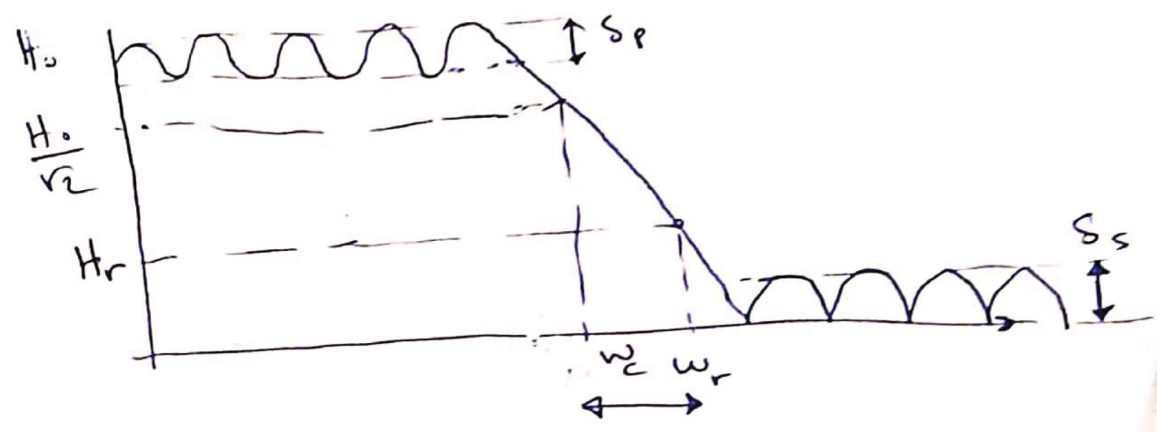
Design of Digital Filters

Given:

Desired Frequency domain characteristics of the Filter. This includes

1. Type of Filter IIR or FIR?
2. LPF or HPF or BPF or BSF ?
3. Type of Approximation
   [Butterworth, Chebyshev I, Chebyshev II, Elliptic]
4. Specifications of Pass Band & Stop Band



Δw: Transition band

δ_p : passband ripples

δ_s : Stop band ripples

w_c : cut off frequency

w_r : rejection frequency

H_r : magnitude at w_r

Δw : Transition B.W

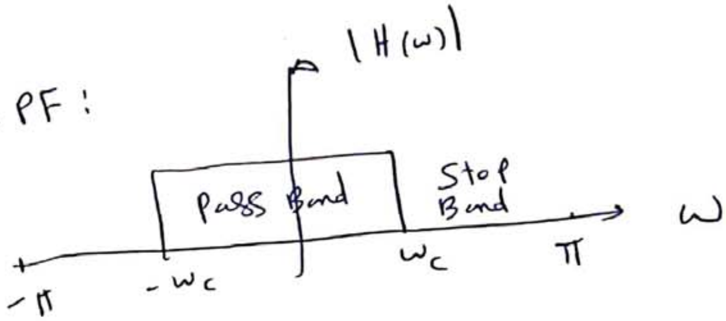
Required

Find the coefficient (a's & b's) of the causal filter that meet the desired frequency domain specification.

This includes:
1. order (N)
2. H(Z) of the filter
3. a's & b's coefficients
4. Realization of filter.

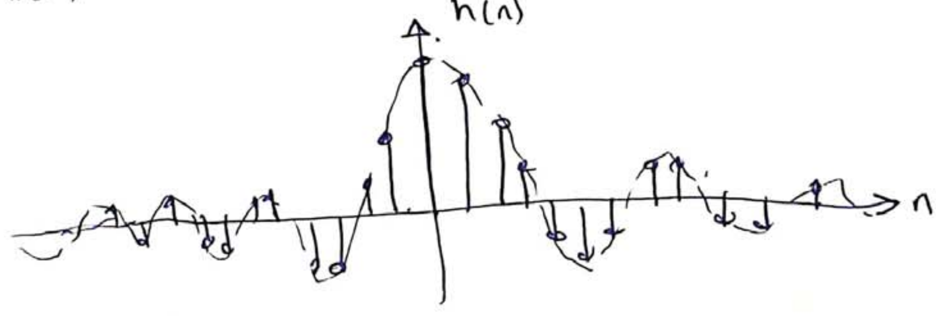
Q Why Ideal Filters can not work?

For Ideal LPF:



rectangular in Frequency Domain $\Rightarrow$ Sinc in time Domain

H(w) h(n)



Non causal why?

$$h(n) \neq 0 , n < 0$$

The Filter depends on future input values

There Fore, Ideal Filters can not be implemented

Q: Compare FIR Filter with IIR Filter?

FIR	IIR
[1] FIR can fullfill <u>Linear phase</u> requirements in pass band	[1] It is very hard To design stable IIR Filter with linear phase.
[2] <u>D.E</u>: $y(n) = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2) - - -$ No Feed back, output depends only on input [more simple]	[2] D.E: $y(n) + b_1 y(n-1) + b_2 y(n-2) = a_0 x(n) + a_1 x(n-1) - - -$ Feed back, out put depends on input & past output
[3] Always Stable	[3] May be Unstable
[4] Unrelated to Continous time Filtering	[4] Mainly derived from Analog prototypes
[5] High order to accomplish different Filtering tasks [<u>dis advantage</u>]	[5] low order to achieve the same objective. typically less than $\frac{1}{10}$ of FIR Filter [<u>Advantage</u>] $N\|_{IIR} < \frac{1}{10} N\|_{FIR}$ [To achieve the same requirements]

Filter Design Techniques

- FIR Design
  - Windowing Method
- IIR Design
  - Continous to Discrete transFormation
    1. Bilinear TransFormation
    2. Impulse Invariance
    3. Step invariance

Design of IIR Digital filters

- In IIR Design, we start the design in the analog plane, then make a transFormation from Analog to digital plane.
- There are many transfer methods but we will study 3 methods [Bilinear TransFormation, Impulse invariance, Step invariance]

Classical Analog Filter $H(s)$	→	Digital Filter $H(z)$
	Bilinear transFormation	
	Impulse invariance	
	Step invariance	

In each type, Filter order is a major concern.

As the order of filter $N \uparrow\uparrow$

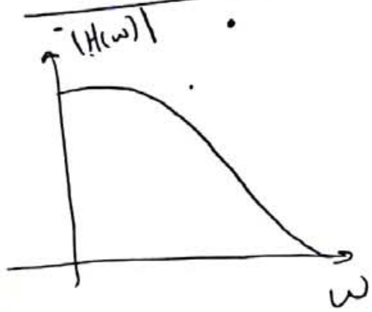
$\Downarrow$

Achieve specifications better but also more complicated & prone to rounding errors.

∴ The First step in IIR design is to get the design of the analog normalized LPF? why?

Analog Filters have a variety of well-established methods for design

Butterworth

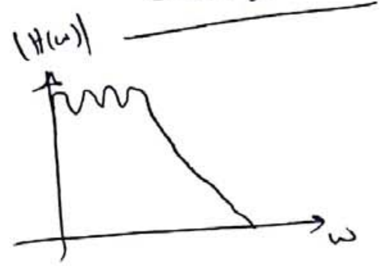


$\Delta\omega$ largest

Flat in both pass & stop Bands

[we will study this only].

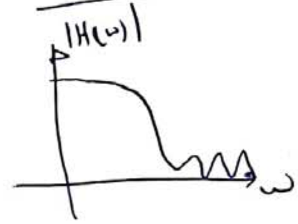
Chebyshev I



$\Delta\omega$ medium

ripples in pass band

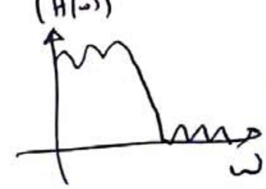
Chebyshev II



$\Delta\omega$ medium

ripples in stop Band

Elliptic

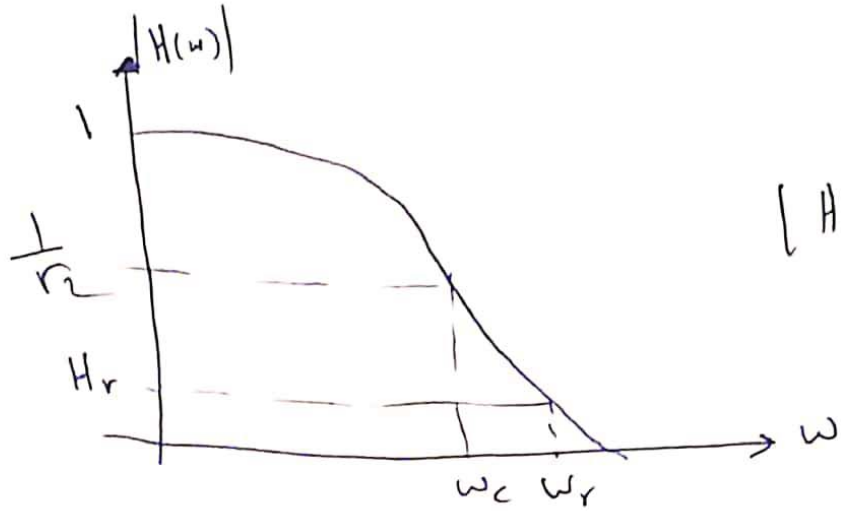


$\Delta\omega$ smallest

ripples in both pass & stop Bands

out of scope of this course.

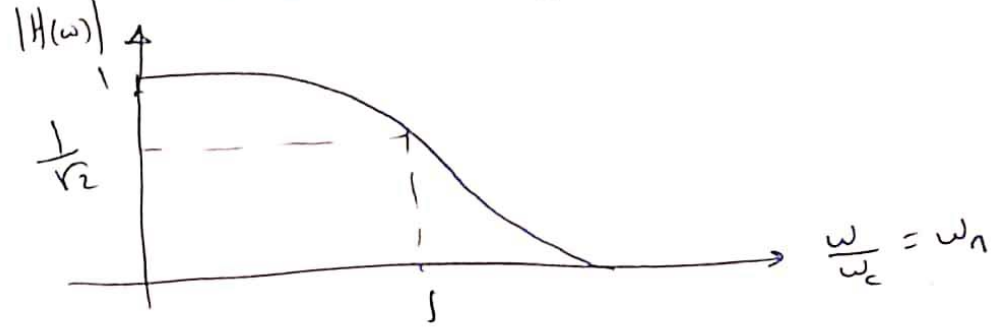
Butler worth Analog Design



$$|H(\omega)|^2 = \frac{1}{1+\left(\frac{w}{w_c}\right)^{2N}}$$

$\Downarrow$

let define $w_n = \frac{w}{w_c}$



$$|H(\omega)|^2 = \frac{1}{1+w_n^{2N}}$$, N: order of the filter.

Note $|H(\omega)|^2 = H(J\omega) \cdot H(-J\omega) = \dfrac{1}{1+(w_n^2)^N}$

$\downarrow$

$H(s) \cdot H(-s) = \dfrac{1}{1+(-s^2)^N}$ becaus $s = J\omega$

$s^2 = (J\omega)^2 = -\omega^2$

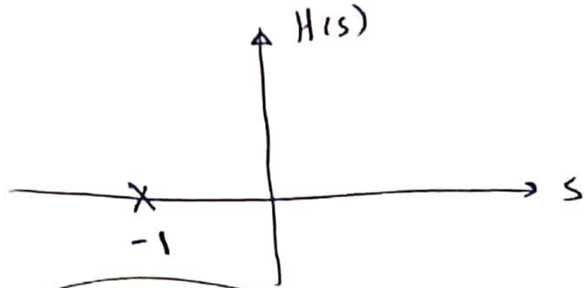
N: is the order " specified according to the specifications in frequency Domain of LPF"

w_c, w_r, H_r

$\boxed{N=1}$ $H(s) \cdot H(-s) = \dfrac{1}{1-s^2} = \dfrac{1}{(1-s)(1+s)}$

This pole is refused as it is not stable

[Poles in LHS ⇒ stable
Poles in RHS ⇒ unstable]



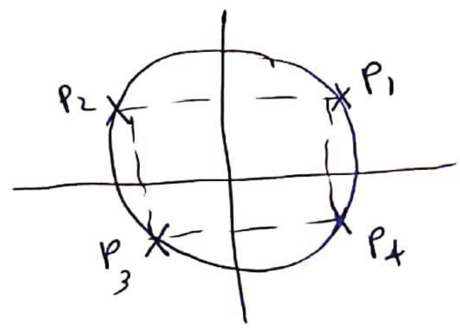
$\therefore \boxed{H(s) = \dfrac{1}{s+1}}$ For $N=1$

$\boxed{N=2}$ $H(s) \cdot H(-s) = \dfrac{1}{1+s^4}$

angle $\frac{\pi}{4}$ between each pole

Four poles: P_1, P_2, P_3, P_4

[P_1, P_4 are refused as they are not stable]



$P_2 = \frac{-1}{\sqrt{2}} + J\frac{1}{\sqrt{2}}$, $P_3 = \frac{-1}{\sqrt{2}} - J\frac{1}{\sqrt{2}}$

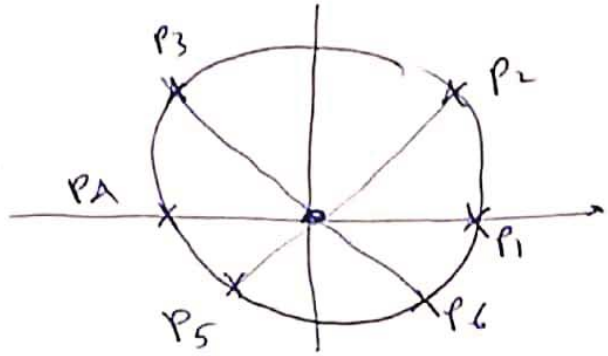
$$H(s) = \frac{1}{(s-P_2)(s-P_3)} = \frac{1}{\left(s-\left(-\frac{1}{\sqrt{2}}+J\frac{1}{\sqrt{2}}\right)\right)\left(s-\left(-\frac{1}{\sqrt{2}}-J\frac{1}{\sqrt{2}}\right)\right)}$$

$$H(s) = \frac{1}{s^2+\sqrt{2}\,s+1}$$

$\boxed{N=3}$ $H(s) \cdot H(-s) = \dfrac{1}{1-s^6}$

Six Poles: $P_1, P_2, P_3, P_A, P_5, P_6$

[P_1, P_2, P_6 are not stable]



$\frac{\pi}{3}$ between each pole.

$$H(s) = \frac{1}{(s-P_3)\,(s-P_A)\,(s-P_5)}$$

where $P_3 = -\frac{1}{2} + J\frac{\sqrt{3}}{2}$

$P_A = -1$

$P_5 = \frac{-1}{2} - J\frac{\sqrt{3}}{2}$

$$\Rightarrow H(s) = \frac{1}{s^3+2s^2+2s+1}$$

Summary

Butterworth Approximation of normalized LPF

$$N=1 \Longrightarrow H(s) = \frac{1}{s+1}$$

$$N=2 \Longrightarrow H(s) = \frac{1}{s^2+\sqrt{2}\,s+1}$$

$$N=3 \Longrightarrow H(s) = \frac{1}{s^3+2s^2+2s+1}$$

Note

N: order of Butterworth filter

[it should be known from frequency specifications of w_c, w_r, H_r]

$$|H(w)|^2 = \frac{1}{1 + \left(\frac{w}{w_c}\right)^{2N}}$$

Given: w_r & H_r beside w_c

$$\therefore \; H_r^2 = \frac{1}{1 + \left(\frac{w_r}{w_c}\right)^{2N}} \Longrightarrow \text{get } N = \_$$

Now, we have the normalized Analog LPF transfer function using Butterworth approximation:

$$N = 1 \Longrightarrow H(s) = \frac{1}{s+1}$$

$$N = 2 \Longrightarrow H(s) = \frac{1}{s^2 + \sqrt{2}\,s + 1}$$

⋮

Q: We need to transform from analog Domain to Digital Domain [3 methods]

- Bilinear Transformation
- Impulse invariance
- Step invariance

I) Bi Linear TransFormation

Put $\boxed{S = \frac{2}{T_s} \frac{Z-1}{Z+1}}$ in $H(s)$ To get $H(z)$

i.e: $H(z) = H(s)\Big|_{S = \frac{2}{T_s}\frac{Z-1}{Z+1}}$

بـ هذا الـ TransFormation بيخلى الـ Digital Freq مرتبطة
بـ الـ analog Frequency ← بيعمل mapping بين
الـ Digital Freq , الـ analog Freq .

$$\boxed{W_a = \frac{2}{T_s} \tan\left(\frac{W_d T_s}{2}\right)}$$ " Derivation in next page "

where W_a: analog Frequency

W_d: digital Freq

T_s: time between 2 successive samples in time domain $= \frac{1}{f_s}$ ← sampling frequency

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→ Recall: $e^{Jx} - e^{-Jx} = 2J \sin x$

$e^{Jx} + e^{-Jx} = 2\cos x$

Q:

Discuss mapping between analog & Digital Frequency in BiLinear TransFormation and prove the relation?

$\boxed{\text{Sol}}$ "prove equation relating analog and digital Freq in BiLinear TransFormation"

Steps

1- in BiLinear TransFormation $S = \frac{2}{T_s} \frac{Z-1}{Z+1}$

2- but S: For Analog Domain $S = jw_a$ ← analog Freq

Z: For Digital Domain $Z = e^{Jw_d T_s}$ ← digital Freq

$$\therefore Jw_a = \frac{2}{T_s} \frac{e^{Jw_d T_s} - 1}{e^{Jw_d T_s} + 1}$$

$$Jw_a = \frac{2}{T_s} \cdot \frac{\cancel{e^{Jw_d \frac{T_s}{2}}}}{\cancel{e^{Jw_d \frac{T_s}{2}}}} \left[\frac{e^{Jw_d \frac{T_s}{2}} - e^{-Jw_d \frac{T_s}{2}}}{e^{Jw_d \frac{T_s}{2}} + e^{-Jw_d \frac{T_s}{2}}} \right]$$

$$Jw_a = \frac{\cancel{2}}{T_s} \frac{2J \sin(w_d \frac{T_s}{2})}{\cancel{2} \cos(w_d \frac{T_s}{2})}$$

$$j w_a = \frac{2j}{T_s} \frac{\sin(w_d T_s/2)}{\cos(w_d \frac{T_s}{2})}$$

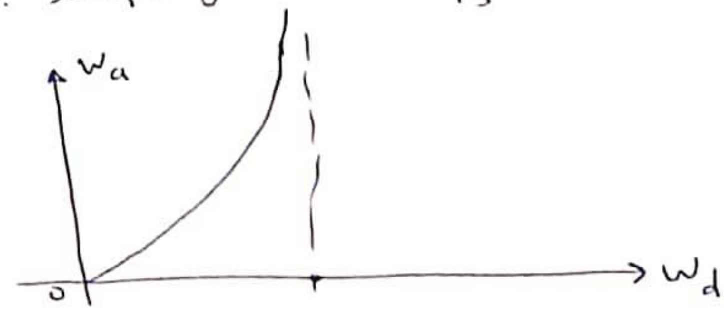
$$W_a = \frac{2}{T_s} \tan\left(\frac{w_d T_s}{2}\right)$$

Non liner relation

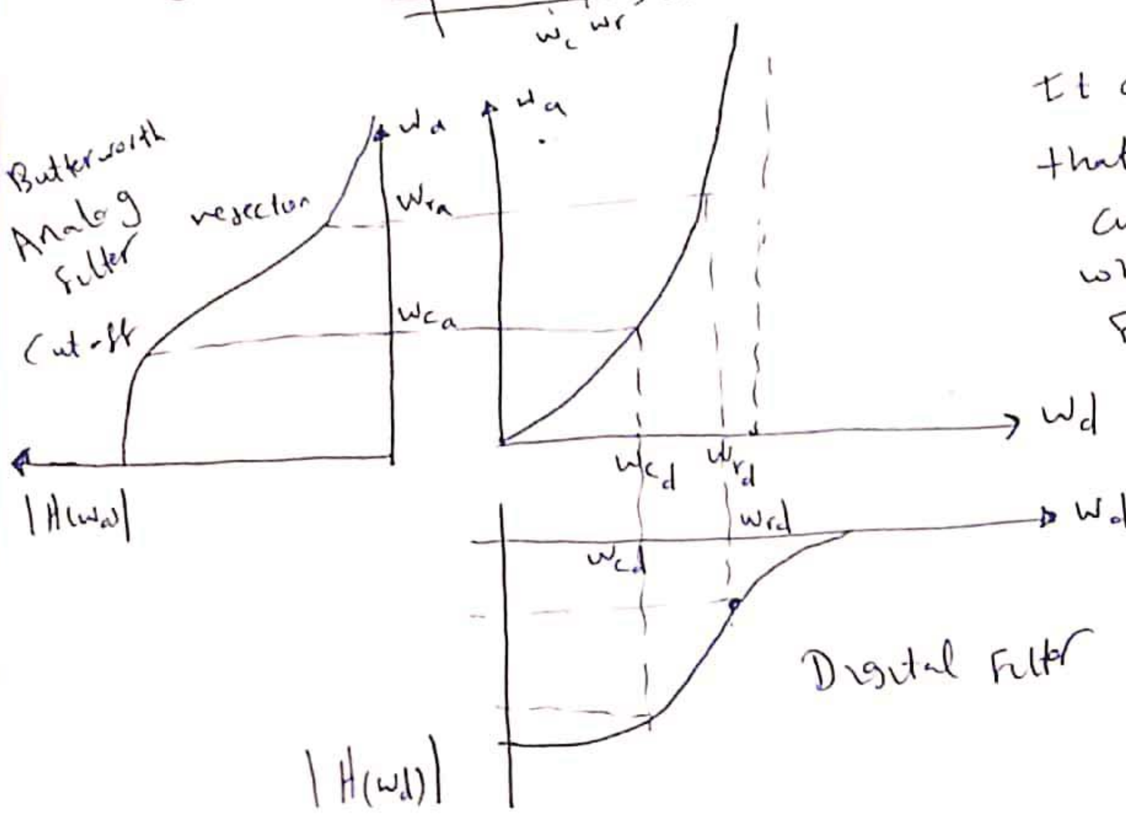
Where: w_a: analog Frequency

w_d: digital Frequency

T_s: Sampling time $= \frac{1}{f_s}$



Now, we can map the analog butterworth approximation curve



It can be seen that the approximation curve is preserved when transfering from analog to Digital..

Procedures for design of IIR digital Filter using BiLinear transformation :-

Steps

1- Find $H(s)$ for a normalized Butterworth LPF محفوظة

$N=1 \Rightarrow H(s) = \frac{1}{s+1}$

$N=2 \Rightarrow H(s) = \frac{1}{s^2+\sqrt{2}s+1}$

2- ∴ Cut off Frequency of digital Filter $\omega_{c_d} = ?$ → given $[\omega_{c_d} = 2\pi f_{c_d}]$

3- Find equivalent analog cut off Frequency

$$\omega_{c_a} = \frac{2}{T_s} \tan\left(\frac{\omega_{c_d} T_s}{2}\right)$$

4- Denormalize analog Transfer Function $H(s)$ as Follow: [Convert normalized LPF to denormalized Filter

① $\underset{\text{normalized}}{\text{LPF}} \longrightarrow \underset{\text{denormalized}}{\text{LPF}}$: $s \rightarrow \frac{s}{\omega_{c_a}}$

② $\underset{\text{normalized}}{\text{LPF}} \longrightarrow \underset{\text{denormalized}}{\text{HPF}}$: $s \rightarrow \frac{\omega_{c_a}}{s}$

(3) LPF normalized $\longrightarrow$ BPF denormalized : $S \longrightarrow \dfrac{S^2 + W_0^2}{S\,(B.W)}$

(4) LPF normalized $\longrightarrow$ BSF denormalized : $S \longrightarrow \dfrac{S\,(B.W)}{S^2 + W_0^2}$

$W_0 = \sqrt{W_{C_a} W_{C_{L_a}}}$, $BW = W_{C_a} - W_{C_a}$

5- Apply BiLinear Transformation

$$S = \frac{2}{T_s} \cdot \frac{Z-1}{Z+1}$$

$$H(Z) = H(S)\Big|_{S = \frac{2}{T_s} \cdot \frac{Z-1}{Z+1}}$$

then we get Transfer Function For Digital Filter $H(Z)$

$$H(Z) = \frac{a_0 + a_1 Z^{-1} + a_2 Z^{-2} - - -}{1 + b_1 Z^{-1} + b_2 Z^{-2} - - -}$$

(6) Implementation
- Direct Form I
- " " II
- Cascade
- Parallel
- ⋮

Example (1)

Design IIR Digital Filter of order = 1 using Bilinear Transformation. The Filter is LPF with $f_c = 30\ HZ$ & $f_s = 150\ HZ$
cut off

Solution

Note that; the Given cut off Frequency is the digital Frequency as the specifications usually are given for the required Digital Filter.

$\therefore\ f_{c_d} = 30\ HZ \quad , f_s = 150\ HZ \Rightarrow T_s = \frac{1}{150}$

Steps:

$\boxed{1}$ $N=1 \Rightarrow H(s)\Big|_{\substack{LPF \\ normalized}} = \frac{1}{S+1}$

$\boxed{2}$ $w_{c_d} = 2\pi f_{c_d} = 2\pi * 30 = \boxed{60\pi}$

$\boxed{3}$ $w_{c_a} = \frac{2}{T_s} \tan\left(\frac{w_{cd}\,T_s}{2}\right) = \frac{2}{T_s} \tan\left(\frac{60\pi\left(\frac{1}{150}\right)}{2}\right)$

$w_{c_a} = \frac{2}{T_s} \tan\left(\frac{\pi}{5}\right) = \frac{2}{T_s} \tan(36^{\circ}) = \boxed{\frac{2}{T_s}\, 0.72}$

4

$$H(s)\Big|_{\text{LPF normalized}} = \frac{1}{s+1}$$

$$\downarrow \; s \rightarrow \frac{s}{\omega_{ca}}$$

$$H(s)\Big|_{\text{LPF denormalized}} = s \rightarrow \frac{s}{\omega_{ca}}$$

$$H(s)\Big|_{\text{Denormalized}} = \frac{1}{\left(\frac{s}{\omega_{ca}}\right)+1} = \frac{\omega_{ca}}{s+\omega_{ca}}$$

$$H(s)\Big|_{\text{Denormalized}} = \frac{\frac{2}{T_s}(0.72)}{s+\frac{2}{T_s}0.72}$$

5 <u>Bilinear Transformation:</u>

put $s = \frac{2}{T_s}\frac{z-1}{z+1}$ in $H(s)\Big|_{\text{Denormalized}}$

$$H(z) = \frac{\left(\frac{2}{T_s}\right)0.72}{\cancel{\left(\frac{2}{T_s}\right)}\frac{z-1}{z+1}+\cancel{\left(\frac{2}{T_s}\right)}0.72} = \frac{0.72}{\frac{z-1}{z+1}+0.72}$$

$$H(z) = \frac{0.72}{\frac{z-1}{z+1}+0.72} * \frac{z+1}{z+1}$$

$$H(z) = \frac{0.72z+0.72}{z-1+0.72z+0.72}$$

$$H(z) = \frac{0.72 + 0.72}{1.72z - 0.28} = \frac{0.72 + 0.72z^{-1}}{1.72 - 0.28z^{-1}}$$

should be (1)

Before implementation

$\Downarrow$

Coefficient of y(n) must be (1)

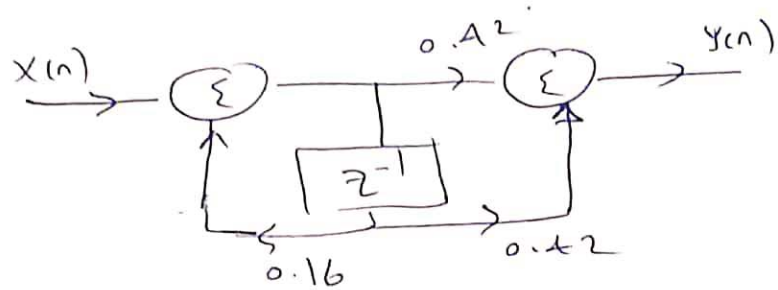
Divide by 1.72

$$H(z) = \frac{0.42 + 0.42z^{-1}}{1 - 0.16z^{-1}}$$

$\boxed{6}$ Implementation (Realization)

Direct Form I or Direct Form II

let's make it using Direct Form II



Example (2)

Design using Bilinear Transformation a 2nd order HPF digital IIR filter having $f_c = 2\ KHZ$ & $f_s = 6\ KHZ$

Solution

Given: $f_{c_d} = 2\ KHZ$, $T_s = \frac{1}{f_s} = \frac{1}{6*10^3}$

Steps:

$\boxed{1}$ $\boxed{N=2} \Rightarrow H(s)\Big|_{\substack{LPF \\ normalized}} = \frac{1}{s^2 + \sqrt{2}\, s + 1}$

$\boxed{2}$ $w_{c_d} = 2\pi f_{c_d} = 2\pi * 2 * 10^3 = 4\pi \quad 10^3$

$\boxed{3}$ $w_{c_a} = \frac{2}{T_s} \tan\left(\frac{w_{cd}\, T_s}{2}\right) = \frac{2}{T_s} \tan\left(\frac{4\pi 10^3\ 1}{2*6*10^3}\right)$

$w_{c_a} = \frac{2}{T_s} \tan\left(\frac{\pi}{6}\right) = \frac{2}{T_s}\sqrt{3}$, $T_s = \frac{1}{6*10^3}$

$\boxed{4}$ $H(s)\Big|_{\substack{LPF \\ normalized}} = \frac{1}{s^2 + \sqrt{2}\, s + 1}$

$\Downarrow \quad s \longrightarrow \frac{w_{ca}}{s}$

$H(s)\Big|_{\substack{HPF \\ Denormalized}} = \quad s \longrightarrow \frac{w_{ca}}{s}$

$$H(s)\Big|_{\text{Denormalized}} = \frac{1}{\left(\frac{w_{ca}}{s}\right)^2 + \sqrt{2}\left(\frac{w_{ca}}{s}\right) + 1} * \frac{s^2}{s^2}$$

$$= \frac{s^2}{s^2 + \sqrt{2}\, w_{ca} s + w_{ca}^2} \quad \text{but } w_{ca} = \frac{2}{T_s}\sqrt{3}$$

$$H(s)\Big|_{\text{Denormalized}} = \frac{s^2}{s^2 + \frac{2}{T_s}\sqrt{6}\, s + \left(\frac{2}{T_s}\right)^2 3}$$

$\boxed{5}$ Bilinear Transformation:

Put $s = \frac{2}{T_s} \frac{z-1}{z+1}$

$$\therefore H(z) = \frac{\left(\frac{2}{T_s}\right)^2 \left(\frac{z-1}{z+1}\right)^2}{\left(\frac{2}{T_s}\right)^2 \left(\frac{z-1}{z+1}\right)^2 + \frac{2}{T_s}\sqrt{6}\,\frac{2}{T_s} \cdot \frac{z-1}{z+1} + \left(\frac{2}{T_s}\right)^2 3}$$

$$H(z) = \frac{\cancel{\left(\frac{2}{T_s}\right)^2} \frac{(z-1)^2}{(z+1)^2}}{\cancel{\left(\frac{2}{T_s}\right)^2} \left(\frac{z-1}{z+1}\right)^2 + \cancel{\left(\frac{2}{T_s}\right)^2}\sqrt{6}\,\frac{z-1}{z+1} + 3\,\cancel{\left(\frac{2}{T_s}\right)^2}}$$

$$H(z) = \frac{\frac{(z-1)^2}{(z+1)^2}}{\frac{(z-1)^2}{(z+1)^2} + \sqrt{6}\,\frac{z-1}{z+1} + 3} * \frac{(z+1)^2}{(z+1)^2}$$

$$H(z) = \frac{(z-1)^2}{(z-1)^2 + \sqrt{6}\,(z^2-1) + 3\,(z+1)^2}$$

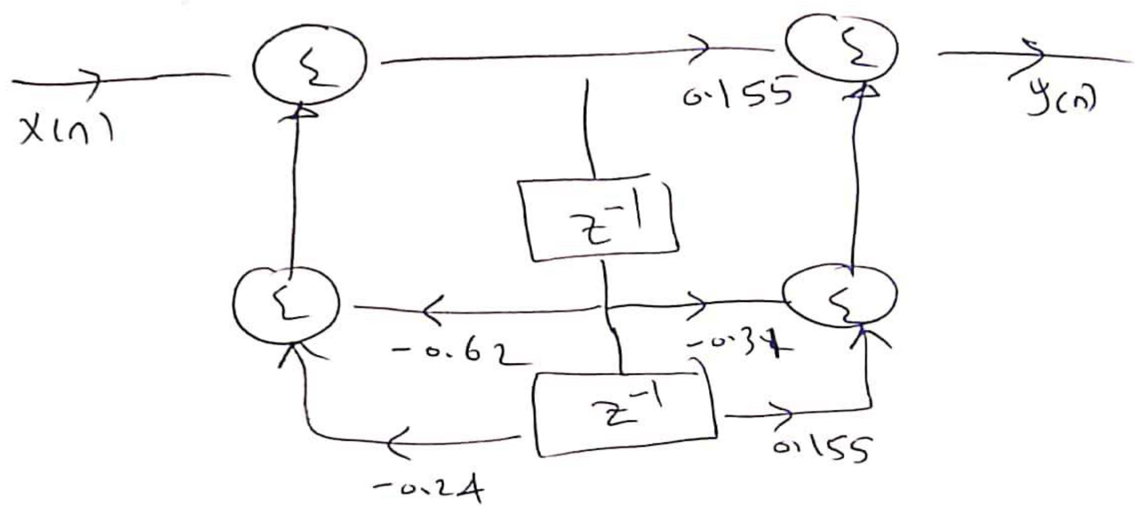
$$H(z) = \frac{z^2 - 2z + 1}{6.45z^2 + 4z + 1.55} = \frac{1 - 2z^{-1} + z^{-2}}{6.45 + 4z^{-1} + 1.55z^{-2}}$$

Divide by (6.45) as coeff of $y(n)$ should be $\underline{1}$

$$H(z) = \frac{0.155 - 0.31z^{-1} + 0.155z^{-2}}{1 + 0.62z^{-1} + 0.24z^{-2}}$$

$\boxed{6}$ Make the realization ⟨ Direct Form I or Direct Form II

let's make it Direct Form II



Example (3)

Using Bilinear transformation, Design IIR Digital HPF Filter having cut off 1 KHZ, Sampling Frequency = 4 KHZ

The Filter order = 3

Solution

Given: $f_{c_d} = 1\,KHZ$, $f_s = 4\,KHZ$, $T_s = \frac{1}{4 * 10^3}$

Steps:

[1] $N = 3 \Rightarrow H(s)\Big|_{\substack{LPF \\ normalized}} = \frac{1}{s^3 + 2s^2 + 2s + 1}$

[2] $w_{c_d} = 2\pi f_{c_d} = 2\pi (1k) = 2\pi\, 10^3$

[3] $W_{c_a} = \frac{2}{T_s} \tan\left(\frac{w_{c_d} T_s}{2}\right)$

$W_{c_a} = \frac{2}{T_s} \tan\left(\frac{2\pi 10^3 \frac{1}{4*10^3}}{2}\right)$

$W_{c_a} = \frac{2}{T_s} \tan\left(\frac{\pi}{4}\right) = \boxed{\frac{2}{T_s} \frac{1}{\sqrt{2}}}$

[4] $\underset{normalized}{LPF} \xrightarrow[s \to \frac{W_{ca}}{s}]{} \underset{denormalized}{HPF}$

$$H(s)\Big|_{\text{Denormalized}} = \frac{1}{s^3+2s^2+2s+1}\Bigg|_{s=\frac{w_{ca}}{s}}$$

$$H(s)\Big|_{\text{Denormalized}} = \frac{1}{\frac{w_{ca}^3}{s^3} + 2\frac{w_{ca}^2}{s^2} + 2\frac{w_{ca}}{s} + 1} * \frac{s^3}{s^3}$$

$$H(s)\Big|_{\text{Denormalized}} = \frac{s^3}{w_{ca}^3 + 2w_{ca}^2 s + 2w_{ca} s^2 + s^3}$$

$$H(s)\Big|_{\text{Denormalized}} = \frac{s^3}{\left(\frac{2}{T_s}\right)^3\left(\frac{1}{\sqrt{2}}\right)^3 + 2\left(\frac{2}{T_s}\right)^2\frac{1}{2}s + 2\left(\frac{2}{T_s}\right)\frac{1}{\sqrt{2}}s^2 + s^3}$$

$\boxed{5}$ Bilinear Transformation

put $s = \frac{2}{T_s}\frac{z-1}{z+1}$ in $H(s)\Big|_{\text{Denormalized}}$

$$H(z) = \frac{\cancel{\left(\frac{2}{T_s}\right)^3}\left(\frac{z-1}{z+1}\right)^3}{\cancel{\left(\frac{2}{T_s}\right)^3}\frac{1}{(2)^{3/2}} + \frac{2}{2}\cancel{\left(\frac{2}{T_s}\right)^3}\left(\frac{z-1}{z+1}\right) + \frac{2}{\sqrt{2}}\cancel{\left(\frac{2}{T_s}\right)^3}\left(\frac{z-1}{z+1}\right)^2 + \cancel{\left(\frac{2}{T_s}\right)^3}\left(\frac{z-1}{z+1}\right)^3}$$

$$H(z) = \frac{(z-1)^3/(z+1)^3}{\frac{1}{(2)^{3/2}} + \frac{(z-1)}{(z+1)} + \frac{2}{\sqrt{2}}\frac{(z-1)^2}{(z+1)^2} + \frac{(z-1)^3}{(z+1)^3}}$$

multiply by $\frac{(z+1)^3}{(z+1)^3}$

$$H(z) = \frac{(z-1)^3}{\frac{1}{(2)^{3/2}}(z+1)^3 + (z-1)(z+1)^2 + \frac{2}{\sqrt{2}}(z-1)^2(z+1) + (z-1)^3}$$

After simplifying

$$H(z) = \frac{z^3 - 3z^2 + 3z - 1}{3.4z^3 - 3.4z^2 + 3.4z - 3.4}$$

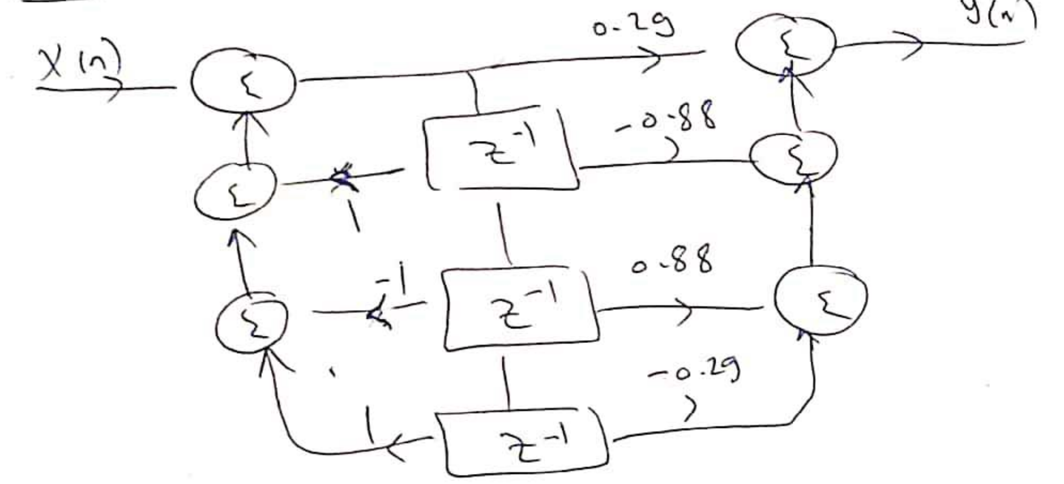
$$H(z) = \frac{1 - 3z^{-1} + 3z^{-1} - z^{-3}}{3.4 - 3.4z^{-1} + 3.4z^{-2} - 3.4z^{-3}}$$

This should be (1)

Divide by 3.4

$$H(z) = \frac{0.29 - 0.88z^{-1} + 0.88z^{-2} - 0.29z^{-3}}{1 - z^{-1} + z^{-2} - z^{-3}}$$

$\boxed{6}$ Implement $\Rightarrow$ Direct form II



Note

للمعرفة فقط

What is the meaning of Denormalization ?

in Step(A) $LPF_{\text{normalized}} \to LPF_{\text{Denorm}}$

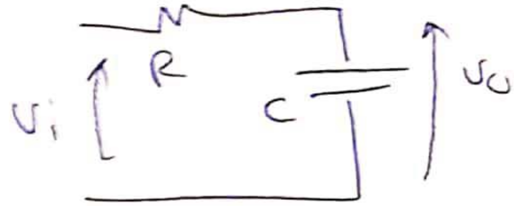
$\to HPF_{\text{Denorm}}$

$\to BPF_{\text{Denorm}}$

$\to BSF_{\text{Denorm}}$

Ex:

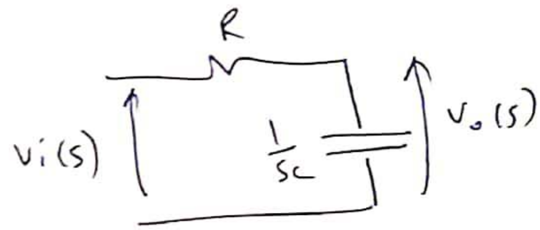
RC LPF



Find $H(s)$ of LPF
- Normalized [Put $R = C = 1$]
- Denormalized in terms of Values of R, C

Sol

$$V_o(s) = V_i(s) \frac{\frac{1}{sC}}{\frac{1}{sC} + R}$$



$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} = \frac{1}{1 + RCs}$$

$$\boxed{H(s)\Big|_{\text{Denormalized}} = \frac{1}{1 + RCs}} \quad , \omega_c = \frac{1}{RC}$$

$$\to H(s)\Big|_{\substack{\text{normalized} \\ R=1, C=1}} = \frac{1}{s+1} \quad \text{[As we know]}$$

to Convert from $LPF_{\text{normalized}} \xrightarrow{s \to \frac{s}{\omega_{ca}}} LPF_{\text{denormalized}}$

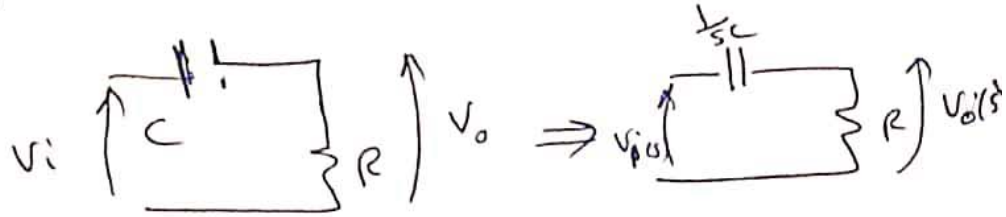
let's check

$$s \longrightarrow \frac{s}{w_{cn}} = \frac{s}{1/RC} = RCs$$

$$\therefore H(s)\Big|_{\text{Denormal}} = \frac{1}{s+1}\Big|_{s \to RCs}$$

$$\boxed{H(s)\Big|_{\text{Denormalized}} = \frac{1}{1+RCs}}$$ ← the same as we got

→ For HPF



$$V_o(s) = V_i(s) \cdot \frac{R}{R + \frac{1}{sC}} \Rightarrow \boxed{H(s)\Big|_{\text{Denormalized}} = \frac{sCR}{1+RCs}}$$

$$w_c = \frac{1}{RC}$$

But For $\boxed{N=1} \Rightarrow$ LPF$_{\text{normalized}}$ $H(s) = \frac{1}{1+s}$

Convert From LPF$_{\text{normalized}}$ $\xrightarrow{s \to \frac{w_{cn}}{s}}$ HPF$_{\text{Denormalized}}$

$$H(s)\Big|_{\substack{\text{HPF}\\ \text{Denormalized}}} = \frac{1}{1+s}\Big|_{s \to \frac{1}{RCs}} = \frac{1}{1+\frac{1}{RCs}} = \frac{RCs}{1+RCs}$$

← The same as we got

and so on.