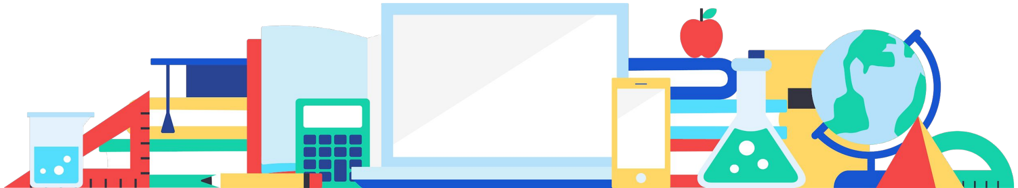
Precalculus Concepts & Vocab

Updated March 2020



Functions

Definition of a Function

Key Vocabulary

A **function** is an equation for which any x that can be plugged into the equation yields exactly one value of y

Functions are generally notated as **y = f(x)**, read as "f of x"

- ➔ Functions can also be notated as g(x), h(x), S(x), etc.

Examples of functions

- ➔ $f(x) = 2x^2 - 5x + 3$
- ➔ $f(x) = \sin(x)$
- ➔ $f(x) = \log(x)$

To evaluate a function at a given value, substitute that value in for x wherever x appears in the equation

- ➔ Ex: $f(x) = 2x^2 - 5x + 3$ evaluated at (-3) is equal to:

$$f(-3) = 2(-3)^2 - 5(-3) + 3 = 2(9) + 15 + 3 = 36$$

Domain and Range

Key Vocabulary

The **domain** of the function is the complete set of all possible values of the input variable *x* for which the function is defined (i.e. for which it will output real y values)

- ➔ Values that lead to division by zero, square roots of negative numbers, logarithms of zero, logarithms of negative numbers, etc. will not be in the domain as they result in imaginary numbers or undefined expressions

The **range** of the function is the set of all possible resulting values of the resultant

Composition of Functions

Key Vocabulary

The **composition** of the two functions $f(x)$ and $g(x)$ is given by

$$(f \circ g)(x) = f(g(x))$$

Compositions are evaluated by plugging the second function, $g(x)$ into the first function, $f(x)$

The order in which one evaluates a composition is important as $f(g(x))$ does not necessarily yield the same value as $g(f(x))$

Inverse Functions

Key Vocabulary

The **inverse function** of f(x), denoted **f-$^{-1}$(x)**, “reverses” the original function so that $f^{-1}(f(x)) = f(f^{-1}(x)) = x$

➔ Note: the −1 in the notation $f^{-1}(x)$ is NOT an exponent and $f^{-1}(x) \neq 1 / f(x)$

Only functions that are one-to-one have inverses. A function is **one-to-one** if if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$ i.e. no two values of x produce the same y

Inverse Functions

Key Steps

To find a function's inverse, $f^{-1}(x)$

1. Replace $f(x)$ with y
2. Replace every x with a y and replace every y with an x
3. Solve the equation from step 2 for y
4. Replace y with $f^{-1}(x)$
5. Check your answer by verifying that $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$

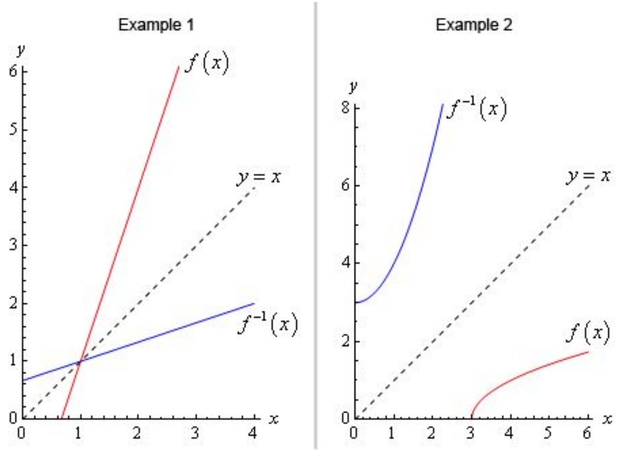
Example: find the inverse of $f(x) = 2x - 2$

1. $y = 2x - 2$
2. $x = 2y - 2$
3. $2y = x + 2 \rightarrow y = x/2 + 1$
4. $f^{-1}(x) = x/2 + 1$
5. $f^{-1}(f(x)) = (2x - 2) / 2 + 1 = x - 1 + 1 = x$
6. $f(f^{-1}(x)) = 2\,(x/2 + 1) - 2 = x + 2 - 2 = x$

Inverse Functions

Key Property

The graph of a function's inverse is a reflection of the actual function about the line y = x



Exponential Equations

Exponential Functions

Key Vocabulary

Let $b > 0$ and $b \neq 1$. An **exponential function** is a function in the form

$$f(x) = b^{x.}$$

We often restrict the value of b because

- ➔ If $b = 0$, this becomes the constant function $f(x) = 1$
- ➔ Iif $b = 1$, this becomes the constant function $f(x) = b$
- ➔ If $b < 0$, the function can lead to an imaginary number for some values where x is odd

A special exponential function is the **natural exponential function**

$$f(x) = e^x, \text{ where } e = 2.71828182845905...$$

- ➔ $e^x \to \infty$ as $x \to \infty$
- ➔ $e^x \to 0$ as $x \to -\infty$

Exponential Functions

Key Properties

Key properties of the exponential function $f(x) = b^x$

1. $f(0) = 1$. The function will always take the value 1 at $x = 0$
2. $f(x) \neq 0$. An exponential function will never be 0
3. $f(x) > 0$. An exponential function is always positive
4. The domain of an exponential function is $(-\infty,\infty)$
5. The range of an exponential function is $(0,\infty)$
6. If $0 < b < 1$, then
    - $f(x) \to 0$ as $x \to \infty$
    - $f(x) \to \infty$ as $x \to -\infty$
7. If $b > 1$,
    - $f(x) \to \infty$ as $x \to \infty$
    - $f(x) \to 0$ as $x \to -\infty$

Exponents

Key Properties

Key rules and properties of exponents that may help you solve exponential functions

1. Product with same base: $b^x \cdot b^y = b^{x+y}$
2. Product with same exponent: $a^x \cdot b^x = (a \cdot b)^x$
3. Quotient rule with same base: $b^x / b^y = b^{x-y}$
4. Quotient rule with same exponent: $a^x / b^x = (a/b)^x$
5. Power rule: $(a^x)^y = a^{x \cdot y}$
6. Power rule with radicals: $\sqrt[x]{b^y} = b^{y/x}$
7. Negative exponents: $b^{-x} = 1 / b^x$
8. Zero rule: $b^0 = 1$
9. One rule: $b^1 = b$

Logarithmic Equations

Logarithmic Functions

Key Vocabulary

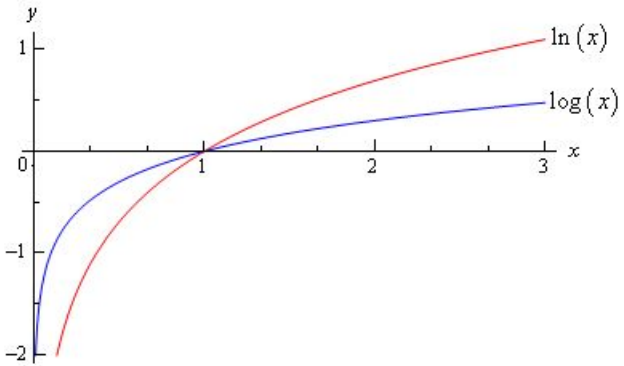
Let $b > 0$ and $b \neq 1$. Then

$$y = \log_b x \text{ is equivalent to } x = b^y$$

- ➔ $y = \log_b x$ is called the **logarithmic form**
- ➔ $x = b^y$ is called the **exponential form**
- ➔ The number b is called the **base**

Two special logarithmic functions are

- ➔ the **natural logarithm**: $\ln x = \log_e x$
- ➔ The **common logarithm:** $\log x = \log_{10} x$



Natural and Common Logarithms

Key Properties

Key properties of the logarithm functions

1. The domain of the logarithmic function is $(0,\infty)$
2. $\log_b b = 1$
3. $\log_b 1 = 0$
4. $\log_b b^x = x$
5. $b^{\log_b x} = x$
6. $\log_b xy = \log_b x + \log_b y$
7. $\log_b (x/y) = \log_b x - \log_b y$
8. $\log_b (x^r) = r\log_b x$

Note: these properties tell us that $f(x) = b^x$ and $g(x) = \log_b x$ are inverses of each other

Note: $\log_b(x+y) \neq \log_b x + \log_b$ and $\log_b(x-y) \neq \log_b x - \log_b$

Change of Base Formula

Key Formula

The **change of base** formula is given by

$$\log_b x = \frac{\log_a x}{\log_a b}$$

The two most common change of base formulas are

- ➔ $\log_b x = \frac{\ln x}{\ln b}$
- ➔ $\log_b x = \frac{\log x}{\log b}$

Solving Exponential Equations

Solving Equations

A key property to use when solving logarithmic equations is

$$\log_b b^x = x$$

Example 1: Solve $7 + 15e^{1-3z} = 10$

- Step 1: Isolate the exponential (with a coefficient of 1)

  $7 + 15e^{1-3z} = 10 \rightarrow 15e^{1-3z} = 3 \rightarrow e^{1-3z} = \frac{1}{5}$

- Step 2: Get the z out of the exponent using the property $\ln e^x = x$, by taking the log of both sides

  $\ln(e^{1-3z}) = \ln(\frac{1}{5}) \rightarrow 1 - 3z = \ln(\frac{1}{5})$

- Solve the equation for z

  $1 - 3z = \ln(\frac{1}{5}) \rightarrow 3z = 1 - \ln(\frac{1}{5}) \rightarrow z = \frac{1}{3} - \frac{1}{3}(\ln(\frac{1}{5})) \approx 0.8698$

Our solution is $z \approx 0.8698$

Solving Exponential Equations

Solving Equations (Continued)

Example 2: Solve $x - xe^{5x+2} = 0$

- ➔ Step 1: Factor the x out of both terms. Do **<u>not</u>** divide an x from both terms, if you do you will miss a solution.
  $x - xe^{5x+2} = 0 \rightarrow x(1 - e^{5x+2}) = 0$
  We can now see there are two solutions
  $x = 0$ **<u>OR</u>** $1 - e^{5x+2} = 0$
- ➔ Step 2: Now we solve for each solution. For $x = 0$, there is nothing more to do. For the second, first isolate the exponential
  $1 - e^{5x+2} = 0 \rightarrow 1 = e^{5x+2}$
- ➔ Step 3: Get the x out of the exponent using the property $\ln e^x = x$
  $1 = e^{5x+2} \rightarrow \ln(1) = \ln(e^{5x+2}) \rightarrow 5x + 2 = \ln(1)$
- ➔ Step 4: Solve the equation for x. Note that $\ln(1) = 0$
  $5x + 2 = \ln(1) \rightarrow 5x + 2 = 0 \rightarrow 5x = -2 \rightarrow x = -\frac{2}{5}$

Our two solutions are $x = 0$ and $x = -\frac{2}{5}$

Solving Exponential Equations

Solving Equations (Continued)

Example 3: Solve $4e^{1+3x} - 9e^{5-2x} = 0$

- Step 1: Get one exponential on each side of the equation

  $4e^{1+3x} - 9e^{5-2x} = 0 \rightarrow 4e^{1+3x} = 9e^{5-2x}$

- Step 2: Divide both sides by one of the exponentials (it doesn't matter which one)

  $4e^{1+3x} = 9e^{5-2x} \rightarrow \dfrac{e^{1+3x}}{e^{5-2x}} = \dfrac{9}{4}$

- Step 3: Use the quotient rule for exponents to combine the exponentials

  $\dfrac{e^{1+3x}}{e^{5-2x}} = \dfrac{9}{4} \rightarrow e^{1+3x-(5-2x)} = 9/4 \rightarrow e^{5x-4} = 9/4$

- Step 4: Get the x out of the exponent using the property $\ln e^x = x$

  $5x - 4 = \ln(9/4)$

- Step 5: Solve for x

  $5x = 4 + \ln(9/4) \rightarrow x = \frac{1}{5}(4 + \ln(9/4)) \rightarrow x = 0.962186\ldots$

Our solution is $x \approx 0.9622$

Solving Logarithmic Equations

Solving Equations (Continued)

A key property to use when solving logarithmic equations is

$$b^{\log_b x} = x$$

Example 4: Solve 3 + 2ln(x/7 + 3) = −4

- ➔ Step 1: Get the logarithm by itself on one side of the equation with a coefficient of 1

  $2\ln(x/7 + 3) = -7 \rightarrow \ln(x/7 + 3) = -7/2$
- ➔ Step 2: Get the x out of the logarithm by using e to "exponentiate" both sides, using the property $e^{\ln(x)} = x$

  $\ln(x/7 + 3) = -7/2 \rightarrow e^{\ln(x/7 + 3)} = e^{-7/2} \rightarrow x/7 + 3 = e^{-7/2}$
- ➔ Step 3: Solve for x

  $x/7 + 3 = e^{-7/2} \rightarrow x/7 = -3 + e^{-7/2} \rightarrow x = 7\,(-3 + e^{-7/2}) \rightarrow x = -20.7886$
- ➔ Step 4: Verify that our solution does not lead to plugging a negative number into a logarithm by making sure x/7 + 3 is positive

  $x/7 + 3 = -20.7886/7 + 3 = 0.030197 > 0$ ✔️

Since we verified our solution does not lead to plugging in a negative number into a logarithm, our solution is $x \approx -20.7886$

Additional Resources

Pre-Calculus

Additional Resources

- ➔ https://www.khanacademy.org/math/precalculus
- ➔ https://www.themathpage.com/aPreCalc/precalculus.htm
- ➔ http://faculty.bard.edu/bloch/precalculus_review_gray.pdf
- ➔ https://math.tntech.edu/machida/MATH_GO/Precalculus_Review/