S.Aleem.A

CHAPTER - 9

Physical Optics

OPTICS :- It is the branch of Physics which deals with the study of production, propagation and properties of light.

LIGHT :- Light is a type of energy which produces sensation of vision.

WAVE THEORY OF LIGHT

In about 1690, Huygen's, a Dutch scientist, proposed that light energy from a luminous source travels in space by means of wave motion. The experimental results in support of wave theory in Huygen's time was not convincing. Because in those days many scientists believed on the Newton's corpuscular theory of light. However, in 1801, Young discovered a wave characteristic, the interference of light which supported the Huygen's wave theory.

9.1 WAVEFRONTS

1- **Definition :-** Such a surface on which all the points have the same phase of vibration is known as wavefront.

2- **Explanation :-**

(a)- Consider a point source of light at 'S'. Waves emitted from this source will propagate outwards in all directions with speed c. After time 't', they will reach the surface of a sphere with centre as S and radius as ct. ($\because s = vt$). Every point on the surface of this sphere will be set into vibration by the waves reaching there. As the distance of all these points from the source is the same, so their state of vibration will be identical. In other words we can say that all the points on the surface of the sphere will have the same phase.

(b)- Types :-

There are two types of the wavefronts

(i)- Spherical Wavefront :-

In case of point source of light in a homogeneous medium, the wavefronts will be concentric spheres with centre at source.

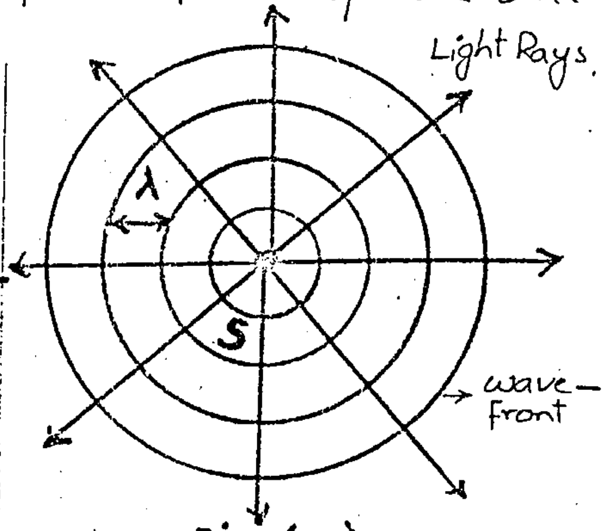


fig.(a)

Such wavefronts are known as spherical wavefronts, as shown in figure (a).

(ii)- Plane Wavefront :-

At very large distance from the source, a small portion of spherical wavefront becomes nearly a plane. This type of wavefront is called as plane wavefront as shown in figure (b).

e.g :- Light from the sun reaches the earth in plane wavefronts. as shown in fig.(c).

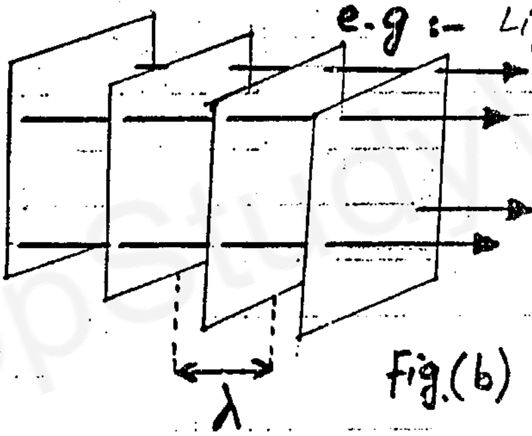


Fig.(b)

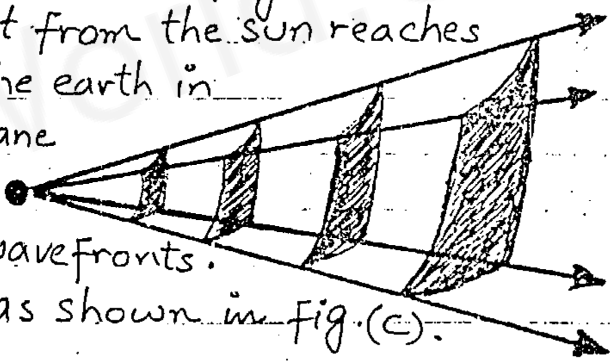


Fig.(c)

➔ The wave propagates in space by the motion of the wavefronts. The distance between the consecutive wavefront is one wave length.

(c)- Ray of Light :- A line normal to the wavefront including the direction of motion is called a ray of light.

(d)- Plane Wave

Small segments of large spherical wavefronts

approximate a plane wave.
In the study of interference and diffraction, plane waves and plane wavefronts are considered.
A usual way to obtain a plane wave is to place point source of light at the focus of a convex lens.
The rays coming out of the lens will constitute plane waves.

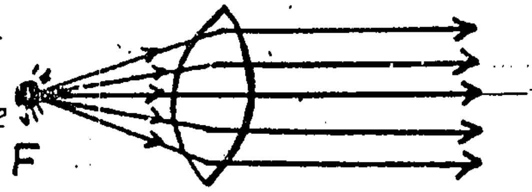


Fig.(d).

9.2 HUYGEN'S PRINCIPLE

1- **Introduction** :- Knowing the shape and location of a wavefront at any time instant 't', Huygen's principle enables us to determine the shape and location of the new wavefront at a later time '$t + \Delta t$'.

2- **Statement** :- This principle consists of two parts:

(a)- Every point of a wavefront may be considered as a source of secondary wavelets which spread out in forward direction with a speed equal to the speed of propagation of the wave.

(b)- The new position of the wavefront after a certain interval of time can be found by constructing a surface that touches all the secondary wavelets.

3- **Explanation** :-
Consider a point source 's'. AB represents the wavefront at any instant 't'. To determine the wavefront at time $t + \Delta t$, draw secondary wavelets with centre at various points on the wavefront AB and radius as '$c\Delta t$' ($s = vt$) where 'c' is the speed of the propagation of

of the wave as shown in fig (a). The location of new wavefront at Time $t + \Delta t$ is $A'B'$ which is a tangent envelope to all the secondary wavelets.

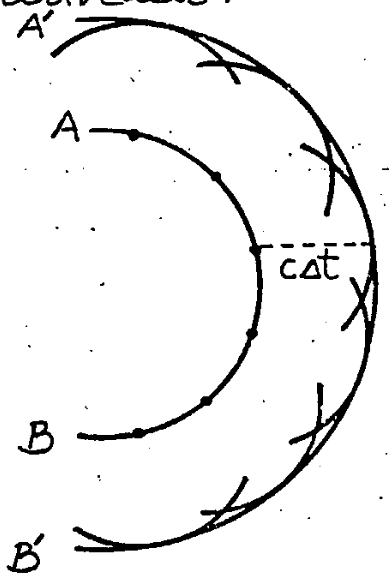


fig. (a)

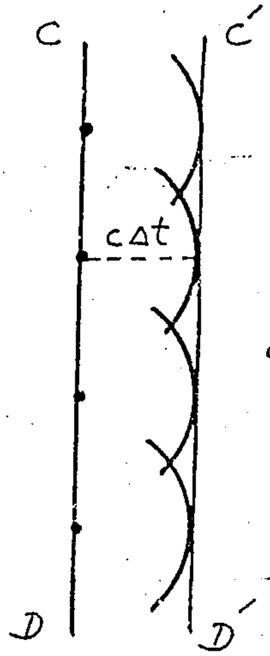


fig (b)

Figure (b) shows a similar construction for a plane wavefront.

In the fig (a) and fig (b) $A'B'$ and $C'D'$ are the new positions of wavefronts.

9.3 INTERFERENCE OF LIGHT WAVES

1- Introduction:-

An oil film floating on water surface exhibits beautiful colour patterns. This happens due to interference of light waves.

As we know that when two waves travel in the same medium, they would interfere constructively or destructively. The amplitude of the resultant wave will be greater than either of the individual wave, if they interfere constructively. In the case of destructive interference, the amplitude of the resultant wave will be less than that of either of the individual waves.

2 Conditions for Detectable Interference

In order to observe the interference of light waves the following conditions must be applied.

(a) The interfering beams must be monochromatic, that is, of a single wavelength.
(b) The interfering beams of light must be coherent.
(c) The principle of linear superposition should be applicable.

3. **Explanation:-**

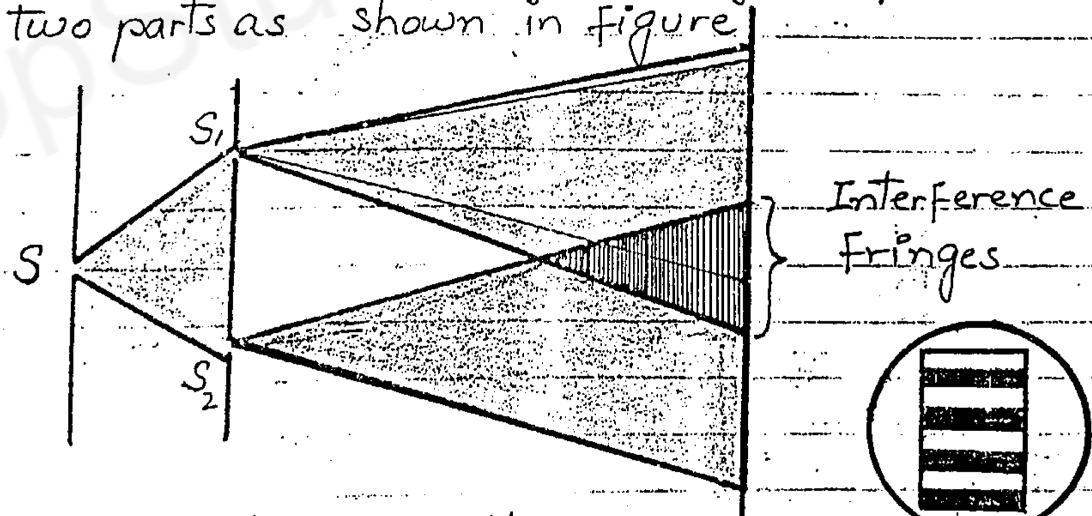
(a). Consider two or more sources of light waves of the same wavelength. If the sources send out crests or troughs at the same instant, the individual waves maintain a constant phase difference with one another. Such waves will interfere constructively or destructively.

(b). **Coherent Sources**

(i) **Def:-** The monochromatic sources of light which emit waves, having a constant phase difference, are called Coherent Sources.

(ii). **Method to produce Coherent Sources**

A common method of producing two coherent light beams is to use a monochromatic source to illuminate a screen containing two small holes, usually in the shape of slits. The light emerging from the two slits is coherent because a single source produces the original beam and the two slits serve only to split it into two parts as shown in figure



→ The points on a Huygen's wavefront which send out secondary wavelets are also coherent sources of light.

9.4 YOUNG'S DOUBLE SLIT EXPERIMENT

1- Introduction:- In 1801, Thomas Young a British Physicist performed an experiment to study the interference effect of light, which supported the Huygen's wave theory.

2. Experimental arrangement:-

A screen having two narrow slits is illuminated by a beam of monochromatic light. The portions of the wavefronts incident on the slits behave as sources of secondary wavelets (Huygen's principle). The secondary wavelets leaving the slits are coherent. Superposition of these wavelets result in a series of bright and dark bands (fringes) which are observed on a second screen placed at some distance parallel to the first screen.

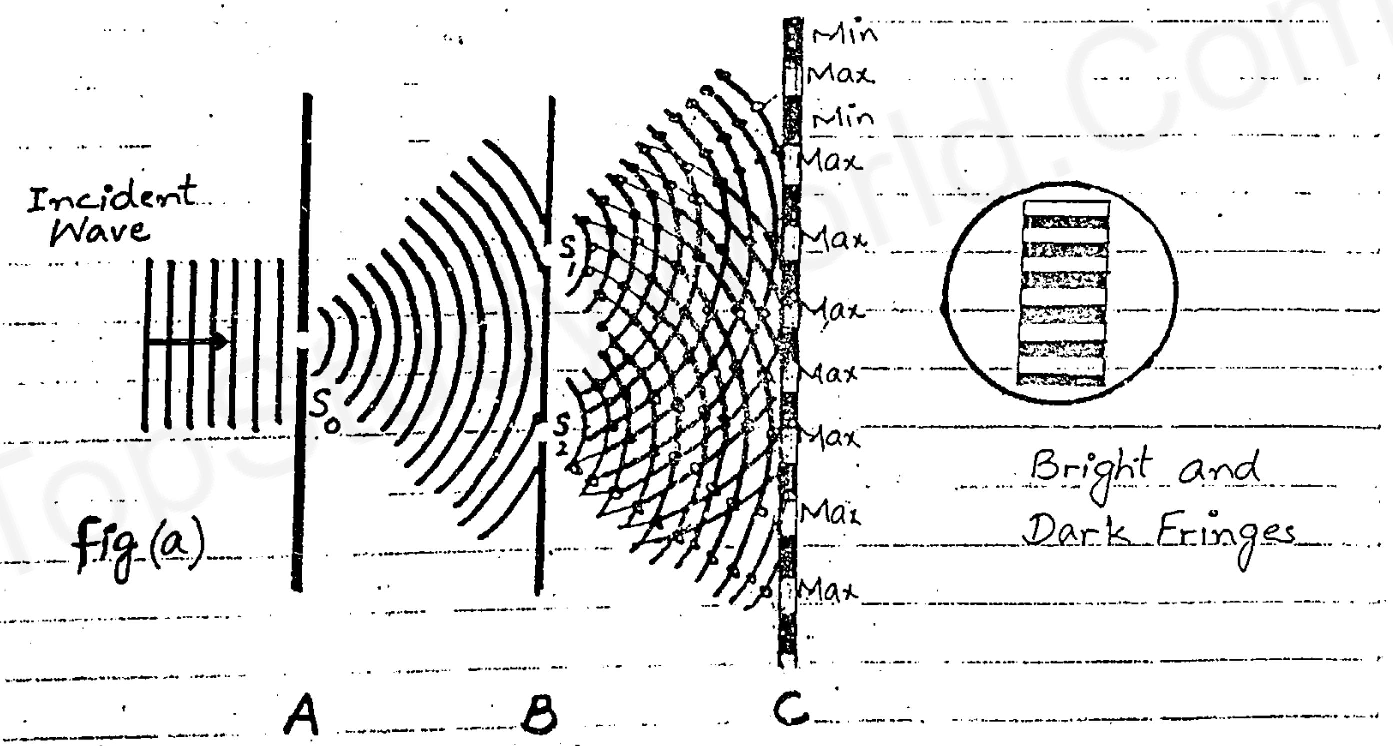


fig (a)

3. FORMATION OF BRIGHT AND DARK BANDS

Let us consider the formation of bright and dark bands (frings). As we know the two slits behave as coherent sources of secondary wavelets.

The wavelets arrive at the screen in such a way that at some points crests fall on crests and troughs on troughs resulting in constructive interference and bright frings are formed. There are some points on the screen where crests meet troughs giving rise to destructive interference and dark frings are formed.

Maxima :- The bright frings are termed as maxima.

Minima :- The dark frings are termed as minima.

4. Equations for Maxima and Minima

Take an arbitrary point 'P' on the screen on one side of the central point 'O' as shown in figure (b). AP and BP are the paths of the rays reaching P. The line AD is drawn such that AP = DP. Let the separation between the centres of two slits S_1 and S_2 be $AB = d$. The distance of second screen from the slits is $CO = L$. The angle between CP and CO is θ. Therefore

$\angle B\hat{A}D = \theta$ (Angle b/w two lines will be the angle b/w their normals).

The path difference between wavelets leaving the slits and arriving at 'P' is BD.

i-e P. Diff. $= BD = BP - AP$

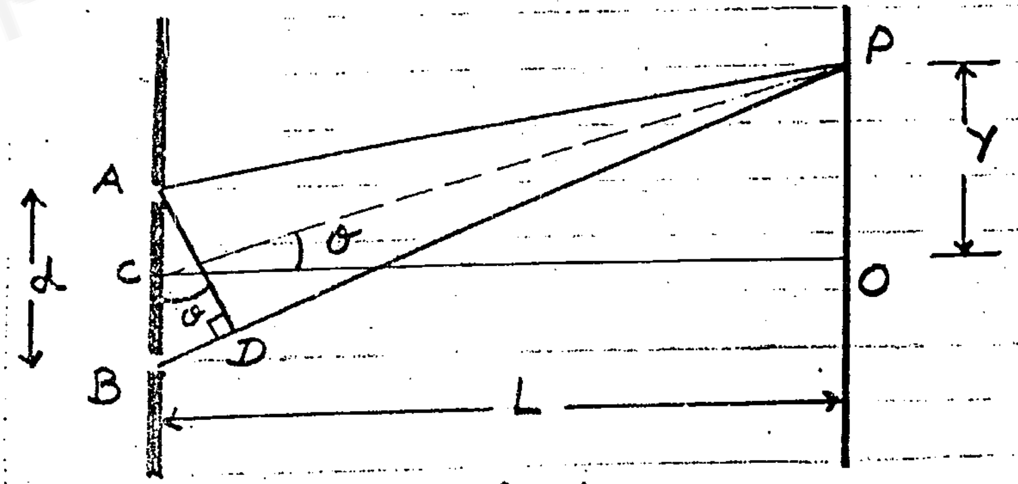


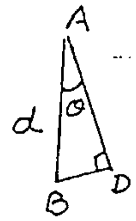
fig. (b)

FOR MAXIMA :- If point P is to have bright fring the path difference must be an integral multiple of wavelength.

i-e $\quad P.diff = BD = m\lambda \quad$ where $m = 0, 1, 2, \ldots$

From geometry of rt $\angle$d ΔABD

$$BD = d \sin\theta$$



So

$$\boxed{d \sin\theta = m\lambda} \quad \text{--- (1)}$$

It is obsereved that each bright fringe one one side of O has symmetrically located bright fringe on the other side of 'O'. The central bright fringe is obtained when $m = 0$.

FOR MINIMA :- If point P is to have dark fringe the path difference must contain odd integral multiple of half of the wavelength.

i-e $\quad P.diff. = BD = (m + \frac{1}{2})\lambda$

where $m = 0, 1, 2, \ldots$

So

$$\boxed{d \sin\theta = (m + \tfrac{1}{2})\lambda} \quad \text{--- (2)}$$

In this case, the first dark fringe will appear for $m = 0$ and second dark fringe $m = 1$. The interference pattern obtained in the Young's experiment is shown in fig (c).

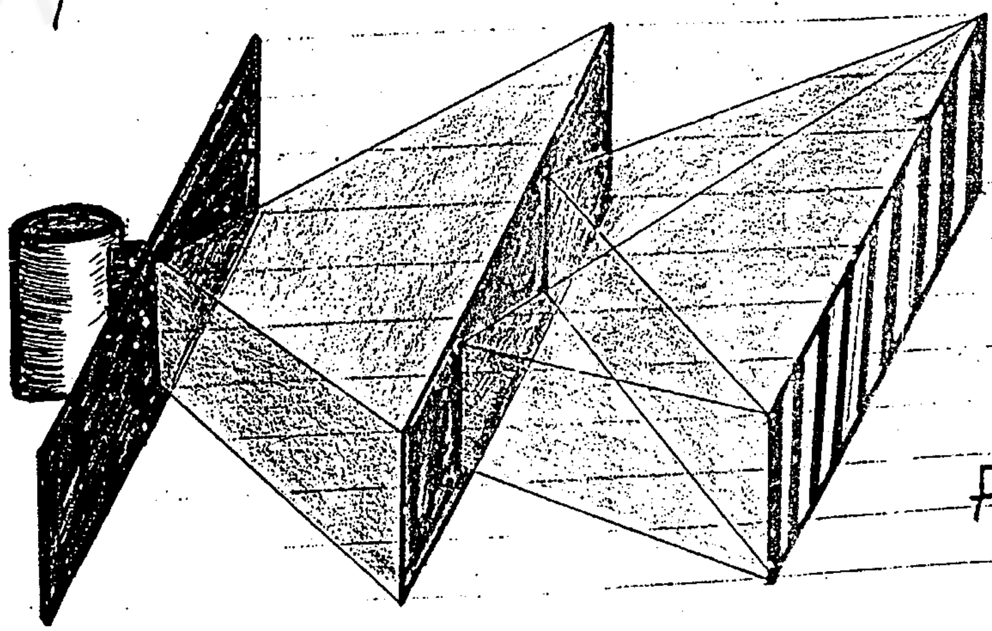


fig (c)

(a) POSITION OF BRIGHT FRINGE

From equation ①

$$d\sin\theta = m\lambda$$

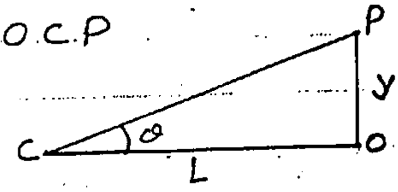
If 'θ' is very small, then

$$\sin\theta \simeq \tan\theta$$

So $$d\tan\theta = m\lambda$$

As from Fig (b) in rt <d △ OCP

$$\tan\theta = \frac{OP}{OL} = \frac{y}{L}$$



above equation becomes

$$\frac{dy}{L} = m\lambda$$

$$\boxed{y = m\frac{\lambda L}{d}} \quad ③$$

(b) - POSITION OF DARK FRINGE

If P is to have dark fringe then from eq ②

$$d\tan\theta = (m+\tfrac{1}{2})\lambda$$

Again

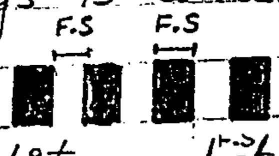
$$\tan\theta = y/L$$

$$\therefore \quad \frac{dy}{L} = (m+\tfrac{1}{2})\lambda$$

So $$\boxed{y = (m+\tfrac{1}{2})\frac{\lambda L}{d}} \quad ④$$

6- FRINGE WIDTH OR FRINGE SPACING

(a) Def:- The distance between two adjacent bright or two adjacent dark frings is called Fringe Width or Fringe Spacing.



(b) Bright Fringe Width:- In order to determine the distance between two adjacent bright fringes on the screen mth and $(m+1)$th fringes are considered

Position of mth bright fringe $y_m = m\frac{\lambda L}{d}$

" " $(m+1)$th " $y_{m+1} = (m+1)\frac{\lambda L}{d}$

If the distance between the adjacent bright fringe (Fringe width) is Δy then

$$\Delta y = y_{m+1} - y_m$$

$$= (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d}$$

$$\boxed{\Delta y = \frac{\lambda L}{d}} \text{ ——— } (5)$$

(c) Dark Fringe Width:-

Position of mth dark fringe $y_m = (m+1)\frac{\lambda L}{d}$

" " (m+1)th " $y_{m+1} = (m+2)\frac{\lambda L}{d}$

So

$$\Delta y = y_{m+1} - y_m$$

$$= (m+2)\frac{\lambda L}{d} - (m+1)\frac{\lambda L}{d}$$

$$\boxed{\Delta y = \frac{\lambda L}{d}}$$

(d) Factors:-

This shows that the bright and dark fringes are of equal width and are equally spaced. Equation (5) shows that fringe spacing increases if red light (long wave length) is used as compared to blue light (short wave length). (i.e $\Delta y \propto \lambda$) The fringe spacing varies directly with distance L between the slits and screen. It varies inversely with the seperation 'd' of the slits.

Monochromatic Light:-

Sodium Chloride in a flame gives out pure yellow light. This light is not a mixture of red and green colours.

EXAMPLE 9.1: The distance between the slits in Young's double slit experiment is 0.25 cm. Interference fringes are formed on a screen placed at a distance of 100 cm from the slits. The distance of the third dark fringe from the central bright fringe is 0.059 cm. Find the wavelength of the incident light.

SOLUTION:-

$d = 0.25\ cm = 2.5 \times 10^{-3}\ m$

$y = 0.059\ cm = 5.9 \times 10^{-4}\ m$

$L = 100\ cm = 1\ m$ and $\lambda = ?$

For the 3rd dark fringe $m = 2$

Using

$$y = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d}$$

$$\lambda = \frac{yd}{(m+\frac{1}{2})L} = \frac{5.9\times10^{-4}\ m \times 2.5\times10^{-3}\ m}{(2+\frac{1}{2})\times 1\ m}$$

$$\lambda = 5.19\times10^{-7}\ m$$

$\therefore$ or

$$\boxed{\lambda = 519\ nm}$$

EXAMPLE 9.2: Yellow sodium light of wavelength 589 nm, emitted by a single source passes through two narrow slits 1 mm apart. The interference pattern is observed on a screen 225 cm away. How far apart are two adjacent bright fringes?

SOLUTION:-

$\lambda = 589\ nm = 589\times10^{-9}\ m$

$d = 1\ mm = 10^{-3}\ m$

$L = 225\ cm = 2.25\ m$

$\Delta y = ?$

Using

$$\Delta y = \frac{\lambda L}{d}$$

$$= \frac{589\times10^{-9}\ m \times 2.25\ m}{10^{-3}\ m}$$

$$\Delta y = 1.33\times10^{-3}\ m$$

$$\boxed{\Delta y = 1.33\ mm}$$

9.5 INTERFERENCE IN THIN FILM

1- **THIN FILM:-** A thin film is a transparent medium whose thickness is comparable with the wavelength of light.

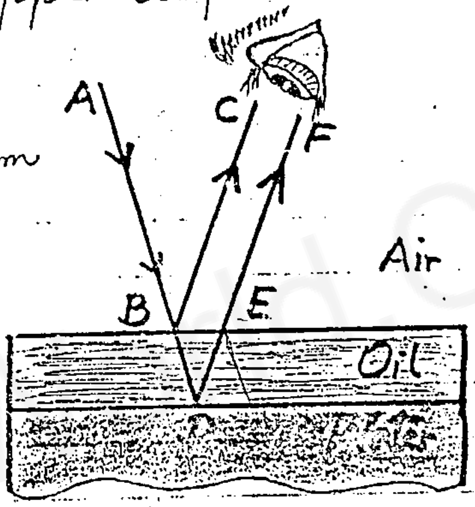
2- **EXAMPLES:-** Soap bubbles and oil film on the surface of water.

3- **EXPLANATION:-** Brillant and beautiful colours in soap bubbles and oil film on the surface of water are due to interference of light reflected from the two surfaces of the film.

(a)- Film Of Regular Thickness

Consider a thin film of a refracting medium. A beam 'AB' of monochromatic light of wavelength λ is incident on its upper surface.

It is partly reflected along BC and partly refracted into the medium along BD. At 'D' it is again partly reflected inside the medium at E and then along EF as shown in figure.



The beam BC and EF being the parts of the same beam has a phase coherence. As the film is thin so, the separation between the beams BC and EF will be very small and they will superpose and the result of their interference will be detected by the eye. It can be seen from figure that the original beam splits into two parts at point B (BC and BD) and they enter the eye after covering different lengths of paths.

Their path difference depends upon

(i)- Thickness and Nature of the film

(ii)- Angle of incidence.

If the two reflected waves BC and EF reinforce each other, then the film as seen with the help of a parallel beam of monochromatic light will look bright. However, if the thickness of the film and angle of incidence are such that the two reflected waves cancel each other, the film look dark.

(b). Film Of Irregular Thickness

If white light is incident on a film of irregular thickness at all possible angles, we should consider the interference pattern due to each spectral colour separately. It is quite possible that at a certain place on the film, its thickness and the angle of incidence of light are such that the condition of destructive interference of one colour is being satisfied. Hence, that portion of the film will exhibit the remaining constituent colour of the white light.

e.g :- (i) Interference pattern produced by a thin soap film illuminated by white light.

(ii) - The vivid iridescence of peacock feather is due to interference of the light reflected from its complex layered surface.

9.6 NEWTON'S RINGS

1. Introduction :- Newton performed an experiment to observe the interference pattern of monochromatic light, due to which dark and bright rings are produced known as Newton's Rings.

2. Experimental arrangement :- The apparatus consists of a plano convex lens of long focal length is placed in contact on

plane glass plate, a thin air film is enclosed between the upper surface of the glass plate and the lower surface of the lens. The thickness of the air film is almost zero at the point of contact O and gradually increases as one proceeds towards the periphery of the lens. Thus, points where the thickness of air film is constant, will lie on a circle with O as centre.

By means of a double convex lens a parallel beam of monochromatic light is produced. This parallel beam is reflected towards the lens 'L' by using a glass slab 'G'. The Newton's rings are observed by microscope M.

3. Working:-

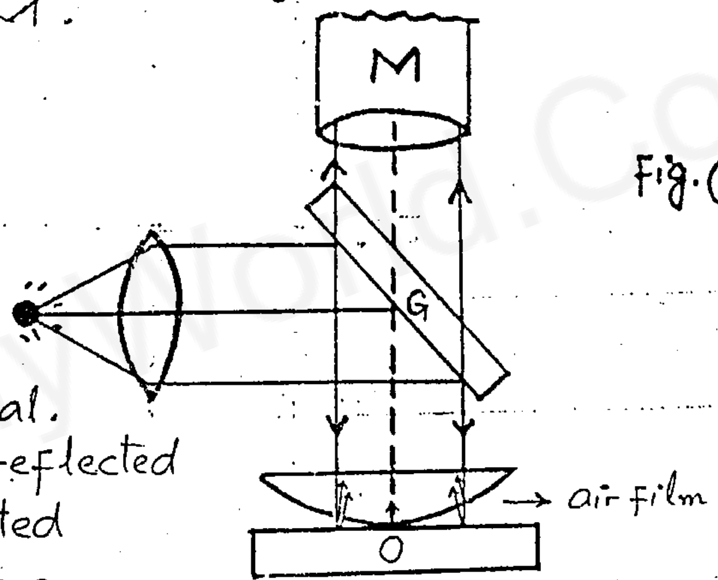


Fig.(a)

(a) Consider a ray of monochromatic light that strikes the upper surface of the air film nearly along normal. The ray is partly reflected and partly refracted as shown in figure. The ray refracted in the air film is also reflected partly at the lower surface of the film as shown in figure. The two reflected rays i-e produced at the upper and lower surfaces of the film are coherent and interfere constructively or destructively. When the light reflected upwards is observed through a microscope M which is focussed on the glass plate, series of

dark and bright rings are seen with centre as O. These concentric rings are known as Newton's rings.

(b). **Central Spot :-**
At the point of contact of the lens and the glass plate, the thickness of the film is effectively zero but due to reflection at the lower surface of air film from denser medium, an additional path difference of $\lambda/2$ and phase difference of 180° is introduced. Consequently, the centre of Newton's rings is dark due to destructive interference.

9.7 MICHELSON'S INTERFEROMETER

1. **Introduction :-** This instrument was invented by an American Physicist Albert A. Michelson in 1881 using the idea of interference of light rays.
2. **Definition :-** It is an instrument which is used for the accurate measurement of wavelength and precise length measurements.
3. **Principle :-** Its working principle is based on interference. When a light beam falls upon it, it splits the light beam into two parts and then recombine them to produce an interference pattern after they have travelled over different paths.
4. **Construction and Working :-**
The essential features of a Michelson's interferometer are shown schematically in figure (a).
Monochromatic light from an external source falls on a half silvered glass plate 'G_1' that partially reflects it and partially transmits it.

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The reflected portion labeled as I in the fig (a) travels a distance L_1 to mirror M_1, which reflects the beam back towards G_1. The half silvered plate G_1 partially Transmits this portion that finally arrives at the observer's eye.

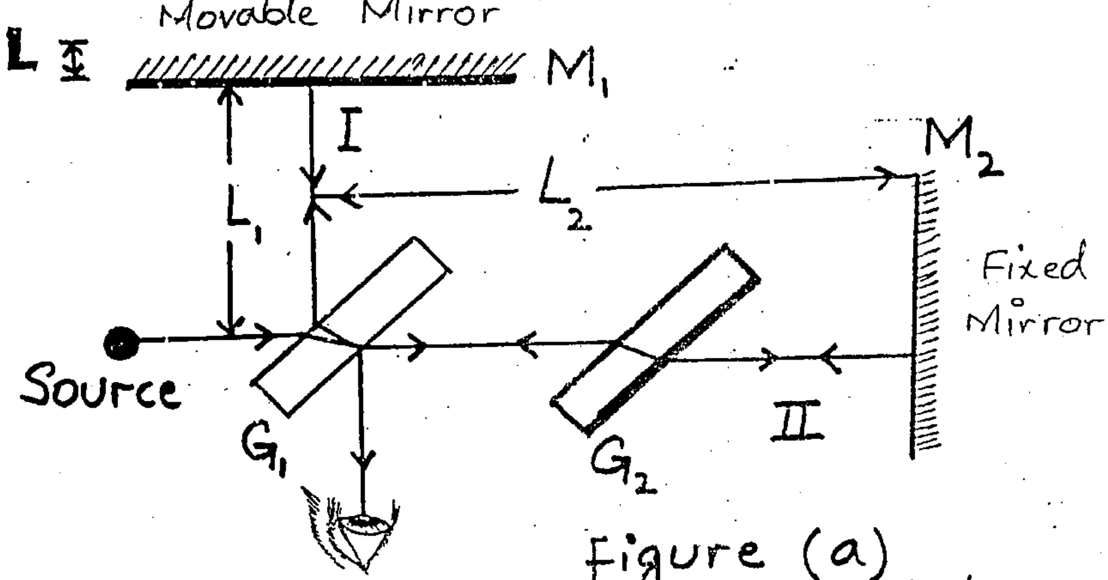


Figure (a)

The Transmitted portion of the original beam labelled as II Travels a distance L_2 to mirror M_2 which reflects the beam back towards G_1. The beam II partially reflected by G_1 also arrives the observer's eye finally. The plate G_2, cut from the same piece of glass as G_1 is introduced in the path of beam II as a compensator plate G_2, therefore equalizes the path length of the beam I and II. The two beams having their different paths are coherent. When they arrive at observers eye, therefore produce interference effects. The observer then sees a series of parallel interference Frings as shown in Fig (b).

5- THEORY:- In a practical interferometer mirror M_1 can be moved along direction perpendicular to its surface by means of a precision screw.

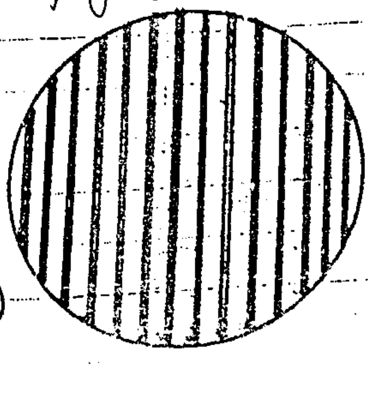


Fig (b)

As length L_1 is changed, the pattern of interference fringes is observed to shift.
If M_1 is displaced through a distance equal to $\lambda/2$, a path difference of double of this displacement is produced i-e equal to 'λ'. Thus a fringe is seen shifted forward across the line of reference of cross wire in the eye piece of the telescope used to view fringes.
A fringe is shifted, each time the mirror is displaced through $\lambda/2$. Hence, by counting the number 'm' of the fringes which are shifted by the displacement L of the mirror, we can write the equation

$$\boxed{L = m\,\lambda/2}$$

6- Uses:-

(a)- Very precise length measurements can be made with an interferometer. The motion of mirror M_1 by only $\lambda/4$ produces a clear difference between brightness and darkness. $\left(\lambda/4 + \lambda/4 = \lambda/2\right)$. For $\lambda = 400\,nm$, this means a high precision of 100 nm or 10^{-4} mm.

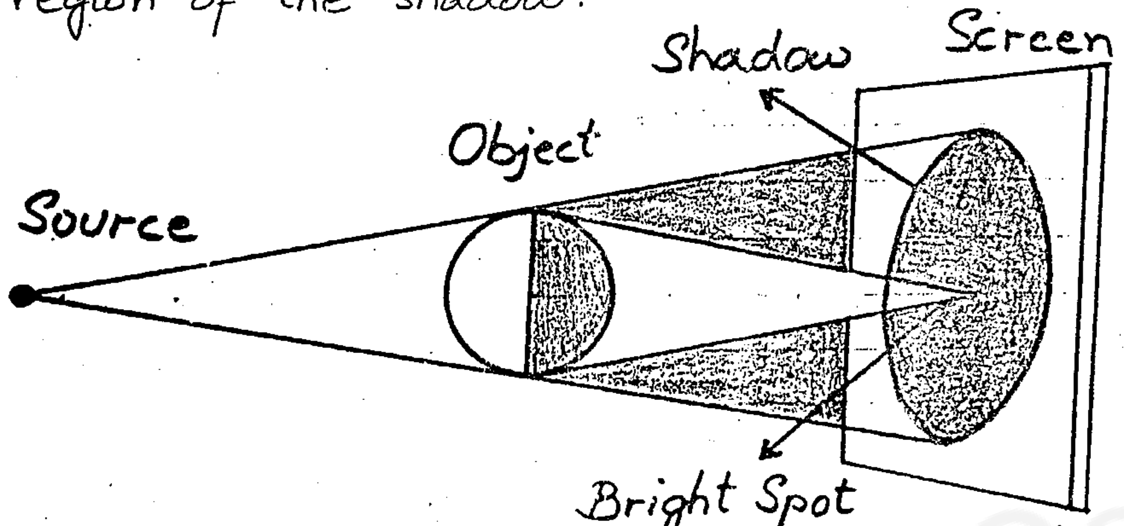
(b)- Michelson measured the length of standard meter interms of the wavelength of red cadmium light and showed that the standard meter was equivalent to 1,553,163.5 wavelengths of this light.

9.8 DIFFRACTION OF LIGHT

1- **Definition** :- The property of bending of light around obstacles and spreading of light waves into the geometrical shadow of an obstacle is called diffraction.

2- **Explanation** :- Consider a small and smooth steel ball of about 3 mm in diameter is illuminated by a point source of light.

The shadow of the object is received on a screen as shown in figure. The figure shows that the shadow of spherical object is not completely dark but has a bright spot at its centre. According to Huygen's principle, each point on the rim of the sphere behaves as a source of secondary wavelets which illuminate the central region of the shadow.

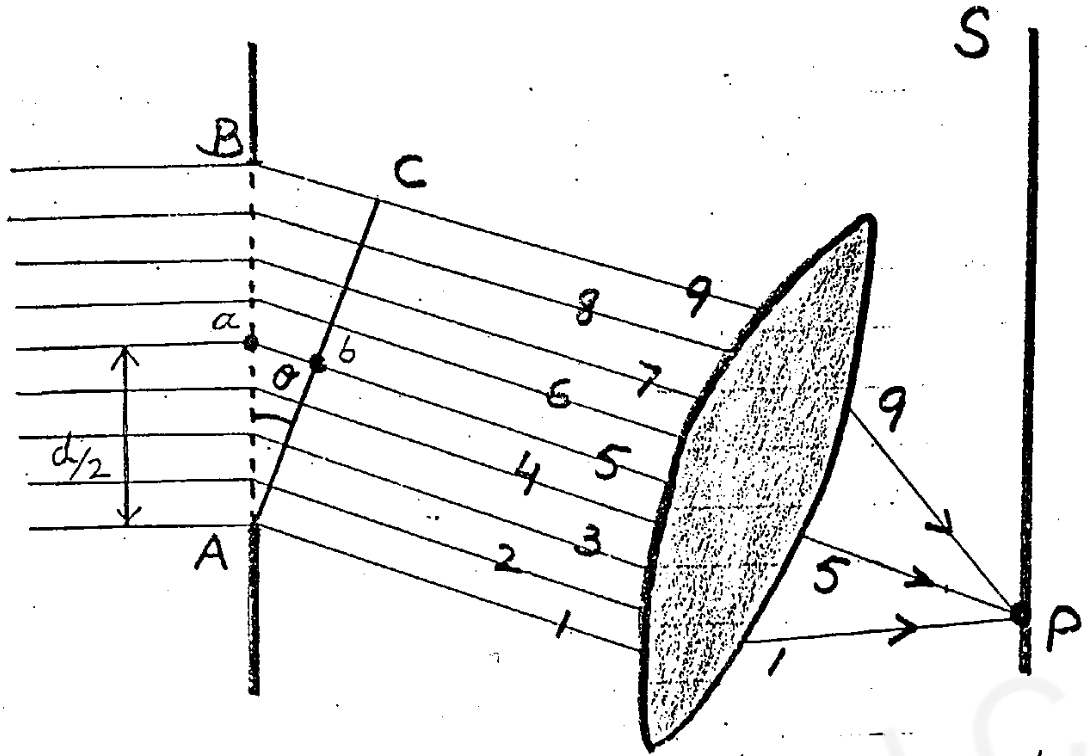


The experiment clearly shows that when light travels past an obstacle, it does not proceed exactly along a straight path, but bends around the obstacle.

3. **Condition:-** The phenomenon is found to be prominent when the wavelength of the light is large as compared with the size of the obstacle or aperture of the slit. The diffraction of the light occurs in effect, due to the interference between rays coming from different parts of the same wavefront.

9.9 DIFFRACTION DUE TO A NARROW SLIT

The experimental arrangement for studing diffraction of light due to a narrow slit is shown in the figure. The slit AB of width 'd' is illuminated by a parallel beam of monochromatic light of wavelength 'λ'. The screen S is placed parallel to the slit for observing the effects of the diffraction of light. A small portion of the incident wavefront passes through the narrow slit.



Fig(a)

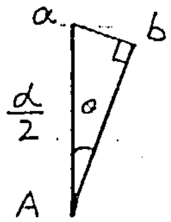
Each point of this section of the wavefront sends out secondary wavelets to the screen. These wavelets then interfere to produce the diffraction pattern. It becomes simple to deal with rays instead of wavefronts as shown in the figure. In this figure, only nine rays have been drawn whereas actually there are a large number of them. Let us consider waves 1 and 5 which are in phase when in the wavefront AB. After these reach the wavefront AC, wave 5 would have a path difference of ab say equal to $\lambda/2$. Thus when these two rays reach point P on the screen; they will interfere destructively. Similarly, each pair 2 and 6, 3 and 7, 4 and 8 differ in path by $\lambda/2$ and will do the same.

The path difference in 1 and 5 $= ab$

From rt ∠d Δ Aab

$$\sin\theta = \frac{ab}{aA} = \frac{ab}{d/2}$$

$$ab = \frac{d}{2}\sin\theta$$



The equation for first minimum is, then

$$\frac{d}{2}\sin\theta = \lambda/2$$ (Path. Diff. = odd integral multiple of $\lambda/2$)

Or $$d\sin\theta = \lambda$$

In general, the conditions for different orders of minima on either side of centre are given as

$$\boxed{d\sin\theta = m\lambda}$$ where $m = 1, 2, 3, \ldots$

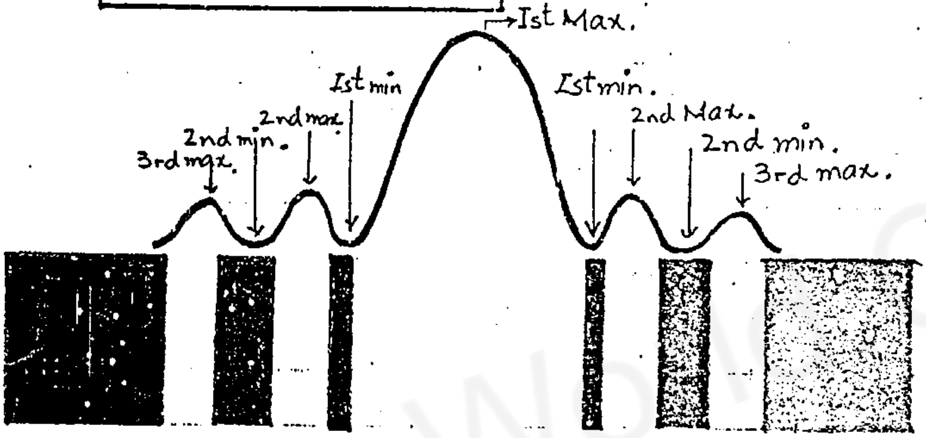


Fig. (b). Diffraction pattern of monochromatic light produced due to a single slit; graphical representation and photograph of the pattern.

The region between any two consecutive minima both above and below O (centre) will be bright. Therefore, a narrow slit produces a series of bright and dark regions with the first bright region at the centre of the pattern. Such a pattern is shown in above figure (b).

9.10 DIFFRACTION GRATING

1- **Definition :-** A diffraction grating is a glass plate having a large number of close parallel equidistant slits mechanically ruled on it. The transparent spacing between the scraches on the glass plate act as slits. A typical diffraction grating has about 400 to 5000 lines per centimetre

2. **Grating Element :-** The distance between two adjacent slits is called Grating Element. It is denoted by 'd'. Its value is obtained by dividing the length 'L' of the grating by the total number N of the lines ruled on it.

i-e $$d = \frac{L}{N} = \frac{1}{N \text{ per metre}}$$

3 - **Diffraction Through Grating :-**

In order to understand how a grating diffracts light consider a parallel beam of monochromatic light illuminating the grating at normal incidence as shown in figure (a). The sections of wavefront that pass through the slits behave as sources of secondary wavelets according to Huygen's principle.

4- **Measurement Of Wavelength By Grating**

Consider the parallel rays which after diffraction through the grating make an angle 'θ' with AB the normal to grating. They are then brought to focus on the screen at P by a convex lens.

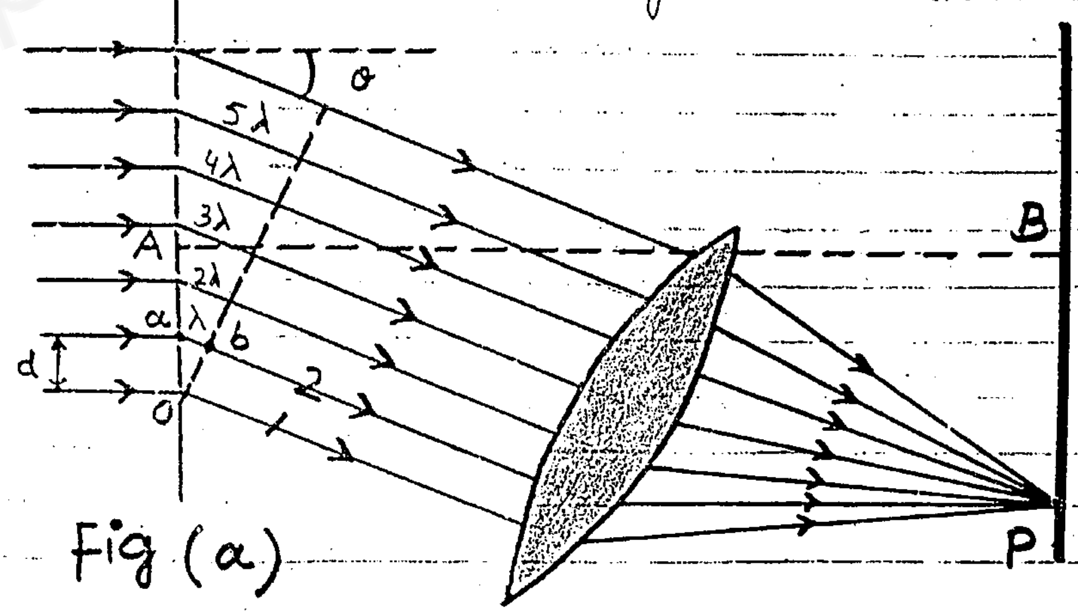


Fig (a)

Now consider rays 1 and 2

From fig.(a), it is clear that the ray 1 covers a distance ab more than ray 2. If path diff ab is equal to 'λ' then they will reinforce each other at P. As incident beam is parallel, so the rays from any two consecutive slits will differ in path by λ when they arrive at P. They will therefore, interfere constructively. Hence condition for constructive interference is that ab, the path difference between two consecutive rays, should be equal to λ i.e.

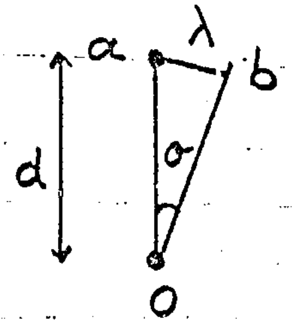


fig (b)

$$ab = \lambda$$

From fig (b). $ab = d\sin\theta$ ——— (1)

$$\therefore \quad d\sin\theta = \lambda \quad \text{——— (2)}$$

According to eq (1), when $\theta = 0$ i.e along the direction of normal to the grating, the path difference between the rays coming out from the slits of the grating will be zero. So we will get a bright image in this direction. This is known as zero order image formed by the grating. If we increase 'θ' on either side of this direction, a value of θ will be arrived at which $d\sin\theta$ will become 'λ' and according to eq (2), we will again get a bright image. This is known as first order image of the grating. In this way if we continue increasing 'θ' we will get the second, third, etc. image on either side of the zero order image according to $d\sin\theta$ becomes equal to 2λ, 3λ etc. Thus the equation (2) can be written in more general form as

$$\boxed{d\sin\theta = n\lambda} \quad \text{——— (3)}$$

where $n = 0, 1, 2, 3, \ldots$ etc.

However, if the incident light contains different wavelengths, the image of each wavelength for a certain value of n is diffracted in a different direction. Thus separate images are obtained corresponding to each wavelength or colour. Equation 3 shows that the value of θ depends upon n, so the images of different colours are much separated in higher orders.

ILLUMINATION OF DISC BY WHITE LIGHT

The fine rulings, each $0.5\,\mu m$ wide on a compact disc function as a diffraction grating. When a small source of white light illuminates a disc, the diffracted light forms coloured 'lanes' that are composite of the diffraction patterns from the rulings.

9.11 DIFFRACTION OF X-RAYS BY CRYSTALS

1- Introduction :-

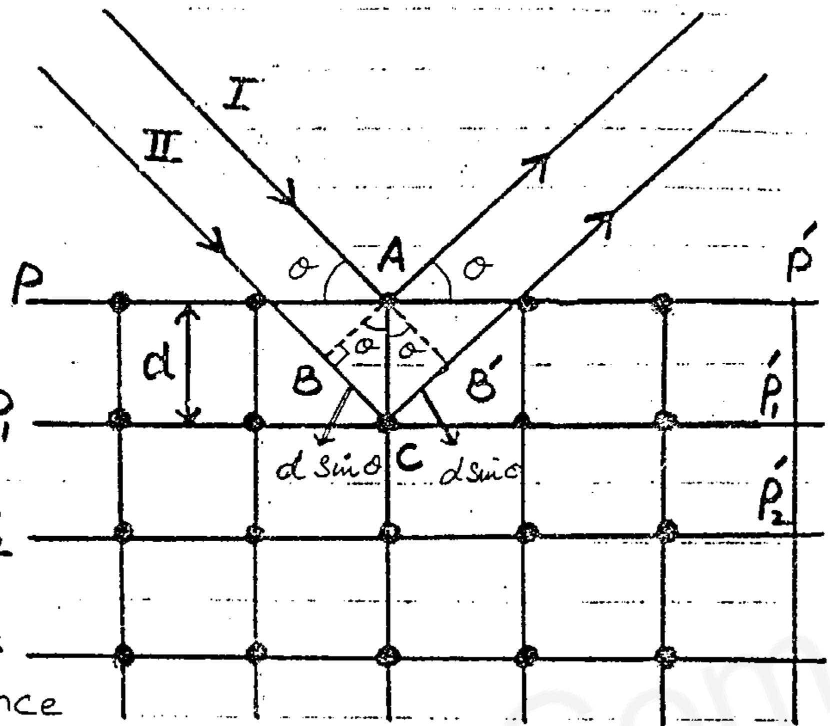
X-rays is a type of electromagnetic waves of very short wavelength i.e about $10^{-10}\,m$.

In 1914, W.H. Bragg and W.L. Bragg studied the atomic structure of crystals by using X-rays. They found that a monochromatic beam of X-rays was reflected from a crystal plane. The crystal plane acted like a mirror. In order to observe the effects of diffraction, the grating spacing must be of the order of the wavelength of radiation used. As the interatomic spacing of crystal is typically of the order of $10^{-10}\,m$, so X-rays are used for observing the effects of diffraction.

2- Bragg's Equation :- As the regular array of atoms in a crystal forms a natural diffraction grating. To understand the diffraction of x-rays by crystals, a series of atomic planes of constant inter planer spacing d parallel to a crystal face are shown by lines PP', P_1P_1', P_2P_2' and so on as shown in figure.

Suppose an x-rays beam is incident at an angle 'θ' on one of the planes.

The beam can be reflected from both the upper and lower planes of atoms. The beam reflected from lower plane travels some extra distance as compared to the beam reflected from the upper plane. (i-e path diff. $= BC + CB'$)



From figure it can be seen that

Effective path difference between the reflected beams $= BC + CB'$

$$= d\sin\theta + d\sin\theta = 2d\sin\theta$$

Therefore, for reinforcement, the path difference should be an integral multiple of the wavelength. Thus

$$\boxed{2d\sin\theta = n\lambda}$$

This equation is known as the Bragg equation. The value of n is referred to as the order of reflection.

3.- **USES**:- (a) The above equation can be used to determine inter planer spacing between similar parallel planes of a crystal if x-rays of known wave-length are allowed to diffract from the crystal.

(b)- X-rays diffraction has been very useful in determining the structure of biologically important molecules such as hemoglobin, which is an important constituent of blood, and double helix structure of DNA.

DIFFERENCE BETWEEN INTERFERENCE AND DIFFRACTION

Interference	Diffraction
1- Interference is the result of superposition of light coming from two different wavefronts originating from the same source.	1- Diffraction is the result of interaction of light coming from different parts of the same wavefront originating from the source.
2- The fringe spacing may or may not be of the same width.	2- Diffraction frings are not of the same width.
3- All bright fringes are of the uniform intensity.	3- All bright frings are not of the same intensity
4. The points of minimum intensity are perfectly dark.	4- The points of minimum intensity are not perfectly dark.

EXAMPLE. 9.3 : Light of wavelength 450 nm is incident on a diffraction grating on which 5000 lines/cm have been ruled.

(i). How many orders of spectrum can be observed on either side of the direct beam?

(ii). Determine the angle corresponding to each other.

SOLUTION :- Given that

$$\lambda = 450\,nm = 450\times10^{-9}\,m$$

$$d = \frac{1}{5000}\,cm = \frac{1}{5000}\times10^{-2}\,m$$

or $$d = \frac{1}{500000}\ metres$$

(i) As we have $\quad d\sin\theta = n\lambda$

For maximum order of spectra $\quad \sin\theta = 1$

Therefore $\quad d(1) = n\lambda$

and $\quad n = d/\lambda$

$$n = \frac{1}{500000}\,m \times \frac{1}{450\times10^{-9}\,m}$$

$$n = 4.4$$

Hence maximum order of spectrum is $\boxed{4}$

(ii)- For first order of spectrum $n=1$

$$d\sin\theta = n\lambda$$

$$\frac{1}{500000}\sin\theta = 1\times 450\times10^{-9}$$

$$\sin\theta = (500000)(450\times10^{-9})$$

$$\sin\theta = 0.225$$

So $\quad \theta = \sin^{-1}(0.225) = \boxed{13^\circ}$

For second order spectrum $n=2$

$$d\sin\theta = n\lambda$$

$$\frac{1}{500000}\,m\ \sin\theta = 2\times 450\times10^{-9}\,m$$

$$\sin\theta = 0.45$$

$$\theta = \sin^{-1}(0.45) = \boxed{26.7^\circ}$$

For third order spectrum $n=3$

$$\sin\theta = 3\times 500000\times 450\times10^{-9}$$

$$\sin\theta = 0.675$$

$$\boxed{\theta = 42.5^\circ}$$

and the fourth order spectrum $n=4$

$$\sin\theta = 4\times500000\times450\times10^{-9}$$

$$= 0.9$$

$$\theta = \sin^{-1}0.9$$

$$\boxed{\theta = 62.2^\circ}$$

9.12 POLARIZATION

Un Polarized Light :- Ordinary light consists of transverse wave motion in which vibration takes place in all directions in a plane perpendicular to the direction of propagation of light. Such light wave is known as unpolarized light

e.g :- Light from incandesent bulb
Light from sun etc.

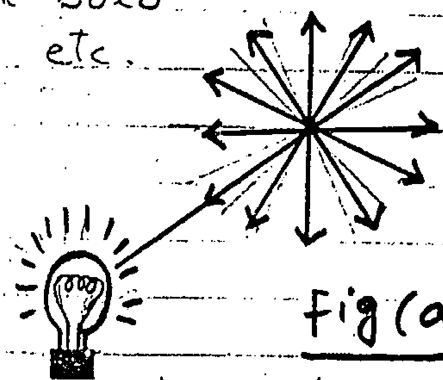


Fig (a)

Plane Polarized Light

1- **Introduction :-** In transverse mechanical waves such as produced in a stretched string, the vibration of the particles of the medium are perpendicular to the direction of propagation of the waves. The vibration can be oriented along vertical, horizontal or any other direction.

In each of these cases, the transverse wave is said to be polarized.

2- **Definition :-** The plane of polarization is the plane containing the direction of vibration of particles of the medium and direction of propagation of the wave.

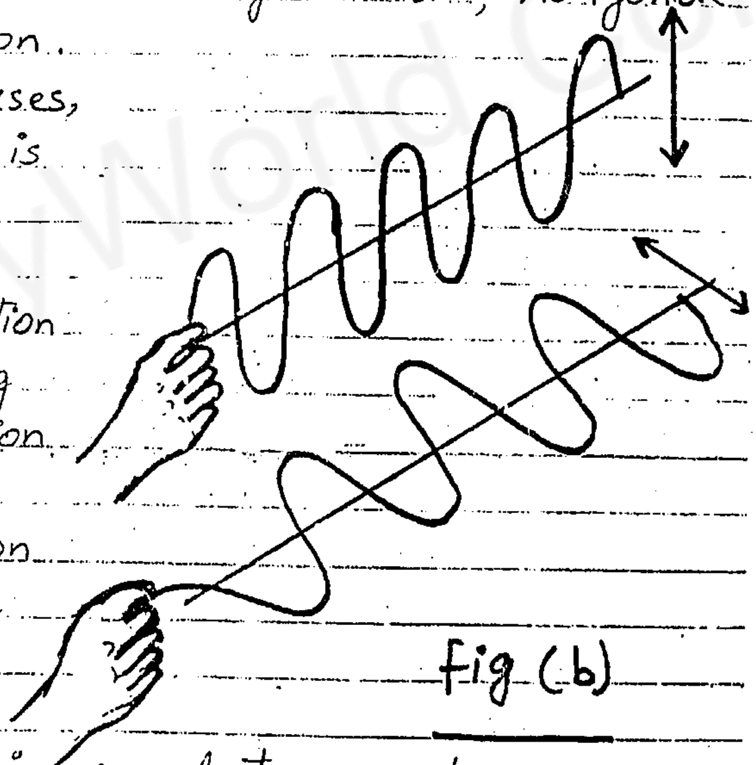


Fig (b)

3. **Explanation :-**

(a)- A light wave is an electromagnetic wave produced by oscillating charge consists of a periodic variation of electric field vector accompanied by the magnetic field vector at right angle to each other. The direction of polarization in a plane polarized light wave is taken as the direction of the electric field vector.

(b)- Production and Detection Of Plane Polarized Light.

An ordinary incandesent light emits un-polarized light, as does the sun, because its (electrical) vibrations are randomly oriented in space (fig (a)). It is possible to obtain plane polarized beam of light from un-polarized light by removing all waves from the beam except those having vibrations along one particular direction.

This can be achieved by various processes

- (i)- Selective absorption
- (ii)- Reflection from different surfaces.
- (iii)- Scattering by small particles.
- (iv)- Refraction through crystals.

(i)- Selective Absorption

The selective absoption method is the most common method to obtain plane polarized light by using certain types of materials called dichroic substances.

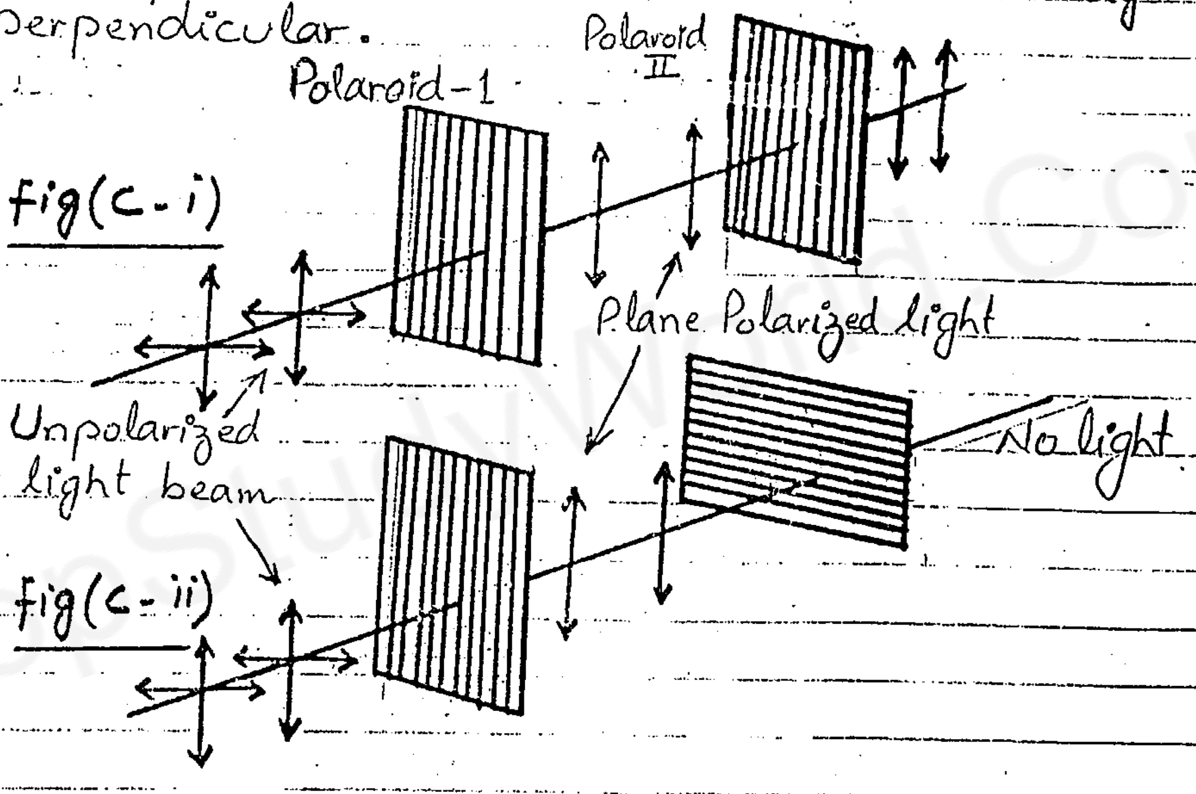
These materials transmit only those waves whose vibrations are parallel to a particular direction and will absorb those waves whose vibrations are in other directions.

One such commercial polarizing material is a polaroid.

LIGHT WAVES ARE TRANSVERSE WAVES

If un-polarized light is made incident on a sheet of polaroid, the transmitted light will be plane polarized. If a second sheet of polaroid is placed in such a way that the axes of the polarid shown by straight lines drawn on them, are parallel as shown in fig. (c-i), the light is transmitted through the second polaroid. If the second polaroid is slowly rotated about the beam of light, as axis of rotation, the light emerging out of the second polaroid gets dimmer and dimmer and disappears when the axes become mutually perpendicular as shown in fig (c-ii). The light reappears on further rotation and becomes brightest when the axes are again parallel to each other.

This experiment proves that light waves are transverse waves. If the light waves were longitudinal, they would never disappear even if the two Polaroid were mutually perpendicular.



(ii). Reflection From Different Surfaces

Reflection of light from water, glass, snow and rough road surfaces, for larger angles of incidences, produces glare. Since the reflected light is partially polarized glare can be considerably reduced by using polaroid sunglasses.

(iii)- Scattering by small particles

Sun light also becomes partially polarized because of scattering by air molecules of the Earth's atmosphere. This effect can be observed by looking directly up through a pair of sunglasses made of polarizing glass. At certain orientations of the lenses, less light passes through than at others.

(iv) Refraction through Crystals

When un-polarized light refracts through crystals then specified plane polarized light is obtained according to crystalline structure.

(C). OPTICAL ROTATION :-

Certain crystals and liquids when placed between polaroids, rotate the plane of polarization of light. Quartz and sodium chlorate crystals are typical examples, which are termed as optically active crystals.

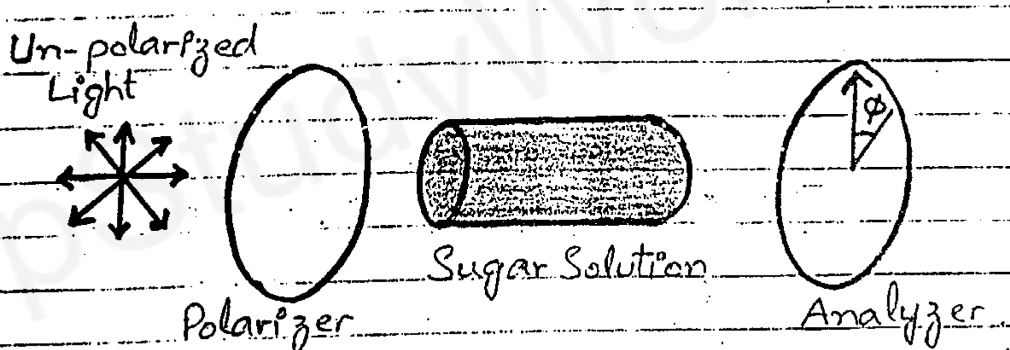


Figure (d). Sugar solution rotates the plane of polarization of incident light so that it is no longer horizontal but at an angle. The analyzer thus stops the light when rotated from the vertical (crossed) positions.

A few millimeter thickness of such crystals will rotate the plane of polarization by many degrees. Certain organic substances, such as sugar and tartaric acid show optical rotation when they are in solution. → This property of optically active substances can be used to determine their concentration in solution.

EXERCISE QUESTIONS

9.1 - Under what conditions two or more sources of light behave as coherent sources?

Answer :- Two or more sources of light can only behave as coherent sources if they have no phase difference or have a constant phase difference. Two independent light sources are never coherent as each source emits waves with random phases.

A common method for producing two coherent light sources is to use monochromatic light source to illuminate a screen containing two slits. The light emerging from both the slits in this way is coherent.

9.2 - How is the distance between interference fringes affected by the separation between the slits of Young's experiment? Can fringes disappear?

Answer :- As we have

$$\Delta y = L\lambda/d$$

$$\Rightarrow \quad \Delta y \propto 1/d$$

This shows that fringe spacing varies inversly with separation between the slits. If separation is increased, the distance between the fringes decreases. They come closer and may eventually disappear producing a general illumination.

9.3 :- Can visible light produce interference fringes? Explain.

Answer :- Yes, visible light or white light can produce interference fringes but each wavelength will produce its own interference fringe and hence the fringe pattern will then be coloured at each point depends on which wavelength is reinforced by interference.

9.4 :- In the Young's experiment, one of the slits is covered with blue filter and other with red filter. What would be the pattern of light intensity on the screen?

Answer :- Since blue and red lights have different wave lengths, so these are not in coherence, hence no interference pattern will be observed.

9.5 :- Explain whether the Young's experiment is an experiment for studing interference or diffraction effects of light.

Answer :- Young's experiment was performed to study the interference of light, although the diffraction can also be studied by this experiment because when light passes through the slit it bends towards the corner.

9.6 :- An oil film spreading over a wet footpath shows colours. Explain how does it happen?

Answer :- An oil film spreading over a wet footpath shows colours due to interference of light through thin film (oil film). When a light beam is incident, a part of it is reflected from the upper surface of thin oil film and other is reflected from the lower surface of thin film. The two reflected beams are coherent being part of same beam. As the sun light (white light) consists of seven colours and each colours refracts differently hence after reflection different colours interfere at different points as compared to others and a wet footpath shows colours.

9.7:- Could you obtain Newton's rings with transmitted light? If yes, would the pattern be different from that obtained with reflected light?

Answer:- Yes, Newton's rings can be observed with transmitted light. The only difference will be that central point will not be dark but bright by transmitted light.

9.8:- In the white light spectrum obtained with a diffraction grating, the third order image of a wavelength coincides with the fourth order image of a second wavelength. Calculate the ratio of the two wavelengths.

Answer:- For a diffraction grating

$$d \sin\theta = n\lambda$$

where $d\sin\theta$ = Path Difference
n = Order of the Image
λ = wave length

For 3rd order image and first wavelength

$$d \sin\theta = 3\lambda_1 \quad \text{---} (1)$$

For 4th order image and second wavelength

$$d \sin\theta = 4\lambda_2 \quad \text{---} (2)$$

As the two wavelengths coincides, so the path diff. will be equal.

Dividing eq (1) by (2)

$$\frac{3\lambda_1}{4\lambda_2} = \frac{d\sin\theta}{d\sin\theta}$$

$$3\lambda_1 = 4\lambda_2$$

$$\boxed{\frac{\lambda_1}{\lambda_2} = \frac{4}{3}}$$

9.9:- How would you manage to get more orders of spectra using a diffraction grating?

Answer:- For a diffraction grating

$$d \sin\theta = n\lambda$$

So for a given wavelength λ, the order 'n' of spectra depends on 'd' the grating element. For maximum value of $\sin\theta$ i-e 1 the angle is 90°. Hence order of spectra depends on grating element (i-e $d \propto n$). The only way to increase the value of grating element or spacing between the lines, we can increase order.

As $$d = \frac{1}{N}$$

Hence a grating with lesser number of lines per centimeter ruled over it can produce more orders of spectra.

9.10:- Why the polaroid sunglasses are better than ordinary sunglasses?

Answer:- The sunlight reflected from smooth surfaces such as wet road, lakes window panels and table tops etc, is horizontally polaroids and produces glare. The glare to the reflected light can be reduced or eliminated by using sunglasses made out polaroid sheet of glass with its select transmission axis vertical. Thus the horizontally polarized light cannot go through.

9.11:- How would you distinguish between un-polarized and plane-polarized lights?

Answer:- The un-polarized and plane-polarized light canbe distinguished by using a polarizer. If a polarizer is rotated infront of incident un-polarized light, a component of light will pass through polarizer in each orientation. In case of polarized light except at a particular orientation no light will pass through at all.

9.12 Fill in the blanks

Answer :-

(i)- According to Huygen's principle, each point on a wavefront act as a source of secondary wavefront

(ii)- In Young's experiment the distance between two adjacent bright fringes for violet light is less than that for green light.

(iii)- The distance between bright fringes in the interference pattern increases as the wavelength of light used increases.

(iv)- A diffraction grating is used to make a diffraction pattern for yellow light and then for red light. The distance between the red spots will be more than that for yellow light

(v)- The phenomenon of polarization of light reveals that light waves are transverse wave.

(vi)- A Polaroid is a commercial polarizer

(vii)- A Polaroid glass eliminates glare of light produced at a road surface

NUMERICAL PROBLEMS

9.1 :- Light of wavelength 546nm is allowed to illuminate the slits of Young's experiment. The separation between the slits is 0.10 mm and the distance of the screen from the slits where interference effects are observed is 20cm. At what angle the first minimum will fall? What will be the linear distance on the screen between adjacent maxima?

SOLUTION :-

Given Data :- $\lambda = 546nm = 546 \times 10^{-9} m$

9.12 Fill in the blanks

Answer :-

(i)- According to Huygen's principle, each point on a wavefront act as a source of secondary wavefront

(ii)- In Young's experiment the distance between two adjacent bright fringes for violet light is less than that for green light.

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SOLUTION :-

Given Data :- $\lambda = 546nm = 546 \times 10^{-9} m$

$d = 0.10\,mm = 0.10 \times 10^{-3}\,m$

$L = 20\,cm = 20 \times 10^{-2}\,m$

To Find

For Ist minimum $= \theta = ?$

Fringe Spacing $= \Delta y = ?$

Calculations:-

For minima

$$d \sin\theta = (m + \tfrac{1}{2})\lambda$$

For Ist minima $m = 0$

$$d \sin\theta = (0 + \tfrac{1}{2})\lambda$$

$$d \sin\theta = \lambda/2$$

$$\sin\theta = \lambda/2d$$

$$\sin\theta = \frac{546 \times 10^{-9}\,m}{2 \times 0.10 \times 10^{-3}\,m} = 0.00273$$

$$\theta = \sin^{-1}(0.00273)$$

$$\boxed{\theta = 0.16^\circ}$$

For fringe spacing

$$\Delta y = L\lambda/d$$

$$\Delta y = \frac{20 \times 10^{-2}\,m \times 546 \times 10^{-9}\,m}{0.10 \times 10^{-3}\,m}$$

$$\Delta y = 1.1 \times 10^{-3}\,m$$

$$\boxed{\Delta y = 1.1\,mm}$$

9.2 :- Calculate the wavelength of light which illuminates two slits 0.5 mm apart and produces an interference pattern on a screen placed 200 cm away from slits. The first bright fringe is observed at a distance of 2.40 mm from central bright image.

Solution:- Data

$d = 0.5\,mm = 0.5 \times 10^{-3}\,m$

$L = 200\,cm = 2\,m$

$\Delta y = 2.40\,mm = 2.4 \times 10^{-3}\,m$

$m = 1$

$\lambda = ?$

Calculations As $y = \frac{mL\lambda}{d}$

$$\Rightarrow \lambda = \frac{dy}{mL}$$

$$= \frac{0.5\times10^{-3}m \times 2.40\times10^{-3}m}{1\times2}$$

$$= 6\times10^{-7}m$$

$$= 600\times10^{-9}m$$

$$\boxed{\lambda = 600\ nm}$$ Answer

9.3:- In a double slit experiment the second order maximum occurs at $\theta = 0.25°$. The wavelength is 650nm. Determine the slit separation.

Solution:-

Data

$m = 2$

$\theta = 0.25°$

$\lambda = 650nm = 650\times10^{-9}m$

$d = ?$

Calculation:-

$$d\sin\theta = m\lambda$$

$$d = m\lambda/\sin\theta$$

$$= \frac{2\times650\times10^{-9}}{\sin 0.25°}$$

$$d = 2.98\times10^{-4}m$$

$$= 3\times10^{-4}m = 0.3\times10^{-3}m$$

$$\boxed{d = 0.3\ mm}$$

9.4:- A monochromatic light of $\lambda = 588\ nm$ is allowed to fall on the half silvered glass plate G_1 in the Michelson Interferometer. If mirror M_1 is moved through 0.233 mm, how many fringes will be observed to shift?

Data:- $\lambda = 588\,nm = 588 \times 10^{-9}\,m$

$L = 0.233\,mm = 0.233 \times 10^{-3}\,m$

$m = ?$

Calculations:-

$$L = m\lambda/2$$

$$m = \frac{2L}{\lambda} = \frac{2 \times 0.233 \times 10^{-3}\,m}{588 \times 10^{-9}\,m}$$

$$\boxed{m = 792}$$

9.5:- A second order spectrum is formed at an angle of 38°, when light falls normally on a diffraction grating have 5400 lines per centimetre. Determine wavelength of the light used.

Solution:-

Data:- $\theta = 38^\circ$

$N = 5400$ lines per cm

$N = 5400 \times 10^{2}$ lines per metre

$n = 2$

$\lambda = ?$

Calculation:-

$$d \sin\theta = n\lambda$$

$$\lambda = \frac{d \sin\theta}{n}$$

As we have $d = \frac{1}{N}$

So

$$\lambda = \frac{1}{N} \frac{\sin\theta}{n}$$

$$= \frac{1}{540000\,m^{-1}} \times \frac{\sin 38^\circ}{2}$$

$$\lambda = 5.7 \times 10^{-7}\,m$$

$$= 570 \times 10^{-9}\,m$$

$$\boxed{\lambda = 570\,nm}$$

9.6:- A light is incident normally on a grating which has 2500 lines per cm^{-1}. Compute the wavelength of a spectral line for which the deviation in second order is 15°.

Solution:-

Data:- $N = 2500$ lines cm^{-1}

$N = 250000$ lines m^{-1}

$n = 2$

$\theta = 15^\circ$

$\lambda = ?$

Calculations:-

$$d \sin\theta = n\lambda$$

$$\lambda = \frac{d \sin\theta}{n}$$

As

$$d = \frac{1}{N}$$

$\therefore$

$$\lambda = \frac{1}{N} \frac{\sin\theta}{n}$$

$$= \frac{1}{250000} \times \frac{\sin 15^\circ}{2} \ m$$

$$= 5.18 \times 10^{-7} \ m$$

$$= 518 \times 10^{-9} \ m$$

$$\boxed{\lambda = 518 \ nm}$$

9.7:- Sodium light ($= 589$ nm) is incident normally on a grating having 3000 lines per centimeter. What is the highest order of the spectrum obtained with this grating.

Solution:-

Data:- $\lambda = 589 \ nm = 589 \times 10^{-9} \ m$

$N = 3000$ lines cm^{-1}

$= 300000$ lines m^{-1}

$n = ?$

$\theta = 90^\circ$

Calculations :- As

$$d \sin\theta = n\lambda$$

$$n = \frac{d \sin\theta}{\lambda}$$

$$= \frac{1}{N} \cdot \frac{\sin\theta}{\lambda} \qquad (\because d = 1/N)$$

$$= \frac{1}{300000\,m^{-1}} \times \frac{\sin 90^\circ}{589 \times 10^{-9}\,m}$$

$$= \frac{1}{0.1767}$$

$$n = 5.66$$

Hence, the highest spectrum obtained with this grating is 5th one (as 6th is incomplete)

9.8 :-

Blue light of wavelength 480 nm illuminates a diffraction grating. The second order image is formed at an angle of 30° from the central image. How many lines in a centimeter of the grating have been ruled?

Solution :- Data :- $\lambda = 480\,nm = 480 \times 10^{-9}\,m$

$n = 2$

$\theta = 30^\circ$

$N = ?$

Calculation :-

$$d \sin\theta = n\lambda$$

As $d = 1/N$

$$\frac{1}{N} \sin\theta = n\lambda$$

$$N = \frac{\sin\theta}{n\lambda}$$

$$N = \frac{\sin 30^\circ}{2 \times 480 \times 10^{-9}\,m}$$

$$N = \frac{0.5}{960 \times 10^{-9}\,m}$$

$N = 5.2 \times 10^{5}$ lines per m

$\boxed{N = 5.2 \times 10^{3} \text{ lines per cm}}$

9.9:- X-rays of wavelength 0.150 nm are observed to undergo a first order reflection at a Bragg angle of 13.3° from a quartz (SiO_2) crystal. What is the interplaner spacing of the reflecting planes in the crystal?

Solution:-

Data:-

$$\lambda = 0.150\ nm = 0.15 \times 10^{-9}\ m$$
$$m = 1$$
$$\theta = 13.3^\circ$$
$$d = ?$$

Calculations:-

According to Bragg's equation

$$2d\sin\theta = m\lambda$$
$$d = \frac{m\lambda}{2\sin\theta}$$
$$= \frac{1 \times 0.15 \times 10^{-9}\ m}{2 \times \sin 13.3^\circ}$$
$$= 3.26 \times 10^{-10}\ m$$
$$= 0.326 \times 10^{-9}\ m$$
$$\boxed{d = 0.326\ nm}$$

9.10:- An X-ray beam of wavelength undergoes a first order reflection from a crystal when is angle of incidence to a crystal face is 26.5°, and an x-ray beam of wavelength 0.097 nm undergoes a third order reflection when its angle of incidence to that face is 60°. Assuming that the two beams reflect from the same family of planes, calculate

(a)- the interplaner spacing of the planes.

(b)- the wavelength.