BS-CS

AHMER AQEEL

15960 ASSIGNMENT 2

Q) Evaluate the following limit by L'Hospital rule

1) $\lim_{x\to\infty} \frac{3x^2+x+4}{5x^2+8x}$

Sol:

$$\lim_{x\to\infty} \frac{d/dx\,(3x^2+x+4)}{d/dx\,(5x^2+8x)}$$

$$\lim_{x\to\infty} \frac{6x+1}{10x+8}$$

again, we get;

$$\lim_{x\to 0} \frac{6}{10}$$

$$= \frac{3}{5} \quad \text{Ans}$$

2) $\lim_{x\to 0} \frac{e^x-1-x-x^2/2}{x^3}$

Sol:

$$\lim_{x\to 0} \frac{d/dx\,(e^x-1x-x^2/2)}{d/dx\,(x^3)}$$

$$\lim_{x\to 0} \frac{e^x-1-x}{3x^2}$$

$= \lim_{x \to 0} \frac{e^x - 1}{6x}$

$= \lim_{x \to 0} \frac{e^x}{6}$

applying limit;

$= \frac{e^0}{6}$

$= \frac{1}{6}$ ans

Q2: Find the derivatives of the following functions.

a) $f(x) = (x^2+2x-5)(x^3-1)$

Sol:

Solving it by product rule, $dy/dx = u.dv + v.du$

$u = x^2+2x-5$ $\quad du = 2x+2$

$v = x^3-1$ $\quad dv = 3x^2$

$f(x) = (x^2+2x-5)(3x^2) + (x^3-1)(2x+2)$

$= 3x^4 + 6x^3 - 15x^2 + 2x^4 + 2x^3 - 2x - 2$

$= 5x^4 + 8x^3 - 15x^2 - 2x - 2$ ans.

(02) $f(x) = \frac{x^2+1}{x-3}$

Sol:- solving it by quotient rule; $= \frac{v\,du - u\,dv}{v^2}$

$u = x^2+1 \qquad du = 2x$

$v = x-3 \qquad dv = 1$

$$f(x) = \frac{[(x-3)(2x)] - [(x^2+1)(1)]}{(x-3)^2}$$

$$= \frac{(2x^2-6x)-(x^2+1)}{(x-3)^2}$$

$$= \frac{2x^2-6x-x^2-1}{(x-3)^2}$$

$$= \frac{x^2-6x-1}{(x-3)^2} \text{ ans.}$$

03) $f(x) = \sin x \cos x$

Sol:- solving it by product rule,

$$= \sin x \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(\sin x)$$

$$= \sin x(-\sin x) + (\cos x)(\cos x)$$

$$= -\sin^2 x + \cos^2 x$$

$$= \cos^2 x - \sin^2 x \qquad (\because \cos^2 x - \sin^2 x = \cos 2x)$$

$$= \cos 2x \text{ ans}$$

04) $f(x) = \frac{e^{-3x}}{x^2+1}$

Sol:- solving it by quotient rule,

$u = e^{-3x}$ $\quad du = -3e^{-3x}$

$v = x^2+1$ $\quad dv = 2x$

$$= \frac{(x^2+1)(-3e^{-3x}) - (e^{-3x})(2x)}{(x^2+1)^2}$$

$$= \frac{-3x^2e^{-3x} - 3e^{-3x} - 2xe^{-3x}}{(x^2+1)^2}$$

$$= \frac{-e^{-3x}(3x^2+2x+3)}{(x^2+1)^2} \text{ ans}$$

05) $f(x) = \frac{\ln(x^2-1)}{\ln(x^3-1)}$

Sol:- solving it by quotient rule;

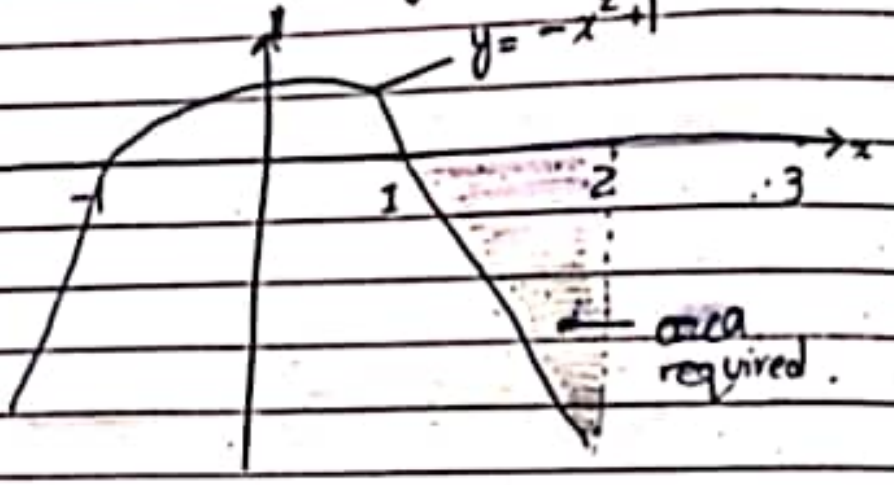
$u = \ln(x^2-1)$ $\quad du = 2x/x^2-1$

$v = \ln(x^3-1)$ $\quad dv = 3x^2/x^3-1$

$$= \frac{\ln(x^3-1)\left(\frac{2x}{x^2-1}\right) - \ln(x^2-1)\left(\frac{3x^2}{x^3-1}\right)}{[\ln(x^3-1)]^2}$$ ans

Q3: Evaluate the area bounded by the curve and x-axis.

a)



Sol:

$$\text{Area} = \int_1^2 (-x^2+1)\,dx$$

$$= \left(\frac{-x^3}{3} + x\right)\Big|_1^2$$

applying limit;

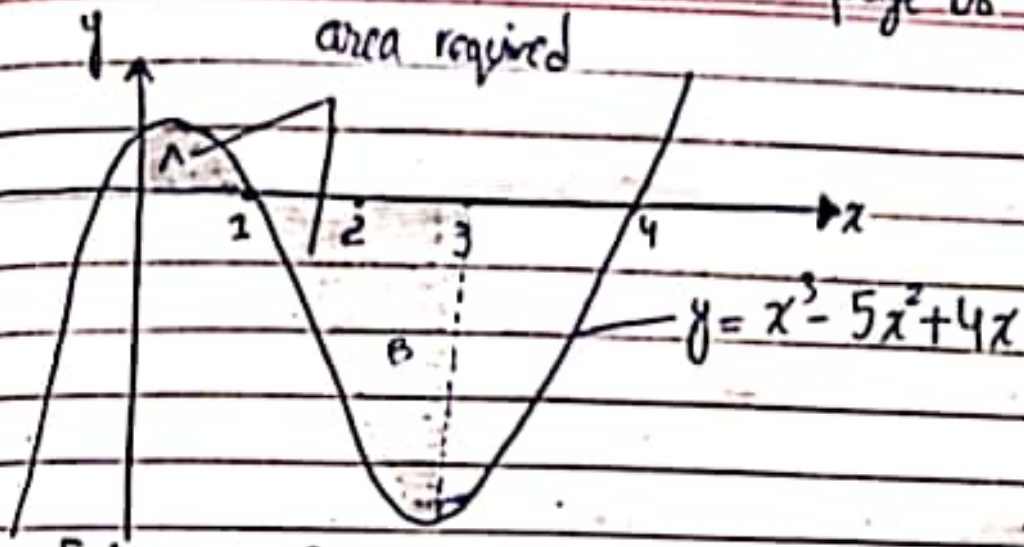
$$= \left[\frac{-(2)^3}{3} + (2)\right] - \left[\frac{-(1)^3}{3} + (1)\right]$$

$$= \left(\frac{-8}{3} + 2\right) - \left(\frac{-1}{3} + 1\right)$$

$$= -2/3 - 2/3$$

$$= \boxed{-4/3}$$

$$= 4/3 \text{ ans}$$



Sol:- finding area of part A;

$$\text{Area} = A = \int_0^1 (x^3 - 5x^2 + 4x)\, dx$$

$$A = \frac{x^4}{4} - \frac{5x^3}{3} + \frac{4x^2}{2} \Bigg|_0^1$$

applying limit;

$$A = \left[\frac{(1)^4}{4} - \frac{5(1)^3}{3} + \frac{4(1)^2}{2}\right] - 0$$

$$= \frac{1}{4} - \frac{5}{3} + \frac{4}{2}$$

$$A = 7/12$$

now finding area of part B;

$$B = \int_1^3 (x^3 - 5x^2 + 4x)\, dx$$

$$B = \frac{x^4}{4} - \frac{5x^3}{3} + \frac{4x^2}{2} \Bigg|_1^3$$

$$B = \left[\frac{(3)^4}{4} - \frac{5(3)^3}{3} + \frac{4(3)^2}{2}\right] - \left[\frac{(1)^4}{4} - \frac{5(1)^3}{3} + \frac{4(1)^2}{2}\right]$$

$$B = \left(\frac{81}{4} - \frac{135}{3} + \frac{36}{2}\right) - \frac{7}{12}$$

$$B = \frac{-27}{4} - \frac{7}{12}$$

$$B = \left|-22/3\right|$$

$$B = 22/3$$

now,

Total area = A + B.

$$= \frac{7}{12} + \frac{22}{3}$$

$$= \frac{95}{12}$$ ans

Q1 Evaluate the following integrals

i) $\int (3x+5)^6 dx$

Sol:-

$u = 3x+5$

$\frac{du}{dx} = 3$

$= \int \frac{u^6 \, du}{3}$

$= \frac{1}{3} \frac{u^{c+1}}{c+1} + C$

$= \frac{1}{3} \times \frac{u^7}{7} + C$

$= \frac{u^7}{21} + C$

$= \frac{(3x+5)^7}{21} + C$ Ans.

2) $\int_0^4 v(x)$ where $v(x) = \begin{cases} 2x & x < 3 \\ -2x & x \geq 3 \end{cases}$

Sol:

$$= \int_0^3 2x\,dx + \int_3^4 -2x\,dx$$

$$= \frac{2x^2}{2}\Big|_0^3 + \left(\frac{-2x^2}{2}\Big|_3^4\right)$$

$$= x^2\Big|_0^3 - x^2\Big|_3^4$$

applying limit;

$$= [(3)^2 - (0)^2] - [(4)^2 - (3)^2]$$

$$= [9 - 0] - [16 - 9]$$

$$= 9 - 7$$

$$= 2 \text{ ans.}$$

3) $\int_{-6}^{1} g(z)\,dz$ where $g(z) = \begin{cases} 2-z & z > -2 \\ 4e^z & z \le -2 \end{cases}$

Sol:-

$$= \int_{-6}^{-2} 4e^z\,dz + \int_{-2}^{1} 2-z\,dz$$

$$= 4e^z \Big|_{-6}^{-2} + 2z - \frac{z^2}{2} \Big|_{-2}^{1}$$

applying limit;

$$= \left[4e^{-2} - 4e^{-6}\right] + \left[2(1) - \frac{(1)^2}{2} - 2(-2) - \frac{(-2)^2}{2}\right]$$

$$= \left[4e^{-2} - 4e^{-6}\right] + \left[\frac{3}{2} + 6\right]$$

$$= 4e^{-2} - 4e^{-6} + \frac{15}{2} \quad \text{ans.}$$

Qs: Check Whether the following vectors are orthogonal or not

1) $a = i + 2j$ and $b = 2i - j$, also evaluate $-2a + 3b$

Sol:

$$a.b = (i + 2j).(2i - j)$$
$$= 2 - 2$$
$$= 0$$

$\vec{a}$ and $\vec{b}$ are orthogonal.

evaluating;

$$= -2a + 3b$$
$$= -2(i + 2j) + 3(2i - j)$$
$$= -2i - 4j + 6i - 3j$$
$$= 4i - 7j$$ ans:

2) $a = 3i + 2j$ and $b = 7i - 5j$

Sol:

$$a.b = (3i + 2j).(7i - 5j)$$
$$= 21 - 10$$
$$= 11 \neq 0$$

$\vec{a}$ and $\vec{b}$ are not orthogonal.

3) $a = i + 2j + 3k$ and $b = 4i + 5j + 6k$, also evaluate $-a - 3b$

Sol:-

$$a \cdot b = (i + 2j + 3k) \cdot (4i + 5j + 6k)$$

$$= 4 + 10 + 18$$

$$= 32 \neq 0$$

a and b are not orthogonal.

evaluating;

$$= -a - 3b$$

$$= -(i + 2j + 3k) - 3(4i + 5j + 6k)$$

$$= -(i + 2j + 3k) - (12i + 15j + 18k)$$

$$= -i - 2j - 3k - 12i - 15j - 18k$$

~~$= -11i - 13j - 15k$~~

$$= -13i - 17j - 21k \quad \text{ans}$$