POLS 5377 Scope & Method of Political Science

Week 13 Hypothesis Testing IV

Chi Square

Healey. (2016) *Statistics: A Tool for Social Research,* Chapter 11

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Key Questions:

- What is a cross-tabulation table?
- What is the logic of chi square?
- How to conduct a chi square hypothesis test, and interpret the results?
- How to conduct chi square tests and interpret the results within SPSS?
- What are the limitations of using chi square tests?

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Outline

- Bivariate (Cross tabulation) Tables
- Logic of Chi Square
- The Computation of Chi Square
- The Chi Square Test within SPSS
- Limitation of the Chi Square Test

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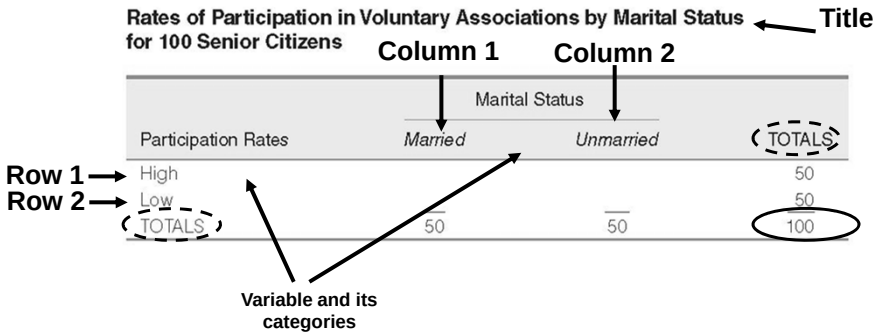
Bivariate Table

- Bivariate table = Cross tabulation table = Contingency table
  - display the scores of cases on two different variables at the same time
  - A researcher is studying membership in voluntary associations and hypothesizes that unmarried people will be more involved because they have fewer family obligations and more free time.



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Bivariate Table

Title			
Rows (**Dependent Variable**)	Columns (**Independent Variable**)		Total
Row 1	cell a	cell b	Row **Marginal 1**
Row 2	cell c	cell d	Row **Marginal 2**
Total	Column **Marginal 1**	Column **Marginal 2**	N

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Bivariate Table

- There are two dimensions: rows and columns
  - Columns are scores of the *independent* variable
    - There will be as many columns as there are categories on the independent variable
  - Rows are scores on the *dependent* variable
    - There will be as many rows as there are categories on the dependent variable
  - There will be as many cells as there are categories on the two variables combined
  - Each cell reports the number of times each combination of categories occurred

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Logic of Chi Square (χ^2)

- Chi Square as a test of statistical significance is a test for **independence**
  - Independence: *Two* variables are independent if the classification of a case into a particular category of one variable has no effect on the probability that the case will fall into any particular category of the second variable" (p. 274 in 9th edition; p. 279 in 10th edition)
- The chi square test is a procedure for evaluating the level of statistical significance of a relationship between *two variables* in a bivariate table or contingency table (also called crosstabs).
- The chi square procedure assumes that there is **no relationship** between two variables in the population (null hypothesis).

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Logic of Chi Square (χ^2)

- We are looking for significant differences between the actual cell frequencies observed in a table (f_o) and those that would be expected by random chance or if cell frequencies were independent (f_e)

$$\chi^2(obtained) = \sum \frac{(f_o - f_e)^2}{f_e}$$

- f_o: the cell frequencies observed in the bivariate table
- f_e : the cell frequencies that would be expected if the variables were independent

$$f_e = \frac{\text{Row marginal} \times \text{Column marginal}}{N}$$

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Logic of Chi Square (χ^2)

- Example
  - Is there a relationship between the **accreditation status** of social work programs and the **employment status** of their students?
  - 100 students were selected and asked 1) if they graduate from a accredited / non-accredited program? 2) if they are working as a social worker?

Employment of 100 Social Work Majors by Accreditation Status of Undergraduate Program

Dependent variable ↓ / Independent variable → Employment Status	Accreditation Status: *Accredited* (Observed Frequencies)	*Not Accredited*	TOTALS
Working as a social worker	30	10	40
Not working as a social worker	25	35	60
TOTALS	55	45	100

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Logic of Chi Square (χ^2)

- Expected frequency (f_e) for the top-left cell

$$f_e = \frac{\text{Row marginal} \times \text{Column marginal}}{N} = \frac{40 \times 55}{100} = 22$$

Expected Frequencies for Table 11.3

Employment Status	Accreditation Status: *Accredited* (Expected Frequency)	*Not Accredited*	TOTALS
Working as a social worker	22	18	40
Not working as a social worker	33	27	60
TOTALS	55	45	100

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Logic of Chi Square (χ^2)

- Compute obtained χ^2

$$\chi^2(obtained) = \sum \frac{(f_o - f_e)^2}{f_e}$$

Computational Table for Table 11.3

(1)	(2)	(3)	(4)	(5)
f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$	$(f_o - f_e)^2/f_e$
30	22	8	64	2.91
10	18	−8	64	3.56
25	33	−8	64	1.94
35	27	8	64	2.37
100	100	0		10.78

Obtained χ^2

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Five-Step Model - Chi Square

Step 1: Make Assumptions and Meet Test Requirements

- Independent random samples
- Level of Measurement is nominal
  - Accredited program / Non-accredited program
  - Working as a social worker / not working as a social worker

(Note: In chi square hypothesis test, we don't need to assume the normal distribution of the population or sampling distribution)

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Five-Step Model - Chi Square

Step 2: State the Null Hypothesis & Alternative Hypothesis

- H_o: $f_o = f_e$ The accreditation status of program and the student employment status are **independent**
  - Null hypothesis always asserts the variables are independent
- H_1: $f_o \neq f_e$ The accreditation status of program and the student employment status are **dependent**
  - Alternative hypothesis always contradicts to null hypothesis, and asserts the variables are dependent

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Five-Step Model - Chi Square

Step 3: Select the Sampling Distribution and Establish the Critical Region

- Sampling Distribution = Chi Square distribution (Appendix C)
- Alpha = 0.05
- Degree of freedom (df) = (Number of rows - 1)* (Number of colums-1) = (2-1)*(2-1)=1
- χ^2(critical) = 3.841
  - The score marks the beginning of the critical region

df	0.99	…	0.05	…
1	…	…	**3.841**	…
2	…	…	5.991	…
3	…	…	7.815	…

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Five-Step Model - Chi Square

Step 4: Calculate the Test Statistic

- Calculate χ^2(obtained) with the formula:

$$\chi^2(obtained) = \sum \frac{(f_o - f_e)^2}{f_e}$$

- In the example of program accreditation status and student employment status, χ^2(obtained) = 10.78

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Five-Step Model - Chi Square

Step 5: Make a Decision and Interpret the Results of the Test

- χ^2(critical) = 3.841
- χ^2(obtained) = 10.78
- The obtained χ^2 score falls in the critical region, so **reject** H_0
  - There is a significant relationship between the two variables.
- Report results:
  - At the significance level of 0.05, the obtained χ^2 score falls in the critical region, so we **reject** H_0. The data suggests there is a significant relationship between employment status and accreditation status in the population from which the sample was drawn.

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Interpreting Chi Square

- Need to interpret chi square test results with the following understanding:
  - The chi square test tells us ONLY if the variables are independent or not
  - It does not tell us the pattern or nature of the relationship
  - To investigate the pattern, we need to compute the percentages within each column and compare across the columns

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Interpreting Chi Square

- Column percentage make the relationship between the two variables more obvious:
  - The students from accredited programs are more likely to be working as social workers.
  - 55% of the students from accredited programs working as social works versus only 22% of the students from non-accredited programs working as social workers
  - According to the test results, we know this relationship is significant. It doesn't occur by chance.

Employment status	Accreditation Status		
	Accredited	Not accredited	
Working as a social worker	30 (54.55%) → **(30/55)*100%**	10 (22.22%) → **(10/45)*100%**	40
Not working as a social worker	25 (45.55%) → **(25/55)*100%**	35 (77.78%) → **(35/45)*100%**	60
	55 (100%)	45 (100%)	100

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Chi Square with SPSS

- In practice, we usually don't calculated a chi square by hand, and we rely on statistical software such as SPSS.
- Now I will show you how to conduct a chi square test with SPSS and interpret the results
- We want to examine the relationships between the **education level** and individuals' **tolerance toward Muslim clergy** in U.S. society.
- Conduct "Crosstabs" procedure in SPSS (pg. 301@10th ed. OR pg. 298 @ 9th ed.)
  - Dataset: GSS2012_Student_B
  - [Analyze] → [Descriptive Statistics] → [Crosstabs]
  - Place dependent variables in the **Row** box
  - Place independent variables in the **Column** box
  - Click the **Statistics** button, select **Chi-square**
  - Click the **Cells** button, select **column** in the **Percentage** box
  - Proceed the analysis

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Chi Square with SPSS

- Dependent variable: Tolerance toward Muslim clergy (muslim_tol)
- Independent variable: Education (educ_2)

Step 1: Meet requirements

- Independent random sample
- Level of measurement: nominal

Step 2: State Null hypothesis and alternative hypothesis

- H_o: $f_o = f_e$ The tolerance toward Muslim clergy and the education level are **independent**
- H_1: $f_o \neq f_e$ The tolerance toward Muslim clergy and the education level are **dependent**

Step 3: Critical Region

- α=0.05

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Chi Square with SPSS

Tolerance twrd Muslim clergy * Education: 2 Cats Crosstabulation

			Education: 2 Cats		
			0-12 yrs	13+ yrs	Total
Tolerance twrd Muslim clergy	Low	Count	230	173	403
		% within Education: 2 Cats	57.2%	32.3%	43.0%
	Middle	Count	97	173	270
		% within Education: 2 Cats	24.1%	32.3%	28.8%
	High	Count	75	190	265
		% within Education: 2 Cats	18.7%	35.4%	28.3%
Total		Count	402	536	938
		% within Education: 2 Cats	100.0%	100.0%	100.0%

- Degree of freedom (df) = (R-1)*(C-1)=(3-1)*(2-1)=2
- χ^2(obtained) = 61.472
- Test result: significance value = .000 < α=0.05
  - Reject H_0
  - There is a significant relationship between the two variables.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	61.472[a]	2	.000
Likelihood Ratio	62.169	2	.000
Linear-by-Linear Association	57.867	1	.000
N of Valid Cases	938		

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 113.57.

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Chi Square with SPSS

- Interpret the result:
  - At the significance level of 0.05, the test of chi square is significant, so we **reject** H_0. The data suggests there is a significant relationship between tolerance toward Muslim clergy and education level in the U.S society.
  - According to the column percentages, the citizens with lower education level are more likely to have a low tolerance toward Muslim clergy.
  - 57.2% of the individuals who received 0 to 12 years education have low tolerance toward Muslim clergy, versus 32.3% of those received 13 years or more education.

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Limitation of Chi Square

- Difficult to interpret when variables have many categories
  - Best when variables have four or fewer categories
- With small sample size, cannot assume that Chi Square sampling distribution will be accurate
  - Small sample: High percentage of cells have expected frequencies of 5 or less
- Like all tests of hypotheses, Chi Square is sensitive to sample size
  - As N increases, obtained Chi Square increases
  - With large samples, trivial relationships may be significant

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After this lecture:

You should learn the following key concepts:

- How to construct and interpret a crosstab table
- The logic of chi square test
- The condition in which conducting a chi square test is appropriate
- How to compute and test chi square manually and with SPSS
- How to interpret the chi square test results
- Limitation of chi square test