Design of IIR Digital Filters

using impulse invariance method & Step Invariance

→ There are 2 more methods to convert Analog filter $H(s)$ to Digital Filter $H(z)$.

- → Impulse invariance
- → Step Invariance
- → Bilinear Transformation (Done ✓)

→ In order to study the 2 methods of Impulse invariance & step invariance, we need to review Laplace table.

$f(t)$	$F(s)$
K	$\frac{K}{s}$
$u(t)$	$\frac{1}{s}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$e^{at}u(t)$	$\frac{1}{s-a}$

Recall

$$Z\{a^n u(n)\} = \frac{z}{z-a}, \ |z|<a$$

I Design of IIR Digital Filter using Impulse Invariance method

In this method, we keep $h(t)$ the same when converting from Analog to Digital Domain

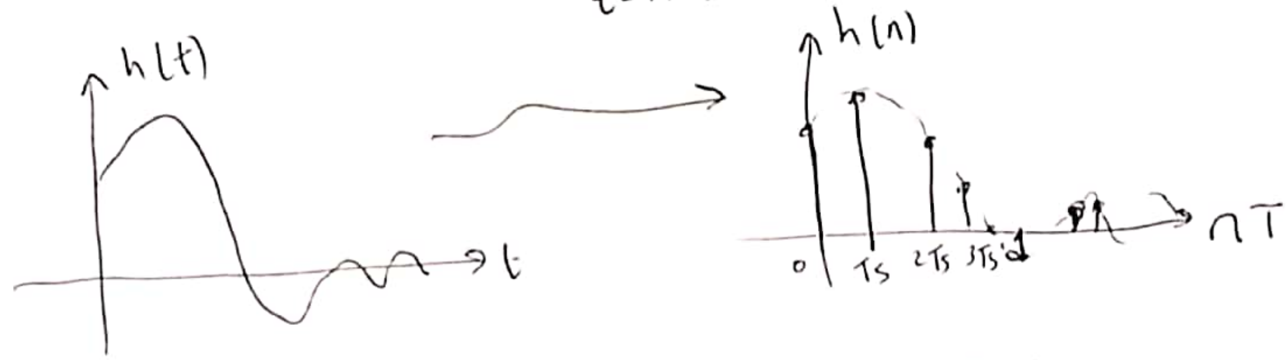
(Just $t \longrightarrow nT_s$) , T_s : Sampling time

Block diagram of Impulse Invariance

$H(s) \rightarrow$ [Laplace inverse] $\xrightarrow{h(t)}$ [Sampling $t \rightarrow nT_s$] $\xrightarrow{h(n)}$ [Z-Transform] $\rightarrow H(z)$

Steps:

1) given $H(s)$ for analog filter
2) get $h(t) = L^{-1}\{H(s)\}$ → impulse response for Analog Filter.
3) Sampling $h(n) = h(t)\Big|_{t=nT_s}$



4) get Transfer Function $H(z)$ for Digital Filter $H(z) = Z\{h(n)\}$
5) Implementation ⟹ usually use parallel

Parallel مستمر، دايماً في مشاكله

أسهل

Q.

Design using Impulse invariance method a digital IIR Filter starting from Analog Transfer Function

$$H(s) = \frac{2}{s^2 + 3s + 2} \quad , \text{Sampling Freq} = 10\,Hz$$

Sol

1) $h(t) = L^{-1}\{H(s)\}$ by partial fractions

$$h(t) = L^{-1}\left\{\frac{2}{(s+1)(s+2)}\right\} = L^{-1}\left\{\frac{A}{s+1} + \frac{B}{s+2}\right\}$$

$$A = \left.\frac{2}{s+2}\right|_{s=-1} = 2 \quad , \quad B = \left.\frac{2}{s+1}\right|_{s=-2} = -2$$

$$h(t) = L^{-1}\left\{\frac{2}{s+1} - \frac{2}{s+2}\right\} = 2e^{-t}u(t) - 2e^{-2t}u(t)$$

$$\boxed{h(t) = 2e^{-t} - 2e^{-2t} \, , t > 0}$$

2) Sampling : $t \longrightarrow nT_s \, , \; T_s = \frac{1}{f_s} = \frac{1}{10} = 0.1$

$t \longrightarrow 0.1\,n$

$$h(n) = h(t)\Big|_{t=0.1n} = 2e^{-0.1n} - 2e^{-0.2n} \quad , n > 0$$

$$\boxed{h(n) = 2e^{-0.1n}u(n) - 2e^{-0.2n}u(n)}$$

(3) $H(z) = Z\{h(n)\}$

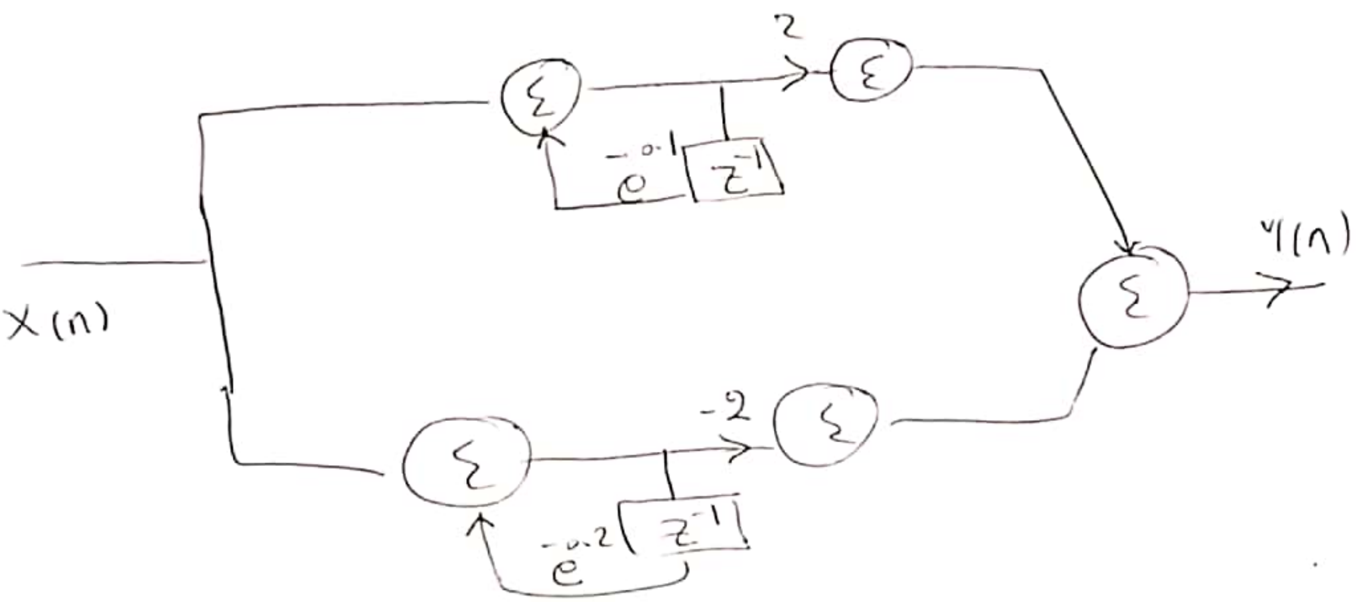
$$= Z\left\{2\left(e^{-0.1}\right)^n u(n) - 2\left(e^{-0.2}\right)^n u(n)\right\}$$

$$H(z) = \frac{2z}{z - e^{-0.1}} - 2\,\frac{z}{z - e^{-0.2}}$$

(4) Implementation : parallel الأول

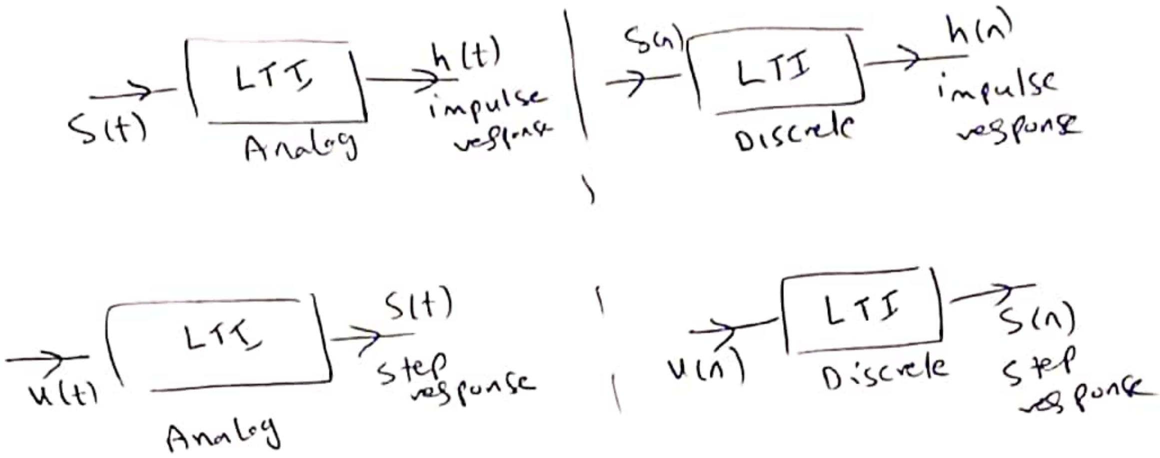
طالما مجموعتين ، [لازم نخليها على الصورة] نضرب $\frac{z^{-1}}{z^{-1}}$

$$H(z) = \frac{2}{1 - e^{-0.1}z^{-1}} - 2\,\frac{1}{1 - e^{-0.2}z^{-1}}$$



2 Design of IIR digital filter using step invariance method

<u>Recall:</u>



→ In this method, we keep the step response $s(t)$ the same when converting from <u>analog</u> to <u>Digital</u>

→ <u>Recall:</u> $s(t) = h(t) * u(t)$

$\downarrow$ Laplace

$$S(s) = H(s) \cdot L\{u(t)\} = H(s) \cdot \frac{1}{s}$$

$$S(s) = \frac{H(s)}{s}$$

→ <u>Similarly in Digital:</u>

$$s(n) = h(n) * u(n)$$

$\downarrow$ Z-Transform

$$S(z) = H(z) \cdot Z\{u(n)\} = H(z) \cdot \frac{z}{z-1}$$

$$\therefore H(z) = \left(\frac{z-1}{z}\right) S(z)$$

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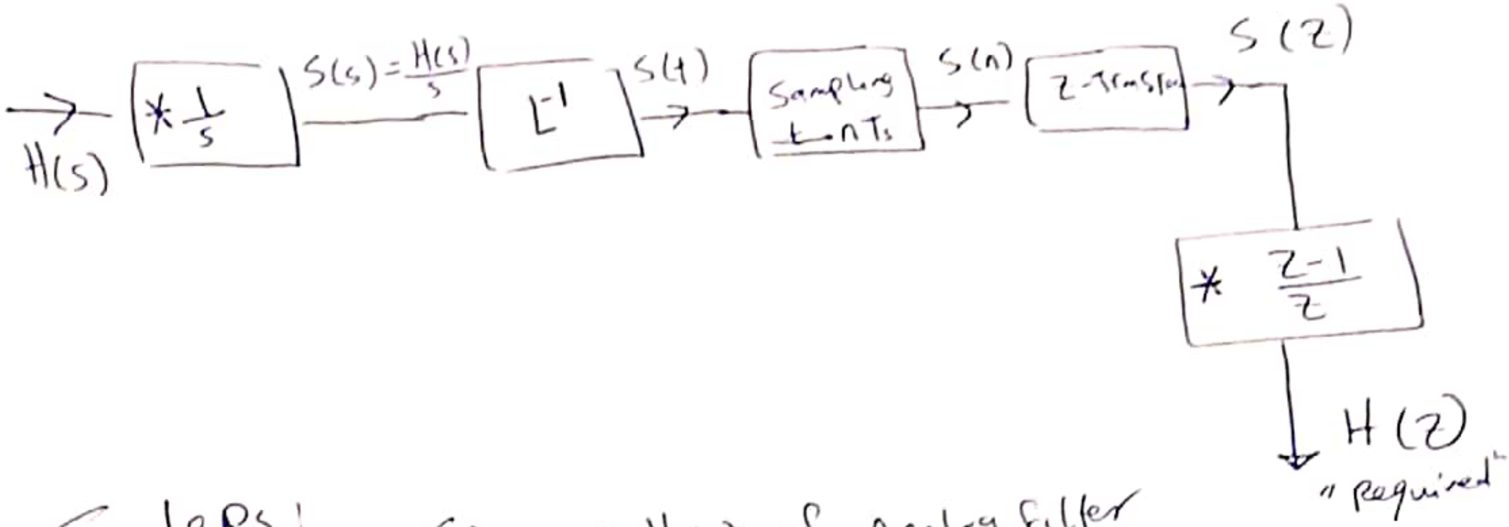
Block diagram of step invariance method



Steps: Given: H(s) of Analog filter

1) Given: $H(s) = \checkmark$

2) get $S(s) = \dfrac{H(s)}{s}$

3) get $S(t) = L^{-1}\{S(s)\} = L^{-1}\left\{\dfrac{H(s)}{s}\right\}$

4) get $S(n) = S(t)\Big|_{t \to nT_s}$, T_s : Sampling time

5) Get $S(z) = Z\{S(n)\}$

6) get $H(z) = S(z) \cdot \dfrac{z-1}{z}$

Implement it using parallel realization

ex

Design using the step invariance method a digital filter IIR starting from

$$H(s) = \frac{2}{s^2+3s+2} \quad , \quad f_s = 10\,Hz$$

Sol

1) get $S(s) = \frac{H(s)}{s} = \frac{2}{s(s^2+3s+2)}$

$$S(s) = \frac{2}{s(s+1)(s+2)}$$

2) get $s(t) = L^{-1}\{S(s)\}$

$$s(t) = L^{-1}\left\{ \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} \right\}$$

$$A = \left.\frac{2}{(s+1)(s+2)}\right|_{s=0} = \frac{2}{2} = 1$$

$$B = \left.\frac{2}{s(s+2)}\right|_{s=-1} = \frac{2}{(-1)(1)} = -2$$

$$C = \left.\frac{2}{s(s+1)}\right|_{s=-2} = 1$$

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$$s(t) = L^{-1}\left\{ \frac{1}{s} + \frac{-2}{s+1} + \frac{1}{s+2} \right\}$$

$$= 1 - 2\bar{e}^{t} + \bar{e}^{2t} \quad , \quad t > 0$$

4) $$s(n) = s(t)\Big|_{t = nT_s = 0.1n} = 1 - 2\bar{e}^{0.1n} + \bar{e}^{0.2n} \quad , \quad n > 0$$

5) $$s(z) = Z\left\{ u(n) - 2\left(\bar{e}^{0.1}\right)^n u(n) + \left(\bar{e}^{0.2}\right)^n u(n) \right\}$$

$$s(z) = \frac{z}{z-1} - 2\,\frac{z}{z - \bar{e}^{0.1}} + \frac{z}{z - \bar{e}^{0.2}}$$

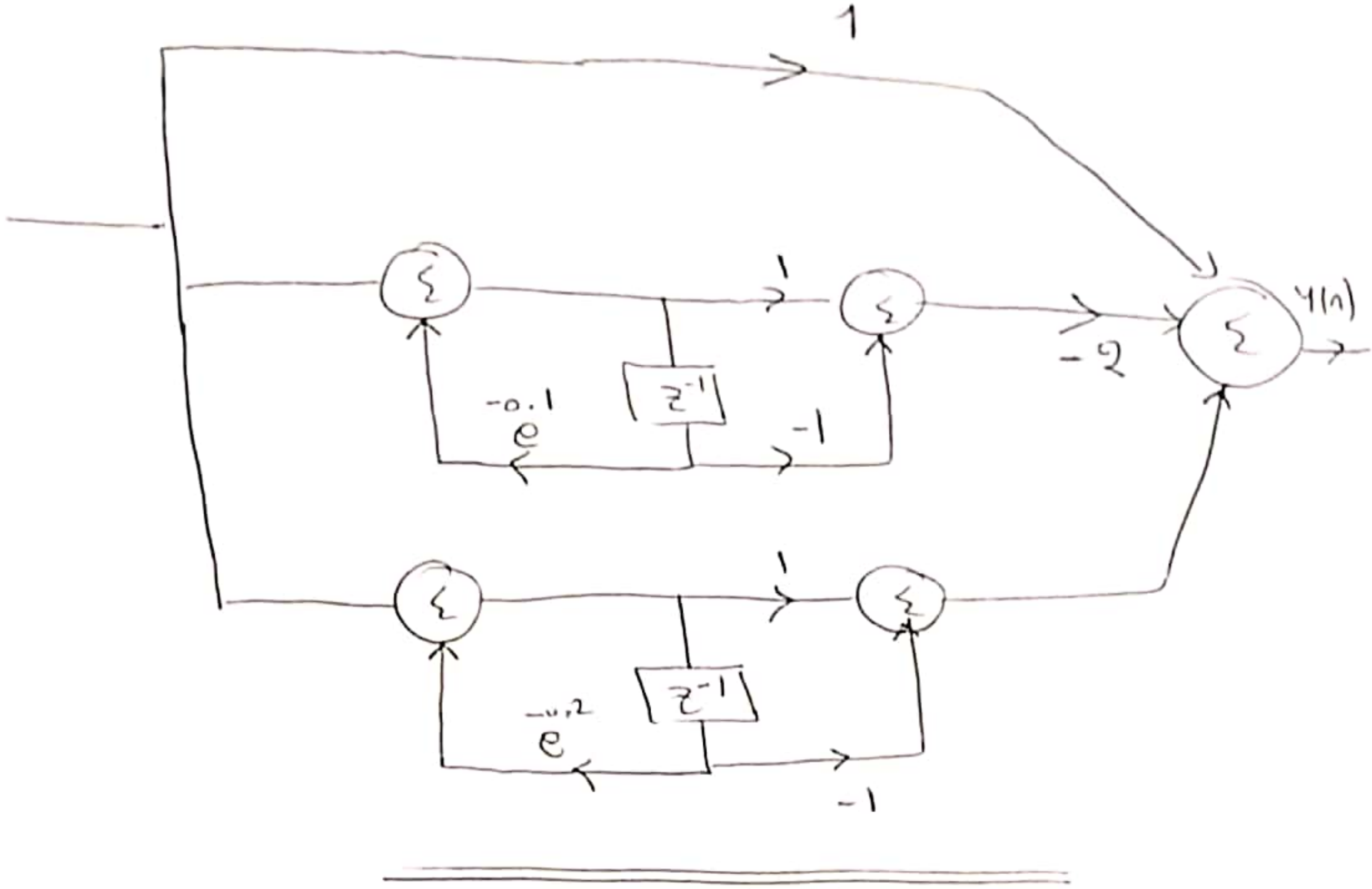
6) $$H(z) = s(z) \cdot \frac{z-1}{z}$$

$$H(z) = \left[1 - 2\,\frac{(z-1)}{(z - \bar{e}^{\,1})} + \frac{(z-1)}{z - \bar{e}^{0.2}} \right]$$

7) Implementation $\longrightarrow$ parallel

نخليها كل الصور، نضرب $\frac{z^{-1}}{z^{-1}}$

$$H(z) = 1 - \frac{2(1 - z^{-1})}{(1 - \bar{e}^{0.1} z^{-1})} + \frac{1 - z^{-1}}{1 - \bar{e}^{0.2} z^{-1}}$$


1
1
-2
y(n)
z⁻¹
e⁻⁰·¹
-1
1
z⁻¹
e⁻⁰·²
-1

Design IIR digital filter using impulse invariance method begining with a prototype

$$H(s) = \frac{1}{s^3 + 9s^2 + 26s + 24}$$

Sampling Frequency = 10 Hz

→ repeat using step invariance method

Sol

I Impulse invariance methode:

Steps

1- given: $H(s) = \dfrac{1}{s^3 + 9s^2 + 26s + 24}$

نحلل المقام بـ Calculator فينتج $-1, -2, -3$

2- $h(t) = L^{-1}\{H(s)\} = L^{-1}\left\{ \dfrac{1}{(s+1)(s+2)(s+3)} \right\}$

$$h(t) = L^{-1}\left\{ \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3} \right\}$$

$$A = \frac{1}{(s+2)(s+3)}\Bigg|_{s=-1} = \frac{1}{(-2)(-1)} = \frac{1}{2}$$

$$B = \frac{1}{(s+1)(s+3)}\Bigg|_{s=-2} = \frac{1}{(2)(1)} = \frac{1}{2}$$

$$C = \frac{1}{(s+1)(s+2)}\Bigg|_{s=-3} = \frac{1}{1(-1)} = -1$$

$$h(t) = \frac{1}{2}e^{-1t} + \frac{1}{2}e^{-2t} - e^{-3t}, \quad t>0$$

3- get $h(n) = h(t)\Big|_{t=nT_s=0.1n}$, $T_s = \frac{1}{f_s} = \frac{1}{10} = 0.1$

$$h(n) = \frac{1}{2}e^{-0.1n} + \frac{1}{2}e^{-0.2n} - e^{-0.3n}, \quad n>0$$

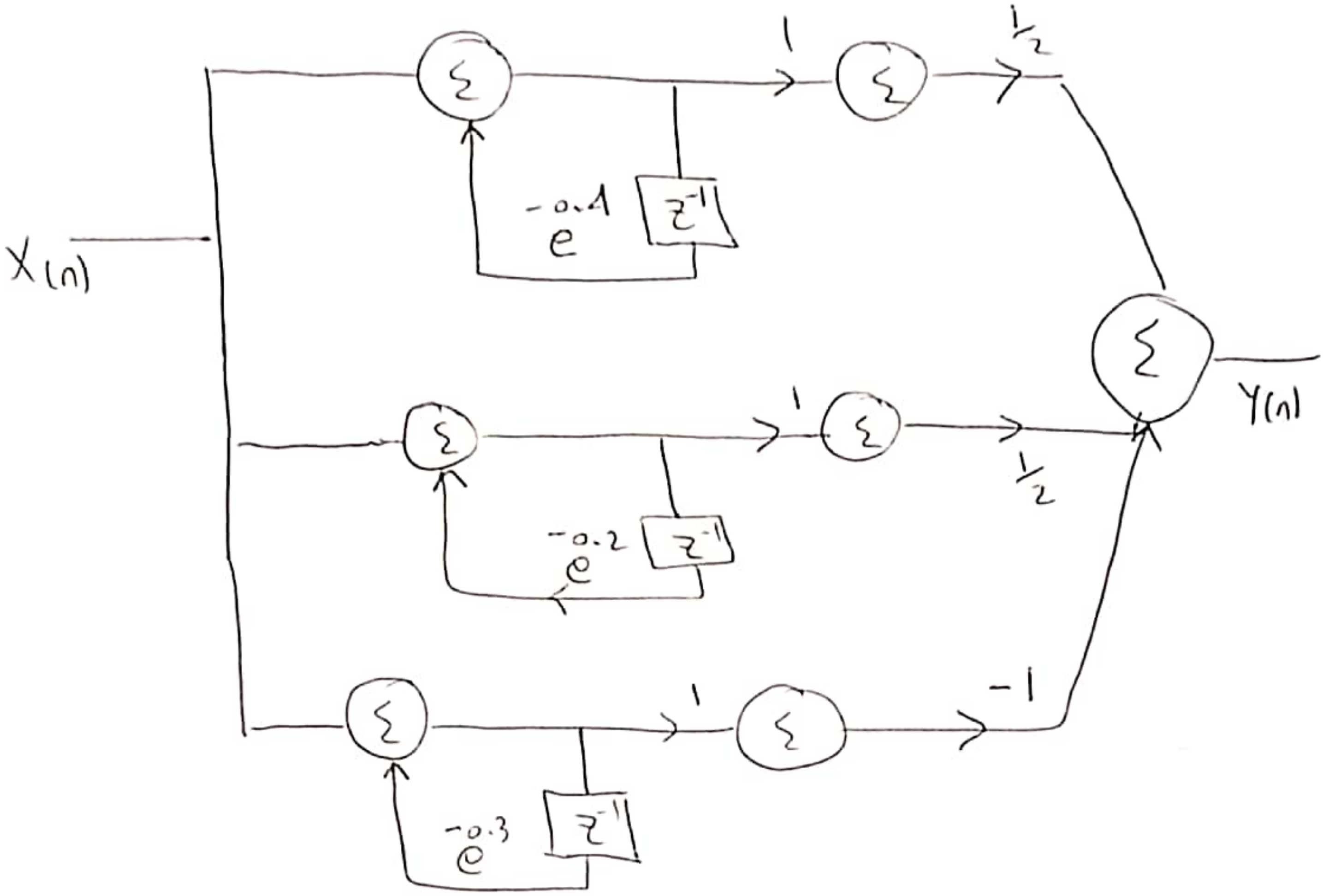
4- get $H(z) = Z\left\{ \frac{1}{2}\left(e^{-0.1}\right)^n u(n) + \frac{1}{2}\left(e^{-0.2}\right)^n u(n) - \left(e^{-0.3}\right)^n u(n) \right\}$

$$H(z) = \frac{1}{2}\frac{z}{z-e^{-0.1}} + \frac{1}{2}\frac{z}{z-e^{-0.2}} - \frac{z}{z-e^{-0.3}}$$

5- Implementation using parallel

- [$\frac{z^{-1}}{z^{-1}}$ بنضرب]

$$H(z) = \frac{1}{2}\,\frac{1}{1-e^{-0.1}z^{-1}} + \frac{1}{2}\,\frac{1}{1-e^{-0.2}z^{-1}} - \frac{1}{1-e^{-0.3}z^{-1}}$$



(II) Step Invariance:

Steps:

1) get $S(s) = \frac{H(s)}{s} = \frac{1}{s(s+2)(s+3)(s+4)}$

2) $s(t) = L^{-1}\left\{ \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3} + \frac{D}{s+4} \right\}$

$A = \frac{1}{2 \times 3 \times 4} = \frac{1}{24}$, $B = \frac{1}{(-2)(1)(2)} = \frac{-1}{4}$

$C = \frac{1}{(-3)(-1)(1)} = \frac{1}{3}$, $D = \frac{1}{(-4)(-2)(-1)} = \frac{-1}{8}$

$s(t) = \frac{1}{24} - \frac{1}{4} e^{-2t} + \frac{1}{3} e^{-3t} - \frac{1}{8} e^{-4t}$, $t > 0$

3) get $s(n)$ $[\ \cancel{t} \longrightarrow n T_s = 0.1 n\ ]$

$s(n) = \frac{1}{24} - \frac{1}{4} e^{-0.2n} + \frac{1}{3} e^{-0.3n} - \frac{1}{8} e^{-0.4n}$, $n > 0$

4) $s(z) = z\left\{ \frac{1}{24} u(n) - \frac{1}{4} \left(e^{-0.2}\right)^n u(n) + \frac{1}{3} \left(e^{-0.3}\right)^n u(n) - \frac{1}{8} \left(e^{-0.4}\right)^n u(n) \right\}$

$s(z) = \frac{1}{24} \frac{z}{z-1} - \frac{1}{4} \frac{z}{z - e^{-0.2}} + \frac{1}{3} \frac{z}{z - e^{-0.3}} - \frac{1}{8} \frac{z}{z - e^{-0.4}}$

⑤ $H(z) = S(z) \cdot \dfrac{z-1}{z}$

$$H(z) = \frac{1}{24} - \frac{(z-1)}{(z - e^{-0.2})} + \frac{1}{3}\,\frac{z-1}{z-e^{-0.3}} - \frac{1}{8}\,\frac{z-1}{z-e^{-0.}}$$

⑥ Implementation ⟶ parallel. $\left[\dfrac{z^{-1}}{z^{-1}} \right.$ نضرب $\left. \right]$

$$H(z) = \frac{1}{24} - \frac{1 - z^{-1}}{(1 - e^{-0.2} z^{-1})} + \frac{1}{3}\,\frac{1 - z^{-1}}{1 - e^{-0.3} z^{-1}} - \frac{1}{8}\,\frac{1 - z^{-1}}{1 - e^{-0.}}$$