BS-CS
SHAHZAR RAZA QADRI
15933

ASSIGNMENT 2

Q) Evaluate the following limit by L'Hospital rule

1) $\lim_{x \to \infty} \frac{3x^2 + x + 4}{5x^2 + 8x}$

Sol:

$\lim_{x \to \infty} \frac{d/dx\,(3x^2 + x + 4)}{d/dx\,(5x^2 + 8x)}$

$\lim_{x \to \infty} \frac{6x + 1}{10x + 8}$

again, we get;

$\lim_{x \to 0} \frac{6}{10}$

$= \frac{3}{5}$ Ans

2) $\lim_{x \to 0} \frac{e^x - 1 - x - x^2/2}{x^3}$

Sol:

$\lim_{x \to 0} \frac{d/dx\,(e^x - 1x - x^2/2)}{d/dx\,(x^3)}$

$\lim_{x \to 0} \frac{e^x - 1 - x}{3x^2}$

$$= \lim_{x \to 0} \frac{e^x - 1}{6x}$$

$$= \lim_{x \to 0} \frac{e^x}{6}$$

applying limit;

$$= \frac{e^0}{6}$$

$$= \frac{1}{6} \quad \text{ans.}$$

Q2: Find the derivatives of the following functions.

1) $f(x) = (x^2+2x-5)(x^3-1)$

Sol:-

Solving it by product rule,

$\boxed{d/dx = u.dv + v.du}$

$u = x^2+2x-5$ $\quad$ $du = 2x+2$

$v = x^3-1$ $\quad$ $dv = 3x^2$

$$f(x) = (x^2+2x-5)(3x^2) + (x^3-1)(2x+2)$$

$$= 3x^4 + 6x^3 - 15x^2 + 2x^4 + 2x^3 - 2x - 2$$

$$= 5x^4 + 8x^3 - 15x^2 - 2x - 2 \quad \text{ans.}$$

02) $f(x) = \frac{x^2+1}{x-3}$

Sol:-

solving it by quotient rule; $= \frac{vdu - udv}{v^2}$

$u = x^2+1$ $\quad du = 2x$

$v = x-3$ $\quad dv = 1$

$$f(x) = \frac{[(x-3)(2x)] - [(x^2+1)(1)]}{(x-3)^2}$$

$$= \frac{(2x^2-6x) - (x^2+1)}{(x-3)^2}$$

$$= \frac{2x^2-6x-x^2-1}{(x-3)^2}$$

$$= \frac{x^2-6x-1}{(x-3)^2} \text{ ans.}$$

03) $f(x) = \sin x \cos x$

Sol:- solving it by product rule,

$$= \sin x \, d/dx (\cos x) + \cos x \, d/dx (\sin x)$$

$$= \sin x(-\sin x) + (\cos x)(\cos x)$$

$$= -\sin^2 x + \cos^2 x$$

$$= \cos^2 x - \sin^2 x \qquad (\because \cos^2 x - \sin^2 x = \cos 2x)$$

$$\boxed{= \cos 2x \text{ ans}} \text{ ans.}$$

04) $f(x) = \dfrac{e^{-3x}}{x^2+1}$

Sol:- solving it by quotient rule,

$u = e^{-3x}$ $\quad du = -3e^{-3x}$

$v = x^2+1$ $\quad dv = 2x$

$$= \frac{(x^2+1)(-3e^{-3x}) - (e^{-3x})(2x)}{(x^2+1)^2}$$

$$= \frac{-3x^2e^{-3x} - 3e^{-3x} - 2xe^{-3x}}{(x^2+1)^2}$$

$$= \frac{-e^{-3x}(3x^2+2x+3)}{(x^2+1)^2} \text{ ans}$$

05) $f(x) = \dfrac{\ln(x^2-1)}{\ln(x^3-1)}$

Sol:- solving it by quotient rule,

$u = \ln(x^2-1)$ $\quad du = 2x/x^2-1$

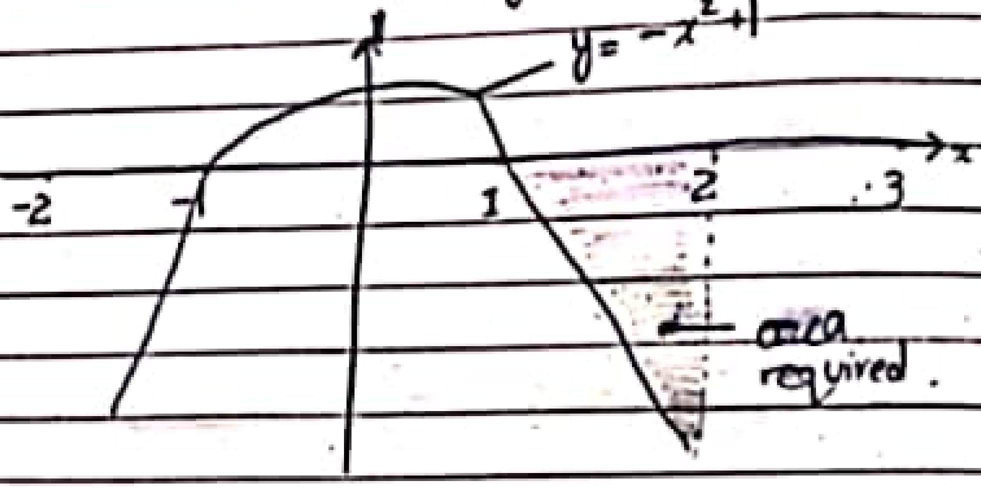
$v = \ln(x^3-1)$ $\quad dv = 3x^2/x^3-1$

$$= \frac{\ln(x^3-1)\left(2x/x^2-1\right) - \ln(x^2-1)\left(3x^2/x^3-1\right)}{[\ln(x^3-1)]^2}$$

ans

Q3: Evaluate the area bounded by the curve and x-axis.

1)



Sol:

$$\text{Area} = \int_1^2 (-x^2+1)\,dx$$

$$= \left(\frac{-x^3}{3} + x\right)\Bigg|_1^2$$

applying limit;

$$= \left[\frac{-(2)^3}{3} + (2)\right] - \left[\frac{-(1)^3}{3} + (1)\right]$$

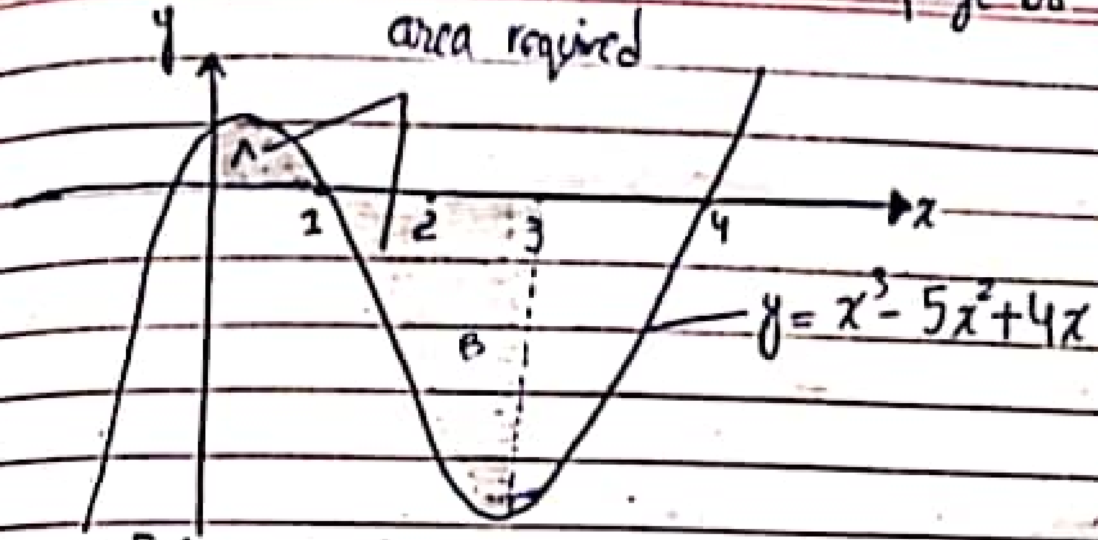
$$= \left(\frac{-8}{3} + 2\right) - \left(\frac{-1}{3} + 1\right)$$

$$= -2/3 - 2/3$$

$$= \boxed{-4/3}$$

$$= 4/3 \text{ ans}$$

2)



Sol:- finding area of part A;

$$\text{Area} = A = \int_0^1 (x^3 - 5x^2 + 4x)\,dx$$

$$A = \frac{x^4}{4} - \frac{5x^3}{3} + \frac{4x^2}{2}\Big|_0^1$$

applying limit;

$$A = \left[\frac{(1)^4}{4} - \frac{5(1)^3}{3} + \frac{4(1)^2}{2}\right] - 0$$

$$= \frac{1}{4} - \frac{5}{3} + \frac{4}{2}$$

$A = 7/12$

now finding area of part B;

$$B = \int_1^3 (x^3 - 5x^2 + 4x)\,dx$$

$$B = \frac{x^4}{4} - \frac{5x^3}{3} + \frac{4x^2}{2}\Big|_1^3$$

2) $\int_0^4 v(x)$ where $v(x) = \begin{cases} 2x & x < 3 \\ -2x & x > 3 \end{cases}$

Sol:-

$$= \int_0^3 2x\,dx + \int_3^4 -2x\,dx$$

$$= \frac{\cancel{2}x^2}{\cancel{2}}\Big|_0^3 + \left(-\frac{\cancel{2}x^2}{\cancel{2}}\Big|_3^4\right)$$

$$= x^2\Big|_0^3 - x^2\Big|_3^4$$

applying limit;

$$= [(3)^2 - (0)^2] - [(4)^2 - (3)^2]$$

$$= [9 - 0] - [16 - 9]$$

$$= 9 - 7$$

$$= 2 \text{ ans.}$$

3) $\int_{-6}^{1} g(z)\,dz$ where $g(z) = \begin{cases} 2-z & z > -2 \\ 4e^{z} & z \leq -2 \end{cases}$

Sol:-

$$= \int_{-6}^{-2} 4e^{z}\,dz + \int_{-2}^{1} 2-z\,dx$$

$$= 4e^{z}\Big|_{-6}^{-2} + 2z - \frac{z^2}{2}\Big|_{-2}^{1}$$

applying limit;

$$= \left[4e^{-2} - 4e^{-6}\right] + \left[2(1) - \frac{(1)^2}{2} - 2(-2) - \frac{(-2)^2}{2}\right]$$

$$= \left[4e^{-2} - 4e^{-6}\right] + \left[\frac{3}{2} + 6\right]$$

$$= 4e^{-2} - 4e^{-6} + \frac{15}{2} \text{ ans.}$$

Q5: Check Whether the following vectors are orthogonal or not

1) $a = i + 2j$ and $b = 2i - j$, also evaluate $-2a + 3b$

Sol:

$a.b = (i + 2j).(2i - j)$
$= 2 - 2$
$= 0$

$\vec{a}$ and $\vec{b}$ are orthogonal.

evaluating;

$= -2a + 3b$
$= -2(i+2j) + 3(2i-j)$
$= -2i - 4j + 6i - 3j$

$= 4i - 7j$ ans.

2) $a = 3i + 2j$ and $b = 7i - 5j$

Sol:

$a.b = (3i+2j).(7i-5j)$

$= 21 - 10$

$= 11 \neq 0$

$\vec{a}$ and $\vec{b}$ are not orthogonal.

3) $a = i + 2j + 3k$ and $b = 4i + 5j + 6k$, also evaluate $-a - 3b$

Sol :-

$$a \cdot b = (i + 2j + 3k) \cdot (4i + 5j + 6k)$$

$$= 4 + 10 + 18$$

$$= 32 \neq 0$$

a and b are not orthogonal.

evaluating;

$$-a - 3b$$

$$= -(i + 2j + 3k) - 3(4i + 5j + 6k)$$

$$= -(i + 2j + 3k) - (12i + 15j + 18k)$$

$$= -i - 2j - 3k - 12i - 15j - 18k$$

$$\cancel{= -11i - 13j - 15k}$$

$$= -13i - 17j - 21k$$ ans.

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