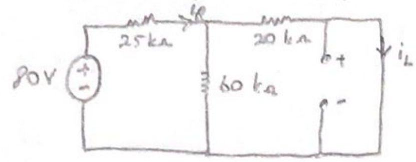
Abdullah Çelik
171044002

CSE 231 - Homework 3

8.2) a) At $t = 0^-$, the equivalent circuit is:



$60 \parallel 20 = 15\ k\Omega$

$i_R(0^-) = 80/(25+15) = 2mA$

$i_L(0^-) = \dfrac{60 \cdot 2mA}{60+20} = 1{,}5\ mA$

$V_C(0^-) = 0$

At $t = 0^+$

$V_C(0^+) = V_C(0^-) = 0$

$i_L(0^+) = i_L(0^-) = 1{,}5\ mA$

$80 = i_R(0^+) \cdot (25+20) + V_C(0^-)$

$i_R(0^+) = \dfrac{80}{45k} = 1{,}778\ mA$

$i_R = i_C + i_L \Rightarrow 1{,}778 = i_C(0^+) + 1{,}5$, $i_C(0^+) = 0{,}278\ mA$

b) $V_L = L \cdot \dfrac{di_L}{dt}$, $\dfrac{di_L(0^+)}{dt} = \dfrac{V_L(0^+)}{L} = 0$

$\dfrac{di(L^+)}{dt} = 0$

$80 = 45\, i_R + V_C$

$0 = 45 \dfrac{di_R}{dt} + \dfrac{dv_C}{dt}$

$\dfrac{dV_C(0^+)}{dt} = \dfrac{i_C(0^+)}{C} = \dfrac{0{,}278\ m\Omega}{1\ uF} = 278\ V/s$

Hence $\dfrac{di_R(0^+)}{dt} = \dfrac{-1}{45} \dfrac{dV_C(0^+)}{dt} = \dfrac{-278}{45}$

$\dfrac{di_R(0^+)}{dt} = -6{,}1778\ A/s$

Also $i_R = i_C + i_L$

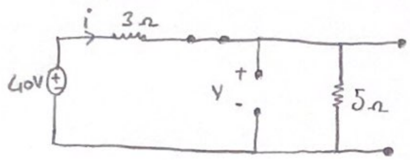
$$\frac{di_R(0^+)}{dt} = \frac{di_C(0^+)}{dt} + \frac{di_L(0^+)}{dt}$$

$$-6{,}1788 = \frac{di_C(0^+)}{dt} + 0 \Rightarrow \frac{di_C(0^+)}{dt} = -6{,}1788 \ A/s$$

c) $i_R(\infty) = i_L(\infty) = \frac{80}{45k} = 1{,}778 \ mA$

$$i_C(\infty) = \frac{C\,dv(\infty)}{dt} = 0$$

8.4) a) At $t = 0^-$, $u(-t) = 1$ and $u(t) = 0$. So that equivalent circuit is:



$$i(0^-) = \frac{40}{3+5} = 5A, \quad V(0) = 5i(0^-) = 25 \ V$$

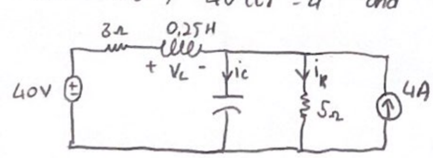
Hence,

$i(0^+) = i(0) = 5A$

$v(0^+) = V(0) = 25V$

b) $i_C = C\frac{dv}{dt}$ or $\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C}$

For $t = 0^+$, $4u(t) = 4$ and $4u(-t) = 0$. The equivalent circuit is:



Since i and V cannot change abruptly,

$$i_R = \frac{V}{5} = \frac{25}{5} = 5A, \quad i(0^+) + 4 = i_C(0^+) + i_R(0^+) \Rightarrow i_C(0^+) = 4A$$

$$\frac{dv(0^+)}{dt} = \frac{4}{0{,}1} = 40 \ V/s$$

$$V_L = L\frac{di}{dt} \Rightarrow \frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

$i(0^+) + V_L(0^+) + V(0^+) = 0$

$15 + U_L(0^+) + 25 = 0 \Rightarrow U_L(0^+) = -40$

$$\frac{di(0^+)}{dt} = \frac{-40}{0{,}25} = -160 \text{ A/s}$$

c) $i(\infty) = \dfrac{-5 \cdot 4}{3+5} = -2{,}5 \text{ A}$

$V(\infty) = 5 \cdot (4 - 2{,}5) = 7{,}5 \text{ V}$

8.6) a) Let i the inductor current. For $t<0$, $v(t) = 0$ so that

$i(0) = 0$ and $V(0) = 0$

For $t>0$, $v(t) = 1$. Since $V(0^+) = V(0^-) = 0$ and $i(0^+) = i(0^-) = 0$

$V_R(0^+) = R \cdot i(0^+) = 0 \text{ V}$

Also, since $V(0^+) = V_R(0^+) + V_L(0^+) = 0 \Rightarrow \underline{\underline{U_L(0^+) = 0 \text{ V}}}$

b) Since $i(0^+) = 0$, $i_c(0^+) = \dfrac{V_s}{R_s}$

But, $i_c = C\dfrac{dv}{dt}$ which leads to $\dfrac{dV(0^+)}{dt} = \dfrac{V_s}{C \cdot R_s}$

$$\frac{dv(0^+)}{dt} = \frac{dV_R(0^+)}{dt} + \frac{dV_L(0^+)}{dt}$$

$V_R = i \cdot R \Rightarrow \dfrac{dv_R}{dt} = R \cdot \dfrac{di}{dt}$

But, $V_L = L \cdot \dfrac{di}{dt}$, $V_L(0^+) = 0 = L \cdot \dfrac{di(0^+)}{dt} \Rightarrow \dfrac{di(0^+)}{dt} = 0$

$$\frac{dv_R(0^+)}{dt} = 0 \text{ V/s}$$

$$\frac{dv_L(0^+)}{dt} = \frac{dv(0^+)}{dt} = \frac{V_s}{CR_s}$$

c) $V_R(\infty) = \dfrac{R}{R+R_s} \cdot V_s$

$V_L(\infty) = 0 \text{ V}$

8.10) $s^2 + 5s + 4 = 0 \Rightarrow s_{1,2} = \frac{-5 \pm \sqrt{25-16}}{2} = -4, -1$

$V(t) = A e^{-4t} + B e^{-t}, \quad V(0) = 0 = A + B$

$\frac{dV}{dt} = -4A e^{-4t} - B e^{-t}$

$\frac{dV(0)}{dt} = 10 = -4A - B \Rightarrow A = \frac{-10}{3}$ and $B = \frac{10}{3}$

So $V(t) = \left(\frac{-10}{3} \cdot e^{-4t} + \frac{10}{3} e^{-t} \right) V$

8.12) a) Overdamped when $C > \frac{4L}{R^2} = \frac{4 \cdot 0{,}6}{400} = 6 \cdot 10^{-3}$, $C > 6mF$

b) Critically damped when $C = 6mF$

c) Underdamped when $C < 6mF$

8.16) At $t=0$, $i(0) = 0$, $V_c(0) = \frac{40 \cdot 30}{50} = 24V$

For $t>0$, we have source-free RLC circuit.

$\alpha = \frac{R}{2L} = \frac{100}{5} = 20$ and $w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \cdot 2{,}5}} = 20$

$w_0 = \alpha$ leads to critical damping

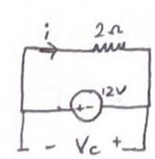
$i(t) = (A + Bt) e^{-20t}, \quad i(0) = 0 = A$

$\frac{di}{dt} = B e^{-20t} + (-20 \cdot B \cdot t \, e^{-20t})$

$\frac{di(0)}{dt} = \frac{-1}{2{,}5} \cdot 24 = \frac{-24}{2{,}5}$

Hence $B = -9{,}6$, $i(t) = -9{,}6 \cdot t \cdot e^{-20t}$

8.13) For $t<0$, the equivalent circuit is:



$V(0) = -12V$, $i(0) = \frac{12}{2} = 6A$

For $t>0$, we have series RLC circuit.

$$a = \frac{R}{2L} = \frac{2}{2 . 0{,}5} = 2$$

$$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0{,}5 . \frac{1}{4}}} = 2\sqrt{2}$$

since $a < w_0$, under damped response

$$w_d = \sqrt{w_0^2 - a^2} = \sqrt{8-4} = 2$$

$$i(t) = (A . \cos 2t + B \sin 2t)\, e^{-2t}$$

$$i(0) = 6 = A$$

$$\frac{di}{dt} = -2 . (6 \cos 2t + \sin 2t)\, e^{-2t} + (-2 . 6 \sin 2t + 2B \cos 2t)\, e^{-at}$$

$$\frac{di(0)}{dt} = -12 + 2B = \frac{-1}{L}\left[R i(0) + V_C(0)\right] = -2 . (12-12) = 0$$

Thus $B = 6$ and $i(t) = (6 \cos 2t + 6 \sin 2t)\, e^{-2t}$

8.24) $s^2 + 2s + 5 = 0$, $\quad s_{12} = \dfrac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j4$

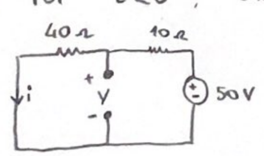
$i(t) = I_s + (A_1 \cos 4t + A_2 \sin 4t) . e^{-t}$, $\quad I_s = \frac{10}{5} = 2$

$i(0) = 2 = 2 + A_1 \Rightarrow A_1 = 0$

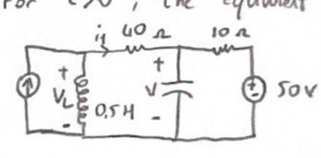
$$\frac{di}{dt} = (A_2 \cos 4t)e^{-t} + (-A_2 \sin 4t)\, e^{-t} = 4 = 4A_2 \Rightarrow A_2 = 1$$

$$i(t) = 2 + \sin 4t e^{-t}$$

8.28) For $t<0$, the equivalent circuit:



For $t>0$, the equivalent circuit:



By KVL

$V(0^+) = V(0^-) = 40$, $\quad i(0^+) = i(0^-) = 1$

By KCL

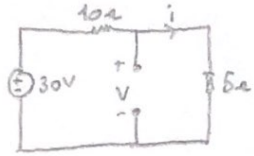
$2 = i(0^+) + i_1 = 1 + i_1 \Rightarrow i_1 = 1$

By KVL

$-V_L + 40 i_1 + V(0^+) = 0 \Rightarrow V_L(0^+) = 40.1 + 40 = 80$

$V_L(0^+) = 80\text{ V}$, $\quad V_C(0^+) = 40\text{V}$

8.30) Transform the current sources to voltage sources



$i(0) = \frac{30}{15} = 2A$, $V(0) = \frac{5.30}{15} = 10\ V$

For $t > 0$, series RLC circuit

$a = \frac{R}{2L} = \frac{5}{2} = 2,5$

$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4}} = 0,5$ $\quad a > w_0$ (overdamped)

$s_{1,2} = -2,5 \pm \sqrt{6.25 - 0,25} = -4,95\ ,\ -0,0505$

$V(t) = V_s + (A_1 . e^{-4,95t} + A_2 . e^{-0,0505 . t})\ ,\quad V_s = 20$

$V(0) = 10 = 20 + A_1 + A_2 \Rightarrow A_2 = -10 - A_1$

$i(0) = C . \frac{dv(0)}{dt}$ $\quad \frac{d(V_0)}{dt} = \frac{2}{4} = \frac{1}{2}$

$0,5 = -4,95 A_1 - 0,0505 A_2$

$A_1 = 0,001125\ ,\ A_2 = -10,001$

$V(t) = 20 + 0,001125 e^{-4,95t} - 10,001 e^{-0,05t}$

8.32) At $t = 0^-$, $i_L(0) = 0$, $v(0) = V_c(0) = 8V$

For $t > 0$, series RLC circuit with step input

$a = \frac{R}{2L} = \frac{2}{2} = 1$, $w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1/5}} = \sqrt{5}$

$s_{1,2} = -1 \pm j.2$

$V(t) = V_s + (A \cos 2t + B \sin 2t) . e^{-t}$, $V_s = 12$

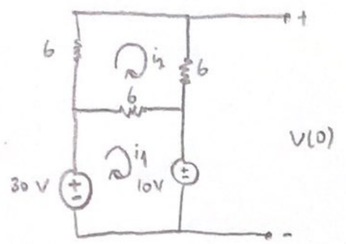
$V(0) = 8 = 12 + A \Rightarrow A = -4$, $i(0) = C\frac{dv(0)}{dt} = 0$

$\frac{dv}{dt} = -(A \cos 2t + B \sin 2t) e^{-t} + 2(-A \sin 2t + B \cos 2t) e^{-t}$

$0 = \frac{dv(0)}{dt} = -A + 2B \Rightarrow 2B = A \Rightarrow B = -2$

$V(t) = 12 - (4 \cos 2t + 2 \sin 2t) e^{-t}$

8.34) For $t=0^-$, the equivalent circuit:



$18i_2 - 6i_1 = 0$

$-30 + 6(i_1 - i_2) + 10 = 0$

$i_1 = 5A, \; i_2 = \frac{5}{3}A$

$i(0) = i_1 = 5A$

$-10 - 6i_2 + V(0) = 0 \Rightarrow V(0) = 20 \text{ V}$

For $t>0$, series RLC circuit

$R = 6 \parallel 12 = 4$

$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2}\cdot\frac{1}{8}}} = 4$

$a = \frac{R}{2L} = \frac{4}{2.\frac{1}{2}} = 4$ $\quad a = w_0$ (critically damped)

$V(t) = V_s + (A + Bt)\, e^{-4t}$, $V_s = 10$

$V(0) = 20 = 10 + A \Rightarrow A = 10$

$i_c = \frac{C.dv}{dt} = C\,[-4.(10+Bt)\,e^{-4t}] + C.(B.e^{-4t})$

$i_c(0^+) = -i_L(0) = -5A$

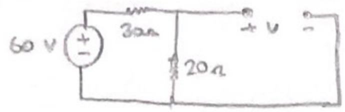
$i_c(0) = -5 = C(-40+B) \Rightarrow -40 = -40 + B \Rightarrow B=0$

$i_c = \frac{C.dv}{dt} = \frac{1}{8}.(-4(10+0.t)e^{-4t}) + \frac{1}{8}.(0.e^{-4t})$

$i_c(t) = \frac{-1}{2} . 10e^{-4t}$

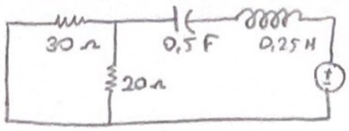
$i(t) = -i_c(t) = 5.e^{-4t}$

8.36) For $t = 0^-$, the equivalent circuit is:



$$V(0) = \frac{20}{50} \cdot 60 = 24 \quad \text{and} \quad i(0) = 0$$

For $t>0$, the equivalent circuit is:



$$R = 20 \| 30 = 12\ \Omega$$

$$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2} \cdot \frac{1}{4}}} = \sqrt{8}$$

$$a = \frac{R}{2L} = 12 \cdot 0,5 = 24$$

$a > w_0$ (overdamped response)

$$s_{1,2} = -47,833 \ , \ -0,167$$

$$V(t) = V_S + (A \cdot e^{-47,833t} + B e^{-0,167t}), \quad V_S = 30$$

$$V(0) = 24 = 30 + A + B$$

$$i(0) = C \cdot \frac{dv(0)}{dt} = 0$$

$$\frac{dv(0)}{dt} = -47,833A - 0,167B = 0 \Rightarrow B = -286,43\ A$$

$A = 0,021$

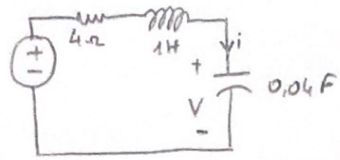
$B = -6,021$

$$V(t) = 30 + (0,021 e^{-47,833t} - 6,021 e^{-0,167t})$$

8.38) At $t=0^-$, the switch is open. $i(0)=0$ and $V(0) = \frac{5.100}{20+5+5} = \frac{50}{3}$

For $t>0$, series RLC circuit

After source transformation



$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.\frac{1}{25}}} = 5$ $\qquad s_{1,2} = -2 \pm j.\,4,583$

$\alpha = \frac{R}{2L} = \frac{4}{2.1} = 2$

$v(t) = V_s + (A\cos w_d t + B\sin w_d t).e^{-2t}$

$w_d = 4,583$, $V_s = 20$

$V(0) = \frac{50}{3} = 20 + A \Rightarrow A = \frac{-10}{3}$

$i(t) = C.\frac{dv}{dt} = C(-2)\,(A\cos w_d t + B\sin w_d t)e^{-2t} + C w_d\,(-A\sin w_d t + B\cos w_d t).e^{-2t}$

$i(0) = 0 = -2A + w_d B$

$B = \frac{2A}{w_d} = -1,455$

$i(t) = 0,7275\sin(4,583)e^{-2t}$

8.40) $w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.0,5}} = \sqrt{2}$

$\alpha = \frac{1}{2RC} = \frac{1}{2.2.0,5} = 0,5$, $\quad \alpha < w_0$ (underdamped response)

$s_{1,2} = -0,5 \pm j.\,1,3229.$

$i(t) = I_s + (A\cos 1,3229t + B\sin 1,3229t).e^{-0,5t}$, $I_s = 4$

$i(0) = 1 = 4 + A \Rightarrow A = -3$

$V = V_C = V_L = L\frac{di(0)}{dt} = 0$

$\frac{di}{dt} = 1,3229.(-A\sin 1,3229t + B\cos 1,3229t)\,e^{-0,5t} + [-0,5(A\cos 1,3229t + B\sin 1,3229t).e^{-0,5t}]$

$\frac{di(0)}{dt} = 0 = 1,3229B - 0,5A \Rightarrow B = -1,1339$

Thus

$i(t) = 4 - (3\cos 1{,}3229t + 1{,}1339 \sin 1{,}1339t)\, e^{-t/2}$

$v(t) = v_L(t) = L \cdot \dfrac{di(t)}{dt}$

$v(t) = (4{,}536 \sin 1{,}1323t) \cdot e^{\frac{-t}{2}}$

8.42) At $t = 0^-$, we obtain

$i_L = 0 = \dfrac{3 \cdot 5}{10+5} = 1A$, $V_o(0) = 0$

For $t > 0$, parallel RLC circuit

$a = \dfrac{1}{2RC} = \dfrac{1}{2 \cdot 5 \cdot 0{,}01} = 10$

$w_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{1 \cdot 0{,}01}} = 10$, $a = w_0$ (critically damped response)

$S_{1,2} = -10$

$i(t) = I_S + (A + Bt)e^{-10t}$, $I_S = 3$

$i(0) = 1 = 3 + A \Rightarrow A = -2$

$V_o = \dfrac{Ldi}{dt} = Be^{-10t} + (-10) \cdot (A + Bt)e^{-10t}$

$V_o(0) = 0 = B - 10A \Rightarrow B = -20$

$V_o(t) = 200te^{-10t}$

8.44) For $t = 0^-$, $i(0) = 3 + \dfrac{12}{4} = 6$, $v(0) = 0$

For $t > 0$, parallel RLC circuit with step input.

$a = \dfrac{1}{2RC} = \dfrac{1}{2 \cdot 5 \cdot 0{,}05} = 2$

$w_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{5 \cdot 0{,}05} = 2$ $a = w_0$ (critically damped response)

$S_{1,2} = -2$

$i(t) = I_S + (A + Bt)\,e^{-2t}$, $I_S = 3$

$i(0) = 6 = 3 + A \Rightarrow A = 3$

$V = L\dfrac{di}{dt} \Rightarrow \dfrac{V}{L} = \dfrac{di}{dt} = Be^{-2t} + (-2)(A + Bt)e^{-2t}$

$\dfrac{V(0)}{L} = 0 = \dfrac{di(0)}{dt} = B - 2 \cdot 3 \Rightarrow B = 6$, $i(t) = 3 + (3 + 6t)e^{-2t}$

8.46) Let i inductor current and v capacitor voltage

At $t=0$, $v(0)=0$, $i(0)=i_0$

For $t>0$, parallel, source-free LC circuit

$$a=\frac{1}{2RC}=0 \quad \text{and} \quad w_0=\frac{1}{\sqrt{LC}}, \quad s_{1,2}=\pm j w_0$$

$$v=A\cos w_0 t+B\sin w_0 t, \quad v(0)=0$$

$$i_C=\frac{C\,dv}{dt}=-i$$

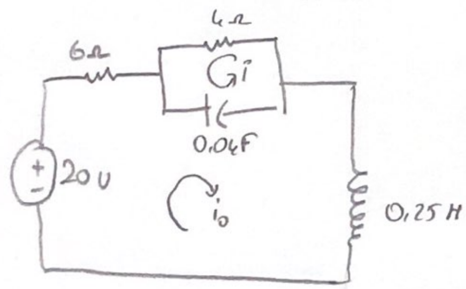
$$\frac{dv}{dt}=w_0 . B\sin w_0 t=\frac{-i}{C}$$

$$\frac{dv(0)}{dt}=w_0 B=\frac{-i_0}{C} \Rightarrow B=\frac{i_0}{w_0 C}$$

$$v(t)=-\left(\frac{i_0}{w_0 . C}\right) . \sin w_0 t$$

8.50) For $t<0$, $i(0)=0$, $v(0)=0$

For $t>0$, the circuit



By KVL

$$-20+6i_0+0{,}25\frac{di}{dt}+25.\int(i_0+i)dt=0$$

Smaller loop: $4i+25\int(i+i_0)\,dt=0 \Rightarrow \int(i+i_0)dt=-0{,}16i$

$$\frac{4di}{dt}+25(i+i_0)=0 \Rightarrow i_0=-0{,}16\frac{di}{dt}-i, \quad \frac{di_0}{dt}=-0{,}16\frac{d^2i}{dt^2}-\frac{di}{dt}$$

$$-20-0{,}96\frac{di}{dt}-6i-0{,}04\frac{d^2i}{dt^2}-0{,}25\frac{di}{dt}-4i=0$$

$$\frac{d^2i}{dt^2}+30.25\frac{di}{dt}+250i=-500$$

$$s^2+30.25s+250=0 \Rightarrow s_{1,2}=\frac{-30.25\pm\sqrt{(30.25)^2-1000}}{2}=-15{,}125\pm j.\,4{,}608$$

(underdamped response)

Thus, $i(t) = I_s + e^{-15,125t}\left(A_1 \cdot \cos(4,608t) + A_2 \cdot \sin(4,608t)\right)$

At $t=0$, $i_o(0)$ and $i(0) = 0 = I_s + A_1 \Rightarrow A_1 = -I_s$

$i(\infty) = \frac{20}{10} = 2A = i(-\infty) \Rightarrow i(\infty) = -2A = I_s \Rightarrow A_1 = 2$

$\frac{di(0)}{dt} = -6,25 i_o(0) - 6,25 i(0) = 0$

$i(t) = -2 + e^{-15,125t} \cdot \left(2\cos(4,608t) + 6,565 \sin(4,608t)\right)$

8.52) At $t = 0^-$, $4u(t) = 0$ so

$i_1(0) = 0 = i_2(0)$

$4 = 0{,}5 \frac{di_1}{dt} + i_1 + i_2$ (nodal analysis) (2)

Also $i_2 = \left(\frac{di_1}{dt} - \frac{di_2}{dt}\right) / 3 \Rightarrow 3i_2 = \frac{di_1}{dt} - \frac{di_2}{dt}$ (3)

Taking derivative of (2), $0 = \frac{d^2 i_1}{dt_2} + 2\frac{di_1}{dt} + 2\frac{di_2}{dt}$ (4)

From (2) and (3)

$\frac{di_2}{dt} = \frac{di_1}{dt} - 3i_2 = \frac{di_1}{dt} - 3\left(4 - i_1 - 0{,}5\frac{di_1}{dt}\right)$

Substituting this into 4, then

$s^2 + 7s + 6 = 0 \Rightarrow s_{1,2} = -1, -6$

$i_1(t) = I_s + A \cdot e^{-t} + B \cdot e^{-6t}$, $6I_s = 24 \Rightarrow I_s = 4$

$i_1(t) = 4 + Ae^{-t} + B \cdot e^{-6t}$, $i_1(0) = 4 + A + B$ (5)

$i_2 = 4 - i_1 - 0{,}5\frac{di_1}{dt} \Leftarrow i_1(t) = 4 + (-4) - (Ae^{-t} + B \cdot e^{-6t}) - (-Ae^{-t} - 6 \cdot Be^{-6t})$

$= (-0{,}5Ae^{-t} + 2Be^{-6t})$, $i_2(0) = 0 = -0{,}5A + 2B$ (6)

From 5 and 6, $A = -3{,}2$, $B = -0{,}8$

$i_2(t) = 1{,}6\, e^{-t} - 1{,}6 \cdot e^{-6t}$

8.54) Parallel RLC circuit

$$a = \frac{1}{2RC} = \frac{1}{2 \cdot 3 \cdot \frac{1}{18}} = 3$$

$$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \cdot \frac{1}{18}}} = 3 \qquad a = w_0 \text{ (critically damped response)}$$

$$s_{1,2} = -3$$

$$V(t) = V_s + (A + Bt)e^{-3t} \quad , \quad V_s = 0$$

$$-10 + V_R + V = 0 \quad \Rightarrow \quad V_R = 10 - V$$

$$V_R = 10 - (A + Bt)e^{-3t}$$