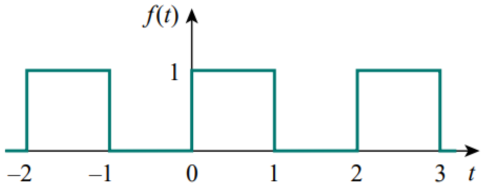
1- Aşağıdaki periyodik fonksiyonun Fourier Seri (*Fourier Series*) hesaplamasını yapınız? Fourier katsayılarını (*Fourier Coefficients*) bulunuz? (*Fourier katsayıları=ao, an ve bn*) (*NOT: Tablo 16.1'den faydalanabilirsiniz.*)



$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \qquad (16.1.1)$$

Our goal is to obtain the Fourier coefficients a_0, a_n, and b_n using Eqs. (16.6), (16.8), and (16.9). First, we describe the waveform as

$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases} \qquad (16.1.2)$$

and $f(t) = f(t+T)$. Since $T = 2$, $\omega_0 = 2\pi/T = \pi$. Thus,

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{2}\left[\int_0^1 1\,dt + \int_1^2 0\,dt\right] = \frac{1}{2}t\Big|_0^1 = \frac{1}{2} \qquad (16.1.3)$$

Using Eq. (16.8) along with Eq. (16.15a),

$$\begin{aligned} a_n &= \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt \\ &= \frac{2}{2}\left[\int_0^1 1\cos n\pi t\,dt + \int_1^2 0\cos n\pi t\,dt\right] \qquad (16.1.4) \\ &= \frac{1}{n\pi}\sin n\pi t\,\Big|_0^1 = \frac{1}{n\pi}\sin n\pi = 0 \end{aligned}$$

From Eq. (16.9) with the aid of Eq. (16.15b),

$$\begin{aligned}
b_n &= \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt \\
&= \frac{2}{2}\left[\int_0^1 1\sin n\pi t\,dt + \int_1^2 0\sin n\pi t\,dt\right] \\
&= -\frac{1}{n\pi}\cos n\pi t\,\Big|_0^1 \qquad\qquad (16.1.5)\\
&= -\frac{1}{n\pi}(\cos n\pi - 1), \qquad \cos n\pi = (-1)^n \\
&= \frac{1}{n\pi}[1-(-1)^n] = \begin{cases}\dfrac{2}{n\pi}, & n = \text{odd}\\ 0, & n = \text{even}\end{cases}
\end{aligned}$$

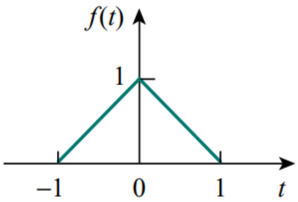
Substituting the Fourier coefficients in Eqs. (16.1.3) to (16.1.5) into Eq. (16.1.1) gives the Fourier series as

$$f(t) = \frac{1}{2} + \frac{2}{\pi}\sin\pi t + \frac{2}{3\pi}\sin 3\pi t + \frac{2}{5\pi}\sin 5\pi t + \cdots \qquad (16.1.6)$$

Since $f(t)$ contains only the dc component and the sine terms with the fundamental component and odd harmonics, it may be written as

$$f(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin n\pi t, \qquad n = 2k-1 \qquad (16.1.7)$$

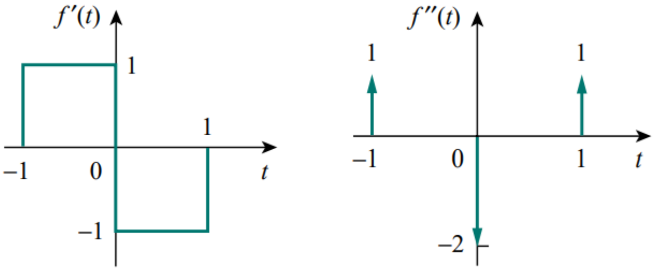
2- Aşağıdaki fonksiyonun Fourier Transform'unu (*Fourier Transform*) hesaplayınız? (*Tablo 17.1 ve Tablo 17.2'den faydalanabilirsiniz.*)



$$f(t) = \begin{cases}1+t, & -1 < t < 0\\ 1-t, & 0 < t < 1\end{cases}$$

Its first derivative is shown in Fig. 17.15(a) and is given by

$$f'(t) = \begin{cases} 1, & -1 < t < 0 \\ -1, & 0 < t < 1 \end{cases}$$



(a) (b)

Its second derivative is in Fig. 17.15(b) and is given by

$$f''(t) = \delta(t+1) - 2\delta(t) + \delta(t-1)$$

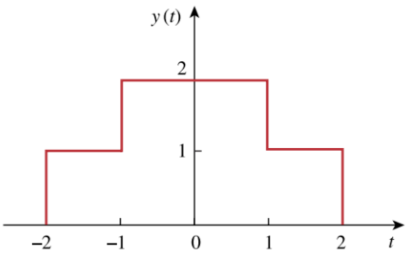
Taking the Fourier transform of both sides,

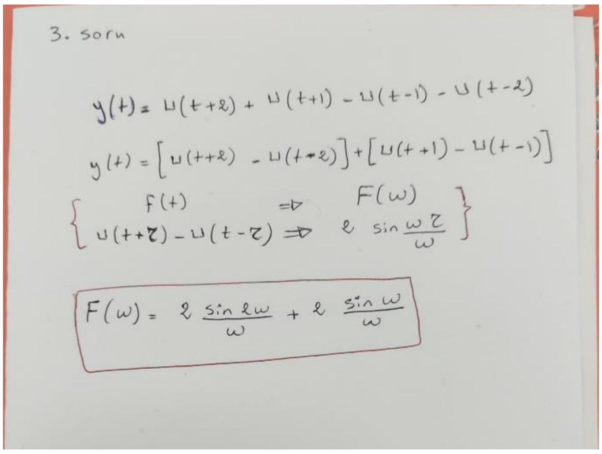
$$(j\omega)^2 F(\omega) = e^{j\omega} - 2 + e^{-j\omega} = -2 + 2\cos\omega$$

or

$$F(\omega) = \frac{2(1-\cos\omega)}{\omega^2}$$

3- Aşağıdaki fonksiyonun Fourier Transform'unu (*Fourier Transform*) hesaplayınız? (*Tablo 17.1 ve Tablo 17.2'den faydalanabilirsiniz*.)




3. soru

$y(t) = u(t+2) + u(t+1) - u(t-1) - u(t-2)$

$y(t) = [u(t+2) - u(t-2)] + [u(t+1) - u(t-1)]$

$\left\{ \begin{array}{lll} f(t) & \Rightarrow & F(\omega) \\ u(t+\tau) - u(t-\tau) & \Rightarrow & 2\,\frac{\sin \omega\tau}{\omega} \end{array} \right\}$

$\boxed{F(\omega) = 2\,\frac{\sin 2\omega}{\omega} + 2\,\frac{\sin\omega}{\omega}}$

4- Analog bir sinyali sayısal bir sinyale (*Analog-to-Digital*) dönüştüren 3 adımı yazınız? Sayısal sistemlerin, analog sistemlere göre avantajları nelerdir?

Sayısal işaret işleme ilk ders notunda cevabı mevcut.

TABLE 17.1 Properties of the Fourier transform.

Property	$f(t)$	$F(\omega)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Scaling	$f(at)$	$\frac{1}{\|a\|} F\left(\frac{\omega}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-j\omega a} F(\omega)$
Frequency shift	$e^{j\omega_0 t} f(t)$	$F(\omega - \omega_0)$
Modulation	$\cos(\omega_0 t)\, f(t)$	$\frac{1}{2}[F(\omega+\omega_0) + F(\omega-\omega_0)]$

TABLE 17.1 *(continued)*

Property	$f(t)$	$F(\omega)$
Time differentiation	$\frac{df}{dt}$	$j\omega F(\omega)$
	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(t)\, dt$	$\frac{F(\omega)}{j\omega} + \pi F(0)\, \delta(\omega)$
Frequency differentiation	$t^n f(t)$	$(j)^n \frac{d^n}{d\omega^n} F(\omega)$
Reversal	$f(-t)$	$F(-\omega)$ or $F^*(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$
Convolution in t	$f_1(t) * f_1(t)$	$F_1(\omega) F_2(\omega)$
Convolution in ω	$f_1(t) f_1(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$

TABLE 16.1 Values of cosine, sine, and exponential functions for integral multiples of π.

Function	Value
$\cos 2n\pi$	1
$\sin 2n\pi$	0
$\cos n\pi$	$(-1)^n$
$\sin n\pi$	0
$\cos \frac{n\pi}{2}$	$\begin{cases} (-1)^{n/2}, & n = \text{even} \\ 0, & n = \text{odd} \end{cases}$
$\sin \frac{n\pi}{2}$	$\begin{cases} (-1)^{(n-1)/2}, & n = \text{odd} \\ 0, & n = \text{even} \end{cases}$
$e^{j2n\pi}$	1
$e^{jn\pi}$	$(-1)^n$
$e^{jn\pi/2}$	$\begin{cases} (-1)^{n/2}, & n = \text{even} \\ j(-1)^{(n-1)/2}, & n = \text{odd} \end{cases}$

TABLE 17.2 Fourier transform pairs.

$f(t)$	$F(\omega)$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$u(t+\tau) - u(t-\tau)$	$2\frac{\sin \omega\tau}{\omega}$
$\|t\|$	$\frac{-2}{\omega^2}$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$
$e^{at}u(-t)$	$\frac{1}{a-j\omega}$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$
$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\sin \omega_0 t$	$j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$
$\cos \omega_0 t$	$\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$
$e^{-at}\sin \omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$
$e^{-at}\cos \omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$