1)

a. $a_n = -2^{n+1}$
$a_{n-1} = -2^{n-1+1} = -2^n$

$a_n = 3a_{n-1} + 2^n \Rightarrow a_n = 3.(-2^n) + 2^n = -2.2^n = -2^{n+1}$

So that $a_n = -2^{n+1}$ is a solution of the given reccurence relation.

b. Step 1- Determine homogeneous and particular part and write general relation.

$$\underbrace{a_n = 3a_{n-1}}_{\text{homogeneous part}} + \underbrace{2^n}_{\text{particular part}}$$

$$\overset{\text{general}}{a_n^{(g)}} = \overset{\text{homogeneous}}{a_n^{(h)}} + a_n^{(p) \rightarrow \text{particular}}$$

Step 2- Solve homogeneous part and find the characteristic equation and root

$a_n^{(h)} \Rightarrow a_n = 3a_{n-1} \Rightarrow \begin{matrix} a_{n-1} = 1 \\ a_n = r \end{matrix} \Rightarrow r = 3.1 = 3$

$a_n^{(h)} = c.3^n$

Step 3- Solve particular part.

$a_n^{(p)} = A.2^n$
$a_{n-1}^{(p)} = A.2^{n-1} = \frac{A}{2}.2^n$

Put it in reccurence relation places and find A.

$a_n = 3.a_{n-1} + 2^n$
$A.\cancel{2^n} = 3.(\frac{A}{2}.\cancel{2^n}) + \cancel{2^n}$
$A = \frac{3A}{2} + 1 \Rightarrow A = -2$
$a_n^{(p)} = A.2^n = -2.2^n = -2^{n+1}$

Step 4- Find general relation. Sum the homogeneous and particular part.

$a_n^{(g)} = a_n^{(h)} + a_n^{(p)}$

$a_n = c.3^n - 2^{n+1}$

Step 5- Find c.

b. $a_n = c.3^n - 2^{n+1}$, $a_0 = 1$

$a_0 = c.3^0 - 2^{0+1} = c - 2 = 1 \Rightarrow c = 3$

$a_n = 3.3^n - 2^{n+1} = 3^{n+1} - 2^{n+1}$

2. Step 1 - Determine homogeneous and particular part and write general relation.

$$\underbrace{f(n) = 4.f(n-1) - 4f(n-2)}_{\text{homogeneous part}} + \underbrace{n^2}_{\text{particular part}}$$

$$\overset{\nearrow \text{general}}{f(n)^{(g)}} = \overset{\nearrow \text{homogeneous}}{f(n)^{(h)}} + f(n)^{(p)} \rightarrow \text{particular}$$

Step 2. Solve homogeneous part and find the characteristic equation and root.

$$f(n)^{(h)} \Rightarrow f(n) = 4.f(n-1) - 4f(n-2) \Rightarrow \left.\begin{array}{l} f(n-2) = 1 \\ f(n-1) = r \\ f(n) = r^2 \end{array}\right\} \Rightarrow r^2 = 4r - 4$$

$r^2 - 4r + 4 = 0$

$(r-2)^2 = 0$

$r = 2$

$f(n)^h = c_1.2^n + c_2.2^n.n$ (The reason for n is that 2 roots are 2.)

Step 3- Solve particular part

$f(n)^{(p)} = A.n^2 + Bn + C$ (This is because the particular part is polynomial.)

$f(n-1)^{(p)} = A.(n-1)^2 + B.(n-1) + C$

$f(n-2)^{(p)} = A.(n-2)^2 + B.(n-2) + C$

Put it reccurence relation places and find A, B and C.

$f(n) = 4f(n-1) - 4f(n-2) + n^2$

$An^2 + Bn + C = 4.[A.(n-1)^2 + B(n-1) + C] - 4[A.(n-2)^2 + B.(n-2) + C] + n^2$

⋮

$An^2 + Bn + C = n^2 + 8An - 12A + 4B$

The coefficients of the terms n^2, n^1 and n^0 must be the same on the right and left of the equation.

$A = 1$

$B = 8A = 8$

$C = -12A + 4B = -12.1 + 4.8 = 20$

$f(n)^{(p)} = An^2 + Bn + C = n^2 + 8n + 20$

Step 4- Find general relation. Sum the homogeneous and particular part.

$$f(n)^{(g)} = f(n)^{(h)} + f(n)^{(p)}$$

$$f(n) = c_1 . 2^n + c_2 . 2^n . n + n^2 + 8n + 20$$

Step 5- Find c_1 and c_2

$f(0) = 2, \quad f(1) = 5$

$$f(0) = c_1 . 2^0 + c_2 . 2^0 . 0 + 0^2 + 8.0 + 20 = c_1 + 20 = 2 \Rightarrow c_1 = -18$$

$$f(1) = -18 . 2^1 + c_2 . 2^1 . 1 + 1^2 + 8.1 + 20 = 2c_2 - 7 = 5 \Rightarrow c_2 = 6$$

$$f(n) = -18 . 2^n + 6 . 2^n . n + n^2 + 8n + 20$$

$$f(n) = -9 . 2^{n+1} + 3 . 2^{n+1} . n + n^2 + 8n + 20$$

3-

a) Step 1- Write general relation.

$$a_n^{(g)} = a_n^{(h)}$$

Step 2- Find the characteristic roots of the reccurence relation.

$$a_n^{(h)} \Rightarrow a_n = 2a_{n-1} - 2a_{n-2} \Rightarrow \left.\begin{array}{l} a_{n-2} = 1 \\ a_{n-1} = r \\ a_n = r^2 \end{array}\right\} \Rightarrow r^2 = 2r - 2$$

$$r^2 - 2r + 2 = 0$$

$\Delta = b^2 - 4ac = (-2)^2 - 4 . 1 . 2 = -4$ (There are complex roots.)

$$r_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-(-2) + \sqrt{-4}}{2.1} = \frac{2 + 2.\sqrt{-1}}{2} = 1 + \sqrt{-1}$$

$$r_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-(-2) - \sqrt{-4}}{2.1} = \frac{2 - 2\sqrt{-1}}{2} = 1 - \sqrt{-1}$$

$$\sqrt{-1} = i \Rightarrow \begin{array}{l} r_1 = 1 + \sqrt{-1} = 1 + i \\ r_2 = 1 - \sqrt{-1} = 1 - i \end{array}$$

$$a_n^{(h)} = c_1 . (1+i)^n + c_2 . (1-i)^n$$

$$a_n^{(g)} = a_n^{(h)}$$

$$a_n = c_1 . (1+i)^n + c_2 . (1-i)^n$$

b) $a_n = c_1 \cdot (1+i)^n + c_2 \cdot (1-i)^n$, $a_0 = 1$, $a_1 = 2$

$$a_0 = c_1 \cdot (1+i)^0 + c_2 \cdot (1-i)^0 = c_1 + c_2 = 1$$

$$a_1 = c_1 \cdot (1+i)^1 + c_2 \cdot (1-i)^1 = c_1 + c_2 + i(c_1 - c_2) = 2$$

$$c_1 + c_2 = 1 \Rightarrow c_1 = 1 - c_2$$

$$\underbrace{c_1 + c_2}_{1} + i \cdot (c_1 - c_2) = 2$$

$$i \cdot (c_1 - c_2) = 1$$

$$i \cdot (1 - c_2 - c_2) = 1$$

$$1 - 2c_2 = \frac{1}{i}$$

$$2c_2 = 1 - \frac{1}{i} \Rightarrow c_2 = \frac{i-1}{2i}$$

$$c_1 = 1 - c_2 \Rightarrow c_1 = 1 - \frac{i-1}{2i} = \frac{i+1}{2i}$$

$$a_n = \frac{i+1}{2i} \cdot (1+i)^n + \frac{i-1}{2i} \cdot (1-i)^n$$

$$a_n = \frac{(1+i)^{n+1} + (i-1) \cdot (1-i)^n}{2i}$$