Discrete time signals

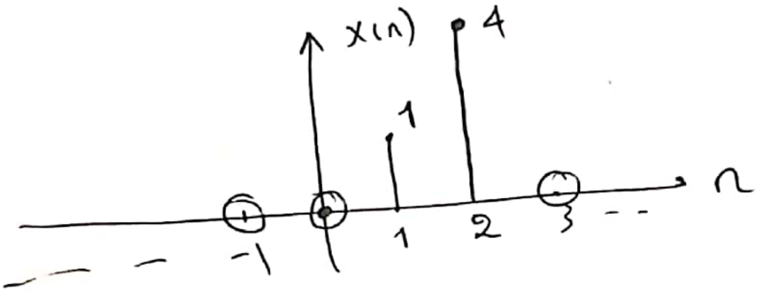
Discrete time signal :-

$x(n)$, $n = 0, \pm 1, \pm 2, ----$

→ it is defined only at discrete instants of time n ($n = 0, \pm 1, \pm 2, ----$)

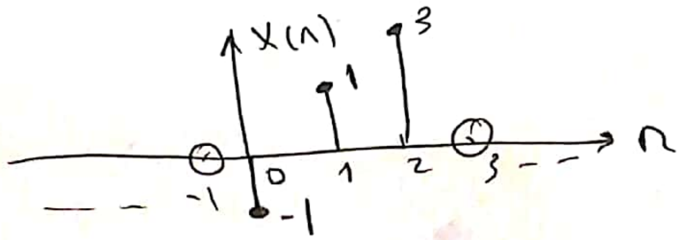
ex: $x(n) = n^2$, $n = 0, 1, 2.$

$x(0) = 0$, $x(1) = (1)^2$, $x(2) = (2)^2$



ex: sketch $x(n) = 2n - 1$, $0 \leq n < 3$

$$x(n) = \begin{cases} -1 , & n = 0 \\ 1 , & n = 1 \\ 3 , & n = 2 \\ 0 , & o.w \end{cases}$$



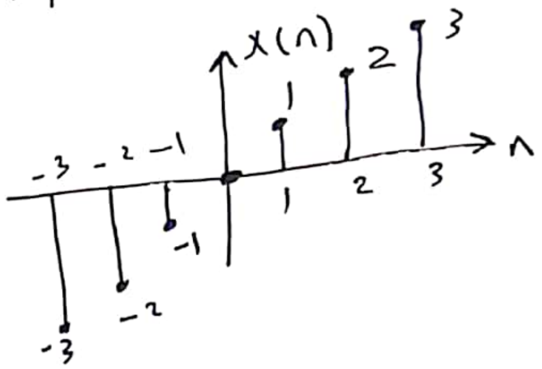
① odd and even signals

Classifications of Discrete signals

odd signal

$x(-n) = -x(n)$

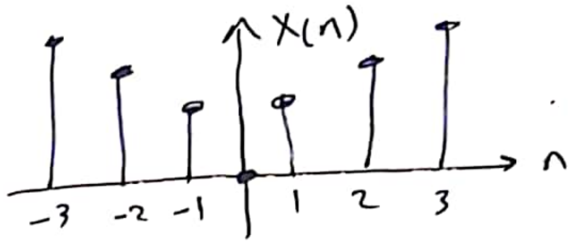
Symm around origin.



even signal

$x(-n) = x(n)$

Symm around y-axis



→ even (+), odd (−) ⟹ $+ * + = +$, $+ * - = -$, $- * - = +$, $\frac{-}{-} = +$, $\frac{-}{+} = -$

→ For any signal $x(n)$, we can write it

$$x(n) = x_{odd}(n) + x_{even}(n)$$

where $x_{odd}(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$

$$x_{even}(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

3) Periodic and non Periodic

* For discrete - time Signal $x[n]$, it is

periodic Signal if :-

1 - $x(n + N) = x[n]$

2 - for all n $[-\infty < n < \infty]$.

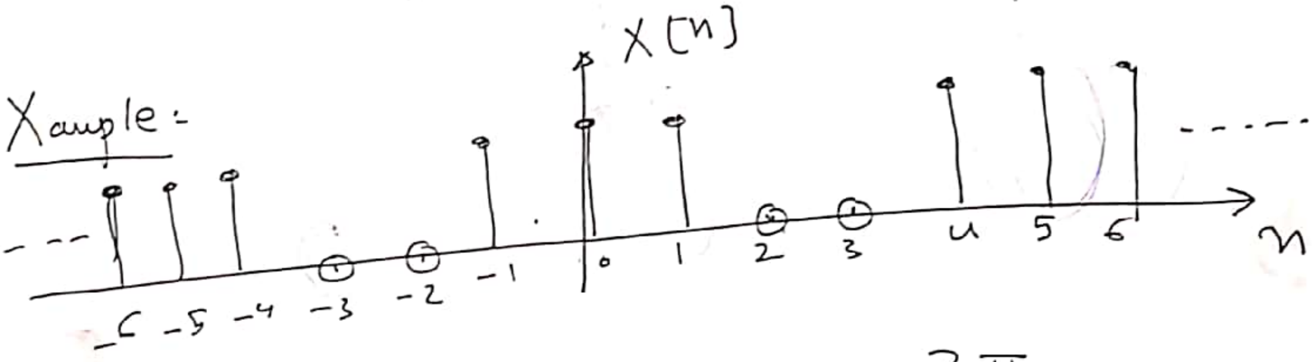
3 - N must be +ve integer

where :- . N : is the fund. period. (Sample)

. f_0 : is the fund. freq $= \frac{1}{N}$

. Ω_0 : the angular freq (rad) $2\pi f_0 = \frac{2\pi}{N}$

Xample :-



Soln $N = 5$, $\Omega_0 = \frac{2\pi}{5}$

* For <u>discrete-time</u> Sinusoidal Signal

$$x(n) = A \cos(\Omega_0 n + \theta)$$

$$\text{or } A \sin(\Omega_0 n + \theta)$$

it can be periodic or non-periodic

<u>Xample:-</u> show that if $x[n]$ is periodic or not, if yes find N.

1- $x(n) = \cos(2n)$

2- $x(n) = 5 \sin\left(\frac{8\pi}{15} n\right)$

<u>Solu</u>

1) $x[n] = \cos(2n)$

step (1) $x[n+N] = \cos(2n + 2N)$

step (2) $\cancel{2}N = \cancel{2}\pi m$ (where $m = 1, 2, 3, 4, \ldots$)

$$\boxed{N = \pi m}$$

We can't find a certain value for (m) to let N be integer

i.e. $N \neq$ integer $\therefore$ $x(n)$ is not periodic.

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2) $x(n) = 5\sin\left(\frac{8\pi}{15} n\right)$

Step(1) $x[n+N] = 5\sin\left(\frac{8\pi}{15} n + \underline{\underline{\frac{8\pi}{15} N}}\right)$

Step(2) $\frac{8\pi}{15} N = 2\pi m \quad (m = 1, 2, 3 \ldots)$

$\therefore N = \frac{15}{4} m$

for $m = 4, 8, 12, 16 \ldots \rightarrow N$ will be integer

$\therefore$ fundumental period $\boxed{N = 15}$ (at $m = 4$)

$\therefore x(n)$ periodic with period $= 15$

<u>special case</u>

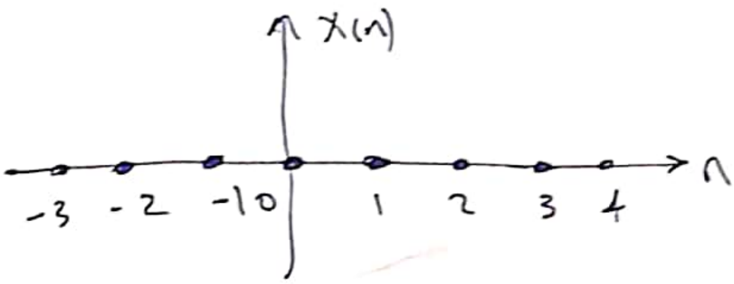
note $x(n) = \sin(m\pi n)$, m: integer

$\Downarrow$

Periodic $N = 1$

ex!

$x(n) = \sin(\pi n)$



$x(0) = 0$

$x(1) = 0$

$x(2) = 0$

$\vdots$

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Note

If we have $x_1(n)$ periodic of period N_1

$x_2(n)$ periodic of period N_2

& $y(n) = x_1(n) + x_2(n)$

$y(n)$ is periodic iff $\frac{N_1}{N_2} = \frac{m}{n}$ ← integer, ← integer } After simplification

& period of $y(n) = N = nN_1 = mN_2$

Example

$$x(n) = 5\sin\left(\frac{8\pi}{15}n\right) + 3\cos\left(\frac{7\pi}{12}n\right)$$

Check if $x(n)$ periodic or not & Find period

Sol

$$x(n) = x_1(n) + x_2(n)$$

$$x_1(n) = 5\sin\left(\frac{8\pi}{15}n\right) \longrightarrow \text{periodic}$$

$\boxed{N_1 = 15}$

(see previous example)

$x_2(n) = 3\cos\left(\frac{7\pi}{12} n\right)$

$x_2(n+N_2) = 3\cos\left(\frac{7\pi}{12} n + \frac{7\pi}{12} N_2\right)$

$\frac{7\pi}{12} N_2 = 2\pi m, \quad m = 1, 2, 3 \ldots$

$N_2 = \frac{24}{7} m, \quad m = 1, 2, 3 \ldots$

$m = 7 \longrightarrow \boxed{N_2 = 24}$ periodic

$\therefore x(n) = x_1(n) + x_2(n)$

periodic $N_1 = 15$ periodic $N_2 = 24$

$\therefore \frac{N_1}{N_2} = \frac{15}{24} = \frac{5}{8} \Longrightarrow N = 5N_2 = 8N_1$

$\therefore$ period of $x(n)$ is $N = 5N_2 = 8N_1$

$\boxed{N = 120}$

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3 Power & Energy signal

Note:

1) total Energy = $E_t = \sum_{n=-\infty}^{\infty} |x(n)|^2$

2) Average power

Periodic of period N_0

$$P_{av} = \frac{1}{N_0} \sum_{n=0}^{N_0-1} x^2(n)$$

or generally

$$P_{av} = \frac{1}{N_0} \sum_{n=\langle N_0 \rangle} x^2(n)$$

ex: if $N_0 = 5$

$$P_{av} = \frac{1}{5} \sum_{n=0}^{4} x^2(n)$$

or

$$\frac{1}{5} \sum_{n=-2}^{2} x^2(n)$$

⋮

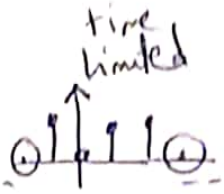
Non periodic

$$P_{av} = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2(n)$$

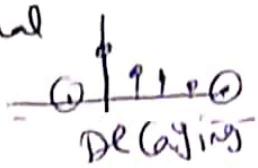
[4] Power & Energy signal:

Energy signal

ex: limited time signal



, Decaying signal



$\Downarrow$

$P_{av} = 0$

$$E_t = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

Power signal

exs: periodic signal

$\Downarrow$

$E_t = \infty$

$$P_{av} = \text{Value} = \frac{1}{N}\sum_{0}^{N-1} |x(n)|^2$$

$\rightarrow$ Note: RMS value of $x(n) = \sqrt{P_{av}}$

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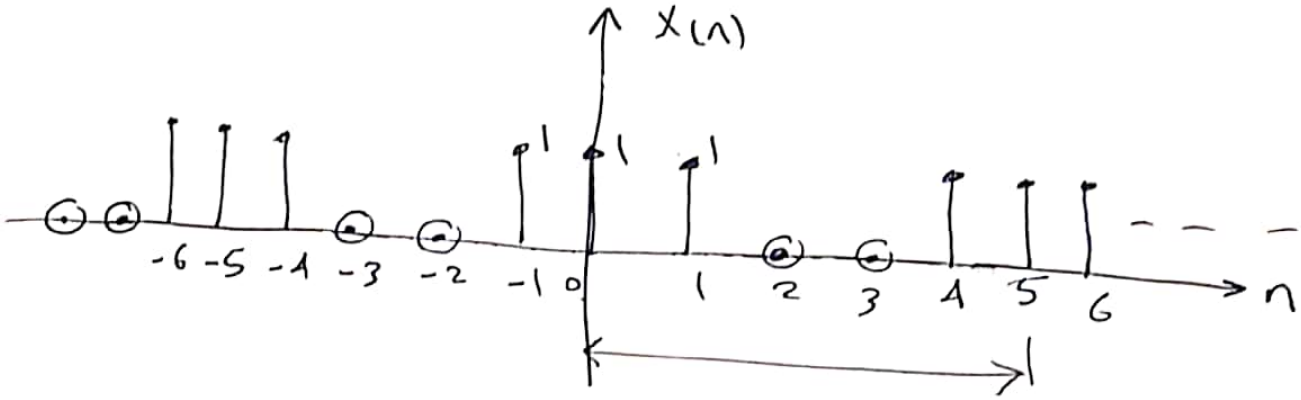
ex Determine whether the following signals are Energy or power signal & get P_{av}, E_t:



periodic

Sol

$N = 5$

2- $\because$ $x(n)$ is a periodic Signal $(N=5)$

$\therefore$ $x(n)$ is power Signal

$\therefore$ $\boxed{E_{tot} = \infty}$

$$P_{av} = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

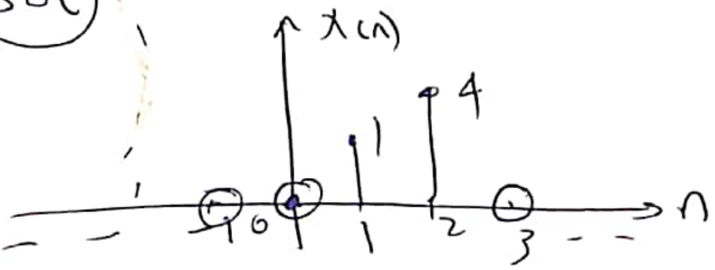
$$= \frac{1}{5} \sum_{n=0}^{4} |x(n)|^2$$

$$= \frac{1}{5} \left[(1)^2 + (1)^2 + 0 + 0 + (1)^2 \right]$$

$$\boxed{P_{av} = \frac{3}{5}}$$

Example: $x(n) = n^2, \ n = 0, 1, 2.$

Sol



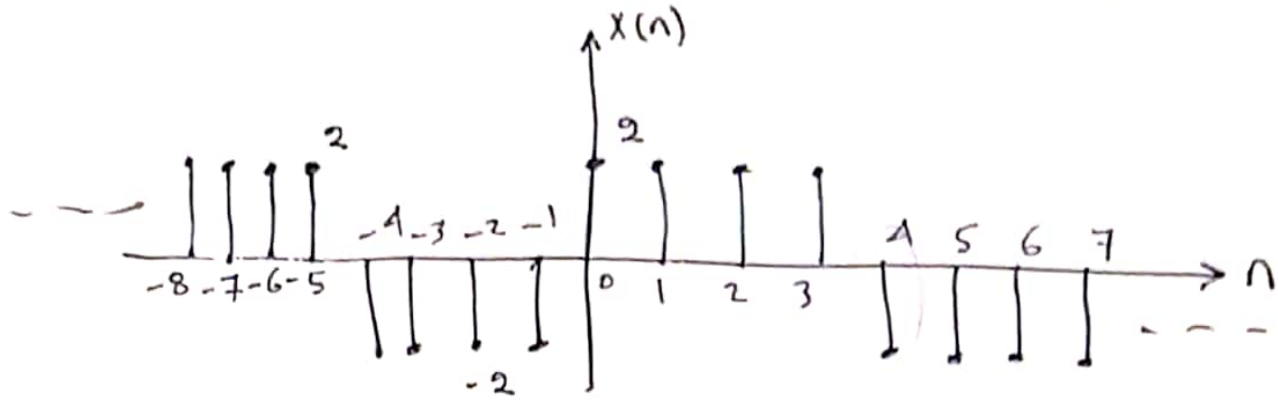
Time limited
$\therefore$
Energy signal

$P_{av} = 0$

$$E_t = \sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=0}^{2} x^2(n) = (0)^2 + (1)^2 + (4)^2$$

$$= 17$$

ex



Find Fundamental Freq & average power

Sol

Periodic discrete signal $N = 8$ [Power Signal]

$\Rightarrow$

Fundamental Freq $= f_0 = \frac{1}{N} = \frac{1}{8}$.

$$\Rightarrow P_{av} = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{8} \left[(2)^2 + (2)^2 + (2)^2 + (2)^2 + (-2)^2 + (-2)^2 + (-2)^2 + (-2)^2 \right]$$

$$= \frac{1}{8.} \sum_{n=0}^{7} (4) = \frac{1}{8} * 4 [7 - 0 + 1]$$

$$= 4$$

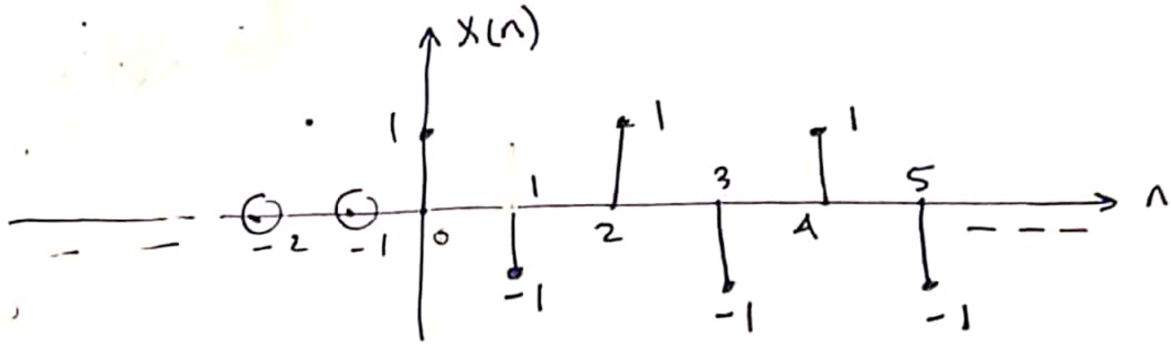
If Req: $\underline{E_t = \infty}$

Note $\sum_{x=a}^{b} K = K [b - a + 1]$

ex: $x(n) = \begin{cases} \cos \pi n & , n \geq 0 \\ 0 & , o.w \end{cases}$

Find P_{av}, E_t

SoL



$$E_t = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} 1 = \boxed{\infty}$$

power signal.

$$P_{av} = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$ "non periodic"

$$P_{av} = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)^2 = \lim_{N \to \infty} \frac{N-0+1}{2N+1}$$

$$= \lim_{N \to \infty} \frac{N+1}{2N+1} = \frac{\infty}{\infty}$$ l' Hopital

$$P_{av} = \frac{1}{2}$$

Note

$$\sum_{x=0}^{N} \underset{\text{ثابت}}{K} = K(N-0+1)$$