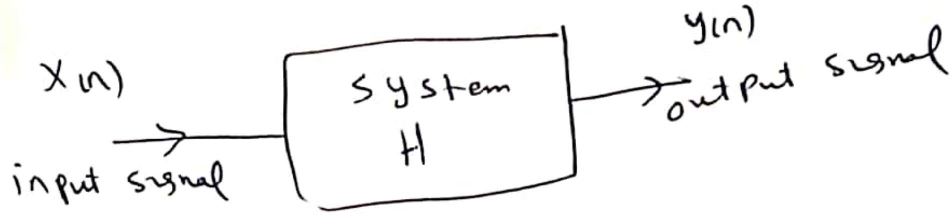
DSP

Systems



$$y(n) = H\{x(n)\}$$

H : operator denoting action of the system on the input

1 Amplitude Scaling

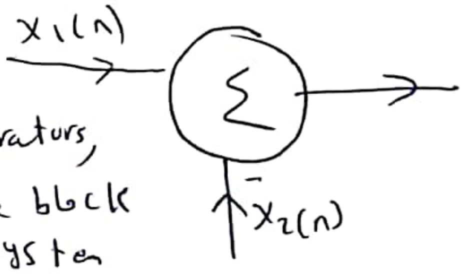
$$y(n) = c\,x(n)$$

2 Time shift

$y(n) = x(n-k)$

S^k = Discrete time shift $S^k[x(n)] = x(n-k)$

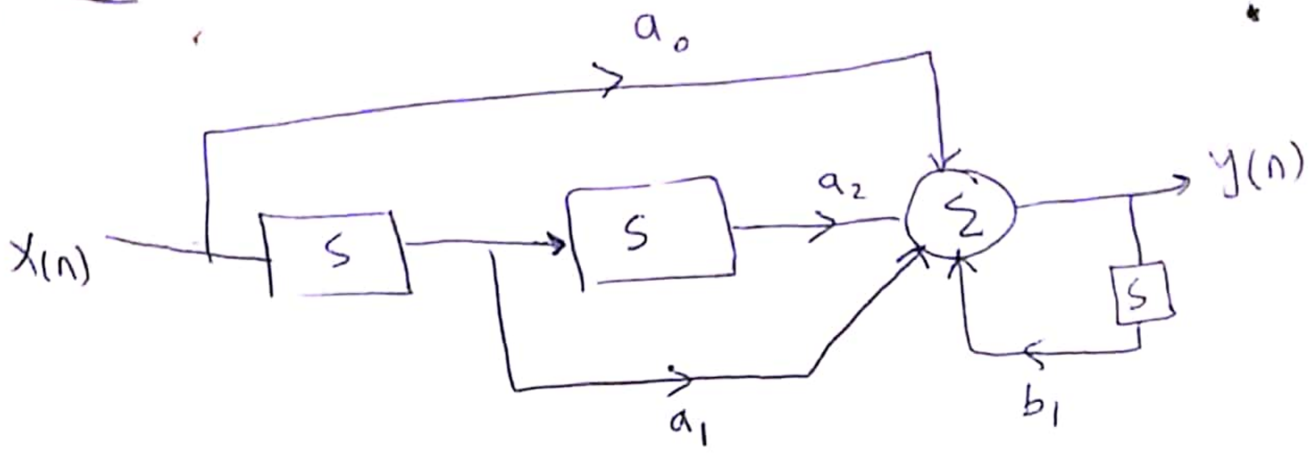
3 Summation / subtraction



$y(n) = x_1(n) - x_2(n)$

Using these operators, we can draw the block diagram of system

ex:



$$y(n) = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2) + b_1 y(n-1)$$

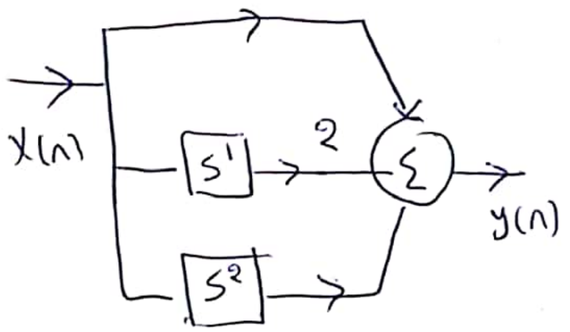
" this Connection is called Cascaded Connection

ex: $y(n) = x(n) + 2x(n-1) + x(n-2)$

Implement this system

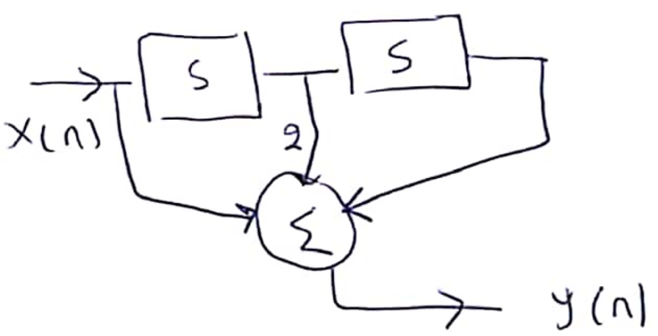
Parallel

Cascaded



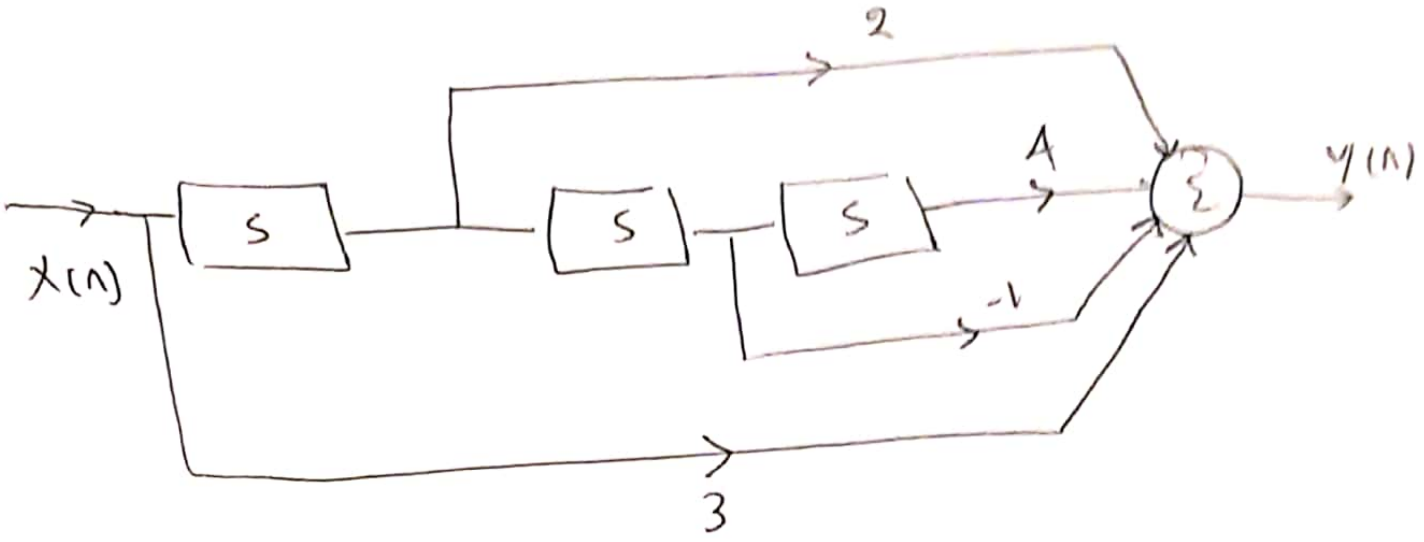
adv: propagation Delay ↓↓

dis adv: expensive



adv: Cheap

dis adv: Propagation delay ↑↑



1) For the shown Block diagram:-

i) Type of Connection ?

ii) Formulate the operator H of this system ?

Sol

i) Cascaded Connection

ii) $y(n) = 3x(n) + 2x(n-1) - x(n-2) + 4x(n-3)$

$$y(n) = \left[3 + 2S^{1} - S^{2} + 4S^{3}\right] x(n)$$

$\therefore$ H = operator of system $= 3 + 2S^{1} - S^{2} + 4S^{3}$

Example

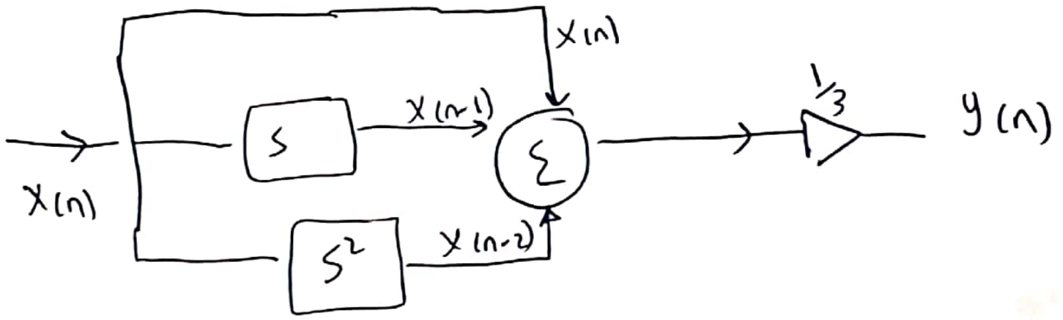
Implement a system whose output signal is the average of the 3 most recent samples

$$y(n) = \frac{1}{3}\left[x(n) + x(n-1) + x(n-2)\right]$$

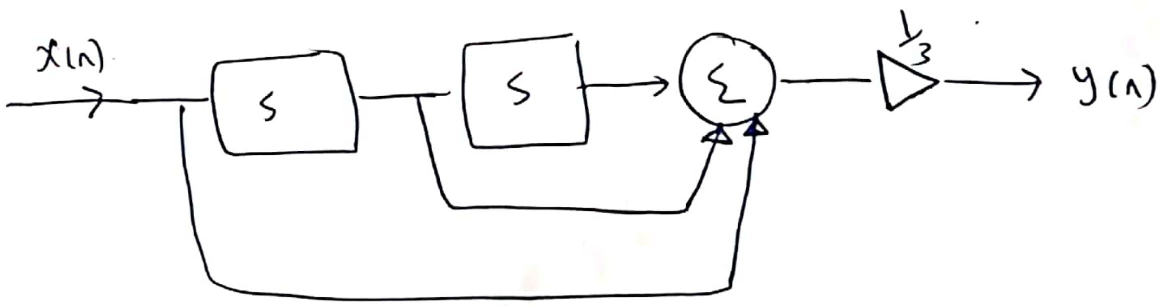
Sol

(i) parallel connection:

$y(n) = \frac{1}{3}\left[x(n) + x(n-1) + x(n-2)\right]$ called Moving Average system



(ii) cascade connection:



Propertues of systems

4 .

1- Stabilty : (BIBO) : Bounded Input Bounded output

System is stable iff every bounded I/P results in bounded O/P.

Steps

1- assume $|x(n)| \leq M_x < \infty$ For all n

maximum value of x(n)

2- get $|y(n)|$

3- if $|y(n)| \leq M_y < \infty \Rightarrow$ stable (y: finite)

max value of y(n)

if $|y(n)| \longrightarrow \infty \Rightarrow$ un stable

where M_x & M_y are +ve finite values

Recall:

$|a+b| \leq |a| + |b|$, $|ab| = |a|\,|b|$

$\boxed{ex}$

$$y(n) = \frac{1}{3}\left[x(n) + x(n-1) + x(n-2)\right]$$

Sol

1- Assume $|x(n)| \leq M_x < \infty$

2- $|y(n)| = \left|\frac{1}{3}\left[x(n) + x(n-1) + x(n-2)\right]\right|$

$$|y(n)| = \frac{1}{3}\,|x(n) + x(n-1) + x(n-2)|$$

$$|y(n)| \leq \frac{1}{3}\,|x(n)| + \frac{1}{3}\,|x(n-1)| + \frac{1}{3}\,|x(n-2)|$$

$$|y(n)| \leq \frac{1}{3}\,[M_x + M_x + M_x]$$

$$|y(n)| \leq M_x < \infty \Longrightarrow \text{Stable.}$$

ex: $y(n) = r^n u(n)\, x(n) \quad , \quad r > 1$

Sol

1- Assume $|x(n)| \leq M_x < \infty$

2- $|y(n)| = |r^n u(n)\, x(n)| = |r^n|\,|x(n)|$

$$= \underbrace{|r^n|}_{\substack{r>1 \\ \text{growing} \\ \to \infty}} \underbrace{M_x}_{\text{Bounded}}$$

$\therefore |y(n)| \longrightarrow \infty$ unstable.

② Memory:

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→ System has a memory IF $y(n)$ depends on Past values of i/p signal.

→ System memoryless IF $y(n)$ depend only on present values of i/p.

exs:

1- $y(n) = \frac{1}{3}[x(n) + x(n-1) + x(n-2)] \Rightarrow$ memory

2- $y(n) = 5x(n) \Rightarrow$ memoryless

3- $y(n) = \sum_{n=-\infty}^{n} x(n) \Rightarrow$ memory ($\text{sum } -\infty \rightarrow n$)

③ Causality:

→ System Causal IF $y(n)$ depends only on present and/or past values of i/p. " لا يوجد $(n+\ldots)$ "

→ System Non Causal IF $y(n)$ depends on Future values of i/p. " يوجد $x(n+\ldots)$ "

exs:

1- $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$

non Causal

2- $y(n) = \frac{1}{3}[x(n) + x(n-1) + x(n-2)]$

Causal

---

④ Invertibility:

$x(n) \longrightarrow \boxed{H} \longrightarrow y(n)$

system (H)

$y(n) \longrightarrow \boxed{H^{-1}} \longrightarrow x(n)$

Inverse system H^{-1}

exs:

1- $x(n) \longrightarrow \underbrace{\boxed{S^{n_0}}}_{H} \longrightarrow y(n) = x(n-n_0)$

$x(n-n_0) \longrightarrow \underbrace{\boxed{S^{-n_0}}}_{H^{-1}} \longrightarrow x(n)$

Invertible

2- ex:

$x(n) \longrightarrow \boxed{(\ )^2} \longrightarrow y(n) = x^2(n)$

$y(n) = x^2(n) \longrightarrow \boxed{\sqrt{\ }} \longrightarrow \pm x(n)$

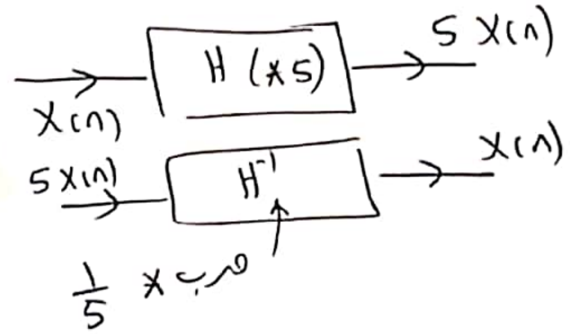
is not invertible!

ex: Is the Following system invertible

1) $y(n) = 5\,x(n)$

$\boxed{\text{Sol}}$

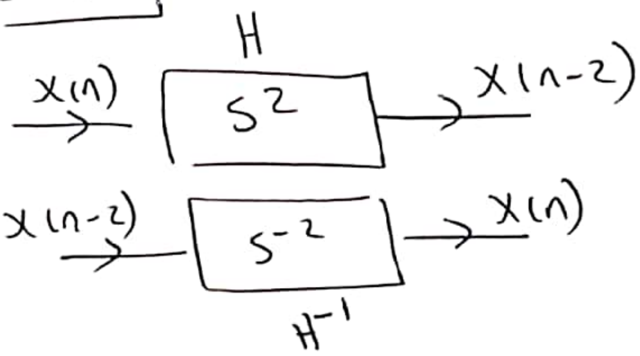
Invertible



2) $y(n) = x(n-2)$

$\boxed{\text{Sol}}$

Invertible

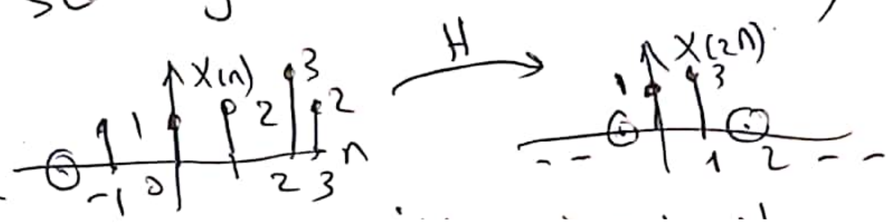


3) $y(n) = x(2n)$

Sol

not Invertible system (because time scaling cancel some samples)

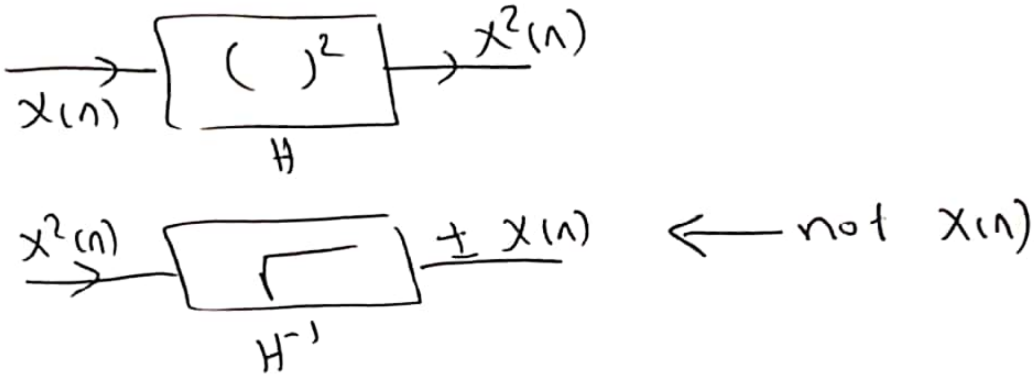
ex:



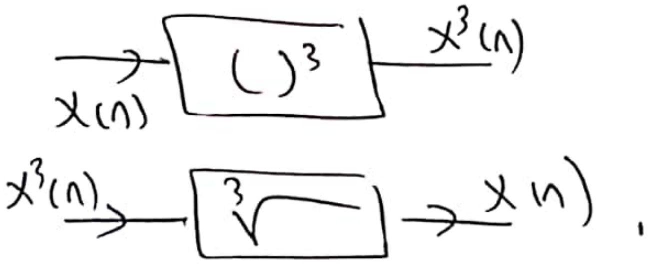
→ $y(n) = 2x(n-2)$ Invertible

→ $y(n) = x(2n-3)$ not Invertible (time scaling)

→ $y(n) = x^2(n)$ [not Invertible]



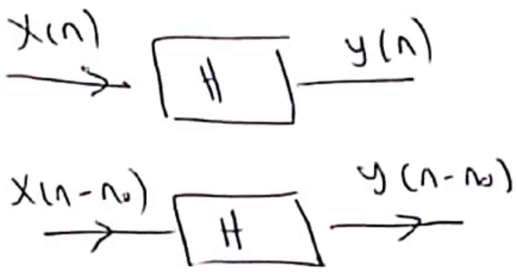
→ $y(n) = x^3(n)$ [Invertible]



(5) Time - Invariance :

→ System time invariance if delay of $x(n)$ leads to same delay in $y(n)$.

i.e:



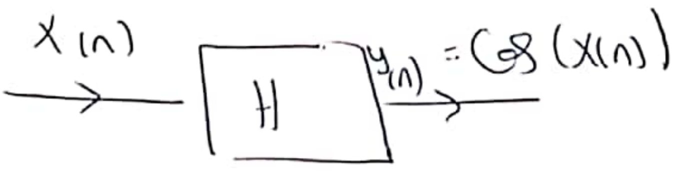
⇒ time invariant

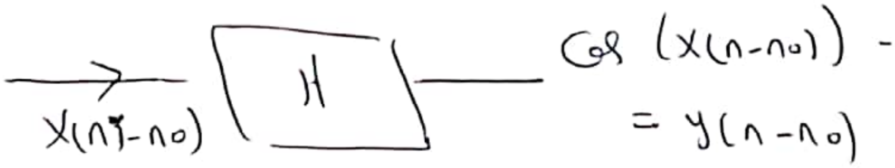
If you find any function of n outside x(n) or y(n) ----------- Time variant

ex: $y(n) = \cos(x(n))$

Sol

Time invariant ⇐ بمجرد النظر.





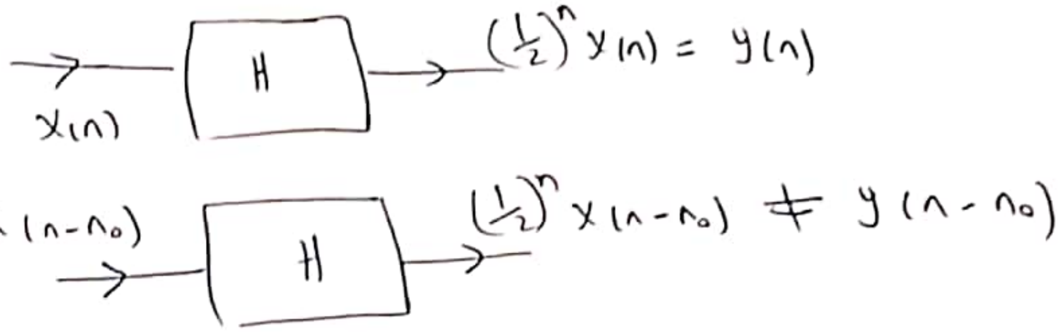
∴ Time invariant

ex:

$$y(n) = \left(\tfrac{1}{2}\right)^n x(n)$$

SoL

There is $\left(\frac{1}{2}\right)^n \Rightarrow$ time variant by inspection.



Time variant.

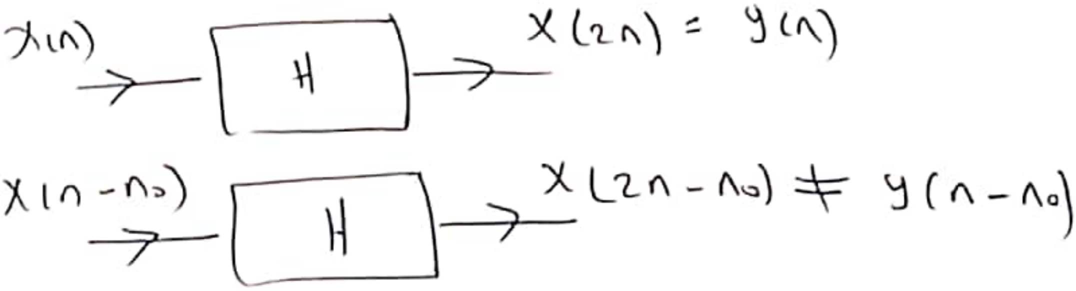
ex:

$$y(n) = x(2n)$$

SoL

integer $*$ n

as time index inside $x(n)$ is "2n" $\Rightarrow$ Time variant



$\therefore$ Time variant.

6 linearity

To check linearity:

IF $H\{x_1(n) + x_2(n)\} = H\{x_1(n)\} + H\{x_2(n)\}$ liner

$\neq H\{x_1(n)\} + H\{x_2(n)\}$ Non liner

$x_1(n) \to \boxed{H} \to y_1(n)$ $\qquad$ $x_2(n) \to \boxed{H} \to y_2(n)$

$x_1(n) + x_2(n) \to \boxed{H} \to y(n)$ $\begin{cases} = y_1(n) + y_2(n) & \text{liner} \\ \neq y_1(n) + y_2(n) & \text{Non liner} \end{cases}$

Summary IF you find non liner operators $\Rightarrow$ Non liner system

IF you Find the following, it is non liner system

1. log, sin, cos, ln, log, | | $\left\{ x(n) \text{ or } y(n) \right\}$
2. $x^2(n)$, $x^3(n)$ --- $x^k(n)$, $y^2(n)$, $y^3(n)$ — —
3. $x(n) \cdot x(n-1)$, $x(n)\, x(n-2)$ — — — — —, $x(n)\, y(n)$ — —
4. $x(n)$ + constant
5. $x(n)$ + function of "n" not $x(n\ldots)$

example

$$y(n) = x(n)\, x(n-1)$$

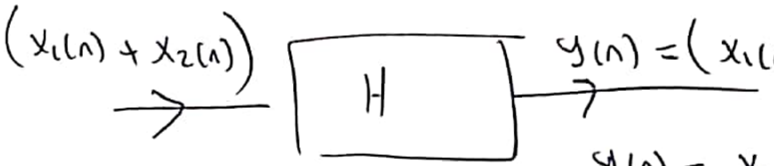
Sol

By inspection $\Rightarrow$ Non liner

why? let's make it

$$y_1(n) = x_1(n)\, x_1(n-1)$$

$$y_2(n) = x_2(n)\, x_2(n-1)$$



$$y(n) = (x_1(n) + x_2(n)) \cdot (x_1(n-1) + x_2(n-1))$$

$$y(n) = x_1(n)\, x_1(n-1) + x_1(n)\, x_2(n-1) + x_2(n)\, x_1(n-1) + x_2(n)\, x_2(n-1)$$

$\therefore$

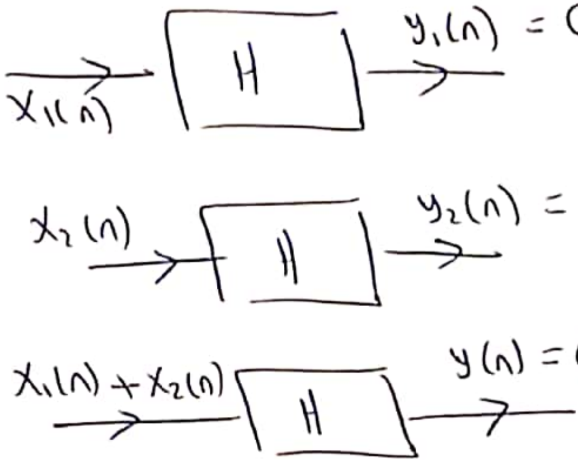
$$y_1(n) + y_2(n) = x_1(n)\, x_1(n-1) + x_2(n)\, x_2(n-1) \neq y(n)$$

$\therefore$ System is Non liner

ex: $y(n) = \cos(x(n))$

Sol

non linear. $\Leftarrow$ بمجرد النظر



$y_1(n) = \cos x_1(n)$

$y_2(n) = \cos(x_2(n))$

$y(n) = \cos(x_1(n) + x_2(n)) \neq y_1(n) + y_2(n)$

$\therefore$ Non linear.

---

ex:

1- $y(n) = 3x(n) + 5 \longrightarrow$ Non linear

2- $y(n) = x^2(n) + 5x(n) \longrightarrow$ Non linear.

3- $y(n) = x(n) \cos n \longrightarrow$ linear

4- $y(n) = \cos(x(n)) \longrightarrow$ non linear.

5- $y(n) = x(n) + \underbrace{\cos n}_{\text{function of "n"}} \longrightarrow$ non linear

6- $y(n) = x(n) + 6 \longrightarrow$ Non linear

7 Passive system

System is passive iff o/p Energy $\leq$ input Energy

$$\sum_{n=-\infty}^{\infty} |y(n)|^2 \leq \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$\therefore$ Energy loss $\Rightarrow$ passive system

8 loss less system

System is loss less if

o/p Energy = input Energy

$$\sum_{n=-\infty}^{\infty} |y(n)|^2 = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

" No Energy loss "

Summary

1 Stability:

$y(n) = ---$ interms of $x(n)$

IF you find **growing** term: $n, n^2, .. n^{\text{Power}>1}, --- (a)^n, a>1$

$\therefore$ System Unstable. Otherwise, Stable

2 Causality:

IF $y(n)$ depends on Future Values of $x(n) \Longrightarrow$ Non Causal

i.e: If you find $x(n+ ---)$ or $x(2n), x(3n) ---$ (integer > 1)

System Non Causal. Otherwise, Causal

3 Memory

If $y(n)$ depends only on $x(n) \Longrightarrow$ Memory less

If you find $x(n \pm --)$ or $x(\text{Const.} n) ---$

$\Downarrow$

Memory

4 Time Varince

→ If you find function of "n" outside $x(n)$ & $y(n)$

or if you find multiplication of "n" inside $x(n)$

⇓

Time variant

ex:

$y(n) = \underline{\cos n}\ x(n)$ Time variant

$y(n) = x(2n)$ Time variant

$y(n) = x(-n)$ Time variant

$y(n) = x(2n-1)$ Time variant

5 linerity

The system is Non liner, if you find the following:

i) $\left.\begin{matrix}\cos \\ \sin \\ \log \\ \ln \\ |\ |\end{matrix}\right\} \{ x(n) \text{ or } y(n) \}$

ii) $x^2(n)$, $x^3(n)$ — —
$y^2(n)$, — — —

iii) $x(n)\,x(n-1)$, $x(n)\,x(n-2)$ — —
$y(n)\,x(n)$, — — —
$y(n)\,y(n-1)$, — — —

iv) $x(n)$ + constant ex: $y(n) = x(n) + 5$

v) $x(n)$ + function of "n" ex: $y(n) = x(n) + \cos n$

---

6 Invertibility

→ Time scaling, $(\ )^2$, $(\ )^{-1}$, - - -
" Non invertible "

→ Amplitude scaling, time shift ⇒ invertible

ex:

$y(n) = 2\,x(n)\,u(n-1)$, Is this system linear, memory, Causal, Stable, - - ?

Sol

1- Time variant $\Rightarrow$ يوجد دالة في (n) بره $x(n)$
وهي $u(n-1)$

2- Causal

4- Linear

5- To check stability

Assume $|x(n)| \leq M_x < \infty$

$$|y(n)| = |2\,x(n)\,u(n-1)|$$

$$= 2\,|\underbrace{\cancel{x(n)}}_{M_x}|\,|\underbrace{\cancel{u(n-1)}}_{1}|$$

$$|y(n)| = 2M_x < \infty$$

$\therefore$ Stable

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[exs] 17

1) $y(n) = x(2n-3)$

Check all system properties ?

Sol

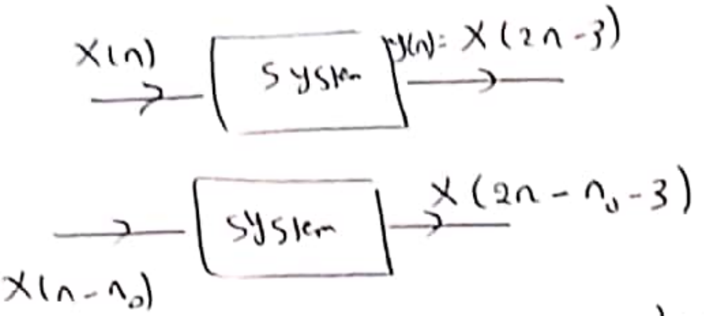
1 → Stable

is Assume $|x(n)| \le M_x$

$|y(n)| = |x(2n-3)| \le M_x$

2 → Linear

3 → Time Variant



$x(2n-n_0-3) \neq y(n-n_0)$

because $y(n-n_0) = x(2(n-n_0)-3)$

$= x(2n-2n_0-3)$

4 → Memory

5 → not invertible

6 → Non Causal

ex

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(2) $y(n) = x(n-1) \cdot x(n)$

Check all properties ?

Sol

1 ⟶ Stable

2 ⟶ memory

3 ⟶ Causal

4 ⟶ Non linear

5 ⟶ time invariant

6- ⟶ not invertible .

Additional

examine the system properties [Causal or not, instantaneous or Dynamic, Linear or not, Stable or not, Time variant or Time invariant]

① $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

→ Dynamic (Memory) لأن $y(n)$ تعتمد على قيم x لـ $-\infty$

→ Linear

→ time invariant

→ Non Causal — لأن الـ summation حتى $(n+1)$
أي $y(n)$ تعتمد على $x(n+1)$

→ Unstable.

$$y(n) = \sum_{k=-\infty}^{n+1} x(k) \quad \text{as } n \to \infty \Rightarrow y(n) \to \infty$$

---

② $y(n) = x(n)\cos(\omega_0 n)$

→ inst (Memoryless), Linear, time variant, Causal, Stable

③ $y(n) = x(-n+2)$

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Sol

$y(n) = x(-(n-2))$

→ Dynamic (Memory)

→ Linear

→ time varient

→ Non causal

لأنه مثلاً لو $n=0$ فيكون $y(0) = x(2)$

→ Stable.

---

④ $y(n) = x(2n)$

→ Dynamic (Memory) لأنه مثلاً $y(1)$ تعتمد على $x(2)$ وهكذا

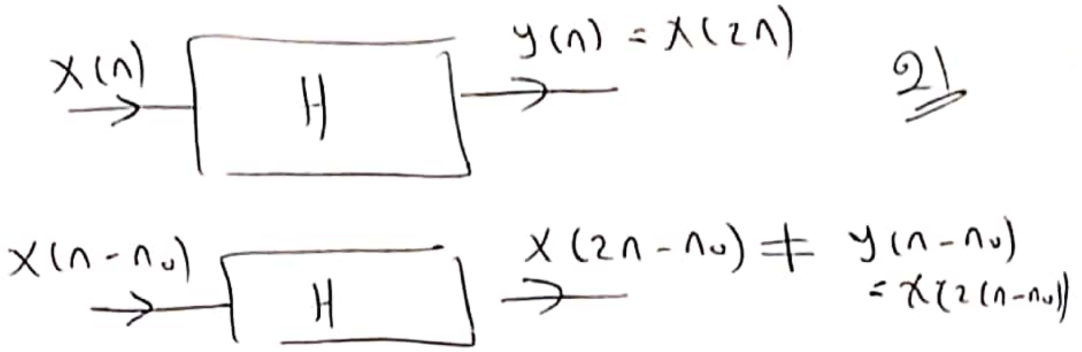
→ Linear

→ Non Causal لأنه $y(1)$ تعتمد على $x(2)$

→ Stable

→ Time varient ←

because



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(5) $y(n) = x(-n)$

→ Dynamic (Memory)
→ Linear
→ time variant
→ Non Causal
→ Stable.

(6) $y(n) = x(n) + n\,x(n+1)$

→ Dynamic
→ Linear
→ time variant
→ Non Causal
→ Un Stable because

IF $x(n)$ bounded
$n \rightarrow \infty$
$y(n) \rightarrow \infty$

(7) $y(n) = |x(n)|$

Sol

$$y(n) = \begin{cases} x(n), & n \geq 0 \\ -x(n), & n < 0 \end{cases}$$

→ instantenous (Memory less), Causal

→ time invarent, stable

→ Non Linear

لأن الـ | | لا يتوزع على الجمع

$|x_1 + x_2| \neq |x_1| + |x_2|$

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(8) $y(n) = 2\,x(2^n)$

Sol

→ Stable

[becase assume $|x(n)| < M_x < \infty$

$|y(n)| = |2\,x(--)| = 2M_x < M_y < \infty$

Stable

→ memory [time index is multiplied

→ time Varient [// //]

→ Non Causal [// //]

→ Linear system

(9) $y(n) = \cos(2\pi\, x(n+1)) + x(n)$

Sol

→ Stable [Assume $|x(n)| < M_x < \infty$

$|y(n)| = |\cos(--) + x(n)|$

$\leq |\cos(--)| + |x(n)|$

$\leq 1 + M_x < M_y < \infty$

→ Non Causal [$x(n+--)$]

→ Non linear [$\cos(--x(n))$]

→ time invarient

(10) $$y(n) = x(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

Sol

$$y(n) = x(n) [\cdots \delta(n+6) + \delta(n+4) + \delta(n+2) + \delta(n) + \delta(n-2) + \delta(n-4) \cdots]$$

$$y(n) = \begin{cases} x(n) \cdot [1], & n = \cdots, -6, -4, -2, 0, 2 \cdots \\ x(n) \cdot [0] = 0, & n = \cdots -3, -1, 1, 3 \cdots \end{cases}$$

$$\boxed{y(n) = \begin{cases} x(n), & n: \text{even} \\ 0, & n: \text{odd} \end{cases}}$$

→ Memoryless

→ Stable

→ Causal

→ Linear

→ Time Variant [لو حدث shift يغير اماكن الsamples والناتج يختلف خالص]

→ Non Invertible

[حيث نضيع الsamples اللى عند n = odd ب zero] فلا نعرف نرجعها.