

The Bresenham's Line Algorithm

The Bresenham's Line Algorithm is a simple and efficient algorithm used for drawing a line on a pixel grid. The algorithm is based on an incremental error approach, making it faster than other methods.

— Derivation of the Algorithm —

The objective is to derive the algorithm to determine which pixel to turn on to approximate a straight line between the two given points: $(x_0, y_0), (x_1, y_1)$

- Calculate the differences between the two points:

$$\Delta x = x_1 - x_0$$

$$\Delta y = y_1 - y_0$$

- The decision parameter (P) is used to determine which pixel to choose at each step of the algorithm.

$$P = 2 \cdot \Delta y - \Delta x$$

→ the initial P is $\Rightarrow P_0 = 2 \cdot \Delta y - \Delta x$

- we set the initial point to (x_0, y_0) and plotting it.

- for each x from $x_0 + 1$ to x_1 :

if $P \geq 0$, increment y and update P as follows:

$$y = y + 1$$

$$P = P + 2 \cdot \Delta y - 2 \cdot \Delta x$$

if $P < 0$, update P as follows:

$$P = P + 2 \cdot \Delta y$$

- for each step, plot the pixel at the current coordinates (x, y)

⇒ after the iteration is complete, you will have a series of plotted points forming a line between (x_0, y_0) and (x_1, y_1)

Example

$\Rightarrow (2,3)$ and $(7,7)$

$$\Delta x = x_1 - x_0 = 7 - 2 = \underline{\underline{5}}$$

$$\Delta y = y_1 - y_0 = 7 - 3 = \underline{\underline{4}}$$

$$P_0 = 2\Delta y - \Delta x = 2 \cdot 4 - 5 = \underline{\underline{3}}$$

$$1) (3,4) \rightarrow P = P_0(3) + \underbrace{2 \cdot \Delta y(4) - 2 \cdot \Delta x(5)}_{-2} = \underline{\underline{1}}$$

$$2) (4,5) \rightarrow P = \underline{\underline{1}} + (-2) = \underline{\underline{-1}}$$

$$3) (5,5) \rightarrow P = -1 + 2\Delta y(4) = 7$$

$$4) (6,6)$$

$$(7,7)$$

