Örnek $\int (x+1)^4 dx = ?$

Çözüm: $x+1 = u \Rightarrow dx = du$

$$\int (x+1)^4 dx = \int u^4 du = \frac{u^5}{5} + c = \frac{(x+1)^5}{5} + c$$

Örnek $\int \frac{(\ln x)^4}{x} dx = ?$

Çözüm: $\ln x = u \Rightarrow \frac{dx}{x} = du$

$$\int \frac{(\ln x)^4}{x} dx = \int u^4 du = \frac{u^5}{5} + c = \frac{(\ln x)^5}{5} + c$$

Örnek $\int \frac{x}{(x^2-1)^4} dx = ?$

Çözüm: $x^2 - 1 = u \Rightarrow 2x dx = du \Rightarrow x dx = \frac{du}{2}$

$$\int \frac{x dx}{(x^2-1)^4} = \int \frac{\frac{du}{2}}{u^4} = \frac{1}{2}\int u^{-4} du = \frac{1}{2}\frac{u^{-4+1}}{-4+1} + c = -\frac{1}{6(x^2-1)^3} + c$$

Örnek $\int \frac{\sin x \, dx}{\cos^3 x} = ?$ III. yol: $\int \frac{\sin x}{\cos x}\frac{dx}{\cos^2 x} = \int \tan x . \frac{dx}{\cos^2 x} = \frac{\tan^2 x}{2} + c$

Çözüm: $u = \cos x \Rightarrow du = -\sin x dx \Rightarrow \sin x dx = -du$

$$\int \frac{\sin x \, dx}{\cos^3 x} = -\int \frac{du}{u^3} = -\int u^{-3} du = \frac{1}{2u^2} + c = \frac{1}{2\cos^2 x} + c$$

II. yol: $u = \sin x \Rightarrow du = \cos x \, dx$

$$\int \frac{\sin x dx}{\cos^3 x} = \int \frac{\sin x . \cos x \, dx}{\cos^4 x} = \int \frac{u \, du}{(1-u^2)^2} \Rightarrow 1-u^2 = t \Rightarrow -2u du = dt$$

$$= -\frac{1}{2}\int \frac{dt}{t^2} = +\frac{1}{2t} + c = \frac{1}{2(1-u^2)} + c = \frac{1}{2(1-\sin^2 x)} + c = \frac{1}{2\cos^2 x} + c$$

Örnek $\int \frac{dx}{\sqrt{2-x}} = ?$

Çözüm: $2 - x = u \Rightarrow -dx = du \Rightarrow dx = -du$

$$\int \frac{dx}{\sqrt{2-x}} = -\int \frac{du}{\sqrt{u}} = -2\sqrt{u} + c = -2\sqrt{2-x} + c$$

Örnek $\int \frac{2x-1}{x^2-x+1}\,dx = ?$ Not: $\int \frac{f'(x)dx}{f(x)} = \ln|f(x)|+c$

Çözüm: $x^2-x+1 = u \Rightarrow (2x-1)dx = du$

$$\int \frac{(2x-1)dx}{x^2-x+1} = \int \frac{du}{u} = \ln|u|+c = \ln|x^2-x+1|+c$$

Örnek $\int \left(\frac{3}{2x-1} - \frac{5}{4x+2}\right)dx = ?$

Çözüm:

$$3\int \frac{dx}{2x-1} - 5\int \frac{dx}{4x+2} = \frac{3}{2}\int \frac{2dx}{2x-1} - \frac{5}{4}\int \frac{4dx}{4x+2}$$

$$= \frac{3}{2}\ln|2x-1| - \frac{5}{4}\ln|4x+2|+c$$

Örnek $\int \frac{e^x dx}{3-2e^x} = ?$

Çözüm: $3-2e^x = u \Rightarrow -2e^x dx = du \Rightarrow e^x dx = -\frac{du}{2}$

$$\int \frac{e^x dx}{3-2e^x} = -\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2}\ln|u|+c = -\frac{1}{2}\ln|3-2e^x|+c$$

Örnek $\int \tan x\,dx = ?$

Çözüm: $\int \frac{\sin x\,dx}{\cos x} = ?$ $\cos x = u \Rightarrow -\sin x\,dx = du$

$$\int \frac{\sin x\,dx}{\cos x} = -\int \frac{du}{u} = -\ln|u|+c = -\ln|\cos x|+c$$

Örnek $\int x e^{x^2} dx = ?$

Çözüm: $x^2 = u \Rightarrow 2x\,dx = du \Rightarrow x\,dx = \frac{du}{2}$

$$\int x e^{x^2} dx = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + c = \frac{1}{2}e^{x^2}+c$$

Örnek $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = ?$

Çözüm: $\sqrt{x} = u \Rightarrow \frac{dx}{2\sqrt{x}} = du \Rightarrow \frac{dx}{\sqrt{x}} = 2du$

$$\int \frac{e^{\sqrt{x}}dx}{\sqrt{x}} = 2\int e^{u}du = 2e^{u} + c = 2e^{\sqrt{x}} + c$$

Örnek $\int e^{3x}dx = ?$

Çözüm: $3x = u \Rightarrow 3dx = du \Rightarrow dx = \frac{du}{3}$

$$\int e^{3x}dx = \frac{1}{3}\int e^{u}du = \frac{1}{3}e^{u} + c = \frac{1}{3}e^{3x} + c$$

Kural $\int e^{ax+b}dx = \frac{1}{a}e^{ax+b} + c$

Örnek $\int e^{2x}dx = \frac{1}{2}e^{2x} + c$

$$\int e^{\frac{x}{2}}dx = \frac{1}{\frac{1}{2}}e^{\frac{x}{2}} + c = 2e^{\frac{x}{2}} + c$$

$$\int e^{-x}dx = -e^{-x} + c$$

Örnek $\int \sin 2x\,dx = ?$

Çözüm: $2x = u \Rightarrow 2dx = du \Rightarrow dx = \frac{du}{2}$

$$\int \sin 2x\,dx = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u + c = -\frac{1}{2}\cos 2x + c$$

Kural $\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + c$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$$

Örnek $\int \cos 3x\,dx = \frac{1}{3}\sin 3x + c$

Örnek: $\int \frac{dx}{\cos^2 3x} = ?$

Çözüm: $3x = u \Rightarrow 3dx = du \Rightarrow dx = du/3$

$$\frac{1}{3}\int \frac{du}{\cos^2 u} = \frac{1}{3}\tan u + c = \frac{1}{3}\tan 3x + c$$

Örnek $\int \frac{x^3dx}{\sqrt{1-9x^8}} = ?$

Çözüm: $\int \frac{x^3dx}{\sqrt{1-(3x^4)^2}} = \frac{1}{12}\int \frac{du}{\sqrt{1-u^2}}$

$3x^4 = u$
$12x^3dx = du$

$= \frac{1}{12}\arcsin u + c = \frac{1}{12}\arcsin(3x^4) + c$

Kural $\int \frac{du}{\sqrt{k^2-u^2}} = \arcsin \frac{u}{k} + c$ (k sbt)

Örnek: $\int \frac{dx}{\sqrt{9-x^2}} = \arcsin \frac{x}{3} + c$

Örnek $\int \frac{dx}{\sqrt{4x-x^2}} = ?$

Çözüm: $4x - x^2 = -(x^2-4x) = -\left[(x-2)^2 - 4\right] = 4-(x-2)^2$

$\int \frac{dx}{\sqrt{4x-x^2}} = \int \frac{dx}{\sqrt{4-(x-2)^2}} = \int \frac{du}{\sqrt{4-u^2}} = \frac{1}{2}\arcsin\frac{u}{2} + c$

$= \frac{1}{2}\arcsin\frac{x-2}{2} + c$

Örnek $\int \frac{3x-2}{\sqrt{4-x^2}}dx = ?$

Çözüm: $\int \frac{3x-2}{\sqrt{4-x^2}}dx = 3\underbrace{\int \frac{xdx}{\sqrt{4-x^2}}}_{I_1} - 2\underbrace{\int \frac{dx}{\sqrt{4-x^2}}}_{I_2}$

I_1 'de $4-x^2 = u \Rightarrow -2xdx = du \Rightarrow xdx = -\frac{du}{2}$

$\Rightarrow 3\int \frac{x\,dx}{\sqrt{4-x^2}} = -\frac{3}{2}\int \frac{du}{\sqrt{u}} = -\frac{3}{2}\cdot 2\sqrt{u} = -3\sqrt{u} = -3\sqrt{4-x^2}$

$\Rightarrow \int \frac{(3x-2)dx}{\sqrt{4-x^2}} = -3\sqrt{4-x^2} - 2\arcsin\frac{x}{2} + c$