PUNJAB GROUP OF COLLEGES FAISALABAD

Chapte # 02

Chapter Name ,, Vectors and Equilibrium

(Short Answers and Numericals)

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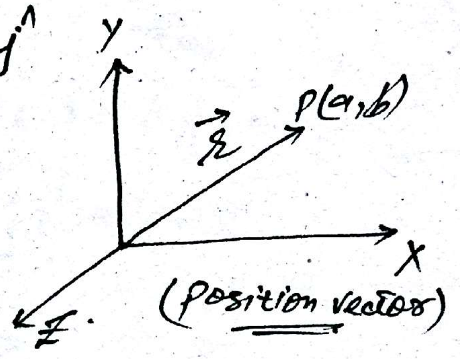
Short Answers

2.1 (i) **Unit vector:-** A vector whose magnitude is one.

$$\hat{A} = \frac{\vec{A}}{A}$$

(ii) **Position vector:-** A vector that describe the location of a particle with respect to origion.
The position vector $\vec{r}$ of a point $P(a,b)$ in xy-plane is;

$$\vec{r} = a\hat{i} + b\hat{j}$$

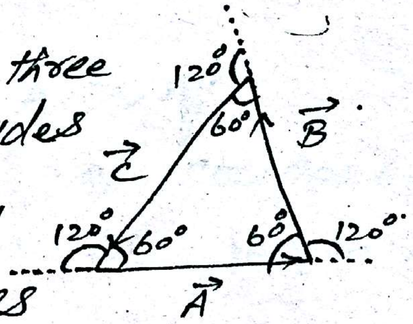


(Position vector)

(iii) **Components of a vector:-**
A component of a vector is its effective value in a given direction. A vector may be considered as the resultant of its components vectors along the specified directors.

$$A_x = A\cos\theta, \quad A_y = A\sin\theta.$$

Q# 2.2 (The resultant vector of three vectors of equal magnitudes is equal to zero.) when they are represented by the three adjacent sides of an equilateral triangle (same angles). (By using head to tail rule, tail of $\vec{c}$ coincides with head of vector $\vec{A}$, so magnitude of vector



$$\vec{R} = \vec{A} + \vec{B} + \vec{c} = 0 \,)$$

$$|\vec{R}| = 0.$$

Orientation: From the fig it can be seen that the orientation b/w $\vec{A}$ and $\vec{B}$ is 120° and b/w $\vec{B}$ and $\vec{c}$ is also 120°. Same for $\vec{c}$ and $\vec{A}$.

Q.4 No, its magnitude cannot be zero, because the magnitude contains the sum of the square of its components. So, If one of component is not zero then the magnitude cannot be zero.

Mathematically,

$$A = \sqrt{A_x^2 + A_y^2}$$

If, $A_x = 0$. then.

$$A = \sqrt{(0)^2 + A_y^2} = \sqrt{A_y^2}$$

$$\boxed{A = A_y}$$

$$A \neq 0.$$

Q.5 No, a vector cannot have the component greater than the magnitude because the components of vectors are always less in magnitude of resultant vector.

$$A = \sqrt{A_x^2 + A_y^2}$$

$$A^2 = A_x^2 + A_y^2$$

$$A^2 \geq A_x^2 \quad \text{or} \quad A^2 \geq A_y^2$$

This shows the magnitude of components can be less or equal than the vector magnitude.

Q.6 No, the magnitude of the vector cannot negative value, because the magnitude contains the sum of square of its components.

$$A = \sqrt{A_x^2 + A_y^2}$$

If one of the component is negative then the square of -ive is always +ive.

2.7 If $\vec{A}+\vec{B}=0$ then;

$$\vec{A}=-\vec{B}$$

we can write their rectangular components;

$$A_x\hat{i}+A_y\hat{j}=-(B_x\hat{i}+B_y\hat{j}).$$

$$A_x\hat{i}+A_y\hat{j}=-B_x\hat{i}-B_y\hat{j}$$

By comparing the co-efficients of $\hat{i}$ and $\hat{j}$

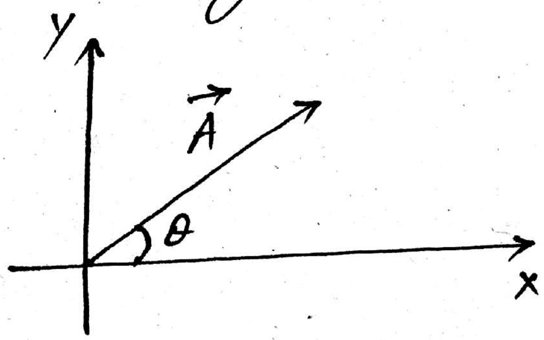
$$A_x=-B_x\,, \qquad A_y=-B_y$$

It Means that If sum of the two vectors is zero then their respective components will be of same Magnitude but in opposite direction.

$\longleftrightarrow$

2.8 Let A_x and A_y are components of vector $\vec{A}$, which makes an angle "θ" with x-axis and then,

$A_x=A\cos\theta$, $A_y=A\sin\theta$



If A_x and A_y are equal in magnitude then;

$$A_x=A_y$$

$$\cancel{A}\cos\theta=\cancel{A}\sin\theta$$

$$\frac{\sin\theta}{\cos\theta}=1 \Rightarrow \tan\theta=1.$$

$$\theta=\tan^{-1}(1) \qquad \because \tan^{-1}(1)=45^\circ$$

$$\theta=45^\circ$$

So, If a vector is Making an angle of 45° with x-axis, then its components will be equal in Magnitude.

$\longleftrightarrow$

(4)

(Q.9) No, it is Not possible to add vector quantity to Scalar quantity because they are totally different things.

* Scalars can be added by simple Mathematical Method. whereas vector can be added by graphical (head to Tail Rule).

* Scalar have only magnitude but vector have both magnitude and direction.

$\longleftrightarrow$

(Q.10) No, we cannot add a zero to a null vector. Because zero is scalar and scalar donot add any vector.

~~This~~ is on the other hand, scalar have only magnitude but vector have both magnitude and direction.

$\longleftrightarrow$

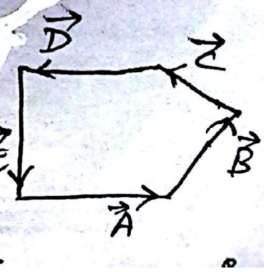
(Q.11) No, their sum cannot be zero.

The sum will be zero if both the vectors have same magnitude and opposite in direction.

So, two vectors of unequal magnitudes can never be combined to give zero resultant, whatever their orientation may be

$\longleftrightarrow$

(2.15) When vector are arranged head to tail, In this case head of last vector coincides with the tail of first vector as shown in Fig.



Their resultant will be zero.

$$\vec{A}+\vec{B}+\vec{C}+\vec{D} = -\vec{E} \quad = \vec{O}$$

$\vec{A}+\vec{B}+\vec{C}+\vec{D}+\vec{E}=0 \Rightarrow \boxed{R=0}$

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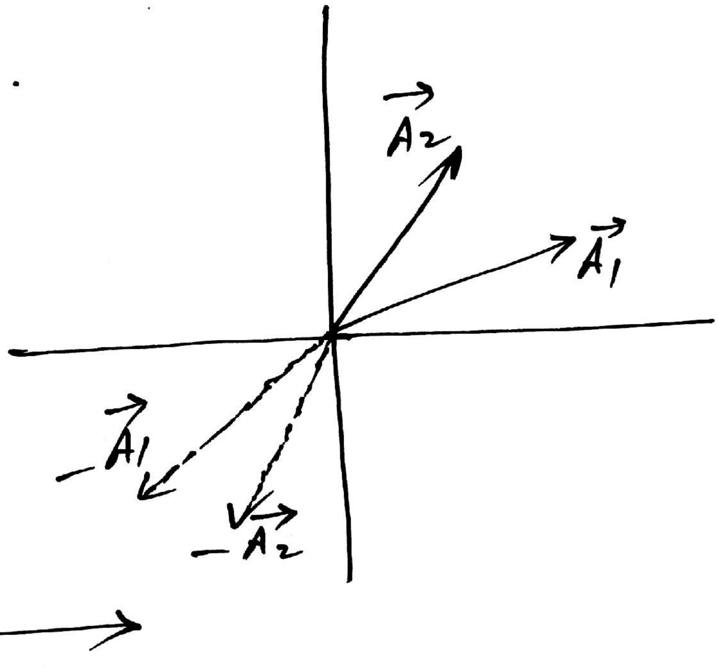
2.17. **Ans:-** It would not change in this case.

From the Fig. 17 all the components of vectors $\vec{A}_1$ and $\vec{A}_2$ are reversed, then again the direction of $(-\vec{A}_1)\times(-\vec{A}_2)$ remains the same.

Mathematically;

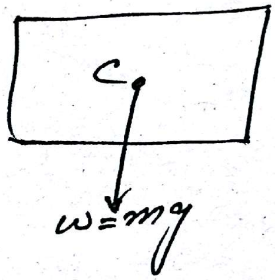
$$\vec{A}_1 \times \vec{A}_2 = (-\vec{A}_1) \times (-\vec{A}_2).$$

$$(\vec{A}_1 \times \vec{A}_2) = \vec{A}_1 \times \vec{A}_2$$



Q.21) No, a body cannot rotate about its centre of Gravity under the action of its weight. Because weight always acts at the centre of Gravity.

In this case line of action of force (weight) passes throught centre of Gravity of the body so,



Moment arm of force $= l = 0$.

Force $= w$

Then $\tau = (F)(l)$

$= (w)(0)$

$$\boxed{\tau = 0}$$

No, turning Effect is Produced. Therefore a body cannot rotate about its centre of gravity under the action of its weight.

UNIT: 2 → Vectors

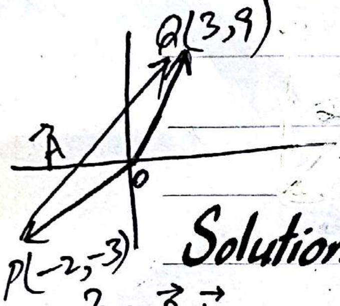
Numericals:-

2.1 Given data:-

Point $P(-2,-3)$

Point $Q(3,9)$

Pictorial representation:-



Solution:- Let In vector form; $\vec{P}=\vec{r_1} = -2\hat{i}-3\hat{j}$

$\vec{Q}=r_2 = 3\hat{i}+9\hat{j}$

[Formula used; $\vec{r}=\vec{r_2}-\vec{r_1}$]

$\vec{r} = ? \rightarrow \vec{r}=(3\hat{i}+9\hat{j})-(-2\hat{i}-3\hat{j})$

$\vec{r} = (3\hat{i}+9\hat{j}+2\hat{i}+3\hat{j})$

$\vec{r} = 5\hat{i}+12\hat{j}$.

$|\vec{r}| = \sqrt{(5)^2+(12)^2}$

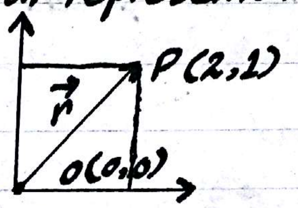
$= \sqrt{169} = 13$ units

2.2 Given data:-

Point $a(0,0)$

Point $b(2,1)$

Pictorial representation:-



Solution:- Let

$\vec{O}=\vec{r_1} = a(0,0)$

$\vec{P}=\vec{r_2} = b(2,1)$

$\vec{r} = ?$

$\vec{r} = \vec{r_2}-\vec{r_1}$

$\vec{r} = (2\hat{i}+\hat{j})-(0\hat{i}+0\hat{j})$

$\vec{r} = 2\hat{i}+\hat{j}$

$|\vec{r}| = \sqrt{(2)^2+(1)^2} = \sqrt{4+1}$

$= \sqrt{5} = 2.2m$

2.3 Given data:-

$\vec{A} = 4\hat{i}+3\hat{j}$

$\hat{A} = ?$

By formula:-

$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

$|\vec{A}| = \sqrt{16+9}$

$= \sqrt{25} = 5$

$\hat{A} = \frac{4\hat{i}+3\hat{j}}{5}$.

2.4 Given data:-

$\vec{r_1} = 3\hat{i}+7\hat{j}$.

$\vec{r_2} = -2\hat{i}+3\hat{j}$.

$\vec{r}=\vec{r_2}-\vec{r_1} = ?$

Orientation $= ?$

Solution:-

$\vec{r} = \vec{r_2}-\vec{r_1}$.

$\vec{r} = (-2\hat{i}+3\hat{j})-(+3\hat{i}+7\hat{j})$.

$\vec{r} = -2\hat{i}+3\hat{j}-3\hat{i}-7\hat{j}$.

$\vec{r} = -5\hat{i}-4\hat{j}$.

$|\vec{r}| = \sqrt{25+16} = \sqrt{41} = 6.4$

As $x=-5$, and $y=-4$

so it lies in IIIrd quadrant.

Here; $\theta = 180-\phi$ —— (I)

and $\tan\phi = \frac{y}{x}$.

$\phi = \tan^{-1}\frac{y}{x}$.

$\phi = \tan^{-1}\left(\frac{-4}{-5}\right)$

$\phi = 38.65°$

$\phi = 39°$ Put in (I).

$\theta = 180°+39°$

$\therefore \theta = 219°$.

2.5 Given data:-

$\vec{B}+\vec{A} = 6\hat{i}+\hat{j}$

$\vec{A}-\vec{B} = -4\hat{i}+7\hat{j}$

$|\vec{A}| = ?$

Solution:-

$\vec{A}+\vec{B} = 6\hat{i}+7\hat{j}$

$\vec{A}-\vec{B} = -4\hat{i}+7\hat{j}$

$2\vec{A} = 2\hat{i}+8\hat{j}$

$\vec{A} = \dfrac{2\hat{i}+8\hat{j}}{2}$

$\vec{A} = \dfrac{\cancel{2}(\hat{i}+4\hat{j})}{\cancel{2}}$

$\vec{A} = \hat{i}+4\hat{j}$

$|\vec{A}| = \sqrt{(1)^2+(4)^2}$

$|\vec{A}| = \sqrt{17}$

$|\vec{A}| = 4.1$

2.6 Given data:-

$\vec{A} = 2\hat{i}+3\hat{j}$

$\vec{B} = 3\hat{i}-4\hat{j}$

(a)- $\vec{C} = \vec{A}+\vec{B}$

(b)- $\vec{D} = 3\vec{A}-2\vec{B}$

Find their magnitude. and angles too.

Solution:-

(a)- $\vec{C} = \vec{A}+\vec{B}$

$\vec{C} = (2\hat{i}+3\hat{j})+(3\hat{i}-4\hat{j})$

$\vec{C} = (2\hat{i}+3\hat{j}+3\hat{i}-4\hat{j})$

$\vec{C} = (5\hat{i}-\hat{j})$

$|\vec{C}| = \sqrt{(5)^2-(1)^2}$

$|\vec{C}| = \sqrt{25+1} = \sqrt{26}$

$|\vec{C}| = 5.1$

As x = +ive, y = −ive so it lies in IV quad.

$\therefore\ \theta = 360-\phi$ — (I)

and $\tan\phi = \dfrac{y}{x}$

$\phi = \tan^{-1}\dfrac{y}{x}$

$\phi = \tan^{-1}\left(\dfrac{1}{5}\right)$

$\phi = 11.30$ Put in (I)

$\therefore\ \theta = 360-11.30$

$\therefore\ \theta = 349°$

(b)- $D = 3\vec{A}-2\vec{B}$

$D = 3(2\hat{i}+3\hat{j})-2(3\hat{i}-4\hat{j})$

$= (\cancel{6\hat{i}}+9\hat{j}-\cancel{6\hat{i}}+8\hat{j})$

$= (0+17\hat{j})$

and $|\vec{B}| = \sqrt{(0)^2+(17)^2}$

$= \sqrt{17^2}$

$|\vec{D}| = 17$

As, x=+ive, y=+ive so θ lies in Ist Q.

where, $\theta = \phi$ — (I)

$\phi = \tan^{-1}\left(\dfrac{17}{0}\right)$

$= \tan^{-1}(\infty)$

$\phi = 90°$ Put in (I)

$\theta = 90°$

here ∞ = infinitive, e.g. $\pi = 3.147...$

2.7 Given data:-

$\vec{A} = 5\hat{i}+\hat{j}$

$\vec{B} = 2\hat{i}+4\hat{j}$

$\theta = ?$

The dot product of A and B

$\vec{A}\cdot\vec{B} = AB\cos\theta$

and $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

$\cos\theta = \dfrac{(5\hat{i}+\hat{j})\cdot(2\hat{i}+4\hat{j})}{(\sqrt{(5)^2+(1)^2})(\sqrt{(2)^2+(4)^2})}$

$\cos\theta = \dfrac{(5)(2)\hat{i}\cdot\hat{i}+(1)(4)\hat{j}\cdot\hat{j}}{\sqrt{26}\times\sqrt{20}}$

$\cos\theta = \dfrac{10+4}{5.1\times4.5} = \dfrac{14}{23}$

$\therefore\ \cos\theta = 0.61$

and $\theta = \cos^{-1}(0.61)$

$\theta = 52°$

2.8. Given data:-

$\vec{F} = 3\hat{i} + 2\hat{j}$.

$\vec{r_A} = 2\hat{i} - \hat{j}$.

$\vec{r_B} = 6\hat{i} + 4\hat{j}$

$\vec{d} = \vec{r_B} - \vec{r_A}$.

Solution:-

$\vec{d} = (6\hat{i} + 4\hat{j}) - (2\hat{i} - \hat{j})$.

$\vec{d} = 6\hat{i} + 4\hat{j} - 2\hat{i} + \hat{j}$.

$\vec{d} = 4\hat{i} + 5\hat{j}$.

By formula:-

$W = \vec{F} \cdot \vec{d}$

$W = (3\hat{i} + 2\hat{j}) \cdot (4\hat{i} + 5\hat{j})$.

$W = 12\hat{i}\cdot\hat{i} + 15\hat{i}\cdot\hat{j} + 18\hat{j}\cdot\hat{i} + 10\hat{j}\cdot\hat{j}$

$W = 12 + 0 + 0 + 10$

$W = 22$ Units.

2.9. Given data:-

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$.

$\vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}$.

$\vec{C} = 4\hat{i} + \hat{j} - 5\hat{k}$.

(The given vectors are mutually perpendicular to each other)

Solution:-

IF ⇒ $\vec{A} \cdot \vec{B} = (\hat{i} \cdot \hat{j} \cdot \hat{k}) \cdot (2\hat{i} - 3\hat{j} + \hat{k})$

$\vec{A} \cdot \vec{B} = 2\hat{i}\cdot\hat{i} - 3\hat{j}\cdot\hat{j} - 5\hat{k}\cdot\hat{k}$

$\vec{A} \cdot \vec{B} = 2 - 3 + 1$

$\vec{A} \cdot \vec{B} = \cancel{3} - \cancel{3} = 0$

and $\vec{B} \cdot \vec{C} = (2\hat{i} - 3\hat{j} + \hat{k}) \cdot (4\hat{i} + \hat{j} - 5\hat{k})$

$\vec{B} \cdot \vec{C} = 8\hat{i}\cdot\hat{i} - 3\hat{j}\cdot\hat{j} - 5\hat{k}\cdot\hat{k}$.

$\vec{B} \cdot \vec{C} = 8 - 3 - 5$.

$= \cancel{8} - \cancel{8}$

$\vec{B} \cdot \vec{C} = 0$.

and $\vec{C} \cdot \vec{A} = (4\hat{i} + \hat{j} - 5\hat{k}) \cdot (\hat{i} \cdot \hat{j} \cdot \hat{k})$

$= 4\hat{i}\cdot\hat{i} + 1\hat{j}\cdot\hat{j} - 5\hat{k}\cdot\hat{k}$.

$= 4 + 1 - 5$

$= 5 - 5$

$\vec{C} \cdot \vec{A} = 0$.

As in dot product, the product of any two mutually perpendicular vectors is equals to zero. So we **"conclude that these three vectors are mutually perpendicular to each other."**

2.10 Given data:-

$\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$.

$\vec{B} = 3\hat{i} + 0\hat{j} - 4\hat{k}$.

$A\cos\theta = ?$

Solution:-

$\vec{A} \cdot \vec{B} = AB\cos\theta \rightarrow AB\cos\theta = \vec{A} \cdot \vec{B}$

$A\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{B}$ —(i)

here; $|\vec{B}| = \sqrt{(3)^2 + (0)^2 + (-4)^2}$

$= \sqrt{9 + 16}$. $= \sqrt{25}$

and $|\vec{B}| = 5$.

Now; $\vec{A} \cdot \vec{B} = (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} + 0\hat{j} - 4\hat{k})$

$\vec{A} \cdot \vec{B} = 3 - 0 - 12$

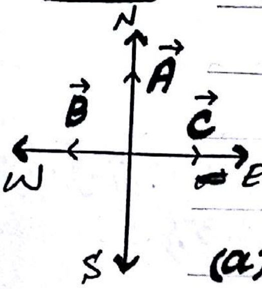
$\vec{A} \cdot \vec{B} = -9$

∴ putting the values in equation (1) we have:-

$A\cos\theta = \dfrac{-9}{5}$. Answer.

2.11

Given data:-



$\vec{A}$ = 4 Units north.

$\vec{B}$ = 3 Units west.

$\vec{C}$ = 8 Units east

(a)- $\vec{A}\times\vec{B} = ?$

(b)- $\vec{A}\times\vec{C} = ?$

(c)- $\vec{B}\times\vec{C} = ?$

Solution:-

(a)- $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$.

(as angle between $\vec{A}$ and $\vec{B}$ is 90° i.e from north to west).

⇒ putting the values, we get:-

$\vec{A}\times\vec{B} = 4\times 3(1)\,\hat{n}$ $\because \sin 90 = 1$

$= 12\hat{n}$.

Direction:- According to right-hand rule, direction of $\vec{A}\cdot\vec{B}$ (i.e $\hat{n}$) is upwards.

(b)- $\vec{A}\times\vec{C} = AC\sin\theta\,\hat{n}$.

(as angle between $\vec{A}$ and $\vec{C}$ is 90°) i.e from north to east.

⇒ So putting the values, we have:-

$\vec{A}\times\vec{C} = 4\times 8(\sin 90)\,\hat{n}$.

$\vec{A}\times\vec{C} = 32(1)\hat{n}$.

$\vec{A}\times\vec{C} = 32\,\hat{n}$.

Direction:- According to right-hand rule, direction of $\vec{A}\times\vec{C}$ i.e ($\hat{n}$) is vertically downwards.

(c):- $\vec{B}\times\vec{C} = BC\sin\theta\,\hat{n}$.

(as angle between $\vec{B}$ and $\vec{C}$ is 180°, from west to east)

⇒ So, putting the values we have:-

$\vec{B}\times\vec{C} = 3\times 8\sin 180\,\hat{n}$.

$\vec{B}\times\vec{C} = 3\times 8(0)\,\hat{n}$.

$\vec{B}\times\vec{C} = 0\hat{n}$.

$\vec{B}\times\vec{C} = \vec{0}$.

Direction:- As $\vec{B}$ and $\vec{C}$ have a resultant equals to zero, so it must have arbitrary direction.

2.12

Given data:-

$\vec{F} = (-3\hat{i}+\hat{j}+5\hat{k})$ (newton)

$\vec{r} = (7\hat{i}+3\hat{j}+\hat{k})$ (meter).

$\vec{\tau} = \vec{r}\times\vec{F} = ?$

Solution:-

As $\vec{\tau} = \vec{r}\times\vec{F}$.

So putting the values, we have:-

$\vec{\tau} = \vec{r}\times\vec{F}$

$\vec{\tau} = (7\hat{i}+3\hat{j}+\hat{k})\times(-3\hat{i}+\hat{j}+5\hat{k})$

In determinant form, we have:-

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$$

$\vec{\tau} = \hat{i}(15-1) - \hat{j}(35+3) + \hat{k}(7+9)$

$\vec{\tau} = 14\hat{i} - 38\hat{j} + 16\hat{k}$ Nm.

2.13 Given data:-

$\vec{F} = \hat{i} - 2\hat{j}$

Position vector →

$\vec{r} = -\hat{j} + \hat{k}$.

Moment of $\vec{F}$ about origin $(\vec{\tau}) = ?$

Moment of $\vec{F}$ about the point of which position vector is $\hat{i} + \hat{k} = \vec{\tau}' = ?$

Solution:-

(a)- $\vec{\tau}$ (about origin)

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} = (-\hat{j} + \hat{k}) \times (\hat{i} - 2\hat{j})$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 1 \\ 1 & -2 & 0 \end{vmatrix}$$

$\vec{\tau} = \hat{i}(0+2) - \hat{j}(0-1) + \hat{k}(0+1)$

$\vec{\tau} = 2\hat{i} + \hat{j} + \hat{k}$.

(b)- $\vec{\tau}$ (about about point having position vector $\hat{i} + \hat{k}$)

In this case:-

$\vec{r_1} = \hat{i} + \hat{k}$.

$\vec{r_2} = -\hat{j} + \hat{k}$.

$\vec{r_2} - \vec{r_1} = (-\hat{j} + \hat{k}) - (\hat{i} + \hat{k})$

$\vec{r_2} - \vec{r_1} = -\hat{j} + \hat{k} - \hat{i} - \hat{k}$.

Now, $\vec{r} = \vec{r_2} - \vec{r_1} = -\hat{i} - \hat{j}$.

Now, putting the values in the formula:-

$\vec{\tau}' = \vec{F} \times \vec{r}$.

$\vec{\tau}' = (-\hat{i} - \hat{j}) \times (\hat{i} - 2\hat{j})$

$$\vec{\tau}' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 0 \\ 1 & -2 & 0 \end{vmatrix}$$

$\vec{\tau}' = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(2+1)$.

$\vec{\tau}' = 3\hat{k}$.

2.14 Given data:-

Let $\vec{A}$ and $\vec{B}$ be two given vectors.

And;

$|\vec{A} \cdot \vec{B}| = 6\sqrt{3}$.

$|\vec{A} \times \vec{B}| = 6$.

Angle between two vectors $\theta = ?$

Solution:-

$|\vec{A} \cdot \vec{B}| = AB\cos\theta = 6\sqrt{3}$ —(i)

$|\vec{A} \times \vec{B}| = AB\sin\theta = 6$ —(ii)

Dividing the two eqs, (as eq (ii) by eq (i)) **we have:-**

$$\frac{AB\sin\theta}{AB\cos\theta} = \frac{\cancel{6}}{\cancel{6}\sqrt{3}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}.$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{\sqrt{3}}\right).$$

$\therefore \theta = 30°$.

2.15

Given data:-

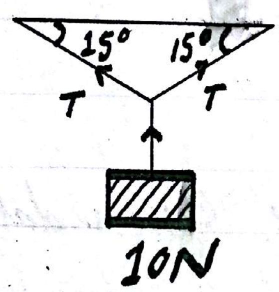
→ Load (W) = 10 N

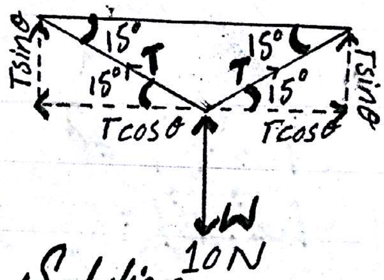
→ $\theta = 15^\circ$.

Tension in cloth line

$T = ?$

Pictorial representation:





Solution:-

Resolving (T) into its rectangular components:-

and

→ Applying 1st cond. of equilibrium;

$\Sigma F_x = 0$.

$T\cos 15^\circ + T\cos 15^\circ = 0$

$T\cos 15^\circ = T\cos 15^\circ$.

Now applying;

$\Sigma F_y = 0$

$T\sin 15^\circ + T\sin 15^\circ = 0$

$2T\sin 15^\circ = W$

$T = \frac{W}{2\sin 15^\circ}$

$T = \frac{10}{2 \times 0.26}$

$T = \frac{10}{0.52}$

$T = 19.3N$.