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BOOK REVIEW

Paulus Gerdes. Sona Geometry from Angola: mathematics of an African tradition

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Since the beginning of the twentieth century, European, and later American, researchers studied *sona*—sand drawings by elders of the Cokwe-Lunda peoples of northeastern Angola, northwestern Zambia, and southwestern Zaire. Researchers and mathematics educators since the 1970s, in fact, included discussions of *sona* in their publications, and even alluded to their mathematical intimations (see Ascher 1991; Gerdes 1994; Zaslavsky 1979). But it was not until Paulus Gerdes's recent book *Sona Geometry from Angola: mathematics of an African tradition* that we find the *sona* tradition to be rife with mathematical ideas and to exhibit highly sophisticated geometric and algebraic concepts. In *Sona Geometry*, Gerdes demonstrates the inextricable links between *sona* and various branches of mathematics. These include, but are by no means limited to, concepts in transformational geometry, abstract algebra, and linear algebra.

Given the exiguous sources on the topic of *sona*, Gerdes, an accomplished, international scholar, weaves together the intricate details that constitute the intellectual foundations of a nearly wiped out tradition. At the same time, Gerdes brings to light the inherent mathematical characteristics of *sona* in a manner that shows respect and dignity to the Cokwe and neighboring peoples who created and developed this highly sophisticated mode of communication, a method of investigation that previous researchers have failed to accomplish.

To be sure, it is rather difficult to do justice when describing the complexities of both the components and the developmental process of a *lusona* (singular form of ''sona'') within the space of a book review, let alone without the inclusion of diagrams. What follows, however, is a concerted attempt to describe the *sona* production process while, at the same time, avoid the risk of oversimplification and possible misrepresentation: the *akwa kuta sona*—the actual individual who is able to draw *sona*—executes the drawing by making impressions in the sand, usually using the index and pointer fingers, simultaneously. The impressions are almost entirely equidistant, and the final *lusona* often consists of a grid with primary and secondary rows and columns, whereby the secondary rows and columns are usually indented. Then, the *akwa kuta sona* uses one finger or possibly some long object such as a stick and creates most often a single path that surrounds each of the impressions—that is, the *lusona* is usually monolinear. *Sona* have culturally specific meanings and tell stories that are of great value to the Cokwe people. It is fascinating to observe the seeming ease at which the *akwa kuta sona* is able to create a single *lusona*—a task, which virtually all visitors would find next to impossible to complete. Creating a *lusona* is no trivial task, and is something that only a select number of Cokwe members can perform adequately.

Fortunately, a number of anthropologists and ethnographers in the last century published *sona* from their own reproductions based on those that the *akwa kuta sona* drew in the sand. At the same time, however, two major factors made it difficult to keep records of *sona*, and hence, contributed to their near obliteration. First, since the late nineteenth century, the Cokwe culture has fragmented as a result of European and American colonization. Second, shortly after its completion, the outlines and sand impressions of the *lusona* would be extinguished, usually by wind

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or some other weather occurrence, within a few hours. So, in order to record and publish *sona*, ethnographers would have to copy them immediately after their completion. For centuries, *sona* were passed on from one generation to the next through the process of oral tradition—that is, younger Cokwe members would learn the process of creating *sona* through observation and experience with elders—not by copying or recording them on paper.

Gerdes shows us that certain tendencies within the *sona* themselves provide indications of specific cultural values. He serves up two clear examples: symmetry, which comprises nearly 80 percent of all *sona*; and monolinearity, which is evident in more than 60 percent of the *sona* (see pp. 25, 26). Symmetry and monolinearity, given their frequency during the production of *sona*, seem to be of great value among the Cokwe people. Gerdes analyzes a complete *lusona* by presenting it in a way that shows its development during the construction process. In doing so, he identifies a *lusona's* algorithm, which I would say is defined as the shortest segment of a constructed path that can be repeated in order to complete the *lusona*. One of the most interesting characteristics of the book has to do with the way in which Gerdes categorizes the *sona* into classes. The class of *sona* related to plaited mat designs is the largest, and is grouped into four subclasses (A, B, C, and D). Each of the subclasses is based on dimensional characteristics of primary and secondary rows and columns. For example, in subclass A, the primary row (r_1) consists of one more dot, or finger impression, than the secondary row (r_2). Likewise, the primary column (c_1) is composed of one more dot than the secondary column (c_2). Gerdes writes this as $r_1 = r_2 + 1$ and $c_1 = c_2 + 1$ (see p. 47). He concludes that, based on 101 *sona* drawings collected by ethnographers in the early and middle twentieth century that fall into subclass A, 26 possess $n \times n$ (i.e., ''square'') grids, which will never be monolinear, and 41 possess $n \times (n + 1)$ or $(n + 1) \times n$ grids, which will always be monolinear (p. 51). This further supports the idea that monolinearity possibly served as a cultural ideal.

Further analysis reveals an embedded mathematical tendency of an *akwa kuta sona* as he connects individual *sona* in order to create a large single *lusona*. Gerdes refers to five algorithm types that undergird these *sona* as chain rules. The first chain rule ''concerns the creation of a new, closed monolinear design by joining two (or more) open monolinear designs in such a way that an endpoint of the first *lusona* connects to an endpoint of the second *lusona*...'' (p. 125). Other chain rules involve connecting two designs with a common point (see p. 128), fusing two or more lines together (see p. 132), or through connection of two designs whose grids have two points in common (see p. 143). It is remarkable and at times mystifying to encounter two comparable *sona*, which have slightly different dimensions, yet one is monolinear and the second is polylinear, that is, the design must be completed by lifting a finger (see pp. 155, 156). Moreover, it is difficult to account for theorems that allow one to generalize patterns relating to dimension and monolinearity or polylinearity. Within the book, however, Gerdes establishes theorems, for example, two theorems having to do with the fourth chain rule (see pp. 147–149), that he believes are known to the *akwa kuta sona* but, of course, have not been previously recorded or published.

Gerdes provides a multifaceted perspective about the *sona* tradition that places the mathematics within it at a comparable level as mathematical developments in European countries at approximately the same time. *Sona Geometry* is not the first publication in which Gerdes demonstrates this. In earlier studies, Gerdes uncovers mathematical traditions in Mozambican artisanship, and shows that various mathematical theorems previously taken for granted to have been solely developed by the early Greeks—for example, the idea that the area of a square is uniquely derived by its diagonal (i.e., the Pythagorean Theorem)—can be traceable to the work of Mozambican artisans and merchants. Moreover, these individuals are able to solve mathematical problems that would be difficult for foreign mathematicians and educators. As Harris (1987) aptly states, ''Part of the newness of Paulus Gerdes' work in Mozambique... is that he offers 'non-standard problems,' easily solved by... Mozambican artisans, to members of the international mathematics education community—who cannot (at first) do them'' (p. 26). In the same manner, Gerdes makes it transparent in *Sona Geometry* that ''non-standard problems'' easily solved by members of the Cokwe people of Angola cannot be solved easily, if at all, by visitors and members of the international mathematics education community.

With regard to mathematics curriculum, *Sona Geometry* can serve as an excellent text for courses in geometry, algebra, survey courses in mathematics, and the history of mathematics. It is true that the study of *sona* can benefit student learning of geometry in primary and secondary school mathematics curricula. But the book does not seem to serve as an introduction to *sona* and geometry for students in the primary and middle-level grades. For this, Gerdes's (1990) earlier book *Vivendo a matemática: Desenhos da África* is an excellent text in that it provides young students with activities that introduce geometric concepts through *sona*. The connection between geometry and *sona*, however, should not be introduced as a mathematical afterthought or as an ancillary device. Rather, it would best serve as a focal point from which students can learn geometric concepts and derive theorems. In short, *sona* geometry is geometry, and much can be derived mathematically from this Angolan sand drawing tradition that, prior to Gerdes's book, was not necessarily thought possible.

Despite the book's perspicuity in not only elucidating on a nearly lost tradition but in revealing embedded abstract mathematical concepts within Angolan sand drawings, it is nearly impossible to overlook traditions and values that seem to exude elements of gender bias. It is known that *sona* were drawn by male elders, and that the tradition has had strong foundations in male circumcision rites. One must wonder, then, why women and girls seemed to have been excluded from these intellectual endeavors, or whether anyone has analyzed female traditions in Cokwe culture, such as farming and the processing of agricultural products, that potentially reveal mathematical propensities. In previous studies, however, Gerdes (1993, 1995) has done a great deal to underscore the rich development of mathematics by women in sub-Saharan Africa. And although *Sona Geometry* does not investigate mathematical products of women of the Cokwe culture, Gerdes has made numerous inroads in unfreezing a nearly wiped out cultural production and method of communication.

That said, *Sona Geometry* provides a detailed, well-articulated, and at the same time, convincing, presentation of a nearly lost, yet highly mathematical, tradition. Further, the volume is filled with more than 350 examples of either actual recorded *sona* or the algorithms that the *akwa kuta sona* may have used to construct them. *Sona Geometry* should be required reading for anyone interested in geometry, the culture and specific traditions of the Cokwe people, mathematics developed in sub-Saharan Africa, or the teaching and learning of geometry and related fields. It is a publication that is sorely long overdue.

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