Digital Signal Processing (DSP)

Lecture 1
Analog to Digital Converter

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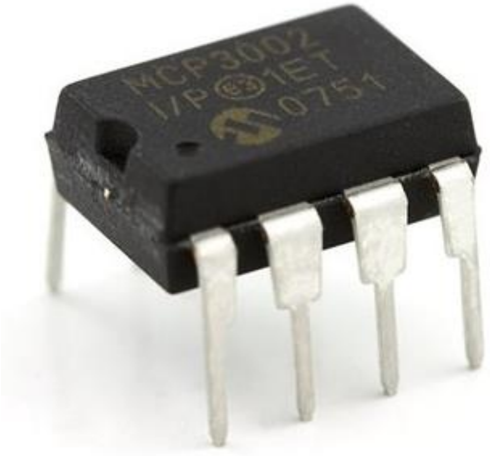
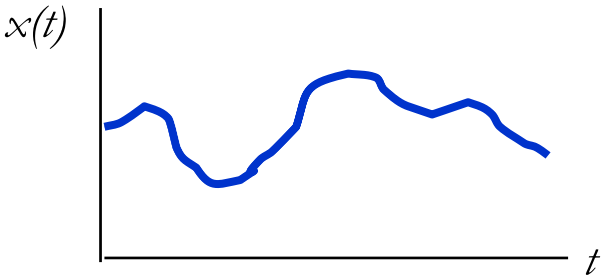
Analog-to-Digital Converter (ADC)



Type of signals

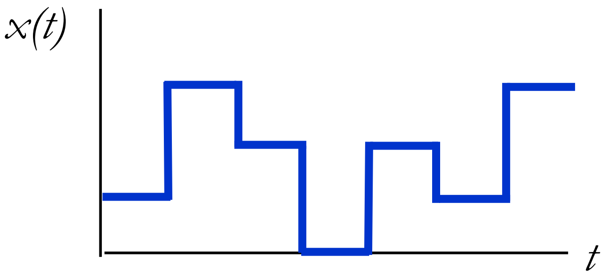
- Analog signals
  - Value varies continuously over a continuous range
  - Examples of analog data

    Video, Audio

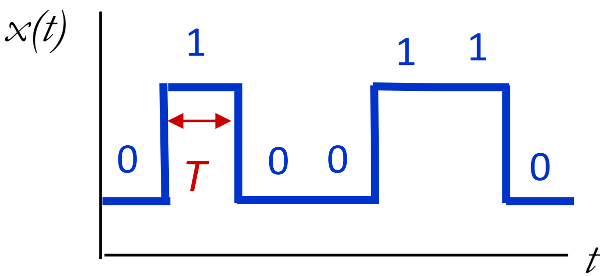


- Quantized signals
  - Value limited to a finite set
  - Examples of digital data

  Text: printed English language (26 letters, 10 numbers, space, and punctuation)

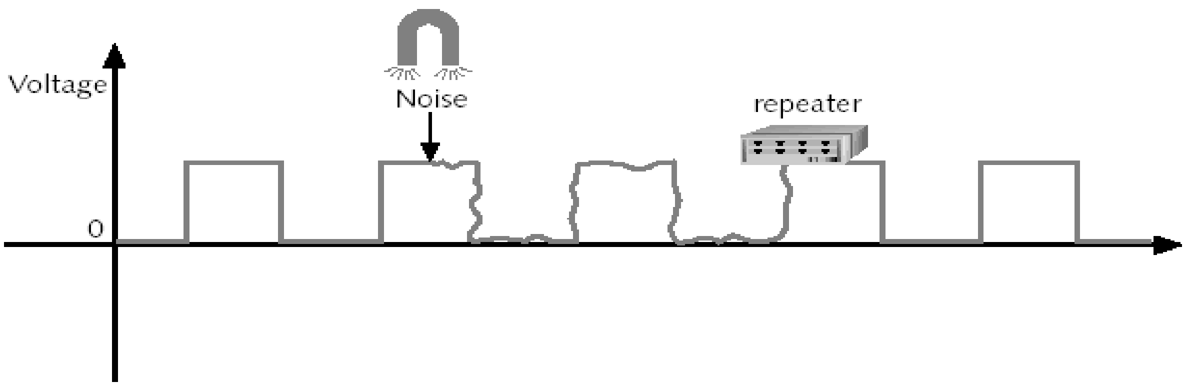


- Binary/Digital signals
  - Has at most 2 values (on and off)
  - Used to represent bit values
  - Computers can only perform processing on digitized signals

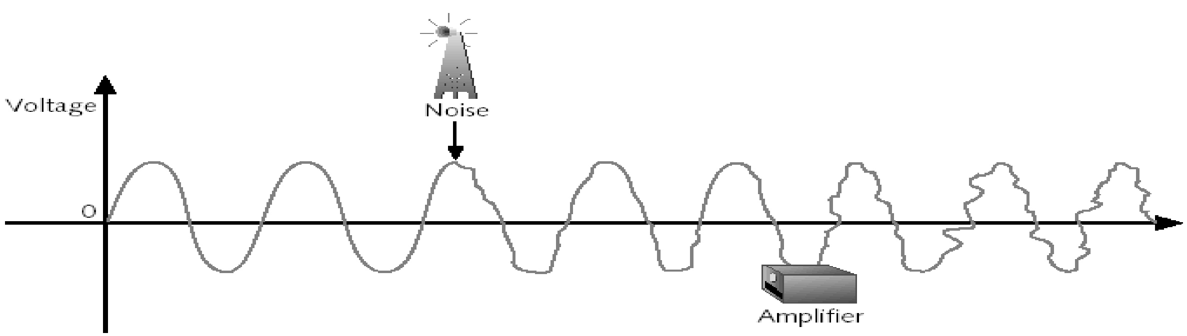


Analog versus digital

- Digital signals can be **regenerated** using repeaters
  - Cleaned up to prevent the accumulation of noise and distortion
  - Allows signal to be transmitted over greater distances



- What happens to analog signals over distances even if they are amplified? Can you reconstruct the original signal



Advantages of Digital Signals

- Digital circuits have only two states so:
  - Changes in value have little effect on digital signals
  - Noise and other forms of interference have little effect on digital signals
  - Little chance of error because voltage in a digital circuit must be in one state or the other
  - Information storage is easy
  - Operation can be readily programmed
  - Can fabricate more digital circuitry onto integrated circuits

Disadvantages of Digital Signals

- The ONE major disadvantage is that ***the real-world is analog in nature***

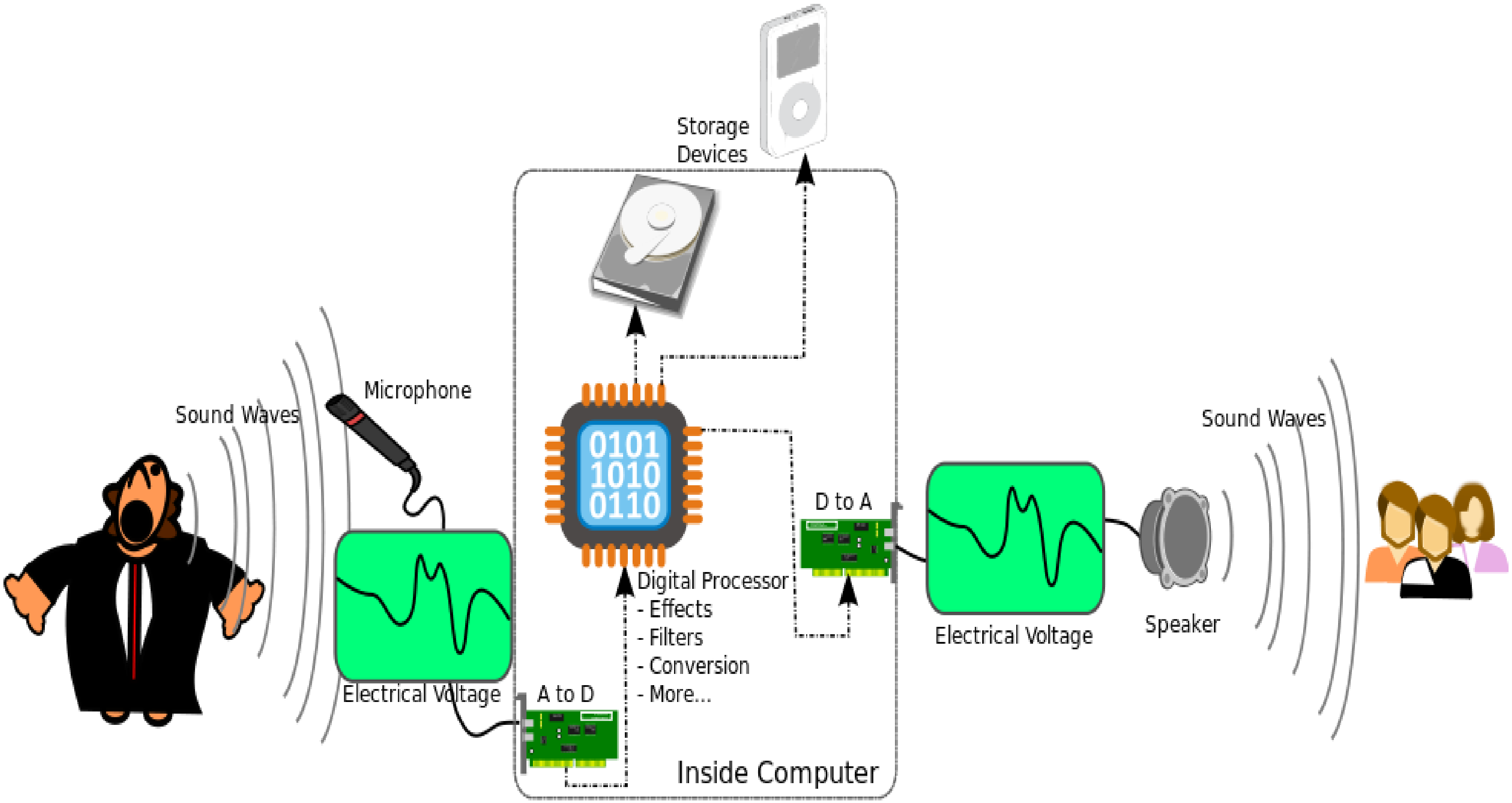
- When dealing with analog inputs and outputs you will always have to:

1) convert analog to digital (ADC)

2) process the digital data

3) convert the digital data back to analog output (DAC)

Example



Storage
Devices
Sound Waves
Microphone
0101
1010
0110
Digital Processor
- Effects
- Filters
- Conversion
- More...
D to A
Sound Waves
Electrical Voltage
A to D
Electrical Voltage
Speaker
Inside Computer

Analog-Digital Converter (ADC)

- An electronic integrated circuit which converts a signal from analog (continuous) to digital (discrete) form
- Provides a link between the analog world of transducers and the digital world of signal processing and data handling

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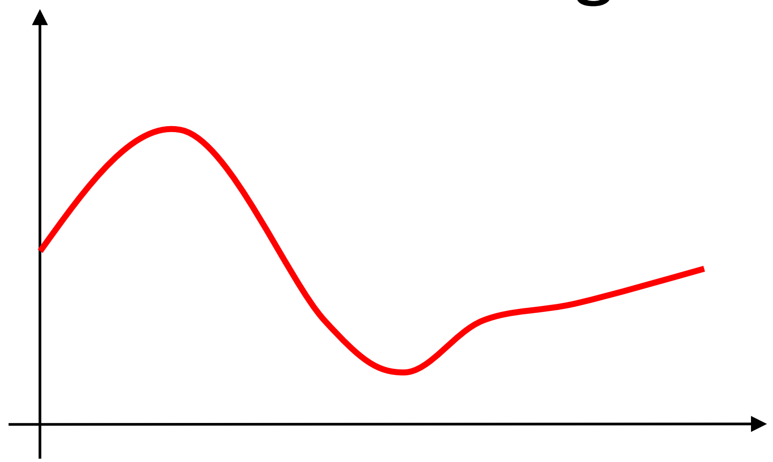

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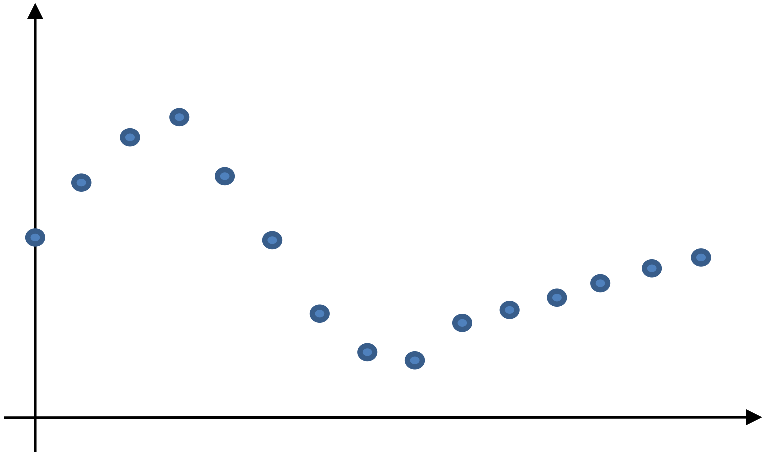
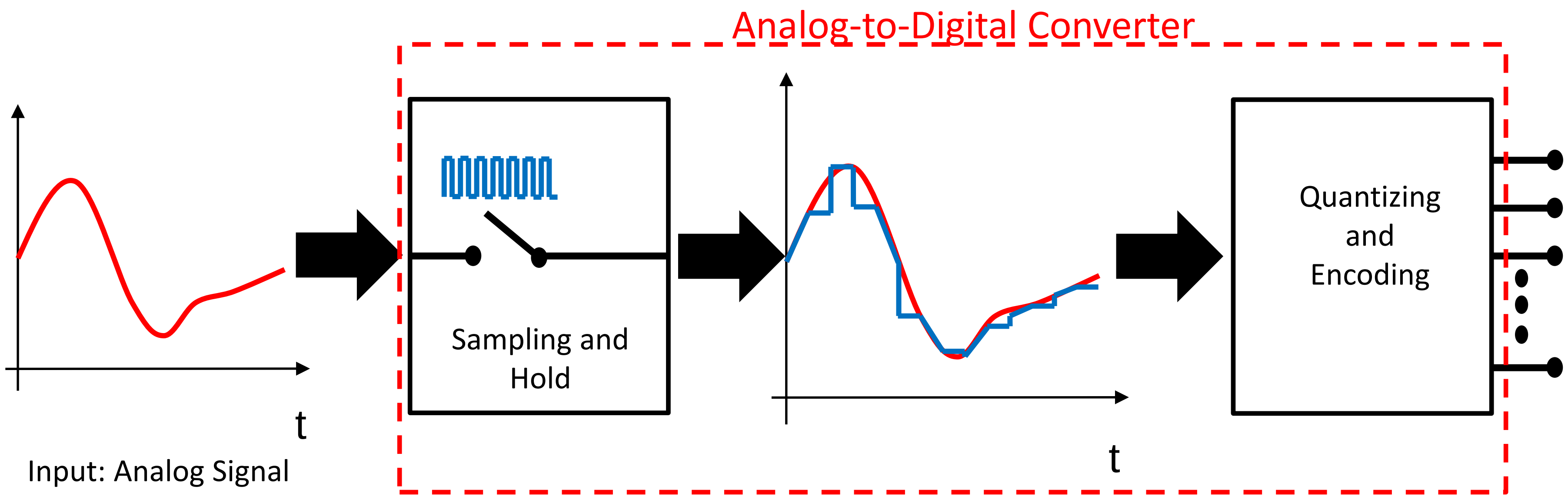


ADC Conversion Process

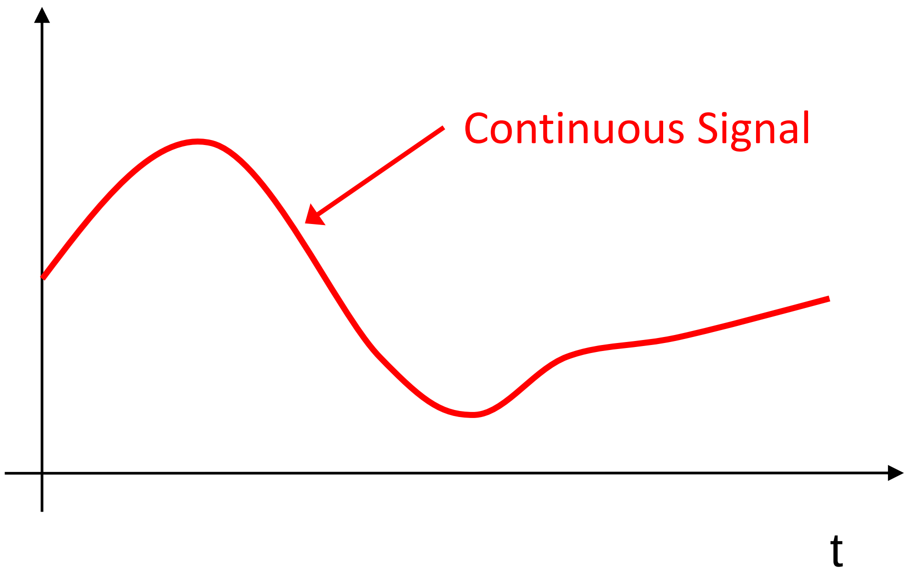
Two main steps of process

1. Sampling and Holding
2. Quantization and Encoding



ADC Process

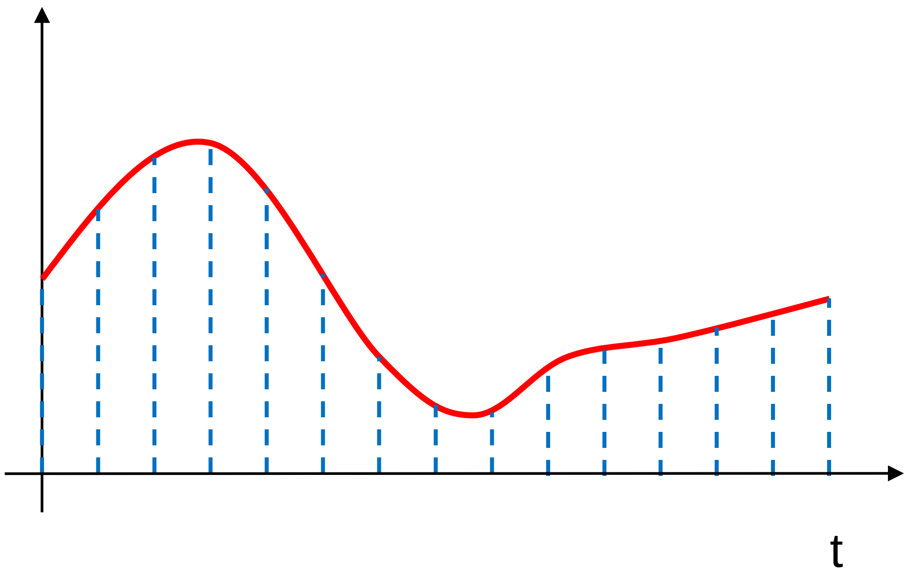
Sampling & Hold



ADC Process

Sampling & Hold

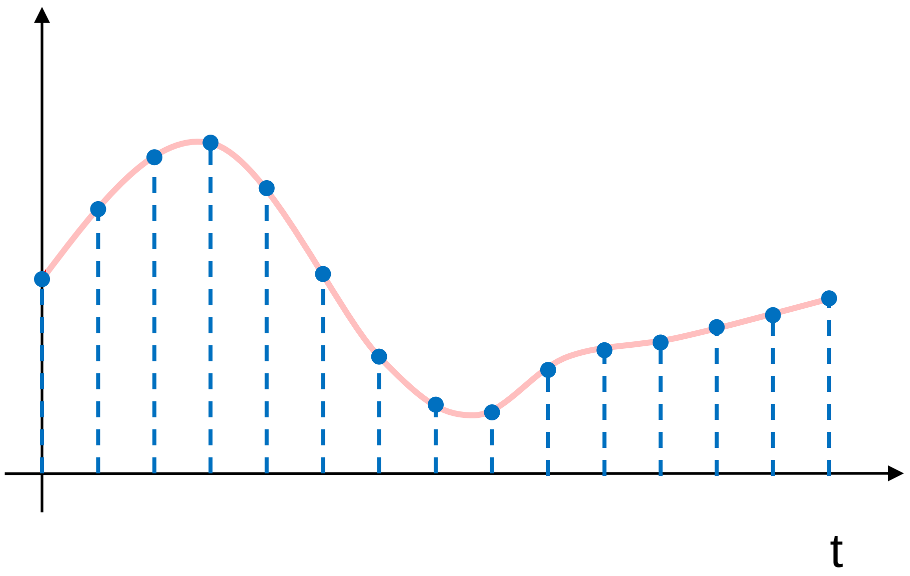
- Measuring analog signals at uniform time intervals
  - Ideally twice as fast as what we are sampling



ADC Process

Sampling & Hold

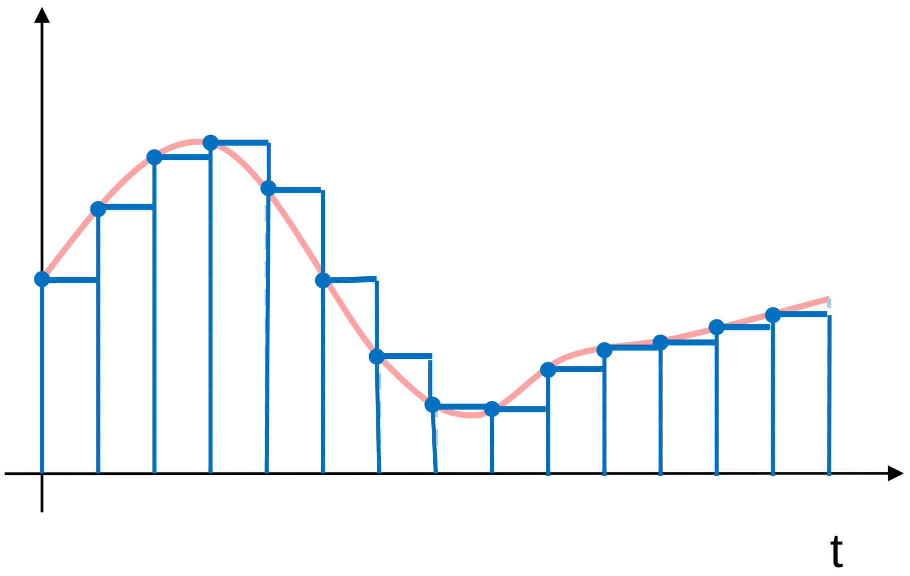
- Measuring analog signals at uniform time intervals
  - Ideally twice as fast as what we are sampling
- Digital system works with discrete states
  - Taking a sample from each location



ADC Process

Sampling & Hold

- Measuring analog signals at uniform time intervals
  - Ideally twice as fast as what we are sampling
- Digital system works with discrete states
  - Taking samples from each location
- Reflects sampled and hold signal
  - Digital approximation



ADC Process

Quantizing

- Separating the input signal into a discrete states with K increments
- $K=2^N$
  - N is the number of bits of the ADC
- Analog quantization size
  - $Q=(V_{max}-V_{min})/2^N$
  - Q is the Resolution

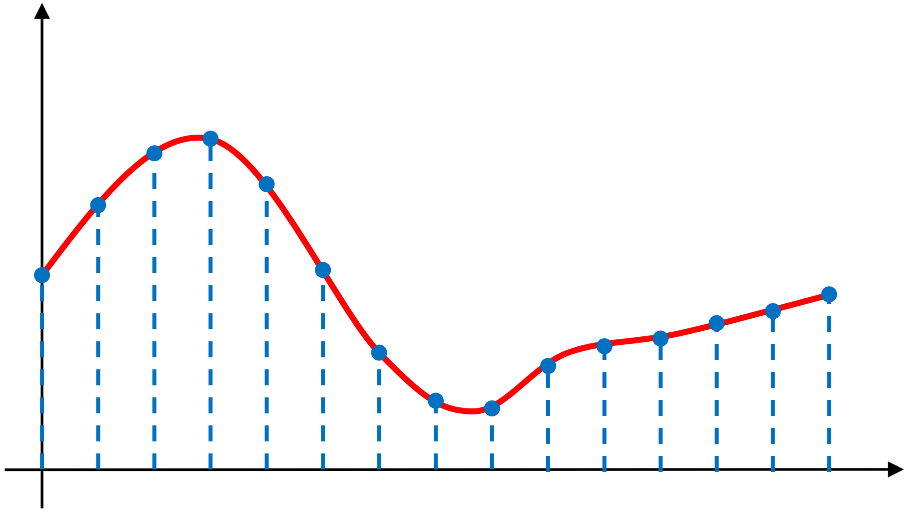
Encoding

- Assigning a unique digital code to each state for input into the microprocessor

ADC Process

Quantization & Coding

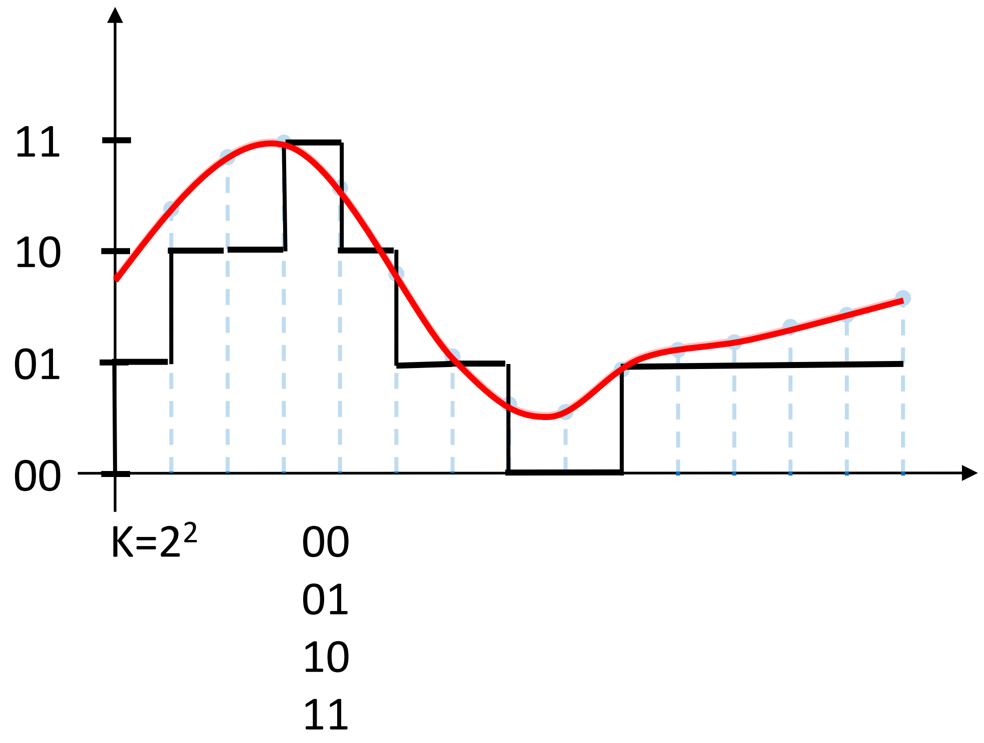
- Use original analog signal



ADC Process

Quantization & Coding

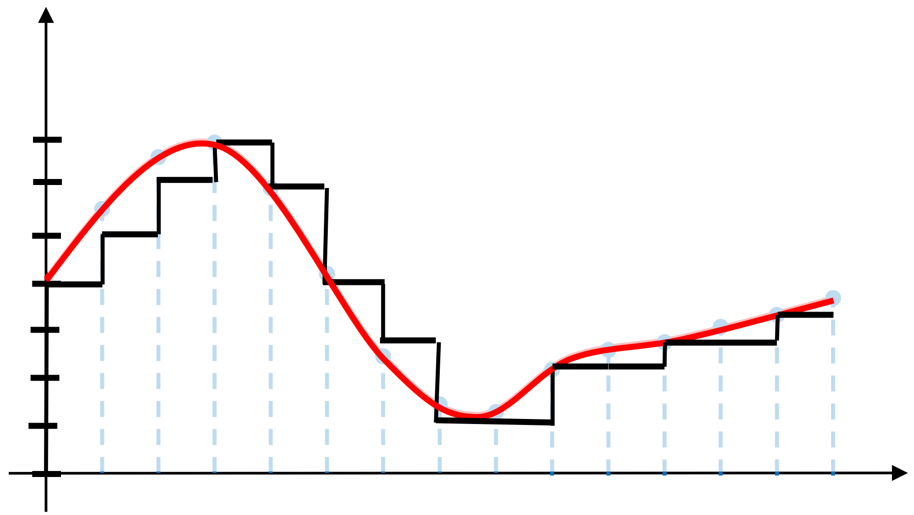
- Use original analog signal
- Apply 2 bit coding



ADC Process

Quantization & Coding

- Use original analog signal
- Apply 3 bit coding



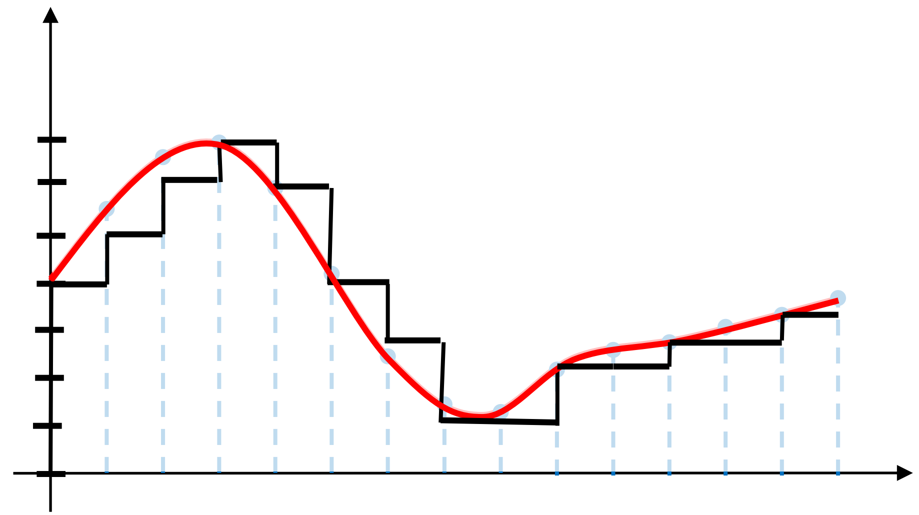
$K=2^3$

000
001
010
011
100
101
110
111

ADC Process

Quantization & Coding

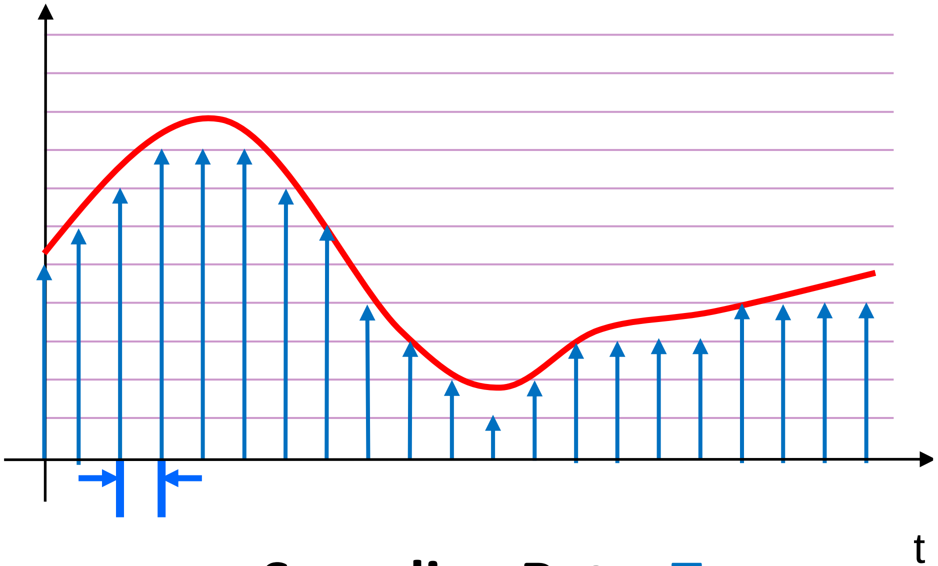
- Use original analog signal
- Apply 3 bit coding
- Better representation of input information with additional bits

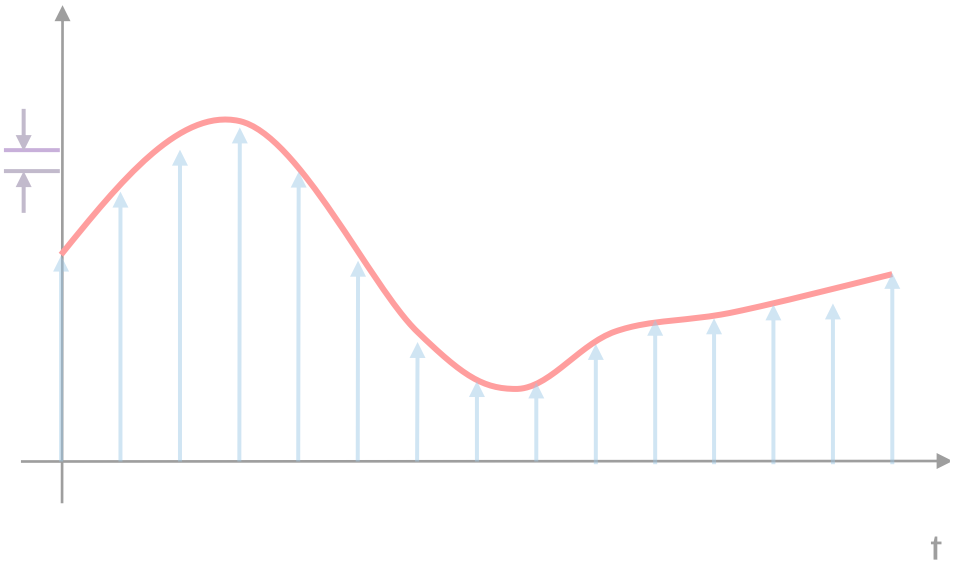


$K=2^3$	000	K=16	0000	K=…
	001		.	
	010		.	
	011		.	
	100		1111	
	101			
	110			
	111			

ADC Process-Accuracy

The accuracy of an ADC can be improved by increasing:



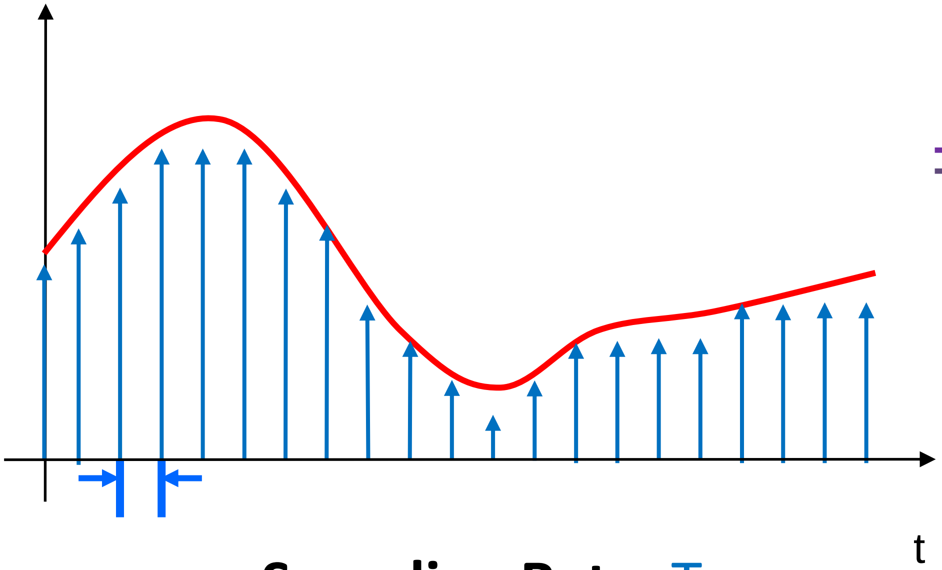


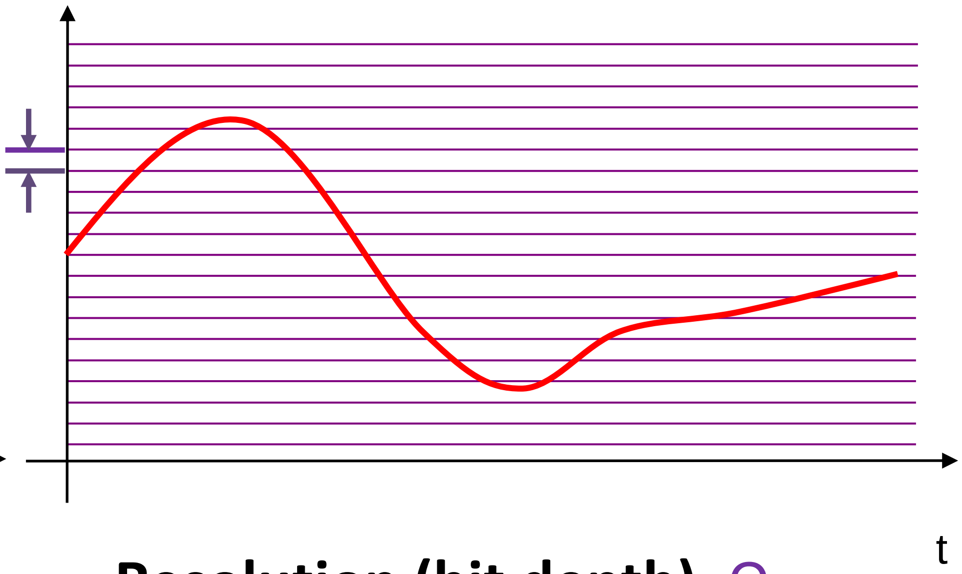
Sampling Rate, Ts

- Based on number of steps required in the conversion process
- Increases the maximum frequency that can be measured

ADC Process-Accuracy

The accuracy of an ADC can be improved by increasing:





Sampling Rate, Ts

- Based on number of steps required in the conversion process
- Increases the maximum frequency that can be measured

Resolution (bit depth), Q

- Improves accuracy in measuring amplitude of analog signal

ADC-Error Possibilities

- Aliasing (sampling)
  - Occurs when the input signal is changing much faster than the sample rate
  - Should follow the Nyquist Rule when sampling
    - Answers question of what sample rate is required
    - Use a sampling frequency at least twice as high as the maximum frequency in the signal to avoid aliasing

$$f_{sample} > 2f_{signal}$$

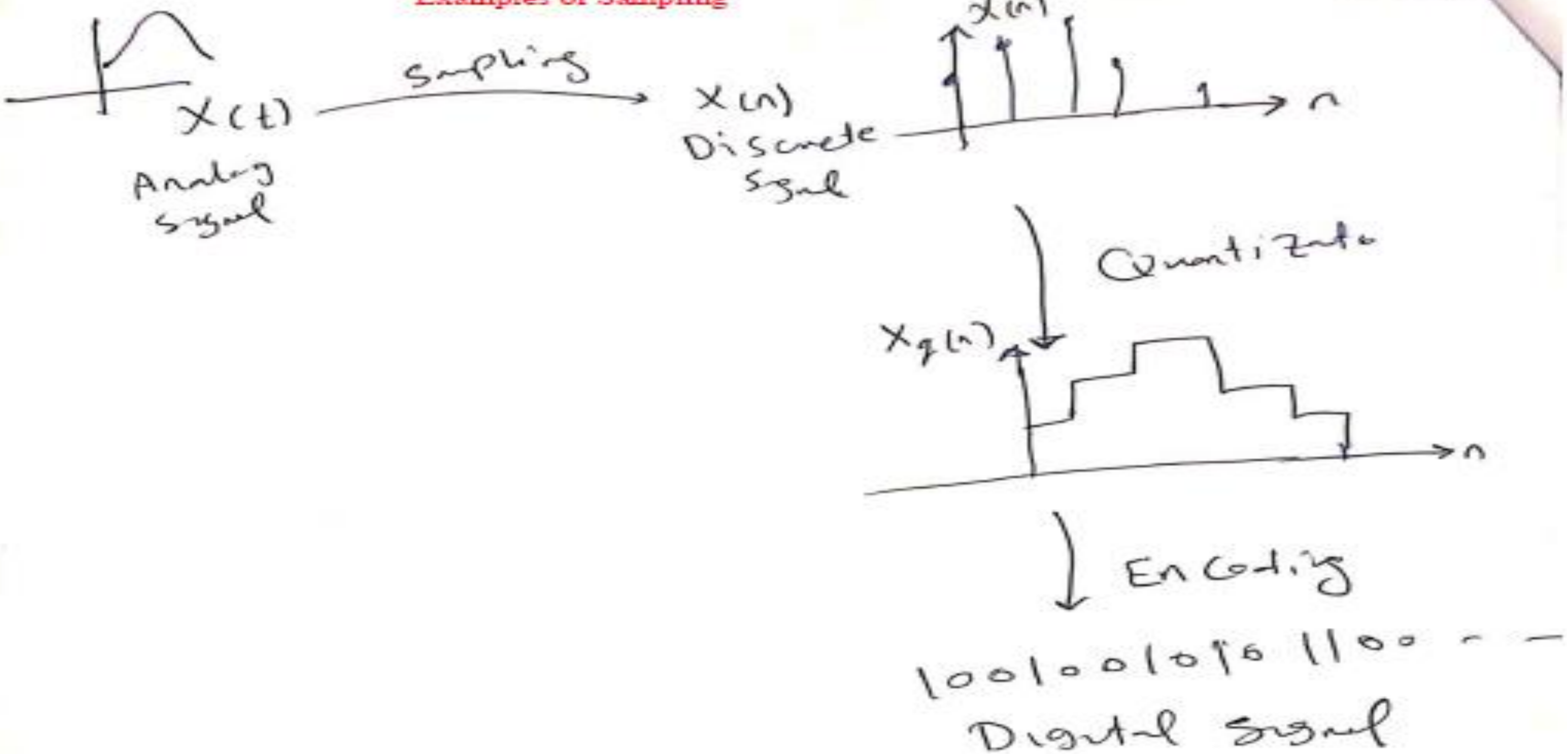
Where f_{signal} is the BW of analog signal

- Quantization Error (resolution)

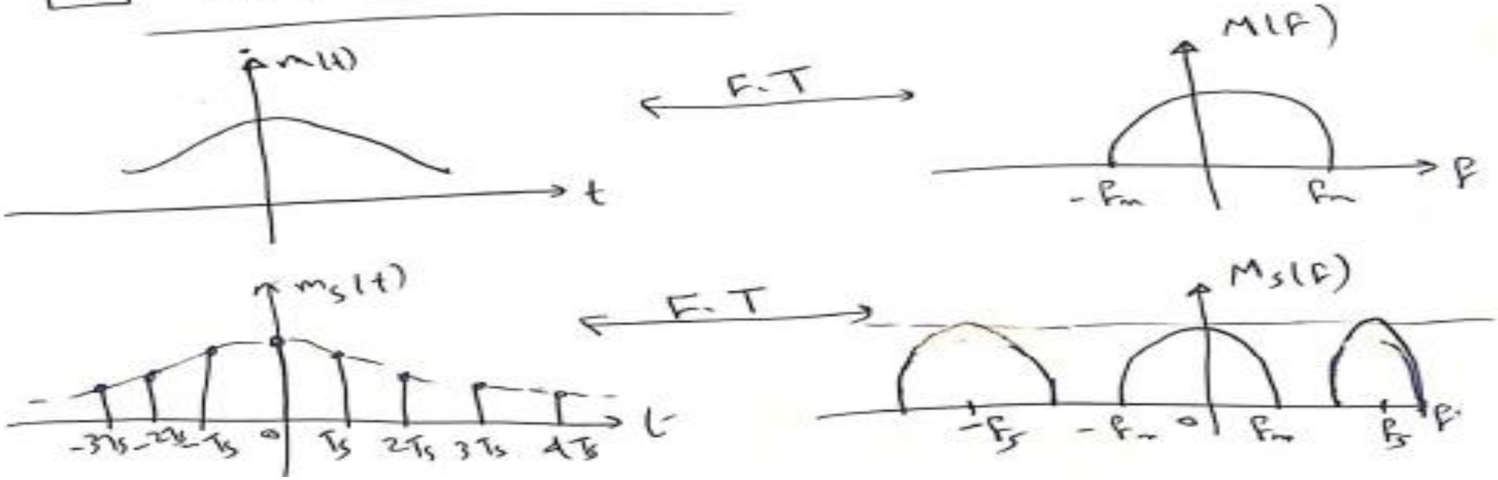
ADC Applications

- ADC are used virtually everywhere where an analog signal has to be processed, stored, or transported in digital form
- Analog data such as voice and video are converted to digital data for transmission over a digital link.
- We can transmit digital data
  - Faster
  - Cheaper
  - With fewer errors

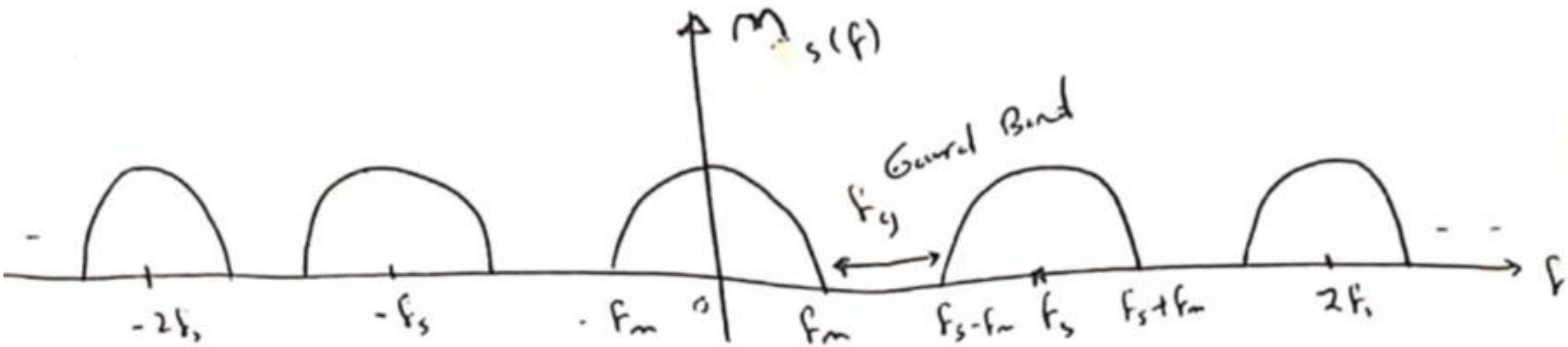
Examples of Sampling



1 Sampling theorem:



From the spectrum of sampled signal



As shown, to avoid aliasing : $f_s - f_m \geq f_m$

$$\boxed{f_s \geq 2f_m}$$

f_s : Sampling frequency
f_m : B.W of $m(t)$

→ Condition of proper Sampling (without aliasing

examples
1- Digital Telephone : $f_m = 3.4$ KHz → $f_s = 8$ KH
2- High Quality Analog music : $f_m = 20$ KHz
$f_s = 44.1$ KHz

Notes

1) $f_{s_{min}}$ = Nyquist rate = $2 f_m$ [minimum sampling frequency]

$T_{s_{max}} = \frac{1}{f_{s_{min}}}$ [maximum sampling time]

2) The condition of proper sampling without aliasing

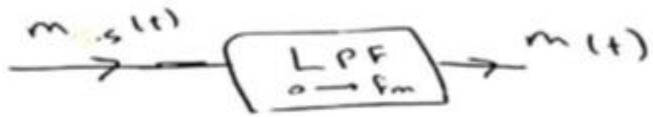
$f_s \geq 2 f_m$

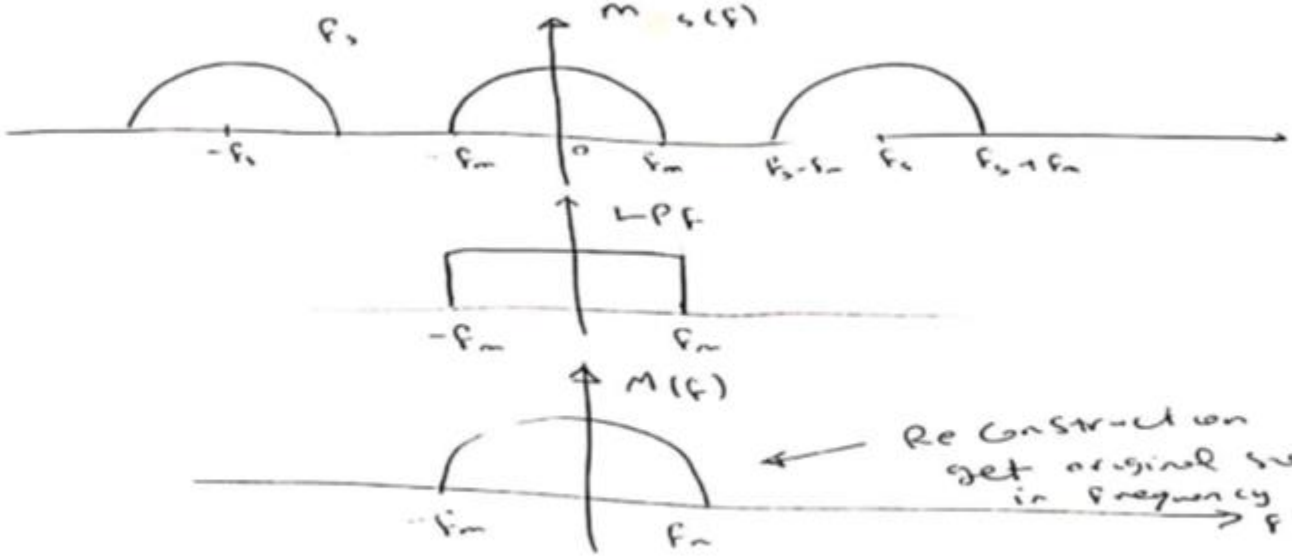
$f_s = 2 f_m + f_g$ ← guard band

3) f_s : sampling frequency = sampling rate

f_s = # of samples / sec

Reconstruction of original signal





Reconstruction
get original signal
in frequency Domain

Condition of Reconstruction

$f_s \geq 2 f_m$ same as proper sampling condition

Notes

- if $f_s > 2 f_m \Rightarrow$ over sampling
- if $f_s = 2 f_m \Rightarrow$ critical (Nyquist) sampling

} No aliasing

- if $f_s < 2 f_m \Rightarrow$ under sampling (aliasing)

Example

Given: $x(t) = 4 \operatorname{sinc}(4000t)$

Apply sampling theorem and then reconstruct the original signal in the following cases:

i) $f_s = 6K$ ii) $f_s = 4K$ iii) $f_s = 3K$

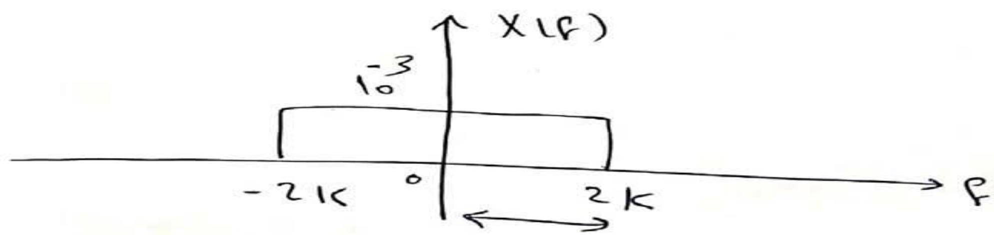
[Hint: $\mathcal{F}\{\operatorname{sinc}(at)\} = \frac{1}{a} \operatorname{rec}(f/a)$]

Solution

$$X(f) = \mathcal{F}\{x(t)\} = \mathcal{F}\{4 \operatorname{sinc}(4000t)\}$$

$$X(f) = \frac{4}{4000} \operatorname{rec}\left(\frac{f}{4000}\right) = 10^{-3} \operatorname{rec}\left(\frac{f}{4k}\right)$$

$$X(f) = 10^{-3} \operatorname{rec}(f/4k)$$



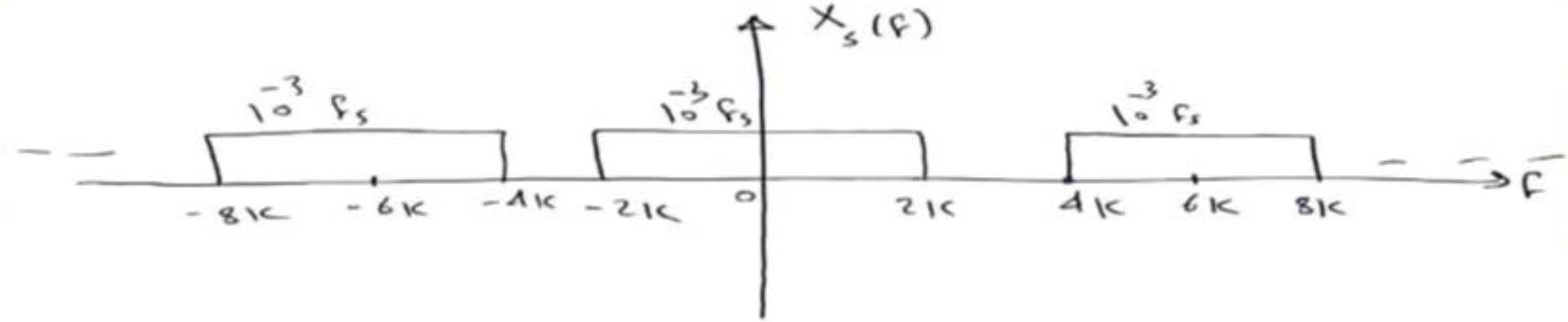
BW = f_m = 2 KHz

$$\boxed{f_{s_{min}} = 2 f_m = 4\ KHz} \Leftarrow \text{Nyquist rate}$$

(i) $f_s = 6\ KHz$

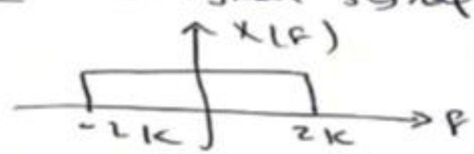
$f_s > f_{s_{min}} \Rightarrow$ No aliasing

Note: spectrum of sampled signal $X_s(f)$ $\rightarrow$ repeat spectrum of $x(t)$ at $0, \pm f_s, \pm 2f_s$ & multiply by f_s



we can reconstruct original signal by LPF ($0 \rightarrow f_m$)

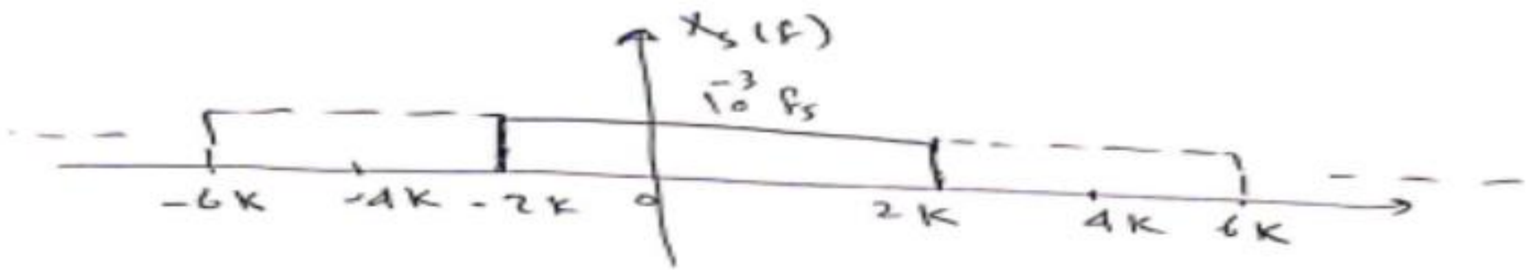
$X_s(f) \rightarrow$ LPF $0 \rightarrow 2\,KHz$ $\rightarrow x(t)$ original signal



No aliasing

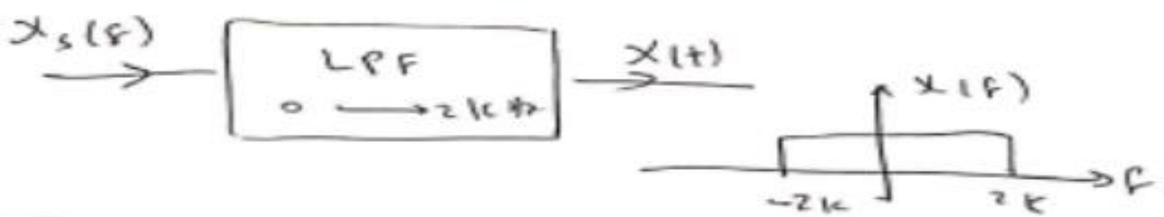
(ii) $f_s = 4\ KHz$

$f_s = f_{s_{min}} = f_{Nyquist} \Longrightarrow$ No aliasing



No aliasing

We can reconstruct original signal by LPF ($0 \rightarrow f_m$)



(iii) $f_s = 3\ K$

$f_s = 3\ KHz < f_{s_{min}} = 4\ KHz$

$\Downarrow$

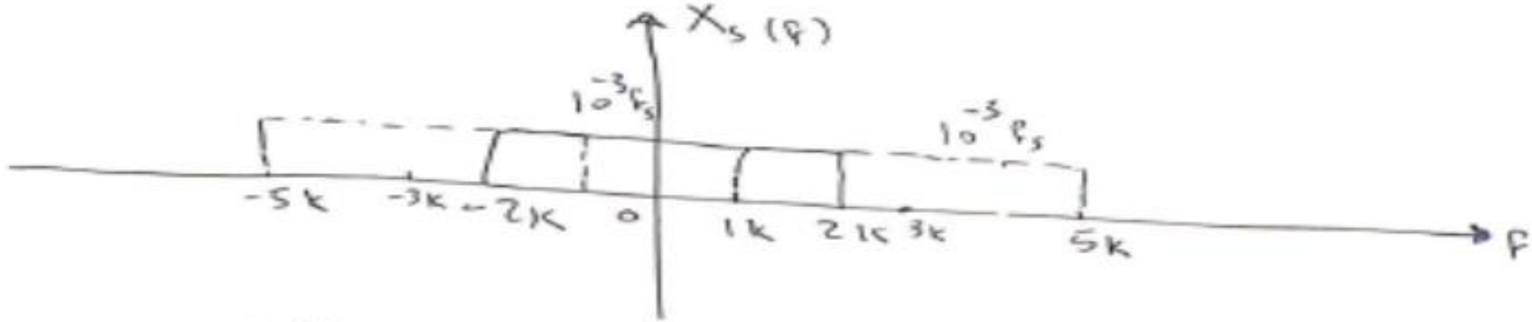
aliasing

Recall:

Spectrum of $X_s(f) \Rightarrow$ repeat spectrum of $x(t)$ at

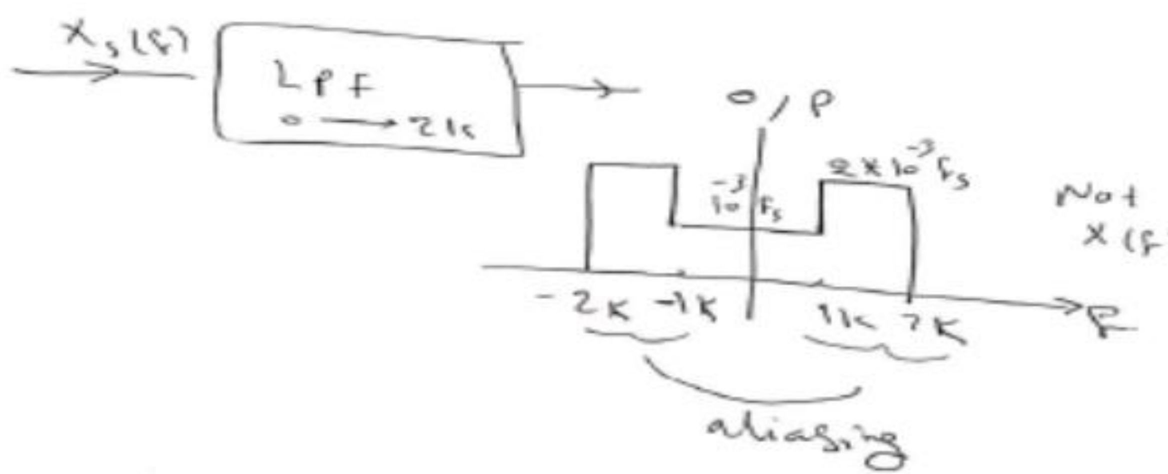
$0, \pm f_s = \pm 3k, \pm 2f_s = \pm 6k$ - - -

& multiply by f_s.



aliasing, we can not reconstruct original signal

Note: if we apply LPF $0 \rightarrow f_m = 2k$



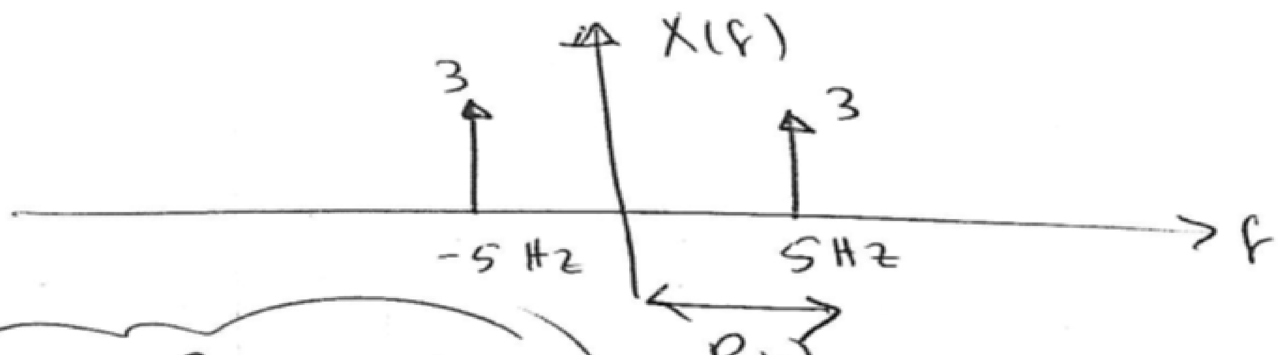
example:

$x(t) = 6\cos(2\pi(5)t)$, apply Ideal sampling for the following sampling frequencies then reconstruct the original signal (1) $f_s = 14$ Hz (2) $f_s = 7$ Hz

Given: $A\cos(2\pi f_c t) \xleftrightarrow{F.T} \frac{A}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right]$

Sol

$x(t) = 6\cos(2\pi(5)t)$

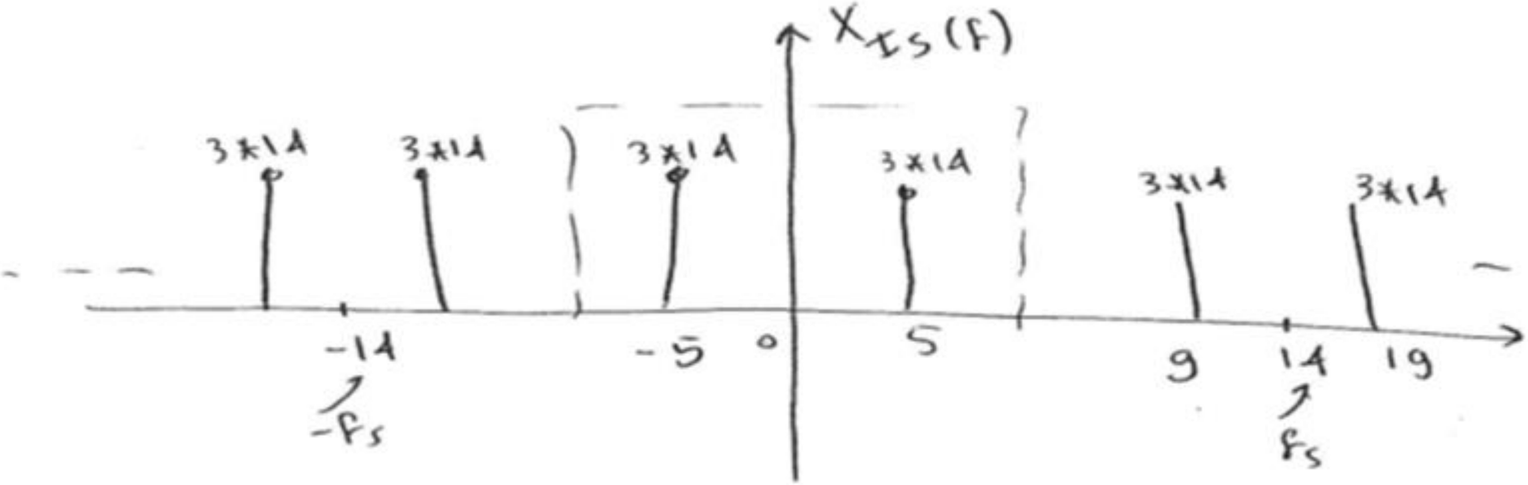


$Bw = f_m = 5$ Hz $\Longrightarrow$ $f_s = 2f_m = 10$ Hz.

① $\boxed{f_s = 14\ Hz} \Rightarrow f_s > 2f_m$ as $2f_m = 10\ Hz$

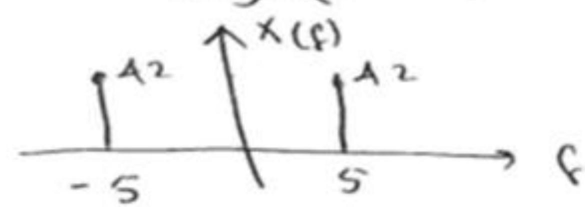
$f_s = 14 > 2f_m$ ✓

$\therefore$ No aliasing



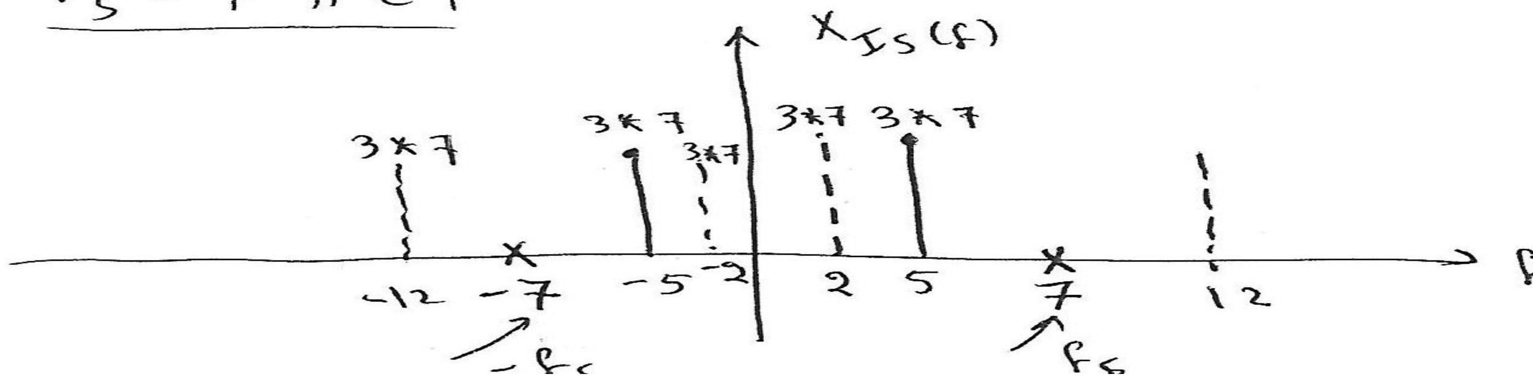
we can reconstruct original signal. by

$\xrightarrow{X_{\delta s}(f)}$ $\boxed{\begin{array}{c}\text{LPF}\\ 0 \rightarrow 5\ Hz\end{array}}$ $\xrightarrow{X(f)}$ original signal.



No aliasing.

② $f_s = 7\ Hz$:



$f_s = 7\ Hz$ not $\geq 2f_m = 10\ Hz$

$\therefore f_s < 2f_m$

$\therefore$ aliasing occurs as shown in the spectrum

→ we can not reconstruct the original signal

→ if we apply LPF (0 → 5 Hz), we will not get the original signal but get 2 tones (2 Hz & 5 Hz) ✗