Quiz 3

Total Marks: $4 \times 3 = 12$

Instructions: You may solve **any three of the first four** questions. The last one is compulsory.

1. Solve the equation $x^2 = 5$ in $\mathbb{Z}_{121}$.
2. Let $p = 12347$, which is a prime. For each of the equations below in $Z_p[x]$, decide whether it has a solution without using a calculator.

   (a) $x^2 = 60$

   (b) $x^2 + x + 1 = 0$.
3. Let p, q be primes. Find an expression for the number of irreducible polynomials of degree q in $\mathbb{Z}_p[x]$.
4. In $\mathbb{Z}_5[x]$, find the remainder when x^{2024} is divided by x^2+x+2. You may use the fact that $x^2 + x + 2$ is irreducible in $\mathbb{Z}_5[x]$.
5. You are given a polynomial $f(x) \in \mathbb{Z}_p[x]$ of degree greater than 1, and the promise that $f(x)$ is of the form $f(x) = g(x)h(x)$, where $g(x)$ and $h(x)$ are irreducible polynomials such that $deg(g) < deg(h)$. Explain how to find $g(x)$ and $h(x)$ efficiently.