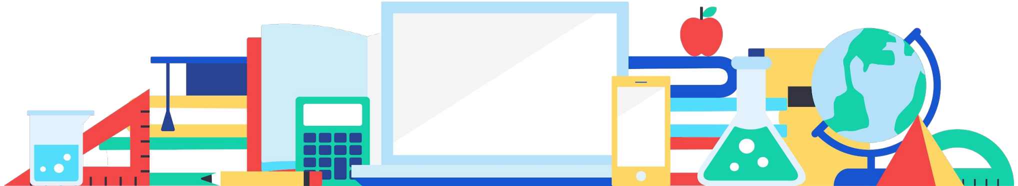
UPchieve

Geometry Concepts & Vocab

Updated March 2020



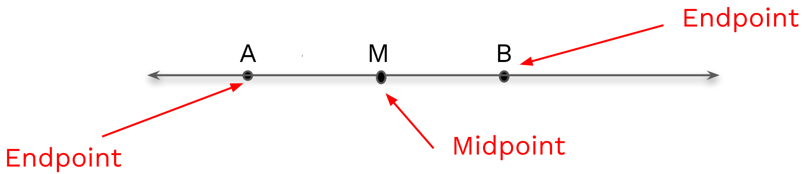
Lines and Angles

Lines

Key Vocabulary

A **line** is a straight one-dimensional figure with no thickness that extends in both directions with no end

Given any two points on a line, a **line segment** is the part of the line that contains those two points (the **endpoints**) and all of the points in between



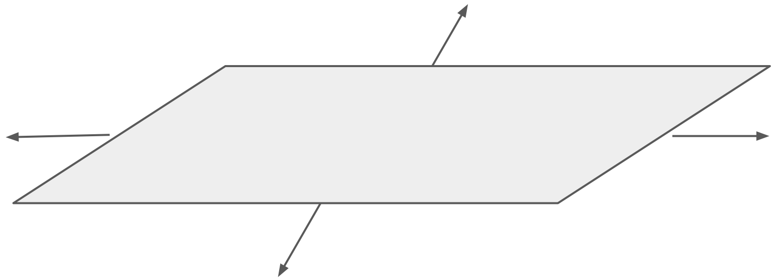
A line segment between two endpoints A and B can be denoted as $\overline{AB}$ or AB (note that sometimes AB denotes the line segment AB and sometimes it denotes the **length** of line segment AB, depending on context). A line that contains points A and B can be denoted as $\overleftrightarrow{AB}$

The point that divides a line segment into two **congruent line segments** (segments that are equal in length) is called the **midpoint** of the line segment

Planes

Key Vocabulary

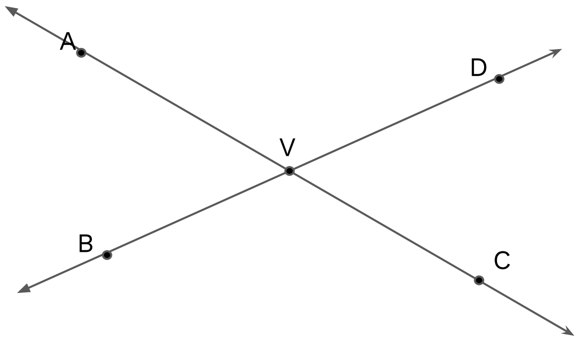
A **plane** is a two-dimensional figure with no thickness that extends in all directions with no end



Angles

Key Vocabulary

When two lines intersect at a point, they form four **angles.** Each angle has a **vertex** at point V (a vertex is the point of intersection of the two lines)



In the above figure, ∠AVB and ∠CVD are **vertical angles** (also called **opposite angles**). Opposite angles are **congruent angles** (they have equal measures).

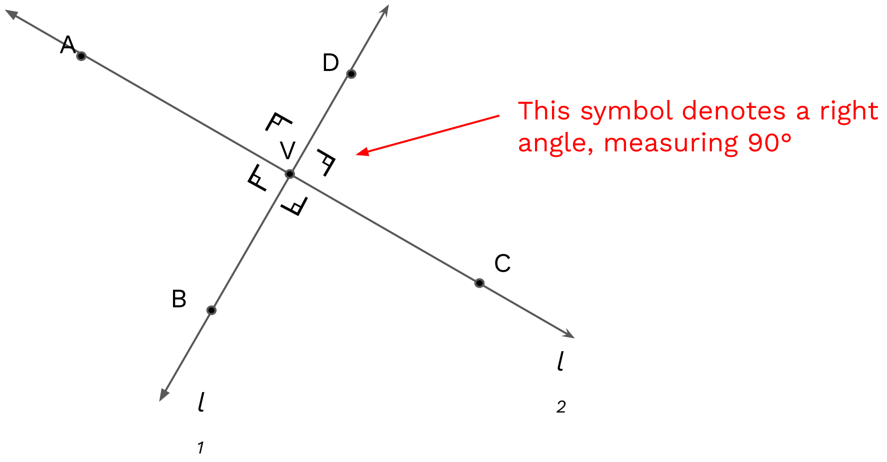
Angles are often measured in **degrees.** A full circle is divided into 360 degrees, and so the sum of the ∠AVB, ∠BVC, ∠CVD, and ∠DVA in the above figure sum to 360°

Perpendicular Lines

Key Vocabulary

Two lines that intersect to form four congruent angles are called **perpendicular lines**.

Each of the four angles has a measure of 90°, called **right angles.**



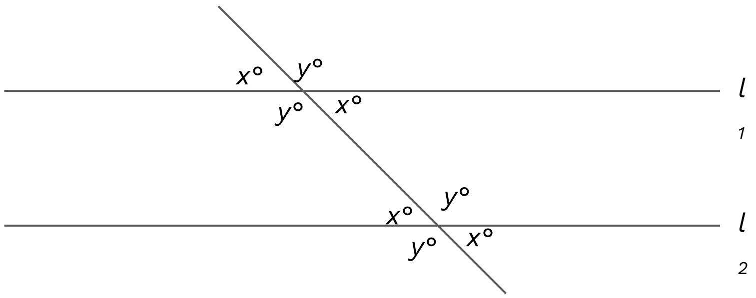
Perpendicular lines l_1 and l_2 can be denoted by $l_1 \perp l_2$

An angle with a measure less than 90° is an **acute angle** and an angle with a measure between 90° and 180° is an **obtuse angle**

Parallel lines

Key Vocabulary

Two lines in the same plane that do not intersect are called parallel lines



Lines l_1 and l_2 in the above figure are parallel, denoted by $l_1 \parallel l_2$

If a third line, l_3 intersects l_1 and l_2 it forms eight angles. Four of these angles measure $x°$ and four measure $y°$, where x + y = 180

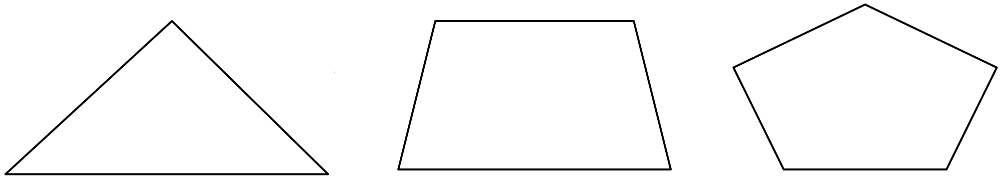
Polygons

Polygons

Key Vocabulary

A **polygon** is a closed figure formed by three or more line segments, called **sides**. Each side is joined to two other sides at its endpoints, called **vertices**. A **convex polygon** is a polygon in which the measure of each interior angle is less than 180°.

<u>Examples of Polygons</u>

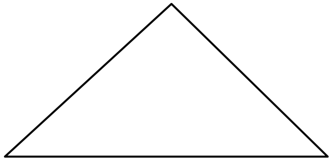


Triangle (3 sides) Quadrilateral (4 sides) Pentagon (5 sides)

Polygons (continued)

Key Vocabulary

A **triangle** is the simplest polygon, with three sides



If a polygon has n sides, then it can be divided into $n - 2$ triangles (ex: a quadrilateral can be divided into $4 - 2 = 2$ triangles and a pentagon can be divided into $5 - 2 = 3$ triangles

Since the sum of the measures of the interior angles of a triangle is 180°, it follows that the sum of the measures of the interior angles of an n-sided polygon is given by $(n - 2)(180°)$. For example, the sum of the interior angles for a quadrilateral ($n = 4$) is $(4 - 2)(180°) = 360°$ and the sum of the interior angles for a **hexagon** ($n = 6$) is given by $(6 - 2)(180°) = 720°$

Polygons (continued)

Key Vocabulary

A polygon in which all sides are congruent and all interior angles are congruent is called a **regular polygon**

Ex: a regular **octagon** has 8 equal sides and 8 equal interior angles. The sum of the measures of the interior angles is equal to $(8 - 2)(180°) = 1{,}080°$, and so each angle is equal to $1{,}080° / 8 = 135°$

The **perimeter** of a polygon is the sum of the lengths of its sides

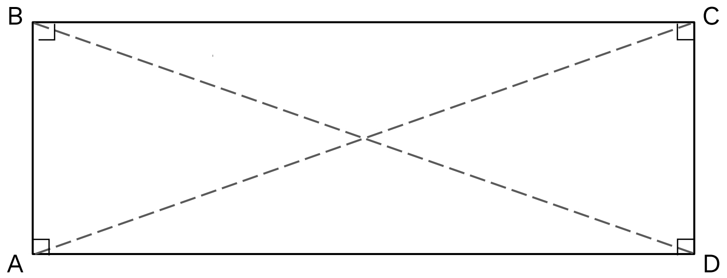
The **area** of a polygon refers to the area of the region enclosed by the polygon

Quadrilaterals

Key Vocabulary

A **quadrilateral** is a polygon with four side and four interior angles. The measure of the interior angles add to 360°. Common quadrilaterals include the rectangle, square, parallelogram, rhombus, and square

A **rectangle** is a quadrilateral with four right angles. Opposite sides of a rectangle are parallel and congruent, and the two diagonals are also congruent



In the above figure, AB // CD and AD // BC, AB = CD and AD = BC, AC = BD

A rectangle with four congruent sides is called a **square**

Quadrilaterals (continued)

Key Vocabulary

A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel. In a parallelogram, opposite sides are congruent and opposite angles are congruent

In the above figure, PQ // SR and PS // QR, PQ = SR and PS = QR

A parallelogram with four congruent sides is called a **rhombus**

*Note that a rectangle is also a parallelogram, but a parallelogram is not necessarily a rectangle

Area of a Quadrilateral

Key Vocabulary

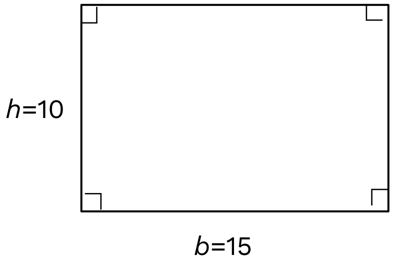
For all parallelograms, including rectangles and squares, the **area** A equals the product of the length of a base b and the corresponding height h; that is,

$$A = bh$$

Any side can be used as a base. The corresponding height is the perpendicular line segment from any point of a base to the opposite side (or to an extension of that side)

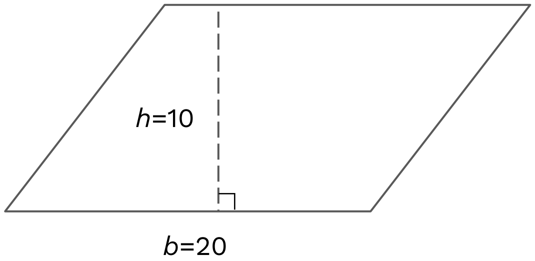
Examples (figures not drawn to scale):

<u>Rectangle</u>



$A = bh = 10 \times 15 = 150$

<u>Parallelogram</u>

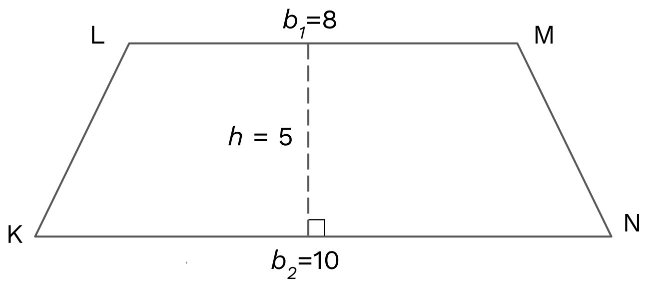


$A = bh = 20 \times 10 = 200$

Quadrilaterals

Key Vocabulary (continued)

A **trapezoid** is a quadrilateral in which two opposite sides are parallel



In the above figure, KN // LM

The area A of a trapezoid equals half the product of the sum of the lengths of the two parallel sides b_1 and b_2 and the corresponding height h; that is,

$$A = ½(b_1 + b_2)(h)$$

For example, in the above trapezoid with bases of length 8 and 10 and a height of 5,

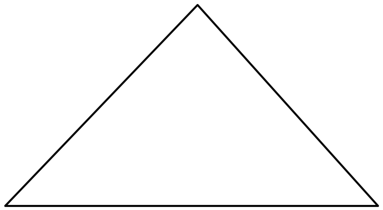
$$A = ½(b_1 + b_2)(h) = ½(8 + 10)(5) = ½\ (18)(5) = (9)(5) = 45$$

Triangles

Triangles

Key Vocabulary

A **triangle** is the simplest polygon, with three sides and three interior angles



The measure of the interior angles of a triangle add up to 180°

The **triangle inequality** states that the length of each side of a triangle must be less than the sum of the lengths of the other two sides. For example, the sides of a triangle could not have lengths of 5, 8, and 14 because 14 is greater than 5 + 8 = 13

Triangle Classifications

Key Vocabulary

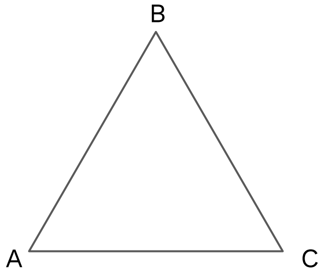
There are several classifications of triangles, with specific characteristics. Ex:

➔ An **equilateral triangle** has three congruent sides and three congruent interior angles (with each angle measuring 60°)

➔ An **isosceles triangle** has at least two congruent sides. If a triangle has two congruent sides, the angles opposite the two sides are also congruent (the converse is also true)

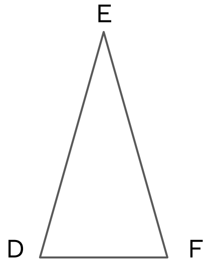
➔ A **scalene triangle** has three sides all different in length (and three angles with different measures)

Equilateral Triangle



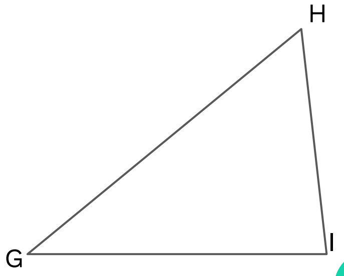
$AB = BC = AC$
$\angle A = \angle B = \angle C = 60°$

Isosceles Triangle



$DE = EF$
$\angle D = \angle F$

Scalene Triangle



$GH \neq HI \neq GI$
$\angle G \neq \angle H \neq \angle I$

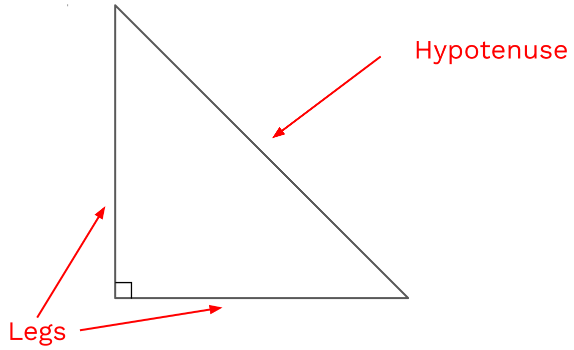
Right Triangles

Key Vocabulary

There are three types of angles that can appear in a triangle:

- ➔ A **right angle** is an angle that measures 90°
- ➔ An **acute angle** is an angle that measure <90°
- ➔ An **obtuse angle** is an angle that measure >90°

A triangle with an interior right angle is called a **right triangle**. The side opposite the right angle is called the **hypotenuse** and the two other sides are called **legs**



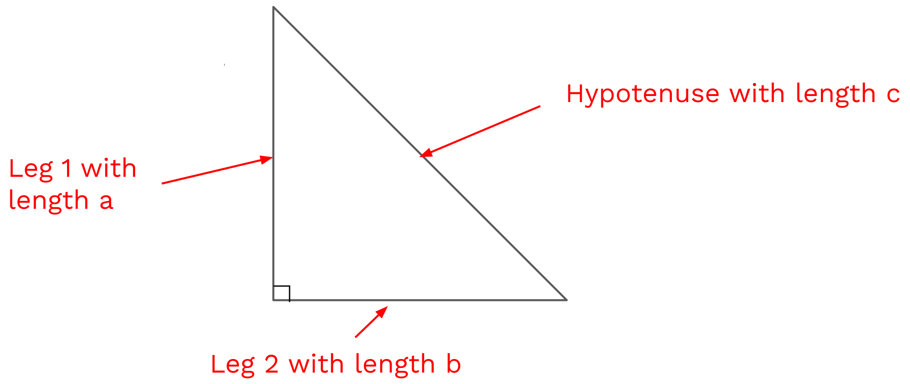
Pythagorean theorem

Key Vocabulary

Right triangles follow certain special properties

The **Pythagorean theorem** states that in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs, i.e. for the triangle below:

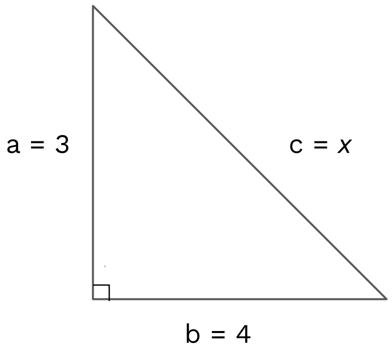
Pythagorean Theorem: $a^2 + b^2 = c^2$



Pythagorean Theorem (continued)

Key Vocabulary

Pythagorean theorem can be used to find the length of one side of a right triangle if the lengths of the other two sides are known, as in the below example



In this case, we can determine the length of the hypotenuse c to be:

$$a^2 + b^2 = c^2 \rightarrow (3)^2 + (4^2) = c^2 \rightarrow 9 + 16 = c^2 \rightarrow 25 = c^2 \rightarrow c = \sqrt{(25)} = 5$$

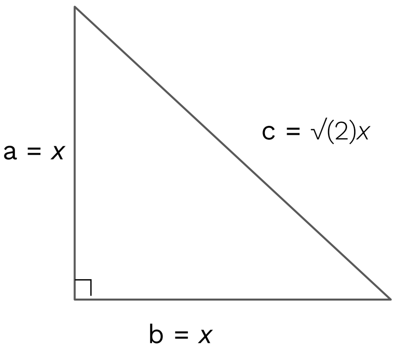
(note that technically the square roots of 25 are +5 and -5, but as we are measuring length and length cannot be negative, we can reject -5 as a solution)

Pythagorean Theorem (continued)

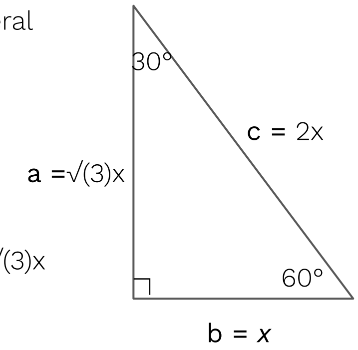
Key Vocabulary

The Pythagorean theorem can be used to determine the ratios of the lengths of the sides of two special right triangles:

- ➔ An **isosceles right triangle** has sides with lengths in the ratio 1:1:√(2), since $x^2 + x^2 = c^2 \rightarrow 2x^2 = c^2 \rightarrow c = +\sqrt{2x^2} \rightarrow c = \sqrt{2}x$



- ➔ A 30°-60°-90° right triangle (half of an equilateral triangle) has sides with lengths in the ratio 1:√(3):2. The length of the shortest side, *x*, is One-half the length of the longest side, 2*x*, and so $a^2 + x^2 = (2x)^2 \rightarrow 4x^2 - x^2 = a^2 \rightarrow a^2 = 3x^2 \rightarrow a = \sqrt{3}x$

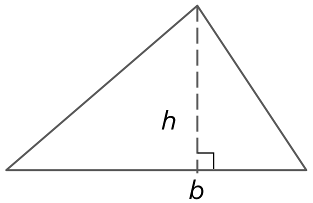


Area of a Triangle

Key Vocabulary

The **area** A of a triangle equals one-half the product of the length of a base b and the corresponding height h; that is,

$$A = (bh)/2$$



Any side of a triangle can be used as a base; the height that corresponds to the base is the perpendicular line segment from the opposite vertex to the base (or to an extension of the base)

The area of all three above triangles is (15)(6) / 2 = 45

Congruent Triangles

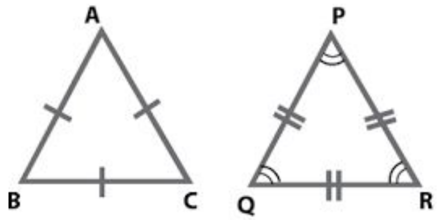
Key Vocabulary

Two triangles are **congruent** if their vertices can be matched up so that the corresponding angles and the corresponding sides are congruent (i.e. two triangles have the same shape and size)

The following three propositions can be used to determine whether two triangles are congruent:

1. If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent
2. If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent
3. If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent

Triangles ABC and PQR to the right are congruent

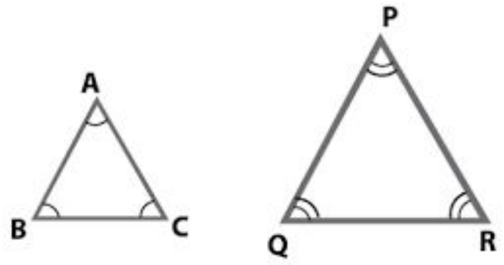


Similar Triangles

Key Vocabulary

Two triangles are **similar** is their vertices can be matched up so that the corresponding angles are congruent or, equivalently, the lengths of the corresponding sides have the same ratio, called the **scale factor of similarity** (i.e. two triangles have the same shape, but not necessarily the same size). For example, 30°-60°-90° triangles are all similar triangles, though they may differ in size

When we say that triangles ABC and PQR below are similar, it is assumed that angles A and P are congruent, angles B and Q are congruent, and angles C and R are congruent, as shown in the below figure



Since triangles ABC and PQR are similar, we have AB / PQ = BC / QR = AC / PR.

By cross multiplication, we can obtain other proportions, such as AB / BC = PQ / QR

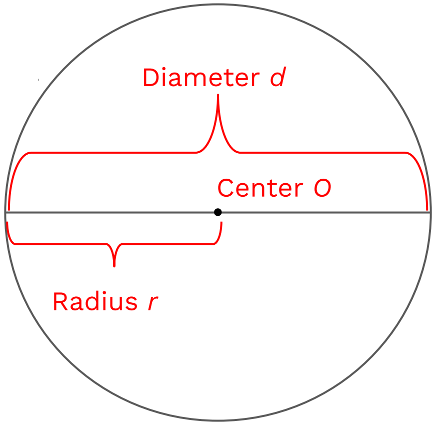
Circles

Circles

Key Vocabulary

Given a point *O* in a plane and a positive number *r*, a **circle** is the set of points in the plane that are a distance of *r* units from *O*

- ➔ The point *O* is the **center** of the circle
- ➔ The distance *r* is the **radius** of the circle
- ➔ Twice the radius of the circle is the **diameter** (i.e. $d = 2r$)

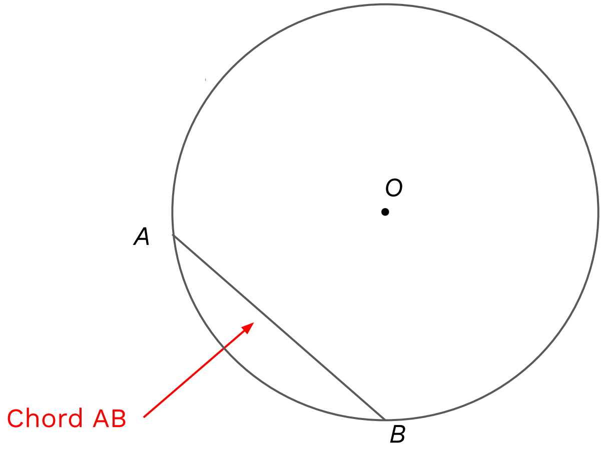


Circles (continued)

Key Vocabulary

Any line segment joining two points on the circle is called a **chord**

- The **diameter** is any chord that passes through the center of the circle
- The **radius** is any line segment joining a point on the circle and the center of the circle



Area and Perimeter of a Circle

Key Vocabulary

The distance around a circle (the perimeter) is called the **circumference** of the circle, where the circumference C is given by the equation

$$C = d\pi$$

In the above equation, d is the diameter of the circle and **π** denotes the value of the ratio of the circumference to the diameter. π is approximately equal to the value $\pi \approx 3.14159$

The **area** A of a circle is given by the equation

$$A = \pi r^2$$

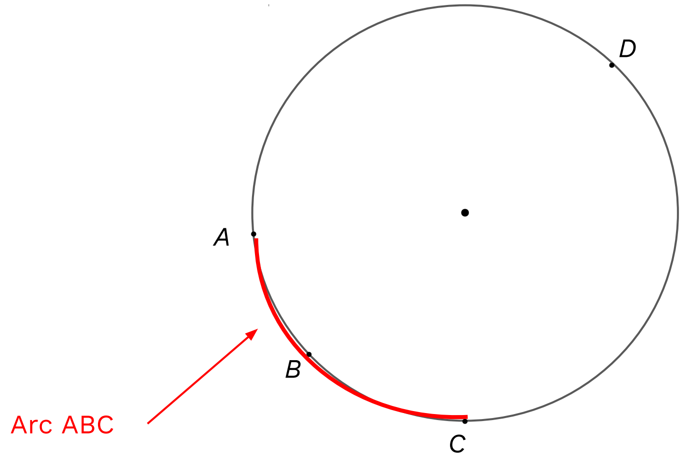
Where r is the measure of the radius of the circle

Arcs

Key Vocabulary

Given any two points on a circle, an **arc** is the part of the circle containing the two points and all the points in between them. Two points on a circle are always the endpoints of two arcs. It is customary to identify an arc by three points (to avoid ambiguity)

In the below circle, arc *ABC* is the shorter arc between *A* and *C* and arc *ADC* is the longer arc between *A* and *C*



Arcs (continued)

Key Vocabulary

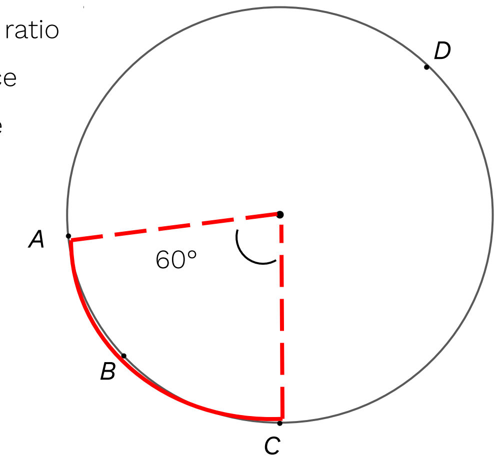
The **measure of an arc** is the measure of its central angle, which is the angle formed by two radii that connect the center of the circle to the two endpoints of the arc (more generally, a **central angle** of a circle is an angle with its vertex at the center of the circle). An entire circle is considered to be an arc with a measure of 360°

In the below circle, the measure of arc *ABC* is 60° and the measure of arc *ADC* is 300°

To find the **length of an arc**, note that the ratio of the length of an arc to the circumference is equal to the ratio of the degree measure of the arc to 360°. If the circumference of the circle to the right is 12π, the length Of arc *ABC* would be given by:

Length / (12π) = 60° / 360°

Length = (⅙)(12π) = 2π



Arcs (continued)

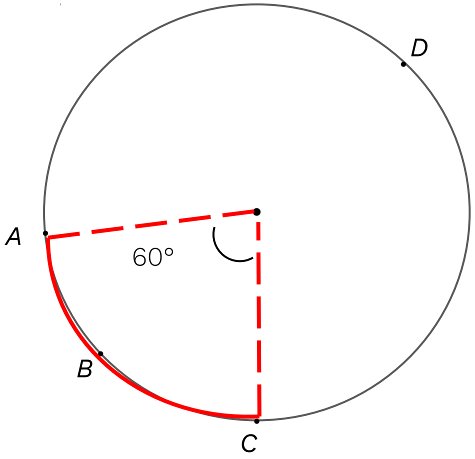
Key Vocabulary

A **sector** of a circle is a region bounded by an arc of the circle and two radii. In the below circle, the region bounded by arc *ABC* and the two dashed radii is a sector with central angle 60°

The ratio of the area of a sector of a circle to the area of the entire circle is equal to the ratio of the degree measure of its arc to 360°. Therefore, if the circle above has a radius of 6 (and so an area of $\pi r^2 = 6^2\pi = 36\pi$), the area of sector *ABC* is given by:

Area / (36π) = 60° / 360°

Area = (60° / 360°) (36π) = (⅙)(36π) = 6π

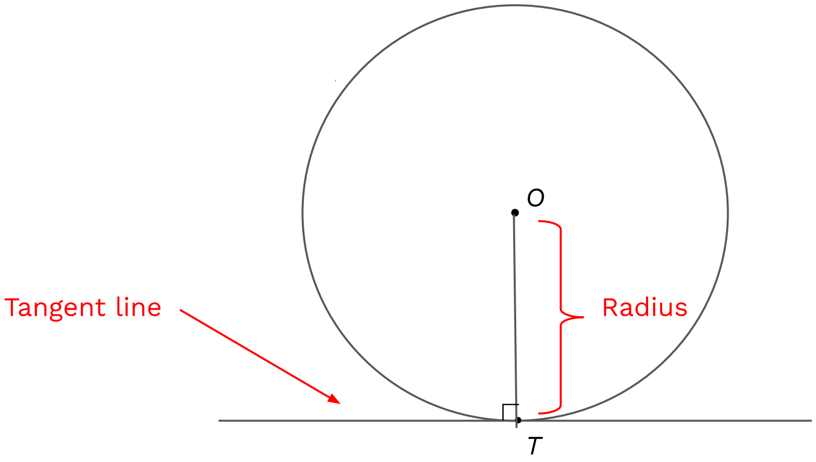


Tangents

Key Vocabulary

A **tangent** to a circle is a line that intersects the circle at exactly one point, called the **point of tangency**, denoted by *T* in the below figure

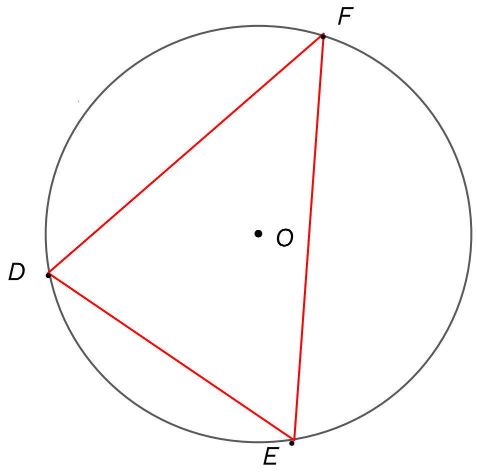
If a line is tangent to a circle, then a radius drawn to the point of tangency is perpendicular to the tangent line. The converse is also true; if a line is perpendicular to a radius at its endpoint on the circle, then the line is a tangent to the circle at that endpoint



Inscribed Polygons

Key Vocabulary

A polygon is **inscribed** in a circle if all its vertices lie on the circle, or equivalently, the circle is **circumscribed** about the polygon. In the below figure, triangle *DEF* is inscribed in the circle with center *O*

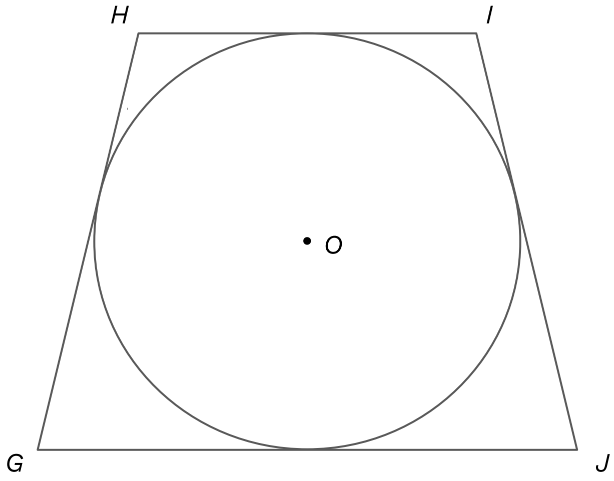


If one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle. Conversely, if an inscribed triangle is a right triangle, then one of its sides is a diameter of the circle

Inscribed Polygons (continued)

Key Vocabulary

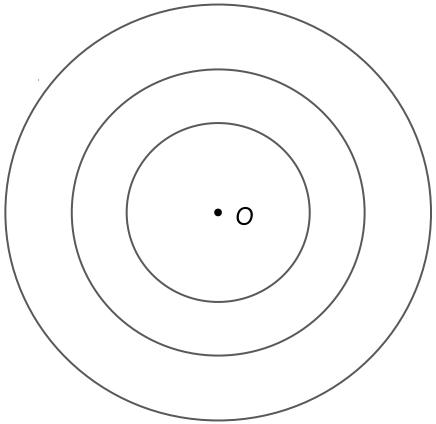
A polygon is **circumscribed** about a circle if each side of the polygon is tangent to the circle, or equivalently, the circle is **inscribed** in the polygon. In the below figure, polygon *GHIJ* is circumscribed about the circle with center *O*



Concentric and Congruent Circles

Key Vocabulary

Two or more circles with the same center are called **concentric circles**, like in the below figure



Two circles with equal radii are called **congruent circles**

Additional Resources

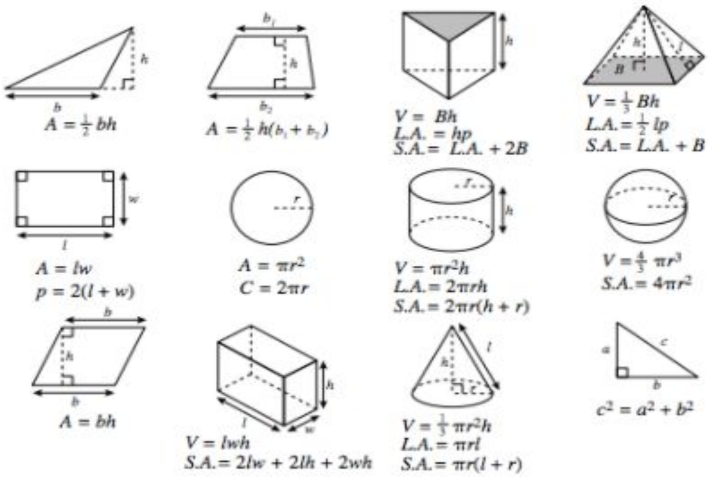
Geometry

Additional Resources

- ➔ https://www.khanacademy.org/math/geometry
- ➔ https://www.mathplanet.com/education/geometry
- ➔ https://www.sparknotes.com/math/
- ➔ http://mathguy.us/Handbooks/GeometryHandbook.pdf
- ➔ http://teachers.dadeschools.net/mcooper/GeometryReviewEOC.html
- ➔ https://www.mathsisfun.com/geometry/index.html
- ➔ https://www.nysmathregentsprep.com/uploads/6/2/3/2/62326735/final_3_-_geometry_[common_core]_regents_review_sheet_-_facts_you_must_know_cold.pdf
- ➔ https://www.radford.edu/~wacase/Geometry%20Notes%201.pdf

Geometry

Useful Formulas



Geometric Symbols

Example	Meaning
$\angle A$	angle A
$m\angle A$	measure of angle A
$\overline{AB}$	line segment AB
AB	measure of line segment AB
$\overleftrightarrow{AB}$	line AB
$\triangle ABC$	triangle ABC
$\square ABCD$	rectangle $ABCD$
$\varDelta ABCD$	parallelogram $ABCD$

Example	Meaning
$\overrightarrow{AB}$	vector AB
∟	right angle
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	Line AB is parallel to line CD.
$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$	Line AB is perpendicular to line CD.
$\angle A \cong \angle B$	Angle A is congruent to angle B.
$\triangle A \sim \triangle B$	Triangle A is similar to triangle B.
	Similarly marked segments are congruent.
	Similarly marked angles are congruent.

Abbreviations

Volume	V
Lateral Area	$L.A.$
Total Surface Area	$S.A.$
Area of Base	B

Pi

$\pi \approx 3.14$

$\pi \approx \frac{22}{7}$