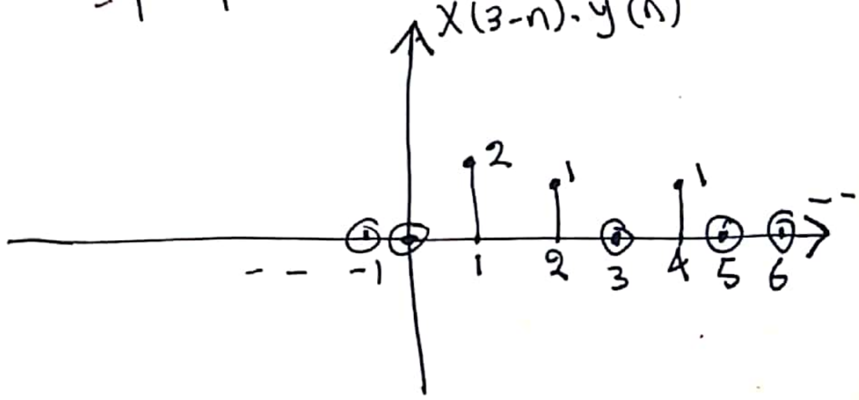


shift

reflection

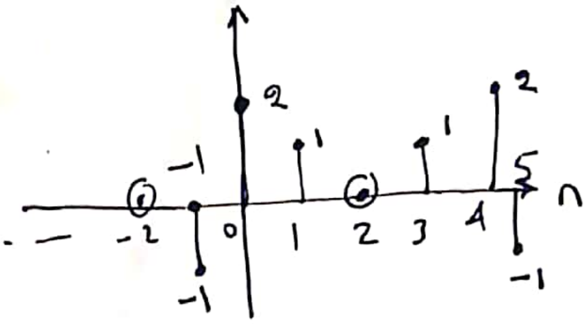






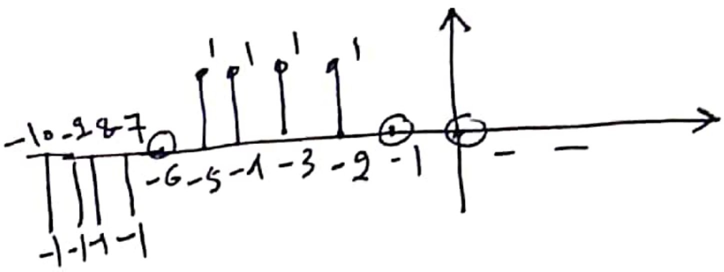
$\boxed{2}$ Req: $x(n-2) \cdot y(6-n)$

$x(n-2)$



$y(6-n) = y(-n+6)$

1- shift 6 right



2- reflection