



# IMAGE MANIPULATION WITH AFFINE MATRIX

Homework #1

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# Image Manipulation with Affine Matrix

## 1. What is the Affine Matrix and Image Manipulation Methods

This homework assignment includes 4 operations these scale, zoom, shear, and rotate. We will do this operations with using 3 approach these forward mapping, backward mapping, and backward mapping by using bilinear interpolation. We will compare each operations result according to the 3 methods.

For each operation I will share original image and different 3 result of the solving methods. Firstly and shortly, I want to explain what is the forward mapping, backward mapping, and interpolation. And also explain how to work their logic. After that we can compare each operation with our methods.

Affine Matrix, is used for manipulate image doing calculate each pixel new position. It is 2x3 matrix and their each values have different effect on the image. It is useful because our coordinate change linearly after calculation.

Forward Mapping, is direct operation of finding new image pixel coordinate. So we **multiply** affine matrix with source image coordinate and it result give us new coordinate of the new image.

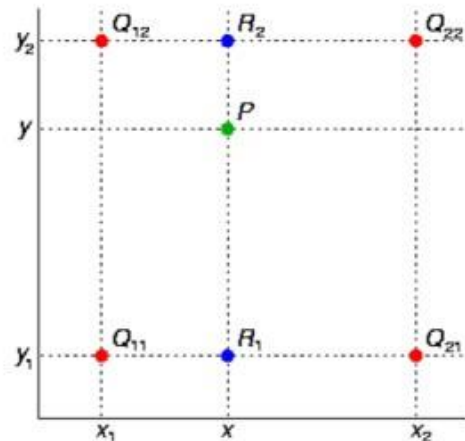
Backward Mapping, is inverse of the forward mapping. Backward Mapping, try to find of each coordinate of the new image, it is **inverse of the affine matrix** operation. I will be explain in the example of operations. We make inverse operation because of that  $(\text{AffineMatrix}^{-1} * \text{tagetImage} = \text{sourcelImage} * \text{AffineMatrix} * \text{AffineMatrix}^{-1})$  with this approach we gain target result coordinate.

Bilinear interpolation is kind of interpolation method. With this method, we avoid bluer image result that comes backward mapping. Idea is not getting target coordinate as a backward mapping, instead taking average of adjacent pixel. Here are bilinear interpolation formula.

Figure 1: Bilinear Interpolation

# Bilinear interpolation

$$\begin{aligned}
 f(x, y) \approx & \frac{f(Q_{11})}{(x_2 - x_1)(y_2 - y_1)}(x_2 - x)(y_2 - y) \\
 & + \frac{f(Q_{21})}{(x_2 - x_1)(y_2 - y_1)}(x - x_1)(y_2 - y) \\
 & + \frac{f(Q_{12})}{(x_2 - x_1)(y_2 - y_1)}(x_2 - x)(y - y_1) \\
 & + \frac{f(Q_{22})}{(x_2 - x_1)(y_2 - y_1)}(x - x_1)(y - y_1).
 \end{aligned}$$



[http://en.wikipedia.org/wiki/Bilinear\\_interpolation](http://en.wikipedia.org/wiki/Bilinear_interpolation)

Meanwhile, I wanna show you difference between bilinear interpolation and without using bilinear interpolation. We get smooth pixel if we use bilinear interpolation. But you should zoom the image. Figure 2 and Figure 3 show that different.

Figure 2: Backward with interpolation



Figure 3: Backward without interpolation



We manipulate image that given below(Figure 3).

Figure 4: Original image



## 2. Zoom Operation



Zoom operation actually is a scale operation but new image size is same as a original image size. Our method will be change but affine matrix values are same.

For doing this operation our affine matrix operation values are:

$a = 1.4, b = 0, tx = 1;$

$c = 0, d = 1.4, ty = 1;$

### 2.1. Forward Mapping Method:

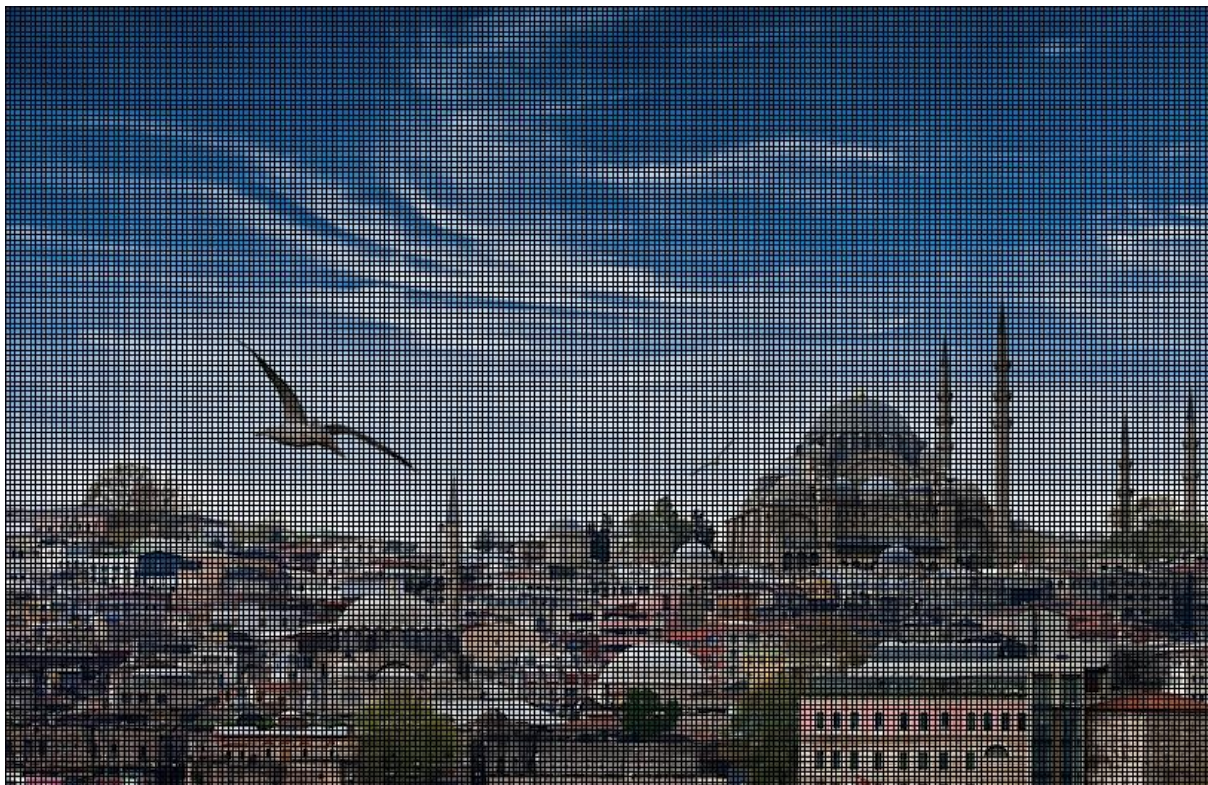
Forward Operation get original image y and x values and multiply with d and a value. After gain these result, I will get newY and newX positions. Forward mapping result have a lot blocks inside the image because of some of (y,x) coordinate lost when multiply each pixel coordinate.

Here are some explanation about how to find new position of each pixel as a psudeu code;

$newImageY = a * y;$

$newImageX = d * x;$

Figure 5: Zoom operation with Forward Mapping



### 2.2. Backward Mapping Method:

When we use backward mapping, method don't create squares as forward mapping. Because of we don't use directly multiply source coordinate with our affine matrix. We want to investigate each of pixel of new image.

Therefore we use same multiply logic but with different approach. Here about calculation;

If  $(newImageY = a * y) \Rightarrow (y = newImageY / a)$ . With this approach, we search for each pixel of new image. Of course some result don't come as a integer, therefore we round it.

Figure 6: Zoom operation with Backward Mapping



### 2.3. Backward Mapping with Bilinear Interpolation:



Actually, we get a better result with backward mapping, instead of using forward mapping. But the image has some blurring and in detail some pixel escape. For solving this issue, we use bilinear interpolation. This method calculates adjacent coordinates and we get a better result. To see the difference between without using bilinear interpolation, you should zoom the image. You will find smooth pixels.

I also share how to do the calculation at the top of the page with an image.

Figure 7: Zoom operation with Backward Mapping with Bilinear Interpolation



### 3. Scale Operation



Our second operation is scale operation. It is different from zoom operation because we should change new image size after scaling. Otherwise we lose some part of image. But of course our affine matrix same with zoom affine matrix.

Here are our affine matrix is same with zoom affine matrix;

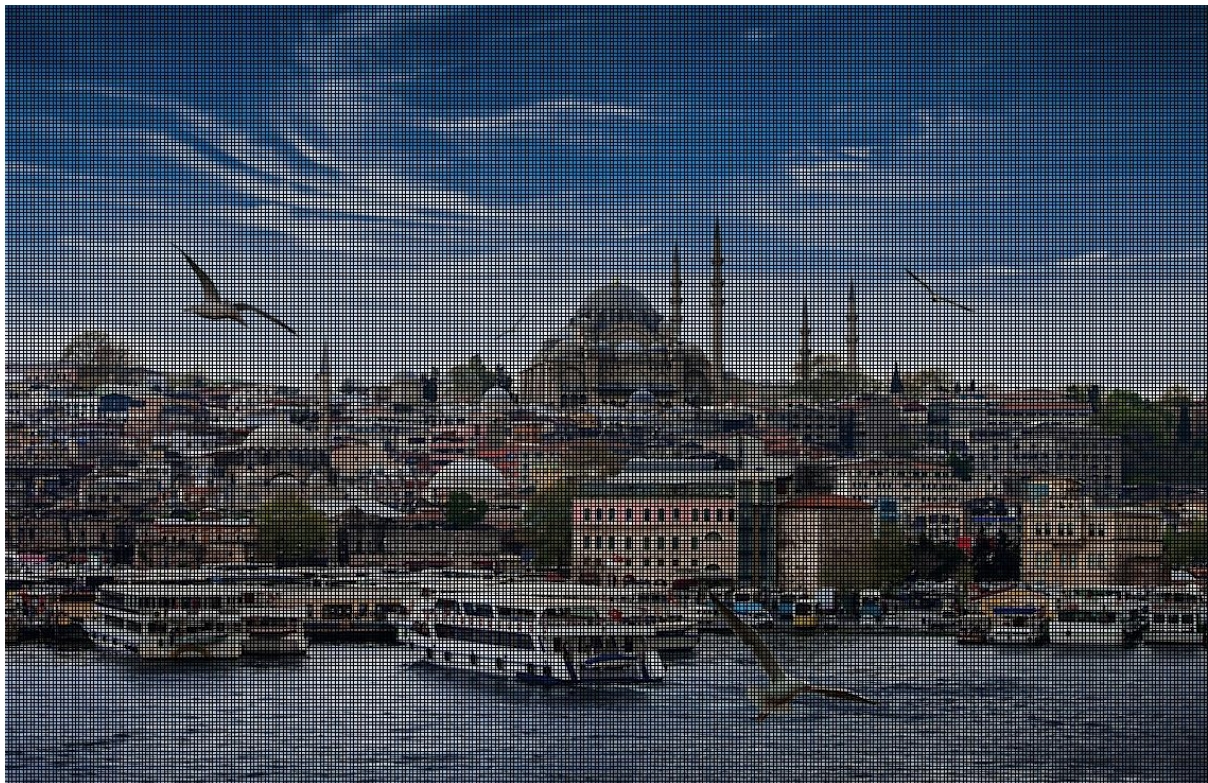
$a = 1.4, b = 1, tx = 0;$

$c = 0, d = 1.4, ty = 0;$

### 3.1. Forward Mapping Method:

We get full size of picture with little square blocks. Because same strategy applies to image and we lost some pixel coordinate of the new image size.

Figure 8: Scale Operation with forward mapping



### 3.2. Backward Mapping Method:

We get better result according to forward mapping and bluer look like less than also zoom image. Because we don't see details. We don't use interpolation and we look image far away. Therefore we cannot see approximated pixel filling.

Figure 9: Scale Operation with backwar mapping



### 3.3. Backward Mapping with Bilinear Interpolation Method:

When we use interpolation with backward Mapping method, we can see clear image. Also if we want to look detail of the new image, we will see a smooth slice of pixels instead of copying before coordinate.

Figure 10: Scale Operation with backward mapping with bilinear interpolation





#### **4. Horizontal Shear Operation**

Our third operation is shear. Horizontal shear means iterate pixels through x position.

For shear operation we are manipulate b and d values of the affine matrix. When increase we b value of the affine matrix our image bottom goes right. Here are our affine matrix;

$a = 1, b = 1.4, t_x = 1;$

$c = 0, d = 1, t_y = 1;$

#### 4.1. Forward Mapping Method

Unlike of the other operations, when we do shear operation our new image doesn't create a pattern inside the image. Because we are already transform the image, so maybe still we have black holes, but there are no any patten. Our image just look like broken.

Figure 11: Horizontal Shear Operation with forward mapping



#### 4.2. Backward Mapping and Backward Mapping with Bilinear Interpolation

Actually, both result look like similar like image that used forward mapping. We talked about how to calculate new position of new image and other differences. So I just visulise image.

Figure 12: Horizontal Shear Operation with backward mapping





Figure 14: Horizontal Shift Operation with backward mapping and bilinear interpolation



## 5. Rotation Operation

Our last operation is rotation. Rotation is more complex than other operations. Because actually, we don't rotate the image, we just shear image horizontally and vertically. For doing proper rotation we should shear both sides simultaneously. In addition for doing this operation image will be increased. For solving scale problem we should decrease size of image simultaneously, too. For doing that we use wave transform of sin and cos.

### 5.1. Images of Rotation Operations

There are images of forward mapping, backward mapping, and backward mapping with bilinear interpolation. I just share the images because of their calculations are same. Here are rotation of affine matrix:

```
int centerX0 = width / 2;  
int centerY0 = height / 2;  
a = cos(radian), b = -sin(radian);  
c = sin(radian), d = cos(radian);  
tx = (newWidth / 2.0) - (centerX0 * a + centerY0 * b);  
ty = (newHeight / 2.0) - (centerX0 * c + centerY0 * d);
```

Figure 15: Rotation Operation with forward mapping

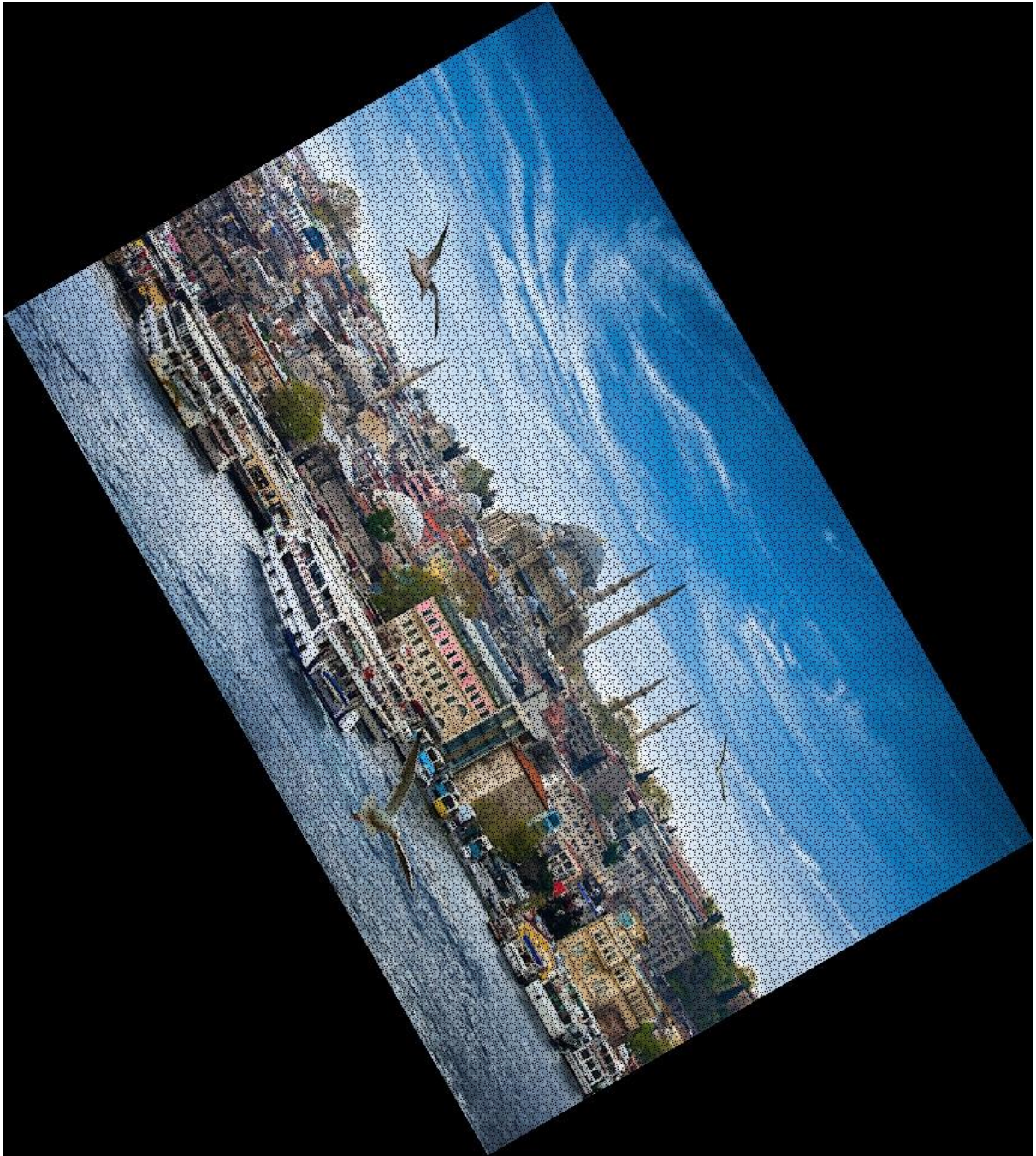


Figure 16: Rotation Operation with backward mapping





Figure 17: Rotation Operation with backward mapping and bilinear interpolation





## 6. Conclusion

To sum up, we manipulate the image with 4 different methods these scale, zoom (type of scale), shear, and rotation. For doing these operations we use 3 methods.

One of them is “Forward Mapping”; this method simply find new coordinate of image multiplying coordinates with affine matrix. Therefore, image have some hole.

Secondly we use “Backward Mapping”; this method better than first one. Because it is reach own pixel density from source image. Therefore, there are not any hole.

Lastly, we add bilinear interpolation method to “Backward Mapping”. This method increases the image quality. Because, normally we just getting nearest coordinate of the image. But with bilinear interpolation we calculate adjacent of the coordinate.

When we compare 3 result, we clearly see difference between forward and backward mappings. But if we don't look at details of image, interpolation it comes a lot cost.

## **7. Appendix**

## 7.1. Forward Mapping Function:

```
8. void forwardMapping(const Mat& image) {
9.     int width = image.cols;
10.    int height = image.rows;
11.    // affine matrix values
12.    double a, b, tx;
13.    double c, d, ty;
14.
15.    #if 0 // zoom image 1.4x
16.
17.        a = 1.4, b = 0, tx = 1;
18.        c = 0, d = 1.4, ty = 1;
19.        Mat zoomAffineMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
20.        // new Image
21.        Mat zoomImage(height, width, CV_8UC3, cv::Scalar(0, 0, 0));
22.
23.        for (int y = 0; y < height; ++y) {
24.            for (int x = 0; x < width; ++x) {
25.                int newX = static_cast<int>(x * zoomAffineMax.at<double>(0, 0));
26.                int newY = static_cast<int>(y * zoomAffineMax.at<double>(1, 1));
27.                if (newX > 0 && newX < width && newY > 0 && newY < height) {
28.                    zoomImage.at<cv::Vec3b>(newY, newX) = image.at<cv::Vec3b>(y, x); // Forward mapping
29.                }
30.            }
31.        }
32.
33.        imshow("zoomFoward", zoomImage);
34.        if (!imwrite("zoomFoward.jpg", zoomImage)) {
35.            std::cerr << "write error!" << std::endl;
36.        }
37.    #endif
38.
39.    #if 0 // horizotanl shear image 1.4x
40.
41.        a = 1, b = 1.4, tx = 1;
42.        c = 0, d = 1, ty = 1;
43.        int newWidth = width + (height * b);
44.        Mat horizontalShearAffMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
45.        // new Image
46.        Mat shearImage(height, newWidth, CV_8UC3, cv::Scalar(0, 0, 0));
47.
48.        for (int y = 0; y < height; ++y) {
49.            for (int x = 0; x < width; ++x) {
50.                int newX = static_cast<int>(x + (y * horizontalShearAffMax.at<double>(0, 1)));
51.                int newY = static_cast<int>(y);
52.                if (newX >= 0 && newX < newWidth && newY >= 0 && newY < height) {
53.                    // Forward mapping
54.                    shearImage.at<cv::Vec3b>(newY, newX) = image.at<cv::Vec3b>(y, x);
55.                }
56.            }
57.        }
58.
59.        if (!imwrite("HorizontalShearForward.jpg", shearImage)) {
60.            std::cerr << "write error!" << std::endl;
61.        }
62.    #endif
63.
64.    #if 0 // scale image 1.4x
65.
66.        a = 1.4, b = 1, tx = 0;
```

```

67.  c = 0, d = 1.4, ty = 0;
68.  int newHeight = a * height;
69.  int newWidth = d * width;
70.  Mat scaleAffMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
71.  // new Image
72.  Mat scaleImage(newHeight, newWidth, CV_8UC3, cv::Scalar(0, 0, 0));
73.
74.  for (int y = 0; y < height; ++y) {
75.      for (int x = 0; x < width; ++x) {
76.          int newX = static_cast<int>(x * scaleAffMax.at<double>(0, 0));
77.          int newY = static_cast<int>(y * scaleAffMax.at<double>(1, 1));
78.          if (newX >= 0 && newX < newWidth && newY >= 0 && newY < newHeight) {
79.              // Forward mapping
80.              scaleImage.at<cv::Vec3b>(newY, newX) = image.at<cv::Vec3b>(y, x);
81.          }
82.      }
83.  }
84.
85.  if (!imwrite("scaledImageForward.jpg", scaleImage)) {
86.      std::cerr << "write error!" << std::endl;
87.  }
88. #endif
89.
90. #if 1 // rotate image 60 degree
91.
92.  int centerX0 = width / 2;
93.  int centerY0 = height / 2;
94.  int degree = 60;
95.  double radyan = degree * 3.14159 / 180;
96.
97.  a = cos(radyan), b = -sin(radyan);
98.  c = sin(radyan), d = cos(radyan);
99.
100.  int newHeight = static_cast<int>(height * fabs(a) + static_cast<int>(width * fabs(c));
101.  int newWidth = static_cast<int>(width * fabs(a) + static_cast<int>(height * fabs(c));
102.
103.  tx = (newWidth / 2.0) - (centerX0 * a + centerY0 * b);
104.  ty = (newHeight / 2.0) - (centerX0 * c + centerY0 * d);
105.
106.  Mat scaleAffMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
107.  // new Image
108.  Mat rotateImage(newHeight, newWidth, CV_8UC3, cv::Scalar(0, 0, 0));
109.
110.  // rotate
111.  for (int y = 0; y < height; ++y) {
112.      for (int x = 0; x < width; ++x) {
113.          int newX = static_cast<int>((x * scaleAffMax.at<double>(0, 0)) + (y *
scaleAffMax.at<double>(0, 1)) + scaleAffMax.at<double>(0, 2));
114.          int newY = static_cast<int>((x * scaleAffMax.at<double>(1, 0)) + (y *
scaleAffMax.at<double>(1, 1)) + scaleAffMax.at<double>(1, 2));
115.          if (newX >= 0 && newX < newWidth + 200 && newY >= 0 && newY < newHeight) {
116.              // Forward mapping
117.              rotateImage.at<Vec3b>(newY, newX) = image.at<Vec3b>(y, x);
118.          }
119.      }
120.  }
121.
122.
123.  if (!imwrite("rotated_imageForward.jpg", rotateImage)) {
124.      std::cerr << "write error!" << std::endl;
125.  }
126. #endif
127.  return;

```



128.     }

## 7.2. Backward Mapping Function:

```
2. void backwardMapping(const Mat& image) {
3.     int width = image.cols;
4.     int height = image.rows;
5.     // affine matrix values
6.     double a, b, tx;
7.     double c, d, ty;
8.
9.     #if 0 // zoom image 1.4x
10.
11.     a = 1.4, b = 0, tx = 1;
12.     c = 0, d = 1.4, ty = 1;
13.     Mat zoomAffineMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
14.     // new Image wiht same size
15.     Mat zoomImage(height, width, CV_8UC3, cv::Scalar(0, 0, 0));
16.
17.     for (int y = 0; y < height; ++y) {
18.         for (int x = 0; x < width; ++x) {
19.             int newX = static_cast<int>(x / zoomAffineMax.at<double>(0, 0));
20.             int newY = static_cast<int>(y / zoomAffineMax.at<double>(1, 1));
21.             if (newX >= 0 && newX < width && newY >= 0 && newY < height) {
22.                 // Backward mapping
23.                 zoomImage.at<cv::Vec3b>(y, x) = image.at<cv::Vec3b>(newY, newX);
24.             }
25.         }
26.     }
27.
28.     if (!imwrite("zoomBackward.jpg", zoomImage)) {
29.         std::cerr << "write error!" << std::endl;
30.     }
31. #endif
32.
33.     #if 0 // horizotanl shear image 1.4x
34.
35.     a = 1, b = 1.4, tx = 1;
36.     c = 0, d = 1, ty = 1;
37.     int newWidth = width + (height * b);
38.     Mat horizontalShearAffMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
39.     // new Image
40.     Mat shearImage(height, newWidth, CV_8UC3, cv::Scalar(0, 0, 0));
41.
42.     for (int y = 0; y < height; ++y) {
43.         for (int x = 0; x < newWidth; ++x) {
44.             int imageX = static_cast<int>(x - (y * horizontalShearAffMax.at<double>(0, 1)));
45.             int imageY = static_cast<int>(y);
46.             if (imageX >= 0 && imageX < width && imageY >= 0 && imageY < height) {
47.                 shearImage.at<cv::Vec3b>(y, x) = image.at<cv::Vec3b>(imageY, imageX); // Backward
mapping
48.             }
49.         }
50.     }
51.
52.     if (!imwrite("HorizontalShearBackward.jpg", shearImage)) {
53.         std::cerr << "write error!" << std::endl;
54.     }
55. #endif
```

```

56.
57. #if 0 // scale image 1.4x
58.
59.     a = 1.4, b = 1, tx = 0;
60.     c = 0, d = 1.4, ty = 0;
61.     int newHeight = a * height;
62.     int newWidth = d * width;
63.     Mat scaleAffMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
64.     Mat scaleImage(newHeight, newWidth, CV_8UC3, cv::Scalar(0, 0, 0)); // new Image
65.
66.     for (int y = 0; y < newHeight; ++y) {
67.         for (int x = 0; x < newWidth; ++x) {
68.             int imageX = static_cast<int>(x / scaleAffMax.at<double>(0, 0));
69.             int imageY = static_cast<int>(y / scaleAffMax.at<double>(1, 1));
70.             if (imageX >= 0 && imageX < width && imageY >= 0 && imageY < height) {
71.                 scaleImage.at<cv::Vec3b>(y, x) = image.at<cv::Vec3b>(imageY, imageX); // Backward
mapping
72.             }
73.         }
74.     }
75.     if (!imwrite("scaledImageBackward.jpg", scaleImage)) {
76.         std::cerr << "write error!" << std::endl;
77.     }
78. #endif
79.
80. #if 0 // rotate image 60 degree
81.
82.     int centerX0 = width / 2;
83.     int centerY0 = height / 2;
84.     int degree = 60;
85.     double radyan = degree * 3.14159 / 180;
86.
87.     a = cos(radyan), b = -sin(radyan);
88.     c = sin(radyan), d = cos(radyan);
89.
90.     int newHeight = static_cast<int>(height * fabs(a)) + static_cast<int>(width * fabs(c));
91.     int newWidth = static_cast<int>(width * fabs(a)) + static_cast<int>(height * fabs(c));
92.
93.     tx = (newWidth / 2.0) - (centerX0 * a + centerY0 * b);
94.     ty = (newHeight / 2.0) - (centerX0 * c + centerY0 * d);
95.
96.     Mat scaleAffMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
97.     Mat rotateImage(newHeight, newWidth, CV_8UC3, cv::Scalar(0, 0, 0)); // new Image
98.
99.     if (inverseOfAffineMax(scaleAffMax) == false) {
100.         std::cerr << "Affine matrix didnt turn inverse itself" << std::endl;
101.         return ;
102.     }
103.
104.     // rotate
105.     for (int y = 0; y < newHeight; ++y) {
106.         for (int x = 0; x < newWidth; ++x) {
107.             int imageX = static_cast<int>((x * scaleAffMax.at<double>(0, 0)) + (y *
scaleAffMax.at<double>(0, 1)) + scaleAffMax.at<double>(0, 2));
108.             int imageY = static_cast<int>((x * scaleAffMax.at<double>(1, 0)) + (y *
scaleAffMax.at<double>(1, 1)) + scaleAffMax.at<double>(1, 2));
109.             //cout << "image x " << imageX;
110.             if (imageX >= 0 && imageX < width && imageY >= 0 && imageY < height) {
111.                 rotateImage.at<cv::Vec3b>(y, x) = image.at<cv::Vec3b>(imageY, imageX); // Backward
mapping
112.             }
113.         }
114.     }

```

```

115.
116.
117. if(!imwrite("rotated_imageBackward.jpg", rotateImage)) {
118.     std::cerr << "write error!" << std::endl;
119. }
120. #endif
121. return;
122. }

```

### 7.3. Backward Mapping with Bilinear Interpolation

```

8. void backwardMappingWithInterpolation(const Mat& image) {
9.     int width = image.cols;
10.    int height = image.rows;
11.    // affine matrix values
12.    double a, b, tx;
13.    double c, d, ty;
14.
15.    #if 0 // zoom image 1.4x
16.
17.        a = 1.4, b = 0, tx = 1;
18.        c = 0, d = 1.4, ty = 1;
19.        Mat zoomAffineMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
20.
21.        Mat zoomImage(height, width, CV_8UC3, Scalar(0, 0, 0)); // new Image wiht new size
22.
23.        for (int y = 0; y < height; ++y) {
24.            for (int x = 0; x < width; ++x) {
25.                double newX = x / (zoomAffineMax.at<double>(0, 0));
26.                double newY = y / (zoomAffineMax.at<double>(1, 1));
27.                if (newX >= 0 && newX < width && newY >= 0 && newY < height) {
28.                    zoomImage.at<Vec3b>(y, x) = calculateBilinearValue(image, newY, newX); // Backward mapping
with interpolation
29.                }
30.            }
31.        }
32.
33.        //imshow("zoomed image", zoomImage);
34.        if (!imwrite("zoomBackwardInterpol.jpg", zoomImage)) {
35.            std::cerr << "write error!" << std::endl;
36.        }
37.
38.    #endif
39.
40.    #if 0 // horizotanl shear image 1.4x
41.
42.        a = 1, b = 1.4, tx = 1;
43.        c = 0, d = 1, ty = 1;
44.        int newWidth = width + (height * b);
45.        Mat horizontalShearAffMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
46.        Mat shearImage(height, newWidth, CV_8UC3, cv::Scalar(0, 0, 0)); // new Image
47.
48.        for (int y = 0; y < height; ++y) {
49.            for (int x = 0; x < newWidth; ++x) {
50.                int newX = static_cast<int>(x - (y * horizontalShearAffMax.at<double>(0, 1)));
51.                int newY = static_cast<int>(y);
52.                if (newX >= 0 && newX < width && newY >= 0 && newY < height) {
53.                    shearImage.at<cv::Vec3b>(y, x) = calculateBilinearValue(image, newY, newX); // Backward
mapping with interpolation
54.                }
55.            }
56.        }

```

```

57.
58.     if (!imwrite("HorizontalShearInterpolation.jpg", shearImage)) {
59.         std::cerr << "write error!" << std::endl;
60.     }
61. #endif
62.
63. #if 0 // scale image 1.4x
64.
65.     a = 1.4, b = 1, tx = 0;
66.     c = 0, d = 1.4, ty = 0;
67.     int newHeight = a * height;
68.     int newWidth = d * width;
69.     Mat scaleAffMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
70.     Mat scaleImage(newHeight, newWidth, CV_8UC3, cv::Scalar(0, 0, 0)); // new Image
71.
72.     for (int y = 0; y < newHeight; ++y) {
73.         for (int x = 0; x < newWidth; ++x) {
74.             int newX = static_cast<int>(x / scaleAffMax.at<double>(0, 0));
75.             int newY = static_cast<int>(y / scaleAffMax.at<double>(1, 1));
76.             if (newX >= 0 && newX < width && newY >= 0 && newY < height) {
77.                 scaleImage.at<cv::Vec3b>(y, x) = calculateBilinearValue(image, newY, newX); // Backward mapping
78.             }
79.         }
80.     }
81.
82.     if (!imwrite("scaledImageBackwardInterpolation.jpg", scaleImage)) {
83.         std::cerr << "write error!" << std::endl;
84.     }
85.
86. #endif
87.
88. #if 0 // rotate image 60 degree
89.
90.     int centerX0 = width / 2;
91.     int centerY0 = height / 2;
92.     int degree = 60;
93.     double radyan = degree * 3.14159 / 180;
94.
95.     a = cos(radyan), b = -sin(radyan);
96.     c = sin(radyan), d = cos(radyan);
97.
98.     int newHeight = static_cast<int>(height * fabs(a)) + static_cast<int>(width * fabs(c));
99.     int newWidth = static_cast<int>(width * fabs(a)) + static_cast<int>(height * fabs(c));
100.
101.     tx = (newWidth / 2.0) - (centerX0 * a + centerY0 * b);
102.     ty = (newHeight / 2.0) - (centerX0 * c + centerY0 * d);
103.
104.     Mat scaleAffMax = (Mat_<double>(2, 3) << a, b, tx, c, d, ty);
105.     Mat rotateImage(newHeight, newWidth, CV_8UC3, cv::Scalar(0, 0, 0)); // new Image
106.
107.     // before inverse
108.
109.     if (inverseOfAffineMax(scaleAffMax) == false) {
110.         std::cerr << "Affine matrix didnt turn inverse itself" << std::endl;
111.         return;
112.     }
113.
114.     // rotate
115.     for (int y = 0; y < newHeight; ++y) {
116.         for (int x = 0; x < newWidth; ++x) {
117.             int newX = static_cast<int>((x * scaleAffMax.at<double>(0, 0)) + (y * scaleAffMax.at<double>(0, 1)) +
scaleAffMax.at<double>(0, 2));

```



```

118.         int newY = static_cast<int>((x * scaleAffMax.at<double>(1, 0)) + (y * scaleAffMax.at<double>(1, 1)) +
scaleAffMax.at<double>(1, 2));
119.
120.         if (newX >= 0 && newX < width && newY >= 0 && newY < height) {
121.             rotateImage.at<cv::Vec3b>(y, x) = calculateBilinearValue(image, newY, newX); // Backward
mapping
122.         }
123.     }
124. }
125.
126.
127. if (!imwrite("rotated_imageBackwardInterpol.jpg", rotateImage)) {
128.     std::cerr << "write error!" << std::endl;
129. }
130. #endif
131. return;
132. }

```

## 7.4. Bilinear Interpolation Function

```

8. Vec3b calculateBilinearValue(const Mat& image, const double &y, const double& x) {
9.     int x1 = static_cast<int>(x); // min round x
10.    int y1 = static_cast<int>(y); // min round y
11.    int x2 = min(x1 + 1, image.cols - 1); // x1 + 1 is top border
12.    int y2 = min(y1 + 1, image.rows - 1); // y1 + 1 is top border
13.
14.
15.    Vec3b Q11 = image.at<Vec3b>(y1, x1);
16.    Vec3b Q21 = image.at<Vec3b>(y1, x2);
17.    Vec3b Q12 = image.at<Vec3b>(y2, x1);
18.    Vec3b Q22 = image.at<Vec3b>(y2, x2);
19.
20.
21.    double div = ((x2 - x1) * (y2 - y1));
22.
23.    if (div <= 0) {
24.        return Q11;
25.    }
26.
27.    Vec3b resultDens;
28.    for (int i = 0; i < 3; ++i) {
29.        resultDens[i] = static_cast<uchar>(
30.            (Q11[i] * (x2 - x) * (y2 - y) / div) +
31.            (Q21[i] * (x - x1) * (y2 - y) / div) +
32.            (Q12[i] * (x2 - x) * (y - y1) / div) +
33.            (Q22[i] * (x - x1) * (y - y1) / div)
34.        );
35.    }
36.    return resultDens;
37. }

```

## 7.5. Inverse Affine Matrix Function:

```

5. bool inverseOfAffineMax(Mat& affineMax) {
6.     double a = affineMax.at<double>(0, 0);

```

```

7.   double b = affineMax.at<double>(0, 1);
8.   double tx = affineMax.at<double>(0, 2);
9.   double c = affineMax.at<double>(1, 0);
10.  double d = affineMax.at<double>(1, 1);
11.  double ty = affineMax.at<double>(1, 2);
12.
13.  // calcukale determinant
14.  double det = a * d - b * c;
15.  if(det == 0) {
16.      return false; // not found deteeterminant
17.  }
18.
19.
20.  Mat inverseMat = (Mat_<double>(2, 3) <<
21.      d * (1 / det), -b * (1 / det), (b * ty - d * tx) * (1/det),
22.      -c * (1 / det), a * (1 / det), (c * tx - a * ty) * (1 / det)
23.      );
24.
25.  inverseMat.copyTo(affineMax);
26.  return true;
27.}

```

## 8. Sources

1. <https://chatgpt.com/>
2. <https://www.algorithm-archive.org/contents/bitlogic/bitlogic.html>
3. Gonzalez, Rafael C. Digital Image 3.ed., Person, Processing-Prentice Hall, 2008