Pre-Algebra Concepts & Vocab

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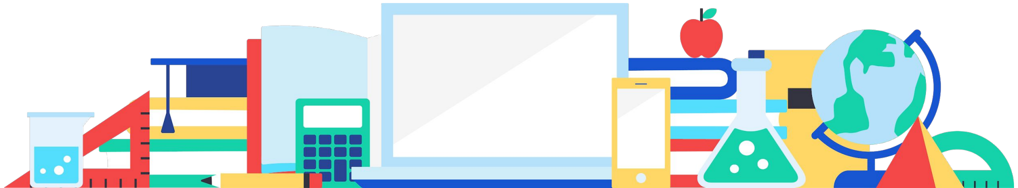

Arithmetic Properties

Order of Operations

Key Vocabulary

Order of operations is the order in which arithmetic operations need to be performed to arrive at the correct answer. It follows the acronym **PEMDAS:**

1. **P**(arenthesis): always perform operations within parentheses first
2. **E**(xponents): powers and roots come next
3. **M/D:** multiplication and division rank equally, and should be performed from left to right
4. **A/S**: addition and subtraction rank equally, and should be performed from left to right

Properties of Addition and Multiplication

Key Properties

Let a, b and c be any numbers. Then the following properties hold true:

Properties of Addition

1. **Commutative** property of addition: $a + b = b + a$
2. **Associative** property of addition: $(a + b) + c = a + (b + c)$
3. **Identity** property of addition: $a + 0 = a$
4. **Distributive** property: $a \times (b + c) = a \times b + a \times c$

Properties of Multiplication

5. **Commutative** property of multiplication: $a \times b = b \times a$
6. **Associative** property of multiplication: $(a \times b) \times c = a \times (b \times c)$
7. **Identity** property of multiplication: $a \times 1 = a$
8. **Distributive** property: $a \times (b + c) = a \times b + a \times c$

Exponents and Radicals

Definition of an Exponent

Key Vocabulary

Positive integer exponent: If b is any number and n is a positive integer then,

$$b^n = \underbrace{b \cdot b \cdot b \cdot b \cdots\cdots b}_{\text{n times}}$$

where n is the **exponent.**

Negative integer exponent: If b is any non-zero number and n is a positive integer, then

$$b^{-n} = 1 / b^n$$

Zero exponent: If b is any non-zero number, then

$$b^0 = 1$$

Rational exponent: A rational exponent is one in the form $b^{(m / n)}$ and is equivalent to

$$b^{(m / n)} = (b^{(1 / n)})^m = (b^m)^{(1 / n)}$$

I.e. $(b^m)^{(1 / n)}$ is the **nth root of b^m**

Properties of Exponents

Key Properties

Let a and b be any numbers and n and m be positive integers, then we have the following properties:

- ➔ $b^n b^m = b^{n+m}$
- ➔ $(b^n)^m = b^{nm}$
- ➔ $b^n / b^m = b^{n-m} = 1 / b^{m-n}$, where $b \neq 0$
- ➔ $(ab)^n = a^n b^n$
- ➔ $(a / b)^n = a^n / b^n$, $b \neq 0$
- ➔ $(a / b)^{-n} = (b / a)^n = b^n / a^n$, $a \neq 0$
- ➔ $(ab)^{-n} = 1 / (ab)^n$, $a \neq 0$, $b \neq 0$
- ➔ $1 / a^{-n} = a^n$
- ➔ $a^{-n} / b^{-m} = b^m / a^n$, $a \neq 0$
- ➔ $(a^n b^m)^k = a^{nk} b^{mk}$
- ➔ $(a^n / b^m)^k = a^{nk} / b^{mk}$, $b \neq 0$

Common Mistakes - Exponents

Common Mistakes

It is only the quantity that is immediately to the left of the exponent that gets the power, unless there are parentheses.

- Example 1
  - $(-2)^4 \neq -2^4$
- Example 2
  - Correct: $ab^{-2} = a / b^2$
  - Incorrect: $ab^{-2} \neq 1 / ab^2$
- Example 3
  - Correct: $1 / (3a^{-5}) = (1 / 3)\ (1 / a^{-5}) = (1 / 3)\ (a^5)$
  - Incorrect: $1 / (3a^{-5}) \neq (3a)^5$

Be careful not to confuse negative exponents with rational exponents:

- Correct: $b^{-n} = 1 / b^n$
- Incorrect: $b^{-n} \neq b^{(1 / n)}$

Definition of a Radical

Key Vocabulary

If n is a positive integer that is greater than 1 and b is a real number, then

$$\sqrt[n]{b} = b^{(1\,/\,n)}$$

Where n is called the **index**, b is called the **radicand**, and the symbol √ is called the **radical.** The left side is often called the **radical form** and the right side is called the **exponent form**

Note: the index is required EXCEPT in the case of the square root, where the index is often dropped such that

$$\sqrt[2]{b} = \sqrt{b}$$

Note: evaluating the square root of a number actually has two answers:

$$\sqrt{16} = 4 \text{ AND } \sqrt{16} = -4$$

Properties of Radicals

Key Properties

If n is a positive integer greater than 1 and both a and b are positive real numbers, then

- $\sqrt[n]{a^n} = a$
- $\sqrt[n]{ab} = \sqrt[n]{a}\ \sqrt[n]{b}$
- $\sqrt[n]{(a / b)} = \sqrt[n]{a} / \sqrt[n]{b}$

A radical is said to be in **simplified radical form** if each of the following is true:

1. All exponents in the radicand must be less than the index
2. Any exponents in the radicand can have no factors in common with the index
3. No fractions appear under a radical
4. No radicals appear in the denominator of a fraction

Common Mistakes - Radicals

Common Mistakes

You can't "break-up" sums and differences under a radical, i.e.

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$
$$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$$

You can only add / subtract radicals if they have the same number / expressions underneath the radical sign. Example:

$$4\sqrt{x} + 9\sqrt{x} = (4+9)\sqrt{x} = 13\sqrt{x}$$

You CAN, however, multiply and divide two radicals with different number / expressions underneath the radical sign:

$$\sqrt{a}\sqrt{b} = \sqrt{ab}$$
$$\sqrt{a} / \sqrt{b} = \sqrt{a/b}$$

Rationalizing the Denominator

Problem Solving

An radical expression in simplified radical form does not have any radicals in the denominator of a fraction. The process to get rid of the radicals in the denominator is called **rationalizing the denominator.**

Example: Simplify $1 / (3 - \sqrt{x})$

Using the property that $(a + b)(a - b) = a^2 - b^2$, we can do the following to get rid of the radical in the denominator:

$$\frac{1}{3-\sqrt{x}} = \frac{1}{(3-\sqrt{x})}\,\frac{(3+\sqrt{x})}{(3+\sqrt{x})} = \frac{3+\sqrt{x}}{(3-\sqrt{x})\ (3+\sqrt{x})} = \frac{3+\sqrt{x}}{9-x}$$

Polynomials

Definition of a Polynomial

Key Vocabulary

Polynomials in one variable are algebraic expressions that consist of terms in the form $\mathbf{ax^n}$

- ➔ n is the exponent and must be a non-negative integer
- ➔ a is the **coefficient** and is a real number
- ➔ The **degree** of a polynomial is the largest exponent in the polynomial

Polynomials in two variables are algebraic expressions that consist of terms in the form ax^ny^m

- ➔ The **degree** of a polynomial in two variables is equal to the largest of the sums of the exponents in each term of the polynomial

A **monomial** is a polynomial that consists of one term, a **binomial** is a polynomial that consists of two terms and a **trinomial** is a polynomial that consists of three terms

Working with Polynomials

Problem Solving

Adding / Subtracting Polynomials

- ➔ When adding or subtracting polynomials, add any like terms together (**like terms** are those whose variables AND their exponents match *exactly*)
- ➔ This often requires use of the **distributive property:** $a(b + c) = ab + bc$
- ➔ Example: Add $6x^5 + 3x^2 + 2$ and $2x^5 - 9x + 4$
  - ◆ Use the distributive property to add together coefficients of like terms: $(6 + 2)x^5 + 3x^2 - 9x + 2 + 4$
  - ◆ Simplify: $8x^5 + 3x^2 - 9x + 6$

Multiplying / Dividing Polynomials

- ➔ When multiplying terms in a polynomial, you multiply coefficients and add exponents. Ex: $3x^2 \cdot 2x^5 = 6x^7$. When dividing, you divide coefficients and subtract exponents. Ex: $6x^3 / 2x^2 = 3x$
- ➔ This also often requires use of the **distributive property** and **FOIL (First, Outer, Inner, Last)**
- ➔ Example: Multiply $(3x + 5)(2x + 1)$
  - ◆ Use FOIL: $6x^2 + 3x + 10x + 5$
  - ◆ Simplify by adding like terms: $6x^2 + 13x + 5$

Additional Resources

Pre-Algebra

Additional Resources

- ➔ https://www.khanacademy.org/math/pre-algebra
- ➔ https://www.mathplanet.com/education/pre-algebra
- ➔ http://www.usd417.net/pages/uploaded_files/PrealgebraParentStudyGuide.pdf
- ➔ http://www.ppstest2.com/PreAlgebraBook.pdf
- ➔ https://www.ptechnyc.org/site/handlers/filedownload.ashx?moduleinstanceid=199&dataid=179&FileName=Pre-Algebra%20student-parent%20study%20guide.pdf
- ➔ http://www.mycompasstest.com/study-guide/pre-algebra/
- ➔ http://tutorial.math.lamar.edu/Classes/Alg/Alg.aspx
- ➔ http://www.evergreenusd.com/files/Pre-Algebra%20Study%20Guide.pdf