

Causal Inference Book: Exercises – R code

*Book by M. A. Hernán and J. M. Robins, R code by Joy Shi and Roger Logan, R
Markdown code and tweaks by Tom Palmer*

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Preface

This book presents code examples from the Causal Inference Book by Hernán and Robins, which is available in draft form from the following webpage.

<https://www.hsph.harvard.edu/miguel-hernan/causal-inference-book/>

The R code is based on the code by Joy Shi and Sean McGrath given here.

Packages to install

To install the R packages required for this book please copy/fork the repository and run:

```
# install.packages('devtools') # uncomment if devtools not  
# installed  
devtools::install_deps()
```

Downloading the datasets

We assume that you have downloaded the data from the Causal Inference Book website and saved it to a `data` subdirectory. You can do this manually or with the following code (nb. we use the `here` package to reference the data subdirectory).

```
library(here)  
  
dataurls <- list()  
dataurls[[1]] <- "https://cdn1.sph.harvard.edu/wp-content/uploads/sites/1268/2012/10/nhefs_sas.zip"  
dataurls[[2]] <- "https://cdn1.sph.harvard.edu/wp-content/uploads/sites/1268/2012/10/nhefs_stata.zip"  
dataurls[[3]] <- "https://cdn1.sph.harvard.edu/wp-content/uploads/sites/1268/2017/01/nhefs_excel.zip"  
dataurls[[4]] <- "https://cdn1.sph.harvard.edu/wp-content/uploads/sites/1268/1268/20/nhefs.csv"  
  
temp <- tempfile()  
for (i in 1:3) {  
  download.file(dataurls[[i]], temp)  
  unzip(temp, exdir = "data")  
}  
  
download.file(dataurls[[4]], here("data", "nhefs.csv"))
```


R code

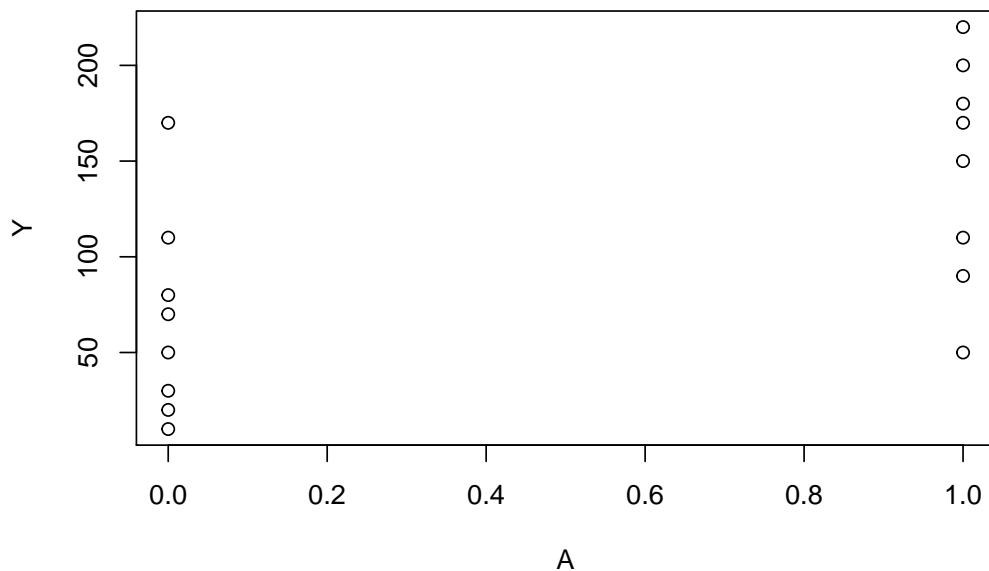
11. Why model?

Program 11.1

- Sample averages by treatment level
- Data from Figures 11.1 and 11.2

```
A <- c(1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0)
Y <- c(200, 150, 220, 110, 50, 180, 90, 170, 170, 30,
      70, 110, 80, 50, 10, 20)

plot(A, Y)
```



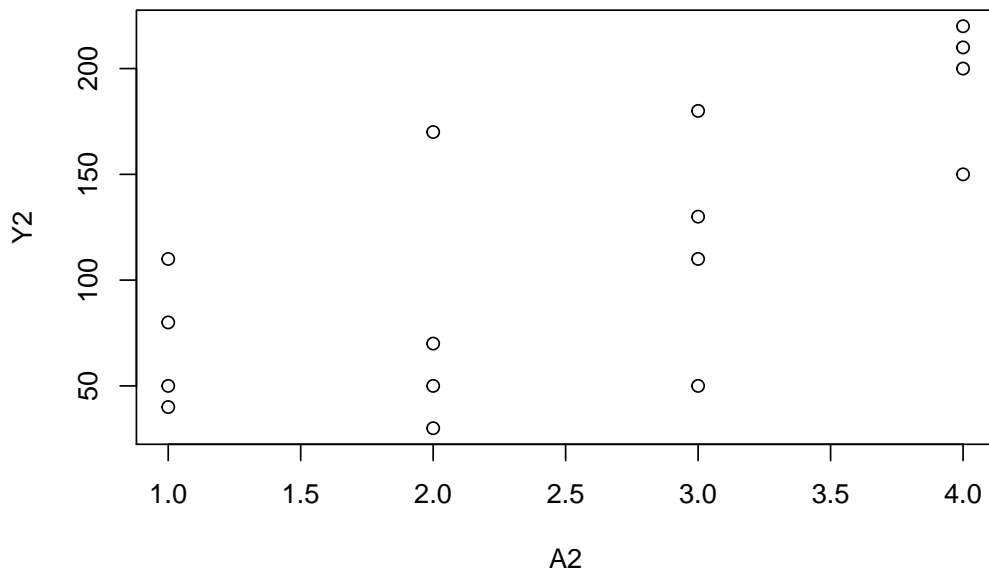
```
summary(Y[A == 0])
```

##	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
##	10.0	27.5	60.0	67.5	87.5	170.0

```
summary(Y[A == 1])
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      50.0   105.0   160.0   146.2   185.0   220.0
A2 <- c(1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4)
Y2 <- c(110, 80, 50, 40, 170, 30, 70, 50, 110, 50, 180,
        130, 200, 150, 220, 210)

plot(A2, Y2)
```



```
summary(Y2[A2 == 1])
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      40.0   47.5   65.0   70.0   87.5   110.0
```

```
summary(Y2[A2 == 2])
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       30     45     60     80     95    170
```

```
summary(Y2[A2 == 3])
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      50.0   95.0  120.0  117.5  142.5  180.0
```

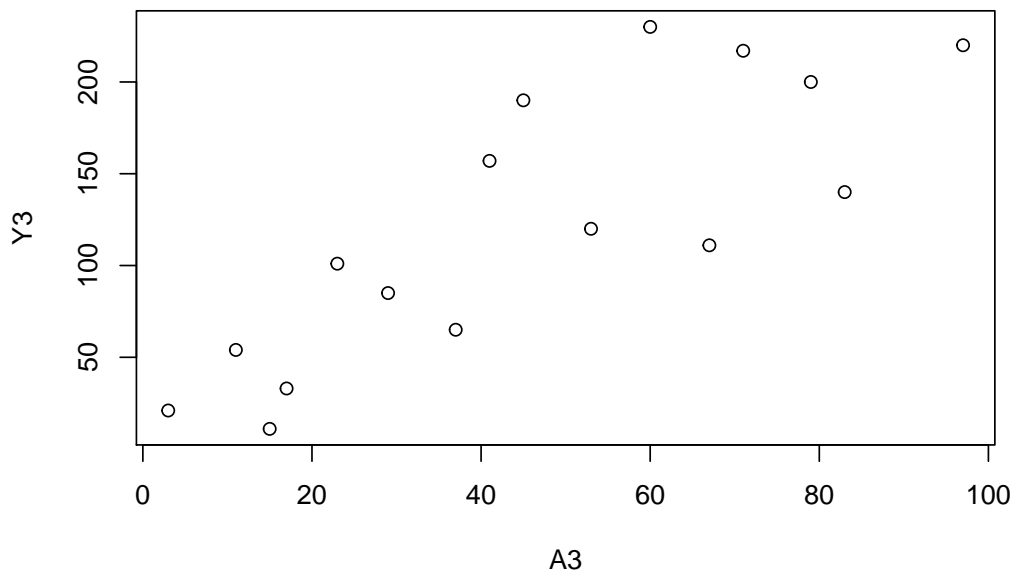
```
summary(Y2[A2 == 4])
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     150.0  187.5  205.0  195.0  212.5  220.0
```

Program 11.2

- 2-parameter linear model
- Data from Figures 11.3 and 11.1

```
A3 <-  
  c(3, 11, 17, 23, 29, 37, 41, 53, 67, 79, 83, 97, 60, 71, 15, 45)  
Y3 <-  
  c(21, 54, 33, 101, 85, 65, 157, 120, 111, 200, 140, 220, 230, 217,  
    11, 190)  
  
plot(Y3 ~ A3)
```



```
summary(glm(Y3 ~ A3))  
  
##  
## Call:  
## glm(formula = Y3 ~ A3)  
##  
## Deviance Residuals:  
##      Min       1Q   Median       3Q      Max   
## -61.930  -30.564   -5.741   30.653   77.225   
##  
## Coefficients:  
##              Estimate Std. Error t value Pr(>|t|)      
## (Intercept)  24.5464    21.3300   1.151  0.269094      
## A3           2.1372     0.3997   5.347  0.000103 ***  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## (Dispersion parameter for gaussian family taken to be 1944.109)
##
##      Null deviance: 82800  on 15  degrees of freedom
## Residual deviance: 27218  on 14  degrees of freedom
## AIC: 170.43
##
## Number of Fisher Scoring iterations: 2
predict(glm(Y3 ~ A3), data.frame(A3 = 90))

##      1
## 216.89
summary(glm(Y ~ A))

##
## Call:
## glm(formula = Y ~ A)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -96.250  -40.000   3.125   35.938  102.500
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    67.50     19.72    3.424  0.00412 **
## A              78.75     27.88    2.824  0.01352 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 3109.821)
##
##      Null deviance: 68344  on 15  degrees of freedom
## Residual deviance: 43538  on 14  degrees of freedom
## AIC: 177.95
##
## Number of Fisher Scoring iterations: 2
```

Program 11.3

- 3-parameter linear model
- Data from Figure 11.3

```
Asq <- A3 * A3
mod3 <- glm(Y3 ~ A3 + Asq)
summary(mod3)
```

```
##
## Call:
```

```

## glm(formula = Y3 ~ A3 + Asq)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -65.27  -34.41   13.21   26.11   64.36
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -7.40688    31.74777  -0.233   0.8192
## A3           4.10723     1.53088   2.683   0.0188 *
## Asq         -0.02038     0.01532  -1.331   0.2062
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 1842.697)
##
##      Null deviance: 82800  on 15  degrees of freedom
## Residual deviance: 23955  on 13  degrees of freedom
## AIC: 170.39
##
## Number of Fisher Scoring iterations: 2
predict(mod3, data.frame(cbind(A3 = 90, Asq = 8100)))

##           1
## 197.1269

```


12. IP Weighting and Marginal Structural Models

Program 12.1

- Descriptive statistics from NHEFS data (Table 12.1)

```
library(here)

# install.packages("readxl") # install package if required
library("readxl")

nhefs <- read_excel(here("data", "NHEFS.xls"))
nhefs$cens <- ifelse(is.na(nhefs$wt82), 1, 0)

# provisionally ignore subjects with missing values for weight in 1982
nhefs.nmv <-
  nhefs[which(!is.na(nhefs$wt82)),]

lm(wt82_71 ~ qsmk, data = nhefs.nmv)

##
## Call:
## lm(formula = wt82_71 ~ qsmk, data = nhefs.nmv)
##
## Coefficients:
## (Intercept)          qsmk
##      1.984         2.541

# Smoking cessation
predict(lm(wt82_71 ~ qsmk, data = nhefs.nmv), data.frame(qsmk = 1))

##      1
## 4.525079

# No smoking cessation
predict(lm(wt82_71 ~ qsmk, data = nhefs.nmv), data.frame(qsmk = 0))

##      1
## 1.984498
```

```

# Table
summary(nhefs.nmv[which(nhefs.nmv$qsmk == 0),]$age)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    25.00  33.00   42.00   42.79  51.00   72.00

summary(nhefs.nmv[which(nhefs.nmv$qsmk == 0),]$wt71)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    40.82  59.19   68.49   70.30  79.38  151.73

summary(nhefs.nmv[which(nhefs.nmv$qsmk == 0),]$smokeintensity)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      1.00   15.00   20.00   21.19   30.00   60.00

summary(nhefs.nmv[which(nhefs.nmv$qsmk == 0),]$smokeyrs)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      1.00   15.00   23.00   24.09   32.00   64.00

summary(nhefs.nmv[which(nhefs.nmv$qsmk == 1),]$age)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    25.00  35.00   46.00   46.17   56.00   74.00

summary(nhefs.nmv[which(nhefs.nmv$qsmk == 1),]$wt71)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    39.58  60.67   71.21   72.35   81.08  136.98

summary(nhefs.nmv[which(nhefs.nmv$qsmk == 1),]$smokeintensity)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      1.0   10.0    20.0    18.6   25.0    80.0

summary(nhefs.nmv[which(nhefs.nmv$qsmk == 1),]$smokeyrs)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      1.00   15.00   26.00   26.03   35.00   60.00

table(nhefs.nmv$qsmk, nhefs.nmv$sex)

##
##      0      1
##    0 542  621
##    1 220  183

prop.table(table(nhefs.nmv$qsmk, nhefs.nmv$sex), 1)

##
##      0      1
##    0 0.4660361 0.5339639
##    1 0.5459057 0.4540943

table(nhefs.nmv$qsmk, nhefs.nmv$race)

```



```
##
##      0    1
##    0 993 170
##    1 367  36

prop.table(table(nhefs.nmv$qsmk, nhefs.nmv$race), 1)

##
##      0      1
##    0 0.85382631 0.14617369
##    1 0.91066998 0.08933002

table(nhefs.nmv$qsmk, nhefs.nmv$education)

##
##      1    2    3    4    5
##    0 210 266 480  92 115
##    1  81  74 157  29  62

prop.table(table(nhefs.nmv$qsmk, nhefs.nmv$education), 1)

##
##      1      2      3      4      5
##    0 0.18056750 0.22871883 0.41272571 0.07910576 0.09888220
##    1 0.20099256 0.18362283 0.38957816 0.07196030 0.15384615

table(nhefs.nmv$qsmk, nhefs.nmv$exercise)

##
##      0    1    2
##    0 237 485 441
##    1  63 176 164

prop.table(table(nhefs.nmv$qsmk, nhefs.nmv$exercise), 1)

##
##      0      1      2
##    0 0.2037833 0.4170249 0.3791917
##    1 0.1563275 0.4367246 0.4069479

table(nhefs.nmv$qsmk, nhefs.nmv$active)

##
##      0    1    2
##    0 532 527 104
##    1 170 188  45

prop.table(table(nhefs.nmv$qsmk, nhefs.nmv$active), 1)

##
##      0      1      2
##    0 0.4574377 0.4531384 0.0894239
##    1 0.4218362 0.4665012 0.1116625
```

Program 12.2

- Estimating IP weights
- Data from NHEFS

```
# Estimation of ip weights via a logistic model
fit <- glm(
  qsmk ~ sex + race + age + I(age ^ 2) +
    as.factor(education) + smokeintensity +
    I(smokeintensity ^ 2) + smokeyrs + I(smokeyrs ^ 2) +
    as.factor(exercise) + as.factor(active) + wt71 + I(wt71 ^ 2),
  family = binomial(),
  data = nhefs.nmv
)
summary(fit)

##
## Call:
## glm(formula = qsmk ~ sex + race + age + I(age^2) + as.factor(education) +
##      smokeintensity + I(smokeintensity^2) + smokeyrs + I(smokeyrs^2) +
##      as.factor(exercise) + as.factor(active) + wt71 + I(wt71^2),
##      family = binomial(), data = nhefs.nmv)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.5127  -0.7907  -0.6387   0.9832   2.3729
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -2.2425191   1.3808360  -1.624 0.104369
## sex             -0.5274782   0.1540496  -3.424 0.000617 ***
## race            -0.8392636   0.2100665  -3.995 6.46e-05 ***
## age              0.1212052   0.0512663   2.364 0.018068 *
## I(age^2)        -0.0008246   0.0005361  -1.538 0.124039
## as.factor(education)2 -0.0287755   0.1983506  -0.145 0.884653
## as.factor(education)3  0.0864318   0.1780850   0.485 0.627435
## as.factor(education)4  0.0636010   0.2732108   0.233 0.815924
## as.factor(education)5  0.4759606   0.2262237   2.104 0.035384 *
## smokeintensity    -0.0772704   0.0152499  -5.067 4.04e-07 ***
## I(smokeintensity^2)   0.0010451   0.0002866   3.647 0.000265 ***
## smokeyrs         -0.0735966   0.0277775  -2.650 0.008061 **
## I(smokeyrs^2)        0.0008441   0.0004632   1.822 0.068398 .
## as.factor(exercise)1  0.3548405   0.1801351   1.970 0.048855 *
## as.factor(exercise)2  0.3957040   0.1872400   2.113 0.034571 *
## as.factor(active)1    0.0319445   0.1329372   0.240 0.810100
## as.factor(active)2    0.1767840   0.2149720   0.822 0.410873
## wt71             -0.0152357   0.0263161  -0.579 0.562625
## I(wt71^2)          0.0001352   0.0001632   0.829 0.407370
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 1786.1  on 1565  degrees of freedom
## Residual deviance: 1676.9  on 1547  degrees of freedom
## AIC: 1714.9
##
## Number of Fisher Scoring iterations: 4

p.qsmk.obs <-
  ifelse(nhefs.nmv$qsmk == 0,
        1 - predict(fit, type = "response"),
        predict(fit, type = "response"))

nhefs.nmv$w <- 1 / p.qsmk.obs
summary(nhefs.nmv$w)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      1.054   1.230   1.373   1.996   1.990  16.700

sd(nhefs.nmv$w)

## [1] 1.474787

# install.packages("geepack") # install package if required
library("geepack")
msm.w <- geeglm(
  wt82_71 ~ qsmk,
  data = nehs.nmv,
  weights = w,
  id = seqn,
  corstr = "independence"
)
summary(msm.w)

##
## Call:
## geeglm(formula = wt82_71 ~ qsmk, data = nehs.nmv, weights = w,
##       id = seqn, corstr = "independence")
##
## Coefficients:
##              Estimate Std.err  Wald Pr(>|W|)
## (Intercept)   1.7800   0.2247  62.73 2.33e-15 ***
## qsmk          3.4405   0.5255  42.87 5.86e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Estimated Scale Parameters:
##              Estimate Std.err
## (Intercept)   65.06   4.221
##
## Correlation: Structure = independenceNumber of clusters: 1566 Maximum cluster size: 1
```

```

beta <- coef(msm.w)
SE <- coef(summary(msm.w))[, 2]
lcl <- beta - qnorm(0.975) * SE
ucl <- beta + qnorm(0.975) * SE
cbind(beta, lcl, ucl)

##           beta    lcl    ucl
## (Intercept) 1.780 1.340 2.22
## qsmk        3.441 2.411 4.47

# no association between sex and qsmk in pseudo-population
xtabs(nhefs.nmv$w ~ dhefs.nmv$sex + dhefs.nmv$qsmk)

##           dhefs.nmv$qsmk
## dhefs.nmv$sex      0      1
##           0 763.6 763.6
##           1 801.7 797.2

# "check" for positivity (White women)
table(dhefs.nmv$age[dhefs.nmv$race == 0 & dhefs.nmv$sex == 1],
      dhefs.nmv$qsmk[dhefs.nmv$race == 0 & dhefs.nmv$sex == 1])

##
##      0  1
## 25 24  3
## 26 14  5
## 27 18  2
## 28 20  5
## 29 15  4
## 30 14  5
## 31 11  5
## 32 14  7
## 33 12  3
## 34 22  5
## 35 16  5
## 36 13  3
## 37 14  1
## 38  6  2
## 39 19  4
## 40 10  4
## 41 13  3
## 42 16  3
## 43 14  3
## 44  9  4
## 45 12  5
## 46 19  4
## 47 19  4
## 48 19  4
## 49 11  3
## 50 18  4
## 51  9  3

```

```
## 52 11 3
## 53 11 4
## 54 17 9
## 55 9 4
## 56 8 7
## 57 9 2
## 58 8 4
## 59 5 4
## 60 5 4
## 61 5 2
## 62 6 5
## 63 3 3
## 64 7 1
## 65 3 2
## 66 4 0
## 67 2 0
## 69 6 2
## 70 2 1
## 71 0 1
## 72 2 2
## 74 0 1
```

Program 12.3

- Estimating stabilized IP weights
- Data from NHEFS

```
# estimation of denominator of ip weights
```

```
denom.fit <-
```

```
  glm(
```

```
    qsmk ~ as.factor(sex) + as.factor(race) + age + I(age ^ 2) +
```

```
          as.factor(education) + smokeintensity +
```

```
          I(smokeintensity ^ 2) + smokeyrs + I(smokeyrs ^ 2) +
```

```
          as.factor(exercise) + as.factor(active) + wt71 + I(wt71 ^ 2),
```

```
    family = binomial(),
```

```
    data = nhefs.nmv
```

```
  )
```

```
summary(denom.fit)
```

```
##
```

```
## Call:
```

```
## glm(formula = qsmk ~ as.factor(sex) + as.factor(race) + age +
```

```
##       I(age^2) + as.factor(education) + smokeintensity + I(smokeintensity^2) +
```

```
##       smokeyrs + I(smokeyrs^2) + as.factor(exercise) + as.factor(active) +
```

```
##       wt71 + I(wt71^2), family = binomial(), data = nhefs.nmv)
```

```
##
```

```
## Deviance Residuals:
```

```
##      Min       1Q   Median       3Q      Max
```

```
## -1.513  -0.791  -0.639   0.983   2.373
```

```
##
```

```
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -2.242519   1.380836  -1.62  0.10437
## as.factor(sex)1  -0.527478   0.154050  -3.42  0.00062 ***
## as.factor(race)1 -0.839264   0.210067  -4.00  6.5e-05 ***
## age             0.121205   0.051266   2.36  0.01807 *
## I(age^2)        -0.000825   0.000536  -1.54  0.12404
## as.factor(education)2 -0.028776   0.198351  -0.15  0.88465
## as.factor(education)3  0.086432   0.178085   0.49  0.62744
## as.factor(education)4  0.063601   0.273211   0.23  0.81592
## as.factor(education)5  0.475961   0.226224   2.10  0.03538 *
## smokeintensity    -0.077270   0.015250  -5.07  4.0e-07 ***
## I(smokeintensity^2)   0.001045   0.000287   3.65  0.00027 ***
## smokeyrs         -0.073597   0.027777  -2.65  0.00806 **
## I(smokeyrs^2)        0.000844   0.000463   1.82  0.06840 .
## as.factor(exercise)1  0.354841   0.180135   1.97  0.04885 *
## as.factor(exercise)2  0.395704   0.187240   2.11  0.03457 *
## as.factor(active)1    0.031944   0.132937   0.24  0.81010
## as.factor(active)2    0.176784   0.214972   0.82  0.41087
## wt71             -0.015236   0.026316  -0.58  0.56262
## I(wt71^2)          0.000135   0.000163   0.83  0.40737
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##    Null deviance: 1786.1  on 1565  degrees of freedom
## Residual deviance: 1676.9  on 1547  degrees of freedom
## AIC: 1715
##
## Number of Fisher Scoring iterations: 4

pd.qsmk <- predict(denom.fit, type = "response")

# estimation of numerator of ip weights
numer.fit <- glm(qsmk ~ 1, family = binomial(), data = nhfs.nmv)
summary(numer.fit)

##
## Call:
## glm(formula = qsmk ~ 1, family = binomial(), data = nhfs.nmv)
##
## Deviance Residuals:
##    Min       1Q   Median       3Q      Max
## -0.771  -0.771  -0.771   1.648   1.648
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -1.0598     0.0578  -18.3  <2e-16 ***
## ---
```

```

## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 1786.1  on 1565  degrees of freedom
## Residual deviance: 1786.1  on 1565  degrees of freedom
## AIC: 1788
##
## Number of Fisher Scoring iterations: 4

pn.qsmk <- predict(numer.fit, type = "response")

nhefs.nmv$sw <-
  ifelse(nhefs.nmv$qsmk == 0, ((1 - pn.qsmk) / (1 - pd.qsmk)),
        (pn.qsmk / pd.qsmk))

summary(nhefs.nmv$sw)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      0.331   0.867   0.950   0.999   1.079   4.298

msm.sw <- geeglm(
  wt82_71 ~ qsmk,
  data = nhefs.nmv,
  weights = sw,
  id = seqn,
  corstr = "independence"
)
summary(msm.sw)

##
## Call:
## geeglm(formula = wt82_71 ~ qsmk, data = nhefs.nmv, weights = sw,
##        id = seqn, corstr = "independence")
##
## Coefficients:
##              Estimate Std.terr Wald Pr(>|W|)
## (Intercept)    1.780    0.225 62.7  2.3e-15 ***
## qsmk           3.441    0.525 42.9  5.9e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Estimated Scale Parameters:
##              Estimate Std.terr
## (Intercept)    60.7    3.71
##
## Correlation: Structure = independenceNumber of clusters: 1566 Maximum cluster size: 1

beta <- coef(msm.sw)
SE <- coef(summary(msm.sw))[, 2]
lcl <- beta - qnorm(0.975) * SE

```

```

ucl <- beta + qnorm(0.975) * SE
cbind(beta, lcl, ucl)

##           beta  lcl  ucl
## (Intercept) 1.78 1.34 2.22
## qsmk        3.44 2.41 4.47

# no association between sex and qsmk in pseudo-population
xtabs(nhefs.nmv$sw ~ dhefs.nmv$sex + dhefs.nmv$qsmk)

##           dhefs.nmv$qsmk
## dhefs.nmv$sex    0    1
##                0 567 197
##                1 595 205

```

Program 12.4

- Estimating the parameters of a marginal structural mean model with a continuous treatment Data from NHEFS

```

# Analysis restricted to subjects reporting <=25 cig/day at baseline
nhefs.nmv.s <- subset(nhefs.nmv, smokeintensity <= 25)

# estimation of denominator of ip weights
den.fit.obj <- lm(
  smkintensity82_71 ~ as.factor(sex) +
    as.factor(race) + age + I(age ^ 2) +
    as.factor(education) + smokeintensity + I(smokeintensity ^ 2) +
    smokeyrs + I(smokeyrs ^ 2) + as.factor(exercise) + as.factor(active) + wt71 +
    I(wt71 ^ 2),
  data = dhefs.nmv.s
)
p.den <- predict(den.fit.obj, type = "response")
dens.den <-
  dnorm(nhefs.nmv.s$smkintensity82_71,
    p.den,
    summary(den.fit.obj)$sigma)

# estimation of numerator of ip weights
num.fit.obj <- lm(smkintensity82_71 ~ 1, data = dhefs.nmv.s)
p.num <- predict(num.fit.obj, type = "response")
dens.num <-
  dnorm(nhefs.nmv.s$smkintensity82_71,
    p.num,
    summary(num.fit.obj)$sigma)

nhefs.nmv.s$sw.a <- dens.num / dens.den
summary(nhefs.nmv.s$sw.a)

```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
```



```
##      0.19      0.89      0.97      1.00      1.05      5.10
msm.sw.cont <-
  geeglm(
    wt82_71 ~ smkintensity82_71 + I(smkintensity82_71 * smkintensity82_71),
    data = nhefs.nmv.s,
    weights = sw.a,
    id = seqn,
    corstr = "independence"
  )
summary(msm.sw.cont)

##
## Call:
## geeglm(formula = wt82_71 ~ smkintensity82_71 + I(smkintensity82_71 *
##      smkintensity82_71), data = nhefs.nmv.s, weights = sw.a, id = seqn,
##      corstr = "independence")
##
## Coefficients:
##
##              Estimate Std.err Wald Pr(>|W|)
## (Intercept)      2.00452  0.29512 46.13  1.1e-11
## smkintensity82_71 -0.10899  0.03154 11.94  0.00055
## I(smkintensity82_71 * smkintensity82_71)  0.00269  0.00242  1.24  0.26489
##
## (Intercept)          ***
## smkintensity82_71      ***
## I(smkintensity82_71 * smkintensity82_71)
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Estimated Scale Parameters:
##              Estimate Std.err
## (Intercept)    60.5      4.5
##
## Correlation: Structure = independenceNumber of clusters: 1162 Maximum cluster size: 1

beta <- coef(msm.sw.cont)
SE <- coef(summary(msm.sw.cont))[, 2]
lcl <- beta - qnorm(0.975) * SE
ucl <- beta + qnorm(0.975) * SE
cbind(beta, lcl, ucl)

##              beta      lcl      ucl
## (Intercept)    2.00452  1.42610  2.58295
## smkintensity82_71 -0.10899 -0.17080 -0.04718
## I(smkintensity82_71 * smkintensity82_71)  0.00269 -0.00204  0.00743
```

Program 12.5

- Estimating the parameters of a marginal structural logistic model

- Data from NHEFS

```
table(nhefs.nmv$qsmk, nehefs.nmv$death)
```

```
##
##      0    1
## 0 963 200
## 1 312  91
```

```
# First, estimation of stabilized weights sw (same as in Program 12.3)
# Second, fit logistic model below
```

```
msm.logistic <- geeglm(
  death ~ qsmk,
  data = nehefs.nmv,
  weights = sw,
  id = seqn,
  family = binomial(),
  corstr = "independence"
)
```

```
## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!
```

```
summary(msm.logistic)
```

```
##
## Call:
## geeglm(formula = death ~ qsmk, family = binomial(), data = nehefs.nmv,
##        weights = sw, id = seqn, corstr = "independence")
##
## Coefficients:
##              Estimate Std.err   Wald Pr(>|W|)
## (Intercept)  -1.4905  0.0789 356.50  <2e-16 ***
## qsmk          0.0301  0.1573   0.04    0.85
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Estimated Scale Parameters:
##              Estimate Std.err
## (Intercept)      1  0.0678
##
## Correlation: Structure = independenceNumber of clusters: 1566 Maximum cluster size: 1
```

```
beta <- coef(msm.logistic)
SE <- coef(summary(msm.logistic))[, 2]
lcl <- beta - qnorm(0.975) * SE
ucl <- beta + qnorm(0.975) * SE
cbind(beta, lcl, ucl)
```

```
##              beta    lcl    ucl
## (Intercept) -1.4905 -1.645 -1.336
## qsmk         0.0301 -0.278  0.338
```

Program 12.6

- Assessing effect modification by sex using a marginal structural mean model
- Data from NHEFS

```
table(nhefs.nmv$sex)

##
##  0  1
## 762 804

# estimation of denominator of ip weights
denom.fit <-
  glm(
    qsmk ~ as.factor(sex) + as.factor(race) + age + I(age ^ 2) +
      as.factor(education) + smokeintensity +
      I(smokeintensity ^ 2) + smokeyrs + I(smokeyrs ^ 2) +
      as.factor(exercise) + as.factor(active) + wt71 + I(wt71 ^ 2),
    family = binomial(),
    data = nhefs.nmv
  )
summary(denom.fit)

##
## Call:
## glm(formula = qsmk ~ as.factor(sex) + as.factor(race) + age +
##      I(age^2) + as.factor(education) + smokeintensity + I(smokeintensity^2) +
##      smokeyrs + I(smokeyrs^2) + as.factor(exercise) + as.factor(active) +
##      wt71 + I(wt71^2), family = binomial(), data = nhefs.nmv)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.513  -0.791  -0.639   0.983   2.373
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -2.242519   1.380836  -1.62  0.10437
## as.factor(sex)1    -0.527478   0.154050  -3.42  0.00062 ***
## as.factor(race)1   -0.839264   0.210067  -4.00  6.5e-05 ***
## age              0.121205   0.051266   2.36  0.01807 *
## I(age^2)         -0.000825   0.000536  -1.54  0.12404
## as.factor(education)2 -0.028776   0.198351  -0.15  0.88465
## as.factor(education)3  0.086432   0.178085   0.49  0.62744
## as.factor(education)4  0.063601   0.273211   0.23  0.81592
## as.factor(education)5  0.475961   0.226224   2.10  0.03538 *
## smokeintensity    -0.077270   0.015250  -5.07  4.0e-07 ***
## I(smokeintensity^2)  0.001045   0.000287   3.65  0.00027 ***
## smokeyrs         -0.073597   0.027777  -2.65  0.00806 **
## I(smokeyrs^2)      0.000844   0.000463   1.82  0.06840 .
## as.factor(exercise)1  0.354841   0.180135   1.97  0.04885 *
## as.factor(exercise)2  0.395704   0.187240   2.11  0.03457 *
```

```

## as.factor(active)1      0.031944   0.132937    0.24  0.81010
## as.factor(active)2      0.176784   0.214972    0.82  0.41087
## wt71                    -0.015236   0.026316   -0.58  0.56262
## I(wt71^2)               0.000135   0.000163    0.83  0.40737
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 1786.1  on 1565  degrees of freedom
## Residual deviance: 1676.9  on 1547  degrees of freedom
## AIC: 1715
##
## Number of Fisher Scoring iterations: 4

```

```

pd.qsmk <- predict(denom.fit, type = "response")

# estimation of numerator of ip weights
numer.fit <-
  glm(qsmk ~ as.factor(sex), family = binomial(), data = nhefs.nmv)
summary(numer.fit)

```

```

##
## Call:
## glm(formula = qsmk ~ as.factor(sex), family = binomial(), data = nhefs.nmv)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.825  -0.825  -0.719   1.576   1.720
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -0.9016    0.0799  -11.28  <2e-16 ***
## as.factor(sex)1 -0.3202    0.1160   -2.76   0.0058 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 1786.1  on 1565  degrees of freedom
## Residual deviance: 1778.4  on 1564  degrees of freedom
## AIC: 1782
##
## Number of Fisher Scoring iterations: 4

```

```

pn.qsmk <- predict(numer.fit, type = "response")

nhefs.nmv$sw.a <-
  ifelse(nhefs.nmv$qsmk == 0, ((1 - pn.qsmk) / (1 - pd.qsmk)),
        (pn.qsmk / pd.qsmk))

```

```
summary(nhefs.nmv$sw.a)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      0.29   0.88   0.96   1.00   1.08   3.80
```

```
sd(nhefs.nmv$sw.a)
```

```
## [1] 0.271
```

```
# Estimating parameters of a marginal structural mean model
```

```
msem.emm <- geeglm(
  wt82_71 ~ as.factor(qsmk) + as.factor(sex)
  + as.factor(qsmk):as.factor(sex),
  data = nhefs.nmv,
  weights = sw.a,
  id = seqn,
  corstr = "independence"
)
```

```
summary(msem.emm)
```

```
##
```

```
## Call:
```

```
## geeglm(formula = wt82_71 ~ as.factor(qsmk) + as.factor(sex) +
##       as.factor(qsmk):as.factor(sex), data = nhefs.nmv, weights = sw.a,
##       id = seqn, corstr = "independence")
```

```
##
```

```
## Coefficients:
```

```
##              Estimate Std.err Wald Pr(>|W|)
## (Intercept)      1.78445  0.30984 33.17  8.5e-09 ***
## as.factor(qsmk)1      3.52198  0.65707 28.73  8.3e-08 ***
## as.factor(sex)1     -0.00872  0.44882  0.00   0.98
## as.factor(qsmk)1:as.factor(sex)1 -0.15948  1.04608  0.02   0.88
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
```

```
## Estimated Scale Parameters:
```

```
##              Estimate Std.err
## (Intercept)      60.8    3.71
```

```
##
```

```
## Correlation: Structure = independenceNumber of clusters: 1566 Maximum cluster size: 1
```

```
beta <- coef(msem.emm)
```

```
SE <- coef(summary(msem.emm))[, 2]
```

```
lcl <- beta - qnorm(0.975) * SE
```

```
ucl <- beta + qnorm(0.975) * SE
```

```
cbind(beta, lcl, ucl)
```

```
##              beta    lcl    ucl
## (Intercept)      1.78445  1.177  2.392
## as.factor(qsmk)1      3.52198  2.234  4.810
## as.factor(sex)1     -0.00872 -0.888  0.871
```

```
## as.factor(qsmk)1:as.factor(sex)1 -0.15948 -2.210 1.891
```

Program 12.7

- Estimating IP weights to adjust for selection bias due to censoring
- Data from NHEFS

```
table(nhefs$qsmk, nehefs$cens)
```

```
##
##      0      1
##  0 1163    38
##  1   403    25
```

```
summary(nhefs[which(nhefs$cens == 0),]$wt71)
```

```
##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
##      39.6   59.5    69.2    70.8   79.8   151.7
```

```
summary(nhefs[which(nhefs$cens == 1),]$wt71)
```

```
##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
##      36.2   63.1    72.1    76.6   87.9   169.2
```

```
# estimation of denominator of ip weights for A
```

```
denom.fit <-
```

```
  glm(
    qsmk ~ as.factor(sex) + as.factor(race) + age + I(age ^ 2) +
      as.factor(education) + smokeintensity +
      I(smokeintensity ^ 2) + smokeyrs + I(smokeyrs ^ 2) +
      as.factor(exercise) + as.factor(active) + wt71 + I(wt71 ^ 2),
    family = binomial(),
    data = nehefs
  )
```

```
summary(denom.fit)
```

```
##
## Call:
## glm(formula = qsmk ~ as.factor(sex) + as.factor(race) + age +
##      I(age^2) + as.factor(education) + smokeintensity + I(smokeintensity^2) +
##      smokeyrs + I(smokeyrs^2) + as.factor(exercise) + as.factor(active) +
##      wt71 + I(wt71^2), family = binomial(), data = nehefs)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.465  -0.804  -0.646   1.058   2.355
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -1.988902   1.241279  -1.60  0.10909
## as.factor(sex)1 -0.507522   0.148232  -3.42  0.00062 ***
## as.factor(race)1 -0.850231   0.205872  -4.13  3.6e-05 ***
```

```

## age                0.103013    0.048900    2.11  0.03515 *
## I(age^2)           -0.000605    0.000507   -1.19  0.23297
## as.factor(education)2 -0.098320    0.190655   -0.52  0.60607
## as.factor(education)3  0.015699    0.170714    0.09  0.92673
## as.factor(education)4 -0.042526    0.264276   -0.16  0.87216
## as.factor(education)5  0.379663    0.220395    1.72  0.08495 .
## smokeintensity      -0.065156    0.014759   -4.41  1.0e-05 ***
## I(smokeintensity^2)   0.000846    0.000276    3.07  0.00216 **
## smokeyrs            -0.073371    0.026996   -2.72  0.00657 **
## I(smokeyrs^2)         0.000838    0.000443    1.89  0.05867 .
## as.factor(exercise)1  0.291412    0.173554    1.68  0.09314 .
## as.factor(exercise)2  0.355052    0.179929    1.97  0.04846 *
## as.factor(active)1    0.010875    0.129832    0.08  0.93324
## as.factor(active)2    0.068312    0.208727    0.33  0.74346
## wt71                 -0.012848    0.022283   -0.58  0.56423
## I(wt71^2)             0.000121    0.000135    0.89  0.37096
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 1876.3  on 1628  degrees of freedom
## Residual deviance: 1766.7  on 1610  degrees of freedom
## AIC: 1805
##
## Number of Fisher Scoring iterations: 4

```

```

pd.qsmk <- predict(denom.fit, type = "response")

# estimation of numerator of ip weights for A
numer.fit <- glm(qsmk ~ 1, family = binomial(), data = nhefs)
summary(numer.fit)

```

```

##
## Call:
## glm(formula = qsmk ~ 1, family = binomial(), data = nhefs)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.781  -0.781  -0.781   1.635   1.635
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -1.0318      0.0563  -18.3   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 1876.3  on 1628  degrees of freedom

```

```
## Residual deviance: 1876.3 on 1628 degrees of freedom
## AIC: 1878
##
## Number of Fisher Scoring iterations: 4
pn.qsmk <- predict(numer.fit, type = "response")

# estimation of denominator of ip weights for C
denom.cens <- glm(
  cens ~ as.factor(qsmk) + as.factor(sex) +
    as.factor(race) + age + I(age ^ 2) +
    as.factor(education) + smokeintensity +
    I(smokeintensity ^ 2) + smokeyrs + I(smokeyrs ^ 2) +
    as.factor(exercise) + as.factor(active) + wt71 + I(wt71 ^ 2),
  family = binomial(),
  data = nhfs
)
summary(denom.cens)

##
## Call:
## glm(formula = cens ~ as.factor(qsmk) + as.factor(sex) + as.factor(race) +
##      age + I(age^2) + as.factor(education) + smokeintensity +
##      I(smokeintensity^2) + smokeyrs + I(smokeyrs^2) + as.factor(exercise) +
##      as.factor(active) + wt71 + I(wt71^2), family = binomial(),
##      data = nhfs)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.097  -0.287  -0.207  -0.157   2.996
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)      4.014466   2.576106    1.56   0.1192
## as.factor(qsmk)1    0.516867   0.287716    1.80   0.0724 .
## as.factor(sex)1     0.057313   0.330278    0.17   0.8622
## as.factor(race)1    -0.012271   0.452489   -0.03   0.9784
## age               -0.269729   0.117465   -2.30   0.0217 *
## I(age^2)           0.002884   0.001114    2.59   0.0096 **
## as.factor(education)2 -0.440788   0.419399   -1.05   0.2933
## as.factor(education)3 -0.164688   0.370547   -0.44   0.6567
## as.factor(education)4  0.138447   0.569797    0.24   0.8080
## as.factor(education)5 -0.382382   0.560181   -0.68   0.4949
## smokeintensity       0.015712   0.034732    0.45   0.6510
## I(smokeintensity^2) -0.000113   0.000606   -0.19   0.8517
## smokeyrs            0.078597   0.074958    1.05   0.2944
## I(smokeyrs^2)       -0.000557   0.001032   -0.54   0.5894
## as.factor(exercise)1 -0.971471   0.387810   -2.51   0.0122 *
## as.factor(exercise)2 -0.583989   0.372313   -1.57   0.1168
## as.factor(active)1  -0.247479   0.325455   -0.76   0.4470
```



```

## as.factor(active)2      0.706583   0.396458    1.78   0.0747 .
## wt71                    -0.087887   0.040012   -2.20   0.0281 *
## I(wt71^2)               0.000635   0.000226    2.81   0.0049 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 533.36  on 1628  degrees of freedom
## Residual deviance: 465.36  on 1609  degrees of freedom
## AIC: 505.4
##
## Number of Fisher Scoring iterations: 7

```

```

pd.cens <- 1 - predict(denom.cens, type = "response")

# estimation of numerator of ip weights for C
numer.cens <-
  glm(cens ~ as.factor(qsmk), family = binomial(), data = nhefs)
summary(numer.cens)

```

```

##
## Call:
## glm(formula = cens ~ as.factor(qsmk), family = binomial(), data = nhefs)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.347  -0.254  -0.254  -0.254   2.628
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -3.421     0.165  -20.75  <2e-16 ***
## as.factor(qsmk)1  0.641     0.264   2.43   0.015 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 533.36  on 1628  degrees of freedom
## Residual deviance: 527.76  on 1627  degrees of freedom
## AIC: 531.8
##
## Number of Fisher Scoring iterations: 6

```

```

pn.cens <- 1 - predict(numer.cens, type = "response")

nhefs$sw.a <-
  ifelse(nhefs$qsmk == 0, ((1 - pn.qsmk) / (1 - pd.qsmk)),
        (pn.qsmk / pd.qsmk))
nhefs$sw.c <- pn.cens / pd.cens

```

```

nhefs$sw <- nehs$sw.c * nehs$sw.a

summary(nhefs$sw.a)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      0.33   0.86   0.95    1.00   1.08    4.21

sd(nhefs$sw.a)

## [1] 0.284

summary(nhefs$sw.c)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      0.94   0.98   0.99    1.01   1.01    7.58

sd(nhefs$sw.c)

## [1] 0.178

summary(nhefs$sw)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      0.35   0.86   0.94    1.01   1.08   12.86

sd(nhefs$sw)

## [1] 0.411

msm.sw <- geeglm(
  wt82_71 ~ qsmk,
  data = nehs,
  weights = sw,
  id = seqn,
  corstr = "independence"
)
summary(msm.sw)

##
## Call:
## geeglm(formula = wt82_71 ~ qsmk, data = nehs, weights = sw,
##        id = seqn, corstr = "independence")
##
## Coefficients:
##              Estimate Std.err Wald Pr(>|W|)
## (Intercept)    1.662    0.233  51.0  9.3e-13 ***
## qsmk           3.496    0.526  44.2  2.9e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Estimated Scale Parameters:
##              Estimate Std.err
## (Intercept)    61.8    3.83
##

```

```
## Correlation: Structure = independenceNumber of clusters: 1566 Maximum cluster size: 1
```

```
beta <- coef(msm.sw)
SE <- coef(summary(msm.sw))[, 2]
lcl <- beta - qnorm(0.975) * SE
ucl <- beta + qnorm(0.975) * SE
cbind(beta, lcl, ucl)
```

```
##           beta  lcl  ucl
## (Intercept) 1.66 1.21 2.12
## qsmk        3.50 2.47 4.53
```


13. Standardization and the parametric G-formula

Program 13.1

- Estimating the mean outcome within levels of treatment and confounders
- Data from NHEFS

```
library(here)

#install.packages("readxl") # install package if required
library("readxl")
nhefs <- read_excel(here("data", "NHEFS.xls"))

# some preprocessing of the data
nhefs$cens <- ifelse(is.na(nhefs$wt82), 1, 0)

fit <-
  glm(
    wt82_71 ~ qsmk + sex + race + age + I(age * age) + as.factor(education)
    + smokeintensity + I(smokeintensity * smokeintensity) + smokeyrs
    + I(smokeyrs * smokeyrs) + as.factor(exercise) + as.factor(active)
    + wt71 + I(wt71 * wt71) + qsmk * smokeintensity,
    data = nhefs
  )
summary(fit)

##
## Call:
## glm(formula = wt82_71 ~ qsmk + sex + race + age + I(age * age) +
##      as.factor(education) + smokeintensity + I(smokeintensity *
##      smokeintensity) + smokeyrs + I(smokeyrs * smokeyrs) + as.factor(exercise) +
##      as.factor(active) + wt71 + I(wt71 * wt71) + qsmk * smokeintensity,
##      data = nhefs)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -42.056  -4.171  -0.343   3.891  44.606
##
```

```

## Coefficients:
##
## Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.5881657 4.3130359 -0.368 0.712756
## qsmk 2.5595941 0.8091486 3.163 0.001590
## sex -1.4302717 0.4689576 -3.050 0.002328
## race 0.5601096 0.5818888 0.963 0.335913
## age 0.3596353 0.1633188 2.202 0.027809
## I(age * age) -0.0061010 0.0017261 -3.534 0.000421
## as.factor(education)2 0.7904440 0.6070005 1.302 0.193038
## as.factor(education)3 0.5563124 0.5561016 1.000 0.317284
## as.factor(education)4 1.4915695 0.8322704 1.792 0.073301
## as.factor(education)5 -0.1949770 0.7413692 -0.263 0.792589
## smokeintensity 0.0491365 0.0517254 0.950 0.342287
## I(smokeintensity * smokeintensity) -0.0009907 0.0009380 -1.056 0.291097
## smokeyrs 0.1343686 0.0917122 1.465 0.143094
## I(smokeyrs * smokeyrs) -0.0018664 0.0015437 -1.209 0.226830
## as.factor(exercise)1 0.2959754 0.5351533 0.553 0.580298
## as.factor(exercise)2 0.3539128 0.5588587 0.633 0.526646
## as.factor(active)1 -0.9475695 0.4099344 -2.312 0.020935
## as.factor(active)2 -0.2613779 0.6845577 -0.382 0.702647
## wt71 0.0455018 0.0833709 0.546 0.585299
## I(wt71 * wt71) -0.0009653 0.0005247 -1.840 0.066001
## qsmk:smokeintensity 0.0466628 0.0351448 1.328 0.184463
##
## (Intercept)
## qsmk **
## sex **
## race
## age *
## I(age * age) ***
## as.factor(education)2
## as.factor(education)3
## as.factor(education)4 .
## as.factor(education)5
## smokeintensity
## I(smokeintensity * smokeintensity)
## smokeyrs
## I(smokeyrs * smokeyrs)
## as.factor(exercise)1
## as.factor(exercise)2
## as.factor(active)1 *
## as.factor(active)2
## wt71
## I(wt71 * wt71) .
## qsmk:smokeintensity
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 53.5683)

```

```
##
## Null deviance: 97176 on 1565 degrees of freedom
## Residual deviance: 82763 on 1545 degrees of freedom
## (63 observations deleted due to missingness)
## AIC: 10701
##
## Number of Fisher Scoring iterations: 2
nhefs$predicted.meanY <- predict(fit, nhefs)

nhefs[which(nhefs$seqn == 24770), c(
  "predicted.meanY",
  "qsmk",
  "sex",
  "race",
  "age",
  "education",
  "smokeintensity",
  "smokeyrs",
  "exercise",
  "active",
  "wt71"
)]

## # A tibble: 1 x 11
## predicted.meanY qsmk sex race age education smokeintensity smokeyrs
## <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 0.342 0 0 0 26 4 15 12
## # ... with 3 more variables: exercise <dbl>, active <dbl>, wt71 <dbl>

summary(nhefs$predicted.meanY[nhefs$cens == 0])

## Min. 1st Qu. Median Mean 3rd Qu. Max.
## -10.876 1.116 3.042 2.638 4.511 9.876

summary(nhefs$wt82_71[nhefs$cens == 0])

## Min. 1st Qu. Median Mean 3rd Qu. Max.
## -41.280 -1.478 2.604 2.638 6.690 48.538
```

Program 13.2

- Standardizing the mean outcome to the baseline confounders
- Data from Table 2.2

```
id <- c(
  "Rheia",
  "Kronos",
  "Demeter",
  "Hades",
  "Hestia",
  "Poseidon",
```

```

"Hera",
"Zeus",
"Artemis",
"Apollo",
"Leto",
"Ares",
"Athena",
"Hephaestus",
"Aphrodite",
"Cyclope",
"Persephone",
"Hermes",
"Hebe",
"Dionysus"
)
N <- length(id)
L <- c(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)
A <- c(0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1)
Y <- c(0, 1, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0)
interv <- rep(-1, N)
observed <- cbind(L, A, Y, interv)
untreated <- cbind(L, rep(0, N), rep(NA, N), rep(0, N))
treated <- cbind(L, rep(1, N), rep(NA, N), rep(1, N))
table22 <- as.data.frame(rbind(observed, untreated, treated))
table22$id <- rep(id, 3)

glm.obj <- glm(Y ~ A * L, data = table22)
summary(glm.obj)

```

```

##
## Call:
## glm(formula = Y ~ A * L, data = table22)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.66667  -0.25000   0.04167   0.33333   0.75000
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.500e-01  2.552e-01   0.980   0.342
## A           -4.164e-16  3.608e-01   0.000   1.000
## L            4.167e-01  3.898e-01   1.069   0.301
## A:L           3.237e-16  4.959e-01   0.000   1.000
##
## (Dispersion parameter for gaussian family taken to be 0.2604167)
##
##      Null deviance: 5.0000  on 19  degrees of freedom
## Residual deviance: 4.1667  on 16  degrees of freedom
## (40 observations deleted due to missingness)

```



```
## AIC: 35.385
##
## Number of Fisher Scoring iterations: 2
table22$predicted.meanY <- predict(glm.obj, table22)

mean(table22$predicted.meanY[table22$interv == -1])

## [1] 0.5
mean(table22$predicted.meanY[table22$interv == 0])

## [1] 0.5
mean(table22$predicted.meanY[table22$interv == 1])

## [1] 0.5
```

Program 13.3

- Standardizing the mean outcome to the baseline confounders:
- Data from NHEFS

```
# create a dataset with 3 copies of each subject
nhefs$interv <- -1 # 1st copy: equal to original one

interv0 <- nhefs # 2nd copy: treatment set to 0, outcome to missing
interv0$interv <- 0
interv0$qsmk <- 0
interv0$wt82_71 <- NA

interv1 <- nhefs # 3rd copy: treatment set to 1, outcome to missing
interv1$interv <- 1
interv1$qsmk <- 1
interv1$wt82_71 <- NA

onesample <- rbind(nhefs, interv0, interv1) # combining datasets

# linear model to estimate mean outcome conditional on treatment and confounders
# parameters are estimated using original observations only (nhefs)
# parameter estimates are used to predict mean outcome for observations with
# treatment set to 0 (interv=0) and to 1 (interv=1)

std <- glm(
  wt82_71 ~ qsmk + sex + race + age + I(age * age)
  + as.factor(education) + smokeintensity
  + I(smokeintensity * smokeintensity) + smokeyrs
  + I(smokeyrs * smokeyrs) + as.factor(exercise)
  + as.factor(active) + wt71 + I(wt71 * wt71) + I(qsmk * smokeintensity),
  data = onesample
)
summary(std)
```

```
##
## Call:
## glm(formula = wt82_71 ~ qsmk + sex + race + age + I(age * age) +
##      as.factor(education) + smokeintensity + I(smokeintensity *
##      smokeintensity) + smokeyrs + I(smokeyrs * smokeyrs) + as.factor(exercise) +
##      as.factor(active) + wt71 + I(wt71 * wt71) + I(qsmk * smokeintensity),
##      data = onesample)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -42.056  -4.171  -0.343   3.891  44.606
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -1.5881657   4.3130359   -0.368  0.712756
## qsmk             2.5595941   0.8091486    3.163  0.001590
## sex            -1.4302717   0.4689576   -3.050  0.002328
## race             0.5601096   0.5818888    0.963  0.335913
## age             0.3596353   0.1633188    2.202  0.027809
## I(age * age)    -0.0061010   0.0017261   -3.534  0.000421
## as.factor(education)2    0.7904440   0.6070005    1.302  0.193038
## as.factor(education)3    0.5563124   0.5561016    1.000  0.317284
## as.factor(education)4    1.4915695   0.8322704    1.792  0.073301
## as.factor(education)5   -0.1949770   0.7413692   -0.263  0.792589
## smokeintensity    0.0491365   0.0517254    0.950  0.342287
## I(smokeintensity * smokeintensity) -0.0009907   0.0009380   -1.056  0.291097
## smokeyrs         0.1343686   0.0917122    1.465  0.143094
## I(smokeyrs * smokeyrs)  -0.0018664   0.0015437   -1.209  0.226830
## as.factor(exercise)1     0.2959754   0.5351533    0.553  0.580298
## as.factor(exercise)2     0.3539128   0.5588587    0.633  0.526646
## as.factor(active)1      -0.9475695   0.4099344   -2.312  0.020935
## as.factor(active)2      -0.2613779   0.6845577   -0.382  0.702647
## wt71             0.0455018   0.0833709    0.546  0.585299
## I(wt71 * wt71)        -0.0009653   0.0005247   -1.840  0.066001
## I(qsmk * smokeintensity)  0.0466628   0.0351448    1.328  0.184463
##
## (Intercept)
## qsmk              **
## sex              **
## race
## age              *
## I(age * age)     ***
## as.factor(education)2
## as.factor(education)3
## as.factor(education)4
## as.factor(education)5
## smokeintensity
## I(smokeintensity * smokeintensity)
```

```

## smokeyrs
## I(smokeyrs * smokeyrs)
## as.factor(exercise)1
## as.factor(exercise)2
## as.factor(active)1 *
## as.factor(active)2
## wt71
## I(wt71 * wt71) .
## I(qsmk * smokeintensity)
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 53.5683)
##
## Null deviance: 97176  on 1565  degrees of freedom
## Residual deviance: 82763  on 1545  degrees of freedom
## (3321 observations deleted due to missingness)
## AIC: 10701
##
## Number of Fisher Scoring iterations: 2
onesample$predicted_meanY <- predict(std, onesample)

# estimate mean outcome in each of the groups interv=0, and interv=1
# this mean outcome is a weighted average of the mean outcomes in each combination
# of values of treatment and confounders, that is, the standardized outcome
mean(onesample[which(onesample$interv == -1), ]$predicted_meanY)

## [1] 2.56319

mean(onesample[which(onesample$interv == 0), ]$predicted_meanY)

## [1] 1.660267

mean(onesample[which(onesample$interv == 1), ]$predicted_meanY)

## [1] 5.178841

```

Program 13.4

- Computing the 95% confidence interval of the standardized means and their difference
- Data from NHEFS

```

#install.packages("boot") # install package if required
library(boot)

# function to calculate difference in means
standardization <- function(data, indices) {
  # create a dataset with 3 copies of each subject
  d <- data[indices, ] # 1st copy: equal to original one`
  d$interv <- -1
  d0 <- d # 2nd copy: treatment set to 0, outcome to missing

```

```

d0$interv <- 0
d0$qsmk <- 0
d0$wt82_71 <- NA
d1 <- d # 3rd copy: treatment set to 1, outcome to missing
d1$interv <- 1
d1$qsmk <- 1
d1$wt82_71 <- NA
d.onesample <- rbind(d, d0, d1) # combining datasets

# linear model to estimate mean outcome conditional on treatment and confounders
# parameters are estimated using original observations only (interv= -1)
# parameter estimates are used to predict mean outcome for observations with set
# treatment (interv=0 and interv=1)
fit <- glm(
  wt82_71 ~ qsmk + sex + race + age + I(age * age) +
    as.factor(education) + smokeintensity +
    I(smokeintensity * smokeintensity) + smokeyrs + I(smokeyrs *
      smokeyrs) +
    as.factor(exercise) + as.factor(active) + wt71 + I(wt71 *
      wt71),
  data = d.onesample
)

d.onesample$predicted_meanY <- predict(fit, d.onesample)

# estimate mean outcome in each of the groups interv=-1, interv=0, and interv=1
return(c(
  mean(d.onesample$predicted_meanY[d.onesample$interv == -1]),
  mean(d.onesample$predicted_meanY[d.onesample$interv == 0]),
  mean(d.onesample$predicted_meanY[d.onesample$interv == 1]),
  mean(d.onesample$predicted_meanY[d.onesample$interv == 1]) -
    mean(d.onesample$predicted_meanY[d.onesample$interv == 0])
))
}

# bootstrap
results <- boot(data = nhfs,
  statistic = standardization,
  R = 5)

# generating confidence intervals
se <- c(sd(results$t[, 1]),
  sd(results$t[, 2]),
  sd(results$t[, 3]),
  sd(results$t[, 4]))
mean <- results$t0
ll <- mean - qnorm(0.975) * se
ul <- mean + qnorm(0.975) * se

```

```
bootstrap <-
  data.frame(cbind(
    c(
      "Observed",
      "No Treatment",
      "Treatment",
      "Treatment - No Treatment"
    ),
    mean,
    se,
    ll,
    ul
  ))
bootstrap
```

```
##           V1           mean           se
## 1      Observed 2.56188497106103 0.143746528513805
## 2      No Treatment 1.65212306626746 0.196707928034904
## 3      Treatment 5.11474489549347 0.509234029529476
## 4 Treatment - No Treatment 3.46262182922601 0.623247033505222
##           ll           ul
## 1 2.28014695227132 2.84362298985075
## 2 1.26658261184555 2.03766352068937
## 3 4.11666453791349 6.11282525307345
## 4 2.24108009008434 4.68416356836767
```


14. G-estimation of Structural Nested Models

Program 14.1

- Preprocessing, ranks of extreme observations, IP weights for censoring
- Data from NHEFS

```
library(here)

#install.packages("readxl") # install package if required
library("readxl")
nhefs <- read_excel(here("data", "NHEFS.xls"))

# some processing of the data
nhefs$cens <- ifelse(is.na(nhefs$wt82), 1, 0)

# ranking of extreme observations
#install.packages("Hmisc")
library(Hmisc)

## Loading required package: lattice
## Loading required package: survival
## Loading required package: Formula
## Loading required package: ggplot2

##
## Attaching package: 'Hmisc'

## The following objects are masked from 'package:base':
##
##      format.pval, units

describe(nhefs$wt82_71)

## nhefs$wt82_71
##      n missing distinct      Info      Mean      Gmd      .05      .10
##  1566      63     1510         1    2.638    8.337   -9.752   -6.292
##    .25     .50     .75     .90     .95
```

```
##      -1.478      2.604      6.690     11.117     14.739
##
## lowest : -41.28047 -30.50192 -30.05007 -29.02579 -25.97056
## highest:  34.01780  36.96925  37.65051  47.51130  48.53839

# estimation of denominator of ip weights for C
cw.denom <- glm(cens==0 ~ qsmk + sex + race + age + I(age^2)
                + as.factor(education) + smokeintensity + I(smokeintensity^2)
                + smokeyrs + I(smokeyrs^2) + as.factor(exercise)
                + as.factor(active) + wt71 + I(wt71^2),
                data = nhefs, family = binomial("logit"))
summary(cw.denom)

##
## Call:
## glm(formula = cens == 0 ~ qsmk + sex + race + age + I(age^2) +
##      as.factor(education) + smokeintensity + I(smokeintensity^2) +
##      smokeyrs + I(smokeyrs^2) + as.factor(exercise) + as.factor(active) +
##      wt71 + I(wt71^2), family = binomial("logit"), data = nhefs)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -2.9959   0.1571   0.2069   0.2868   1.0967
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -4.0144661   2.5761058  -1.558  0.11915
## qsmk           -0.5168674   0.2877162  -1.796  0.07242 .
## sex            -0.0573131   0.3302775  -0.174  0.86223
## race            0.0122715   0.4524887   0.027  0.97836
## age            0.2697293   0.1174647   2.296  0.02166 *
## I(age^2)       -0.0028837   0.0011135  -2.590  0.00961 **
## as.factor(education)2  0.4407884   0.4193993   1.051  0.29326
## as.factor(education)3  0.1646881   0.3705471   0.444  0.65672
## as.factor(education)4 -0.1384470   0.5697969  -0.243  0.80802
## as.factor(education)5  0.3823818   0.5601808   0.683  0.49486
## smokeintensity    -0.0157119   0.0347319  -0.452  0.65100
## I(smokeintensity^2)   0.0001133   0.0006058   0.187  0.85171
## smokeyrs         -0.0785973   0.0749576  -1.049  0.29438
## I(smokeyrs^2)        0.0005569   0.0010318   0.540  0.58938
## as.factor(exercise)1  0.9714714   0.3878101   2.505  0.01224 *
## as.factor(exercise)2  0.5839890   0.3723133   1.569  0.11675
## as.factor(active)1    0.2474785   0.3254548   0.760  0.44701
## as.factor(active)2   -0.7065829   0.3964577  -1.782  0.07471 .
## wt71             0.0878871   0.0400115   2.197  0.02805 *
## I(wt71^2)         -0.0006351   0.0002257  -2.813  0.00490 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
```



```
##
## Null deviance: 533.36 on 1628 degrees of freedom
## Residual deviance: 465.36 on 1609 degrees of freedom
## AIC: 505.36
##
## Number of Fisher Scoring iterations: 7
nhefs$pd.c <- predict(cw.denom, nhefs, type="response")
nhefs$wc <- ifelse(nhefs$cens==0, 1/nhefs$pd.c, NA) # observations with cens=1 only contribute to cens
```

Program 14.2

- G-estimation of a 1-parameter structural nested mean model
- Brute force search
- Data from NHEFS

G-estimation: Checking one possible value of ψ

```
#install.packages("geepack")
library("geepack")

nhefs$psi <- 3.446
nhefs$Hpsi <- nhefs$wt82_71 - nhefs$psi*nhefs$qsmk

fit <- geeglm(qsmk ~ sex + race + age + I(age*age) + as.factor(education)
             + smokeintensity + I(smokeintensity*smokeintensity) + smokeyrs
             + I(smokeyrs*smokeyrs) + as.factor(exercise) + as.factor(active)
             + wt71 + I(wt71*wt71) + Hpsi, family=binomial, data=nhefs,
             weights=wc, id=seqn, corstr="independence")

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

summary(fit)

##
## Call:
## geeglm(formula = qsmk ~ sex + race + age + I(age * age) + as.factor(education) +
## smokeintensity + I(smokeintensity * smokeintensity) + smokeyrs +
## I(smokeyrs * smokeyrs) + as.factor(exercise) + as.factor(active) +
## wt71 + I(wt71 * wt71) + Hpsi, family = binomial, data = nhefs,
## weights = wc, id = seqn, corstr = "independence")
##
## Coefficients:
##
## Estimate Std.err Wald Pr(>|W|)
## (Intercept) -2.403e+00 1.329e+00 3.269 0.070604
## sex -5.137e-01 1.536e-01 11.193 0.000821
## race -8.609e-01 2.099e-01 16.826 4.10e-05
## age 1.152e-01 5.020e-02 5.263 0.021779
## I(age * age) -7.593e-04 5.296e-04 2.056 0.151619
```

```

## as.factor(education)2      -2.894e-02  1.964e-01  0.022 0.882859
## as.factor(education)3      8.771e-02  1.726e-01  0.258 0.611329
## as.factor(education)4      6.637e-02  2.698e-01  0.061 0.805645
## as.factor(education)5      4.711e-01  2.247e-01  4.395 0.036036
## smokeintensity             -7.834e-02  1.464e-02 28.635 8.74e-08
## I(smokeintensity * smokeintensity) 1.072e-03 2.650e-04 16.368 5.21e-05
## smokeyrs                   -7.111e-02  2.639e-02  7.261 0.007047
## I(smokeyrs * smokeyrs)      8.153e-04  4.490e-04  3.298 0.069384
## as.factor(exercise)1       3.363e-01  1.828e-01  3.384 0.065844
## as.factor(exercise)2       3.800e-01  1.889e-01  4.049 0.044187
## as.factor(active)1         3.412e-02  1.339e-01  0.065 0.798778
## as.factor(active)2         2.135e-01  2.121e-01  1.012 0.314308
## wt71                       -7.661e-03  2.562e-02  0.089 0.764963
## I(wt71 * wt71)             8.655e-05  1.582e-04  0.299 0.584233
## Hpsi                       -1.903e-06  8.839e-03  0.000 0.999828
##
## (Intercept)                .
## sex                        ***
## race                       ***
## age                        *
## I(age * age)
## as.factor(education)2
## as.factor(education)3
## as.factor(education)4
## as.factor(education)5      *
## smokeintensity             ***
## I(smokeintensity * smokeintensity) ***
## smokeyrs                   **
## I(smokeyrs * smokeyrs)      .
## as.factor(exercise)1       .
## as.factor(exercise)2       *
## as.factor(active)1
## as.factor(active)2
## wt71
## I(wt71 * wt71)
## Hpsi
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Estimated Scale Parameters:
##           Estimate Std.err
## (Intercept)  0.9969 0.06717
##
## Correlation: Structure = independenceNumber of clusters: 1566 Maximum cluster size: 1

```

G-estimation: Checking multiple possible values of psi

```

#install.packages("geepack")
grid <- seq(from = 2,to = 5, by = 0.1)

```

```

j = 0
Hpsi.coefs <- cbind(rep(NA,length(grid)), rep(NA, length(grid)))
colnames(Hpsi.coefs) <- c("Estimate", "p-value")

for (i in grid){
  psi = i
  j = j+1
  nhefs$Hpsi <- nhefs$wt82_71 - psi * nhefs$qsmk

  gest.fit <- geeglm(qsmk ~ sex + race + age + I(age*age) + as.factor(education)
    + smokeintensity + I(smokeintensity*smokeintensity) + smokeyrs
    + I(smokeyrs*smokeyrs) + as.factor(exercise) + as.factor(active)
    + wt71 + I(wt71*wt71) + Hpsi, family=binomial, data=nhefs,
    weights=wc, id=seqn, corstr="independence")
  Hpsi.coefs[j,1] <- summary(gest.fit)$coefficients["Hpsi", "Estimate"]
  Hpsi.coefs[j,2] <- summary(gest.fit)$coefficients["Hpsi", "Pr(>|W|)"]
}

```

```

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

```

```

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

```

```

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

```

```

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

```

```

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

```

```

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

```

```

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

```

```

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

```

```

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

```

```

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

```

```

## Warning in eval(family$initialize): non-integer #successes in a binomial

```

[illegible]

```
## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!
```

```
## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!
```

```
## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!
```

```
## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!
```

```
Hpsi.coefs
```

##		Estimate	p-value
##	[1,]	0.0267219	0.001772
##	[2,]	0.0248946	0.003580
##	[3,]	0.0230655	0.006963
##	[4,]	0.0212344	0.013026
##	[5,]	0.0194009	0.023417
##	[6,]	0.0175647	0.040430
##	[7,]	0.0157254	0.067015
##	[8,]	0.0138827	0.106626
##	[9,]	0.0120362	0.162877
##	[10,]	0.0101857	0.238979
##	[11,]	0.0083308	0.337048
##	[12,]	0.0064713	0.457433
##	[13,]	0.0046069	0.598235
##	[14,]	0.0027374	0.755204
##	[15,]	0.0008624	0.922101
##	[16,]	-0.0010181	0.908537
##	[17,]	-0.0029044	0.744362
##	[18,]	-0.0047967	0.592188
##	[19,]	-0.0066950	0.457169
##	[20,]	-0.0085997	0.342360
##	[21,]	-0.0105107	0.248681
##	[22,]	-0.0124282	0.175239
##	[23,]	-0.0143523	0.119841
##	[24,]	-0.0162831	0.079580
##	[25,]	-0.0182206	0.051347
##	[26,]	-0.0201649	0.032218
##	[27,]	-0.0221160	0.019675
##	[28,]	-0.0240740	0.011706
##	[29,]	-0.0260389	0.006792
##	[30,]	-0.0280106	0.003847
##	[31,]	-0.0299893	0.002129

Program 14.3

- G-estimation for 2-parameter structural nested mean model
- Closed form estimator
- Data from NHEFS

G-estimation: Closed form estimator linear mean models

```
logit.est <- glm(qsmk ~ sex + race + age + I(age^2) + as.factor(education)
               + smokeintensity + I(smokeintensity^2) + smokeyrs
               + I(smokeyrs^2) + as.factor(exercise) + as.factor(active)
               + wt71 + I(wt71^2), data = nhefs, weight = wc,
               family = binomial())
```

```
## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!
```

```
summary(logit.est)
```

```
##
```

```
## Call:
```

```
## glm(formula = qsmk ~ sex + race + age + I(age^2) + as.factor(education) +
##      smokeintensity + I(smokeintensity^2) + smokeyrs + I(smokeyrs^2) +
##      as.factor(exercise) + as.factor(active) + wt71 + I(wt71^2),
##      family = binomial(), data = nhefs, weights = wc)
```

```
##
```

```
## Deviance Residuals:
```

```
##      Min       1Q   Median       3Q      Max
## -1.529  -0.808  -0.650   1.029   2.417
```

```
##
```

```
## Coefficients:
```

```
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -2.40e+00  1.31e+00  -1.83  0.06743 .
## sex           -5.14e-01  1.50e-01  -3.42  0.00062 ***
## race          -8.61e-01  2.06e-01  -4.18  2.9e-05 ***
## age            1.15e-01  4.95e-02   2.33  0.01992 *
## I(age^2)       -7.59e-04  5.14e-04  -1.48  0.13953
## as.factor(education)2 -2.89e-02  1.93e-01  -0.15  0.88079
## as.factor(education)3  8.77e-02  1.73e-01   0.51  0.61244
## as.factor(education)4  6.64e-02  2.66e-01   0.25  0.80301
## as.factor(education)5  4.71e-01  2.21e-01   2.13  0.03314 *
## smokeintensity  -7.83e-02  1.49e-02  -5.27  1.4e-07 ***
## I(smokeintensity^2)  1.07e-03  2.78e-04   3.85  0.00012 ***
## smokeyrs       -7.11e-02  2.71e-02  -2.63  0.00862 **
## I(smokeyrs^2)      8.15e-04  4.45e-04   1.83  0.06722 .
## as.factor(exercise)1  3.36e-01  1.75e-01   1.92  0.05467 .
## as.factor(exercise)2  3.80e-01  1.82e-01   2.09  0.03637 *
## as.factor(active)1    3.41e-02  1.30e-01   0.26  0.79337
## as.factor(active)2    2.13e-01  2.06e-01   1.04  0.30033
## wt71           -7.66e-03  2.46e-02  -0.31  0.75530
```

```
## I(wt71^2)          8.66e-05   1.51e-04   0.57  0.56586
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 1872.2  on 1565  degrees of freedom
## Residual deviance: 1755.6  on 1547  degrees of freedom
## (63 observations deleted due to missingness)
## AIC: 1719
##
## Number of Fisher Scoring iterations: 4
```

```
nhefs$pqsmk <- predict(logit.est, neufs, type = "response")
describe(nhefs$pqsmk)
```

```
## neufs$pqsmk
##      n missing distinct      Info      Mean      Gmd      .05      .10
##  1629      0      1629        1  0.2622  0.1302  0.1015  0.1261
##    .25    .50    .75    .90    .95
##  0.1780  0.2426  0.3251  0.4221  0.4965
##
## lowest : 0.05145 0.05157 0.05438 0.05583 0.05931
## highest: 0.67208 0.68643 0.71391 0.73330 0.78914
```

```
summary(nhefs$pqsmk)
```

```
##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
##  0.0514  0.1780  0.2426  0.2622  0.3251  0.7891
```

```
# solve sum(w_c * H(psi) * (qsmk - E[qsmk | L])) = 0
# for a single psi and H(psi) = wt82_71 - psi * qsmk
# this can be solved as psi = sum(w_c * wt82_71 * (qsmk - pqsmk)) / sum(w_c * qsmk * (qsmk - pqsmk))
```

```
nhefs.c <- neufs[which(!is.na(nhefs$wt82)),]
with(nhefs.c, sum(wc*wt82_71*(qsmk-pqsmk)) / sum(wc*qsmk*(qsmk - pqsmk)))
```

```
## [1] 3.446
```

G-estimation: Closed form estimator for 2-parameter model

```
diff = with(nhefs.c, qsmk - pqsmk)
diff2 = with(nhefs.c, wc * diff)

lhs = matrix(0,2,2)
lhs[1,1] = with(nhefs.c, sum(qsmk * diff2))
lhs[1,2] = with(nhefs.c, sum(qsmk * smokeintensity * diff2))
lhs[2,1] = with(nhefs.c, sum(qsmk * smokeintensity * diff2))
lhs[2,2] = with(nhefs.c, sum(qsmk * smokeintensity * smokeintensity * diff2))

rhs = matrix(0,2,1)
```

```

rhs[1] = with(nhefs.c, sum(wt82_71 * diff2))
rhs[2] = with(nhefs.c, sum(wt82_71 * smokeintensity * diff2))

psi = t(solve(lhs,rhs))
psi

##          [,1]      [,2]
## [1,] 2.859 0.03004

```


15. Outcome regression and propensity scores

Program 15.1

- Estimating the average causal effect within levels of confounders under the assumption of effect-measure modification by smoking intensity ONLY
- Data from NHEFS

```
library(here)

#install.packages("readxl") # install package if required
library("readxl")

nhefs <- read_excel(here("data", "NHEFS.xls"))
nhefs$cens <- ifelse(is.na(nhefs$wt82), 1, 0)

# regression on covariates, allowing for some effect modification
fit <- glm(wt82_71 ~ qsmk + sex + race + age + I(age*age) + as.factor(education)
          + smokeintensity + I(smokeintensity*smokeintensity) + smokeyrs
          + I(smokeyrs*smokeyrs) + as.factor(exercise) + as.factor(active)
          + wt71 + I(wt71*wt71) + I(qsmk*smokeintensity), data=nhefs)
summary(fit)

##
## Call:
## glm(formula = wt82_71 ~ qsmk + sex + race + age + I(age * age) +
##      as.factor(education) + smokeintensity + I(smokeintensity *
##      smokeintensity) + smokeyrs + I(smokeyrs * smokeyrs) + as.factor(exercise) +
##      as.factor(active) + wt71 + I(wt71 * wt71) + I(qsmk * smokeintensity),
##      data = nhefs)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -42.056  -4.171  -0.343   3.891  44.606
##
## Coefficients:
##                                Estimate Std. Error t value Pr(>|t|)
## (Intercept)                -1.5881657   4.3130359  -0.368  0.712756
```

```

## qsmk                2.5595941  0.8091486   3.163 0.001590
## sex                 -1.4302717  0.4689576  -3.050 0.002328
## race                0.5601096  0.5818888   0.963 0.335913
## age                 0.3596353  0.1633188   2.202 0.027809
## I(age * age)        -0.0061010  0.0017261  -3.534 0.000421
## as.factor(education)2  0.7904440  0.6070005   1.302 0.193038
## as.factor(education)3  0.5563124  0.5561016   1.000 0.317284
## as.factor(education)4  1.4915695  0.8322704   1.792 0.073301
## as.factor(education)5 -0.1949770  0.7413692  -0.263 0.792589
## smokeintensity       0.0491365  0.0517254   0.950 0.342287
## I(smokeintensity * smokeintensity) -0.0009907  0.0009380  -1.056 0.291097
## smokeyrs            0.1343686  0.0917122   1.465 0.143094
## I(smokeyrs * smokeyrs) -0.0018664  0.0015437  -1.209 0.226830
## as.factor(exercise)1  0.2959754  0.5351533   0.553 0.580298
## as.factor(exercise)2  0.3539128  0.5588587   0.633 0.526646
## as.factor(active)1    -0.9475695  0.4099344  -2.312 0.020935
## as.factor(active)2    -0.2613779  0.6845577  -0.382 0.702647
## wt71                 0.0455018  0.0833709   0.546 0.585299
## I(wt71 * wt71)        -0.0009653  0.0005247  -1.840 0.066001
## I(qsmk * smokeintensity) 0.0466628  0.0351448   1.328 0.184463
##
## (Intercept)
## qsmk                **
## sex                 **
## race
## age                 *
## I(age * age)        ***
## as.factor(education)2
## as.factor(education)3
## as.factor(education)4 .
## as.factor(education)5
## smokeintensity
## I(smokeintensity * smokeintensity)
## smokeyrs
## I(smokeyrs * smokeyrs)
## as.factor(exercise)1
## as.factor(exercise)2
## as.factor(active)1  *
## as.factor(active)2
## wt71
## I(wt71 * wt71)      .
## I(qsmk * smokeintensity)
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 53.5683)
##
## Null deviance: 97176  on 1565  degrees of freedom
## Residual deviance: 82763  on 1545  degrees of freedom

```

```

## (63 observations deleted due to missingness)
## AIC: 10701
##
## Number of Fisher Scoring iterations: 2
# (step 1) build the contrast matrix with all zeros
# this function builds the blank matrix
# install.packages("multcomp") # install packages if necessary
library("multcomp")

## Loading required package: mvtnorm
## Loading required package: survival
## Loading required package: TH.data
## Loading required package: MASS
##
## Attaching package: 'TH.data'
## The following object is masked from 'package:MASS':
##
##      geyser
makeContrastMatrix <- function(model, nrow, names) {
  m <- matrix(0, nrow = nrow, ncol = length(coef(model)))
  colnames(m) <- names(coef(model))
  rownames(m) <- names
  return(m)
}
K1 <- makeContrastMatrix(fit, 2, c('Effect of Quitting Smoking at Smokeintensity of 5',
                                   'Effect of Quitting Smoking at Smokeintensity of 40'))
# (step 2) fill in the relevant non-zero elements
K1[1:2, 'qsmk'] <- 1
K1[1:2, 'I(qsmk * smokeintensity)'] <- c(5, 40)

# (step 3) check the contrast matrix
K1

##
##                                     (Intercept) qsmk sex
## Effect of Quitting Smoking at Smokeintensity of 5          0   1   0
## Effect of Quitting Smoking at Smokeintensity of 40          0   1   0
##
##                                     race age I(age * age)
## Effect of Quitting Smoking at Smokeintensity of 5          0   0          0
## Effect of Quitting Smoking at Smokeintensity of 40          0   0          0
##
##                                     as.factor(education)2
## Effect of Quitting Smoking at Smokeintensity of 5          0
## Effect of Quitting Smoking at Smokeintensity of 40          0
##
##                                     as.factor(education)3
## Effect of Quitting Smoking at Smokeintensity of 5          0
## Effect of Quitting Smoking at Smokeintensity of 40          0
##
##                                     as.factor(education)4

```

```

## Effect of Quitting Smoking at Smokeintensity of 5 0
## Effect of Quitting Smoking at Smokeintensity of 40 0
## as.factor(education)5
## Effect of Quitting Smoking at Smokeintensity of 5 0
## Effect of Quitting Smoking at Smokeintensity of 40 0
## smokeintensity
## Effect of Quitting Smoking at Smokeintensity of 5 0
## Effect of Quitting Smoking at Smokeintensity of 40 0
## I(smokeintensity * smokeintensity)
## Effect of Quitting Smoking at Smokeintensity of 5 0
## Effect of Quitting Smoking at Smokeintensity of 40 0
## smokeyrs
## Effect of Quitting Smoking at Smokeintensity of 5 0
## Effect of Quitting Smoking at Smokeintensity of 40 0
## I(smokeyrs * smokeyrs)
## Effect of Quitting Smoking at Smokeintensity of 5 0
## Effect of Quitting Smoking at Smokeintensity of 40 0
## as.factor(exercise)1
## Effect of Quitting Smoking at Smokeintensity of 5 0
## Effect of Quitting Smoking at Smokeintensity of 40 0
## as.factor(exercise)2
## Effect of Quitting Smoking at Smokeintensity of 5 0
## Effect of Quitting Smoking at Smokeintensity of 40 0
## as.factor(active)1
## Effect of Quitting Smoking at Smokeintensity of 5 0
## Effect of Quitting Smoking at Smokeintensity of 40 0
## as.factor(active)2 wt71
## Effect of Quitting Smoking at Smokeintensity of 5 0 0
## Effect of Quitting Smoking at Smokeintensity of 40 0 0
## I(wt71 * wt71)
## Effect of Quitting Smoking at Smokeintensity of 5 0
## Effect of Quitting Smoking at Smokeintensity of 40 0
## I(qsmk * smokeintensity)
## Effect of Quitting Smoking at Smokeintensity of 5 5
## Effect of Quitting Smoking at Smokeintensity of 40 40

```

```

# (step 4) estimate the contrasts, get tests and confidence intervals for them
estimates1 <- glht(fit, K1)
summary(estimates1)

```

```

##
## Simultaneous Tests for General Linear Hypotheses
##
## Fit: glm(formula = wt82_71 ~ qsmk + sex + race + age + I(age * age) +
## as.factor(education) + smokeintensity + I(smokeintensity *
## smokeintensity) + smokeyrs + I(smokeyrs * smokeyrs) + as.factor(exercise) +
## as.factor(active) + wt71 + I(wt71 * wt71) + I(qsmk * smokeintensity),
## data = nhefs)
##
## Linear Hypotheses:

```

```

##                                     Estimate
## Effect of Quitting Smoking at Smokeintensity of 5 == 0    2.7929
## Effect of Quitting Smoking at Smokeintensity of 40 == 0    4.4261
##                                     Std. Error z value
## Effect of Quitting Smoking at Smokeintensity of 5 == 0      0.6683    4.179
## Effect of Quitting Smoking at Smokeintensity of 40 == 0      0.8478    5.221
##                                     Pr(>|z|)
## Effect of Quitting Smoking at Smokeintensity of 5 == 0  5.84e-05 ***
## Effect of Quitting Smoking at Smokeintensity of 40 == 0  3.56e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)

confint(estimates1)

##
## Simultaneous Confidence Intervals
##
## Fit: glm(formula = wt82_71 ~ qsmk + sex + race + age + I(age * age) +
##      as.factor(education) + smokeintensity + I(smokeintensity *
##      smokeintensity) + smokeyrs + I(smokeyrs * smokeyrs) + as.factor(exercise) +
##      as.factor(active) + wt71 + I(wt71 * wt71) + I(qsmk * smokeintensity),
##      data = nhefs)
##
## Quantile = 2.2281
## 95% family-wise confidence level
##
## Linear Hypotheses:
##                                     Estimate lwr
## Effect of Quitting Smoking at Smokeintensity of 5 == 0  2.7929    1.3039
## Effect of Quitting Smoking at Smokeintensity of 40 == 0  4.4261    2.5372
##                                     upr
## Effect of Quitting Smoking at Smokeintensity of 5 == 0  4.2819
## Effect of Quitting Smoking at Smokeintensity of 40 == 0  6.3151

# regression on covariates, not allowing for effect modification
fit2 <- glm(wt82_71 ~ qsmk + sex + race + age + I(age*age) + as.factor(education)
            + smokeintensity + I(smokeintensity*smokeintensity) + smokeyrs
            + I(smokeyrs*smokeyrs) + as.factor(exercise) + as.factor(active)
            + wt71 + I(wt71*wt71), data=nhefs)

summary(fit2)

##
## Call:
## glm(formula = wt82_71 ~ qsmk + sex + race + age + I(age * age) +
##      as.factor(education) + smokeintensity + I(smokeintensity *
##      smokeintensity) + smokeyrs + I(smokeyrs * smokeyrs) + as.factor(exercise) +
##      as.factor(active) + wt71 + I(wt71 * wt71), data = nhefs)
##

```

```

## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -42.332   -4.216   -0.318    3.807   44.668
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -1.6586176   4.3137734   -0.384 0.700666
## qsmk             3.4626218   0.4384543    7.897 5.36e-15
## sex            -1.4650496   0.4683410   -3.128 0.001792
## race             0.5864117   0.5816949    1.008 0.313560
## age             0.3626624   0.1633431    2.220 0.026546
## I(age * age)    -0.0061377   0.0017263   -3.555 0.000389
## as.factor(education)2  0.8185263   0.6067815    1.349 0.177546
## as.factor(education)3  0.5715004   0.5561211    1.028 0.304273
## as.factor(education)4  1.5085173   0.8323778    1.812 0.070134
## as.factor(education)5 -0.1708264   0.7413289   -0.230 0.817786
## smokeintensity  0.0651533   0.0503115    1.295 0.195514
## I(smokeintensity * smokeintensity) -0.0010468   0.0009373   -1.117 0.264261
## smokeyrs        0.1333931   0.0917319    1.454 0.146104
## I(smokeyrs * smokeyrs) -0.0018270   0.0015438   -1.183 0.236818
## as.factor(exercise)1  0.3206824   0.5349616    0.599 0.548961
## as.factor(exercise)2  0.3628786   0.5589557    0.649 0.516300
## as.factor(active)1   -0.9429574   0.4100208   -2.300 0.021593
## as.factor(active)2   -0.2580374   0.6847219   -0.377 0.706337
## wt71              0.0373642   0.0831658    0.449 0.653297
## I(wt71 * wt71)      -0.0009158   0.0005235   -1.749 0.080426
##
## (Intercept)
## qsmk            ***
## sex            **
## race
## age            *
## I(age * age)    ***
## as.factor(education)2
## as.factor(education)3
## as.factor(education)4 .
## as.factor(education)5
## smokeintensity
## I(smokeintensity * smokeintensity)
## smokeyrs
## I(smokeyrs * smokeyrs)
## as.factor(exercise)1
## as.factor(exercise)2
## as.factor(active)1  *
## as.factor(active)2
## wt71
## I(wt71 * wt71)    .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```
##
## (Dispersion parameter for gaussian family taken to be 53.59474)
##
## Null deviance: 97176 on 1565 degrees of freedom
## Residual deviance: 82857 on 1546 degrees of freedom
## (63 observations deleted due to missingness)
## AIC: 10701
##
## Number of Fisher Scoring iterations: 2
```

Program 15.2

- Estimating and plotting the propensity score
- Data from NHEFS

```
fit3 <- glm(qsmk ~ sex + race + age + I(age*age) + as.factor(education)
           + smokeintensity + I(smokeintensity*smokeintensity) + smokeyrs
           + I(smokeyrs*smokeyrs) + as.factor(exercise) + as.factor(active)
           + wt71 + I(wt71*wt71), data=nhefs, family=binomial())
summary(fit3)
```

```
##
## Call:
## glm(formula = qsmk ~ sex + race + age + I(age * age) + as.factor(education) +
##      smokeintensity + I(smokeintensity * smokeintensity) + smokeyrs +
##      I(smokeyrs * smokeyrs) + as.factor(exercise) + as.factor(active) +
##      wt71 + I(wt71 * wt71), family = binomial(), data = dhefs)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.4646  -0.8044  -0.6460   1.0578   2.3550
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -1.9889022    1.2412792   -1.602  0.109089
## sex             -0.5075218    0.1482316   -3.424  0.000617
## race            -0.8502312    0.2058720   -4.130  3.63e-05
## age              0.1030132    0.0488996    2.107  0.035150
## I(age * age)    -0.0006052    0.0005074   -1.193  0.232973
## as.factor(education)2 -0.0983203    0.1906553   -0.516  0.606066
## as.factor(education)3  0.0156987    0.1707139    0.092  0.926730
## as.factor(education)4 -0.0425260    0.2642761   -0.161  0.872160
## as.factor(education)5  0.3796632    0.2203947    1.723  0.084952
## smokeintensity    -0.0651561    0.0147589   -4.415  1.01e-05
## I(smokeintensity * smokeintensity)  0.0008461    0.0002758    3.067  0.002160
## smokeyrs        -0.0733708    0.0269958   -2.718  0.006571
## I(smokeyrs * smokeyrs)  0.0008384    0.0004435    1.891  0.058669
## as.factor(exercise)1   0.2914117    0.1735543    1.679  0.093136
## as.factor(exercise)2   0.3550517    0.1799293    1.973  0.048463
```

```

## as.factor(active)1          0.0108754  0.1298320   0.084 0.933243
## as.factor(active)2          0.0683123  0.2087269   0.327 0.743455
## wt71                        -0.0128478  0.0222829  -0.577 0.564226
## I(wt71 * wt71)              0.0001209  0.0001352   0.895 0.370957
##
## (Intercept)
## sex                        ***
## race                       ***
## age                        *
## I(age * age)
## as.factor(education)2
## as.factor(education)3
## as.factor(education)4
## as.factor(education)5      .
## smokeintensity             ***
## I(smokeintensity * smokeintensity) **
## smokeyrs                   **
## I(smokeyrs * smokeyrs)     .
## as.factor(exercise)1       .
## as.factor(exercise)2       *
## as.factor(active)1
## as.factor(active)2
## wt71
## I(wt71 * wt71)
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 1876.3  on 1628  degrees of freedom
## Residual deviance: 1766.7  on 1610  degrees of freedom
## AIC: 1804.7
##
## Number of Fisher Scoring iterations: 4

```

```

nhefs$ps <- predict(fit3, nhefs, type="response")

summary(nhefs$ps[nhefs$qsmk==0])

```

```

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.05298 0.16949 0.22747 0.24504 0.30441 0.65788

```

```

summary(nhefs$ps[nhefs$qsmk==1])

```

```

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.06248 0.22046 0.28897 0.31240 0.38122 0.79320

```

```

# # plotting the estimated propensity score
# install.packages("ggplot2") # install packages if necessary
# install.packages("dplyr")
library("ggplot2")

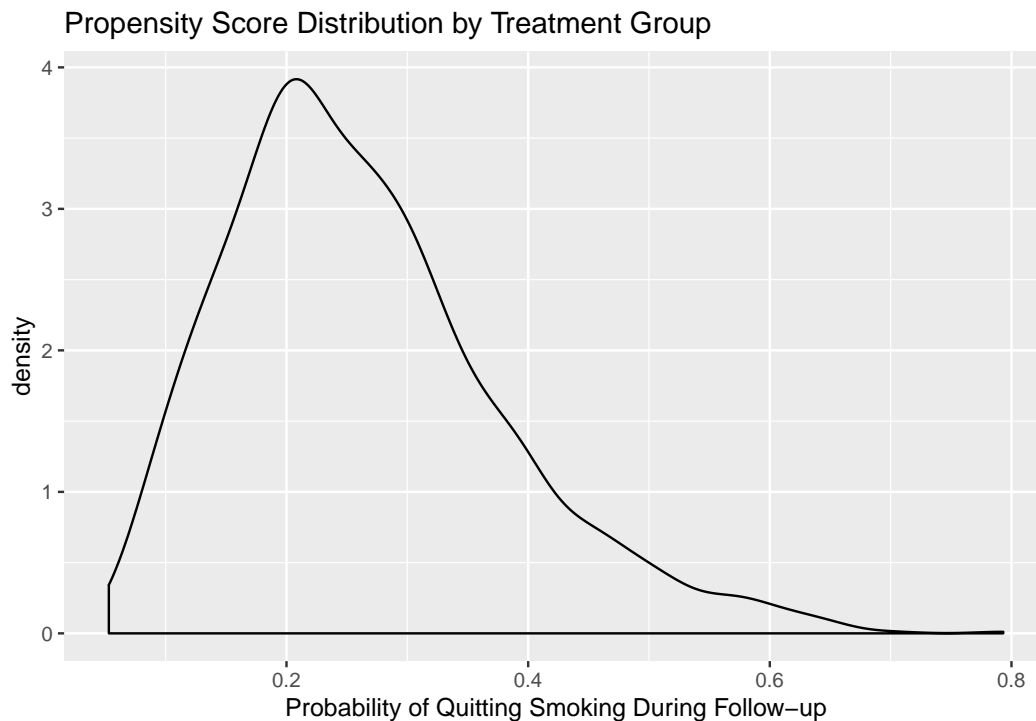
```



```
library("dplyr")
```

```
##  
## Attaching package: 'dplyr'  
  
## The following object is masked from 'package:MASS':  
##  
##   select  
  
## The following objects are masked from 'package:stats':  
##  
##   filter, lag  
  
## The following objects are masked from 'package:base':  
##  
##   intersect, setdiff, setequal, union
```

```
ggplot(nhefs, aes(x = ps, fill = qsmk)) + geom_density(alpha = 0.2) +  
  xlab('Probability of Quitting Smoking During Follow-up') +  
  ggtitle('Propensity Score Distribution by Treatment Group') +  
  scale_fill_discrete('') +  
  theme(legend.position = 'bottom', legend.direction = 'vertical')
```

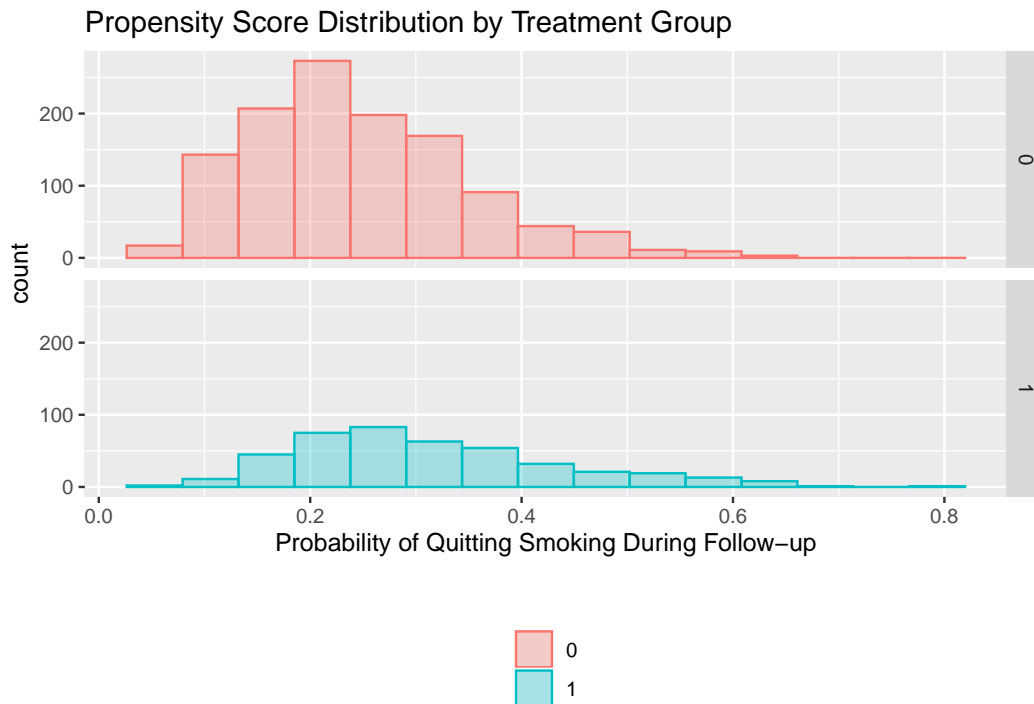


```
# alternative plot with histograms  
nhefs <- nhefs %>% mutate(qsmklabel = ifelse(qsmk == 1,  
  yes = 'Quit Smoking 1971-1982',  
  no = 'Did Not Quit Smoking 1971-1982'))  
ggplot(nhefs, aes(x = ps, fill = as.factor(qsmk), color = as.factor(qsmk))) +  
  geom_histogram(alpha = 0.3, position = 'identity', bins=15) +  
  facet_grid(as.factor(qsmk) ~ .) +
```

```

xlab('Probability of Quitting Smoking During Follow-up') +
ggtitle('Propensity Score Distribution by Treatment Group') +
scale_fill_discrete('') +
scale_color_discrete('') +
theme(legend.position = 'bottom', legend.direction = 'vertical')

```



attempt to reproduce plot from the book

```

nhefs %>%
  mutate(ps.grp = round(ps/0.05) * 0.05) %>%
  group_by(qsmk, ps.grp) %>%
  summarize(n = n()) %>%
  ungroup() %>%
  mutate(n2 = ifelse(qsmk == 0, yes = n, no = -1*n)) %>%
  ggplot(aes(x = ps.grp, y = n2, fill = as.factor(qsmk))) +
  geom_bar(stat = 'identity', position = 'identity') +
  geom_text(aes(label = n, x = ps.grp, y = n2 + ifelse(qsmk == 0, 8, -8))) +
  xlab('Probability of Quitting Smoking During Follow-up') +
  ylab('N') +
  ggtitle('Propensity Score Distribution by Treatment Group') +
  scale_fill_discrete('') +
  scale_x_continuous(breaks = seq(0, 1, 0.05)) +
  theme(legend.position = 'bottom', legend.direction = 'vertical',
        axis.ticks.y = element_blank(),
        axis.text.y = element_blank())

```

Program 15.3

- Stratification on the propensity score
- Data from NHEFS

```
# calculation of deciles
nhefs$ps.dec <- cut(nhefs$ps,
                    breaks=c(quantile(nhefs$ps, probs=seq(0,1,0.1))),
                    labels=seq(1:10),
                    include.lowest=TRUE)

#install.packages("psych") # install package if required
library("psych")

##
## Attaching package: 'psych'

## The following objects are masked from 'package:ggplot2':
##
##   %+%, alpha

describeBy(nhefs$ps, list(nhefs$ps.dec, nehs$qsmk))

##
## Descriptive statistics by group
## : 1
## : 0
##   vars   n mean   sd median trimmed  mad  min  max range  skew kurtosis
## X1      1 151  0.1 0.02   0.11     0.1 0.02 0.05 0.13  0.08 -0.55   -0.53
##   se
## X1      0
## -----
## : 2
## : 0
##   vars   n mean   sd median trimmed  mad  min  max range  skew kurtosis
## X1      1 136  0.15 0.01   0.15     0.15 0.01 0.13 0.17  0.04 -0.04   -1.23
##   se
## X1      0
## -----
## : 3
## : 0
##   vars   n mean   sd median trimmed  mad  min  max range  skew kurtosis
## X1      1 134  0.18 0.01   0.18     0.18 0.01 0.17 0.19  0.03 -0.08   -1.34
##   se
## X1      0
## -----
## : 4
## : 0
##   vars   n mean   sd median trimmed  mad  min  max range  skew kurtosis
## X1      1 129  0.21 0.01   0.21     0.21 0.01 0.19 0.22  0.02 -0.04   -1.13
##   se
```

```

## X1 0
## -----
## : 5
## : 0
## vars n mean sd median trimmed mad min max range skew kurtosis se
## X1 1 120 0.23 0.01 0.23 0.23 0.01 0.22 0.25 0.03 0.24 -1.22 0
## -----
## : 6
## : 0
## vars n mean sd median trimmed mad min max range skew kurtosis
## X1 1 117 0.26 0.01 0.26 0.26 0.01 0.25 0.27 0.03 -0.11 -1.29
## se
## X1 0
## -----
## : 7
## : 0
## vars n mean sd median trimmed mad min max range skew kurtosis
## X1 1 120 0.29 0.01 0.29 0.29 0.01 0.27 0.31 0.03 -0.23 -1.19
## se
## X1 0
## -----
## : 8
## : 0
## vars n mean sd median trimmed mad min max range skew kurtosis se
## X1 1 112 0.33 0.01 0.33 0.33 0.02 0.31 0.35 0.04 0.15 -1.1 0
## -----
## : 9
## : 0
## vars n mean sd median trimmed mad min max range skew kurtosis se
## X1 1 96 0.38 0.02 0.38 0.38 0.02 0.35 0.42 0.06 0.13 -1.15 0
## -----
## : 10
## : 0
## vars n mean sd median trimmed mad min max range skew kurtosis
## X1 1 86 0.49 0.06 0.47 0.48 0.05 0.42 0.66 0.24 1.1 0.47
## se
## X1 0.01
## -----
## : 1
## : 1
## vars n mean sd median trimmed mad min max range skew kurtosis
## X1 1 12 0.1 0.02 0.11 0.1 0.03 0.06 0.13 0.07 -0.5 -1.36
## se
## X1 0.01
## -----
## : 2
## : 1
## vars n mean sd median trimmed mad min max range skew kurtosis se
## X1 1 27 0.15 0.01 0.15 0.15 0.01 0.13 0.17 0.03 -0.03 -1.34 0

```

```
## -----
## : 3
## : 1
##   vars  n mean   sd median trimmed  mad  min  max range skew kurtosis se
## X1     1 29 0.18 0.01   0.18   0.18 0.01 0.17 0.19  0.03 0.01   -1.34  0
## -----
## : 4
## : 1
##   vars  n mean   sd median trimmed  mad  min  max range skew kurtosis se
## X1     1 34 0.21 0.01   0.21   0.21 0.01 0.19 0.22  0.02 -0.31   -1.23  0
## -----
## : 5
## : 1
##   vars  n mean   sd median trimmed  mad  min  max range skew kurtosis se
## X1     1 43 0.23 0.01   0.23   0.23 0.01 0.22 0.25  0.03 0.11   -1.23  0
## -----
## : 6
## : 1
##   vars  n mean   sd median trimmed  mad  min  max range skew kurtosis se
## X1     1 45 0.26 0.01   0.26   0.26 0.01 0.25 0.27  0.03 0.2   -1.12  0
## -----
## : 7
## : 1
##   vars  n mean   sd median trimmed  mad  min  max range skew kurtosis se
## X1     1 43 0.29 0.01   0.29   0.29 0.01 0.27 0.31  0.03 0.16   -1.25  0
## -----
## : 8
## : 1
##   vars  n mean   sd median trimmed  mad  min  max range skew kurtosis se
## X1     1 51 0.33 0.01   0.33   0.33 0.02 0.31 0.35  0.04 0.11   -1.19  0
## -----
## : 9
## : 1
##   vars  n mean   sd median trimmed  mad  min  max range skew kurtosis se
## X1     1 67 0.38 0.02   0.38   0.38 0.03 0.35 0.42  0.06 0.19   -1.27  0
## -----
## : 10
## : 1
##   vars  n mean   sd median trimmed  mad  min  max range skew kurtosis
## X1     1 77 0.52 0.08   0.51   0.51 0.08 0.42 0.79  0.38 0.88   0.81
##       se
## X1 0.01
```

```
# function to create deciles easily
decile <- function(x) {
  return(factor(quantcut(x, seq(0, 1, 0.1), labels = FALSE)))
}

# regression on PS deciles, allowing for effect modification
for (deciles in c(1:10)) {
```

```
print(t.test(wt82_71~qsmk, data=nhefs[which(nhefs$ps.dec==deciles),]))
}
```

```
##
## Welch Two Sample t-test
##
## data: wt82_71 by qsmk
## t = 0.0060506, df = 11.571, p-value = 0.9953
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -5.283903 5.313210
## sample estimates:
## mean in group 0 mean in group 1
## 3.995205 3.980551
##
## Welch Two Sample t-test
##
## data: wt82_71 by qsmk
## t = -3.1117, df = 37.365, p-value = 0.003556
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -6.849335 -1.448161
## sample estimates:
## mean in group 0 mean in group 1
## 2.904679 7.053426
##
## Welch Two Sample t-test
##
## data: wt82_71 by qsmk
## t = -4.5301, df = 35.79, p-value = 6.317e-05
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -9.474961 -3.613990
## sample estimates:
## mean in group 0 mean in group 1
## 2.612094 9.156570
##
## Welch Two Sample t-test
##
## data: wt82_71 by qsmk
## t = -1.4117, df = 45.444, p-value = 0.1648
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -5.6831731 0.9985715
## sample estimates:
## mean in group 0 mean in group 1
```

```

##          3.474679          5.816979
##
##
## Welch Two Sample t-test
##
## data: wt82_71 by qsmk
## t = -3.1371, df = 74.249, p-value = 0.002446
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -6.753621 -1.507087
## sample estimates:
## mean in group 0 mean in group 1
##          2.098800          6.229154
##
##
## Welch Two Sample t-test
##
## data: wt82_71 by qsmk
## t = -2.1677, df = 50.665, p-value = 0.0349
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -8.7516605 -0.3350127
## sample estimates:
## mean in group 0 mean in group 1
##          1.847004          6.390340
##
##
## Welch Two Sample t-test
##
## data: wt82_71 by qsmk
## t = -3.3155, df = 84.724, p-value = 0.001348
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -6.904207 -1.727590
## sample estimates:
## mean in group 0 mean in group 1
##          1.560048          5.875946
##
##
## Welch Two Sample t-test
##
## data: wt82_71 by qsmk
## t = -2.664, df = 75.306, p-value = 0.009441
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -6.2396014 -0.9005605
## sample estimates:
## mean in group 0 mean in group 1
##          0.2846851          3.8547661

```

```
##
##
## Welch Two Sample t-test
##
## data: wt82_71 by qsmk
## t = -1.9122, df = 129.12, p-value = 0.05806
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -4.68143608 0.07973698
## sample estimates:
## mean in group 0 mean in group 1
## -0.8954482 1.4054014
##
##
## Welch Two Sample t-test
##
## data: wt82_71 by qsmk
## t = -1.5925, df = 142.72, p-value = 0.1135
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -5.0209284 0.5404697
## sample estimates:
## mean in group 0 mean in group 1
## -0.5043766 1.7358528

# regression on PS deciles, not allowing for effect modification
fit.psdec <- glm(wt82_71 ~ qsmk + as.factor(ps.dec), data = nhefs)
summary(fit.psdec)

##
## Call:
## glm(formula = wt82_71 ~ qsmk + as.factor(ps.dec), data = nhefs)
##
## Deviance Residuals:
## Min 1Q Median 3Q Max
## -43.543 -3.932 -0.085 4.233 46.773
##
## Coefficients:
## Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.7505 0.6089 6.159 9.29e-10 ***
## qsmk 3.5005 0.4571 7.659 3.28e-14 ***
## as.factor(ps.dec)2 -0.7391 0.8611 -0.858 0.3908
## as.factor(ps.dec)3 -0.6182 0.8612 -0.718 0.4730
## as.factor(ps.dec)4 -0.5204 0.8584 -0.606 0.5444
## as.factor(ps.dec)5 -1.4884 0.8590 -1.733 0.0834 .
## as.factor(ps.dec)6 -1.6227 0.8675 -1.871 0.0616 .
## as.factor(ps.dec)7 -1.9853 0.8681 -2.287 0.0223 *
## as.factor(ps.dec)8 -3.4447 0.8749 -3.937 8.61e-05 ***
## as.factor(ps.dec)9 -5.1544 0.8848 -5.825 6.91e-09 ***
## as.factor(ps.dec)10 -4.8403 0.8828 -5.483 4.87e-08 ***
```



```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 58.42297)
##
##      Null deviance: 97176  on 1565  degrees of freedom
## Residual deviance: 90848  on 1555  degrees of freedom
##   (63 observations deleted due to missingness)
## AIC: 10827
##
## Number of Fisher Scoring iterations: 2
```

```
confint.lm(fit.psdec)
```

```
##              2.5 %      97.5 %
## (Intercept)    2.556098  4.94486263
## qsmk           2.603953  4.39700504
## as.factor(ps.dec)2 -2.428074  0.94982494
## as.factor(ps.dec)3 -2.307454  1.07103569
## as.factor(ps.dec)4 -2.204103  1.16333143
## as.factor(ps.dec)5 -3.173337  0.19657938
## as.factor(ps.dec)6 -3.324345  0.07893027
## as.factor(ps.dec)7 -3.688043 -0.28248110
## as.factor(ps.dec)8 -5.160862 -1.72860113
## as.factor(ps.dec)9 -6.889923 -3.41883853
## as.factor(ps.dec)10 -6.571789 -3.10873731
```

Program 15.4

- Standardization using the propensity score
- Data from NHEFS

```
#install.packages("boot") # install package if required
library("boot")
```

```
##
## Attaching package: 'boot'

## The following object is masked from 'package:psych':
##
##      logit

## The following object is masked from 'package:survival':
##
##      aml
```

```
# standardization by propensity score, agnostic regarding effect modification
std.ps <- function(data, indices) {
  d <- data[indices,] # 1st copy: equal to original one`
  # calculating propensity scores
  ps.fit <- glm(qsmk ~ sex + race + age + I(age*age)
    + as.factor(education) + smokeintensity
```

```

+ I(smokeintensity*smokeintensity) + smokeyrs
+ I(smokeyrs*smokeyrs) + as.factor(exercise)
+ as.factor(active) + wt71 + I(wt71*wt71),
data=d, family=binomial())
d$pscore <- predict(ps.fit, d, type="response")

# create a dataset with 3 copies of each subject
d$interv <- -1 # 1st copy: equal to original one`
d0 <- d # 2nd copy: treatment set to 0, outcome to missing
d0$interv <- 0
d0$qsmk <- 0
d0$wt82_71 <- NA
d1 <- d # 3rd copy: treatment set to 1, outcome to missing
d1$interv <- 1
d1$qsmk <- 1
d1$wt82_71 <- NA
d.onesample <- rbind(d, d0, d1) # combining datasets

std.fit <- glm(wt82_71 ~ qsmk + pscore + I(qsmk*pscore), data=d.onesample)
d.onesample$predicted_meanY <- predict(std.fit, d.onesample)

# estimate mean outcome in each of the groups interv=-1, interv=0, and interv=1
return(c(mean(d.onesample$predicted_meanY[d.onesample$interv==-1]),
mean(d.onesample$predicted_meanY[d.onesample$interv==0]),
mean(d.onesample$predicted_meanY[d.onesample$interv==1]),
mean(d.onesample$predicted_meanY[d.onesample$interv==1]) -
mean(d.onesample$predicted_meanY[d.onesample$interv==0])))
}

# bootstrap
results <- boot(data=nhefs, statistic=std.ps, R=5)

# generating confidence intervals
se <- c(sd(results$t[,1]), sd(results$t[,2]),
sd(results$t[,3]), sd(results$t[,4]))
mean <- results$t0
ll <- mean - qnorm(0.975)*se
ul <- mean + qnorm(0.975)*se

bootstrap <- data.frame(cbind(c("Observed", "No Treatment", "Treatment",
" Treatment - No Treatment"), mean, se, ll, ul))
bootstrap

```

```

##           V1           mean           se
## 1      Observed 2.63384609228479 0.139460634517311
## 2    No Treatment 1.71983636149843 0.218464734296926
## 3      Treatment 5.35072300362993 0.632937541379902
## 4 Treatment - No Treatment 3.63088664213151 0.729777367173091
##           ll           ul

```

```
## 1 2.36050827136976 2.90718391319983
## 2 1.29165335038434 2.14801937261251
## 3 4.11018821806199 6.59125778919787
## 4 2.20054928573978 5.06122399852323

# regression on the propensity score (linear term)
model6 <- glm(wt82_71 ~ qsmk + ps, data = nhefs) # p.qsmk
summary(model6)

##
## Call:
## glm(formula = wt82_71 ~ qsmk + ps, data = nhefs)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -43.314  -4.006  -0.068   4.244  47.158
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   5.5945     0.4831  11.581 < 2e-16 ***
## qsmk          3.5506     0.4573   7.765 1.47e-14 ***
## ps          -14.8218     1.7576  -8.433 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 58.28455)
##
##      Null deviance: 97176  on 1565  degrees of freedom
## Residual deviance: 91099  on 1563  degrees of freedom
## (63 observations deleted due to missingness)
## AIC: 10815
##
## Number of Fisher Scoring iterations: 2

# standarization on the propensity score
# (step 1) create two new datasets, one with all treated and one with all untreated
treated <- nhefs
  treated$qsmk <- 1

untreated <- nhefs
  untreated$qsmk <- 0

# (step 2) predict values for everyone in each new dataset based on above model
treated$pred.y <- predict(model6, treated)
untreated$pred.y <- predict(model6, untreated)

# (step 3) compare mean weight loss had all been treated vs. that had all been untreated
mean1 <- mean(treated$pred.y, na.rm = TRUE)
mean0 <- mean(untreated$pred.y, na.rm = TRUE)
mean1
```

```

## [1] 5.250824
mean0

## [1] 1.700228
mean1 - mean0

## [1] 3.550596

# (step 4) bootstrap a confidence interval
# number of bootstraps
nboot <- 100
# set up a matrix to store results
boots <- data.frame(i = 1:nboot,
                    mean1 = NA,
                    mean0 = NA,
                    difference = NA)
# loop to perform the bootstrapping
nhefs <- subset(nhefs, !is.na(ps) & !is.na(wt82_71)) # p.qsmk
for(i in 1:nboot) {
  # sample with replacement
  sampl <- nhefs[sample(1:nrow(nhefs), nrow(nhefs), replace = TRUE), ]

  # fit the model in the bootstrap sample
  bootmod <- glm(wt82_71 ~ qsmk + ps, data = sampl) # ps

  # create new datasets
  sampl.treated <- sampl %>%
    mutate(qsmk = 1)

  sampl.untreated <- sampl %>%
    mutate(qsmk = 0)

  # predict values
  sampl.treated$pred.y <- predict(bootmod, sampl.treated)
  sampl.untreated$pred.y <- predict(bootmod, sampl.untreated)

  # output results
  boots[i, 'mean1'] <- mean(sampl.treated$pred.y, na.rm = TRUE)
  boots[i, 'mean0'] <- mean(sampl.untreated$pred.y, na.rm = TRUE)
  boots[i, 'difference'] <- boots[i, 'mean1'] - boots[i, 'mean0']

  # once loop is done, print the results
  if(i == nboot) {
    cat('95% CI for the causal mean difference\n')
    cat(mean(boots$difference) - 1.96*sd(boots$difference),
        ', ',
        mean(boots$difference) + 1.96*sd(boots$difference))
  }
}

```

```
## 95% CI for the causal mean difference  
## 2.585271 , 4.448191
```

A more flexible and elegant way to do this is to write a function to perform the model fitting, prediction, bootstrapping, and reporting all at once.

16. Instrumental variables estimation

Program 16.1

- Estimating the average causal using the standard IV estimator via the calculation of sample averages
- Data from NHEFS

```
library(here)

#install.packages("readxl") # install package if required
library("readxl")
nhefs <- read_excel(here("data", "NHEFS.xls"))

# some preprocessing of the data
nhefs$cens <- ifelse(is.na(nhefs$wt82), 1, 0)
summary(nhefs$price82)

##      Min. 1st Qu.  Median      Mean 3rd Qu.   Max.    NA's
##      1.452   1.740   1.815   1.806   1.868   2.103      92

# for simplicity, ignore subjects with missing outcome or missing instrument
nhefs.iv <- nhefs[which(!is.na(nhefs$wt82) & !is.na(nhefs$price82)),]
nhefs.iv$highprice <- ifelse(nhefs.iv$price82>=1.5, 1, 0)

table(nhefs.iv$highprice, nhefs.iv$qsmk)

##
##           0      1
##  0      33      8
##  1 1065    370

t.test(wt82_71 ~ highprice, data=nhefs.iv)

##
## Welch Two Sample t-test
##
## data:  wt82_71 by highprice
## t = -0.10179, df = 41.644, p-value = 0.9194
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -3.130588  2.830010
## sample estimates:
```

```
## mean in group 0 mean in group 1
##      2.535729      2.686018
```

Program 16.2

- Estimating the average causal effect using the standard IV estimator via two-stage-least-squares regression
- Data from NHEFS

```
#install.packages("sem") # install package if required
library(sem)

model1 <- tsls(wt82_71 ~ qsmk, ~ highprice, data = nhefs.iv)
summary(model1)

##
## 2SLS Estimates
##
## Model Formula: wt82_71 ~ qsmk
##
## Instruments: ~highprice
##
## Residuals:
##      Min.    1st Qu.    Median      Mean    3rd Qu.      Max.
## -43.34863  -4.00206  -0.02712   0.00000   4.17040  46.47022
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.068164   5.085098  0.40671  0.68428
## qsmk         2.396270  19.840037  0.12078  0.90388
##
## Residual standard error: 7.8561141 on 1474 degrees of freedom

confint(model1) # note the wide confidence intervals

##              2.5 %    97.5 %
## (Intercept) -7.898445 12.03477
## qsmk         -36.489487 41.28203
```

Program 16.3

- Estimating the average causal using the standard IV estimator via additive marginal structural models
- Data from NHEFS
- G-estimation: Checking one possible value of ψ
- See Chapter 14 for program that checks several values and computes 95% confidence intervals

```
nhefs.iv$psi <- 2.396
nhefs.iv$Hpsi <- nhefs.iv$wt82_71 - nhefs.iv$psi * nhefs.iv$qsmk

#install.packages("geepack") # install package if required
library("geepack")
```



```

g.est <- geeglm(highprice ~ Hpsi, data=nhefs.iv, id=seqn, family=binomial(),
               corstr="independence")
summary(g.est)

##
## Call:
## geeglm(formula = highprice ~ Hpsi, family = binomial(), data = dhefs.iv,
##       id = seqn, corstr = "independence")
##
## Coefficients:
##             Estimate Std.err Wald Pr(>|W|)
## (Intercept) 3.555e+00 1.652e-01 463.1  <2e-16 ***
## Hpsi        2.748e-07 2.273e-02  0.0      1
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Estimated Scale Parameters:
##             Estimate Std.err
## (Intercept)      1  0.7607
##
## Correlation: Structure = independenceNumber of clusters:   1476   Maximum cluster size: 1

beta <- coef(g.est)
SE <- coef(summary(g.est))[,2]
lcl <- beta-qnrm(0.975)*SE
ucl <- beta+qnrm(0.975)*SE
cbind(beta, lcl, ucl)

##             beta      lcl      ucl
## (Intercept) 3.555e+00  3.23152 3.87917
## Hpsi        2.748e-07 -0.04456 0.04456

```

Program 16.4

- Estimating the average causal using the standard IV estimator with alternative proposed instruments
- Data from NHEFS

```

summary(tsls(wt82_71 ~ qsmk, ~ ifelse(price82 >= 1.6, 1, 0), data = dhefs.iv))

##
## 2SLS Estimates
##
## Model Formula: wt82_71 ~ qsmk
##
## Instruments: ~ifelse(price82 >= 1.6, 1, 0)
##
## Residuals:
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    -55.6   -13.5     7.6     0.0    12.5    56.4
##

```

```
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -7.89      42.25  -0.187   0.852
## qsmk          41.28     164.95   0.250   0.802
##
## Residual standard error: 18.6055 on 1474 degrees of freedom
```

```
summary(tsls(wt82_71 ~ qsmk, ~ ifelse(price82 >= 1.7, 1, 0), data = nhefs.iv))
```

```
##
## 2SLS Estimates
##
## Model Formula: wt82_71 ~ qsmk
##
## Instruments: ~ifelse(price82 >= 1.7, 1, 0)
##
## Residuals:
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   -54.4  -13.4   -8.4     0.0   18.1   75.3
##
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept)    13.16     48.08   0.274   0.784
## qsmk          -40.91    187.74  -0.218   0.828
##
## Residual standard error: 20.591 on 1474 degrees of freedom
```

```
summary(tsls(wt82_71 ~ qsmk, ~ ifelse(price82 >= 1.8, 1, 0), data = nhefs.iv))
```

```
##
## 2SLS Estimates
##
## Model Formula: wt82_71 ~ qsmk
##
## Instruments: ~ifelse(price82 >= 1.8, 1, 0)
##
## Residuals:
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   -49.37  -8.31   -3.44    0.00   7.27   60.53
##
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept)     8.086     7.288   1.110   0.267
## qsmk          -21.103    28.428  -0.742   0.458
##
## Residual standard error: 13.0188 on 1474 degrees of freedom
```

```
summary(tsls(wt82_71 ~ qsmk, ~ ifelse(price82 >= 1.9, 1, 0), data = nhefs.iv))
```

```
##
## 2SLS Estimates
##
## Model Formula: wt82_71 ~ qsmk
##
```

```
## Instruments: ~ifelse(price82 >= 1.9, 1, 0)
##
## Residuals:
##      Min. 1st Qu.  Median      Mean 3rd Qu.     Max.
## -47.24   -6.33    -1.43     0.00    5.52   54.36
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    5.963      6.067   0.983   0.326
## qsmk          -12.811     23.667  -0.541   0.588
##
## Residual standard error: 10.3637 on 1474 degrees of freedom
```

Program 16.5

- Estimating the average causal using the standard IV estimator
- Conditional on baseline covariates
- Data from NHEFS

```
model2 <- tsls(wt82_71 ~ qsmk + sex + race + age + smokeintensity + smokeyrs +
                as.factor(exercise) + as.factor(active) + wt71,
                ~ highprice + sex + race + age + smokeintensity + smokeyrs + as.factor(exercise) +
                as.factor(active) + wt71, data = nhefs.iv)
summary(model2)

##
## 2SLS Estimates
##
## Model Formula: wt82_71 ~ qsmk + sex + race + age + smokeintensity + smokeyrs +
##               as.factor(exercise) + as.factor(active) + wt71
##
## Instruments: ~highprice + sex + race + age + smokeintensity + smokeyrs + as.factor(exercise) +
##               as.factor(active) + wt71
##
## Residuals:
##      Min. 1st Qu.  Median      Mean 3rd Qu.     Max.
## -42.23   -4.29    -0.62     0.00    3.87   46.74
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    17.280330   2.335402   7.399 2.3e-13 ***
## qsmk           -1.042295  29.987369  -0.035  0.9723
## sex            -1.644393   2.630831  -0.625  0.5320
## race           -0.183255   4.650386  -0.039  0.9686
## age            -0.163640   0.240548  -0.680  0.4964
## smokeintensity   0.005767   0.145504   0.040  0.9684
## smokeyrs         0.025836   0.161421   0.160  0.8729
## as.factor(exercise)1  0.498748   2.171239   0.230  0.8184
## as.factor(exercise)2  0.581834   2.183148   0.267  0.7899
## as.factor(active)1   -1.170145   0.607466  -1.926  0.0543 .
## as.factor(active)2   -0.512284   1.308451  -0.392  0.6955
```

```
## wt71          -0.097949   0.036271  -2.701   0.0070 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.7162 on 1464 degrees of freedom
```

17. Causal survival analysis

Program 17.1

- Nonparametric estimation of survival curves
- Data from NHEFS

```
library(here)

library("readxl")
nhefs <- read_excel(here("data", "NHEFS.xls"))

# some preprocessing of the data
nhefs$survtime <- ifelse(nhefs$death==0, 120,
                        (nhefs$yrdrth-83)*12+nhefs$modth) # yrdrth ranges from 83 to 92

table(nhefs$death, nhefs$qsmk)

##
##      0    1
## 0 985 326
## 1 216 102

summary(nhefs[which(nhefs$death==1),]$survtime)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      1.00   35.00   61.00   61.14   86.75  120.00

#install.packages("survival")
#install.packages("ggplot2") # for plots
#install.packages("survminer") # for plots
library("survival")
library("ggplot2")
library("survminer")

## Loading required package: ggpubr

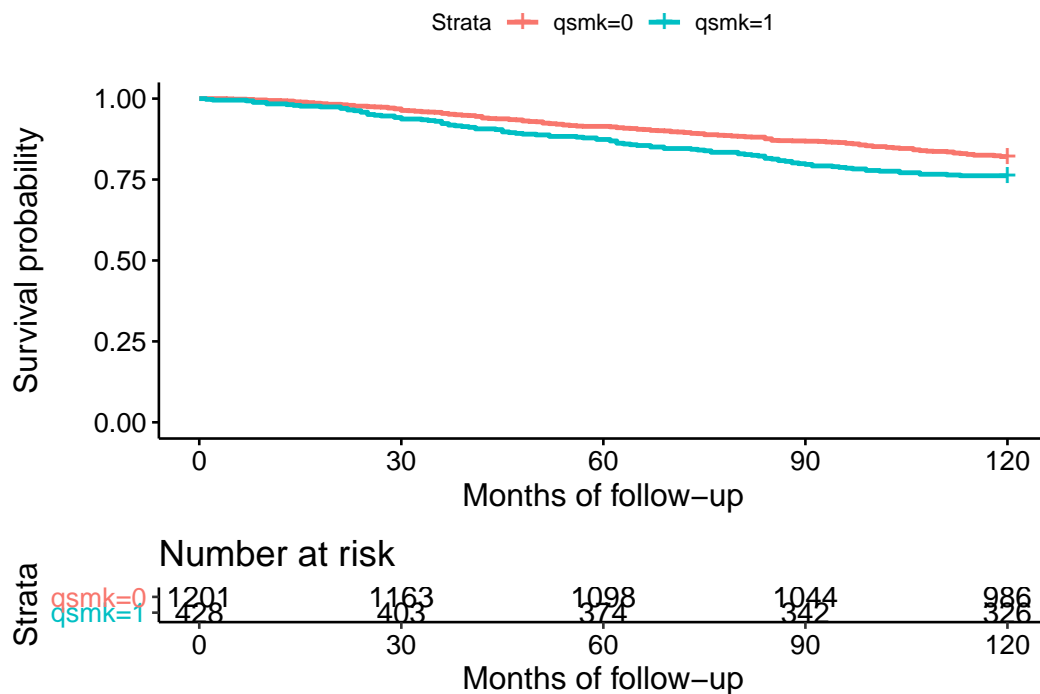
## Loading required package: magrittr

survdif(Surv(survtime, death) ~ qsmk, data=nhefs)

## Call:
## survdif(formula = Surv(survtime, death) ~ qsmk, data = nhefs)
```

```
##
##           N Observed Expected (O-E)^2/E (O-E)^2/V
## qsmk=0 1201      216   237.5      1.95      7.73
## qsmk=1  428      102    80.5      5.76      7.73
##
##  Chisq= 7.7  on 1 degrees of freedom, p= 0.005

fit <- survfit(Surv(survtime, death) ~ qsmk, data=nhefs)
ggsurvplot(fit, data = dhefs, xlab="Months of follow-up",
            ylab="Survival probability",
            main="Product-Limit Survival Estimates", risk.table = TRUE)
```



Program 17.2

- Parametric estimation of survival curves via hazards model
- Data from NHEFS

```
# creation of person-month data
#install.packages("splitstackshape")
library("splitstackshape")
nhefs.surv <- expandRows(nhefs, "survtime", drop=F)
nhefs.surv$time <- sequence(rle(nhefs.surv$seqn)$lengths)-1
nhefs.surv$event <- ifelse(nhefs.surv$time==nhefs.surv$survtime-1 &
                           nhefs.surv$death==1, 1, 0)
nhefs.surv$timesq <- nhefs.surv$time^2

# fit of parametric hazards model
hazards.model <- glm(event==0 ~ qsmk + I(qsmk*time) + I(qsmk*timesq) +
```

```

time + timesq, family=binomial(), data=nhefs.surv)
summary(hazards.model)

```

```

##
## Call:
## glm(formula = event == 0 ~ qsmk + I(qsmk * time) + I(qsmk * timesq) +
##     time + timesq, family = binomial(), data = dhefs.surv)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -3.7253   0.0546   0.0601   0.0625   0.0783
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    6.996e+00  2.309e-01  30.292  <2e-16 ***
## qsmk           -3.355e-01  3.970e-01  -0.845   0.3981
## I(qsmk * time) -1.208e-02  1.503e-02  -0.804   0.4215
## I(qsmk * timesq) 1.612e-04  1.246e-04   1.293   0.1960
## time           -1.960e-02  8.413e-03  -2.329   0.0198 *
## timesq          1.256e-04  6.686e-05   1.878   0.0604 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 4655.3  on 176763  degrees of freedom
## Residual deviance: 4631.3  on 176758  degrees of freedom
## AIC: 4643.3
##
## Number of Fisher Scoring iterations: 9

```

```

# creation of dataset with all time points under each treatment level
qsmk0 <- data.frame(cbind(seq(0, 119),0,(seq(0, 119))^2))
qsmk1 <- data.frame(cbind(seq(0, 119),1,(seq(0, 119))^2))

colnames(qsmk0) <- c("time", "qsmk", "timesq")
colnames(qsmk1) <- c("time", "qsmk", "timesq")

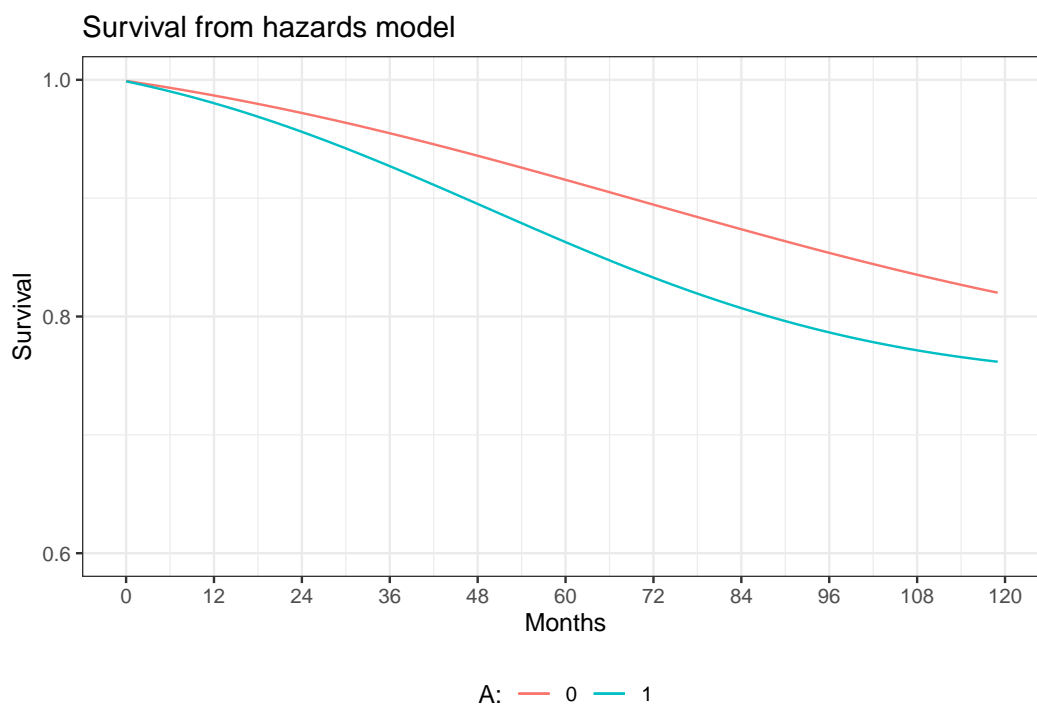
# assignment of estimated (1-hazard) to each person-month */
qsmk0$p.noevent0 <- predict(hazards.model, qsmk0, type="response")
qsmk1$p.noevent1 <- predict(hazards.model, qsmk1, type="response")

# computation of survival for each person-month
qsmk0$urv0 <- cumprod(qsmk0$p.noevent0)
qsmk1$urv1 <- cumprod(qsmk1$p.noevent1)

# some data management to plot estimated survival curves
hazards.graph <- merge(qsmk0, qsmk1, by=c("time", "timesq"))
hazards.graph$survdiff <- hazards.graph$urv1-hazards.graph$urv0

```

```
# plot
ggplot(hazards.graph, aes(x=time, y=surv)) +
  geom_line(aes(y = surv0, colour = "0")) +
  geom_line(aes(y = surv1, colour = "1")) +
  xlab("Months") +
  scale_x_continuous(limits = c(0, 120), breaks=seq(0,120,12)) +
  scale_y_continuous(limits=c(0.6, 1), breaks=seq(0.6, 1, 0.2)) +
  ylab("Survival") +
  ggtitle("Survival from hazards model") +
  labs(colour="A:") +
  theme_bw() +
  theme(legend.position="bottom")
```



Program 17.3

- Estimation of survival curves via IP weighted hazards model
- Data from NHEFS

```
# estimation of denominator of ip weights
p.denom <- glm(qsmk ~ sex + race + age + I(age*age) + as.factor(education)
              + smokeintensity + I(smokeintensity*smokeintensity)
              + smokeyrs + I(smokeyrs*smokeyrs) + as.factor(exercise)
              + as.factor(active) + wt71 + I(wt71*wt71),
              data=nhefs, family=binomial())
nhefs$pd.qsmk <- predict(p.denom, nhefs, type="response")

# estimation of numerator of ip weights
```



```

p.num <- glm(qsmk ~ 1, data=nhefs, family=binomial())
nhefs$pn.qsmk <- predict(p.num, nhefs, type="response")

# computation of estimated weights
nhefs$sw.a <- ifelse(nhefs$qsmk==1, nhefs$pn.qsmk/nhefs$pd.qsmk,
                    (1-nhefs$pn.qsmk)/(1-nhefs$pd.qsmk))
summary(nhefs$sw.a)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.3312 0.8640 0.9504 0.9991 1.0755 4.2054

# creation of person-month data
nhefs.ipw <- expandRows(nhefs, "survtime", drop=F)
nhefs.ipw$time <- sequence(rle(nhefs.ipw$seqn)$lengths)-1
nhefs.ipw$event <- ifelse(nhefs.ipw$time==nhefs.ipw$survtime-1 &
                          nhefs.ipw$death==1, 1, 0)
nhefs.ipw$timesq <- nhefs.ipw$time^2

# fit of weighted hazards model
ipw.model <- glm(event==0 ~ qsmk + I(qsmk*time) + I(qsmk*timesq) +
                  time + timesq, family=binomial(), weight=sw.a,
                  data=nhefs.ipw)

## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!

summary(ipw.model)

##
## Call:
## glm(formula = event == 0 ~ qsmk + I(qsmk * time) + I(qsmk * timesq) +
##      time + timesq, family = binomial(), data = nhefs.ipw, weights = sw.a)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -7.1859   0.0528   0.0595   0.0640   0.1452
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)    6.897e+00  2.208e-01  31.242  <2e-16 ***
## qsmk           1.794e-01  4.399e-01   0.408  0.6834
## I(qsmk * time) -1.895e-02  1.640e-02  -1.155  0.2481
## I(qsmk * timesq) 2.103e-04  1.352e-04   1.556  0.1198
## time          -1.889e-02  8.053e-03  -2.345  0.0190 *
## timesq         1.181e-04  6.399e-05   1.846  0.0649 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##

```

```
##      Null deviance: 4643.9  on 176763  degrees of freedom
## Residual deviance: 4626.2  on 176758  degrees of freedom
## AIC: 4633.5
##
## Number of Fisher Scoring iterations: 9

# creation of survival curves
ipw.qsmk0 <- data.frame(cbind(seq(0, 119),0,(seq(0, 119))^2))
ipw.qsmk1 <- data.frame(cbind(seq(0, 119),1,(seq(0, 119))^2))

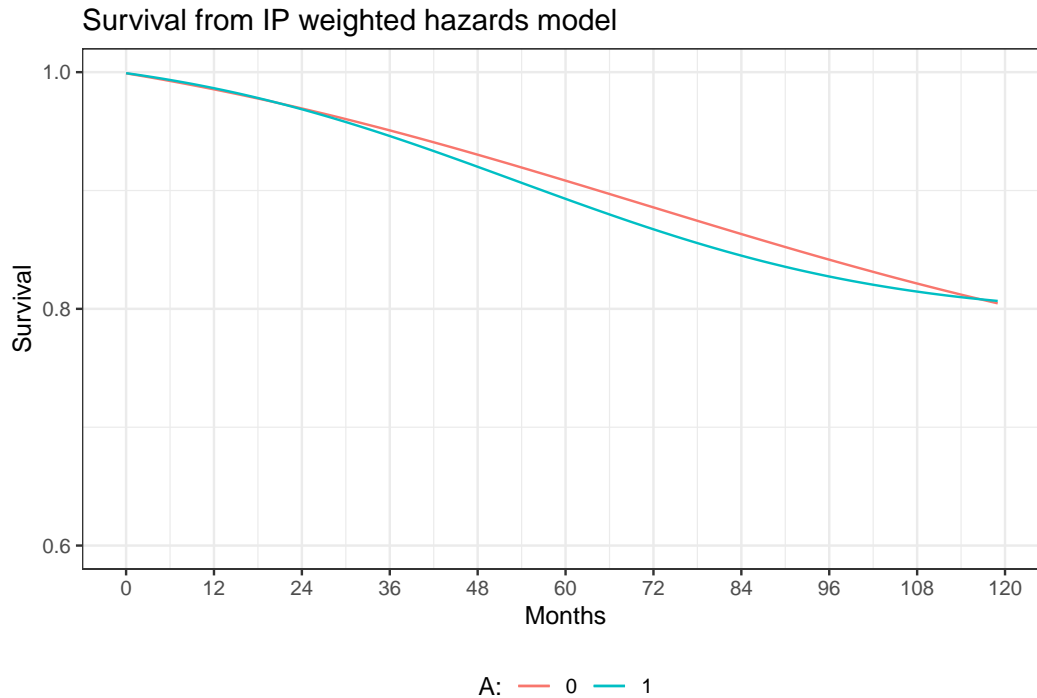
colnames(ipw.qsmk0) <- c("time", "qsmk", "timesq")
colnames(ipw.qsmk1) <- c("time", "qsmk", "timesq")

# assignment of estimated (1-hazard) to each person-month */
ipw.qsmk0$p.noevent0 <- predict(ipw.model, ipw.qsmk0, type="response")
ipw.qsmk1$p.noevent1 <- predict(ipw.model, ipw.qsmk1, type="response")

# computation of survival for each person-month
ipw.qsmk0$urv0 <- cumprod(ipw.qsmk0$p.noevent0)
ipw.qsmk1$urv1 <- cumprod(ipw.qsmk1$p.noevent1)

# some data management to plot estimated survival curves
ipw.graph <- merge(ipw.qsmk0, ipw.qsmk1, by=c("time", "timesq"))
ipw.graph$survdif <- ipw.graph$urv1-ipw.graph$urv0

# plot
ggplot(ipw.graph, aes(x=time, y=surv)) +
  geom_line(aes(y = surv0, colour = "0")) +
  geom_line(aes(y = surv1, colour = "1")) +
  xlab("Months") +
  scale_x_continuous(limits = c(0, 120), breaks=seq(0,120,12)) +
  scale_y_continuous(limits=c(0.6, 1), breaks=seq(0.6, 1, 0.2)) +
  ylab("Survival") +
  ggtitle("Survival from IP weighted hazards model") +
  labs(colour="A:") +
  theme_bw() +
  theme(legend.position="bottom")
```



Program 17.4

- Estimating of survival curves via g-formula
- Data from NHEFS

```
# fit of hazards model with covariates
gf.model <- glm(event==0 ~ qsmk + I(qsmk*time) + I(qsmk*timesq)
               + time + timesq + sex + race + age + I(age*age)
               + as.factor(education) + smokeintensity
               + I(smokeintensity*smokeintensity) + smkintensity82_71
               + smokeyrs + I(smokeyrs*smokeyrs) + as.factor(exercise)
               + as.factor(active) + wt71 + I(wt71*wt71),
               data=nhefs.surv, family=binomial())
summary(gf.model)

##
## Call:
## glm(formula = event == 0 ~ qsmk + I(qsmk * time) + I(qsmk * timesq) +
##      time + timesq + sex + race + age + I(age * age) + as.factor(education) +
##      smokeintensity + I(smokeintensity * smokeintensity) + smkintensity82_71 +
##      smokeyrs + I(smokeyrs * smokeyrs) + as.factor(exercise) +
##      as.factor(active) + wt71 + I(wt71 * wt71), family = binomial(),
##      data = dhefs.surv)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -4.3160   0.0244   0.0395   0.0640   0.3303
```

```

##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)      9.272e+00  1.379e+00   6.724 1.76e-11
## qsmk              5.959e-02  4.154e-01   0.143 0.885924
## I(qsmk * time)    -1.485e-02  1.506e-02  -0.987 0.323824
## I(qsmk * timesq)   1.702e-04  1.245e-04   1.367 0.171643
## time             -2.270e-02  8.437e-03  -2.690 0.007142
## timesq            1.174e-04  6.709e-05   1.751 0.080020
## sex               4.368e-01  1.409e-01   3.101 0.001930
## race             -5.240e-02  1.734e-01  -0.302 0.762572
## age              -8.750e-02  5.907e-02  -1.481 0.138536
## I(age * age)       8.128e-05  5.470e-04   0.149 0.881865
## as.factor(education)2 1.401e-01  1.566e-01   0.895 0.370980
## as.factor(education)3 4.335e-01  1.526e-01   2.841 0.004502
## as.factor(education)4 2.350e-01  2.790e-01   0.842 0.399750
## as.factor(education)5 3.750e-01  2.386e-01   1.571 0.116115
## smokeintensity     -1.626e-03  1.430e-02  -0.114 0.909431
## I(smokeintensity * smokeintensity) -7.182e-05  2.390e-04  -0.301 0.763741
## smkintensity82_71    -1.686e-03  6.501e-03  -0.259 0.795399
## smokeyrs           -1.677e-02  3.065e-02  -0.547 0.584153
## I(smokeyrs * smokeyrs) -5.280e-05  4.244e-04  -0.124 0.900997
## as.factor(exercise)1  1.469e-01  1.792e-01   0.820 0.412300
## as.factor(exercise)2 -1.504e-01  1.762e-01  -0.854 0.393177
## as.factor(active)1    -1.601e-01  1.300e-01  -1.232 0.218048
## as.factor(active)2    -2.294e-01  1.877e-01  -1.222 0.221766
## wt71                 6.222e-02  1.902e-02   3.271 0.001073
## I(wt71 * wt71)       -4.046e-04  1.129e-04  -3.584 0.000338
##
## (Intercept)      ***
## qsmk
## I(qsmk * time)
## I(qsmk * timesq)
## time            **
## timesq           .
## sex              **
## race
## age
## I(age * age)
## as.factor(education)2
## as.factor(education)3      **
## as.factor(education)4
## as.factor(education)5
## smokeintensity
## I(smokeintensity * smokeintensity)
## smkintensity82_71
## smokeyrs
## I(smokeyrs * smokeyrs)
## as.factor(exercise)1

```

```

## as.factor(exercise)2
## as.factor(active)1
## as.factor(active)2
## wt71 **
## I(wt71 * wt71) ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 4655.3 on 176763 degrees of freedom
## Residual deviance: 4185.7 on 176739 degrees of freedom
## AIC: 4235.7
##
## Number of Fisher Scoring iterations: 10
# creation of dataset with all time points for
# each individual under each treatment level
gf.qsmk0 <- expandRows(nhefs, count=120, count.is.col=F)
gf.qsmk0$time <- rep(seq(0, 119), nrow(nhefs))
gf.qsmk0$timesq <- gf.qsmk0$time^2
gf.qsmk0$qsmk <- 0

gf.qsmk1 <- gf.qsmk0
gf.qsmk1$qsmk <- 1

gf.qsmk0$p.noevent0 <- predict(gf.model, gf.qsmk0, type="response")
gf.qsmk1$p.noevent1 <- predict(gf.model, gf.qsmk1, type="response")

#install.packages("dplyr")
library("dplyr")

##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
## filter, lag

## The following objects are masked from 'package:base':
##
## intersect, setdiff, setequal, union

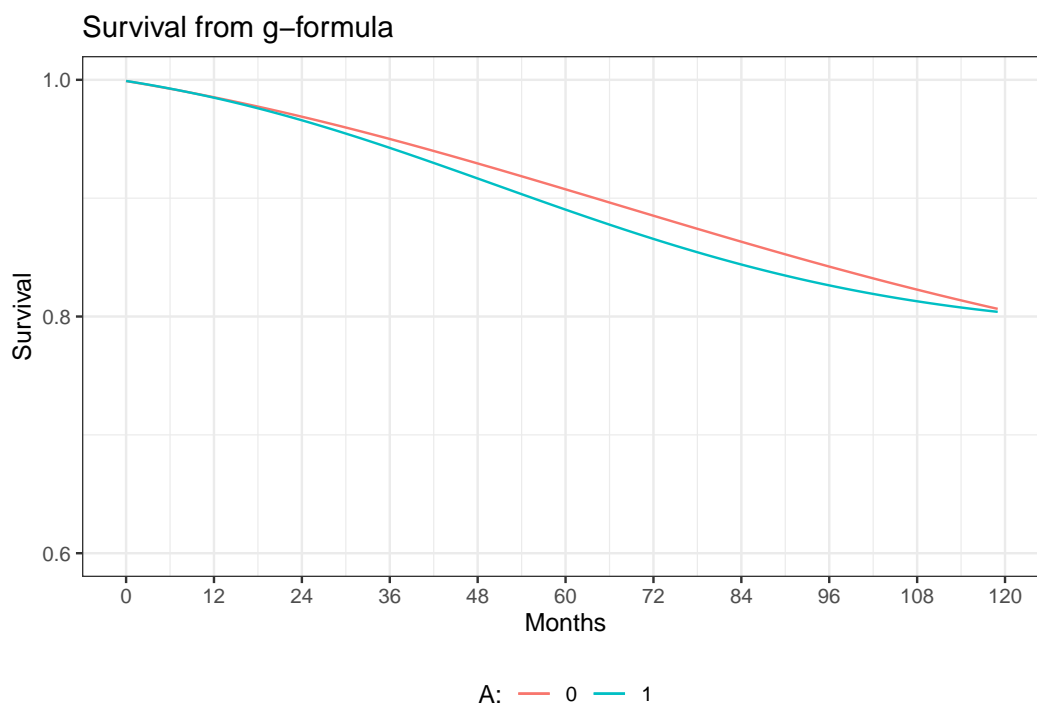
gf.qsmk0.surv <- gf.qsmk0 %>% group_by(seqn) %>% mutate(surv0 = cumprod(p.noevent0))
gf.qsmk1.surv <- gf.qsmk1 %>% group_by(seqn) %>% mutate(surv1 = cumprod(p.noevent1))

gf.surv0 <- aggregate(gf.qsmk0.surv, by=list(gf.qsmk0.surv$time), FUN=mean)[c("qsmk", "time", "surv0")]
gf.surv1 <- aggregate(gf.qsmk1.surv, by=list(gf.qsmk1.surv$time), FUN=mean)[c("qsmk", "time", "surv1")]

gf.graph <- merge(gf.surv0, gf.surv1, by=c("time"))
gf.graph$survdif <- gf.graph$surv1-gf.graph$surv0

```

```
# plot
ggplot(gf.graph, aes(x=time, y=surv)) +
  geom_line(aes(y = surv0, colour = "0")) +
  geom_line(aes(y = surv1, colour = "1")) +
  xlab("Months") +
  scale_x_continuous(limits = c(0, 120), breaks=seq(0,120,12)) +
  scale_y_continuous(limits=c(0.6, 1), breaks=seq(0.6, 1, 0.2)) +
  ylab("Survival") +
  ggtitle("Survival from g-formula") +
  labs(colour="A:") +
  theme_bw() +
  theme(legend.position="bottom")
```



Program 17.5

- Estimating of median survival time ratio via a structural nested AFT model
- Data from NHEFS

```
# some preprocessing of the data
nhefs <- read_excel(here("data", "NHEFS.xls"))
nhefs$survtime <- ifelse(nhefs$death==0, NA, (nhefs$yrdth-83)*12+nhefs$modth) # * yrdth ranges from 83

# model to estimate E[A/L]
modelA <- glm(qsmk ~ sex + race + age + I(age*age)
  + as.factor(education) + smokeintensity
  + I(smokeintensity*smokeintensity) + smokeyrs
  + I(smokeyrs*smokeyrs) + as.factor(exercise))
```

```

      + as.factor(active) + wt71 + I(wt71*wt71),
      data=nhefs, family=binomial())

nhefs$p.qsmk <- predict(modelA, nhefs, type="response")
d <- nhefs[!is.na(nhefs$survtime),] # select only those with observed death time
n <- nrow(d)

# define the estimating function that needs to be minimized
sumeef <- function(psi){

  # creation of delta indicator
  if (psi>=0){
    delta <- ifelse(d$qsmk==0 |
                    (d$qsmk==1 & psi <= log(120/d$survtime)),
                    1, 0)
  } else if (psi < 0) {
    delta <- ifelse(d$qsmk==1 |
                    (d$qsmk==0 & psi > log(d$survtime/120)), 1, 0)
  }

  smat <- delta*(d$qsmk-d$p.qsmk)
  sval <- sum(smat, na.rm=T)
  save <- sval/n
  smat <- smat - rep(save, n)

  # covariance
  sigma <- t(smat) %*% smat
  if (sigma == 0){
    sigma <- 1e-16
  }
  estimateq <- sval*solve(sigma)*t(sval)
  return(estimateq)
}

res <- optimize(sumeef, interval = c(-0.2,0.2))
psi1 <- res$minimum
objfunc <- as.numeric(res$objective)

# Use simple bisection method to find estimates of lower and upper 95% confidence bounds
incred <- 0.1
for_conf <- function(x){
  return(sumeef(x) - 3.84)
}

if (objfunc < 3.84){
  # Find estimate of where sumeef(x) > 3.84

```

```

# Lower bound of 95% CI
psilow <- psi1
testlow <- objfunc
countlow <- 0
while (testlow < 3.84 & countlow < 100){
  psilow <- psilow - increm
  testlow <- sumeef(psilow)
  countlow <- countlow + 1
}

# Upper bound of 95% CI
psihigh <- psi1
testhigh <- objfunc
counthigh <- 0
while (testhigh < 3.84 & counthigh < 100){
  psihigh <- psihigh + increm
  testhigh <- sumeef(psihigh)
  counthigh <- counthigh + 1
}

# Better estimate using bisection method
if ((testhigh > 3.84) & (testlow > 3.84)){

  # Bisection method
  left <- psi1
  fleft <- objfunc - 3.84
  right <- psihigh
  fright <- testhigh - 3.84
  middle <- (left + right) / 2
  fmiddle <- for_conf(middle)
  count <- 0
  diff <- right - left

  while (!(abs(fmiddle) < 0.0001 | diff < 0.0001 | count > 100)){
    test <- fmiddle * fleft
    if (test < 0){
      right <- middle
      fright <- fmiddle
    } else {
      left <- middle
      fleft <- fmiddle
    }
    middle <- (left + right) / 2
    fmiddle <- for_conf(middle)
    count <- count + 1
    diff <- right - left
  }
}

```



```

psi_high <- middle
objfunc_high <- fmiddle + 3.84

# lower bound of 95% CI
left <- psilow
fleft <- testlow - 3.84
right <- psi1
fright <- objfunc - 3.84
middle <- (left + right) / 2
fmiddle <- for_conf(middle)
count <- 0
diff <- right - left

while(!(abs(fmiddle) < 0.0001 | diff < 0.0001 | count > 100)){
  test <- fmiddle * fleft
  if (test < 0){
    right <- middle
    fright <- fmiddle
  } else {
    left <- middle
    fleft <- fmiddle
  }
  middle <- (left + right) / 2
  fmiddle <- for_conf(middle)
  diff <- right - left
  count <- count + 1
}
psi_low <- middle
objfunc_low <- fmiddle + 3.84
psi <- psi1
}
c(psi, psi_low, psi_high)

```

```
## [1] -0.05041591 -0.22312099 0.33312901
```


R session information

For reproducibility.

```
# install.packages("sessioninfo")
sessioninfo::session_info()
```

```
## - Session info -----
## setting value
## version R version 3.6.1 (2019-07-05)
## os      Windows 10 x64
## system  x86_64, mingw32
## ui      RTerm
## language (EN)
## collate English_United Kingdom.1252
## ctype   English_United Kingdom.1252
## tz      Europe/London
## date    2019-10-01
##
## - Packages -----
## package      * version date      lib source
## assertthat   0.2.1   2019-03-21 [1] CRAN (R 3.6.0)
## bookdown     0.13    2019-08-21 [1] CRAN (R 3.6.1)
## cli          1.1.0   2019-03-19 [1] CRAN (R 3.6.0)
## crayon       1.3.4   2017-09-16 [1] CRAN (R 3.6.0)
## digest       0.6.21  2019-09-20 [1] CRAN (R 3.6.1)
## evaluate     0.14    2019-05-28 [1] CRAN (R 3.6.0)
## htmltools    0.3.6   2017-04-28 [1] CRAN (R 3.6.0)
## knitr        1.25    2019-09-18 [1] CRAN (R 3.6.1)
## magrittr     1.5     2014-11-22 [1] CRAN (R 3.6.0)
## Rcpp         1.0.2   2019-07-25 [1] CRAN (R 3.6.1)
## rmarkdown    1.15    2019-08-21 [1] CRAN (R 3.6.1)
## sessioninfo  1.1.1   2018-11-05 [1] CRAN (R 3.6.0)
## stringi      1.4.3   2019-03-12 [1] CRAN (R 3.6.0)
## stringr      1.4.0   2019-02-10 [1] CRAN (R 3.6.0)
## withr        2.1.2   2018-03-15 [1] CRAN (R 3.6.0)
## xfun         0.9     2019-08-21 [1] CRAN (R 3.6.1)
## yaml         2.2.0   2018-07-25 [1] CRAN (R 3.6.0)
##
## [1] C:/Users/palmertm/library
## [2] C:/Program Files/R/R-3.6.1/library
```

