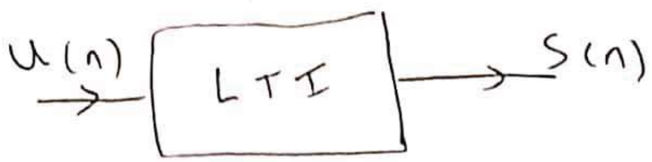
Step response of LTI system



s(n): step response (o/p of the system due to unit step input).

$\boxed{s(n) = u(n) * h(n)}$ i.e: when $x(n) = u(n)$

$y(n) = s(n)$

→ if given s(n), we can get h(n)

$$\boxed{h(n) = s(n) - s(n-1)}$$

<u>ex:</u> Find step response of a system

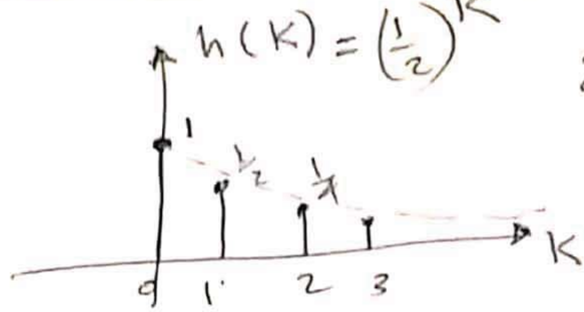
$h(n) = \left(\frac{1}{2}\right)^n u(n)$

Solution

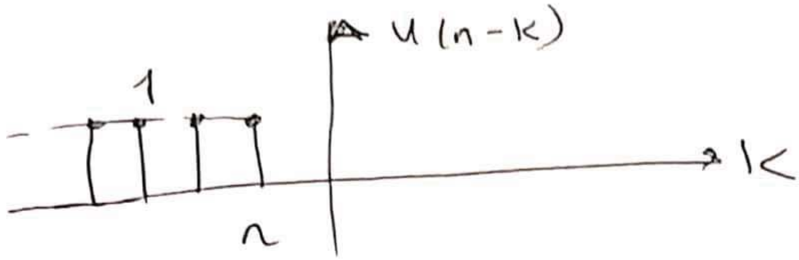
$$s(n) = u(n) * h(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

1) $n < 0$,

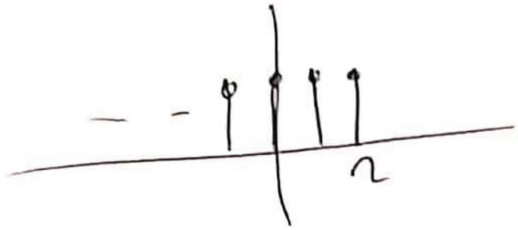
$$\boxed{S(n) = 0}$$



2) $n \geq 0$,



$$s(n) = \sum_{K=0}^{n} \left(\tfrac{1}{2}\right)^{K} \cdot 1$$



$$s(n) = \frac{1 - \left(\frac{1}{2}\right)^{n+1}}{1 - \frac{1}{2}}$$

$$s(n) = 2\left(1 - \left(\tfrac{1}{2}\right)^{n+1}\right)$$

$$s(n) = \begin{cases} 0, & n < 0 \\ 2\left(1 - \left(\frac{1}{2}\right)^{n+1}\right), & n \geq 0 \end{cases} = \left[2\left(1 - \left(\tfrac{1}{2}\right)^{n+1}\right)\right] u(n)$$

Properties of LTI system

given: $h(n)$

(1) Memory less :-

LTI system is memory less if $\boxed{h(n) = c\,\delta(n)}$

غير ذلك يكون memory

ex: $h(n) = (2)^n u(n) \longrightarrow$ memory

$h(n) = 2\,\delta(n) \longrightarrow$ memoryless

$h(n) = 2\,\delta(n-1) \longrightarrow$ memory

$h(n) = 2\,[u(n) - u(n-1)] = 2\,\delta(n) \longrightarrow$ memoryless

3/

2 - Causality

LTI system Causal if

$$h(n) = 0 \quad , n < 0$$

otherwise non Causal

exs: $h(n) = (\frac{1}{2})^n u(n) \longrightarrow$ Causal

$h(n) = (\frac{1}{2})^n u(n+2) \longrightarrow$ non Causal

3 - Stability

LTI system is stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

exs: $h(n) = (\frac{1}{2})^n u(n)$

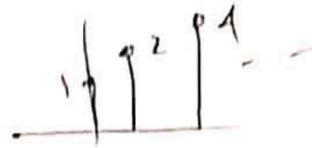
Sol

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=0}^{\infty} (\tfrac{1}{2})^n = \frac{1}{1-\frac{1}{2}} = 2 < \infty$$

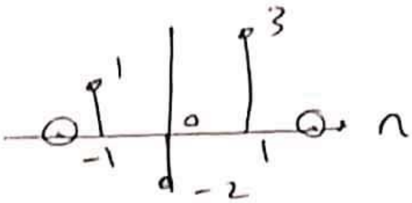
Stable.

ex: $h(n) = (2)^n u(n) \longrightarrow \sum h(n) = \sum_{n=0}^{\infty} (2)^n = \infty$

(unstable)



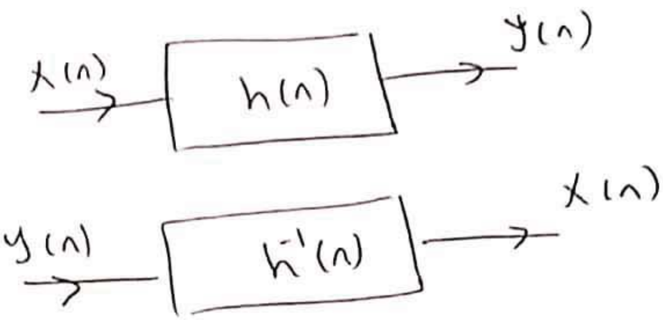
ex: $h(n) = \delta(n+1) - 2\delta(n) + 3\delta(n-1)$



$\sum |h(n)| = 1 + 2 + 3 = 6 < \infty$

Stable

(4) Invertibility:



$h^{-1}(n)$: Impulse response of inverse system.

$\boxed{\text{Condition: } h(n) * h^{-1}(n) = \delta(n)}$

Condition of inverse system.

example:

LTI system: has step response

$s(n) = 2\delta(n) - \delta(n-1)$

1- Find Impulse response

2- Is system Causal ?

3- Is system stable ?

4- Is system memory ?

Solution

given:

Step response $s(n) = 2\delta(n) - \delta(n-1)$

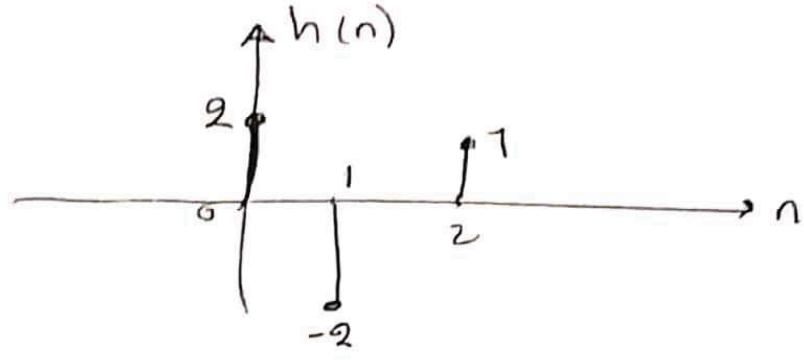
$\boxed{1}$ $h(n) = s(n) - s(n-1)$

$h(n) = [2\delta(n) - \delta(n-1)] - [2\delta(n-1) - \delta(n-2)]$

$h(n) = [2\delta(n) - 3\delta(n-1) - \delta(n-2)]$

Impulse response.

$$\boxed{h(n) = 2\,\delta(n) - 3\,\delta(n-1) + \delta(n-2)}$$



→ Causal, Memory.

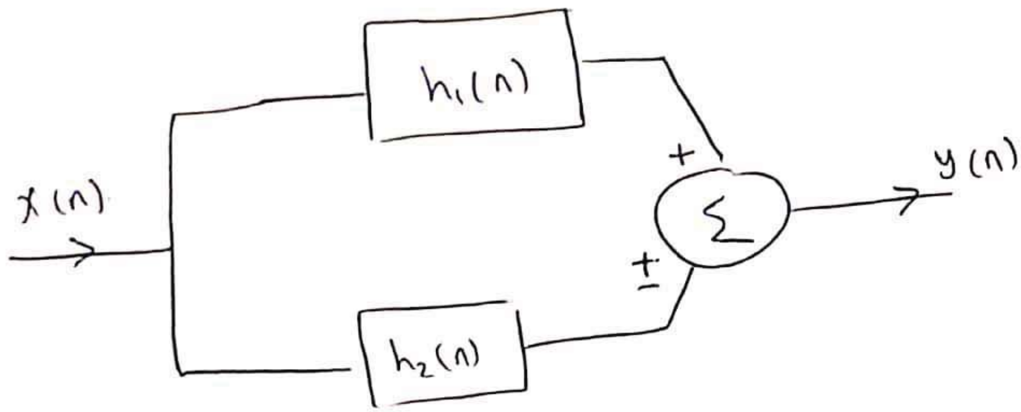
→ To check stability :-

$$\sum_{n=-\infty}^{\infty} |h(n)| = |2| + |-3| + |1| = 6$$

Stable

Properties of LTI systems

1) Parallel - Connected system :-



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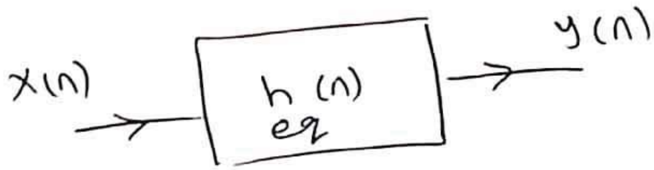
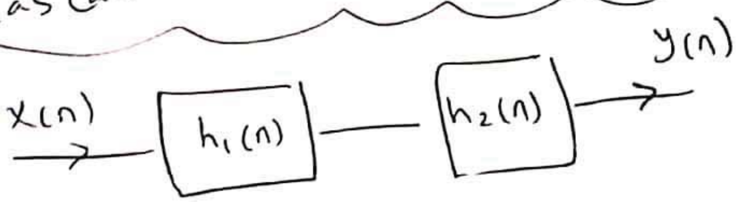


$$h_{eq}(n) = h_1(n) \pm h_2(n)$$

2) Cascaded - Connected system



$\equiv$

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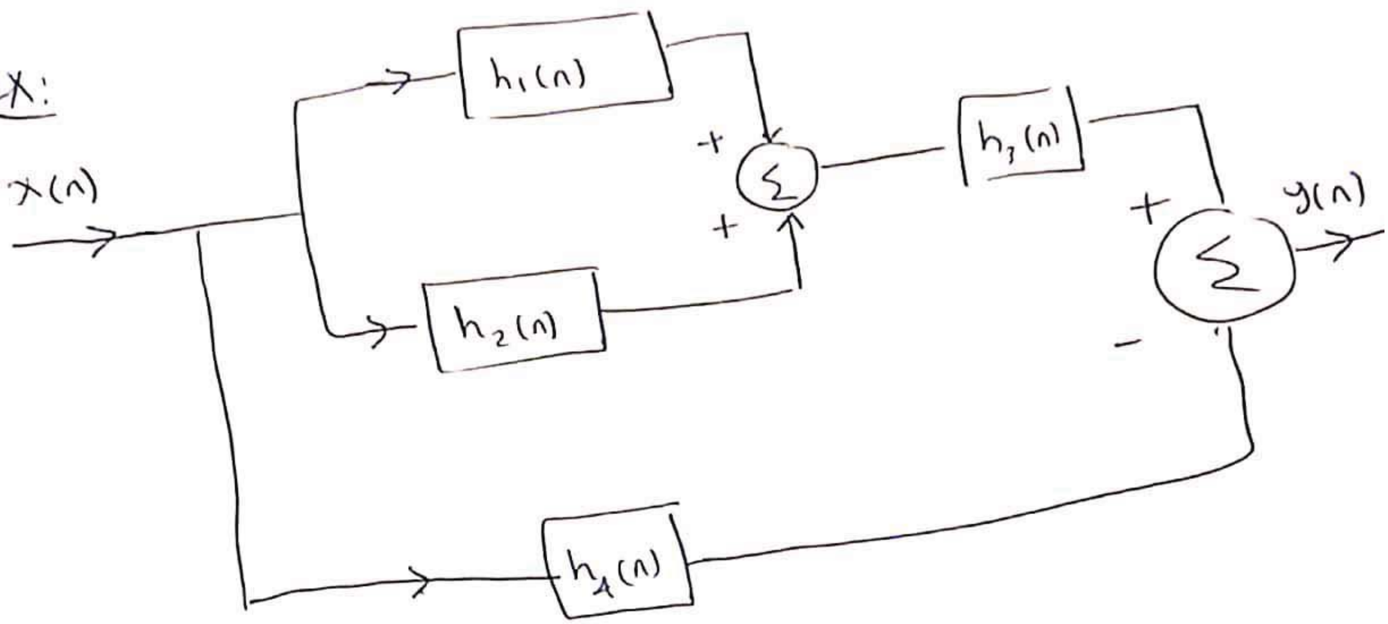
$$h_{eq}(n) = h_1(n) * h_2(n)$$

ex:

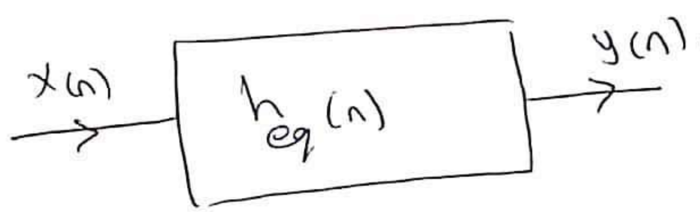


where:-

$h_1(n) = u(n)$, $h_2(n) = u(n+2) - u(n)$

$h_3(n) = \delta(n-2)$, $h_4(n) = \alpha^n u(n)$, Find $h(n)$?

Sol



$$h_{eq}(n) = \left[\left(h_1(n) + h_2(n)\right) * h_3(n) - h_4(n) \right].$$

$$h_{eq}(n) = \left[\left[\cancel{u(n)} + u(n+2) - \cancel{u(n)} \right] * \delta(n-2) - \alpha^n u(n) \right]$$

$$= \left[u(n+2) * \delta(n-2) - \alpha^n u(n) \right]$$

$$= u(n-2+1) - \alpha^n u(n)$$

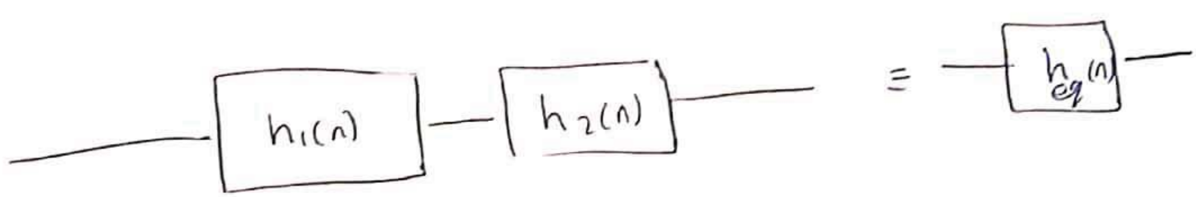
$$h_{eq}(n) = u(n) - \alpha^n u(n) = u(n) \left[1 - \alpha^n \right]$$

$\boxed{\text{ex}}$

2 Systems are Connected in series
with $h_1(n) = \frac{1}{2} \left[\delta(n) + \delta(n-1) \right]$
$h_2(n) = \frac{1}{2} \left[\delta(n) - \delta(n-1) \right]$

Find and plot $h_{eq}(n)$

Sol



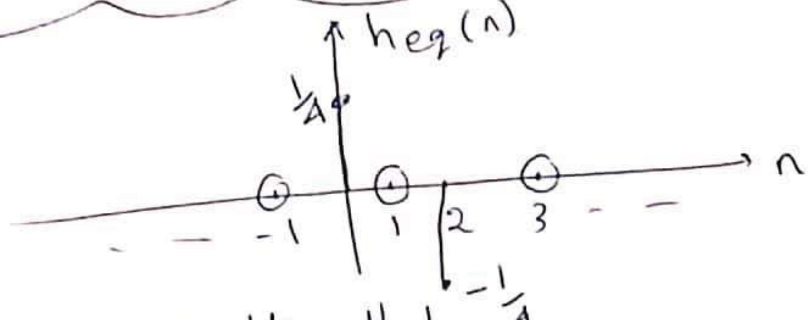
$$h_{eq}(n) = h_1(n) * h_2(n)$$

$$= \frac{1}{2}\left[\delta(n) + \delta(n-1)\right] * \frac{1}{2}\left[\delta(n) - \delta(n-1)\right]$$

$$= \frac{1}{4}\left[\delta(n) * \delta(n) + \delta(n-1) * \delta(n) - \delta(n) * \delta(n-1) - \delta(n-1) * \delta(n-1)\right]$$

$$= \frac{1}{4}\left[\delta(n) + \cancel{\delta(n-1)} - \cancel{\delta(n-1)} - \delta(n-1-1)\right]$$

$$h_{eq}(n) = \frac{1}{4}\left[\delta(n) - \delta(n-2)\right]$$



You can use the Table method if you want.

ex:

LTI system with $x(n) = 3\delta(n)$, the o/p is

$$y(n) = \begin{cases} 3, & n = 0 \\ -3, & n = 1 \\ 6, & n = 2 \\ 0, & o.w. \end{cases}$$

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a) Find Energy of output signal?

b) Is the system Causal? Stable?

c) Find the Impulse reponse $h(n)$?

d) Find the output produced by the input $x(n) = 2\delta(n) - \delta(n-1) + \delta(n-2)$

Sol

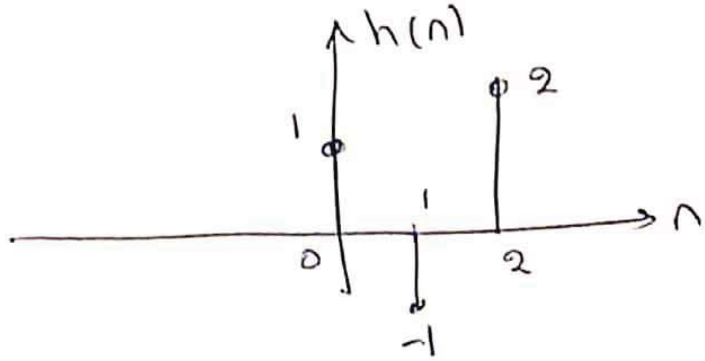
$$x(n) = 3\delta(n) \longrightarrow y(n) = \begin{cases} 3, & n=0 \\ -3, & n=1 \\ 6, & n=2 \\ 0, & o.w \end{cases}$$

$$x(n) = \delta(n) \longrightarrow y(n) = h(n) = \begin{cases} 1, & n=0 \\ -1, & n=1 \\ 2, & n=2 \end{cases}$$

$$\therefore h(n) = \begin{cases} 1, & n=0 \\ -1, & n=1 \\ 2, & n=2. \end{cases}$$

(a) $$E_t\Big|_{y(n)} = \sum_{n=-\infty}^{\infty} |y(n)|^2 = (3)^2 + (-3)^2 + (6)^2 = \_\_$$

(b) To check Causality



$\because h(n) = 0 \quad , n < 0 \Rightarrow$ Causal.

→ To check stability

get $\sum_{n=-\infty}^{\infty} |h(n)| = 1 + 1 + 2 = 4 < \infty$

Stable

(d) $x(n) = 2\delta(n) - \delta(n-1) + \delta(n-2)$

$$h(n) = \begin{cases} 1 , & n = 0 \\ -1 , & n = 1 \\ 2 , & n = 2 \end{cases} \quad , \quad x(n) = \begin{cases} 2 , & n = 0 \\ -1 , & n = 1 \\ 1 , & n = 2 \end{cases}$$

using short method (Table) $\quad y(n) = ?$

1) range of $y(n)$

$$0 + 0 \leq n \leq 2 + 2$$

$$0 \leq n \leq 4$$

2) $$y(n) = x(n) * h(n) = \sum_{k=0}^{4} x(k)\, h(n-k)$$

n	0	1	2	3	4	x(k)
	1	-1	2			2
		1	-1	2		-1
			1	-1	2	1
y(n)	2	-3	6	-3	2	

$$y(n) = \begin{cases} 2, & n = 0 \\ -3, & n = 1 \\ 6, & n = 2 \\ -3, & n = +3 \\ 2, & n = 4 \end{cases}$$

$\boxed{ex}$

the impulse response of 2 cascaded LTI systems are,

$$h_1(n) = (n+1)\{u(n) - u(n-3)\}$$

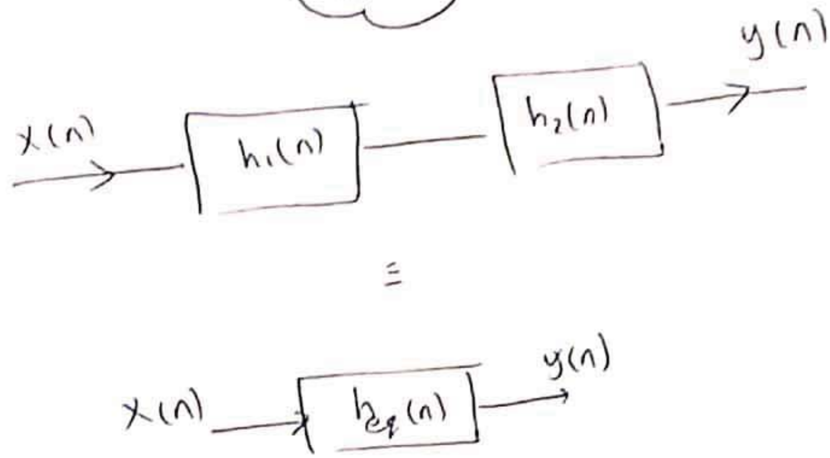
$$h_2(n) = 2\sin\left(\frac{n\pi}{2}\right)\{u(n+1) - u(n-2)\}$$

Find and sketch

1) total impulse response
2) the response of total system To input

$$x(n) = 3\delta(n-2)$$

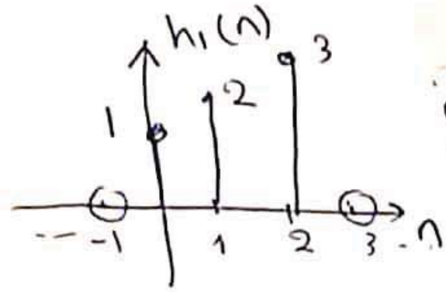
Sol



$$h_{eq}(n) = h_1(n) * h_2(n)$$

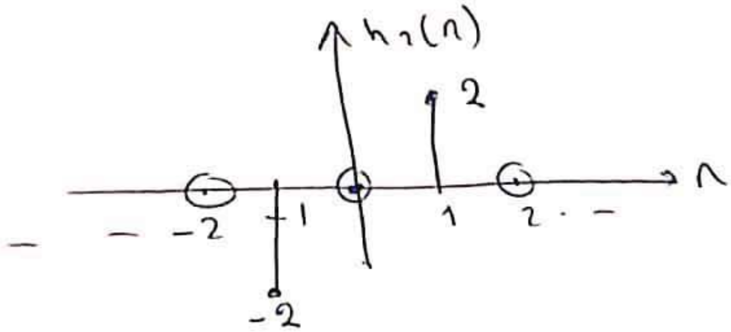
$$\text{Where}\quad h_1(n) = (n+1)\cdot\begin{cases} 1, & n=0 \\ 1, & n=1 \\ 1, & n=2 \\ 0, & o.w. \end{cases} \Leftarrow u(n)-u(n-3)$$

$$h_1(n) = \begin{cases} 1 \cdot 1 = \boxed{1} & , n = 0 \\ 2 \cdot 1 = \boxed{2} & , n = 1 \\ 3 \cdot 1 = \boxed{3} & , n = 2 \\ 0 & , o.w \end{cases}$$



$$h_2(n) = 2\sin\left(\frac{n\pi}{2}\right) \begin{cases} 1 & , n = -1 \\ 1 & , n = 0 \\ 1 & , n = 1 \\ 0 & , o.w \end{cases}$$

$$h_2(n) = \begin{cases} 2\sin\left(-\frac{\pi}{2}\right) & , n = -1 \\ 2\sin(0) & , n = 0 \\ 2\sin\frac{\pi}{2} & , n = 1 \\ 0 & , o.w \end{cases} = \begin{cases} -2 & , n = -1 \\ 0 & , n = 0 \\ 2 & , n = 1 \\ 0 & , o.w \end{cases}$$



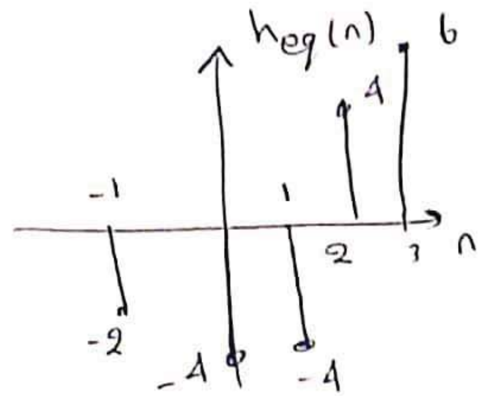
$\rightarrow$ $h_{eq}(n) = h_1(n) * h_2(n)$ // "Short methode" Table.

1) $-1+0 \leq n \leq 1+2 \rightsquigarrow -1 \leq n \leq 3$

2) $h_{eq}(n) = \sum_{k=-1}^{3} h_1(k)\, h_2(n-k)$

n	-1	0	1	2	3	$h_1(n)$
$h_2(n)$	-2	0	2			1
		-2	0	2		2
			-2	0	2	3
	-2	-4	-4	4	6	$h_{eq}(n)$

$$h_{eq}(n) = \begin{cases} -2, & n=-1 \\ -4, & n=0 \\ -4, & n=1 \\ 4, & n=2 \\ 6, & n=3 \\ 0, & o.w \end{cases}$$

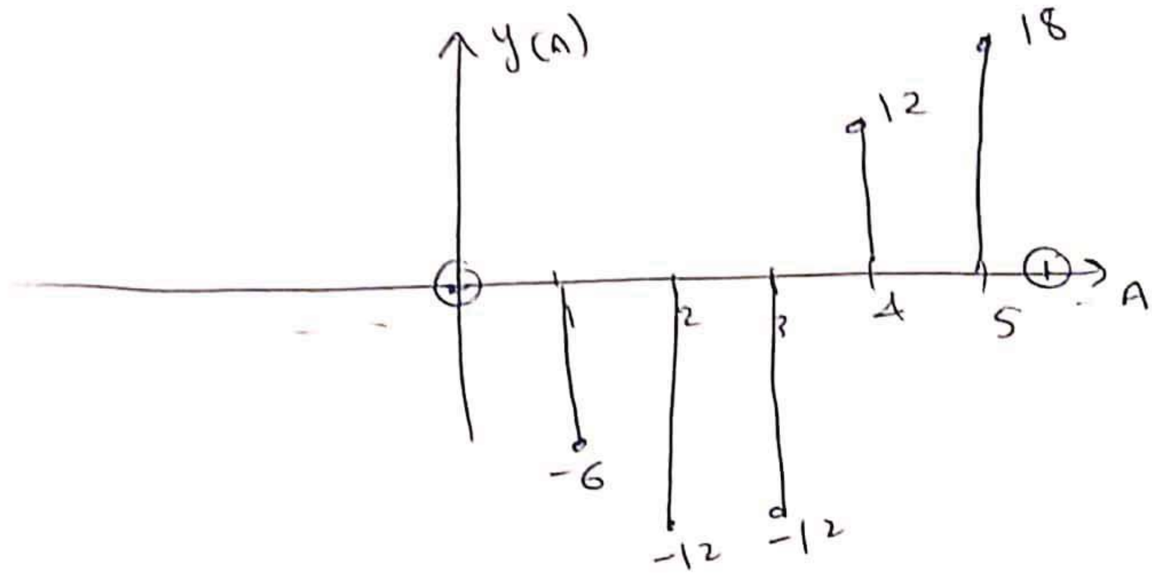


i) $h_{eq}(n) = -2\,\delta(n+1) - 4\,\delta(n) - 4\,\delta(n-1) + 4\,\delta(n-2) + 6\,\delta(n-3)$

ii) $x(n) = 3\,\delta(n-2) \Rightarrow y(n) = x(n) * h(n)$

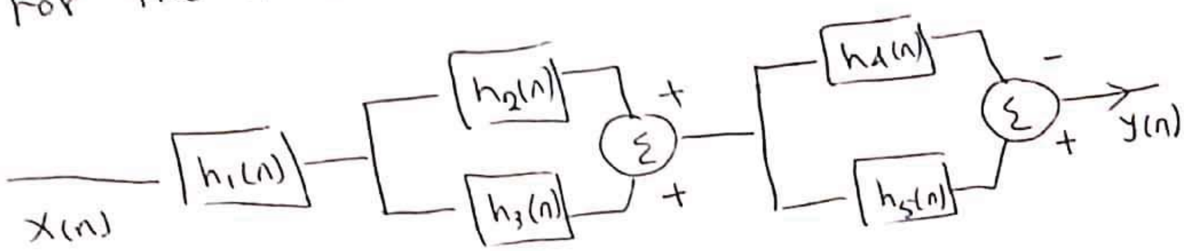
$$y(n) = 3\,\delta(n-2) * \left[-2\,\delta(n+1) - 4\,\delta(n) - 4\,\delta(n-1) + 4\,\delta(n-2) + 6\,\delta(n-3)\right]$$

$$y(n) = -6\,\delta(n-1) - 12\,\delta(n-2) - 12\,\delta(n-3) + 12\,\delta(n-4) + 18\,\delta(n-5)$$



ex:

For the discrete system shown



1) Sketch each Impulse response.
2) Find total Impulse response
3) Is System Stable, memoryless, Causal

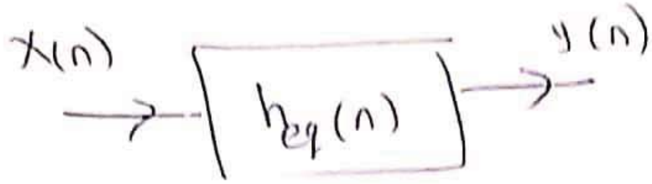
Where $h_1(n) = (3)^n\,[u(n) - u(n-2)]$

$h_2(n) = u(n+2) - u(n-1)$

$h_3(n) = -\delta(n+1) + 2\delta(n-1)$

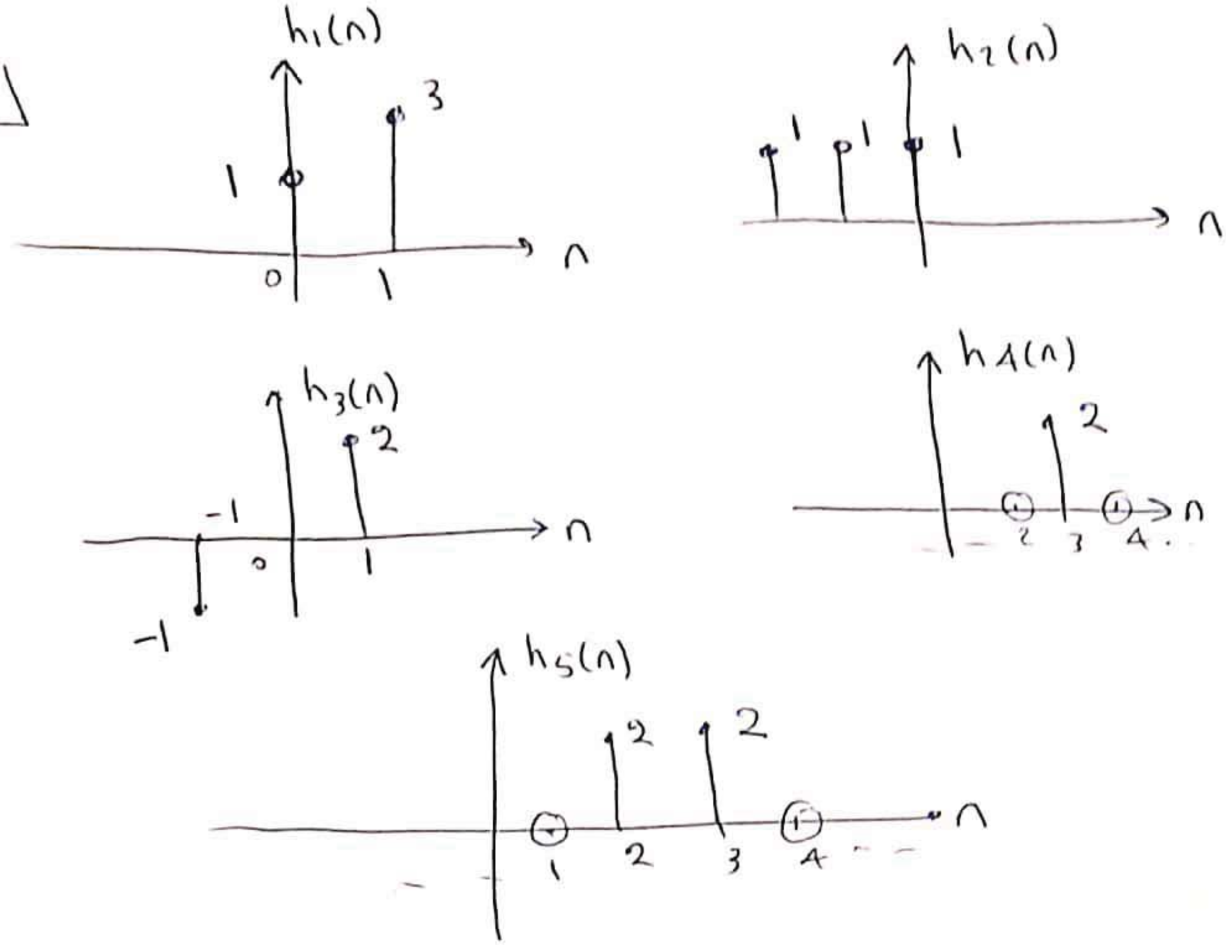
$h_4(n) = 2\delta(n-3)$, $h_5(n) = 2[u(n-2) - u(n-4)]$.

Sol



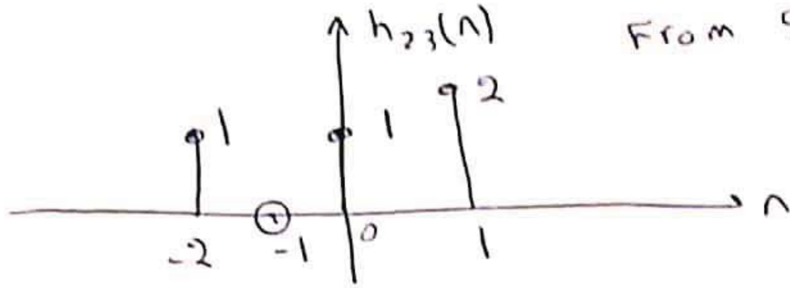
where $h_{eq}(n) = h_1(n) * [h_2(n) + h_3(n)] * [h_5(n) - h_4(n)]$

1



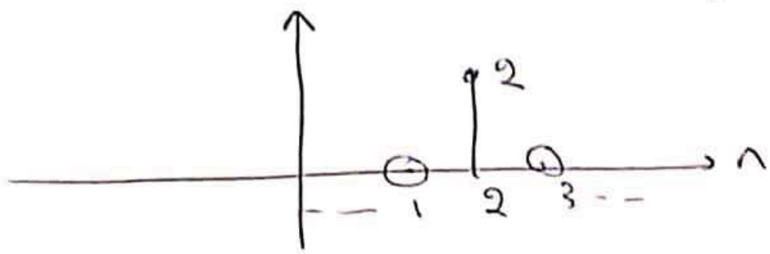
$\therefore$ let $h_{23}(n) = h_2(n) + h_3(n)$

From graphs



let $h_{54}(n) = h_5(n) - h_4(n) = 2\,\delta(n-2)$

From graph



$$\therefore h_{eq}(n) = \underbrace{h_1(n) * h_{23}(n)}_{h_{eq_1}(n)} * \underbrace{h_{54}(n)}_{2\delta}$$

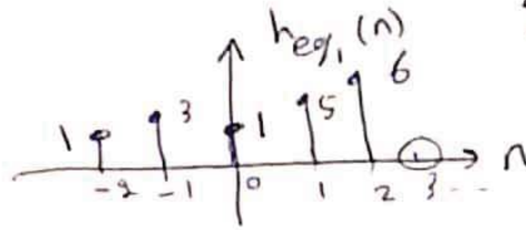
$$h_{eq}(n) = h_{eq_1}(n) * 2\,\delta(n-2) \quad \text{—— (1)}$$

i) $\underline{h_{eq_1}(n)}$:- $h_1(n) * h_{23}(n)$ using table

i) $0+-2 \leq n \leq 1+2 \rightarrow -2 \leq n \leq 2$

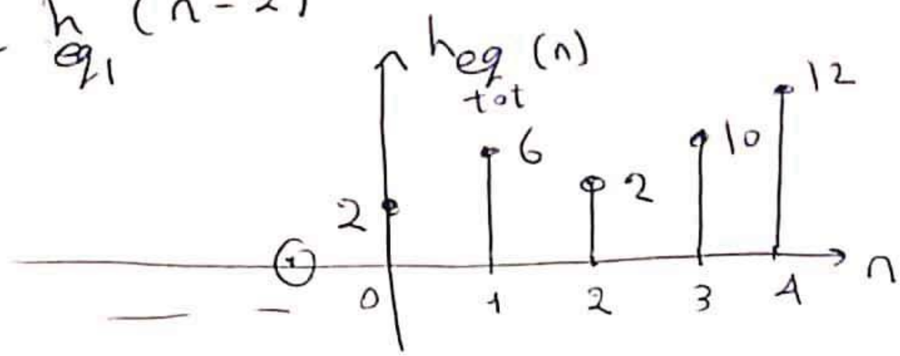
n	-2	-1	0	1	2	$h_1(n)$
$h_{23}(n)$	1	0	1	2		1
		1	0	1	2	3
	1	3	1	5	6	$h_{eq_1}(n)$

$$\therefore h_{eq_1}(n) = \begin{cases} 1, & n=-2 \\ 3, & n=-1 \\ 1, & n=0 \\ 5, & n=1 \\ 6, & n=2. \end{cases}$$



ii) $h_{eq_{tot}}(n) = h_{eq_1}(n) * 2\delta(n-2)$

$$h_{eq_{tot}}(n) = 2\, h_{eq_1}(n-2)$$



$$\therefore h_{eq_{tot}}(n) = 2\delta(n) + 6\delta(n-1) + 2\delta(n-2) + 10\delta(n-3) + 12\delta(n-4)$$

II

* System $\rightarrow$ memory
* System $\rightarrow$ Causal
* $\sum |h_{eq}(n)| = 2+6+2+10+12 = 32 < \infty$

Stable

Example

LTI system with $h(n) = 2\,\delta(n-1)$

Find the Impulse response of the inverse system if exists

Solution

Condition of Invertibility:

$$h(n) * h^{-1}(n) = \delta(n)$$

$$2\,\delta(n-1) * [\ ?\ ] = \delta(n)$$

Recall:

$\delta(n) * x(n) = x(n)$ $\Rightarrow$ $\delta(n) * \delta(n) = \delta(n)$

$\delta(n-n_0) * x(n) = x(n-n_0)$ $\quad$ $\delta(n-1) * \delta(n+1) = \delta(n)$

$$\boxed{\therefore h^{-1}(n) = \frac{1}{2}\,\delta(n+1)}$$

because $\underbrace{2\,\delta(n-1)}_{h(n)} * \underbrace{\frac{1}{2}\,\delta(n+1)}_{h^{-1}(n)} = \delta(n)$