Time Domain Representation of LTI System

1. Impulse Response Representation (previous lecture)
2. Difference Equation Representation
3. Block Diagram Representation

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2 Difference Equation Representation

The general form of Difference equation is:

$$y(n) = \sum_{k=1}^{N} a_K\, y(n-k) + \sum_{K=0}^{M} b_K\, x(n-k)$$

$$y(n) = \cdots - a_2\, y(n-2) - a_1\, y(n-1) + b_0\, x(n) + b_1\, x(n-1) - -$$

$$\therefore\ y(n) + a_1\, y(n-1) + a_2\, y(n-2) \cdots = b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2) - -$$

There are 2 Types of Difference equations

Difference equation

Recurrsive LTI system

IIR
"infinite Impulse response"

" If there are $y(n-1)$ or $y(n-2)$ - - - "

$\therefore y(n) + a_1 y(n-1) - - - = b_0 x(n) + b_1 x(n-1) - -$

Non Recurrsive LTI system

FIR
"Finite Impulse response"

$y(n)$ depends only on $x(n)$, $x(n-1)$, - - -

[No $y(n-0)$]

$y(n) = b_0 x(n) + b_1 x(n-1)$ - - -

order of Difference Equation

is the largest shift in $y(n)$

examples

$\boxed{1}$ $y(n) + 2y(n-1) = x(n) \Rightarrow$ order $= 1$

$\boxed{2}$ $y(n) + 3y(n-1) = 2x(n) + x(n-2) \Rightarrow$ order $= 1$

$\boxed{3}$ $y(n) + 3y(n-1) + 4y(n-2) = 2x(n-1) \Rightarrow$ order $= 2$

and so on ✓

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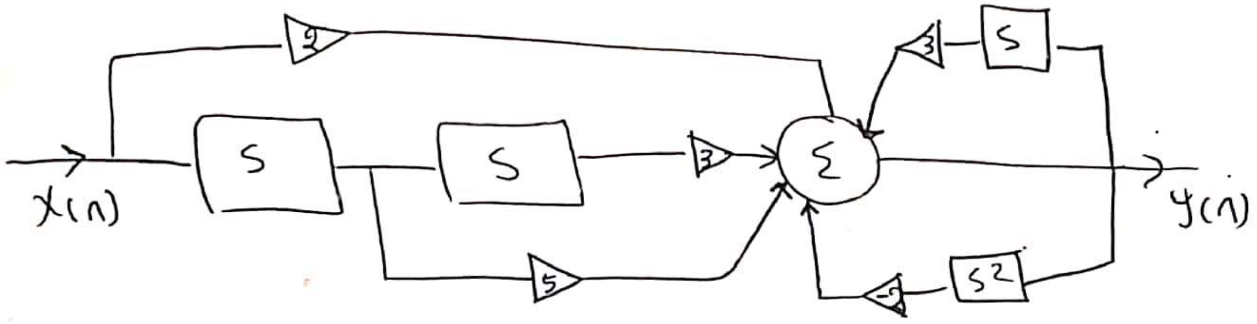
we can get the D.E From a given Block diagram :-

ex

1) Find the Difference equation For the Following Block diagram

2) Indicate Type of system



SoL

$$y(n) = 2x(n) + 5x(n-1) + 3x(n-2) + 3y(n-1) - 2y(n-2)$$

$$y(n) - 3y(n-1) + 2y(n-2) = 2x(n) + 5x(n-1) + 3x(n-2)$$

Recurrsive system (IIR)

How to Solve the Difference equation ?

In order to solve D.E, we need x(n) & D.E & initial values. The required is y(n) ?

$$y(n) = y_{C.F}(n) + y_{P.I}(n)$$

Total response

Natural response
at $x(n) = 0$
+ initial Conditions

Forced response
at $x(n) = $ Value

Steps of Solution

$\boxed{1}$ **Get Natural response;** $y_{C.F}(n)$

put $x(n) = 0$ & replace $y(n) \longrightarrow \lambda^{n}$

$y(n-1) \longrightarrow \lambda^{n-1}$

$y(n-2) \longrightarrow \lambda^{n-2}$

& get characteristics (Complementry) equation

$\Downarrow$

2nd degree if order = 2

get roots [2 roots iff order = 2]

repeated roots

$\lambda_1 = \lambda_2 = \lambda$

$y_{C.F} = C_1 \lambda^n + C_2 n \lambda^n$

distinct roots

$\lambda_1, \lambda_2 \quad (\lambda_1 \neq \lambda_2)$

$y_{C.F} = C_1 \lambda_1^n + C_2 \lambda_2^n$

$\boxed{2}$ get Forced response: $y_{P.I}(n)$

$y_{P.I}(n)$ depends on the input $x(n)$ and take the same form of $x(n)$ as shown in the following table:

i/p signal $x(n)$	$y_{P.I}$
Constant: A	K
$A\,u(n)$	$K\,u(n)$
$A\,(a)^n$	$K\,(a)^n$

Substitute in the D.E to get K

$\boxed{3}$ $y(n)$ = total response = $y_{C.F}(n) + y_{P.I}(n)$

get Constants (C_1, C_2) from initial conditions ex:

$$y(n) - \frac{3}{4}\, y(n-1) + \frac{1}{8}\, y(n-2) = 2\, x(n).$$

where $x(n) = 2\, u(n)$

$y(-1) = 1$, $y(-2) = -1$

Sol

1) Natural response:

$x(n) = 0$, $y(n) = \lambda^n$, $y(n-1) = \lambda^{n-1}$, $y(n-2) = \lambda^{n-}$

$$\lambda^n - \frac{3}{4}\lambda^{n-1} + \frac{1}{8}\lambda^{n-2} = 0$$

الضرب في $\frac{1}{\lambda^{n-2}}$

$$\lambda^2 - \frac{3}{4}\lambda + \frac{1}{8} = 0.$$

$\lambda_1 = \frac{1}{2}$, $\lambda_2 = \frac{1}{4}$ Dif ferent

$$\boxed{y_{C.F} = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{4}\right)^n}$$

2) Forced response:

$y_{P.I} = ?$ $x(n) = 2u(n)$ let $y_{P.I}(n) = K\, u(n)$

To get K : Sub in D.E.

$$K u(n) - \frac{3}{4} K\, u(n-1) + \frac{1}{8} K\, u(n-2) = 2\cdot(2\, u(n))$$

$\boxed{\text{For } n \geq 2}$

$$K(1) - \frac{3}{4}K(1) + \frac{1}{8}K(1) = 4\,(1)$$

$$K\left[1 - \frac{3}{4} + \frac{1}{8}\right] = 4 \longrightarrow \boxed{K = \frac{32}{3}}$$

$$\boxed{y_{p.I} = \frac{32}{3}\,u(n)}$$

$$\boxed{y_{G.S} = y_{C.F} + y_{p.I} = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{4}\right)^n + \frac{32}{3}u(n)}$$

هنحسب C_1, C_2 من ال Initial Conditions، لكن لازم لما $n \geq 0$

لان الحل $y(n)$ لما $n \geq 0$.

So we have $y(-1) = 1$, $y(-2) = -1$

use D.E ; $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 4u(n)$

let $n=0 \longrightarrow y(0) - \frac{3}{4}\underset{1}{y(\cancel{-1})} + \frac{1}{8}\underset{-1}{y(\cancel{-2})} = 4\underset{1}{u(\cancel{0})}$

$$y(0) = \frac{3}{4} + \frac{1}{8} + 4 = \boxed{\frac{7}{8} + 4}$$

let $n=1 \longrightarrow y(1) - \frac{3}{4}\underset{4+\frac{7}{4}}{y(\cancel{0})} + \frac{1}{8}\underset{1}{y(\cancel{-1})} = 4\underset{1}{u(\cancel{1})}$

$y(1) = \frac{3}{4}\left(\frac{39}{4}\right) - \frac{1}{8} + 4 = \checkmark$

$$y_{total}(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{4}\right)^n + \frac{32}{3}u(n)$$

$\rightsquigarrow y(0) = \frac{7}{8} + 4 = c_1 + c_2 + \frac{32}{3}$ —— ①

$\rightarrow y(1) = \checkmark = \frac{1}{2}c_1 + \frac{1}{4}c_2 + \frac{32}{3}$ —— ②

Solve ① & ② to get c_1, c_2

$\boxed{3}$ Block Diagram Representation

Block diagram representation is closely bounded with the Difference equation representation

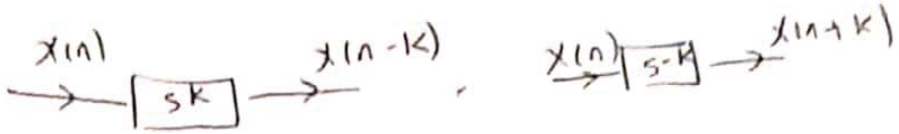
$$y(n) + a_1 y(n-1) + a_2 y(n-2) \ldots = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) \; - - -$$

should be ①

We can implement it directly using block diagram:

1) Direct Form I
2) Direct Form II

Notes:



1- 

" Time shifting "

2 - 

"Scalar multiplication"

or

3 - 

$Y = x(n) \pm w(n)$

"if there is No sign (+) or (-)"

⇓

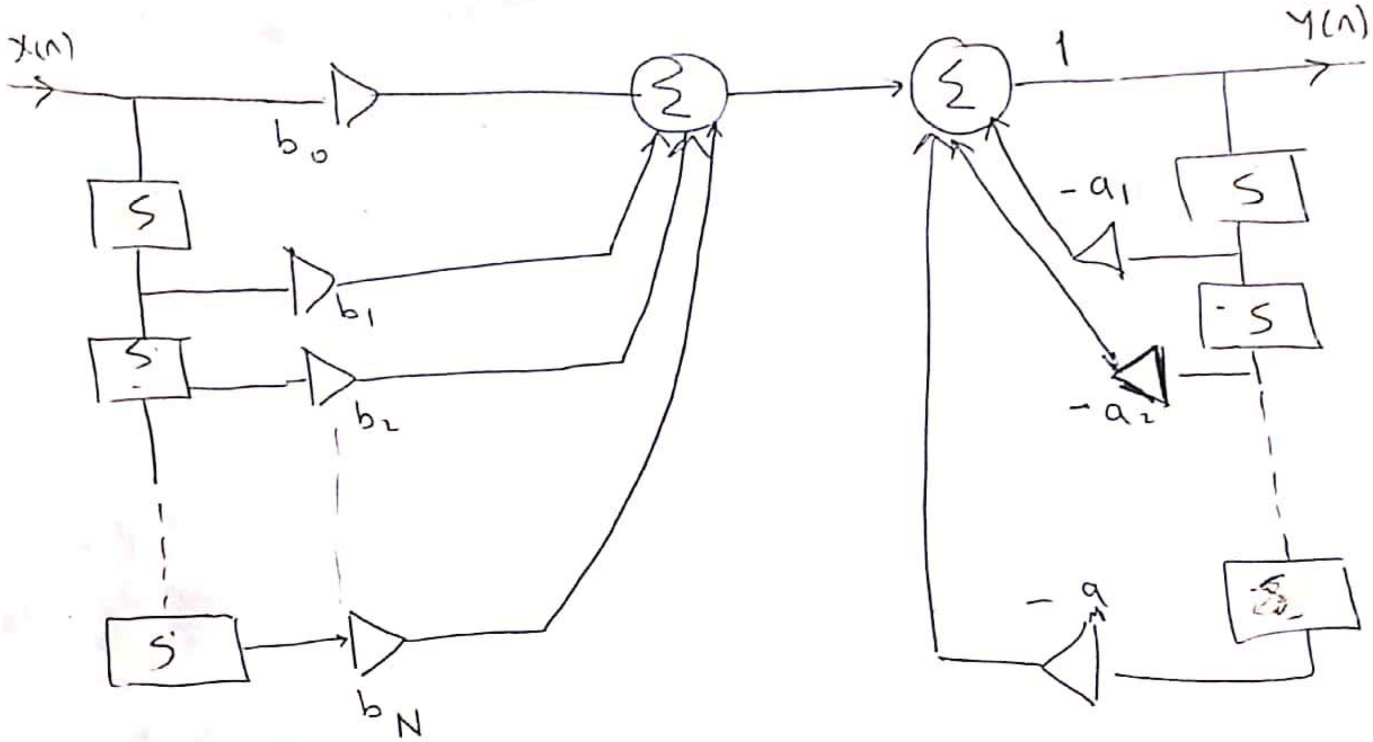
this mean (+)

$\rightarrow y(n) + a_1 y(n-1) - - - = b_0 x(n) + b_1 x(n-1) - -$

Ⓘ Direct Form 1

معاملات X على الشمال

معاملات Y على اليمين بالسالب



Why Direct Form I ?

→ IF an adder Fails, it is easy to detect & fix this error

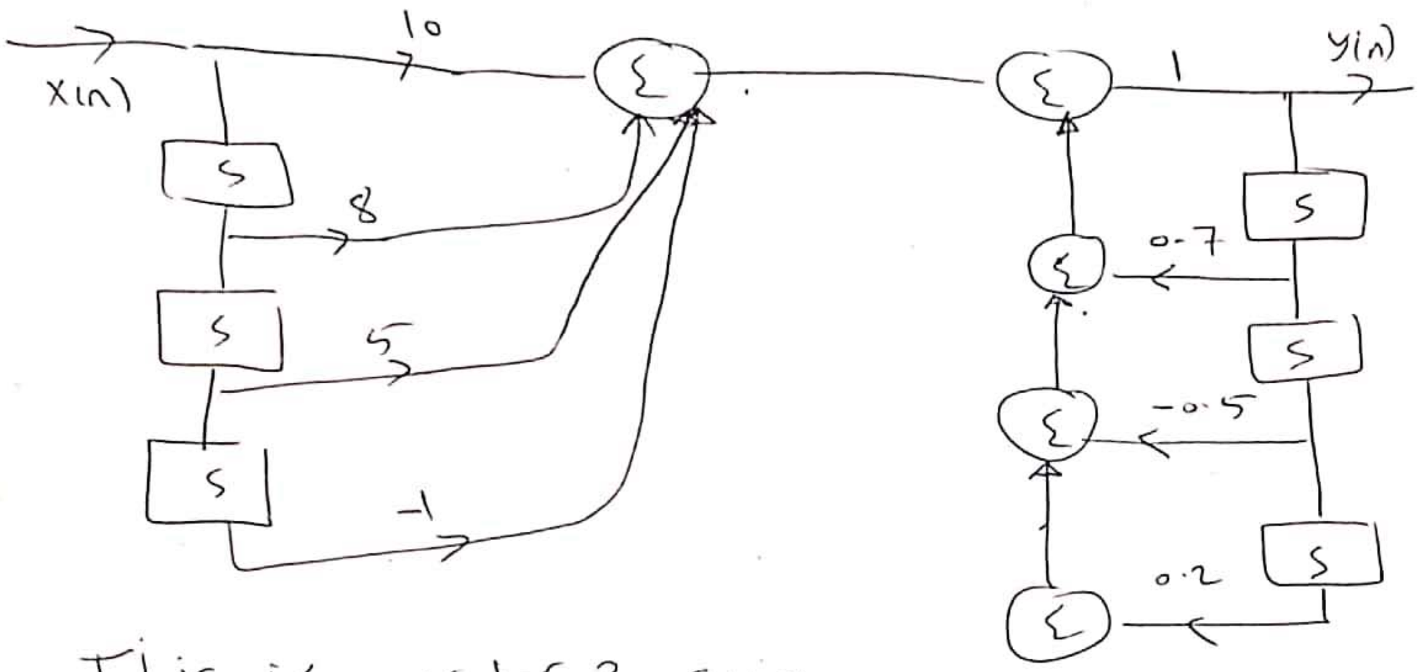
Example

$$y(n) = 10\,x(n) + 8\,x(n-1) + 5\,x(n-2) - x(n-3) + 0.7\,y(n-1) - 0.5\,y(n-2) + 0.2\,y(n-3))$$

sol

Re write it:

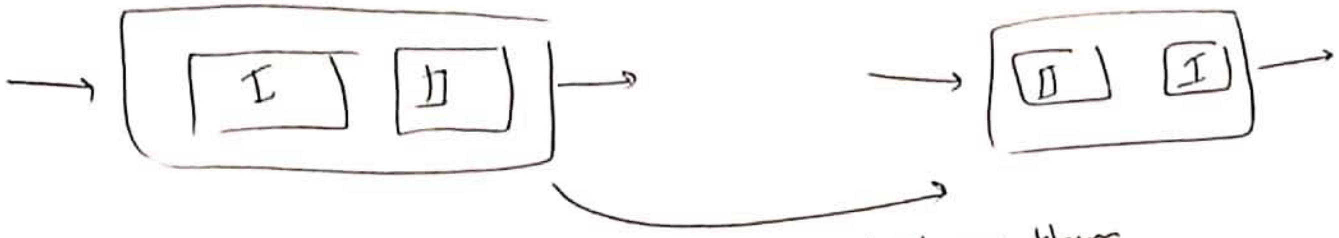
$$y(n) - 0.7\,y(n-1) + 0.5\,y(n-2) - 0.2\,y(n-3)) = 10\,x(n) + 8\,x(n-1) + 5\,x(n-2) - x(n-3))$$



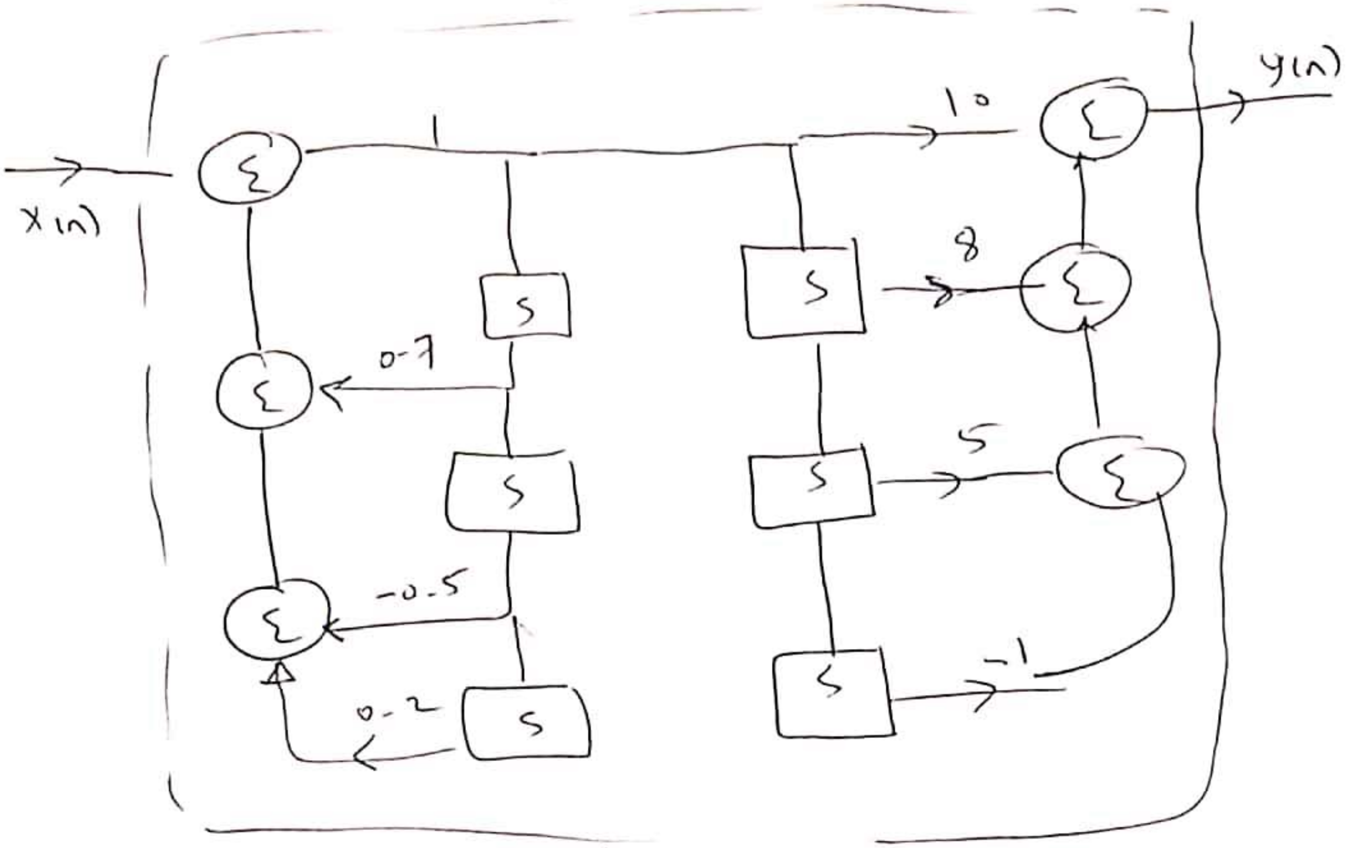
This is order $\underline{3}$ system

Note

We can consider Direct Form I as 2 seprate systems interconnected inside

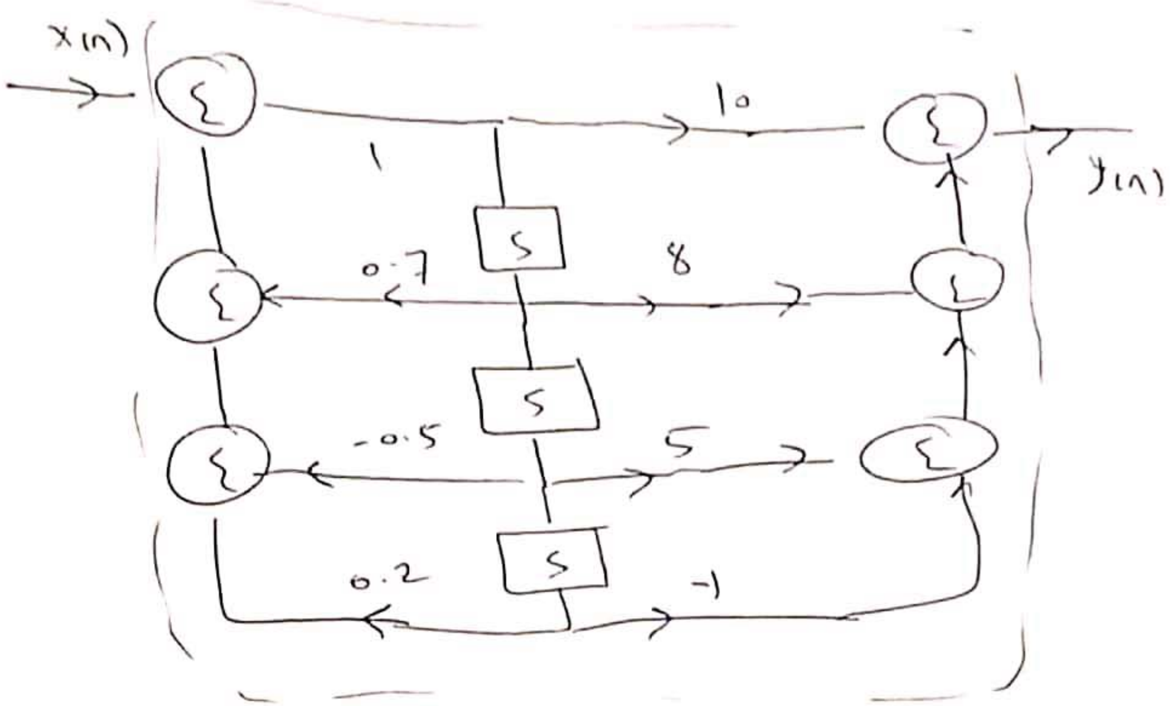


We can switch between them



⇓

This is useful because we can reconstruct the structure to be less costy & more efficient



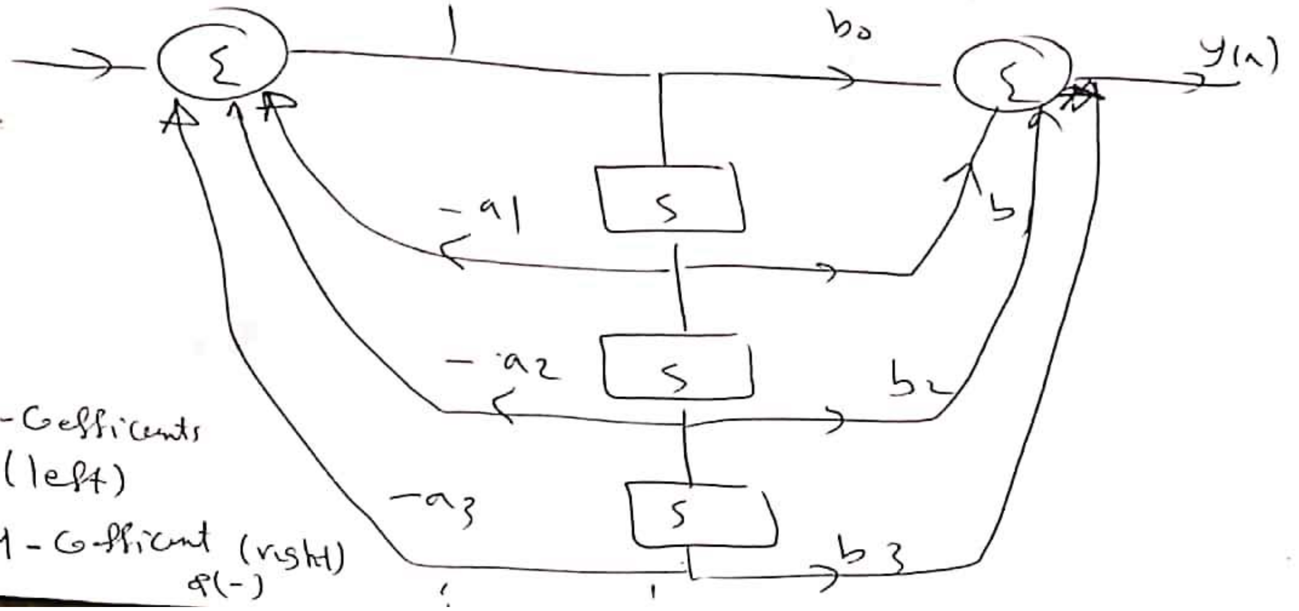
This is called Direct Form II

or Canonic Form.

[It uses minimum number of shift registers]

Generally

$y(n) + a_1 y(n-1) + a_2 y(n-2) - - = b_0 x(n) + b_1 x(n-1) - - -$



*X-Coefficents (left)

* Y-Cofficent (right) & (-)

Example

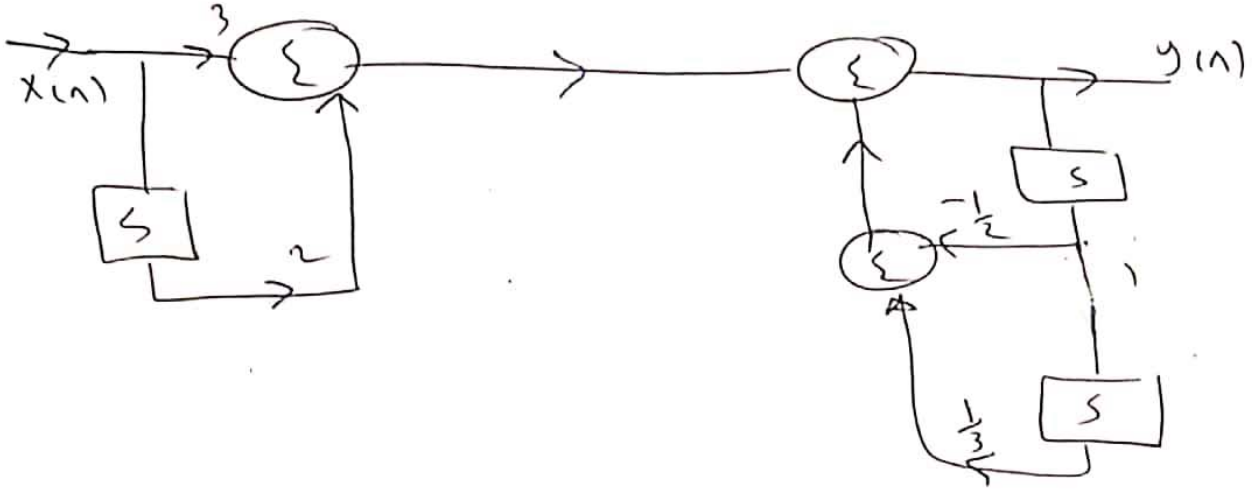
Depict the block diagram of the following LTI system

$$y(n) + \frac{1}{2} y(n-1) - \frac{1}{3} y(n-2) = 3 x(n) + 2 x(n-1)$$

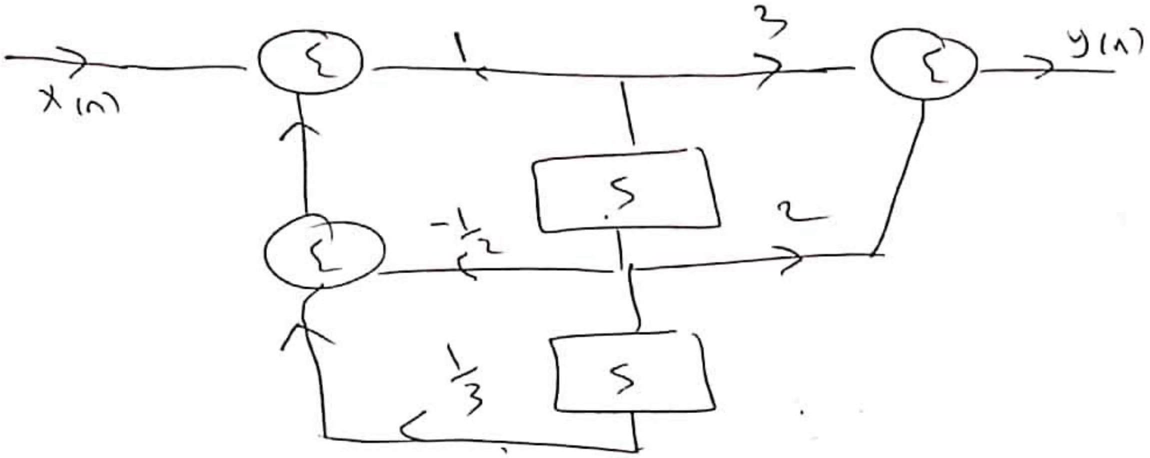
using (1) Direct Form I (2) Direct Form II

Sol

1 Direct Form I ;

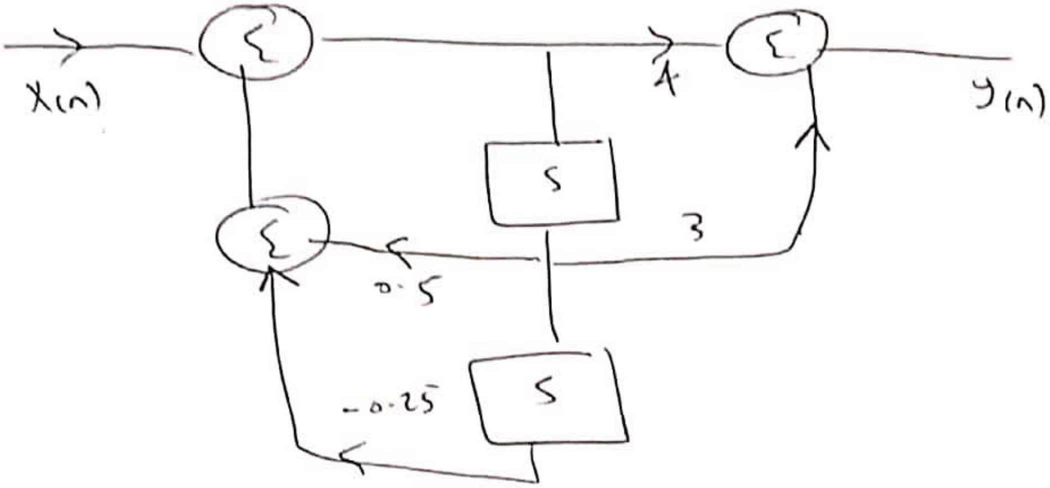


2 Direct Form II ; Canonic form



EXample

Find D.E of the system that has the following Block diagram



Solution

Direct Form II : left Coefficients ⟹ X - Coefficients

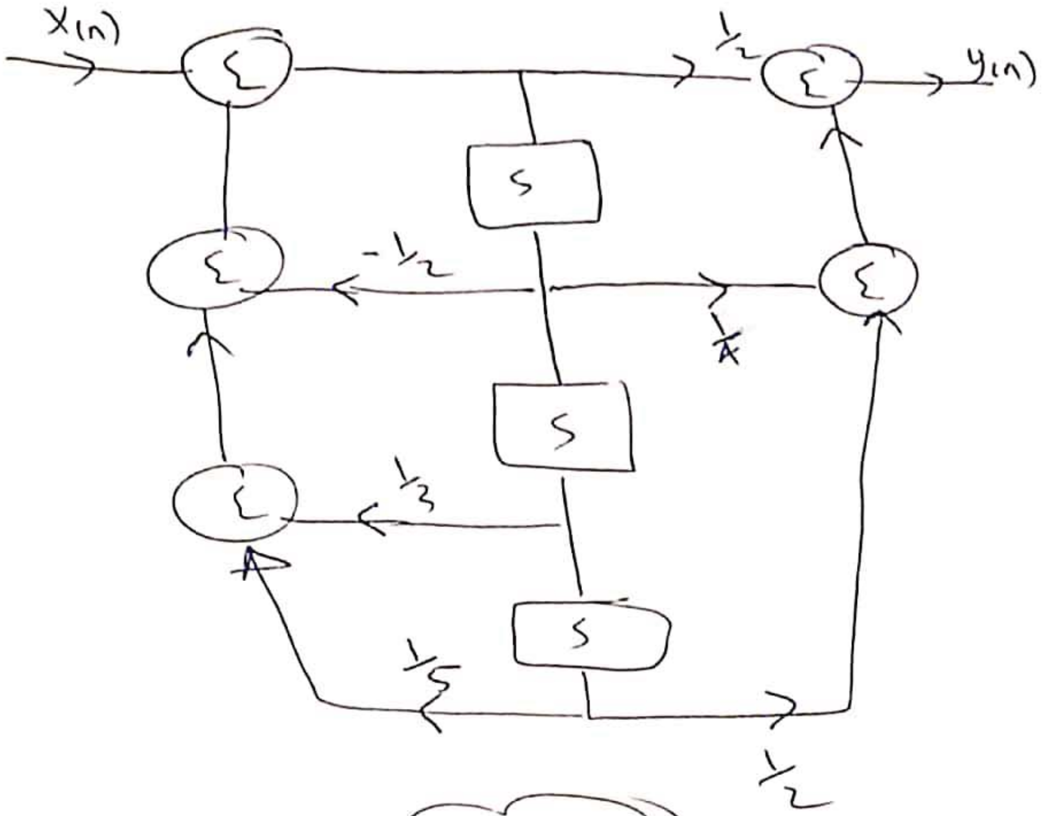
right Coefficients ⟹ y - Coefficients (-ve)

$$y(n) + a_1 y(n-1) + a_2 y(n-2) = b_0 X(n) + b_1 X(n-1)$$

$$\boxed{y(n) - 0.5\, y(n-1) + 0.25\, y(n-2) = 4\, X(n) + 3X(n-1)}$$

Example

Find the Difference Equation



Solution

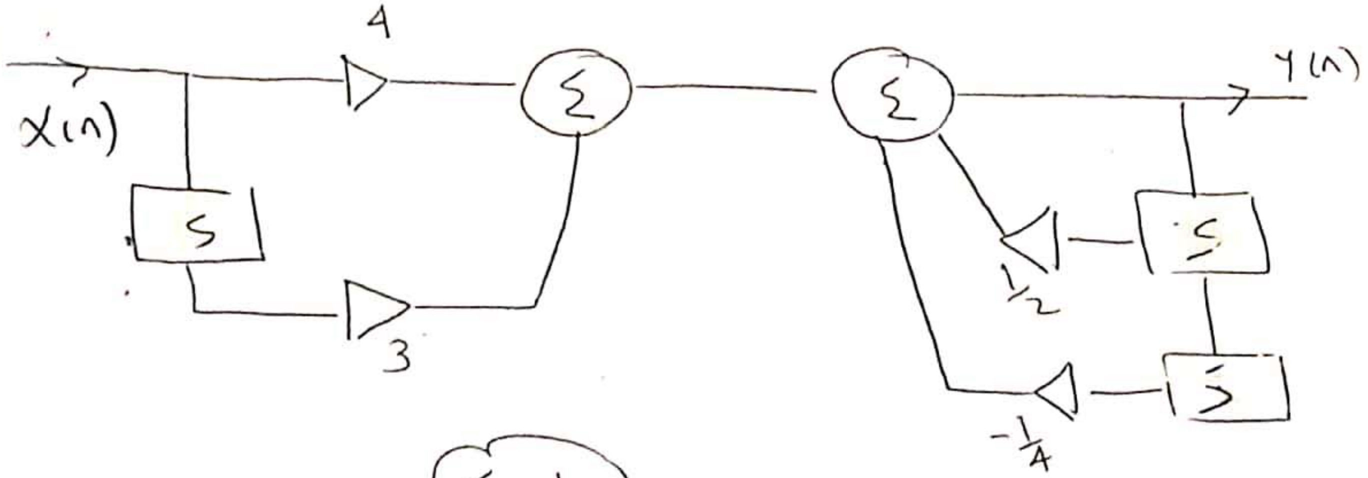
Direct Form II ;

$$y(n) + a_1 y(n-1) + a_2 y(n-2) + a_3 y(n-3) = b_0 x(n) + b_1 x(n-1) + b_3 x(n-3)$$

[Note $b_2 = 0$] ⇒ left ⇒ y - Coeff (opposite sign)
right ⇒ x - Coff

$$y(n) + \frac{1}{2} y(n-1) - \frac{1}{3} y(n-1) - \frac{1}{5} y(n-2) = \frac{1}{2} x(n) + \frac{1}{4} x(n-1) + \frac{1}{2} x(n-3)$$

ex Find Difference equation ,.



Sol

it is Direct Form (I)

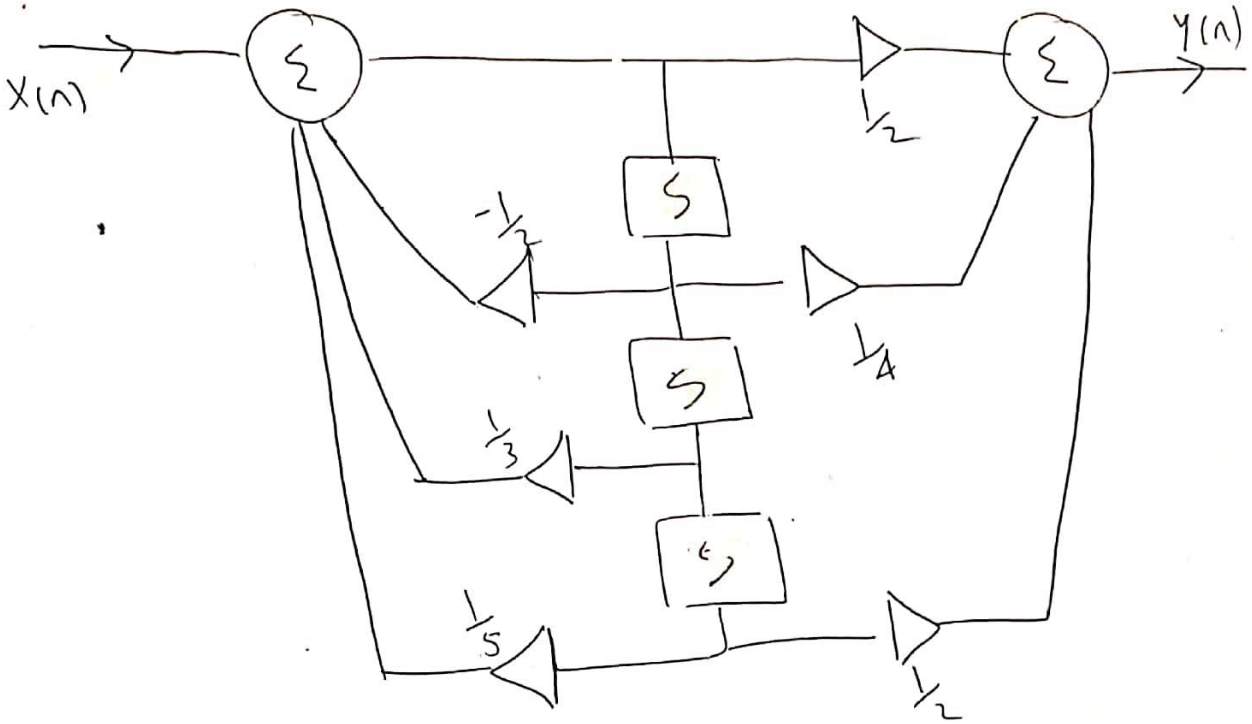
$b_0 = 4$, $b_1 = 3$, $a_1 = -\frac{1}{2}$, $a_2 = \frac{1}{4}$

عكس الاشارة

$\therefore$ D.E : $y(n) + a_1\, y(n-1) + a_2\, y(n-2) = b_0\, x(n) + b_1 x(n-1)$

$$y(n) - \frac{1}{2}\, y(n-1) + \frac{1}{4}\, y(n-2) = 4\, x(n) + 3\, x(n-1)$$

$\boxed{ex}$ Find Difference equation



Sol

it is Direct form (II)

$$\boxed{b_0 = \frac{1}{2},\ b_1 = \frac{1}{4},\ b_2 = 0,\ b_3 = \frac{1}{2}}$$

$$\boxed{a_1 = \frac{1}{2},\ a_2 = -\frac{1}{3},\ a_3 = -\frac{1}{5}}$$

$$\therefore \text{D.E}: \ y(n) + \frac{1}{2} y(n-1) - \frac{1}{3} y(n-2) - \frac{1}{5} y(n-3) = \frac{1}{2} x(n) + \frac{1}{4} x(n-1) + \frac{1}{2} x(n-3)$$