Computational Number Theory
HW 1

Due Date: 12/08/2022

1. A jeweler buys a diamond and a ruby for 2000 ducats. Find the price of the ruby given that its price is the cube root of the price of the diamond. [This was one of the challenge problems proposed by Antonio Fiore in his mathematical duel with Tartaglia in 1535.]

2. A cubic polynomial without a x^2 term is called a *depressed cubic*. Reduce the equation $x^3 + x^2 = 10$ to an equation with a depressed cubic: you do not have to solve the depressed cubic.

3. Find one integer solution of $6x + 10y + 15z = 1$.

4. Show that if a, m, n are natural numbers with $a > 1$, then

$$gcd(a^m - 1, a^n - 1) = a^{gcd(m,n)} - 1.$$

5. Describe all integer solutions of $2x + 3y + 5z = 0$.