CSE 351 - Homework 1

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1.- a) X

$$\frac{dy_1}{dt} + 2y_1(t) = f_1^2(t) \longrightarrow \frac{dk_1 y_1(t)}{dt} + 2k_1 y_1(t) = k_1 f_1^2(t)$$

$$\frac{dy_2}{dt} + 2y_2(t) = f_2^2(t) \longrightarrow \frac{dk_2 y_2(t)}{dt} + 2k_2 y_2(t) = k_2 f_2^2(t)$$

then

$$\frac{d}{dt}\Big[\overbrace{k_1 y_1(t) + k_2 y_2(t)}^{y(t)}\Big] + 2.\Big[\overbrace{k_1 y_1(t) + k_2 y_2(t)}^{y(t)}\Big] = \overbrace{k_1 f_1^2(t) + k_2 f_2^2(t)}^{f_X(t)}$$

$$\boxed{f_X(t) = k_1 f_1^2(t) + k_2 f_2^2(t)} \; *$$

Y Secondly

$$\frac{d}{dt}\Big[\overbrace{k_1 y_1(t) + k_2 y_2(t)}^{y(t)}\Big] + 2\Big[\overbrace{k_1 y_1(t) + k_2 y_2(t)}^{y(t)}\Big] = \Big[\overbrace{k_1 f_1(t) + k_2 f_2(t)}^{f_Y(t)}\Big]^2$$

$f_X(t) \neq f_Y(t)$. So systems are non-linear.

b) X

$$\frac{dy_1(t)}{dt} + 3t y_1(t) = t^2 f_1(t) \longrightarrow \frac{dk_1 y_1(t)}{dt} + 3t k_1 y_1(t) = t^2 k_1 f_1(t)$$

$$\frac{dy_2(t)}{dt} + 3t y_2(t) = t^2 f_2(t) \longrightarrow \frac{dk_2 y_2(t)}{dt} + 3t k_2 y_2(t) = t^2 k_2 f_2(t)$$

then

$$\frac{d}{dt}\Big[\overbrace{k_1 y_1(t) + k_2 y_2(t)}^{y(t)}\Big] + 3t\Big[\overbrace{k_1 y_1(t) + k_2 y_2(t)}^{y(t)}\Big] = t^2\Big[\overbrace{k_1 f_1(t) + k_2 f_2(t)}^{f_X(t)}\Big]$$

$$\boxed{f_X(t) = k_1 f_1(t) + k_2 f_2(t)}$$

Y Secondly

$$\frac{d}{dt}\Big[\overbrace{k_1 y_1(t) + k_2 y_2(t)}^{y(t)}\Big] + 3t\Big[\overbrace{k_1 y_1(t) + k_2 y_2(t)}^{y(t)}\Big] = t^2\Big[\overbrace{k_1 f_1(t) + k_2 f_2(t)}^{f_Y(t)}\Big]$$

$f_X(t) = f_Y(t)$. So systems are linear.

2-

a) Characteristic equation

$$\lambda^2 + 5\lambda + 6 = 0$$

$$(\lambda+2)(\lambda+3) = 0$$

$$\lambda = -2, \quad \lambda = -3$$

$$y_0(t) = C_1 \cdot e^{\lambda_1 t} + C_2 \cdot e^{\lambda_2 t} = c_1 \cdot e^{-2t} + c_2 \cdot e^{-3t}$$

$$y_0(t) = -2c_1 e^{-2t} - 3c_2 e^{-3t}$$

if $t=0$, $y_0(0) = 2$, $y_0(0) = -1$

then,

$$\begin{aligned} c_1 + c_2 &= 2 \\ -2c_1 - 3c_2 &= -1 \end{aligned} \Rightarrow \begin{aligned} c_1 &= 5 \\ c_2 &= -3 \end{aligned}$$

$$\boxed{y_0(t) = 5e^{-2t} - 3e^{-3t}}$$

b) $f(t) = u(t) \rightarrow h(t) = e^{-t} u(t) \rightarrow y(t) = X$

$$y(t) = h(t) + f(t) = e^{-t} u(t) + u(t) = (1+e^{-t})(u(t))$$

3-

a) $y(k+1) + 2y(k) = f(k)$

$$(E+2)\, y(k) = f(k)$$

$$h(k) = \frac{b_0}{a_0} \delta(k) + y_0(k)\, u(k)$$

Characteristic equation : $\gamma + 2 = 0 \rightarrow \gamma = -2$

$$y_0(k) = c \cdot \gamma^k = c \cdot (-2)^k$$

$$a_0 = 2, \quad b_0 = 1$$

So,

$$h(k) = \frac{1}{2} \delta(k) + c \cdot (-2)^k$$

Iterative solution to determine c

$$(E+2)\, h(k) = \delta(k)$$

$$h(k+1) + 2h(k) = \delta(k)$$

$k = -1$ and substitude $h(-1) = \delta(-1) = 0 \rightarrow h(0) = 0$

for $k = 0$, use $h(0) = 0$

$0 = \frac{1}{2} + c(-2)^0 \rightarrow c = \frac{-1}{2}$

$h(k) = \frac{1}{2}\delta(k) - \frac{1}{2}(-2)^k u(k)$

b) $h(k) = (-2)^k u(k)$, $f(k) = e^{-k} u(k) \rightarrow y(k) = f(k) * h(k)$
(pair 4 in table 3.1 of text book)

$f_1(k) = \gamma_1^k u(k)$, $f_2(k) = \gamma_2^k u(k)$, $\gamma_1 \neq \gamma_2$

$$f_1(k) * f_2(k) = \left[\frac{\gamma_1^{k+1} - \gamma_2^{k+1}}{\gamma_1 - \gamma_2}\right] u(k)$$

So

$y(k) = e^{-k} u(k) * (-2)^k u(k) = \left(\frac{1}{e}\right)^k u(k) * (-2)^k u(k)$

$$= \left[\frac{\left(\frac{1}{e}\right)^{k+1} - (-2)^{k+1}}{\left(\frac{1}{e}\right) + 2}\right] \cdot u(k)$$