II FIR Digital Filter Realization

Direct Form — Simplified Direct Form

FIR Filter: $\Rightarrow$ No feed back

D.E: $y(n) = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2) - - -$

$\downarrow$

$$H(z) = \frac{\text{coeff of } x}{\text{coeff of } y} = a_0 + a_1 z^{-1} + a_2 z^{-2} + - - -$$

$$H(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + - - - - -$$

$\downarrow z^{-1}$

$$h(n) = a_0 + a_1 \delta(n-1) + a_2 \delta(n-2) - - - -$$

$$h(n) = \{ a_0, a_1, a_2, - - \}$$

Example

FIR filter with D.E

$$y(n) = 5x(n) + 3x(n-1) - x(n-2) + 0.8x(n-3)$$

Sol

$$H(z) = 5 + 3z^{-1} - z^{-2} + 0.8\,z^{-3}$$

$$\Big\downarrow z^{-1}$$

$$h(n) = \{5, 3, -1, 0.8\}$$

Notes

$\boxed{1}$ FIR Filter is always stable [Advantage]

<u>why?</u>

$h(n)$ is of finite length $[\sum h(n) = \text{value} \neq \infty]$

$\boxed{2}$ FIR Filter structure is simple [No recursive or Feedback]

$\boxed{3}$ FIR Filter can be designed to have <u>linear</u> <u>phase</u>

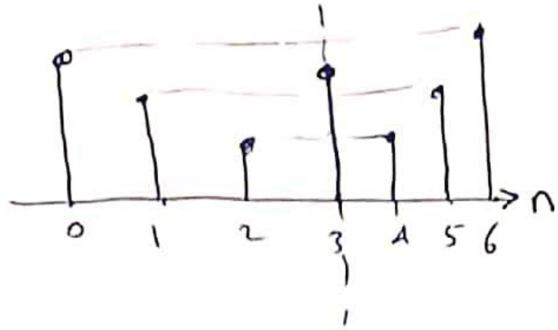
When the impulse response $h(n)$ is <u>symmetric</u> [Advantage]

Condition of symmetry: $h(n) = h(N-1-n)$, $n = 0, 1, 2 \cdots N-1$

Where N: number of samples of $h(n)$

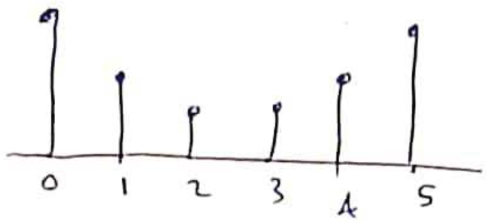
For N = 7

$h(0) = h(6)$
$h(1) = h(5)$
$h(2) = h(4)$
$h(3) = h(3)$



For N = 6

$h(0) = h(5)$
$h(1) = h(4)$
$h(2) = h(3)$



Q: What is the liner phase?

- All Frequency Componants of the input Signal undergo the same time delay. $\boxed{\theta(\omega) = K\omega}$, K: Constant
- Same time delay that any signal with Frequency ω undergoes

Without liner phase response $\Longrightarrow$ Phase distortion [problem]

Example

A FIR digital Filter with $H(z) = 1 + 2.5\, z^{-1} + z^{-2}$

(i) Does this Filter have liner phase?

(ii) Find the phase response

Solution

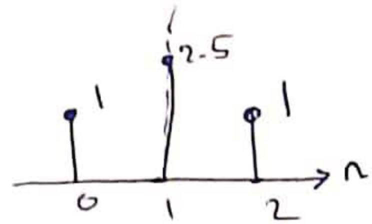
(i) $H(z) = 1 + 2.5\,z^{-1} + z^{-2}$

$\downarrow z^{-1}$

$h(n) = \{ \underset{\uparrow}{1}, 2.5, 1 \}$, $h(0) = 1$, $h(1) = 2.5$, $h(2) = 1$

To know if it has linear phase or not $\Rightarrow$ check symmetry condition

$h(n) = h(N-1-n)$, $N = 3$



$h(0) = h(2)$ ✓

$\therefore$ symmetric

$h(1) = h(1)$ ✓

$\therefore$ This filter has linear phase

(ii) To get the phase response

$$H(\omega) = H(z)\Big|_{z=e^{j\omega}} = 1 + 2.5\,e^{-j\omega} + e^{-j2\omega}$$

$$H(\omega) = e^{-j\omega}\left[e^{j\omega} + 2.5 + e^{-j\omega} \right] = e^{-j\omega}\Big[2.5 + \underbrace{e^{j\omega} + e^{-j\omega}}_{2\cos\omega} \Big]$$

$$H(\omega) = \left[2.5 + 2\cos\omega \right] e^{-j\omega}$$

$\boxed{\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)}$

$\therefore |H(\omega)| = 2.5 + 2\cos\omega$, $\angle H(\omega) = -\omega$

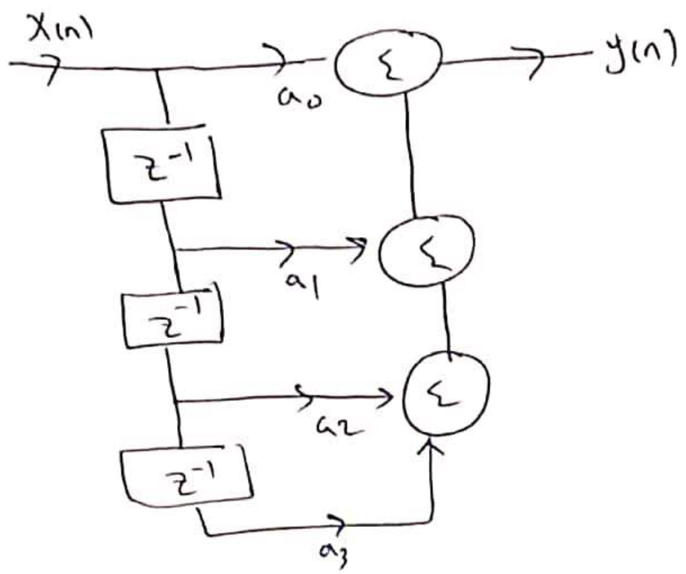
$\therefore$ phase response $\angle H(\omega) = -\omega$ is linear phase

Realizations of FIR Filter

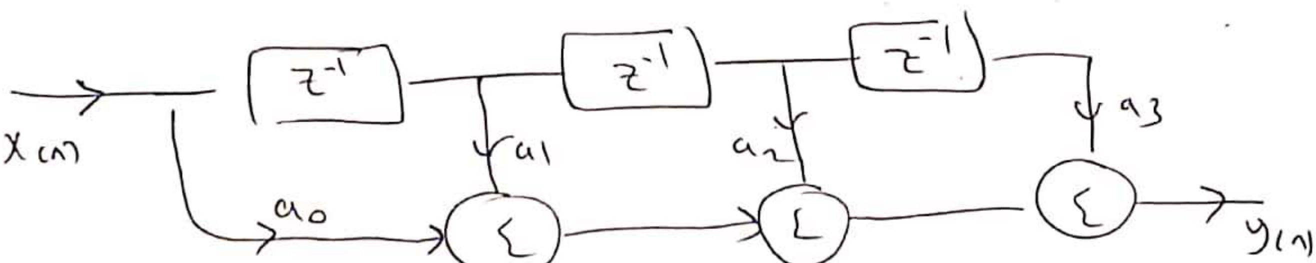
[1] Direct Form

" There is No Direct Form II "

D.E: $y(n) = a_0\, x(n) + a_1\, x(n-1) + a_2\, x(n-2) + a_3\, x(n-3)$



Or It can be written.

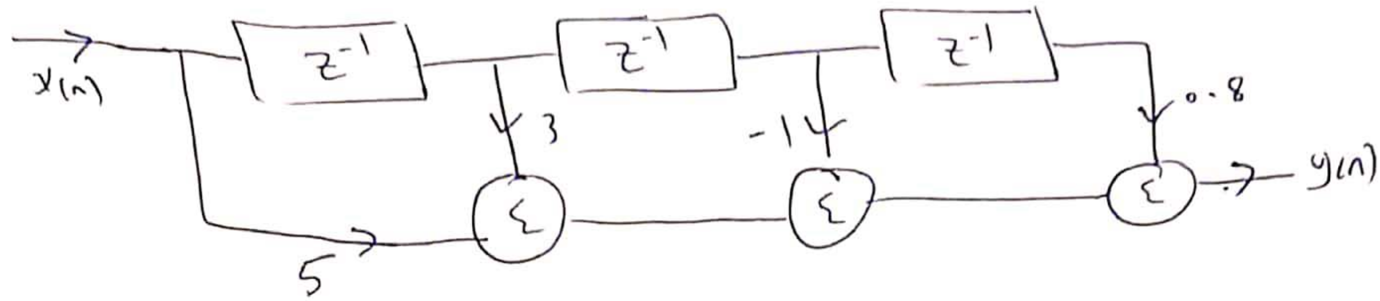


Example

FIR Filter with $H(z) = 5 + 3z^{-1} - z^{-2} + 0.8z^{-3}$

Implement it using Direct Form?

Sol



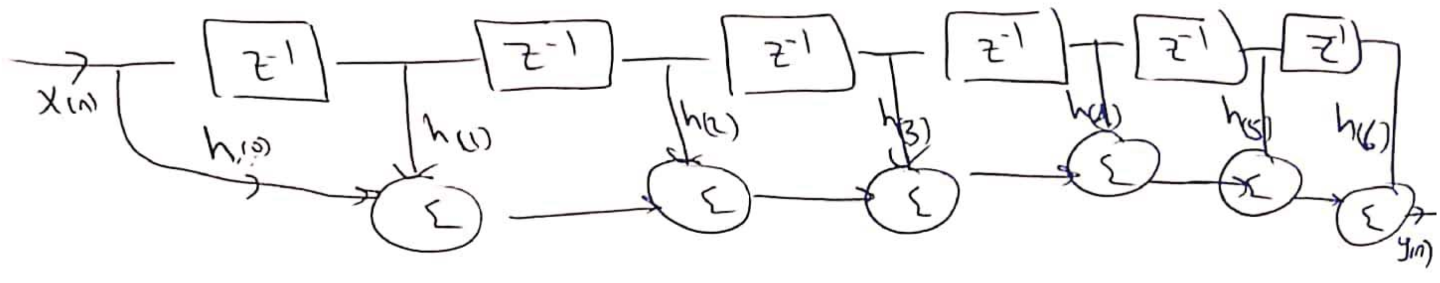
2 Simplified Direct Form

Note that for $h(n) = \{h(0), h(1), h(2), h(3), h(4), h(5), h(6)\}$

$N = 7$ samples.

With implementing it using Direct Form I, we need

6 adders, 6 delay units, 7 multipliers



For high order FIR Filter <u>with liner phase</u>

$\Downarrow$

We can use simplified Direct Form

[Due to liner phase: $h(0) = h(6)$, $h(n) = h(N-1-n)$
$h(1) = h(5)$
$h(2) = h(4)$
$h(3)$

Symmetry Condition

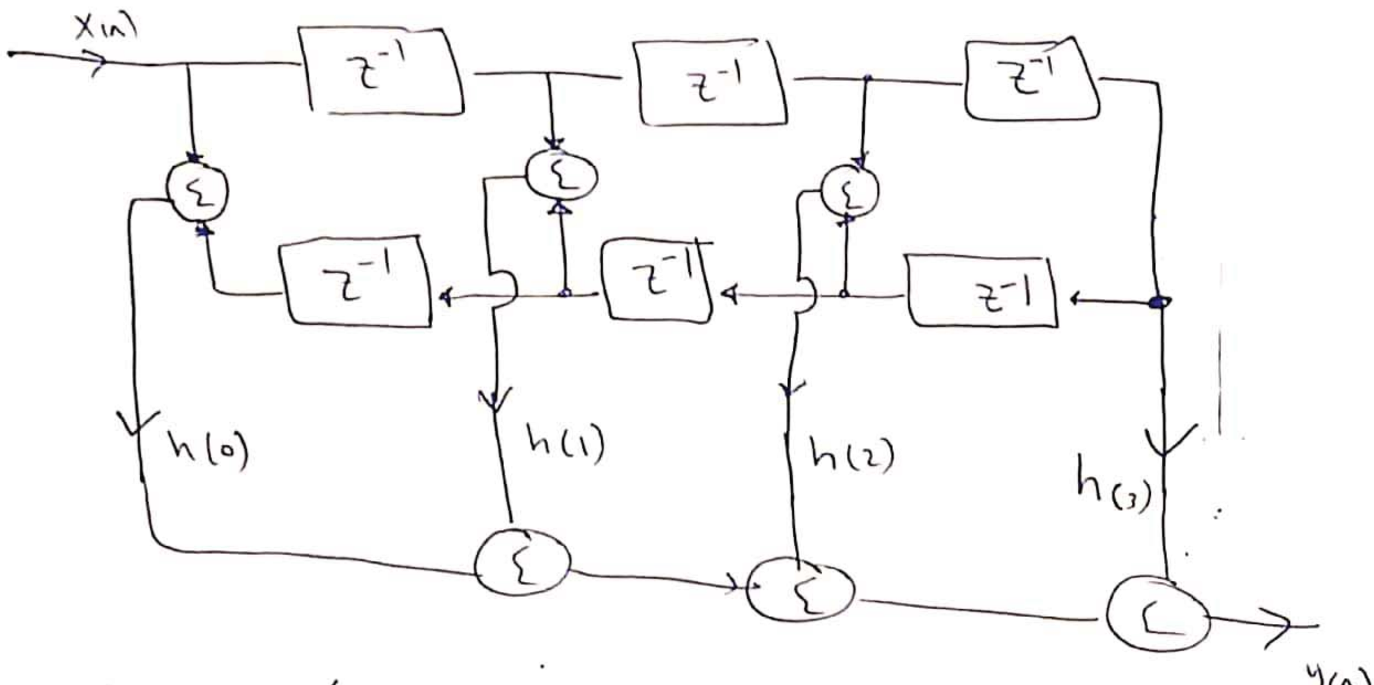
$\therefore$

$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + h(2)\,x(n-2) + h(3)\,x(n-3)$$
$$+ h(4)\,x(n-4) + h(5)\,x(n-5) + h(6)\,x(n-6)$$

Due to symmetry of $h(n)$ [linear phase]

$h(0) = h(6)$ & $h(1) = h(5)$ & $h(2) = h(4)$

$$\therefore y(n) = h(0)\left[x(n) + x(n-6)\right] + h(1)\left[x(n-1) + x(n-5)\right]$$
$$+ h(2)\left[x(n-2) + x(n-4)\right] + h(3)\left[x(n-3)\right]$$



We use 6 adders, 6 delay units, 4 Multipliers.

⇒ at high orders ⇒ no of multipliers is reduced significantly.

⇒ Condition of using Simplified Form ⇒ is to have linear phase (h(n): symmetric)

Example

Digital Filter with $H(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$

i) Type of Filter ?

ii) Find impulse response & Difference equation

iii) Discuss Stability.

iv) Does this filter exhibit linear phase ? why ?

v) Implement this Filter with Direct Form & Simplified Form (if possible)

Solution

(i) $H(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$

[FIR Filter , No recursive]

(ii) $h(n) = Z^{-1}\{H(z)\} = \delta(n) + 2\delta(n-1) + 3\delta(n-2) + 2\delta(n-3) + \delta(n-4)$

→ Impulse Response : $h(n) = \{1, 2, 3, 2, 1\}$

D-E : $y(n) = x(n) + 2x(n-1) + 3x(n-2) + 2x(n-3) + x(n-4)$

(iii) FIR Filter is always stable

(iv) To know if this filter has linear phase or not

⇓ Check: $h(n) = \{\underset{\uparrow}{1}, 2, 3, 2, 1\}$

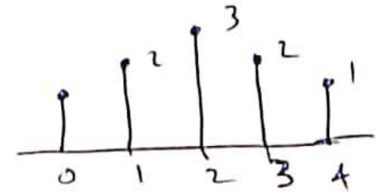
Check if $h(n) = h(N-1-n)$ or not

$N = 5$

$h(0) = h(4) = 1$ ✓

$h(1) = h(3) = 2$ ✓

$h(2) = 3$ ✓



∴ $h(n)$ is symmetric

⇓

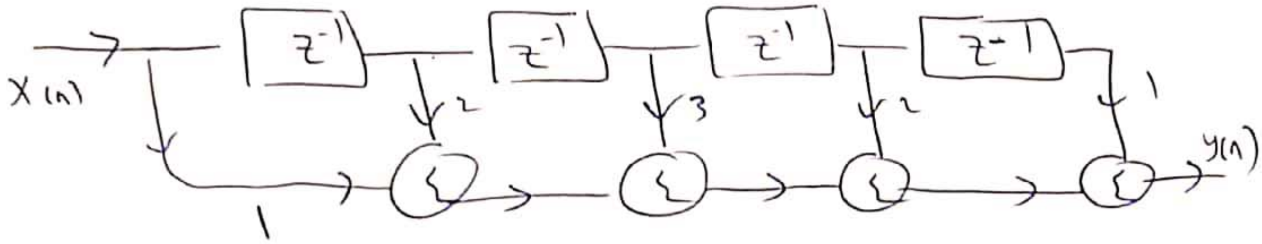
∴ The filter has linear phase.

(v) We can implement this filter

Direct Form
[No Conditions]

Simplified Form
[Condition: linear phase ✓]

→ Direct Form: $H(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$



4 Adders, 4 delay units, 5 Multipliers

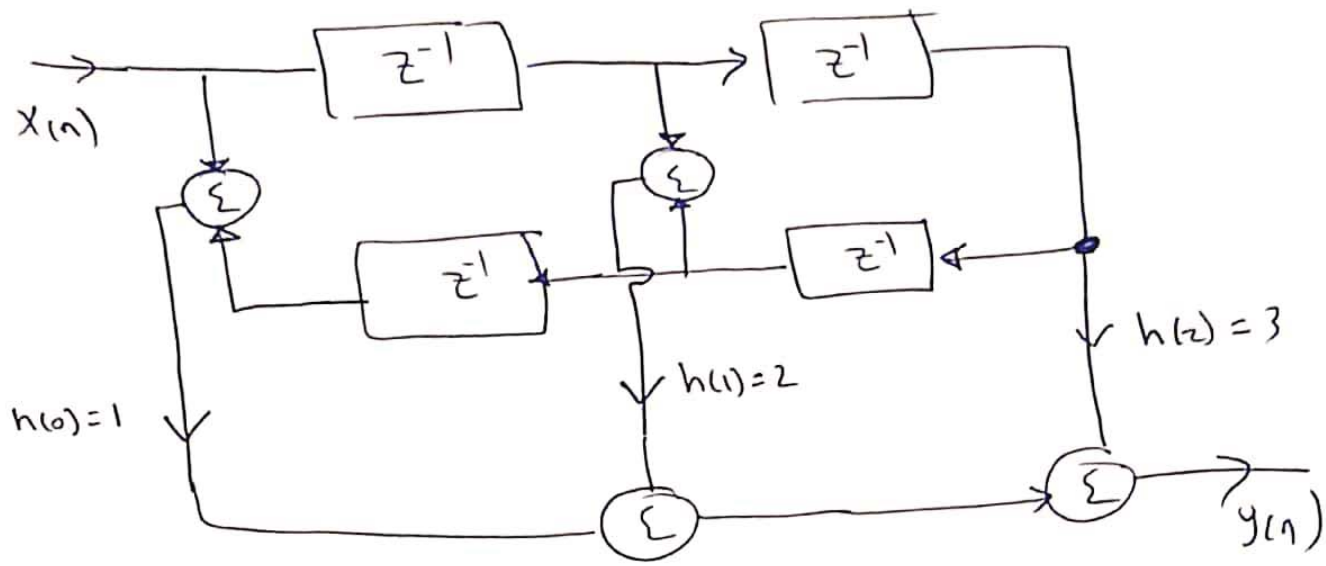
→ Simplified Form: [has linear phase]

$h(0) = h(4) = 1$

$h(1) = h(3) = 2$

$h(2) = 3$

$$y(n) = 1\left[x(n) + x(n-4)\right] + 2\left[x(n-1) + x(n-3)\right] + 3\,x(n-2)$$



We use 4 adders, 4 delay units, $\boxed{3}$ multipliers

Ex: Digital Filter with Difference equation

$$y(n) = B_0 [x(n) + x(n-1)]$$

i) what is the filter type? Find $H(z)$, $h(n)$?

ii) Does it experince linear phase? why?

iii) Find the magnitude frequency response & phase frequency response?

iv) Does this filter represent LPF or HPF or BPF?

SoL

i) FIR Filter [Non recursive] $\longrightarrow y(n) = B_0 x(n) + B_0 x(n-1)$

$H(z) = B_0 + B_0 z^{-1}$, B_0: constant

$\downarrow$

$h(n) = \{B_0, B_0\}$ or $B_0 \delta(n) + B_0 \delta(n-1)$

ii) $h(n) = \{B_0, B_0\} \Longrightarrow$ symmetric

$\therefore$ It has linear phase

iii) to get $|H(\omega)|$ & $\angle H(\omega)$

get First $H(\omega) = H(z)\big|_{z=e^{j\omega}}$

$$H(\omega) = H(z)\Big|_{z=e^{j\omega}} = B_0 + B_0 z^{-1}\Big|_{z=e^{j\omega}} = B_0 + B_0 e^{-j\omega}$$

$$H(\omega) = B_0\left[1 + e^{-j\omega}\right] = B_0\, e^{-j\omega/2}\left[e^{j\omega/2} + e^{-j\omega/2}\right]$$

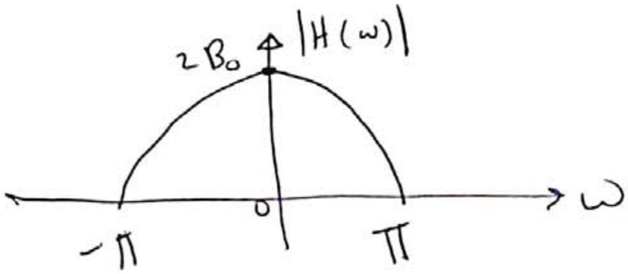
$$H(\omega) = B_0\, e^{-j\omega/2}\, 2\cos(\omega/2)$$

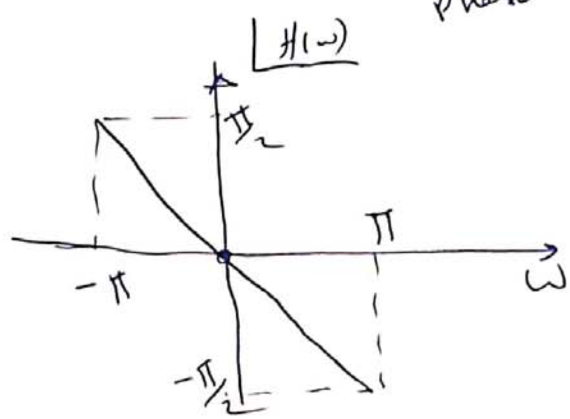
$$\boxed{H(\omega) = 2B_0 \cos(\omega/2)\, e^{-j\omega/2}}$$

$$|H(\omega)| = 2B_0\cos(\omega/2)$$

$$\angle H(\omega) = -\omega/2$$

linear phase





Type: LPF