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Logic of Hypothesis Testing

Two hypotheses:

- Null Hypothesis (H_0)
  - "The difference is caused by random chance" ; H_0 always states there is "no significant difference"
  - H_0: There is no difference between citizens in our sample and all citizens in the State as a whole in terms of flu vaccination rate.
- Alternative hypothesis (H_1)
  - "The difference is real"; H_1 always contradicts H_0
  - H_1 : There is a difference between citizens in our sample and all citizens in the State as a whole in terms of flu vaccination rate.
- One (and only one) of these explanations *must* be true, but which one?

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Logic of Hypothesis Testing

- Assume the H_0 is true
  - What is the probability of getting the sample mean (48%) if the H_0 is true and all citizens received the campaign messages really have a mean of 43% vaccination rate?
  - If the probability is less than 0.05, reject the null hypothesis – which means reject the idea of there is no difference between the two groups
- Use the 0.05 value as a guideline to identify if the difference is rare or not.
- Use the normal curve table to determine the probability of getting the observed difference.
- If the observed value falls into the 0.05 area ($z > 1.96$; $z < -1.96$), the difference is large enough, that only 0.05 chance the H_0 is true.
- Reject H_0

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Logic of Hypothesis Testing

Example:

A researcher is studying the effectiveness of a rehabilitation center that treats alcoholics. The researcher sampled 127 people who were treated in the center.

Community (all alcoholics)	Sample drawn from all treated alcoholics
$\mu = 7.2$ days of absent per year $\sigma = 1.43$	$\bar{X} = 6.8$ days of absent per year N = 127

Question: Does the population of all treated alcoholics have different number of absent day than the community as a whole?

- What is the cause of the difference between 6.8 and 7.2?
  - Real difference or Random chance?

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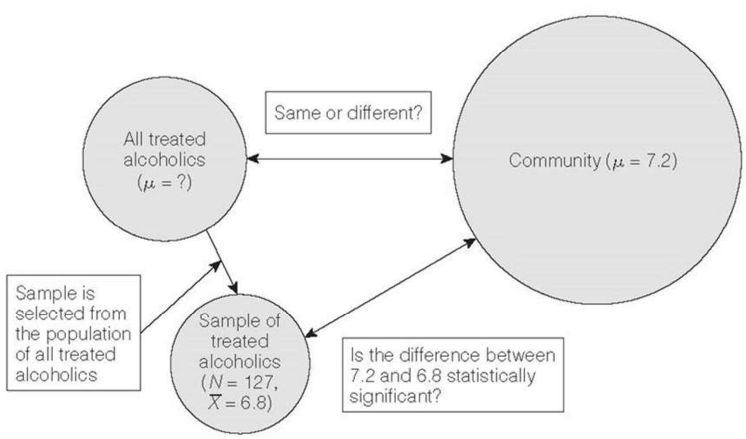
Logic of Hypothesis Testing

A Test of Hypothesis for Single-Sample Means



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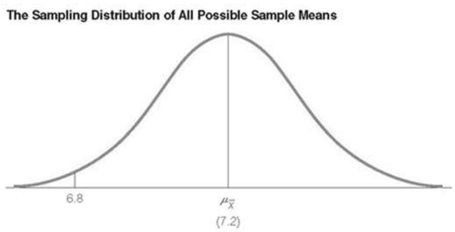
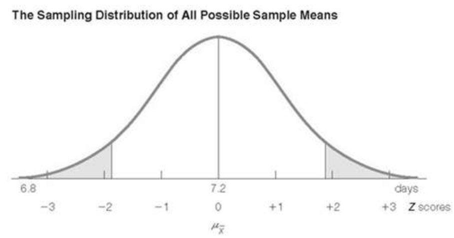
Logic of Hypothesis Testing

- H_0: There is no difference between treated alcoholics in our sample and the whole community in terms of number of absent day.
- H_1 : There is a difference between treated alcoholics in our sample and the whole community in terms of number of absent day.
- If H_0 is true, then the mean of the sampling distribution should be same as the population mean, $\mu = \mu_{\bar{X}} = 7.2$
- What is the probability of getting a sample that its mean ($\bar{X}$) equals 6.8?





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Logic of Hypothesis Testing

- Calculate the z score with the formula:

Sample mean **Population mean**

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{N}}$$

Population Standard Deviation **Standard deviation of sampling distribution**

$$Z = \frac{6.8 - 7.2}{1.4/\sqrt{127}} = \frac{-0.4}{0.127} = -3.15$$

$Reminder: Z = \frac{X_i - \bar{X}}{\sigma}$

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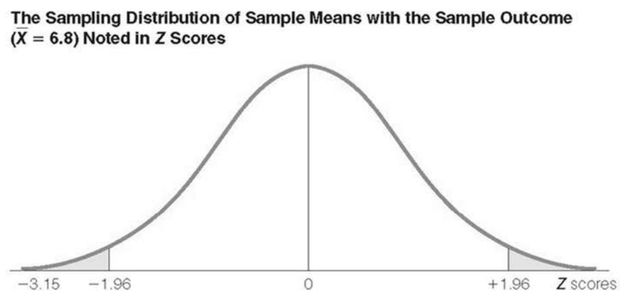
Logic of Hypothesis Testing

- With the sampling error 0.05 ($Z=\pm1.96$), the result $Z = -3.15$ shows the sample mean falls in the shaded area.
- This means, if H_0 is true, the probability of the sample mean to be 6.8 is less than 0.05, which is very low (H_0 is unlikely to be true).
- Therefore, we **reject** H_0 .



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Five-Step Model

- Make assumptions and meet test requirements
- State the null hypothesis (H_0) and alternative hypothesis (H_1)
- Select the sampling distribution and establish the critical region
- Compute the test statistic
- Make a decision and interpret the results of the test

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Five-Step Model - Example

- Citizens in a city complain about the poor design of the transportation system which has caused the long commute-to-work time.
- The nationwide average commute-to-work time is 25.3 minutes (μ).
- A random sample of 182 (N) residents in the city has a mean commute-to-work time 28.0 minutes, with a standard deviation (s) of 9.4 minutes.
- Is there a difference between the city and the nationwide average in terms of commute time?

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Five-Step Model - Example

Step 1: Make Assumptions and Meet Test Requirements

- Random sampling
  - Hypothesis testing assumes samples were selected according to EPSEM
  - The sample of 182 was randomly selected from all residents of the city
- Level of measurement is interval-ratio
  - Commute time measured in minutes is an interval-ratio level variable, so the mean is an appropriate statistic
- Sampling distribution is normal in shape
  - This is a “large” sample ($N \geq 100$)

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Five-Step Model - Example

Step 2: State the Null and Alternative hypothesis

- H_0: There is no difference between the city and the nationwide average in terms of commute-to-work time.
  - The sample of 182 comes from a population (the City citizens) that has an average commute-to-work time of 25.3 minutes
  - The difference between 25.3 and 28 is trivial and caused by random chance
- H_1: There is a difference between the city and the nationwide average in terms of commute-to-work time.
  - The sample of 182 comes from a population that does not have a commute-to-work time of 25.3 minutes
  - The difference between 25.3 and 28 reflects an actual difference of commute time between the city and nationwide.

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Five-Step Model - Example

Step 3: Select Sampling Distribution and Establish the Critical Region

- Sampling Distribution= Z
  - Alpha (α) = 0.05
  - Alpha is the indicator of “rare” events
  - Any difference with a probability less than α is rare and will cause us to reject the H_0
- Critical region begins at ±1.96
  - This is the critical Z score associated with a two-tailed test and α equal to 0.05
  - If the Z score of the sample mean falls in the critical region, reject H_0

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Five-Step Model - Example

Step 4: Compute the Test Statistic

- Sample size: 182 (>100, large sample)
- Population standard deviation (σ) : unknown
- Sample standard deviation (s): 9.4
- Formula:

$$Z = \frac{\bar{X} - \mu}{S/\sqrt{N-1}}$$

$$Z(obtained) = \frac{28.0 - 25.3}{9.4/\sqrt{182-1}} = 3.86$$

Z (obtained): 3.86

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Five-Step Model - Example

Step 5: Make a Decision and Interpret Results

- The obtained Z score fell in the critical region so we reject the H_0
  - If the H_0 were true, a sample outcome of 28.0 would be unlikely
  - Therefore, the H_0 is false and must be rejected
- Report result:
  - With α equals 0.05, the obtained Z score of 3.86, the null hypothesis is rejected. The data suggests that there is a difference between the city and the nationwide in terms of the commute-to-work time. The decision to reject the null hypothesis has a 0.05 probability of being wrong.

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Hypothesis Testing - Summary

- It is important to know that, there are different types of test of significance, and to know when to use what:
  - When comparing samples to the population, use
    - one-sample z test (large sample; N≥100) or
    - one-sample *t*-test (small sample; N<100)
  - When comparing two samples, use a two-sample *t*-test.
- Two formulas of calculating z scores:
  - When σ is known: $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{N}}$
  - When σ is unknown: $Z = \frac{\bar{X}-\mu}{S/\sqrt{N-1}}$

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Hypothesis Testing - Summary

- Each test produces a score, either a z score or a t score depending upon the test. This is called the *test statistic*.
- What the test statistic tells us is whether the difference between the means we are comparing is statistically significant or not.
  - If the score is <u>significant</u>, we **reject** the <u>null hypothesis</u> – there is a difference between the two data sets.
  - If the score is <u>not significant</u>, we **do not reject** the <u>null hypothesis</u> – there is no significant difference between the two data sets.