Abdullah Çelik
171044002

CSE 351 - Signals and Systems, Spring 2020
Homework #2
Solutions

1) a) $f(t) = e^{-3t} u(t)$ so $F(s) = \frac{1}{s+3}$

$$Y(s) = H(s) . F(s) = \frac{(2s+3)}{(s+3)(s^2+5s+6)} \rightarrow \frac{(2s+3)}{(s+3)^2(s+2)}$$

$$= \frac{k}{s+2} + \frac{a_0}{(s+3)^2} + \frac{a_1}{s+3} \qquad k = \left.\frac{2s+3}{(s+3)^2}\right|_{s=-2} \rightarrow \frac{2.(-2)+3}{(-2+3)^2} = -1$$

$$a_0 = \left.\frac{2s+3}{s+2}\right|_{s=-3} \rightarrow \frac{2.(-3)+3}{-3+2} = \frac{-3}{-1} = 3$$

$$Y(s) = \frac{1}{s+2} + \frac{3}{(s+3)^2} + \frac{a_1}{s+3} = \frac{2s+3}{(s+3)^2(s+2)}$$

multiply s;

$$\rightarrow \lim_{s\to\infty} \frac{s}{s+2} + \lim_{s\to\infty} \frac{s.3}{(s+3)^2} + \lim_{s\to\infty} \frac{a_1}{s+3} = \lim_{s\to\infty} \frac{(2s+3).s}{(s+2)(s+3)^2}$$

$$\rightarrow -1 + 0 + a_1 = 0 \rightarrow a_1 = 1$$

$$Y(s) = \frac{-1}{s+2} + \frac{3}{(s+3)^2} + \frac{1}{s+3}$$

$$y(t) = \left[-e^{-2t} + (1+3t)\, e^{-3t}\right] u(t)$$

b)

$$y(s) = \left(\frac{2s+3}{s^2+5s+6}\right) F(s) \qquad y(s).(s^2+5s+6) = (2s+3).F(s)$$

$$\frac{d^2y}{dt^2} + \frac{5dy}{dt} + 6y(t) = 2\frac{df}{dt} + 3f(t)$$

c) $H(s) = \dfrac{(s+2)}{s(s+1)^2} \rightarrow \dfrac{k}{s} + \dfrac{a_0}{(s+1)^2} + \dfrac{a_1}{s+1}$

$$K = \left.\frac{(s+2)}{(s+1)^2}\right|_{s=0} = \frac{2}{1} = 2$$

$$a_0 = \left.\frac{s+2}{s}\right|_{s=-1} = \frac{1}{-1} = -1$$

$$H(s) = \frac{2}{s} + \frac{-1}{(s+1)^2} + \frac{a_1}{s+1}$$

Multiply s;

$$\lim_{s\to\infty} \frac{2.s}{s} - \lim_{s\to\infty} \frac{1.s}{(s+1)^2} + \lim_{s\to\infty} \frac{s.a_1}{s+1} = \lim_{s\to\infty} \frac{s.(s+2)}{s.(s+1)^2}$$

$$2 - 0 + a_1 = 0$$

$$a_1 = -2$$

$$H(s) = \frac{2}{s} - \frac{1}{(s+1)^2} - \frac{2}{s+1}$$

$$h(t) = \left[2 - (2+t)\,e^{-t}\right] u(t)$$

2) Delay forms:

$y[k] \to y[z]$

$y[k-1] \to \frac{1}{z} y[z]$

$y[k-2] \to \frac{1}{z^2} y[z] + 1$

$$2\, y[z] - \frac{3}{z} y[z] + \frac{1}{z^2} y[z] + 1 = \frac{4z}{z-0{,}25} - \frac{3}{z-0{,}25} = \frac{4z-3}{z-0{,}25}$$

$$\left(2 - \frac{3}{z} + \frac{1}{z^2}\right) y[z] = -1 + \frac{4z-3}{z-0{,}25} = \frac{3z-2{,}75}{z-0{,}25}$$

$$\frac{y(z)}{z} = \frac{z.(3z-2{,}75)}{(2z^2-3z+1)(z-0{,}25)} = \frac{z(3z-2{,}75)}{2(z-0{,}5)(z-1)(z-0{,}25)}$$

$$\frac{y(z)}{z} = \frac{\frac{5}{2}}{z-\frac{1}{2}} + \frac{\frac{1}{3}}{z-1} - \frac{\frac{4}{3}}{z-0{,}25}$$

$$y[k] = \left(\frac{1}{3} + \frac{5}{2}.2^{-k} - \frac{1}{3}.4^{-k}\right).u[k]$$

3)

$$\frac{H(z)}{z} = \frac{-5z+22}{(z+1)(z-2)^2} = \frac{3}{z+1} + \frac{k}{z-2} + \frac{4}{(z-2)^2}$$

Limit and multiply z.

$$\lim_{z\to\infty} \frac{3z}{z+1} + \lim_{z\to\infty} \frac{z.k}{z-2} + \lim_{z\to\infty} \frac{4.z}{(z-2)^2} = \lim_{z\to\infty} \frac{(-5z+22).z}{(z+1)(z-2)^2}$$

$3 + k + 0 = 0 \to k = -3$

$$H(z) = 3\frac{z}{z+1} - 3\frac{z}{z-2} + 4\frac{z}{(z-2)^2} \qquad H[z] \to h[k]$$

$$h[k] = (3(-1)^k - 3.2^k + 2k.2^k)\, u[k]$$